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Lattice Parton Collaboration

Pion and Kaon Distribution Amplitudes from Lattice QCD

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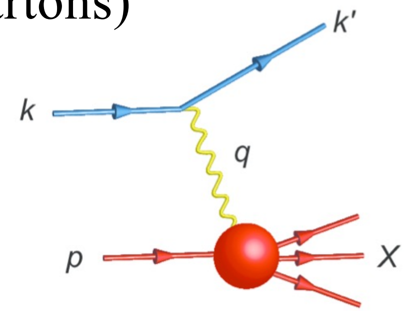
Outline

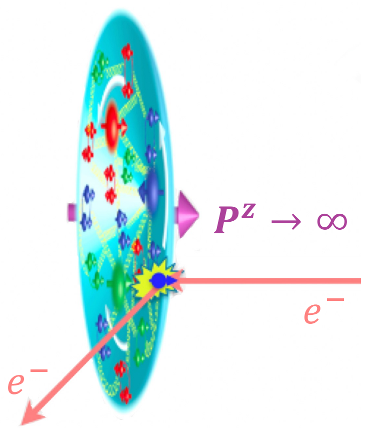
- An introduction to LaMET
- Light cone distribution by LaMET
  - Self renormalization
  - Fourier transform (Extrapolation)
- Numerical results

Introduction

- Our understanding of hadron structure has greatly advanced since deep-inelastic scattering experiments showed that the proton contains much smaller point-like objects (Feynman's partons)



R. Feynman
Nobel prize in 1965



- **Ultra-relativistic (1D, pie shape):**

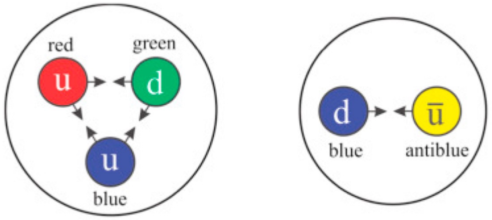
  Quarks and gluons "frozen" in the transverse plane due to the time dilation effect.

- **Parton model (1969, R. Feynman):**

  During a hard collision, the proton can be approximate as a beam of free particles that Feynman called partons.

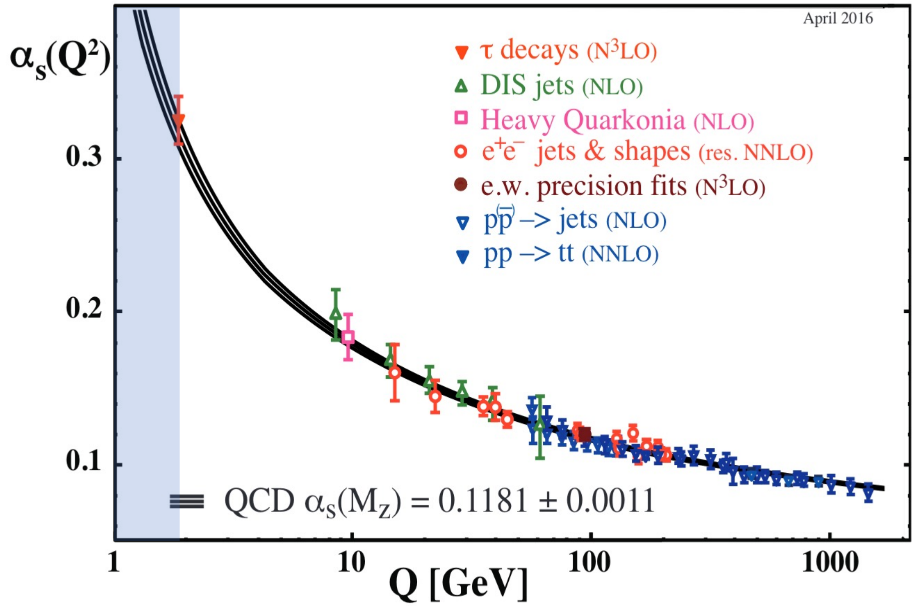
Introduction

➤ **Color confinement and asymptotic freedom**



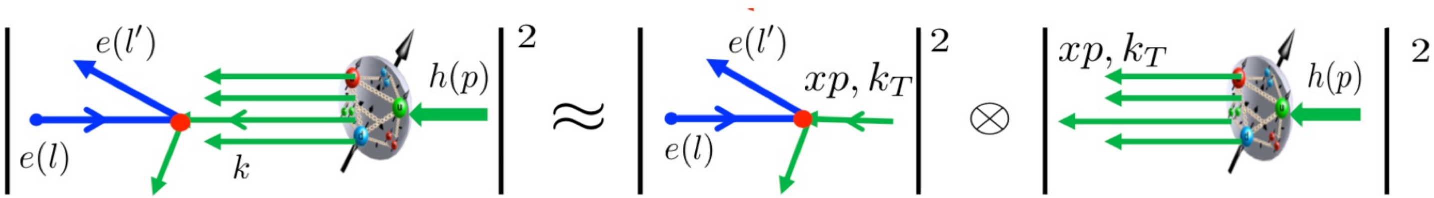
Baryon
(proton, p^+)

Meson
(negative pion, π^-)



➤ **QCD factorization**

1-dimensional parton density (PDF, DA)

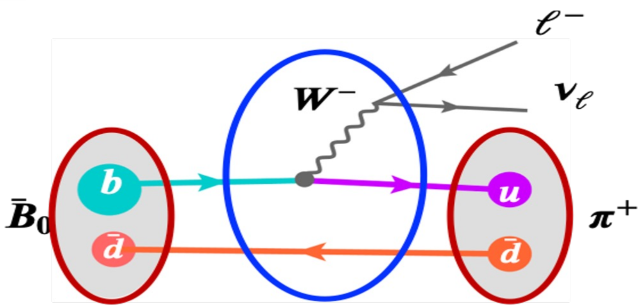


Cross section

Asymptotic freedom
(hard kernel)

Color confinement
(Parton in hadron)

Introduction



- **Hadron distribution amplitudes (DAs) are important inputs in the description of hard exclusive processes**

