Spin alignment of vector mesons in heavy-ion collisions

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Z.-T. Liang, and X.-N. Wang.

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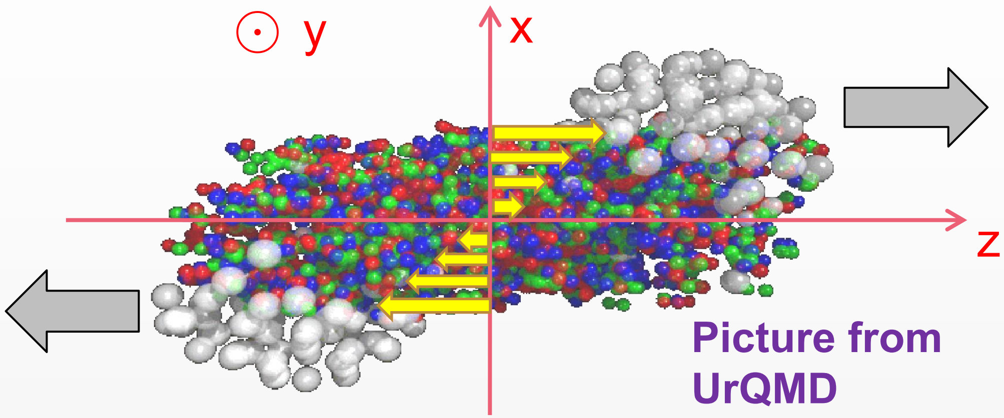
- Introduction
- Boltzmann equation for vector meson
- Numerical results for ϕ meson's spin alignment
- Summary

Heavy-ion collisions



Vorticity, magnetic field

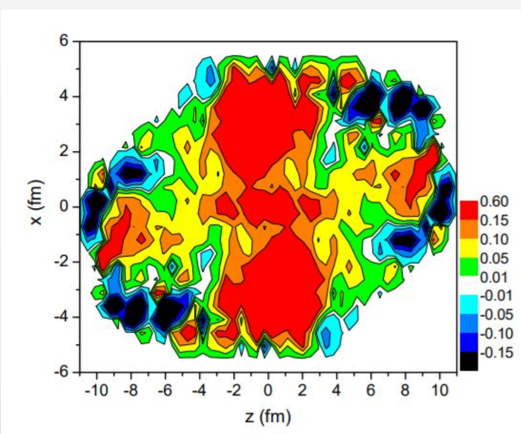
↓ spin-orbit / magnetic coupling

Spin of hadrons

↓ weak / strong p-wave decay

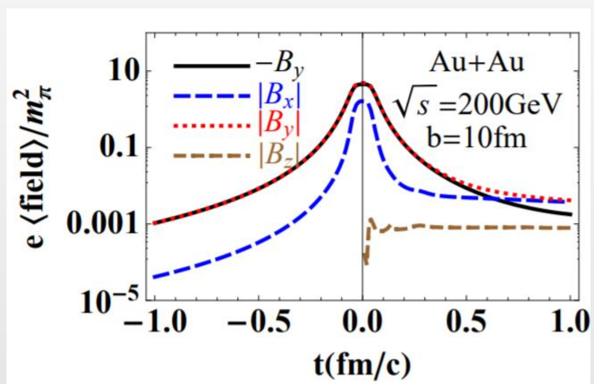
Measurements

Vorticity fields



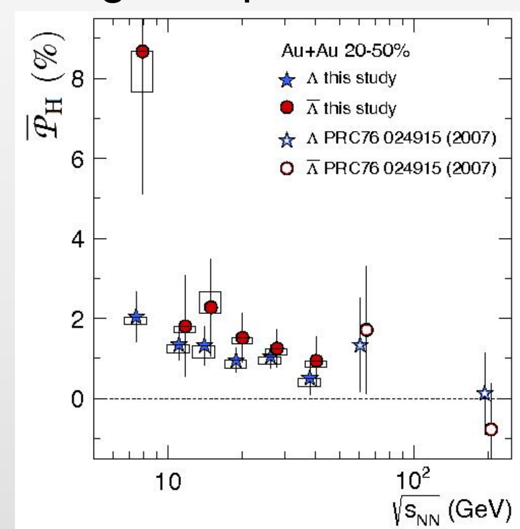
F. Becattini, L. Csernai, D.J. Wang, PRC 88, 034905 (2013); PRC 93, 069901 (2016)

Electromagnetic fields



W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012).

Λ's global polarization



L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.

