# Overview of intermittency analysis in heavy-ion collisions

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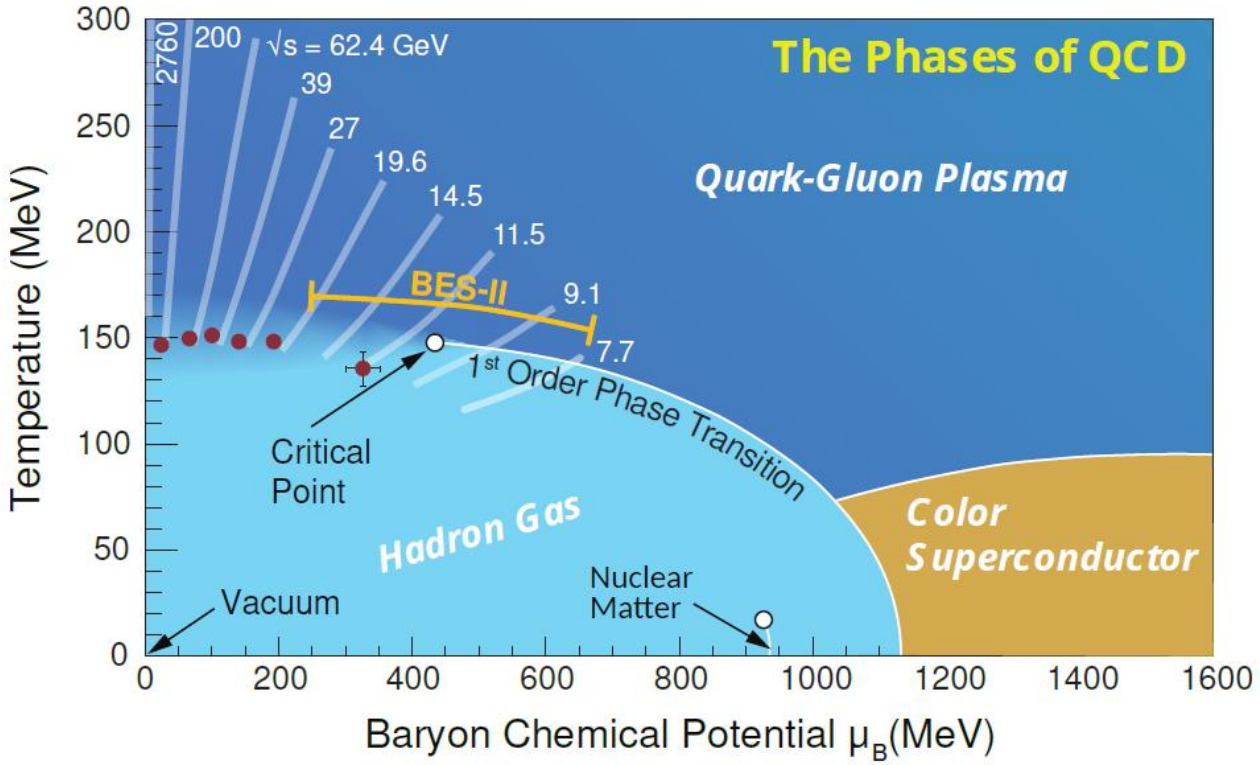
- Introduction
- Measurements from heavy-ion experiments
- Results from phenomenological models
- Summary and outlook

中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会

# Phase diagram of QCD



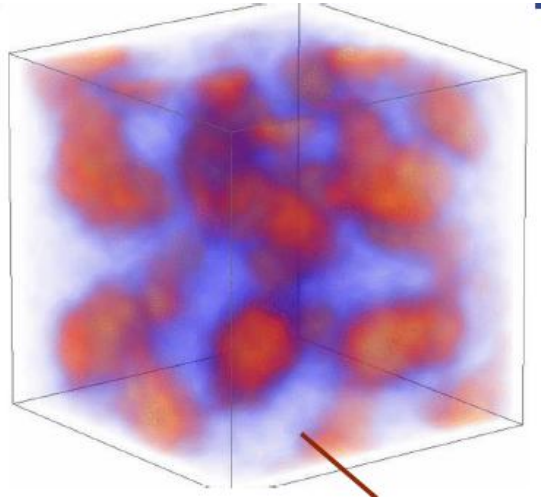

A. Bzdak, et al., Phys. Rep., 853, 1 (2020)

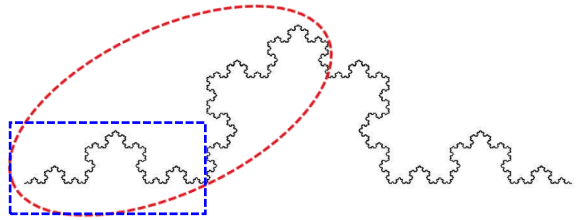
## Experimental observables:

Local density fluctuation of conserved charges

- Intermittency:

  fractal, self-similar, scale-inviarant



Goal: exploring the QCD phase diagram and Critical Point.