- **At leading-twist, they represent the distribution of longitudinal momentum among quarks in the leading Fock state of a hadron**

- **Among these, the simplest one is the pion DA**

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0|\bar{\psi}_1(0) n \cdot \gamma\gamma_5 U(0,\xi^-)\psi_2(\xi^-)|\pi(p)\rangle = i f_\pi \Phi_\pi(x)$$

- **It is important for many phenomenological applications, e.g.**
  - $B \to \pi l \nu_l$, $B \to \pi\pi$, …
  - $\gamma^* \to \gamma\pi$, $\gamma\gamma \to \pi\pi$
  - $eN \to eN\pi$
  - $B \to K^* l^+ l^-$
  - $B \to \phi l^+ l^-$
  - …

Introduction

42 Years

1980 → 2022

- **Asymptotic LCDAs**

  A.V. Efremov et. al., Theor.Math.Phys.42 (1980)

- **Sum rules**

  V.L. Chernyak et. al., Nucl.Phys.B 201 (1982)
  Vladimir M. Braun et. al., Z.Phys.C 44 (1989)
  Patricia Ball et. al., JHEP 08 (2007)

- **Lattice calculation by OPE**

  G. Martinelli et. al., Phys.Lett.B 190 (1987)
  RQCD Collaboration, JHEP 11 (2020)

- **Quantum Computing**

  QuNu Collaboration arXiv:2207.13258(2022)

- **Quark model**

  Choi, Phys.Rev.D 75 (2007)

- **Dyson-Schwinger Equation**

  F. Gao, L. Chang et.al.Phys.Rev.D 90 (2014)
  Craig D.et.al., Prog.Part.Nucl.Phys. (2021)

- **Light-cone sum rule**

  S. Cheng et.al. Phys.Rev.D 102 (2020)

- **Lattice calculation by LaMET**

  Zhang, et. al., Phys.Rev.D 95 (2017)
  R. Zhang et.al., Phys.Rev.D 102 (2020)
  J.Hua et.al(LPC)., Pev.Lett.127 (2021)

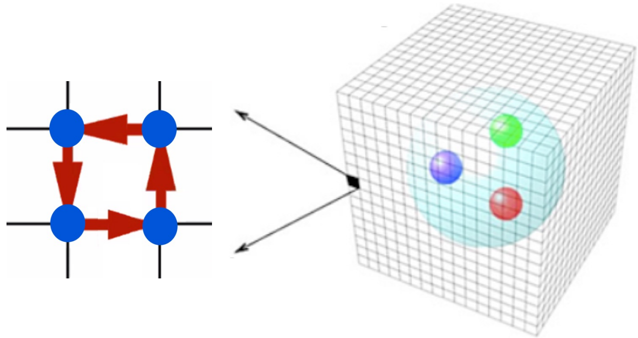
Lattice QCD

- **Numerical simulation in discretized 4D Euclidean space-time;**

- **Lattice QCD: action**

$$S_E^{\mathrm{latt}} = -\sum_{\Box} \frac{6}{g^2} \mathrm{Re}\,\mathrm{tr}_N \left(U_{\Box,\mu\nu}\right) - \sum_q \bar{q} \left(D_\mu^{\mathrm{lat}} \gamma_\mu + a m_q\right) q$$

Wilson gauge action   Lattice fermion action



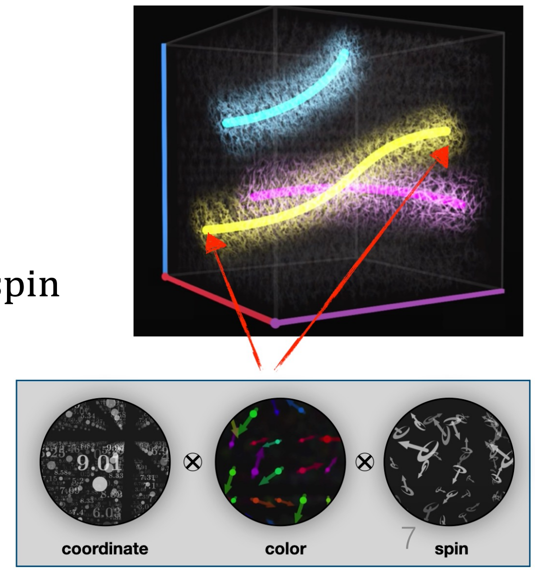
- **Correlation functions:**

$$\langle \mathcal{O}(U, q, \bar{q}) \rangle = \frac{\int [\mathcal{D}U] e^{-S_{\mathrm{glue}}^{\mathrm{latt}}} \prod_q \det\left(D_\mu^{\mathrm{latt}} \gamma_\mu + a m_q\right) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}U] e^{-S_{\mathrm{glue}}^{\mathrm{latt}}} \prod_q \det\left(D_\mu^{\mathrm{latt}} \gamma_\mu + a m_q\right)}$$

- **Monte Carlo simulation:**

- The integration is performed for all link variables: $n_s^3 \times n_t \times N_{\mathrm{color}} \times N_{\mathrm{spin}}$
- Importance sampling: $e^{-S_{\mathrm{glue}}^{\mathrm{latt}}}(U) \prod_q \det\left(D_\mu^{\mathrm{latt}}(U) \gamma_\mu + a m_q\right)$
- Therefore

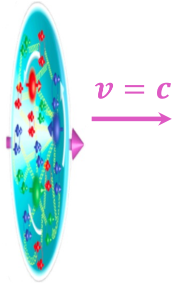
$$\langle \mathcal{O}(U, q, \bar{q}) \rangle = \frac{1}{N_{\mathrm{conf}}} \sum_{k=1}^{N_{\mathrm{conf}}} \tilde{\mathcal{O}}\left(U^{(k)}\right)$$

