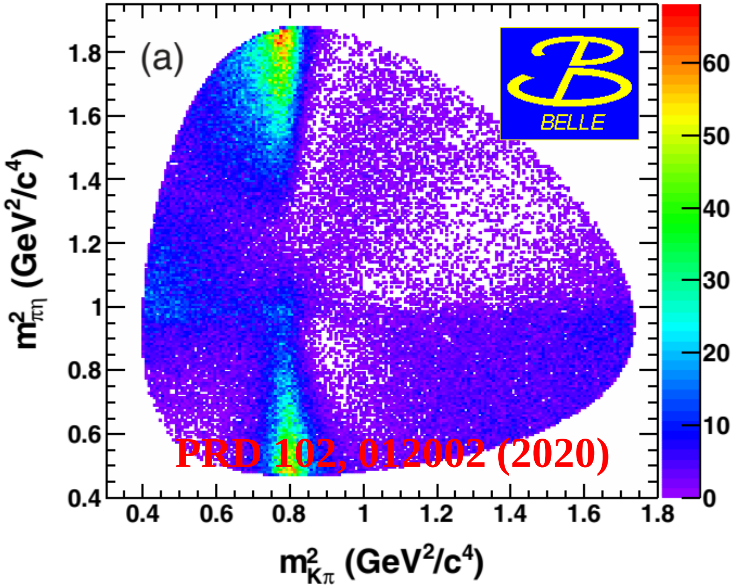
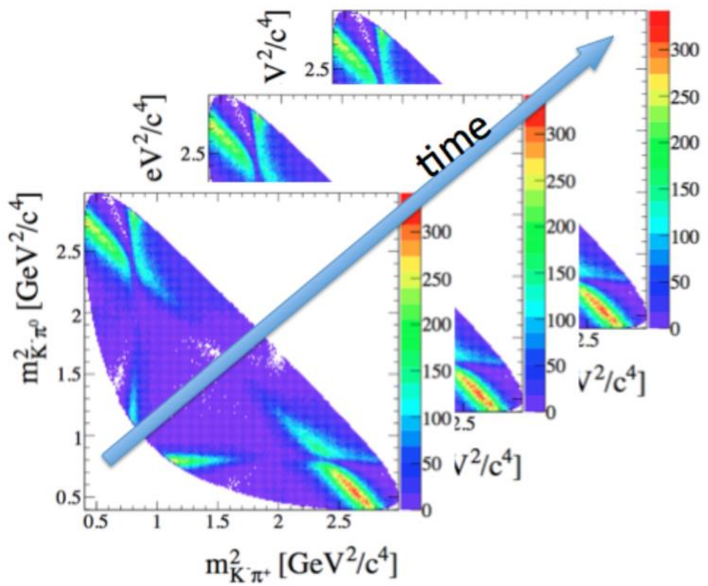
Dalitz analysis of $D^0 \rightarrow K^- \pi^+ \eta$ and Time-(in)dependent Dalitz Analysis package DAFNE

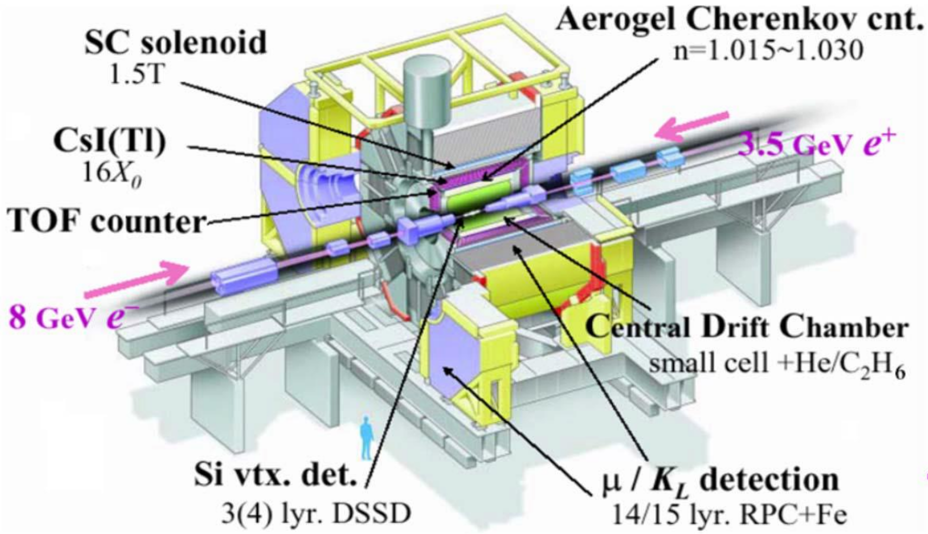
鄢文标 (中国科学技术大学)

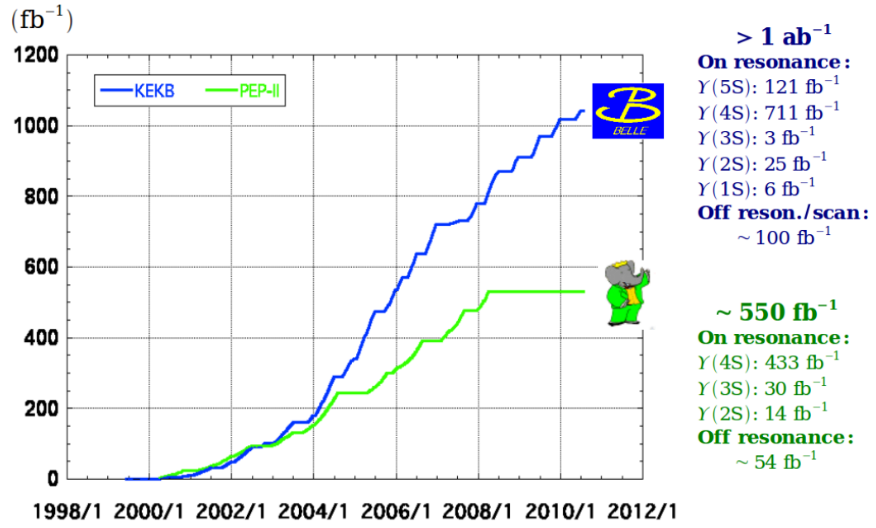




中国物理学会高能物理分会第十一届全国委员代表大会暨学术年会
2022.08.11, 大连

KEKB accelerator & Belle Detector

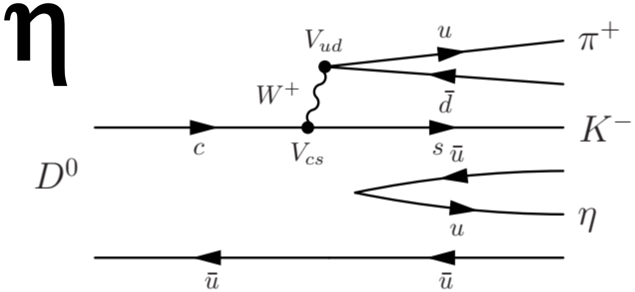




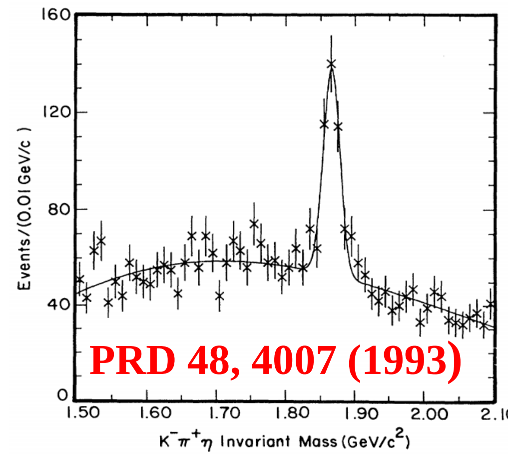
- **Asymmetric e^+e^- collider**
  - ✓ **8 GeV(e^-); 3.5GeV(e^+)**
  - ✓ **Around 10.58GeV ↔ Υ(4S)**
- **B factory, also tau-charm factory**