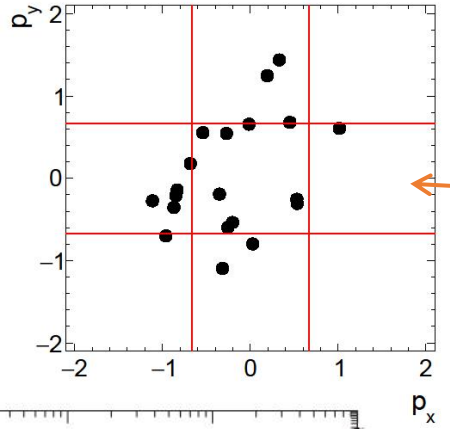
N. Antoniou, et al., Phy. Rev. Lett. 97,032002 (2006)
J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)
Z. Li, Mod. Phys. Lett. A 37, 2230009 (2022)

# Scaled factorial moment (SFM) and intermittency

- Intermittency can be measured via the scaled factorial moments $F_q(M)$ ,

$$F_q(M) = \frac{< \frac{1}{M^D} \sum_{i=1}^{M^D} n_i(n_i-1)\dots(n_i-q+1) >}{< \frac{1}{M^D} \sum_{i=1}^{M^D} n_i >^q}$$

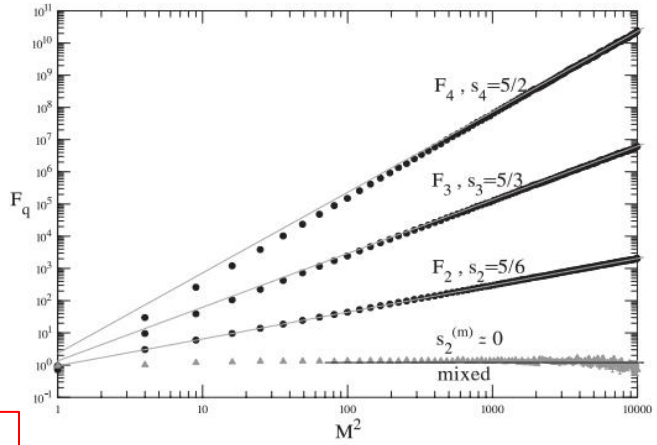
A. Bialas and R. B. Peschanski, Nucl. Phys. B 273, 703 (1986)
A. Bialas and R. B. Peschanski, Nucl. Phys. B 308, 857 (1988)




Particle multiplicity in the *i-th* cell $n_i$

- **$F_q(M)/M$ scaling**:

$$F_q(M) \propto (M^2)^{\phi_q}, M \to \infty$$




$\phi_q^{\text{critical}} = 5q/12(\text{Baryon}),$
$\phi_q^{\text{critical}} = q/3(\text{pion})$

N. Antoniou, et al., Phy. Rev. Lett. 97,032002 (2006)
N. Antoniou, et al., Phys. Rev. D 97, 034015 (2018)

- **$F_q(M)/F_2(M)$ scaling:**

$$F_q(M) \propto F_2(M)^{\beta_q} \qquad \beta_q \propto (q-1)^{\nu}$$

$\nu_{\text{critical}} = 1.304$ (Ginzburg-Landau, entire space phase);

**Scaling exponent, *ν*,** quantitatively describes all the scaling indices $\beta_q$:

R. C. Hwa and M. T. Nazirov, Phys. Rev. Lett. 69, 741 (1992)
R. C. Hwa and C. B. Yang, Phys. Rev. C 85, 044914 (2012)

# Background subtraction in the calculation of SFM

## Mixed event method

$$\Delta F_q(M) = F_q^{data}(M) - F_q^{mix}(M)$$

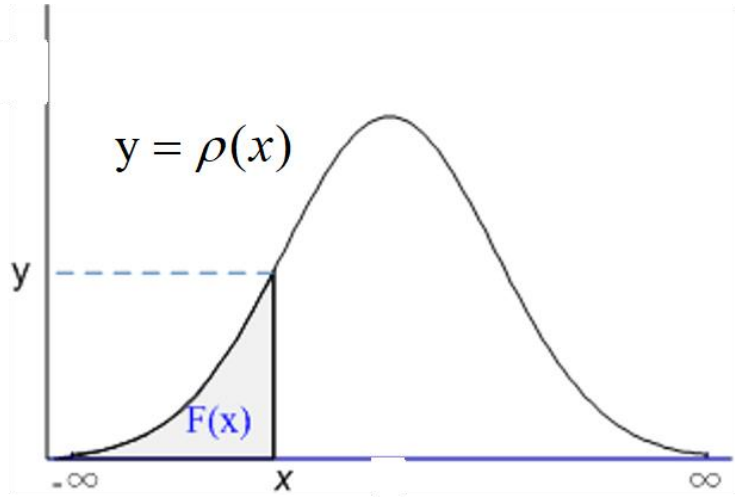
T. Anticic , et al. (NA49 Coll.), Eur. Phys. J. C 75, 587 (2015)
J. Wu (STAR Coll.), ISMD2021

Assuming that multiplicity distributions of measured particles in each partitioned cell can be simply divided into background and critical contributions.