Spin alignment

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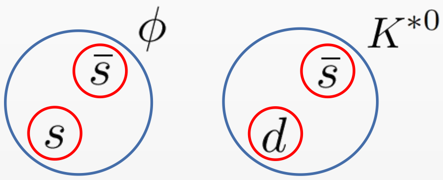
- Spin alignment for a vector meson ($J^P = 1^-$) is 00-element ρ_{00} of its normalized spin density matrix

$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

- Mesons decay to pseudo-scalar mesons (Parity-odd strong decay)

$$\phi \to K^+ + K^-$$

$$K^{*0} \to K^+ + \pi^-$$

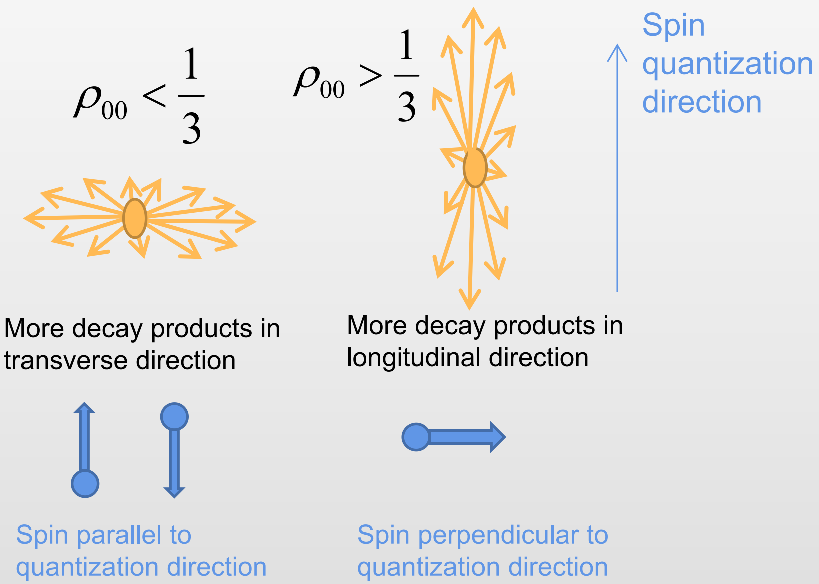


- Spin alignment is measured through polar angle distribution of decay products

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].

$$\frac{dN}{d\theta} = \frac{3}{4}\left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta\right]$$

ρ_{00}=1/3 if spin does not have a preferred direction



Experimental results

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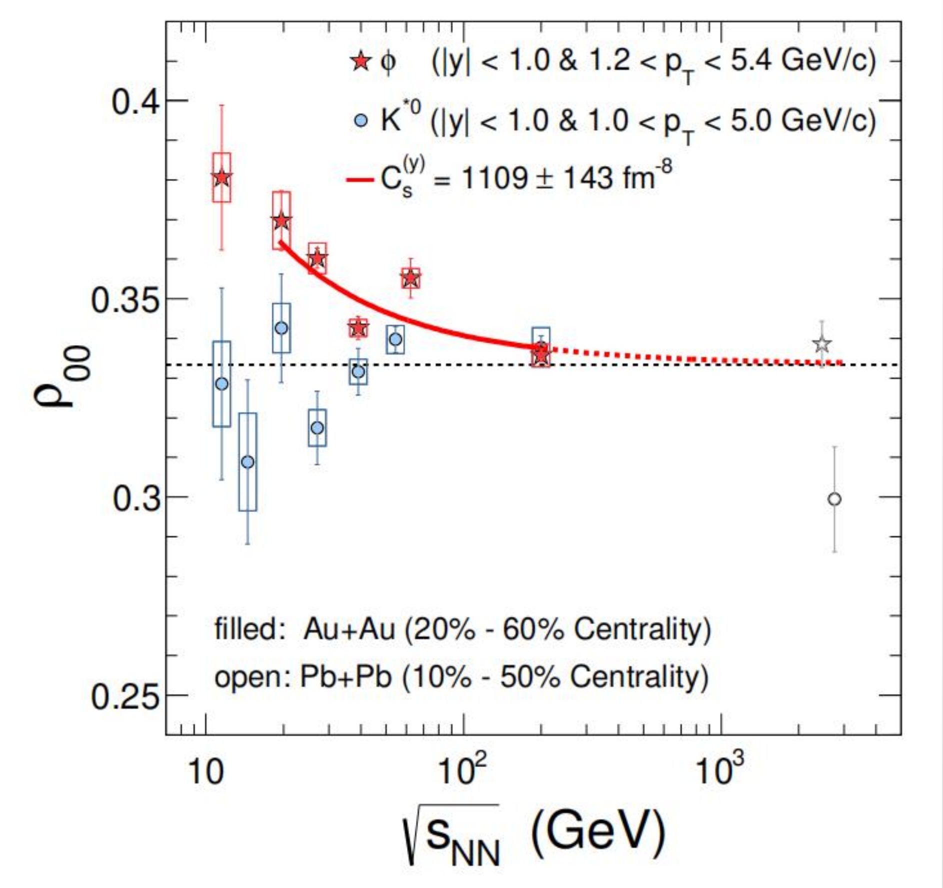
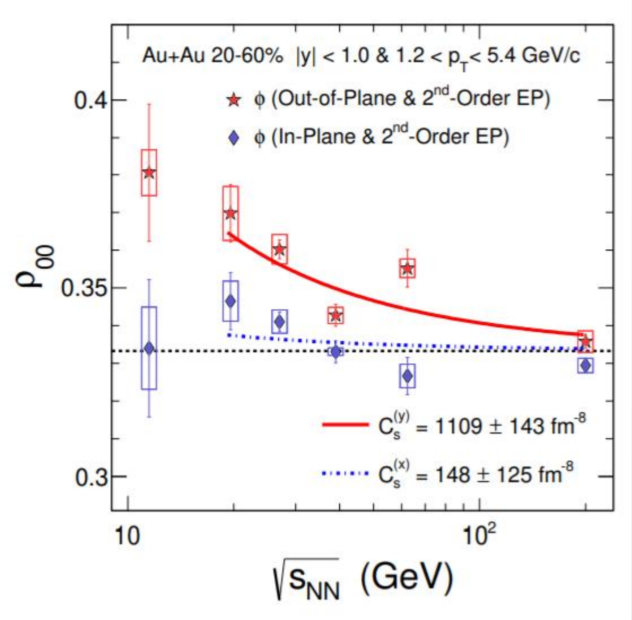
Observation of Global Spin Alignment of ϕ and K^{*0} Vector Mesons in Nuclear Collisions

