Quantum Color Screening in Magnetic Field

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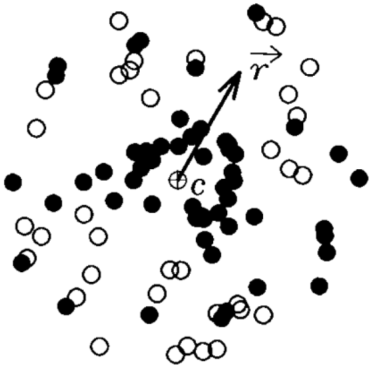
Motivation

Debye screening of a pair of charged particles:

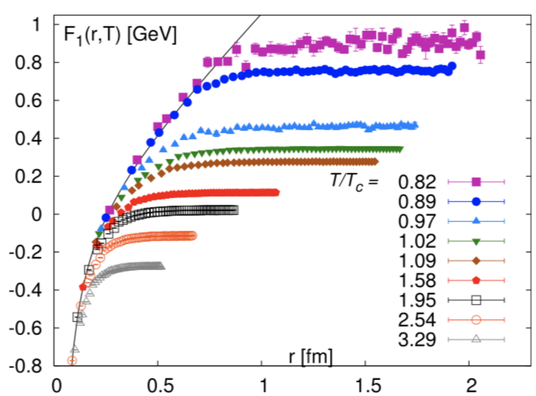


$$\frac{1}{r} \rightarrow \frac{1}{r} e^{-m_D r} \sim \frac{1}{r} e^{-r/r_D}$$

screening mass m_D

screening length r_D

Debye screening of a pair of colored particles:



P.Petreczky, J.Phys.G37, 094009(2010)

$$V_{HTL}(r) = -\tilde{\alpha}_s \left[m_D + \frac{e^{-m_D r}}{r} + iT\phi(m_D r) \right] + \mathcal{O}(g^4)$$

$$\phi(x) = 2 \int_0^\infty \mathrm{d}z \, \frac{z}{(z^2+1)^2} \left(1 - \frac{\sin(xz)}{xz} \right)$$

Kapusta and Gale:
Finite-Temperature Field Theory: Principles and Applications

Motivation

1) Strong magnetic field created in HIC:

$$eB \sim 5m_\pi^2 \ at\ RHIC\ and\ 70m_\pi^2 \ at\ LHC$$

Question: what is the electromagnetic effect on color screening?

Studies in the two limits of weak and strong magnetic field:

[37] M.Hasan and B.Patra, Phys. Rev. **D102**, 036020(2020).
[38] B.Karmakar, A.Bandyopadhyay, N.Haque and M.Mustafa, Eur. Phys. J. **C79**, 658(2019).
[39] C.Bonati, M.Elia, M.Mariti, M.Mesiti, F.Negro, A.Rucci and F.Sanfilippo, Phys. Rev. **D95**, 074515(2017).
[40] B.Singh, L.Thakur and H.Mishra, Phys. Rev. **D97**, 096011(2018).
[41] M.Hasan, B.Patra, B.Chatterjee and P.Bagchi, Nucl. Phys. **A995**, 121688(2020).
[42] M.Hasan, B.Chatterjee and B.Patra, Eur. Phys. J. **C77**, 767(2017)

We will calculate $m_D(T, B)$ in general magnetic field

2) Well-known Landau energy levels for charged particles in an external magnetic field:

$$\varepsilon_n = (2n+1)\frac{|qB|}{2m}$$

What is the quantization effect on color screening?

Quark propagator in magnetic field

$\vec{B} = B\vec{e}_z$

Quark propagator in electromagnetic field:

$$(i\gamma\cdot\partial + q\gamma\cdot A - m)G(x,x') = \delta(x-x')$$

$$G(x,x') = \left\langle x\left|\frac{1}{\gamma\cdot\hat{\Pi}-m}\right|x'\right\rangle$$

$$= -\int_0^\infty ds\left\langle x\left|(\gamma\cdot\hat{\Pi}+m)e^{-\left(m^2-(\gamma\cdot\hat{\Pi})^2\right)s}\right|x'\right\rangle$$

Schwinger propagator:

$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{[m+(\gamma\cdot p)_\parallel][1-isgn(q)\gamma_1\gamma_2\tanh(v)] - \frac{(\gamma\cdot p)_\perp}{cosh^2(v)}\right\}$$

$$\times\, e^{-\frac{v}{|qB|}\left[m^2-p_\parallel^2+\frac{\tanh(v)}{v}p_\perp^2\right]}$$

[45] J.Schwinger, Phys. Rev. **82**, 664(1951).
[46] J.Alexandre, Phys. Rev. **D63**, 073010(2001).