## The cumulative variable method

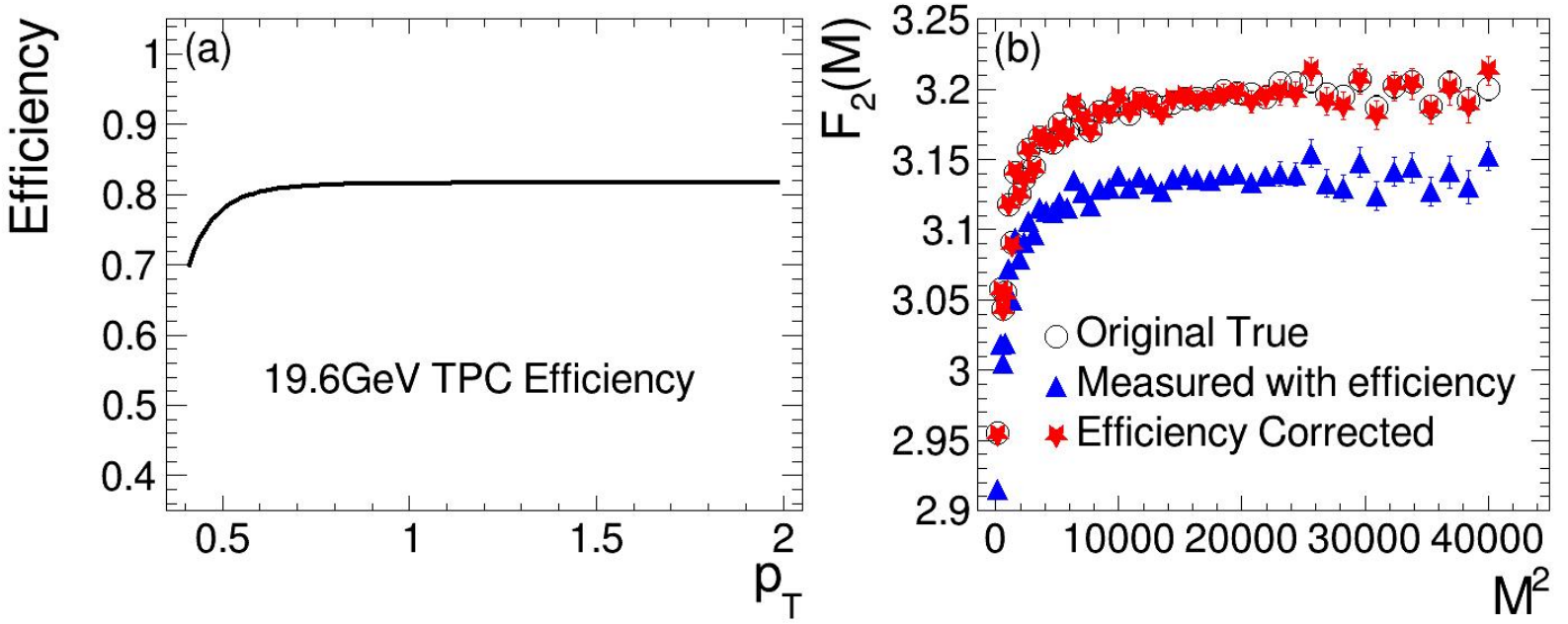
$$X(x) = \frac{\int_{x_{min}}^{x} \rho(x)dx}{\int_{x_{min}}^{x_{max}} \rho(x)dx}$$

It does not depend on the choice of the original variable *x*, but is uniquely determined by the shape of density distribution $\rho(x)$.




A. Bialas , et al., Phys. Lett. B 252, 483 (1990)
W. Ochs, Z. Phys. C 50, 339 (1991)
J. Wu, Y. Lin, Z. Li, X. Luo and Y. Wu, Phys. Rev. C 104, 034902 (2021)

# Efficiency correction for SFM

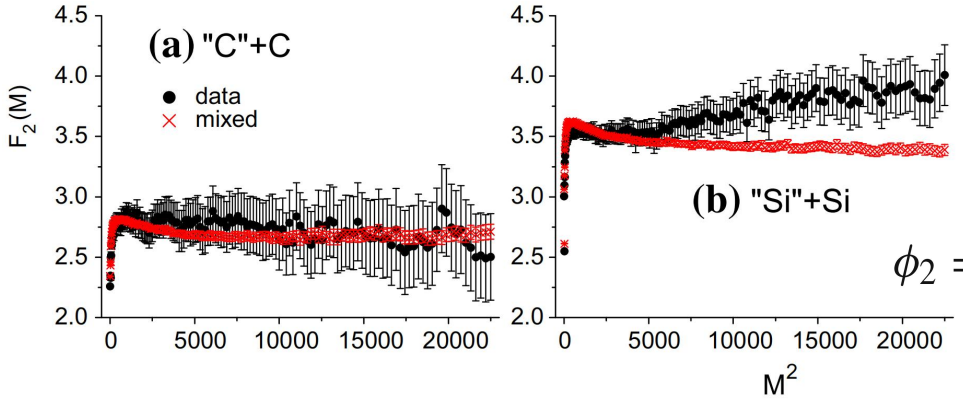



$$F_q^{corrected}(M) = \frac{< \frac{1}{M^2} \sum_{i=1}^{M^2} \frac{n_i(n_i-1)\dots(n_i-q+1)}{\bar{\varepsilon}_i^q} >}{< \frac{1}{M^2} \sum_{i=1}^{M^2} \frac{n_i}{\bar{\varepsilon}_i} >^q}$$

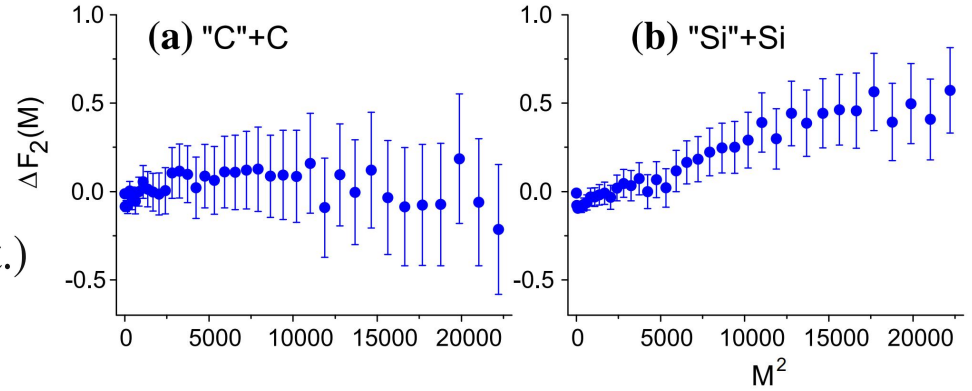
J. Wu, Y. Lin, Z. Li, X. Luo and Y. Wu, Phys. Rev. C 104, 034902 (2021)