Process	σ (nb) @ Υ(4S)
$b\bar{b}$	1.1
$c\bar{c}$	1.3
$q\bar{q}$ (q=u,d,s)	2.1
$\tau^+\tau^-$	0.93

$D^0 \to K^- \pi^+ \eta$

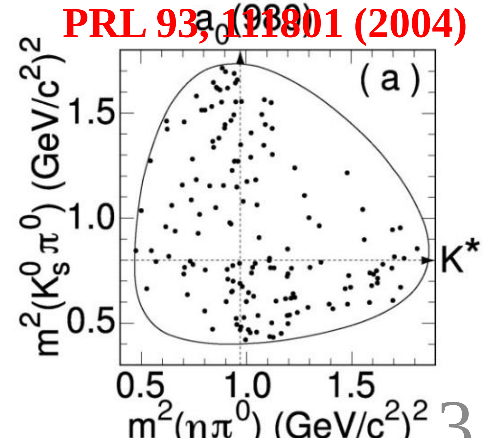


- $D^0 \to K^- \pi^+ \eta$ process
  - ✓ Cabibbo favored via $c \to s\, u\, \bar{d}$
  - ✓ Firstly observed by CLEO
  - ✓ BESIII: Br=(1.853±0.025±0.031)%
  - ✓ Input for D^0-$\bar{D}^0$ mixing via $D^0 \to K^+ \pi^- \eta$
- $D^0 \to K^{*0} \eta$: W-exchange diagram
  - ✓ $D^0 \to K_S{}^0 \pi^0 \eta$: (1.02 ± 0.30)%
  - ✓ Theory calculation: RR D81, 074021; D86, 036012; D89, 054006; (0.51-0.92)%

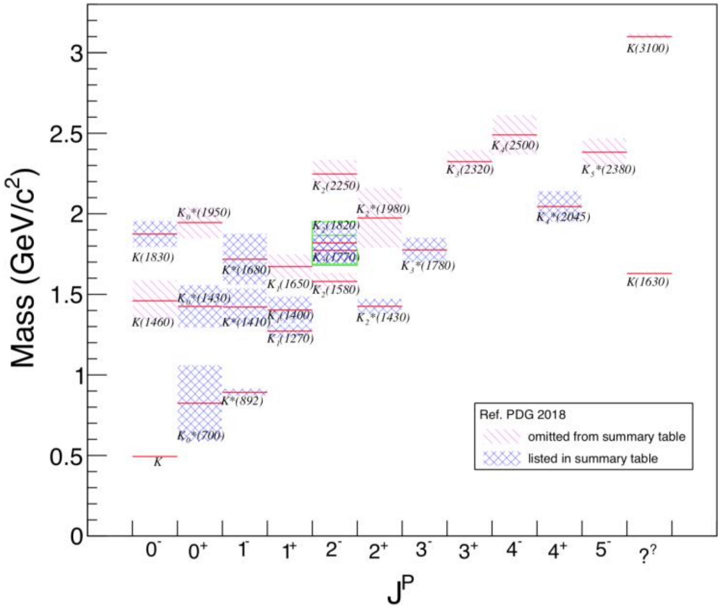


PRD 48, 4007 (1993)

PRL 93, 111801 (2004)

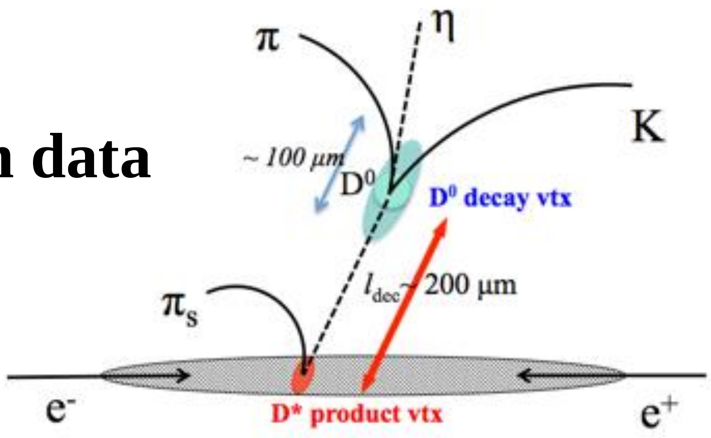


$D^0 \to K^- \pi^+ \eta$



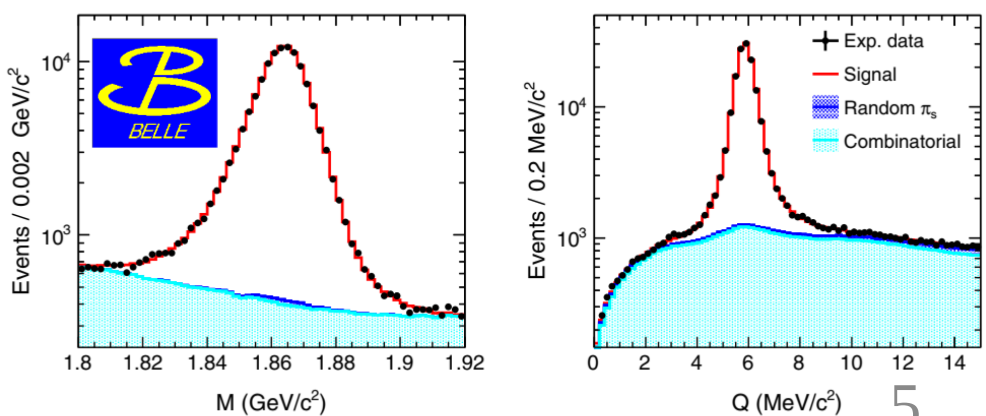
- D three-body hadronic decay
  - ✓ study light hadron spectrum
- Kaon spectrum
  - ✓ PDG2022: 25 kaon states below 3.1 GeV/c^2
  - ✓ Only 13 states well established,
  - ✓ 12 states need confirmation
  - ✓ Little progress in past 30 years
- K* @ Kπ & Kη
  - ✓ Search K*(1410), K*(1680) and K_2*(1980) @ Kη
  - ✓ K*(1680) as pure 1^3D_1 state
  - ✓ K*(1410) & K*(1680): mixture 2^3S_1 and 1^3D_1

$D^0 \rightarrow K^- \pi^+ \eta$ @ Belle



- **Belle data: total 953 fb^{-1}**
  - ✓ **Y(nS) with n=[1, 5] & continuum data**
- **$e^+e^- \rightarrow \gamma^* \rightarrow D^{*+} + X$**

  $D^{*+} \rightarrow D^0 \pi_s^+$ and $D^0 \rightarrow K^- \pi^+ \eta$
  - ✓ **$D^0/\overline{D}^0$ tagged by π_s of D^***
  - ✓ **Veto D^{*+} from B decay**
- **M-Q two-dimension fit:**
  - ✓ **$M \equiv M_{K\pi\eta}$ and $Q \equiv M_{K\pi\eta\pi s} - M_{K\pi\eta} - m_{\pi s}$**
  - ✓ **105197 ± 60 events at signal region, with purity (94.6 ± 0.9)%**



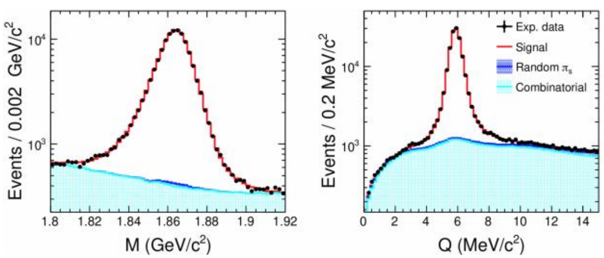
PRD 102, 012002 (2020)