● *no more translation invariance.*

● *the two Schwinger phases for q and $\bar{q}$, which are neglected here, will be cancelled to each other in loop calculation.*

Quark loop

$$\Pi_{\mu\nu}(T,B) = \lim_{\vec{k}\to 0} \Pi_{\mu\nu}\left(k_0 = 0, \vec{k}\right)$$

$$\begin{aligned}\Pi_{\mu\nu}(T,B) \;=\; & \frac{g^2T}{2|qB|^2}\sum_{npv_1v_2}\mathrm{Tr}\Big\{\frac{(\gamma\cdot p)_\perp\gamma_\mu(\gamma\cdot p)_\perp\gamma_\nu}{\cosh^2 v_1\cosh^2 v_2} + [1-i\,\mathrm{sgn}(q)\gamma_1\gamma_2\tanh v_1]\left(m^2\gamma_\mu - \omega_n^2\gamma_0\gamma_\mu\gamma_0 + p_z^2\gamma_3\gamma_\mu\gamma_3\right) \\ & \times [1-i\,\mathrm{sgn}(q)\gamma_1\gamma_2\tanh v_2]\,\gamma_\nu\Big\} e^{-\frac{(v_1+v_2)(m^2+\omega_n^2+p_z^2)+(\tanh v_1+\tanh v_2)p_\perp^2}{|qB|}} \qquad (14)\end{aligned}$$

doing $\int dp_\perp F(p_\perp) \to \sum_{n_1} F(n_1)$ through Legendre expansion:

$$\Pi^{||}_{\mu\mu}(T,B) = g^2T|qB|\sum_{np_zn_1}\frac{(2-\delta_{n_10})\left(\delta^{||}_{\mu\mu}+g^{||}_{\mu\mu}\right)(-\omega_n^2+p_z^2)}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2}$$

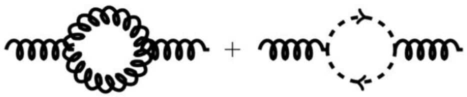
quark energy $p_0 = i\omega_n = i(2n+1)\pi T$
longitudinal momentum p_z
quantized transverse energy $\varepsilon_n^2 = 2n_1|qB|$

Note: normal Landau energy levels are for on-shell fermions, the propagating quarks here are off-shell particles.

$$\Pi^{\perp}_{\mu\mu}(T,B) = 0$$

Color screening mass

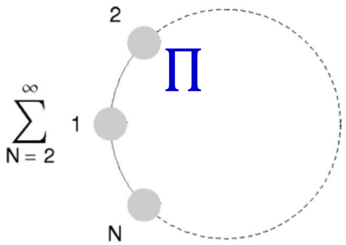
Gluon and ghost loops:



$$\overline{\Pi}_{\mu\nu}(T,B) = \overline{\Pi}_{\mu\nu}(T,0)$$

Kapusta and Gale:
Finite-Temperature Field Theory: Principles and Applications

Summation over ring diagrams → gluon propagator:



Pole of the propagator → screening mass:

$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$

$$m_Q^2(T,B) = -\Pi_{00}^{||}(T,B),$$

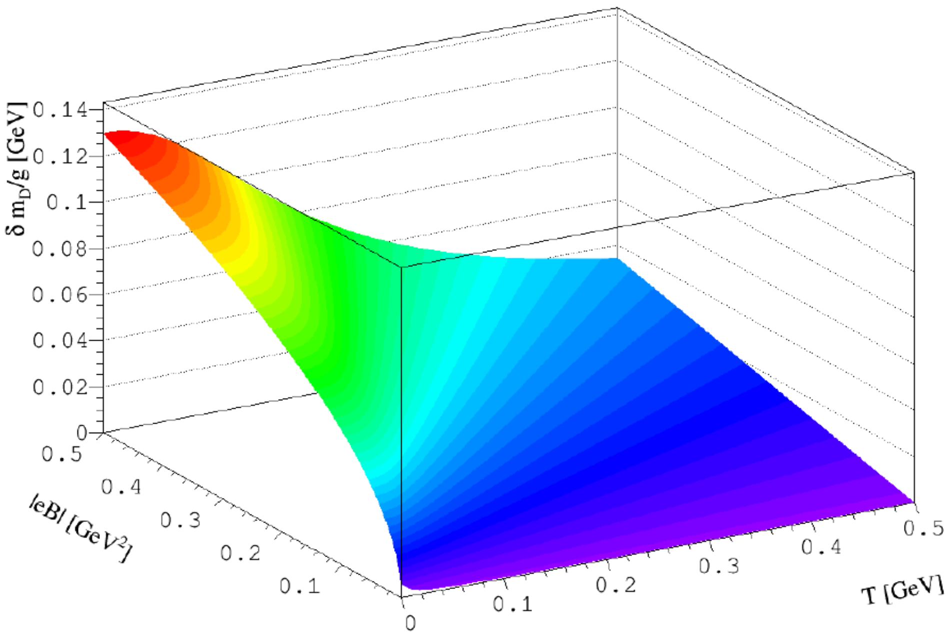
$$m_G^2(T) = -\overline{\Pi}_{00}^{||}(T).$$

$$m_Q^2(T,B) = -g^2T|qB| \sum_{np_zn_1} \left[(2-\delta_{n_1,0}) \frac{m^2 - \omega_n^2 + p_z^2 + 2n_1|qB|}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2} \right] \qquad m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

Result

Magnetic field induced Debye mass shift:

$$\delta m_D^2(T,B) \;=\; m_D^2(T,B) - m_D^2(T,0)$$



- $\delta m_D/g$ *is* g *independent.*
- $m_D(0,B) = 0.13$ GeV *at* $eB = 25m_\pi^2$*, the magnetic effect is gradually washed out by thermal motion.*

Summary

1) *We presented a calculation of color screening mass $m_D(T, B)$ in QCD without restriction to T and B.*

2) *The quantized Landau energy levels in magnetic field are automatically embedded into the color screening.*

3) *The magnetic field effect at LHC looks washed out by the strong thermal motion of quarks and gluons.*