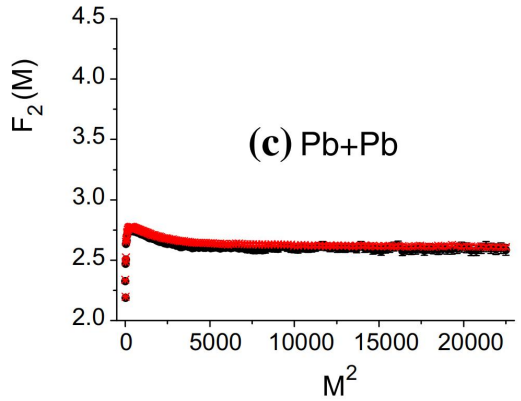
- The STAR TPC tracking efficiency firstly increases with increasing $p_T$ , and then gets saturated in higher $p_T$ regions.
- The measured SFMs with efficiency are systematically smaller than the true ones, especially in the large number of partitioned cells. However, the efficiency corrected SFMs are found to be well consistent with the original true ones.

# Measurements of $F_q(M)/M$ scaling in the NA49 experiment at SPS

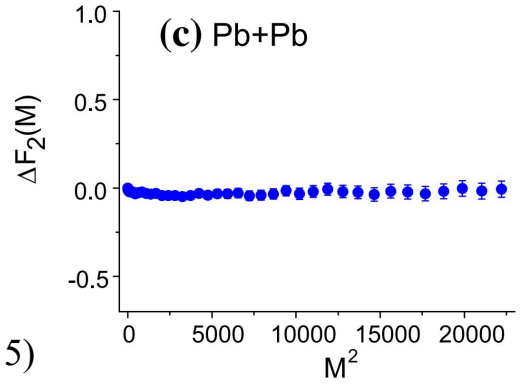



$$\phi_2 = 0.96^{+0.38}_{-0.25}\ (\text{stat.}) \pm 0.16\ (\text{syst.})$$




$$\Delta F_2^{(e)}(M) = F_2^{(d)}(M) - F_2^{(m)}(M)$$

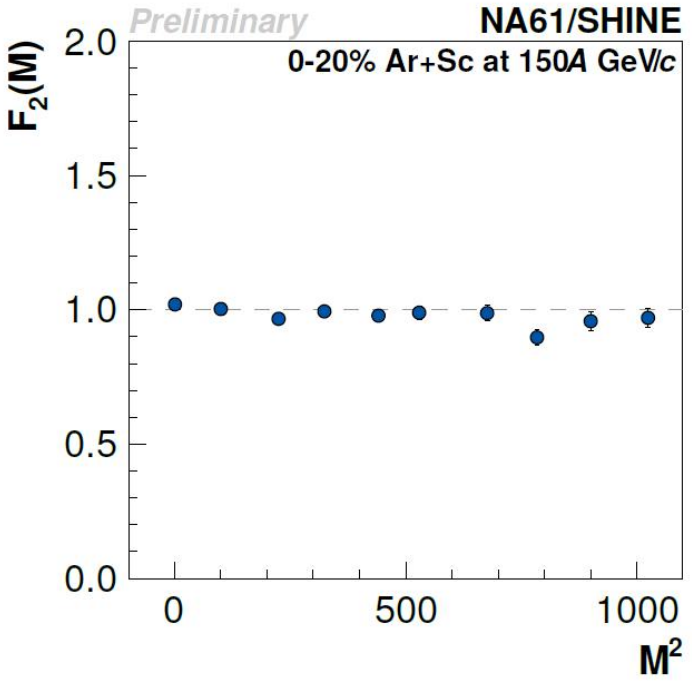


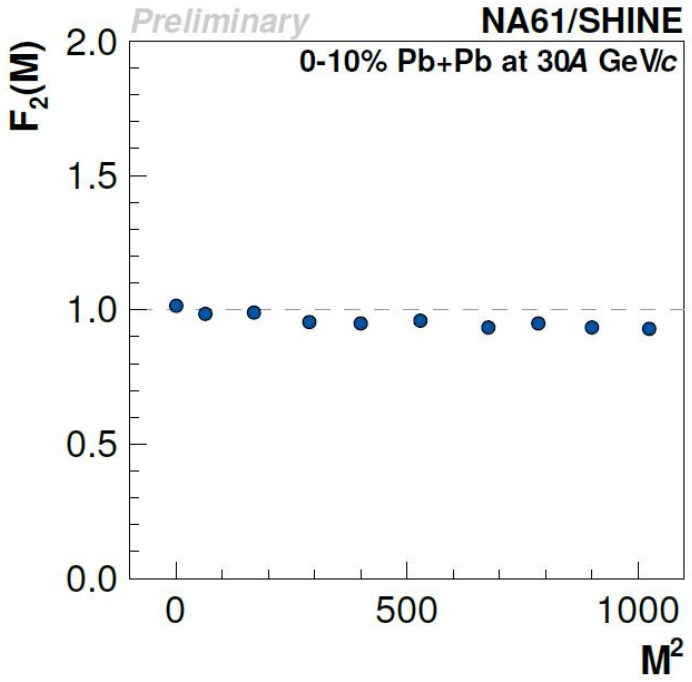

N. Davis, et al. (NA49 Coll.), QM2018
T. Anticic , et al. (NA49 Coll.), Eur. Phys. J. C 75, 587 (2015)

- The second-order SFM in the Si + Si collisions are larger than those calculated from mixed events, but almost overlap with those of mixed events in the C + C and Pb + Pb systems

- Intermittency of NA49 experiment reveals significant power-law fluctuations of proton density in **Si + Si** collisions at $\sqrt{s_{NN}}$ = 17.3 GeV. But no intermittent behavior is visible in C+C or Pb + Pb collisions.

