

# Polarized vector meson production in semi-inclusive DIS

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*Based on: X.S. Jiao, K.B. Chen, Phys. Rev. D105, 054010 (2022)*

*K.B. Chen, Z.T. Liang, Y.K. Song, S.Y. Wei, Phys. Rev. D102, 034001 (2020)*



# Contents

- Introduction
- Kinematics and structure functions for  $eN \rightarrow eVX$
- Parton model results and the spin alignment
- Summary



# Contents

➤ Introduction

➤ Kinematics and structure functions for  $eN \rightarrow eVX$

➤ Parton model results and the spin alignment

➤ Summary



# Introduction

## ■ PDFs and FFs

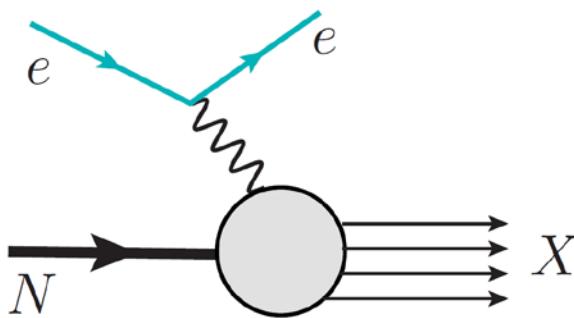
Parton distribution functions (**PDFs**)



Fragmentation functions (**FFs**)

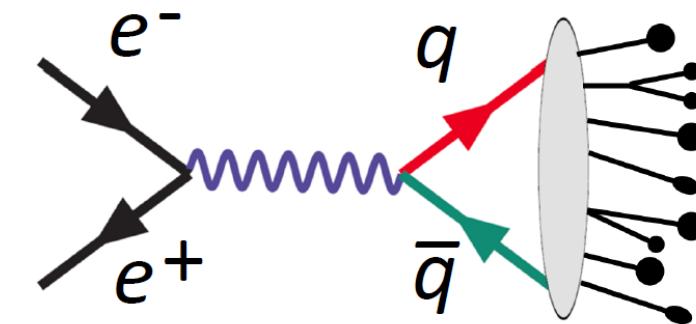


Parton momentum distribution  
inside a hadron



Lepton-nucleon deep inelastic scattering (DIS)

Hadron momentum distribution  
inside a parton jet

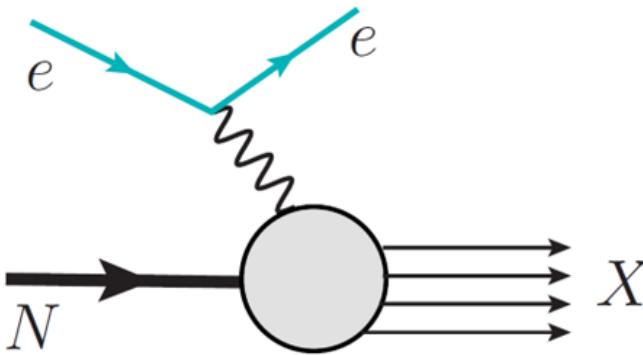


$e^+e^-$  annihilation to hadrons



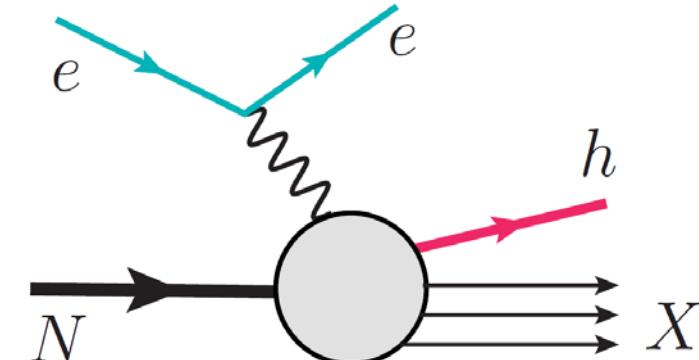
# Introduction

## ■ Semi-inclusive DIS (SIDIS)



Inclusive DIS

- One dimensional imaging of the nucleon
- Without fragmentation functions
- Target spin asymmetries