Generalization of Dalitz analysis

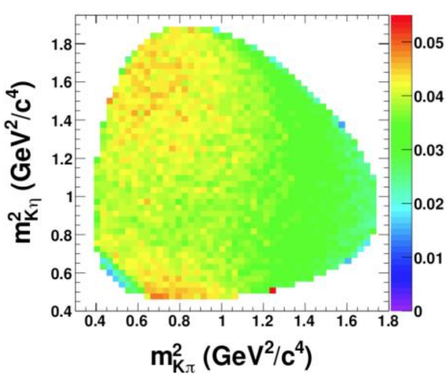
- **Unbinned maximum likelihood method**

$$-2ln\mathcal{L}(m^2_{AB}, m^2_{BC}) = -2\sum_{i=1}^{n} ln[f^i_{sig}p_{sig}(m^2_{AB,i}, m^2_{BC,i}) + f^i_{bkg}p_{bkg}(m^2_{AB,i}, m^2_{BC,i})]$$

M-Q 2D fit

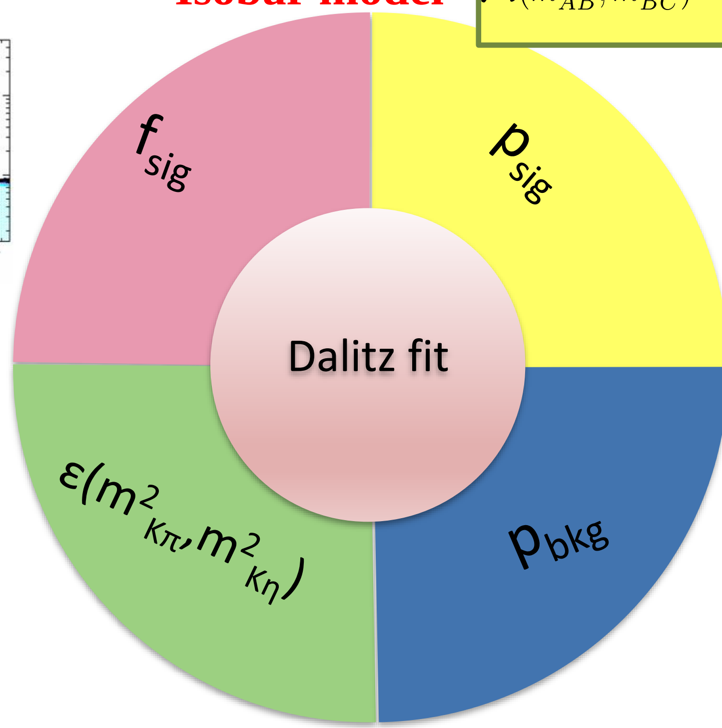


Efficiency plane



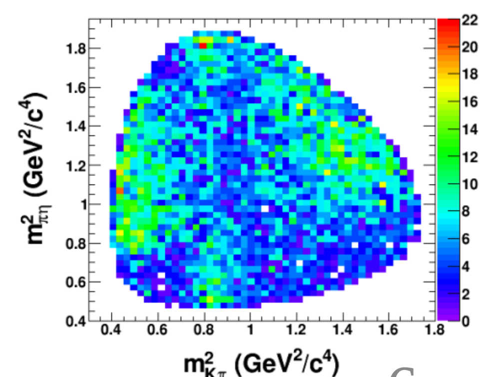
Isobar model

$$\mathcal{M}(m^2_{AB}, m^2_{BC}) = a_{NR}e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m^2_{AB}, m^2_{BC})$$



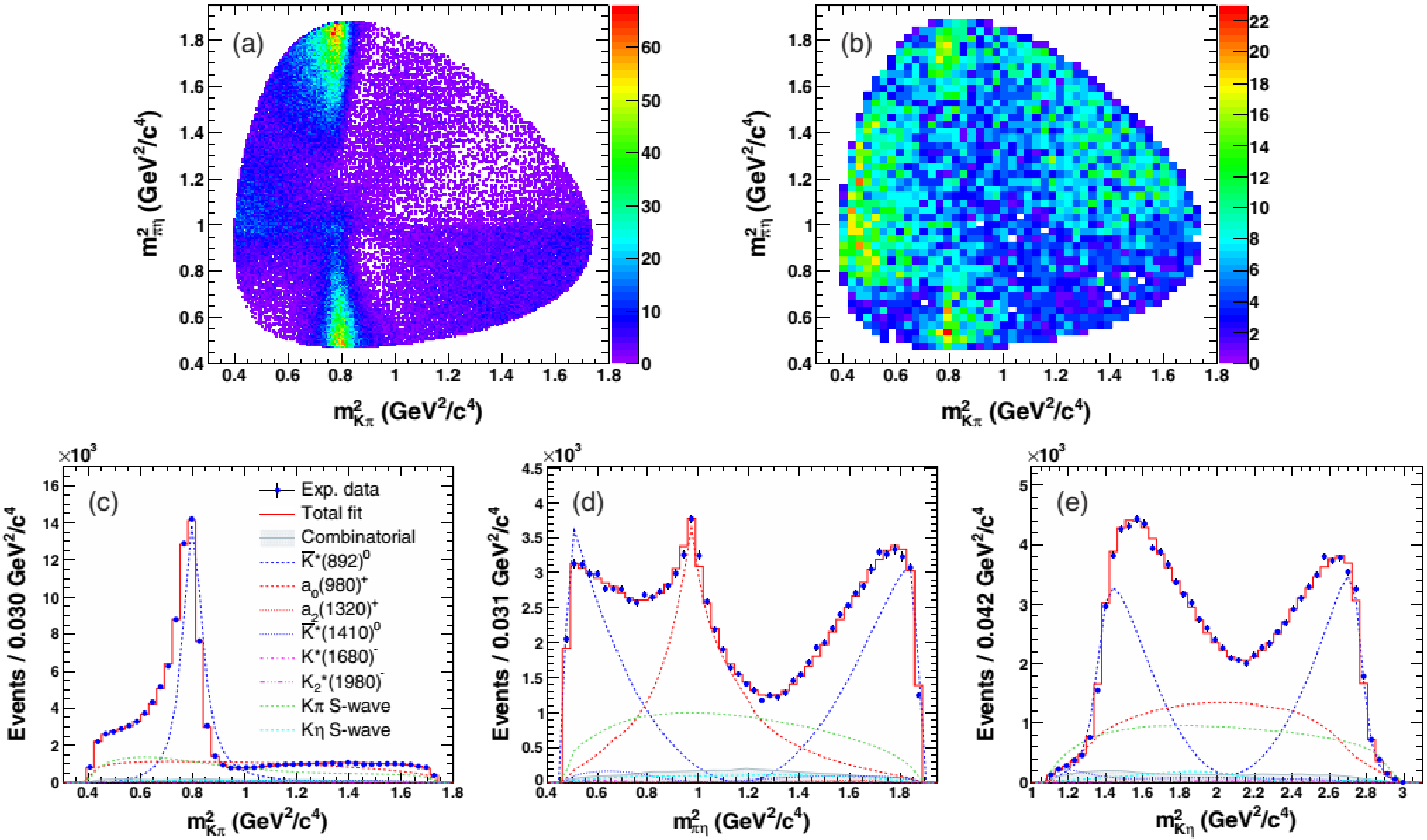
Background PDF

- D^0 √, π_s ×
- D^0 ×, π_s √
- D^0 ×, π_s ×
- M sideband @ Q signal



$$p_{sig} = \frac{\sum|\mathcal{M}(m^2_{12,i}, m^2_{23,i})|^2\epsilon_j(m^2_{12,i}, m^2_{23,i})}{\sum\iint_{DP} dm^2_{12}dm^2_{23}|\mathcal{M}(m^2_{12}, m^2_{23})|^2\epsilon_j(m^2_{12}, m^2_{23})}$$