# $F_q(M)/M$ scaling from the NA61/SHINE Collaboration



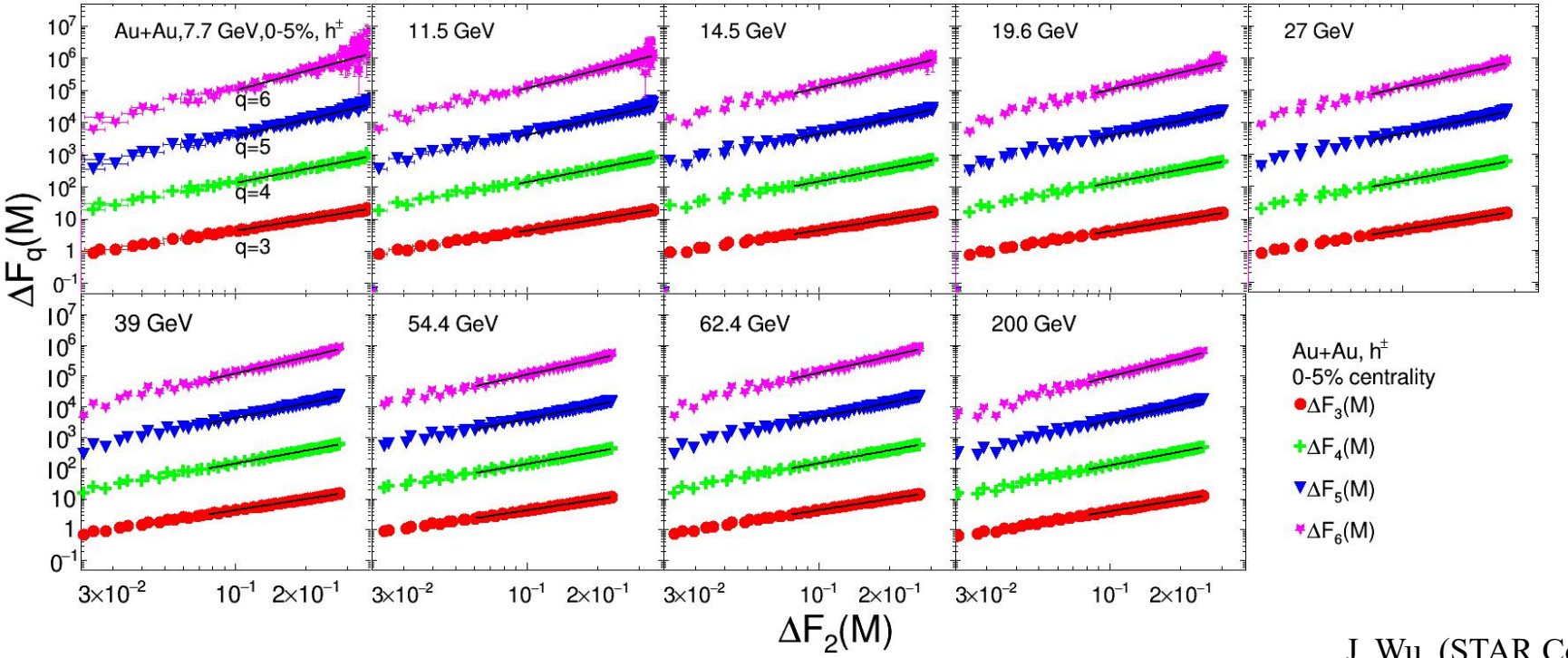



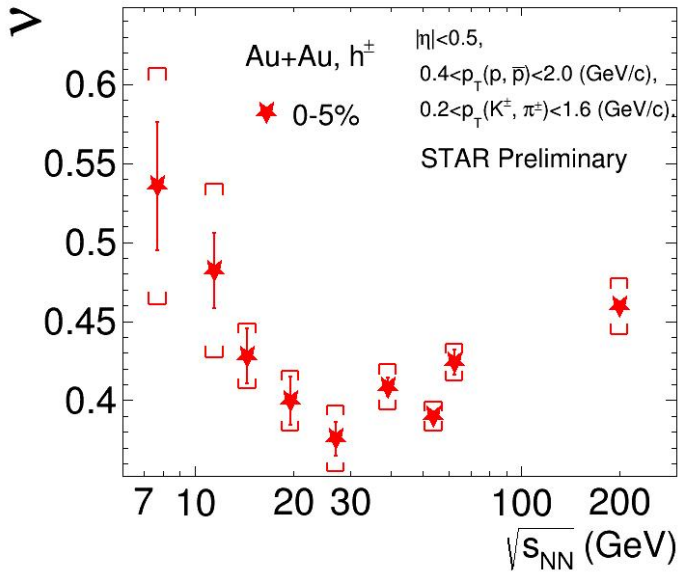

T. Czopowicz (NA61/SHINE Coll.), CPOD2021
M. Kuich, (NA61/SHINE Coll. ), EPJ Web Conf. 259, 01001 (2022)

- $F_2(M)$ is nearly flat with increasing number of division bins for Ar + Sc collisions at 150$A$ GeV/c and Pb + Pb collisions at 30$A$ GeV/c.

- No indication of an intermittency behavior in the NA61/SHINE result.