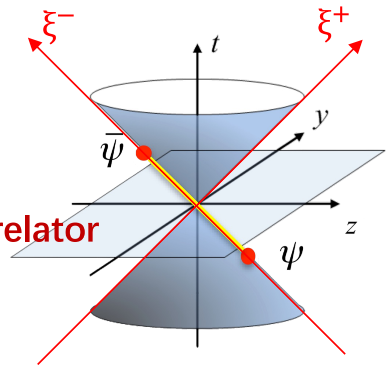


Large-Momentum effective theory

- DAs/PDFs are defined from correlation functions on the light-cone:

infinite-momentum frame





DA/PDF (or more general parton physics):

Minkowski space, real time

infinite momentum frame, on the light-cone

Light-cone DA:

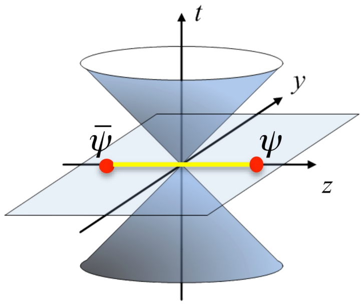
$$q(x) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \left\langle 0 \middle| \bar{\psi}\left(\frac{\xi^-}{2}\right)[\cdots]\psi\left(-\frac{\xi^-}{2}\right) \middle| P^+ \right\rangle \sim \langle 0|\psi^\dagger(xP^+)[\cdots]\psi(xP^+)|P^+\rangle$$

Light-like coordinates: $\xi^\pm = \frac{t \pm z}{\sqrt{2}}$

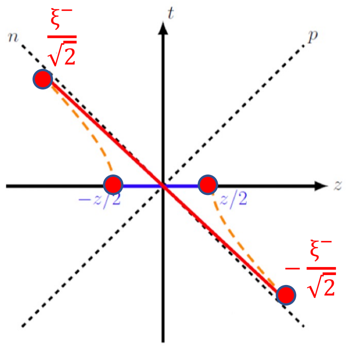
Cannot deal by Lattice QCD directly !

Large-Momentum effective theory

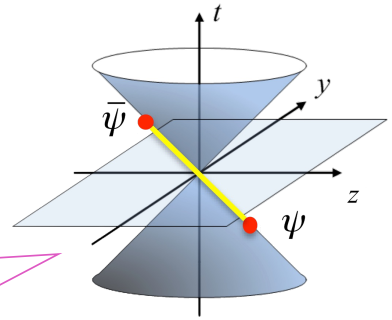
- Define a lattice calculable, equal-time correlation: **quasi-DA**



Boosting P^z



EFT



- Effective field theory:
  - Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C(x, y, P^z, \mu)\, q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)$$

Quasi-DA; Matching kernel; LCDA

Power suppressed by $m^2/(P^z)^2$, $\Lambda^2/(P^z)^2$

Light cone distribution by LaMET

- **Pion LCDA:**

I. Calculate the bare quasi-DA correlation

$$\tilde{h}(z,a,P_z) = \langle 0|\bar{\psi}_1(0)n_z\cdot\gamma\gamma_5 U(0,z)\psi_2(z)|\pi(P)\rangle$$

II. Non-perturbative renormalization

$$\tilde{h}(z,a,P_z) = Z(z,a)\tilde{h}_R(z,a,P_z)$$

III. Fourier transform (Extrapolation)

$$if_\pi\tilde{\phi}_\pi(x,P_z) = \int \frac{d_z}{2\pi} e^{-ixzP_z}\tilde{h}_R(z,a\to 0,P_z)$$

IV. Matching to light clone

$$\tilde{\phi}_\pi(x,P_z) = \int dy\, Z(x,y,Pz,\mu)\phi(x,\mu) + p.c.$$

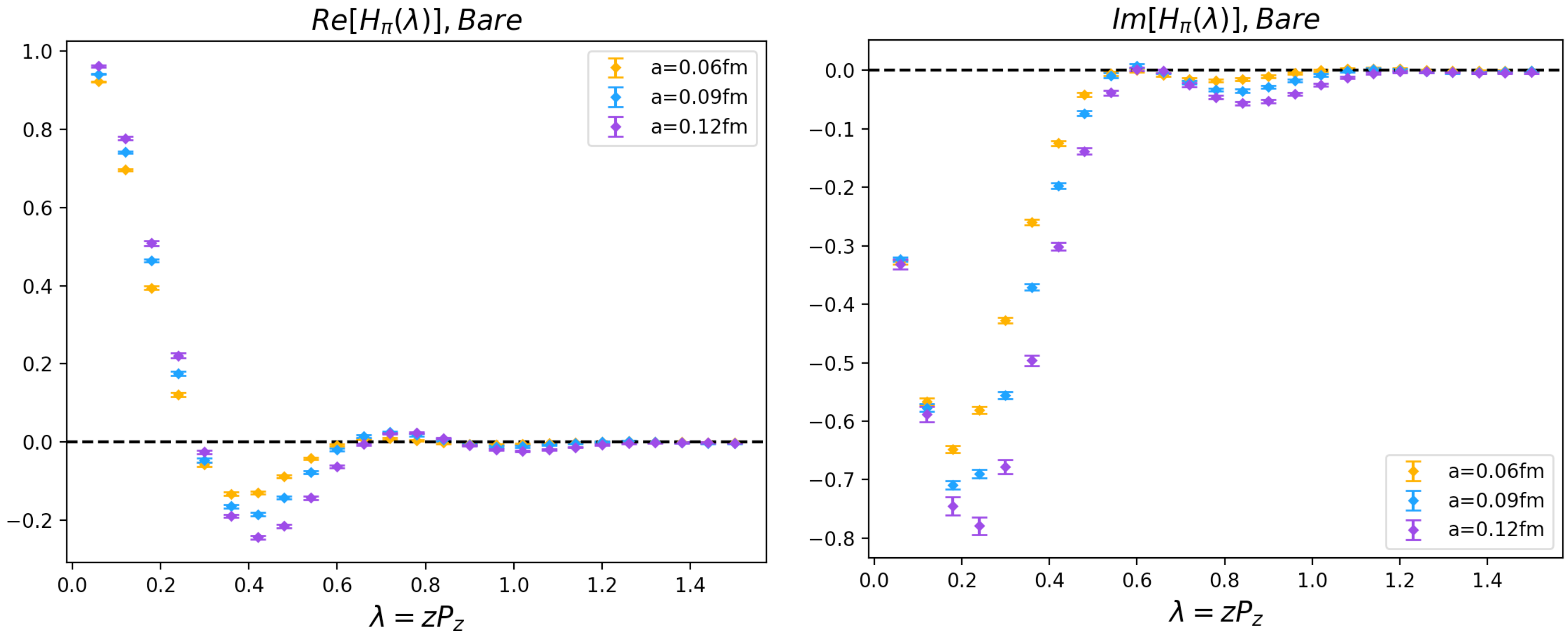
I . Bare quasi-LF correlation

- We simulate on MILC ensembles at 3 lattice spacings: a ≈ 0.12, 0.09, 0.06 fm and physical point