STAR collaboration, arXiv:2204.02302



ϕ meson's ρ_{00} is significantly larger than 1/3 for collision energies of 62 GeV and below (8.4σ !!)



Related studies

- **Spin Alignment of Vector Mesons in Non-central $A+A$ Collisions** PLB 629, 20 (2005).

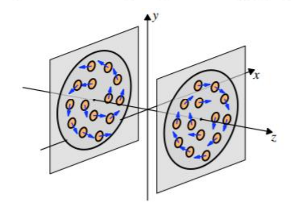
Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]
[1]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*
uclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 947
(Dated: November 5, 2018)

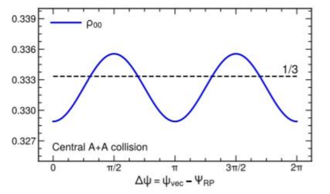
Quark-antiquark recombination: $\rho_{00}^{V(\mathrm{rec})} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}}$, Fragmentation: $\rho_{00}^{\rho\,(\mathrm{frag})} = \frac{1 + \beta P_q^2}{3 - \beta P_q^2}$,

- Contributions from vorticity and magnetic field:
  Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018).

	$\mathscr{P}_\phi$	ρ_{00}^{ϕ}	$\mathscr{P}_\Lambda$	
$\mathscr{P}(\omega)$ or $\rho_{00}^{\phi}(\omega)$	$\frac{2}{3}\beta\omega$	$\frac{1}{3} - \frac{1}{9}(\beta\omega)^2$	$\frac{1}{2}\beta\omega$	
$\mathscr{P}(B)$ or $\rho_{00}^{\phi}(B)$	0	$\frac{1}{3} + \frac{4}{9}(\beta\mu_{\mathrm{ms}}B)^2$	$\beta\mu_{\mathrm{ms}}B$	$\frac{1}{3}\beta B($
$\mathscr{P}(B)$	$\frac{2}{3}\beta\mu_{\mathrm{m}\phi}B$	$\frac{1}{3} - \frac{1}{9}(\beta\mu_{\mathrm{m}\phi}B)^2$	$\beta\mu_{\mathrm{m}\Lambda}B$	

- Local vorticity:
  X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817,136325 (2021).





- Helicity alignment: J.-H. Gao, PRD 104, 076016 (2021).

  Turbulent color fields: B. Mueller, D.-L. Yang, PRD 105, 1 (2022).

  Shear-induced spin alignment: F.Li, S.Liu, arXiv:2206.11890. D.Wagner, N.Weickgenannt, E.Speranza, arXiv:2207.0111.

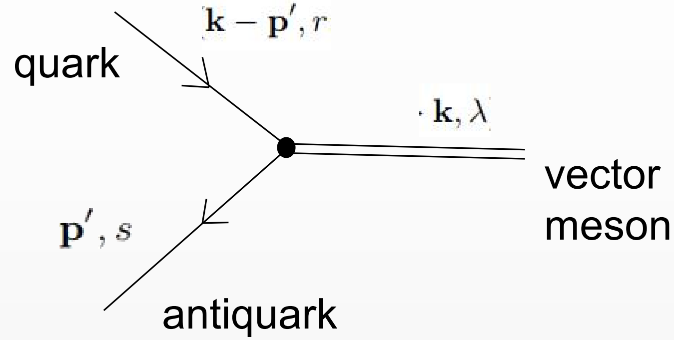
- Vector meson fields: XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv: 2206.05868; arXiv: 2205.15689. XLS, Q.Wang, X.-N.Wang, PRD 102, 056013 (2020). XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020).

Contents

Quark combination

- Boltzmann equation for quark-antiquark combination

$$q+\bar{q}\rightarrow V$$



$$k\cdot\partial_x f_\lambda^V(x,\mathbf{k}) \sim \sum_{r,s=\pm 1/2}\int \frac{d^3\mathbf{p}'}{E_{\mathbf{p}'}^{\bar{q}} E_{\mathbf{k}-\mathbf{p}'}^{q}}\delta\left(E_{\mathbf{k}}^V - E_{\mathbf{k}-\mathbf{p}'}^q - E_{\mathbf{p}'}^{\bar{q}}\right)$$
$$\times \left|M(\mathbf{k}-\mathbf{p}', r; \mathbf{p}', s\rightarrow \mathbf{k},\lambda)\right|^2$$
$$\times\left\{f_r^q(\mathbf{k}-\mathbf{p}')f_s^{\bar{q}}(\mathbf{p}')\left[1+f_\lambda^V(\mathbf{k})\right] - f_\lambda^V(\mathbf{k})\left[1-f_r^q(\mathbf{k}-\mathbf{p}')\right]\left[1-f_s^{\bar{q}}(\mathbf{p}')\right]\right\}$$

energy conservation

gain term

loss term (decay process)