Dalitz plot fit results

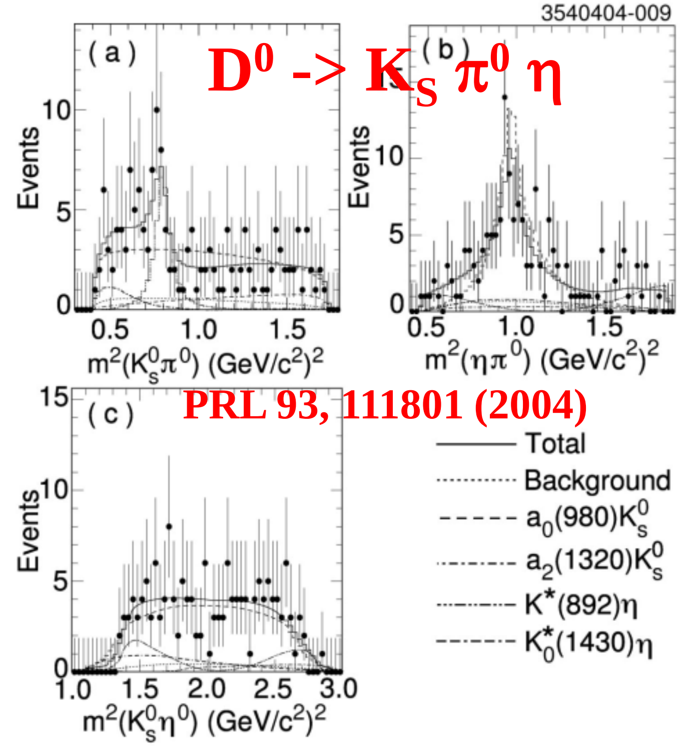


- **The fit quality χ^2/d.o.f = 1638/(1415-24) = 1.18**

Dalitz plot fit results

Component	Magnitude	Phase (°)	Fit fraction (%)
$\bar{K}^*(892)^0$	1	0	$47.61 \pm 1.32^{+0.24+3.64}_{-0.49-2.71}$
$a_0(980)^+$	2.779 ± 0.032	310.3 ± 1.1	$39.28 \pm 1.50^{+1.58+4.38}_{-0.51-3.30}$
$K\pi$ S-wave	10.82 ± 0.23	50.0 ± 5.7	$31.92 \pm 1.21^{+1.47+2.75}_{-0.53-2.87}$
$K\eta$ S-wave	1.70 ± 0.082	113.8 ± 13.6	$3.37 \pm 0.50^{+0.77+3.20}_{-0.27-1.21}$
$a_2(1320)^+$	1.27 ± 0.079	283.4 ± 4.7	$0.74 \pm 0.09^{+0.06+0.37}_{-0.04-0.17}$
$\bar{K}^*(1410)^0$	4.84 ± 0.36	352.7 ± 2.8	$6.94 \pm 0.85^{+0.55+2.37}_{-1.61-3.22}$
$K^*(1680)^-$	2.56 ± 0.18	232.2 ± 6.6	$1.07 \pm 0.16^{+0.11+0.58}_{-0.10-0.36}$
$K_2^*(1980)^-$	9.29 ± 0.69	207.7 ± 4.0	$1.13 \pm 0.15^{+0.05+0.88}_{-0.05-0.98}$
Sum			$132.1 \pm 3.4^{+1.6+8.3}_{-0.7-4.5}$

$D^0 \to K^+ \pi^- \eta$



- **Dominant components: $\bar{K}^*(892)^0$ and $a_0(980)^+$**
- **$K\eta$ S-wave with $K_0^*(1430)$: > 30σ**
- **$K^*(1680)^-/K_2^*(1980)^- \to K^- \eta$ are observed for the first time with 16σ /17σ**

Branching fractions @ $D^0 \to K^- \pi^+ \eta$

- Normalized mode $D^0 \to K^- \pi^+$
- $B(D^0 \to K^*(892)^0\eta) = (1.41 \pm 0.04^{+0.12}_{-0.11} \pm 0.01)\%$
  - ✓ PDG: $B(D^0 \to \bar{K}^*(892)^0 \eta) = (1.02 \pm 0.30)\%$
  - ✓ Theory prediction: (0.51-0.92)%
  - ✓ deviates theory prediction with significance of more than 3σ
- $B(D^0 \to K^*(1680)^-\pi^+ \to K^-\eta\pi^+) = (2.11 \pm 0.32^{+1.16}_{-0.72} \pm 0.02)\times 10^{-4}$
  - ✓ $B(D^0 \to K^*(1680)^-\pi^+ \to K^-\pi^0\pi^+) = (0.19 \pm 0.07)\%$ @ PDG
  - ✓ $\frac{B(K^*(1680)\to K^-\eta)}{B(K^*(1680)\to K^-\pi)} = 0.11 \pm 0.02^{+0.06}_{-0.04} \pm 0.04$
  - ✓ $B(K^*(1680)^- \to K^-\pi^0) = (12.90 \pm 0.83)\%$ @ PDG
  - ✓ $B(K^*(1680)^- \to K^-\eta) = (1.44 \pm 0.21^{+0.79}_{-0.49} \pm 0.54)\%$
- $B(D^0 \to K_2^*(1980)^-\pi^+ \to K^-\pi^+\eta) = (2.2 +1.7-1.9)\times 10^{-4}$
  - ✓ Strongly suppressed do to phase-space and yet allowed due to large width of $K_2^*(1980)$

K*(1680)

- K*(1680) as pure 1^3D_1 state
  - ✓ Kπ : Kη ≈ 1.0 @ theory
  - ✓ Belle: Kπ : Kη ≈ 0.11±0.07
  - ✓ Kπ, Kρ, and K*(892)π
  - ✓ Any idea ?

Mode	EPJC 77, 861	PRD 68, 054014
$\Gamma_{K\pi}$	69.2 MeV	45 MeV
$\Gamma_{K\eta}$	64.4 MeV	53 MeV

±

	EPJC 77, 861	PRD 68, 054014	Experiment
$\Gamma_{K\pi}/\Gamma_{K^*(892)\pi}$	1.66	1.8	2.8±1.1
$\Gamma_{K\rho}/\Gamma_{K\pi}$	0.65	0.58	1.2±0.4
$\Gamma_{K\rho}/\Gamma_{K^*(892)\pi}$	1.07	1.04	1.05+0.27-0.11

- K*(1410) and K*(1680): mixture 2^3S_1 and 1^3D_1

$$\begin{pmatrix} |K^*(1410)\rangle \\ |K^*(1680)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{sd} & \sin\theta_{sd} \\ -\sin\theta_{sd} & \cos\theta_{sd} \end{pmatrix} \begin{pmatrix} |1^3D_1\rangle \\ |2^3S_1\rangle \end{pmatrix}$$

D^0-$\bar{D}^0$ mixing

D^0 and $\bar{D}^0$ are flavor eigenstates, propagate and decays according to

$$i\frac{\partial}{\partial t}\left(\begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array}\right) = \left(M - \frac{i}{2}\Gamma\right)\left(\begin{array}{c} D^0(t) \\ \bar{D}^0(t) \end{array}\right)$$

D^0 and $\bar{D}^0$ are combinations of mass eigenstates

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\ |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle \end{aligned}$$