$\Gamma_1 = \gamma^z\gamma_5, \Gamma_2 = \gamma_5$

Pion two point correlator:

$$C_2^m(z,\vec{P},t) = \int d^3y e^{-i\vec{P}\cdot\vec{y}} \langle 0|\bar{\psi}_1(\vec{y},t)\Gamma_1 U(\vec{y},\vec{y}-z\hat{z})\psi_2(\vec{y}-z\hat{z},t)\bar{\psi}_2(0,0)\Gamma_2\psi_1(0,0)|0\rangle$$



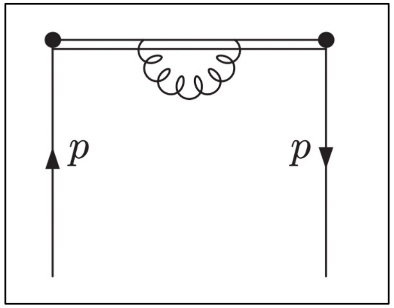
$P_z = 2.15GeV$

I . Bare quasi-LF correlation

- **The main reason for the deviations of different lattice spacings is the linear divergence**

Matrix element:

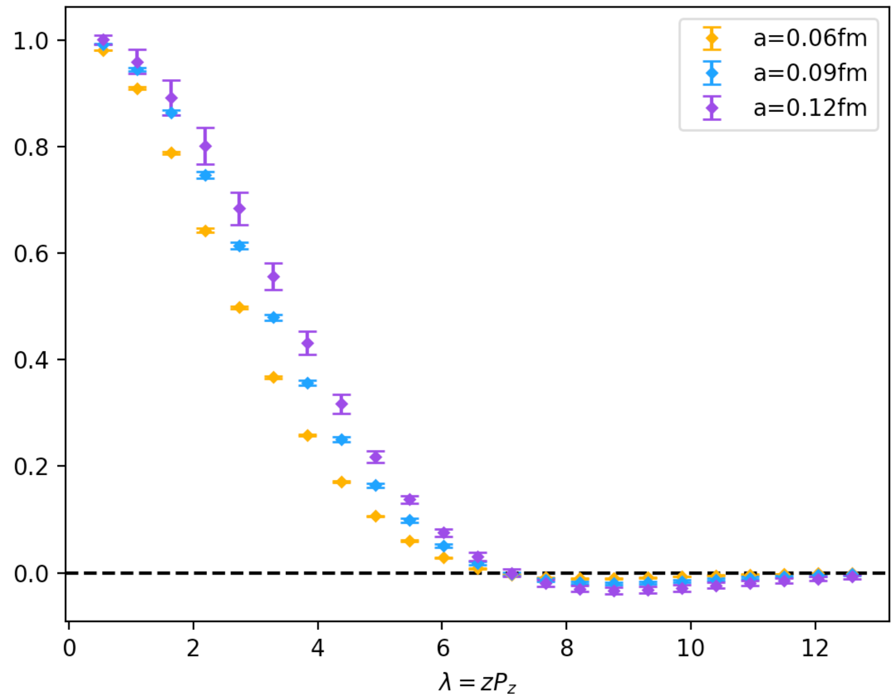
$$\langle P|\bar{\psi}(0)U(0,z)\psi(z)|P\rangle$$



$$M(z) \sim \exp\left(-\frac{C(\alpha)z}{a}\right) f(z)$$

z : non-local separation

$$e^{\frac{izP_z}{2}} H_\pi(z), Bare$$



- **Linear divergence comes from self-energy of gauge link**

NPB969(2021)115443

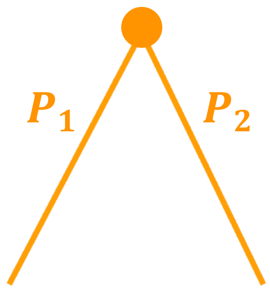
- **The quasi-LF correlation operator is multiplicatively renormalized**

$$[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_B = e^{\delta m|z|}Z\,[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_R$$

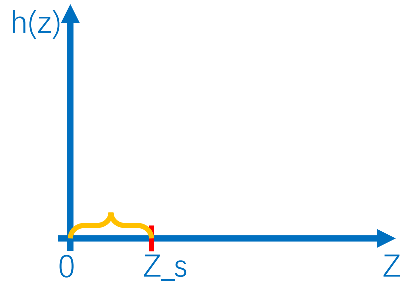
$\langle P|\bar{\psi}(z)\,\Gamma\,W(z,0)\,\psi(0)|P\rangle / \langle X|\bar{\psi}(z)\,\Gamma\,W(z,0)\,\psi(0)|X\rangle$ is UV finite

Some proposals:

- RI/MOM: Alexandrou et al, NPB 17', Stewart, Zhao, PRD 18'
  $|X\rangle$ is chosen as a single off-shell quark state
  Free from power-divergent mixings
- Ratio: Radyushkin, PRD 17'
  $|X\rangle$ is chosen as a zero momentum hadron state
  Cancellation of discretization effects
- VEV: Braun et al, PRD 19'
  $|X\rangle$ is chosen as the vacuum
  Without compilicated external state



➢ **To solve to problem:** **Undesired IR effects(Residual linear divergence) at large distances**



Possible Solution: **self renormalization LPC** (Huo, Su et al (LPC), NPB 21')