- $f_\lambda^V(x,\mathbf{k})$ is distribution in phase space for a vector meson with spin λ along spin quantization direction

- General form: Matrix valued spin dependent distribution

$$\begin{pmatrix} f_1^V & 0 & 0 \\ 0 & f_0^V & 0 \\ 0 & 0 & f_{-1}^V \end{pmatrix} \rightarrow \begin{pmatrix} f_{1,1}^V & f_{1,0}^V & f_{1,-1}^V \\ f_{0,1}^V & f_{00}^V & f_{0,-1}^V \\ f_{-1,1}^V & f_{-1,0}^V & f_{-1,-1}^V \end{pmatrix} \equiv f_{rs}^V(x,\mathbf{k})$$

Boltzmann equation

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- Matrix valued spin dependent distribution

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \equiv \int \frac{d^4u}{2(2\pi\hbar)^3}\delta(k\cdot u)e^{-iu\cdot x/\hbar} \times \left\langle a^\dagger_V\left(\lambda_2,\mathbf{k}-\frac{\mathbf{u}}{2}\right)a_V\left(\lambda_1,\mathbf{k}+\frac{\mathbf{u}}{2}\right)\right\rangle$$

3×3, Hermitian, related to Wigner function

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = 2E^V_{\mathbf{k}}\int\frac{dk^0}{2\pi\hbar}\epsilon^{*\mu}(\lambda_1,\mathbf{k})G^<_{\mu\nu}(x,k)\epsilon^\nu(\lambda_2,\mathbf{k})$$

Spin vectors for vector meson

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V},\ \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right)$$

- Dyson-Schwinger equation
  - ➡ Kadanoff-Baym equation for Wigner function
  - ➡ Matrix-form Boltzmann equation

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv: 2206.05868

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) - \mathcal{C}_{\mathrm{diss}}(\mathbf{k})f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

$$\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k}) = \int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\bar{q}}_{\mathbf{p}'}-E^q_{\mathbf{k}-\mathbf{p}'}\right) \times \mathrm{Tr}\left\{\Gamma^\nu(p'\cdot\gamma-m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right] \times\Gamma^\mu\left[(k-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot P^q(x,\mathbf{k}-\mathbf{p}')\right]\right\} \times f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}'),$$

polarizations of quark/antiquark

- Spin alignment only depend on coalescence process

$$\rho_{00} \equiv \frac{f^V_{00}}{f^V_{+1,+1}+f^V_{00}+f^V_{-1,-1}} = \frac{\epsilon^*_\mu(0,\mathbf{k})\epsilon_\nu(0,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})}$$

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Polarization of quarks

- Polarizations of strange quark/antiquark in a thermal equilibrium system

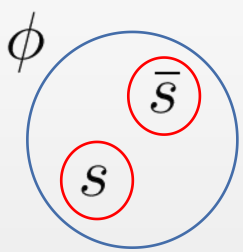
$$P^{\mu}_{s}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\left[\omega_{\rho\sigma}+\frac{Q_s}{(u\cdot p)T}F_{\rho\sigma}+\frac{g_{\phi}}{(u\cdot p)T}F^{\phi}_{\rho\sigma}\right]$$

$$P^{\mu}_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\left[\omega_{\rho\sigma}-\frac{Q_s}{(u\cdot p)T}F_{\rho\sigma}-\frac{g_{\phi}}{(u\cdot p)T}F^{\phi}_{\rho\sigma}\right]$$

F.Becattini, V.Chandra, L.Del Zanna, E.Grossi, Annals Phys. 338, 32 (2013).

XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020);

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.



thermal vorticity fields (rotation and acceleration)

classical electromagnetic fields

$$\frac{e^2}{4\pi}\sim\frac{1}{137}$$

vector ϕ field coupled to $s/\bar{s}$ (strong force field)

$$\frac{g_{\phi}^2}{4\pi}\sim\mathcal{O}(1)\gg\frac{e^2}{4\pi}$$

- At low energies, strong interaction is mediated by mesons, which was proposed by Yukawa in 1935.

H. Yukawa, Proc. Phys. Math. Soc. Jap. 17, 48 (1935)

- Vector meson field has been used to explain the difference between polarizations of Λ and $\overline{\Lambda}$

L.P.Csernai, J.I.Kapusta, T.Welle, PRC 99, 021901 (2019)

Spin alignment in lab frame

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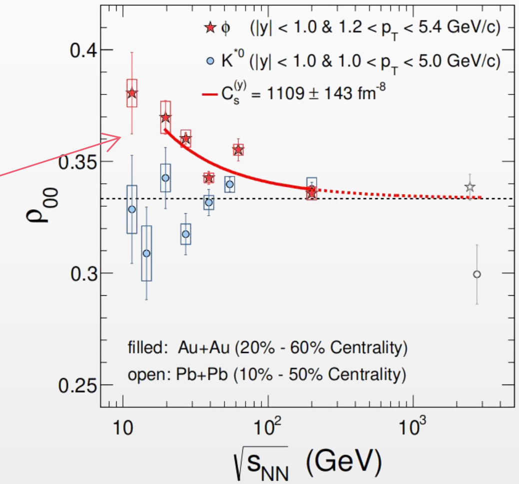
- Spin alignment of the ϕ meson in its rest frame measuring along the direction of ϵ_0

$$\rho_{00}(x,\mathbf{0}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right],$$

larger at lower energies ~ $1/T^2$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}.$$