The mass eigenstates develop in time as

$$\begin{aligned} |D_{1,2}(t)\rangle &= e_{1,2}(t)|D_{1,2}(0)\rangle \\ e_{1,2}(t) &\equiv e^{\left[-i\left(M_{1,2}-\frac{i}{2}\Gamma_{1,2}\right)t\right]} \end{aligned}$$

Two parameters describe D^0 and $\bar{D}^0$ mixing

$$x \equiv \frac{\Delta M}{\Gamma} \qquad \Delta M \equiv M_1 - M_2$$

$$y \equiv \frac{\Delta\Gamma}{2\Gamma} \qquad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$

If either x or y are not zero, mixing occurs

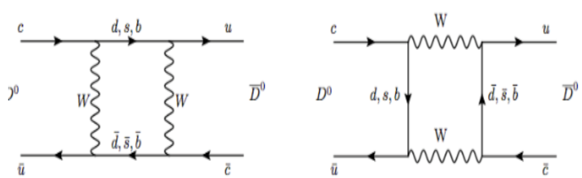
$$|\langle\bar{D}^0|D^0(t)\rangle|^2 = \frac{1}{2}\left|\frac{q}{p}\right|^2 e^{-\Gamma t}\left[cosh(y\Gamma t) - cos(x\Gamma t)\right]$$

$$|\langle D^0|\bar{D}^0(t)\rangle|^2 = \frac{1}{2}\left|\frac{p}{q}\right|^2 e^{-\Gamma t}\left[cosh(y\Gamma t) - cos(x\Gamma t)\right]$$

D^0-$\bar{D}^0$ mixing

- D^0-$\bar{D}^0$ mixing: only up-type quark meson system

  $K^0 \Leftrightarrow \bar{K}^0$, $B_d^0 \Leftrightarrow \bar{B}_d^0$ and $B_s^0 \Leftrightarrow \bar{B}_s^0$
- In Standard model (SM) , D^0-$\bar{D}^0$ mixing is
  - ✓ GIM & CKM
- The SM predicts: $|x|$, $|y|$ ~O(1%)



short distance (<0.1%)

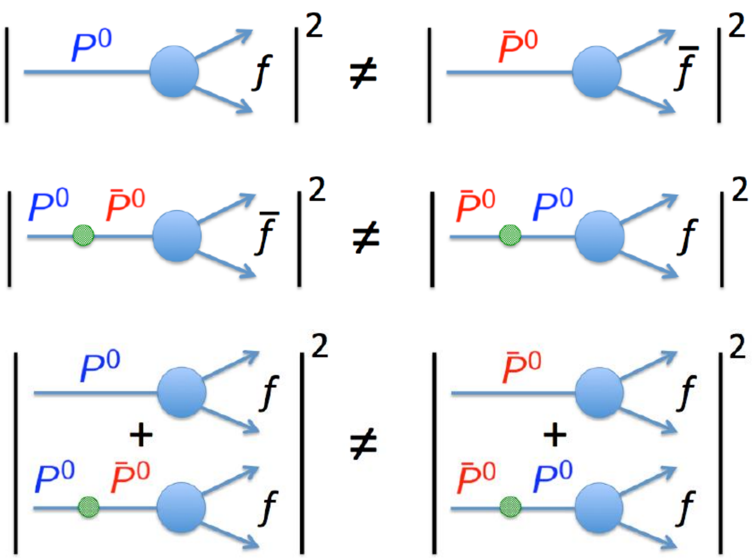
long distance (~1%)

- Precisely measured x and y
  - ✓ Test SM prediction
  - ✓ Sensitive to new physics

CP violation

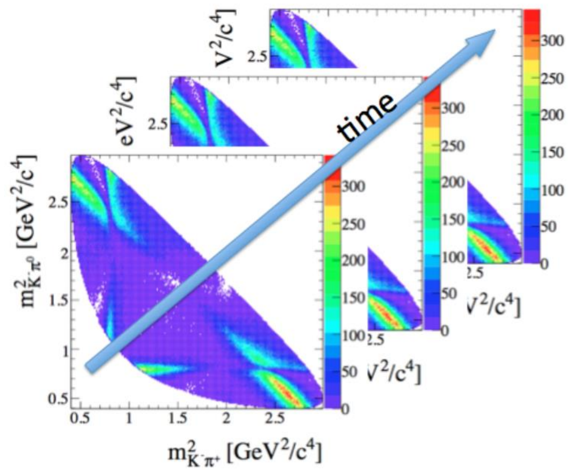
- CP Violation @SM: phase in CKM
  - ✓ @ charm sector: ~$O(10^{-3})$
  - ✓ ~1% exp. sensitivity to observe NP
- Time integrated CP asymmetry A_{CP}
  - ✓ Decay @ D^+ & D_s^+: direct CPV
  - ✓ Decay @ D^0: direct and indirect CP Violation combined

$$A_{CP} = \frac{\Gamma(D \to f) - \Gamma(\bar{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\bar{D} \to \bar{f})}$$



- **Direct** CPV, $|\bar{A}_{\bar{f}}/A_f| \neq 1$
- CPV in mixing, $|q/p| \neq 1$
- CPV in interference, $\mathrm{Arg}(q/p)=\phi\neq 0$

Time-dependent Dalitz Analysis (TDDA)



- **Three (four) dimensional fit**
  - ✓ **Dalitz analysis (two variable)**
  - ✓ **$D^0/\bar{D}^0$ decay time**
  - ✓ **(with) decay time error**
- **Extract mixing parameters in $D^0/\bar{D}^0$ decay**
  - ✓ **(x, y), |q/p| and Arg(q/p)= ϕ**
  - ✓ **e.g. self-conjugated $D^0 \rightarrow K_S\, \pi^+\, \pi^-$**

$$|M(\boldsymbol{f},t)|^2 = \frac{e^{-\Gamma t}}{2}\Big[\left(|A_f|^2 + \left|\frac{q}{p}\right|^2 |A_{\bar{f}}|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - \left|\frac{q}{p}\right|^2 |A_{\bar{f}}|^2\right)\cos(x\Gamma t)$$

$$+2\mathrm{Re}\left[\frac{q}{p}A_{\bar{f}}A_f^*\right]\sinh(y\Gamma t) + 2\mathrm{Im}\left[\frac{q}{p}A_{\bar{f}}A_f^*\right]\sin(x\Gamma t)\Big]$$

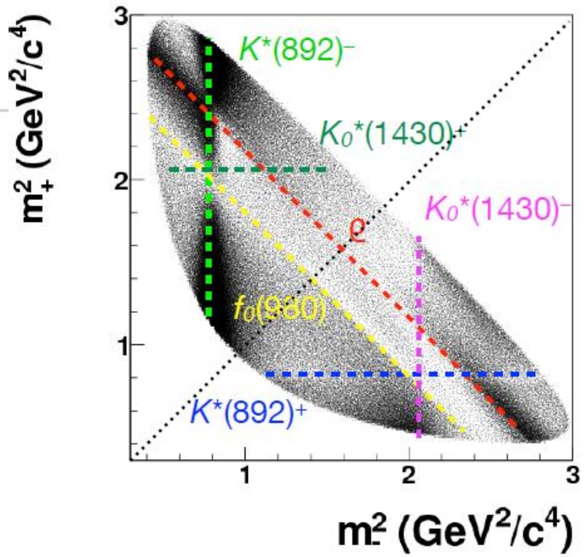
Could be represented by Dalitz plot

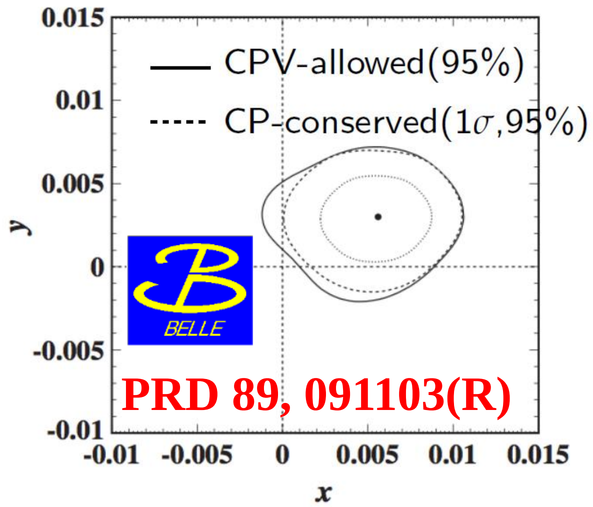
- **Measure CP violation in pure mixing or in interference of decay amplitudes with and without mixing**