# $F_q(M)/F_2(M)$ scaling in the RHIC/STAR experiment

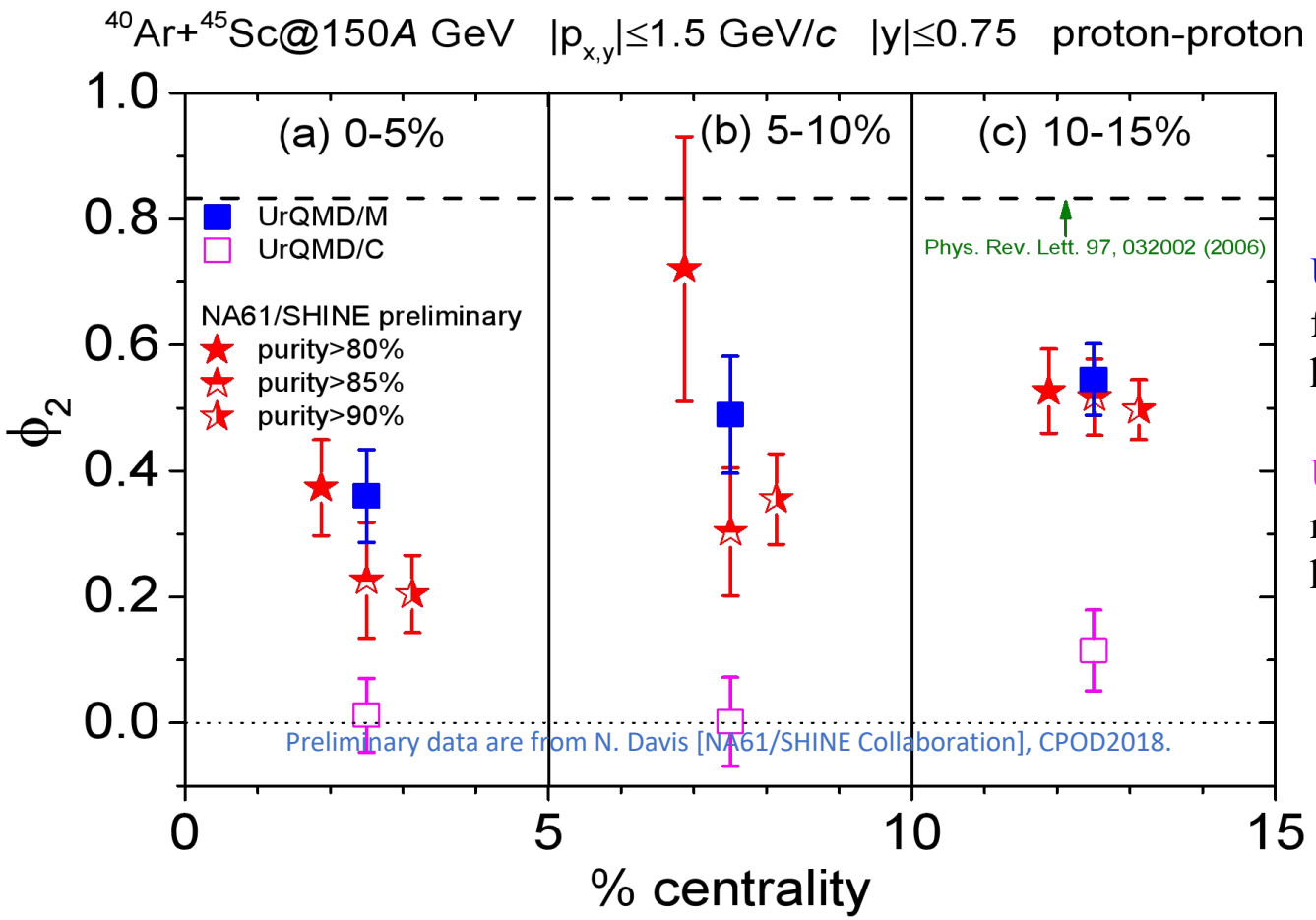



J. Wu (STAR Coll.), ISMD2021

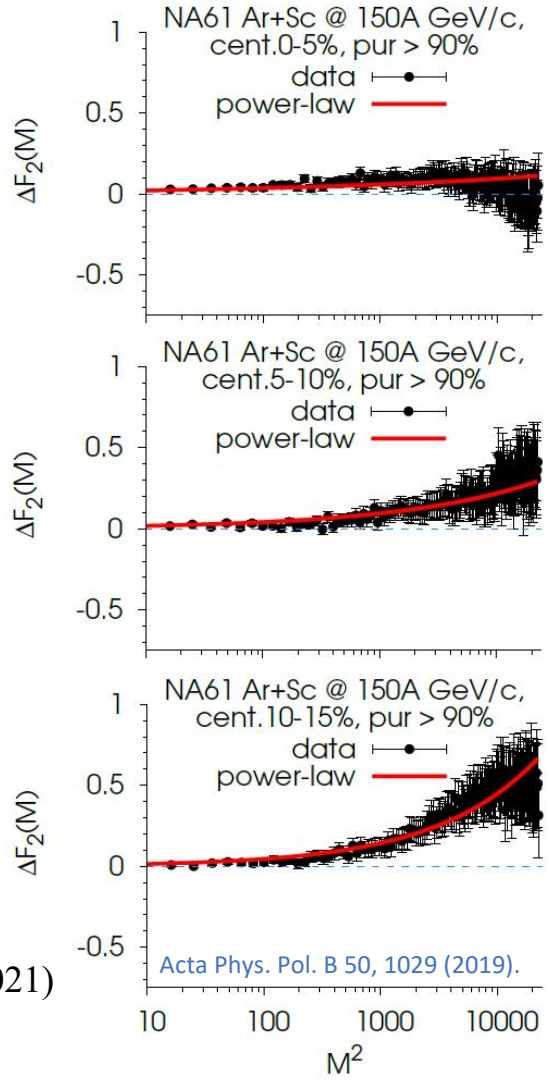
- $\Delta F_q(M)/\Delta F_2(M)$ scaling behaviors are clearly visible in Au+Au collisions at RHIC BES-I energies.
- In the most central collisions, the scaling exponent ν exhibits a non-monotonic behavior on collision energy and seems to reach a minimum around $\sqrt{s_{NN}}$ = 20-30 GeV.