STAR collaboration, arXiv:2204.02302

- Momentum dependence is recovered by taking a Lorentz boost

$$\mathbf{B}'_\phi = \gamma\mathbf{B}_\phi - \gamma\mathbf{v}\times\mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$

$$\mathbf{E}'_\phi = \gamma\mathbf{E}_\phi + \gamma\mathbf{v}\times\mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v},$$

$$\boldsymbol{\omega}' = \gamma\boldsymbol{\omega} - \gamma\mathbf{v}\times\boldsymbol{\varepsilon} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\omega}}{v^2}\mathbf{v},$$

$$\boldsymbol{\varepsilon}' = \gamma\boldsymbol{\varepsilon} + \gamma\mathbf{v}\times\boldsymbol{\omega} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\varepsilon}}{v^2}\mathbf{v},$$

$$\gamma = \frac{E_{\mathbf{k}}^\phi}{m_\phi}, \quad \mathbf{v} = \frac{\mathbf{k}}{E_{\mathbf{k}}^\phi}$$

spin alignment in lab frame $\rho_{00}(\mathbf{k})$

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

Numerical set-up

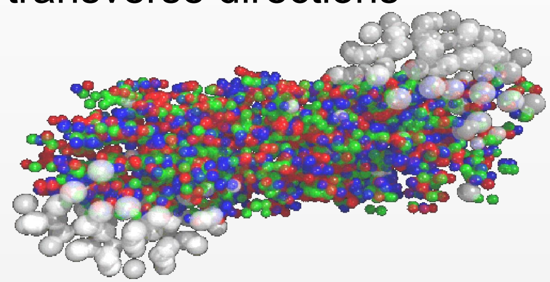
- Set-up for our numerical calculation

$$\langle(\omega_i)^2\rangle = \langle(\varepsilon_i)^2\rangle = 0$$

$$\left\langle(g_\phi \mathbf{B}^\phi_{x(y)})^2\right\rangle = \left\langle(g_\phi \mathbf{E}^\phi_{x(y)})^2\right\rangle = F^2$$

$$\langle(g_\phi \mathbf{B}^\phi_z)^2\rangle = \langle(g_\phi \mathbf{E}^\phi_z)^2\rangle = r_z F^2 < F^2$$

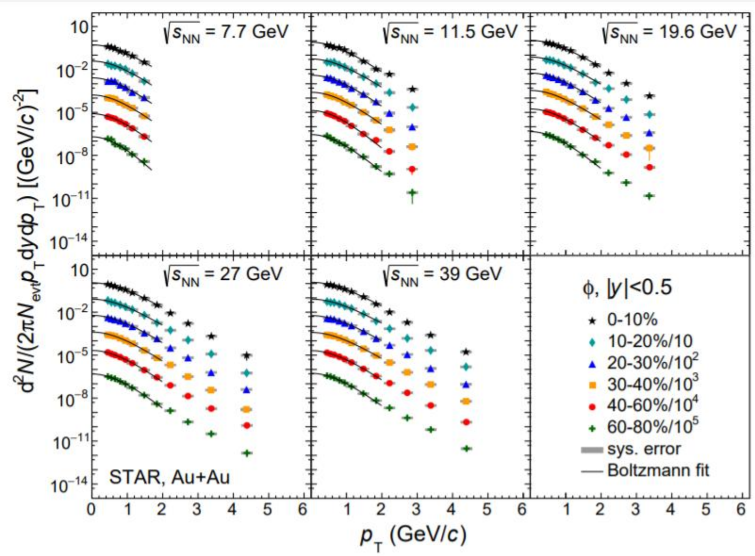
r_z denotes the anistropy between longitudinal and transverse directions