$D^0 \to K_S \pi^+ \pi^-$

- $1.2M\ D^0 \to K_S \pi^+ \pi^-$ events @ Belle

  - ✓ RS: $K^{*-} \pi^+$
  - ✓ WS: $K^{*+} \pi$-
  - ✓ CP+: $K_S f_0$
  - ✓ CP-: $K_S \rho$





PRD 89, 091103(R)

- Belle II vertex resolution, ~2 better than BaBar
- Decay time resolution 0.14ps, ~2 better than Belle,
- Increased tracking volume in SVD & CDC ⇒ ~30% higher K_S efficiency
- Improved PID with better K/π separation relative to Belle
- Belle II: 50 ab^{-1} data (plan) → $\sim 80\ M\ D^0 \to K_S \pi^+ \pi^-$

No TDDA package @ market & Speed !!!

The DAFNE

- **The DAFNE (DAlitz Fitter aNd Event generator)**
  - ✓ **C++ Time-(in)dependent Dalitz analysis framework for $P \rightarrow PPP$ decay**
  - ✓ **Running on multithread CPU based on Hydra framework**

DAFNE
(DAlitz Fitter aNd Event generator):
https://stash.desy.de/users/dicanto/repos/dafne/browse

Computing Support:
HYDRA https://github.com/MultithreadCorner/Hydra/

CUDA (Compute Unified Device Architecture)	OpenMP (Open Multi-Processing)	TBB (Threading Building Blocks)
Nvidia GPU	CPU	CPU

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Reference for formula & logic:
cfit (original from Babar):
https://github.com/cfit/cfit/tree/master/include/cfit

The DAFNE Framework

- **Time-(in)dependent Dalitz analysis**
- **Supported amplitutudes**
  - ✓ **BW, GS, LASS, generated LASS**
  - ✓ **Flatte, K-matrix**

$$\left|T_f(m_+^2, m_-^2; t)\right|^2 = \left|A_f g_+(t) + \bar{A}_f \frac{q}{p} g_-(t)\right|^2 = \left|A_f g_+(t) + A_{\bar{f}} \frac{q}{p} g_-(t)\right|^2$$

$$g_\pm(t) = \theta(t) e^{-imt} e^{-t/2} {}^{\cosh(zt/2)}_{\sinh(zt/2)}, z = -(y + ix)$$

- **MC generation**
  - ✓ **Efficiency plane**
  - ✓ **Dalitz variable & decay time smearing background, exponential sampling method**

$$\left|T_f(m_+^2, m_-^2; t)\right|^2 \otimes G(t, b, s\sigma_t)$$
$$= \left\{ \left|\frac{A_f + A_{\bar{f}}}{2}\right|^2 e^{-(1-x)\Gamma t} + \left|\frac{A_f^* + A_{\bar{f}}^*}{2}\right|^2 e^{-(1+x)\Gamma t} + 2Re\left[\frac{A_f + A_{\bar{f}}}{2} \frac{A_f^* + A_{\bar{f}}^*}{2}\right] e^{-(1-iy)\Gamma t} \right\} \otimes G(t, b, s\sigma_t)$$

- **Fitting**
  - ✓ **Improved Minuit2 interference, time resolution Semi-analytical normalization, contour**
  - ✓ **Efficiency plan, background**
  - ✓ **Future feature: KDE efficiency, automatic scan plots generation**
- **Auxiliary functions:**
  - ✓ **configuration file interface, components plotting time-independent & time dependent (not yet)**

Dalitz analysis

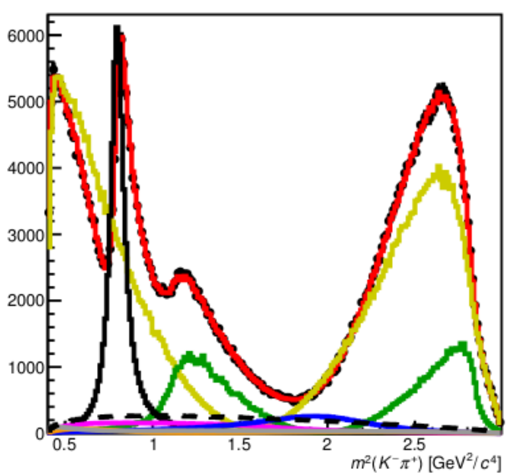
- **Dalitz analysis of $D^0 \rightarrow K^- \pi^+ \pi^0$**
  - ✓ **0.5 M $D^0 \rightarrow K^- \pi^+ \pi^0$ toy MC with CLEO results**

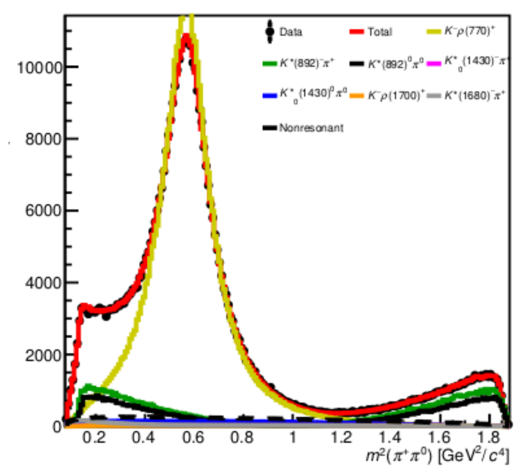
CLEO

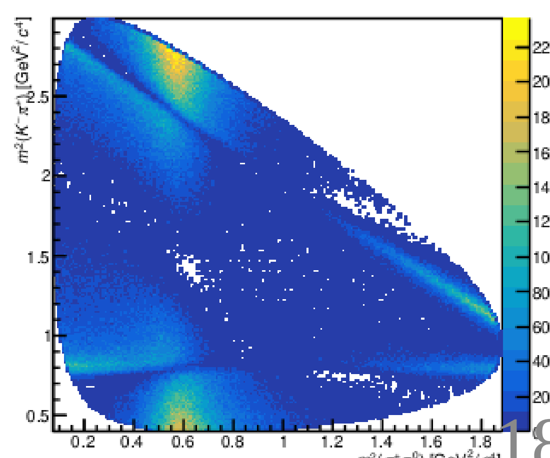
Mode	Fit fraction
$\rho(770)^+ K^-$	$0.788 \pm 0.019 \pm 0.013 \pm 0.046$
$K^*(892)^- \pi^+$	$0.161 \pm 0.007 \pm 0.007^{+0.026}_{-0.008}$
$\overline{K}^*(892)^0 \pi^0$	$0.127 \pm 0.009 \pm 0.005 \pm 0.015$
$\rho(1700)^+ K^-$	$0.057 \pm 0.008 \pm 0.007 \pm 0.006$
$\overline{K}^*_0(1430)^0 \pi^0$	$0.041 \pm 0.006 \pm 0.007^{+0.031}_{-0.005}$
$K^*_0(1430)^- \pi^+$	$0.033 \pm 0.006 \pm 0.007 \pm 0.012$
$K^*(1680)^- \pi^+$	$0.013 \pm 0.003 \pm 0.003 \pm 0.003$
Non-resonant	$0.075 \pm 0.009 \pm 0.006^{+0.056}_{-0.009}$

```
***** Fit fractions:
RHO_770_P = 0.769072 +- 0.003002
KST_892_M = 0.164155 +- 0.000950
KST_892_0 = 0.126109 +- 0.000728
RHO_1700_P = 0.009295 +- 0.000029
K0_1430_0 = 0.039910 +- 0.000118
K0_1430_M = 0.032692 +- 0.000096
KST_1680_M = 0.014659 +- 0.000048
Nonresonant = 0.072988 +- 0.000200
```