- **Fitting the bare matrix elements at multiple lattice spacings to**

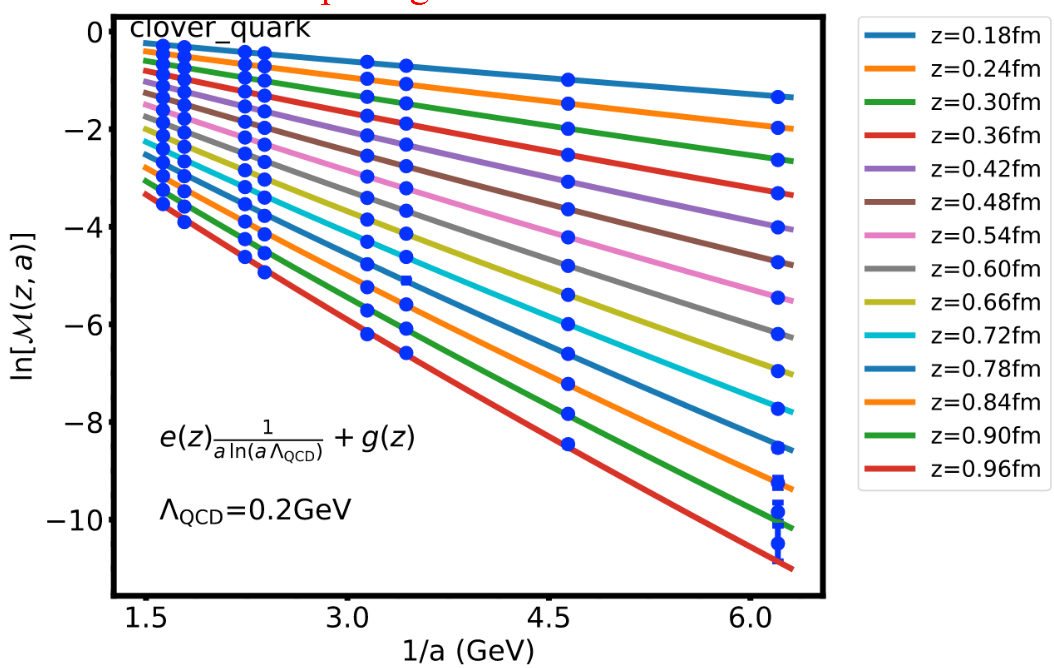
$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z + g'(z) + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1+\frac{d}{\ln(a\Lambda_{\rm QCD})}\right]$$

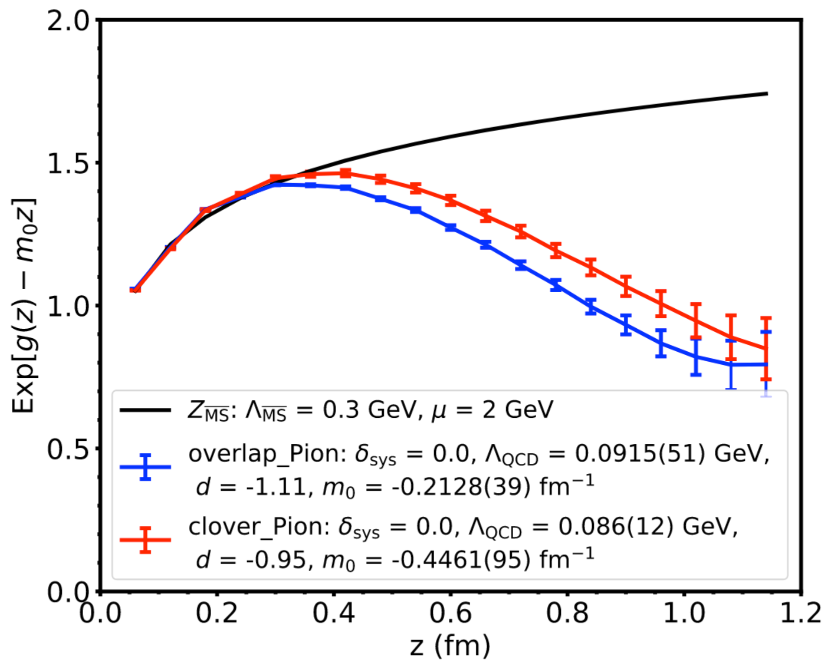
- **The pieces other than g′(z) are renormalization factors**
- **Renormalon ambiguity m_0 can be determined by matching the renormalized matrix element to the continuum $\overline{\rm MS}$ result at short distance**
- **Such renormalized matrix element can then, in principle, be matched to the light-cone distribution using the $\overline{\rm MS}$ matching**

- **To solve to problem:** Undesired **IR effects(Residual linear divergence)** at **large distances**

The only possible solution : self renormalization LPC(Huo, Su et al, NPB 21')