- Spectra of ϕ meson

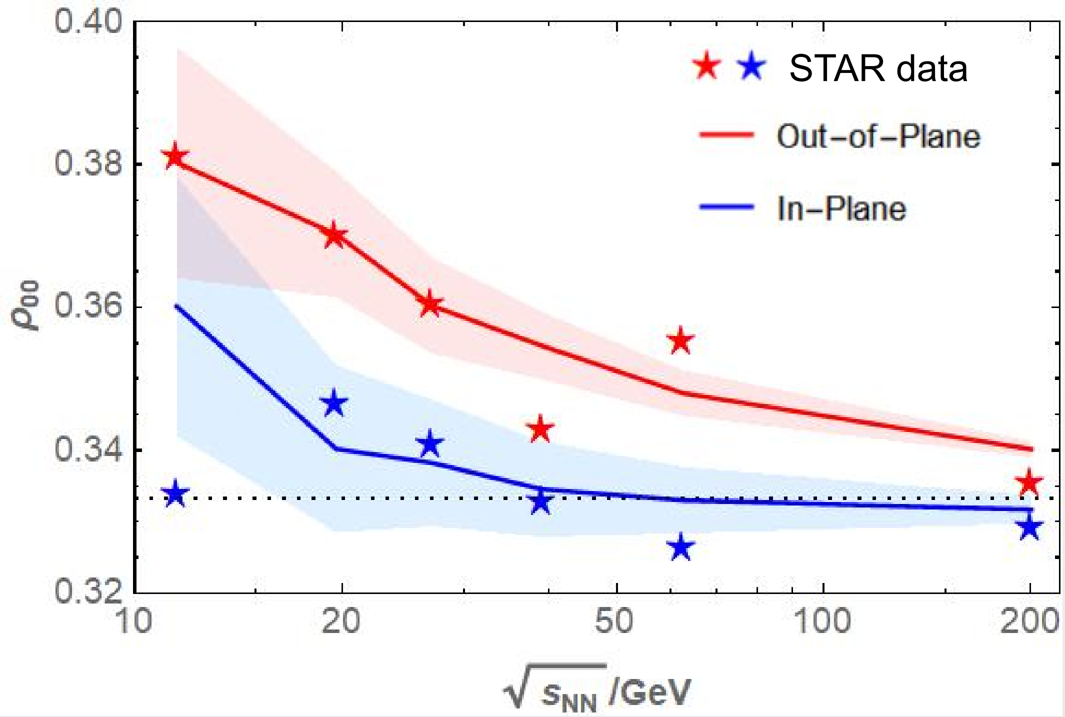
$$\frac{dN}{d^2\mathbf{k}_T dy} = \frac{1}{4\pi}\left[1 + 2v_2(k_T)\cos(2\phi)\right]\frac{dN}{k_T dk_T dy}$$

STAR collaboration, PRL 99,112301 (2007); PRC 79, 064903 (2009); PRC 88,014902 (2013); PRC 102, 034909(2020).



Spin alignment

- Spin alignment as a function of collision energy



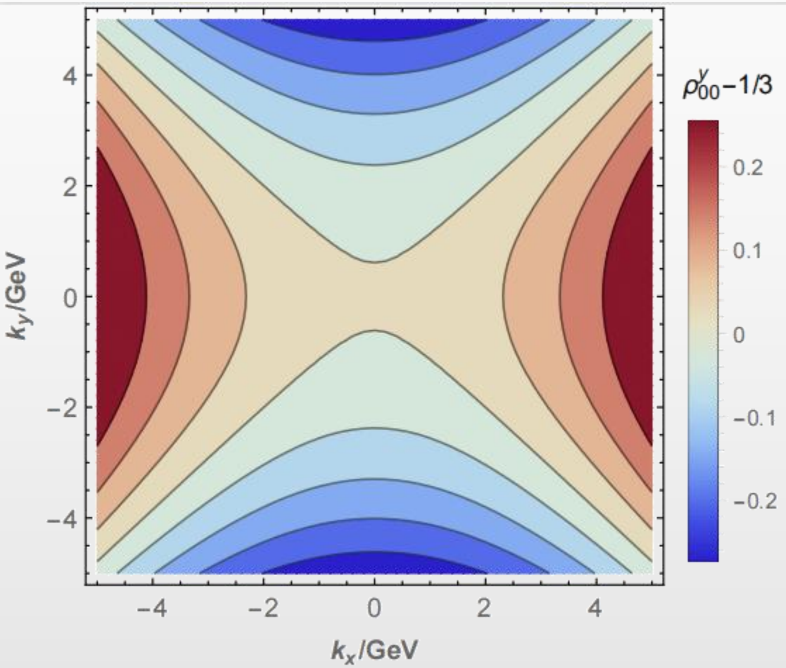
$F^2 = 0.45\, m_\pi^4$ if $m_s = 170$ MeV
$F^2 = 5.02\, m_\pi^4$ if $m_s = 530$ MeV

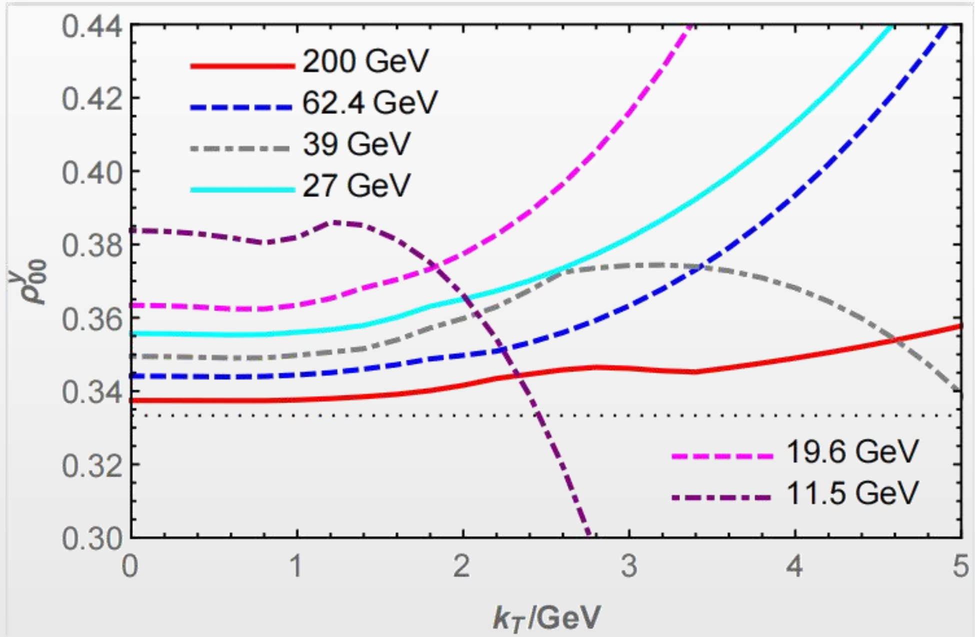
Solid line: $r_z = 0.79$
Upper bound $r_z = 0.59$
Lower bound $r_z = 0.99$