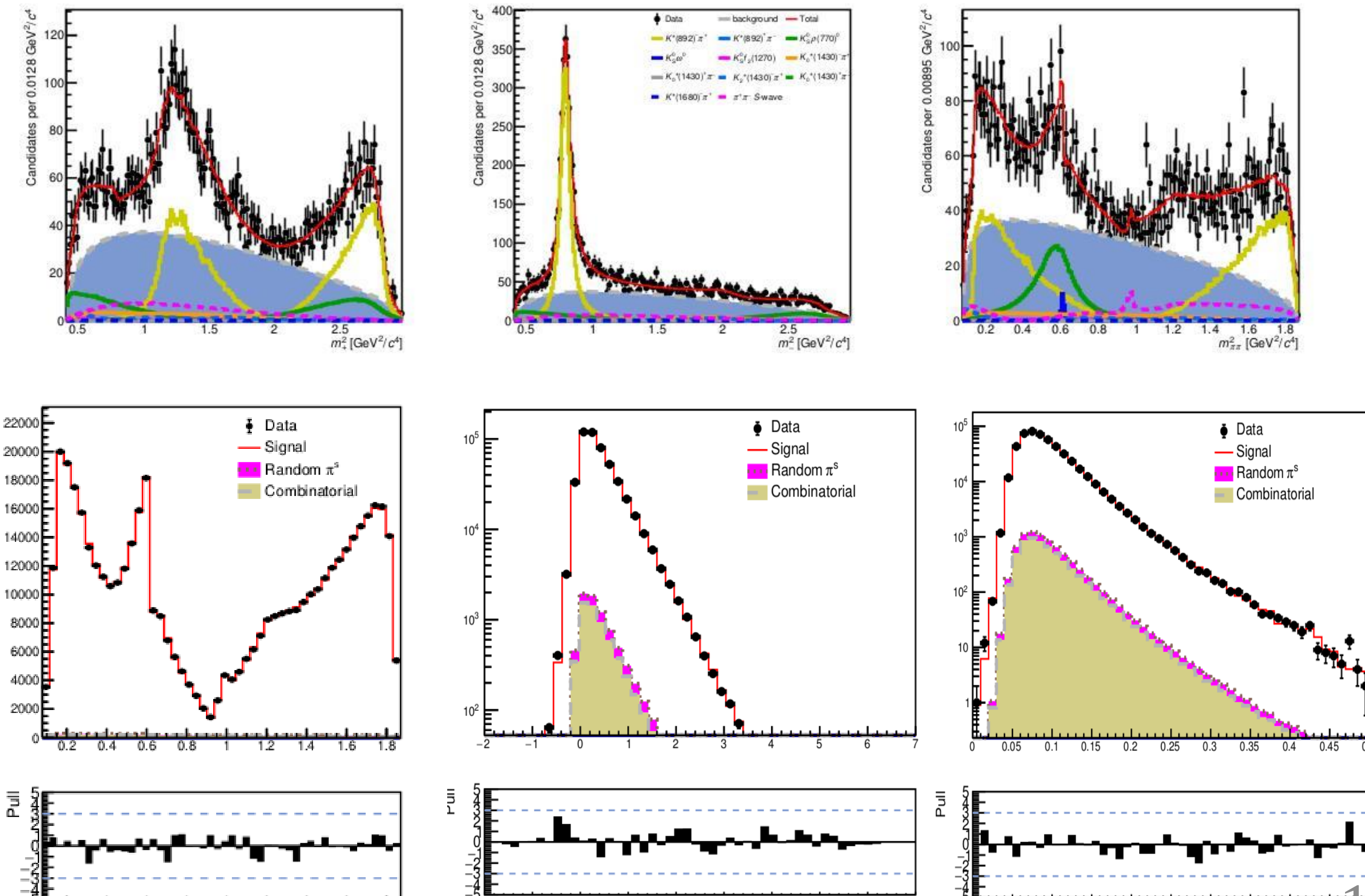
DAFNE







Time-dependent Dalitz analysis



MC results with DAFNE

- Use 1.2M $D^0 \rightarrow K_S \pi^+ \pi^-$ events, x = 0.004, y = 0.006

τ	x	y	b	s
0.4108 ± 0.0006	0.0027 ± 0.0012	0.0061 ± 0.0010	-0.0007 ± 0.0005	1.2000 ± 0.0019

BELLE

x $[10^{-3}]$	$5.6 \pm 1.9\,^{+0.7}_{-1.1}$
y $[10^{-3}]$	$3.0 \pm 1.5\,^{+0.5}_{-0.9}$
$\|q/p\|$	$0.90\,^{+0.16}_{-0.15}\,^{+0.08}_{-0.06}$
ϕ [rad]	$0.10 \pm 0.19\,^{+0.07}_{-0.09}$

$$\sqrt{\sigma_{fit}^2 + \sigma_{float\ dalitz}^2} = \sqrt{0.0012^2 + 0.0010^2} = 0.0016$$

Operate with 1M events	Single-thread Program	DAFNE, in a 48 threads CPU
Time-independent Dalitz Fit	~2 hours	~8 minutes
Time-independent toy generation	~14 minutes	~1 minutes
Time-dependent Dalitz Fit	~10 minutes	<1 minutes
Time-dependent toy generation	~4 hours	~10 minutes