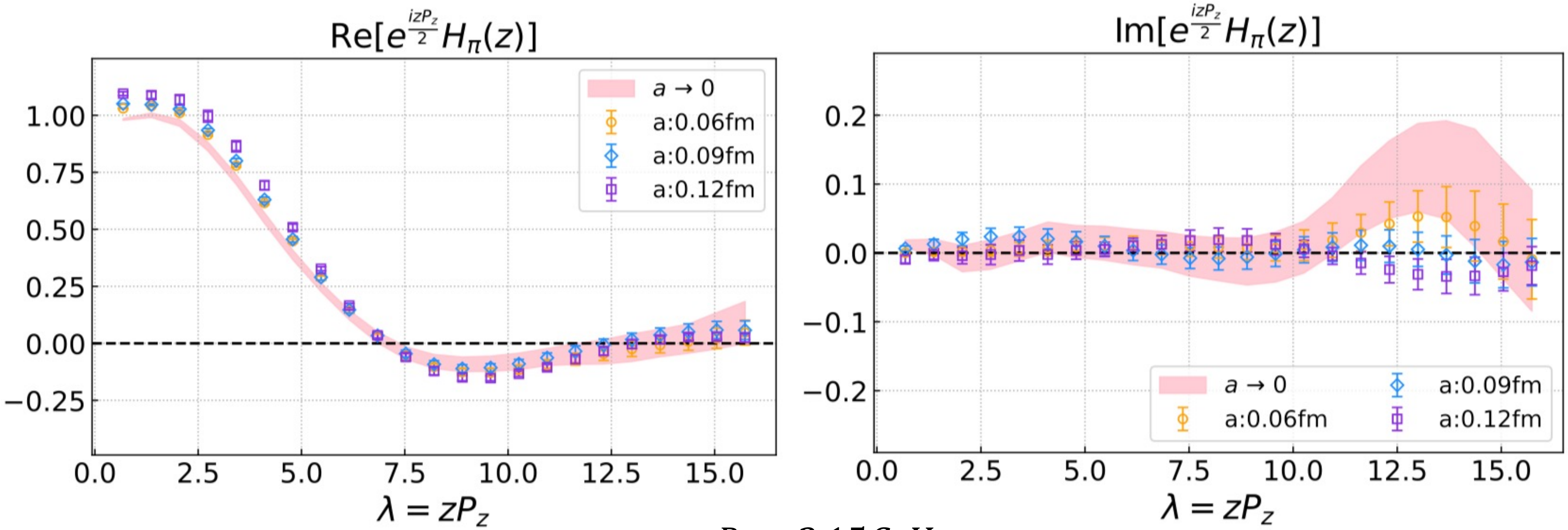
8 lattice spacings: 0.12fm ~ 0.03fm





➢ Application of Self renormalization: pion DA

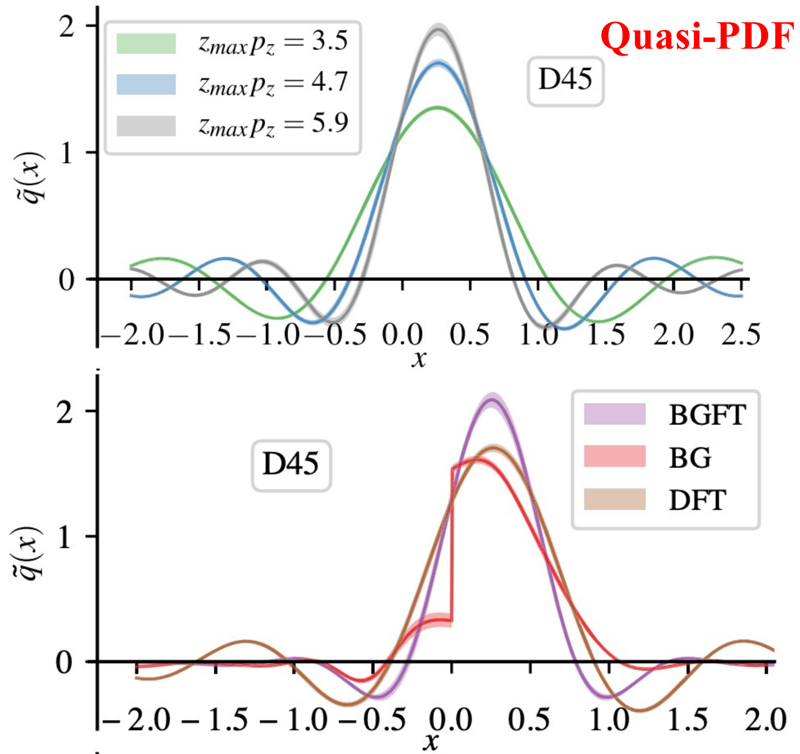
- **Renormalized quasi-DA of difference lattice spacings**



$P_z = 2.15GeV$

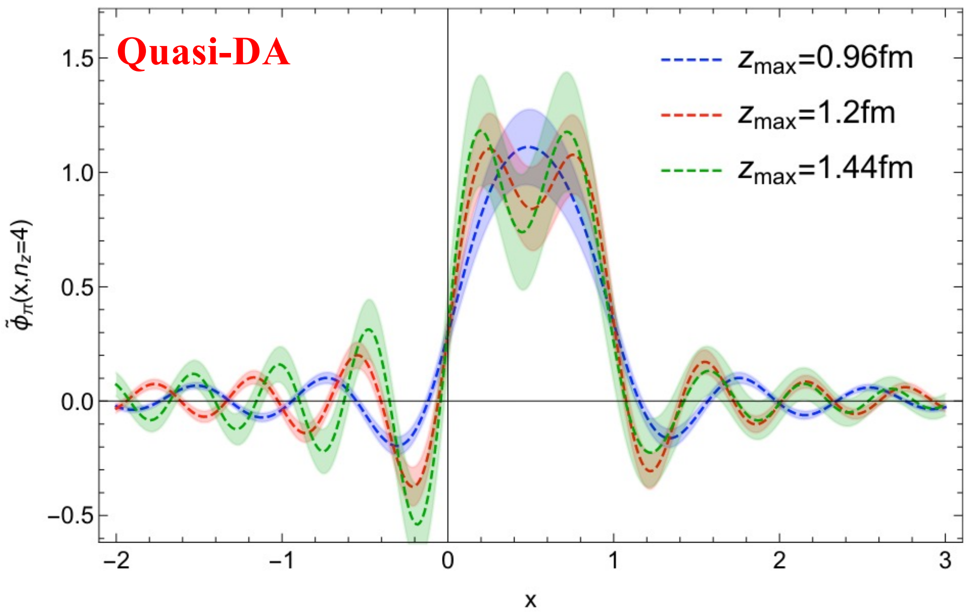
Ⅲ. FT and Extrapolation

➢ **Truncated FT introduces high frequency oscillations(systematic) uncertainty**



e . g . Backus − Gilbert method & Bayes − Gauss − FT

Alexandrou et al, PRD 21'



R. Zhang et al, PRD 20'

Ⅲ. FT and Extrapolation

- **Extrapolation to asymptotic distance** Ji et al, NPB 21'
- **Lattice data (available up to limited z_{max}/λ_{max}) may be supplemented with physics-based extrapolation**

Application in Vector DA:

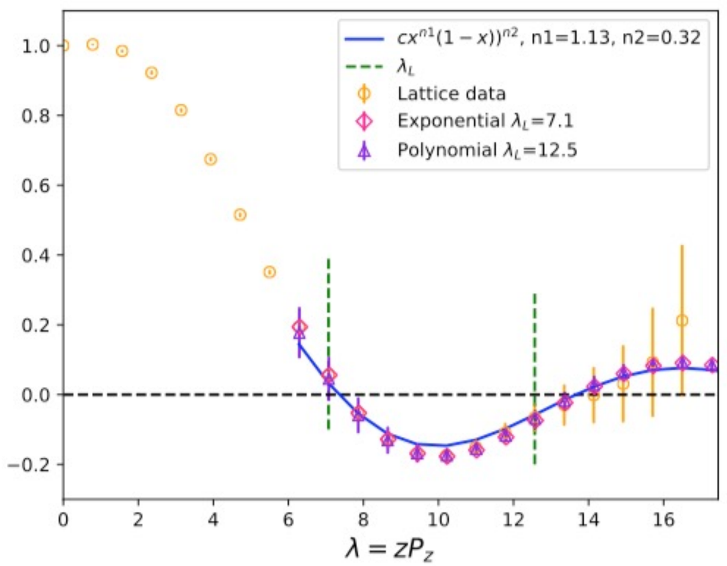
- Asymptotic behavior at x~0,1:

  $\psi(x) \sim x^a(1-x)^b$

- Two extrapolation form:

$$\tilde{H}(z,P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right]e^{-\frac{\lambda}{\lambda_0}},$$

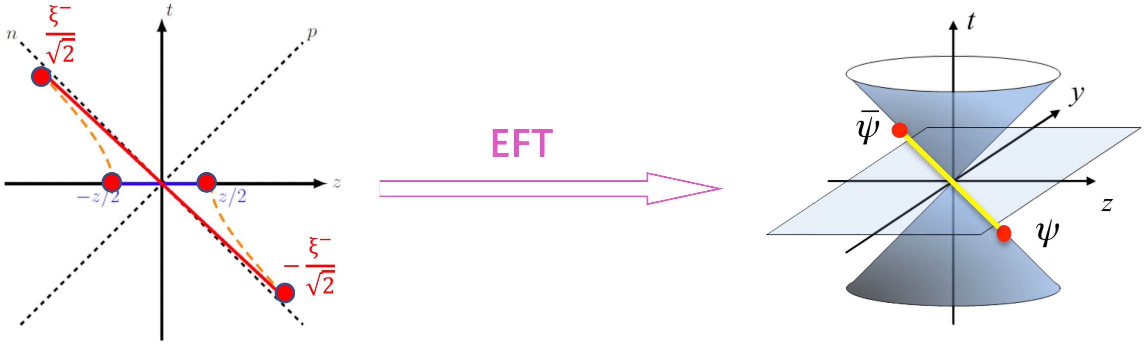
$$\tilde{H}(z,P_z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b},$$



Jun et al (LPC), PRL 21'

IV. Factorization / matching

- **From quasi → light cone**



- **Factorization or matching formular**

$$q(x,P^z,\mu)=\int dy\,C^{-1}(x,y,P^z,\mu)\tilde{q}(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(xP^z)^2},\frac{\Lambda_{QCD}^2}{((1-x)P^z)^2}\right)$$

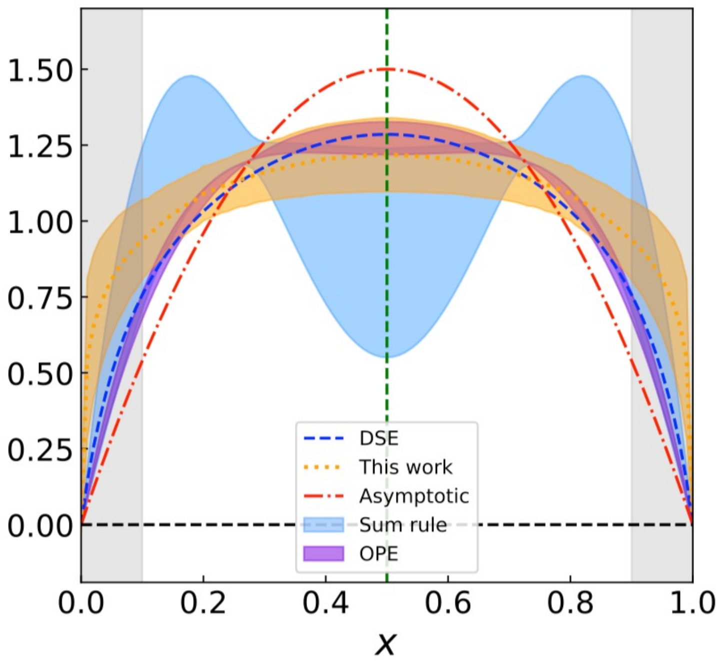
LCDA — **Perturbative calculation** — Quasi-DA

Large momentum expansion breaks done in end point region:

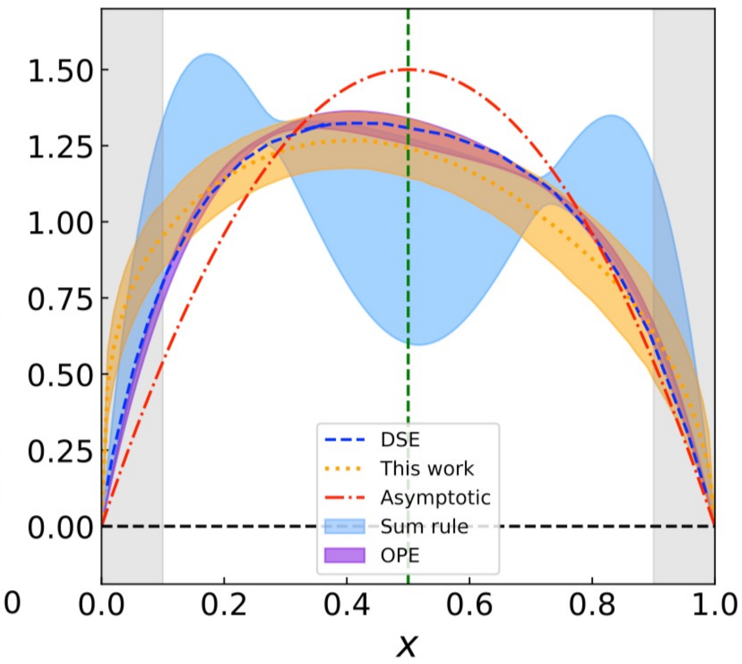
$xP^z\sim\Lambda_{QCD}$; $(1-x)P^z\sim\Lambda_{QCD}$; For $P^z_{max}=2.15GeV$, reliable region: (0.1, 0.9)