Reproduce out-of-plane data and in-plane data at the same time!

- Agree with STAR's recent data, arXiv:2204.02302.
- Difference between red line and blue line is attribute to v_2

- Spin alignment as a functions of transverse momentum
  (k_z is integrated out by taking an average over rapidity range |y|<1)





Spin alignment

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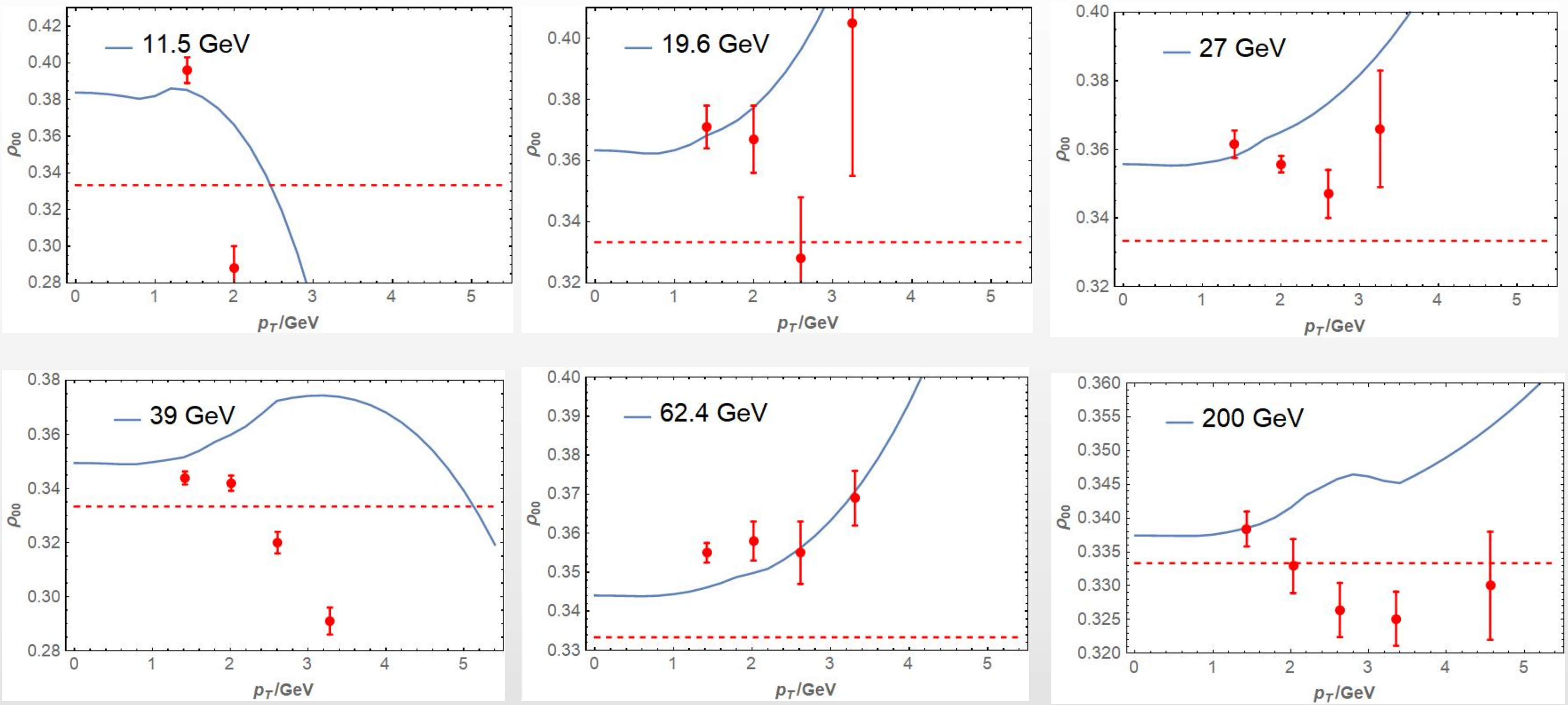
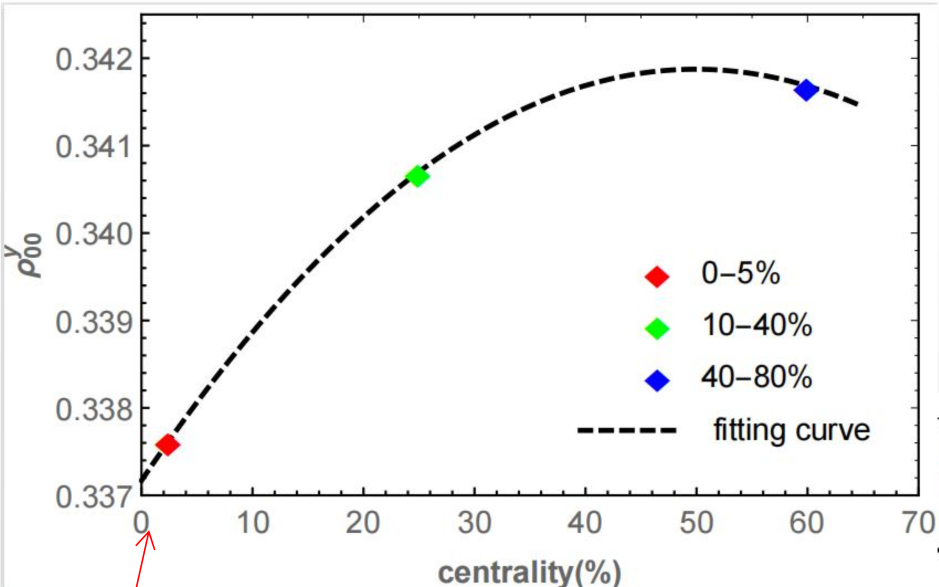
- Spin alignment as a functions of transverse momentum