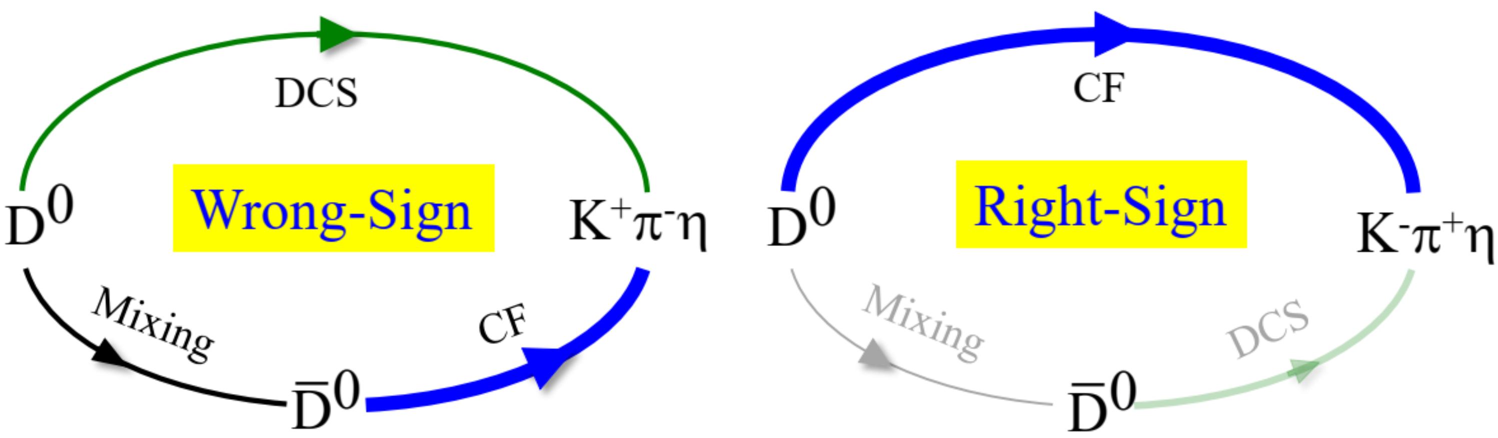
~x10 faster! ***For the moment***

Summary and outlook

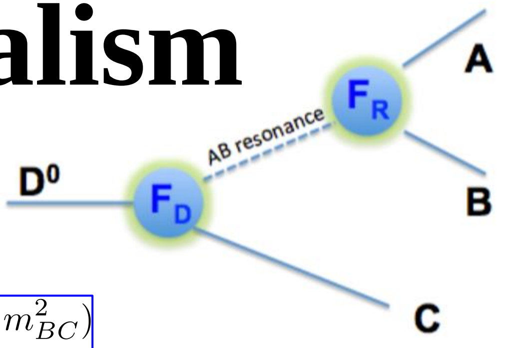
- **Based on 953 fb^{-1} Belle data, Dalitz analysis of $D^0 \to K^- \pi^+ \eta$ is performed for the first time.**
  - ✓ **$B(D^0 \to K^- \pi^+ \eta)$ is consistent with BESIII's result.**
  - ✓ **$B(D0 \to \bar{K}^*(892)^0 \eta)$=(1.41 +0.13-0.12)%, deviates theory prediction with significance of more than 3σ**
  - ✓ **$K^*(1680)^-/K_2^*(1980)^- \to K^- \eta$ are observed.**
  - ✓ **$\frac{B(K^*(1680)\to K^-\eta)}{B(K^*(1680)\to K^-\pi)}=0.11\pm0.02(\text{stat})^{+0.06}_{-0.04}(\text{sys})\pm0.04(\text{PDG})$, is not consistent with theory prediction (≈1) under assumption K*(1680) as pure 1^3D_1 state.**
- **A c++ package DAFNE for time-(in)dependent Dalitz analysis**
  - ✓ **Paper in preparation**

$D^0 \to K^- \pi^+ \eta$

- **Wrong-sign decays have important role on D^0-$\bar{D}^0$ mixing & CP violation**
- **Wrong-sign $D^0 \to K^+ \pi^- \eta$ for D^0-$\bar{D}^0$ mixing: not yet**
- **With 50 ab^{-1} data @ Belle II, Time-dependent Dalitz analysis of $D^0 \to K^+ \pi^- \eta$: possible**
- **Right-sign $D^0 \to K^- \pi^+ \eta$: necessary input for $D^0 \to K^+ \pi^- \eta$**



Dalitz analysis formalism



- **Dalitz standard form** $d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{M}|^2 dm^2_{AB}dm^2_{BC}$
- **Isobar model** $\mathcal{M}(m^2_{AB}, m^2_{BC}) = a_{NR}e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m^2_{AB}, m^2_{BC})$
- **Matrix element A_r** $\mathcal{A}(ABC|r) = F_D \times F_r \times T_r \times \Omega_J$
  - **Blatt-Weisskopf centrifugal barrier factor: F_r, F_D**
  - **Angular distribution function Ω_J by Zemach tensor**
  - **Dynamical function T_r**
    - ✓ **most of resonances by relativistic Breit-Wigner**
    - ✓ **a_0(980) by Flatte model**

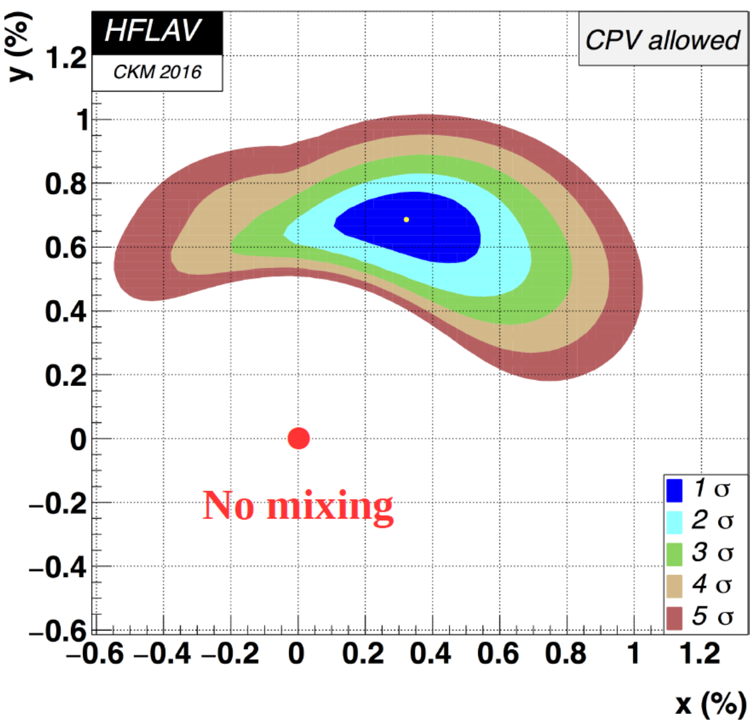
$$T_R(s) = \frac{1}{m^2_{a_0} - s - i(g^2_{\pi\eta}\rho_{\pi\eta} + g^2_{\bar{K}^0K}\rho_{\bar{K}^0K} + g^2_{\pi\eta'}\rho_{\pi\eta'})}$$

    - ✓ **Kπ & Kη S-wave contribution by LASS model**

$$\mathcal{A}_{g\mathrm{LASS}}(s) = \frac{\sqrt{s}}{2q}\cdot[B\sin(\delta_B+\phi_B)e^{i(\delta_B+\phi_B)} + \sin(\delta_R)e^{i(\delta_R+\phi_R)}e^{2i(\delta_B+\phi_B)}],$$

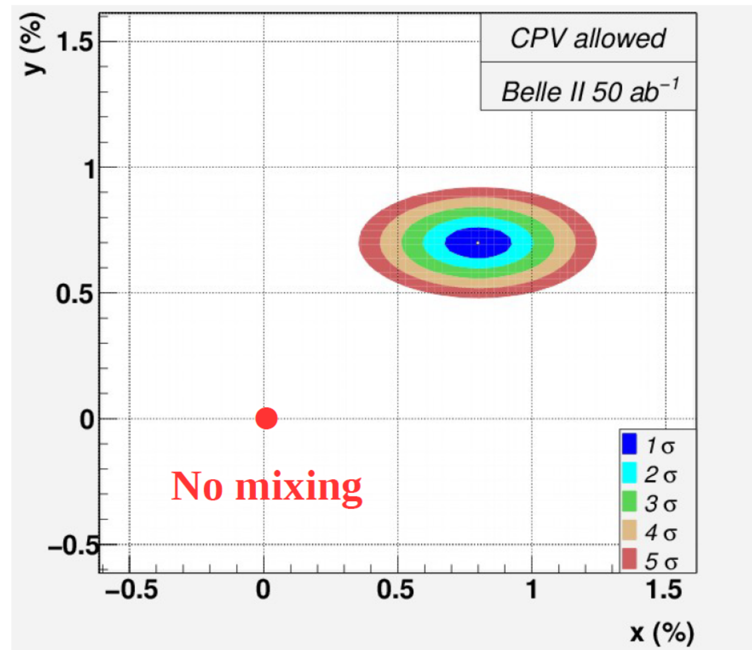
Non-resonant and K_0*(1430) components

Expected Belle II precision



World average (mixing):

$x = (0.32 \pm 0.14)\%$, $y = (0.69^{+0.06}_{-0.07})\%$



Belle II (50 ab^{-1})

$x = 0.8 \pm 0.09\%$, $y = 0.7 \pm 0.04\%$

(result is conservative, does not include modes: $K^+\pi^-\pi^0$, $K_SK^+K^-$ etc.)