SIDIS

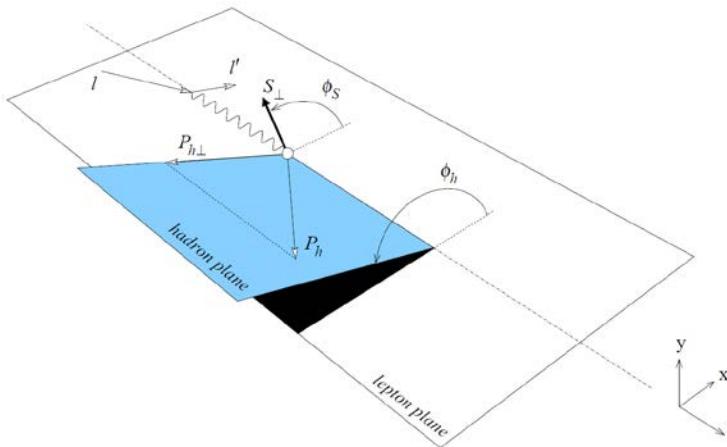
- Three dimensional imaging of the nucleon
- Accessing fragmentation functions
- Azimuthal and/or spin asymmetries

**More abundant physical contents!**



# Introduction

## ■ Semi-inclusive DIS (SIDIS)



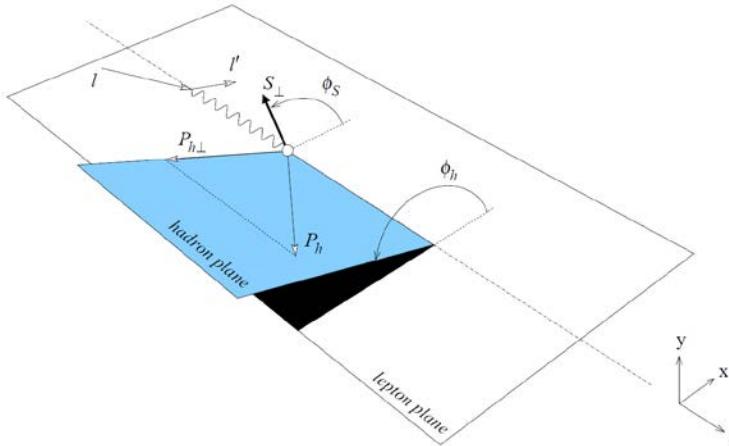
A. Bacchetta *et al.*, JHEP 02, 093 (2007)

$$\frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{aligned} & F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \\ & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \\ & + S_{\parallel} \left[ \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\ & + S_{\parallel} \lambda_e \left[ \sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\ & + |\mathbf{S}_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\ & \quad \left. + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right. \\ & \quad \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\ & + |\mathbf{S}_{\perp}| \lambda_e \left[ \sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\ & \quad \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \end{aligned} \right\}$$



# Introduction

## ■ Semi-inclusive DIS (SIDIS)



A. Bacchetta *et al.*, JHEP 02, 093 (2007)

$P_{h\perp} \ll Q$ : TMD factorization applies.

Structure functions are expressed by the convolution of TMD PDFs and FFs.

$$F_{UU,T} \sim f_1 \otimes D_1$$

$$F_{LL} \sim g_{1L} \otimes D_1$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^\perp \otimes D_1$$

$$F_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T} \otimes D_1$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} \sim h_1 \otimes H_1^\perp$$

$$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$

$$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

$$F_{UT}^{\sin(3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$$

⋮

|                |           |                |
|----------------|-----------|----------------|
| Number density | $\otimes$ | Unpolarized FF |
| Helicity       |           |                |
| Sivers         |           |                |
| Worm-gear      |           |                |
| Transversity   | $\otimes$ | Collins FF     |
| Boer-Mulders   |           |                |
| Worm-gear      |           |                |
| Pretzelosity   |           |                |