# Intermittency in the UrQMD model with hadronic potentials




**UrQMD/M** (mean-field mode): with the hadronic potentials

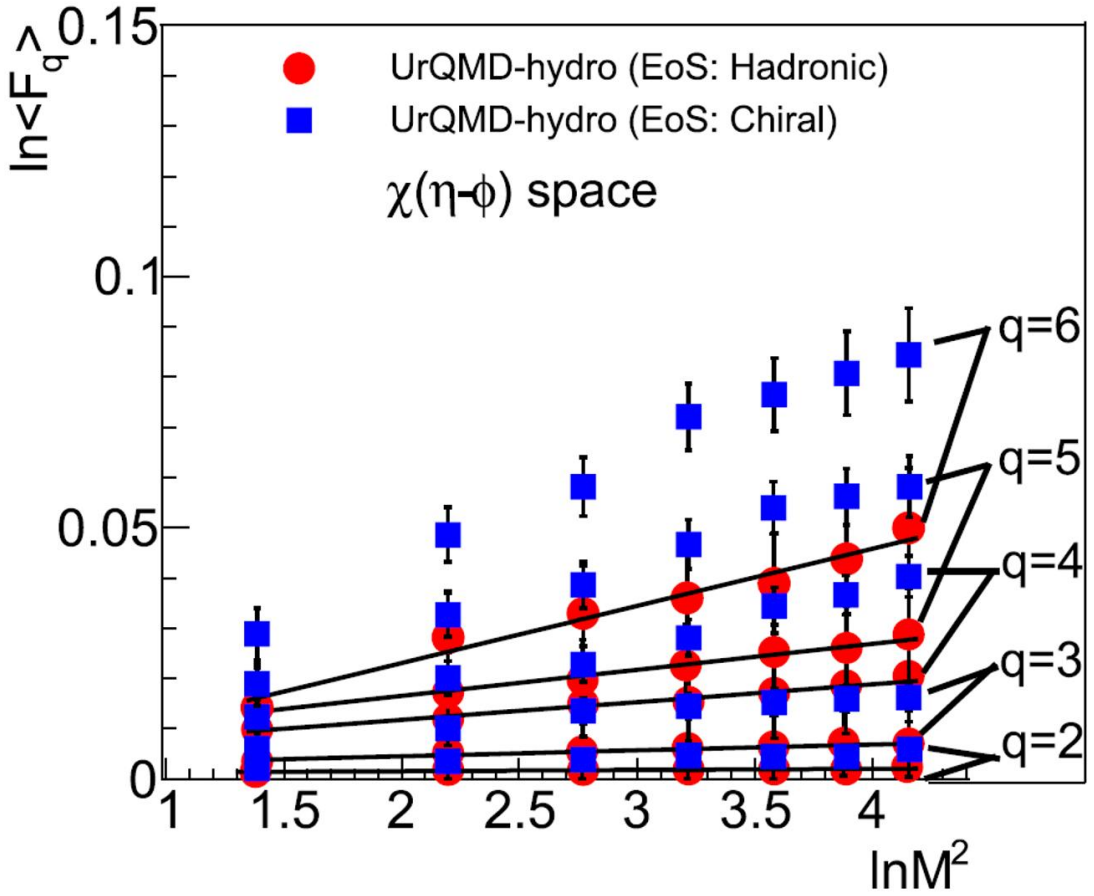
**UrQMD/C** (cascade mode): without the hadronic potentials




P. Li, Q. Li, et al., Phys. Lett. B 818, 136393 (2021)

- The intermittency index, $\phi_2$, calculated by the UrQMD/C model is around zero.
- With the inclusion of hadronic potentials, it is found that $\phi_2$ from the UrQMD/M model is comparable to the NA61/SHINE data in different centrality bins.