LCDA Numerical Results

π LCDA:



K LCDA:



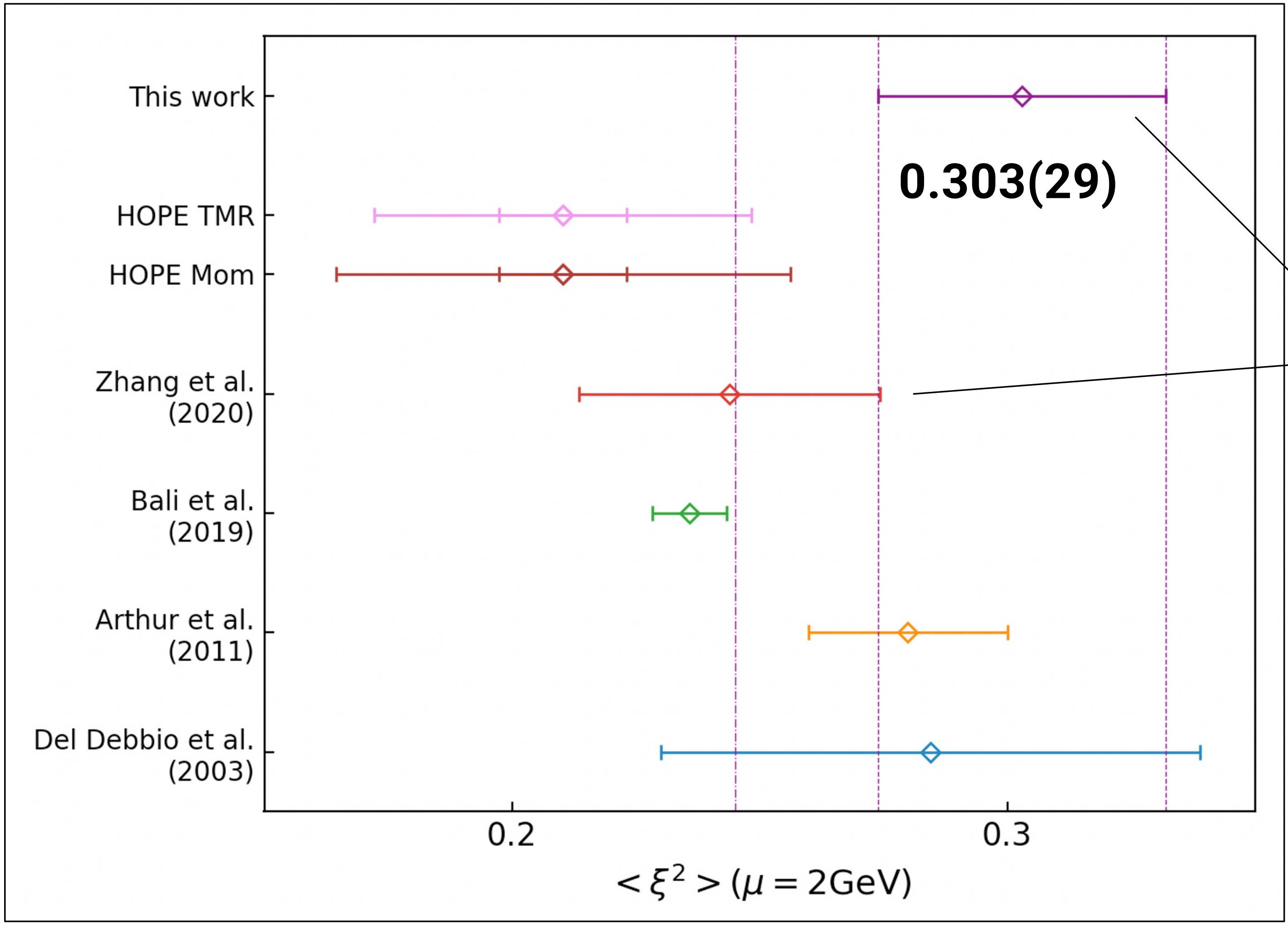
- MILC, 3 lattice spacings: (0.12, 0.09, 0.06) fm, Largest assemble ($96^3\times192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: π: 0.13GeV, K: 0.49GeV

Summary

- **Precise knowledge of meson LCDAs are important for understanding various hard exclusive processes**
- **LaMET and Lattice QCD now allow to do ab initio calculations of these meson DAs and make a comparison with measurements**
- **Self renormalization scheme have been applied to avoid undesired IR effects**
- **Extrapolation strategies have been applied to facilitate FT to momentum space**

Thank you for your attentions!

Back up slides



$$\langle \xi^n \rangle = \int_0^1 dx\, (2x-1)^n \phi_\pi(x, \mu^2)\,,$$

- **Renormalization method**
- **Extrapolation in the coordinate space**

Linear divergence

Renormalon ambiguity

Residual

Discretization error

$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln\left[a\Lambda_{QCD}\right]} + m_0 z + r(z) + f_z a + \frac{3C_F}{b_0}\ln\left[\frac{\ln\left[1/\left(a\Lambda_{QCD}\right)\right]}{\ln\left[\mu/\Lambda_{QCD}\right]}\right] + \ln\left[1 + \frac{d}{\ln\left(a\Lambda_{QCD}\right)}\right]$$

Resummation of log divergence