Red dots and error bars are read from STAR's paper arXiv:2204.02302

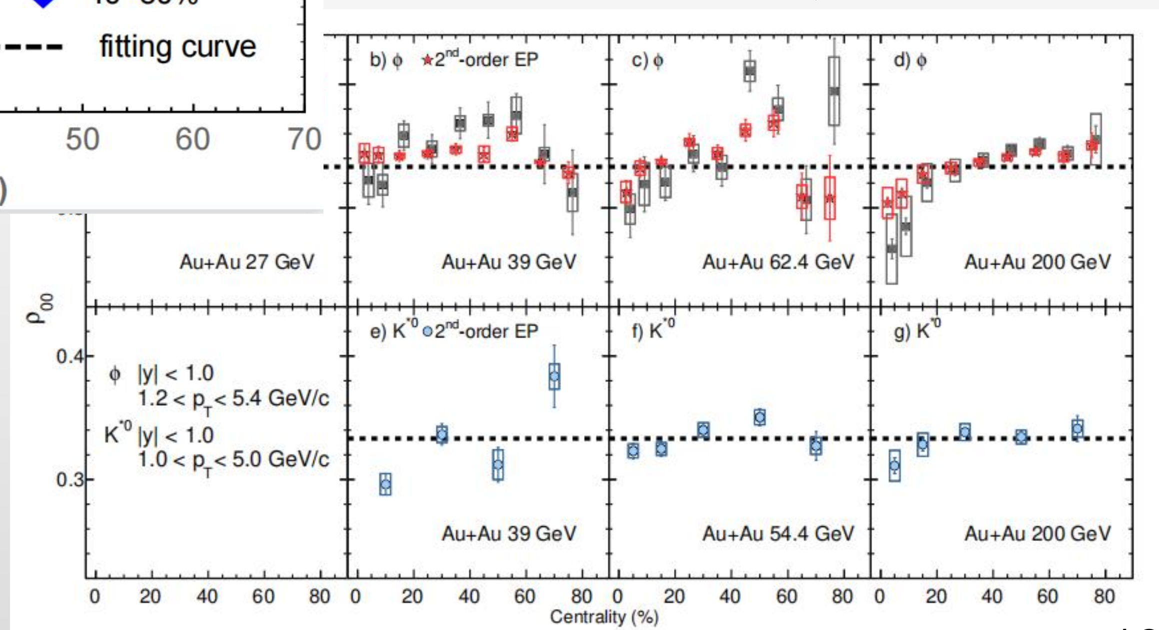
Centrality dependence

- Spin alignment as a function of centrality



>1/3 in most central collisions, may be contributions from local vorticity

STAR collaboration, arXiv:2204.02302



Summary

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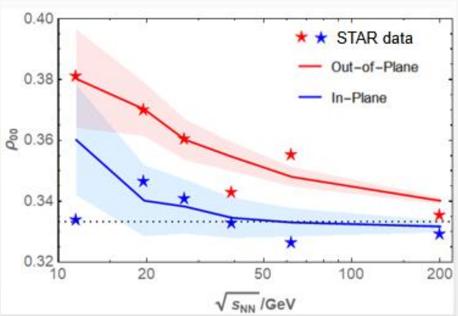
- We derive a relativistic Boltzmann equation for quark-antiquark combination and form vector meson
- Using two parameters (fluctuations for transverse and longitudinal components of meson field), we can reproduce most of recent STAR data for ϕ meson spin alignment

$$\left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle = F^2$$

$$\langle (g_\phi \mathbf{B}^\phi_z)^2 \rangle = \langle (g_\phi \mathbf{E}^\phi_z)^2 \rangle = r_z F^2 < F^2$$



Thank you!