# Intermittency in the hybrid UrQMD-hydro model

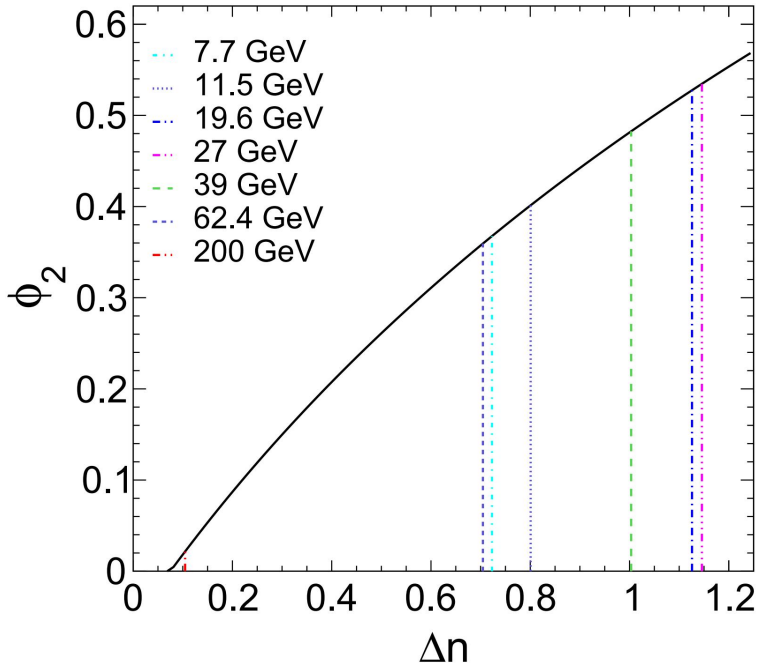



S. Gope and B. Bhattacharjee, Eur. Phys. J. A 57, 44 (2021)

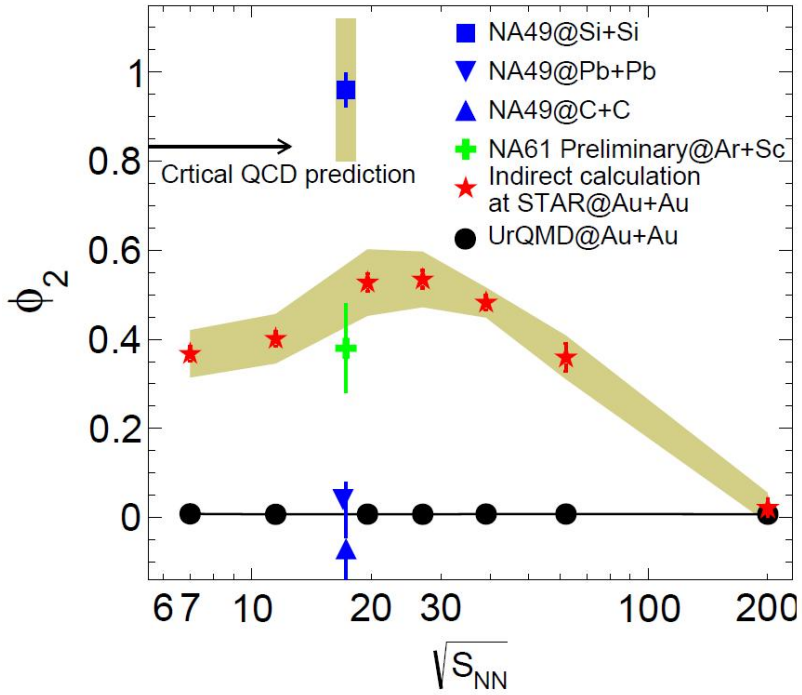
- $lnF_2(M)$ is found to linearly develop with increasing $ln\ M^2$ for both hadronic EoS and chiral one in the (0–5%) most central collisions in Au + Au collisions at 10A GeV/c from the UrQMD-hydro model.

- The intermittency indices of chiral EoS are found to be larger than those of hadronic one. It accounts for the production of cascading particles in partonic level and the hydrodynamic evolution in the chiral EoS case.

# Intermittency in the CMC model belonging to the 3D Ising universality class




$$\Delta n = \frac{\langle (\delta n)^2 \rangle}{\langle n \rangle^2} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2}$$




J. Wu, Z. Li, et al., Phys. Lett. B 801, 135186 (2020)

- $\phi_2$ monotonically increases with $\Delta n$. It infers that great intermittency can be achieved if large baryon density fluctuations are developed for the system near the QCD critical point.

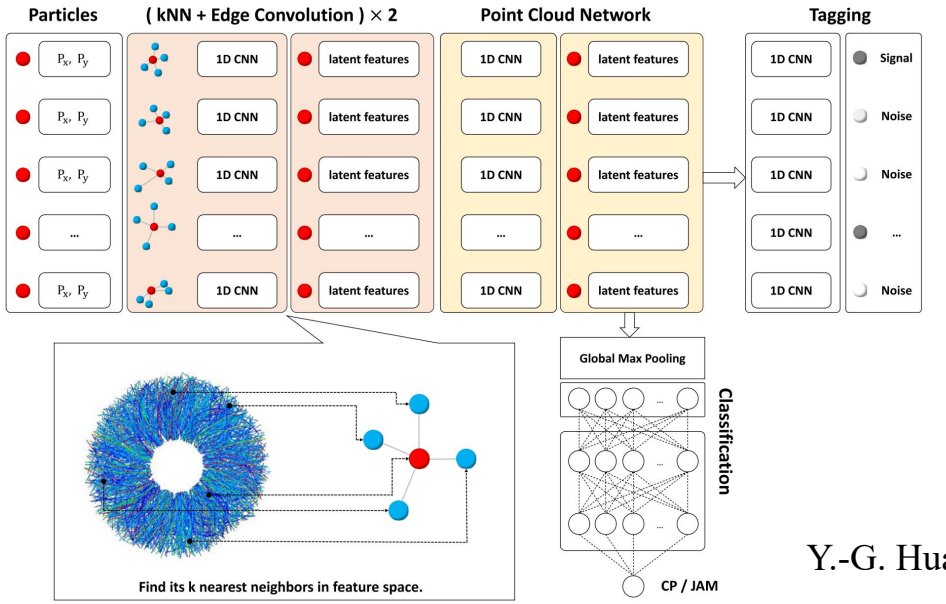
- The energy dependence of $\phi_2$ displays a non-monotonic behavior with a peak at energy around 20-30GeV, indicating that the strength of intermittency becomes the largest in this region.

# Summary and outlook

- The main results from both experimental measurements and phenomenological model exploration of the QCD critical point via intermittency analysis in heavy-ion collisions are reported.

- Further investigations should be done to compare results coming from STAR, NA49 and NA61 heavy-ion experimental data despite the differences between the detectors.

- It is interesting to investigate whether the cutting-edge machine learning method would be helpful to pick out possibly existing weak intermittency signals associated with critical phenomena.




Thank you!

Y.-G. Huang, L.-G. Pang, X. Luo and X.-N. Wang, Phys. Lett. B 827, 137001 (2022)