# Introduction

- The polarization of the produced hadron**

Vector meson production

⇒ New polarization dependent TMD FFs

| Quark pol. | Hadron pol.  | Chiral-even     |                 |                | Chiral-odd      |                 |
|------------|--------------|-----------------|-----------------|----------------|-----------------|-----------------|
|            |              | T-even          |                 | T-odd          | T-even          | T-odd           |
| <b>U</b>   | <b>U</b>     | $D_1$           | $D^\perp$       | $D_3$          |                 |                 |
|            | <b>L</b>     |                 |                 |                | $D_L^\perp$     |                 |
|            | <b>T</b>     |                 |                 |                | $D_{AT}^\perp$  | $D_T^\perp$     |
|            | <b>LL</b>    | $D_{1LL}$       | $D_{2LL}^\perp$ | $D_{3LL}$      |                 | $E_{LL}$        |
|            | <b>LT</b>    | $D_{1LT}^\perp$ | $D_{LT}^\perp$  | $D_{LT}^\perp$ | $D_{3LT}^\perp$ | $E_{LT}^\perp$  |
|            | <b>TT</b>    | $D_{1TT}^\perp$ | $D_{TT}^\perp$  | $D_{TT}^\perp$ | $D_{3TT}^\perp$ | $E_{TT}^\perp$  |
| <b>L</b>   | <b>U</b>     |                 |                 |                | $G^\perp$       |                 |
|            | <b>L</b>     | $G_{1L}$        | $G_L^\perp$     | $G_{3L}$       |                 | $E_L$           |
|            | <b>T</b>     | $G_{1T}^\perp$  | $G_T^\perp$     | $G_T^\perp$    | $G_{3T}^\perp$  | $E_T^\perp$     |
|            | <b>LL</b>    |                 |                 |                | $G_{LL}^\perp$  |                 |
|            | <b>LT</b>    |                 |                 |                | $G_{1LT}^\perp$ | $G_{LT}^\perp$  |
|            | <b>TT</b>    |                 |                 |                | $G_{1TT}^\perp$ | $G_{TT}^\perp$  |
| <b>T</b>   | <b>U</b>     |                 |                 |                |                 | $H_1^\perp$     |
|            | <b>L</b>     |                 |                 |                |                 | $H_L$           |
|            | <b>T(//)</b> |                 |                 |                |                 | $H_{3L}^\perp$  |
|            | <b>T(⊥)</b>  |                 |                 |                |                 | $H_{1T}$        |
|            | <b>LL</b>    |                 |                 |                |                 | $H_T^\perp$     |
|            | <b>LT</b>    |                 |                 |                |                 | $H_{3T}^\perp$  |
|            | <b>TT</b>    |                 |                 |                |                 | $H_{1T}^\perp$  |
|            |              |                 |                 |                |                 | $H_{LT}^\perp$  |
|            |              |                 |                 |                |                 | $H_{1LT}^\perp$ |
|            |              |                 |                 |                |                 | $H_{LT}^\perp$  |
|            |              |                 |                 |                |                 | $H_{3LT}^\perp$ |
|            |              |                 |                 |                |                 | $H_{1TT}^\perp$ |
|            |              |                 |                 |                |                 | $H_{TT}^\perp$  |
|            |              |                 |                 |                |                 | $H_{3TT}^\perp$ |
|            |              |                 |                 |                |                 | $H_{3LT}^\perp$ |

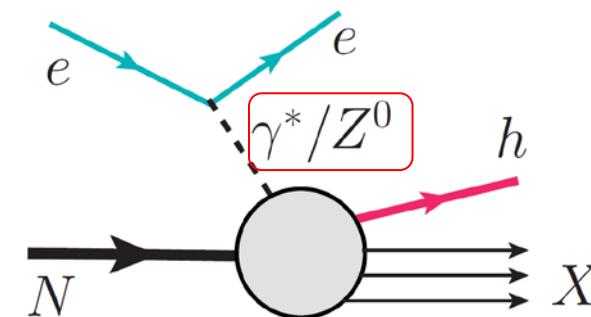
72 TMD FFs for polarized vector meson production

Chen, Yang, Wei and Liang, PRD94, 034003 (2016)

- Electroweak contributions at high energy**

Parity violating SIDIS

⇒ New structure functions



See e.g.,  
Boer, Jakob and Mulders,  
NPB 564, 471 (2000)



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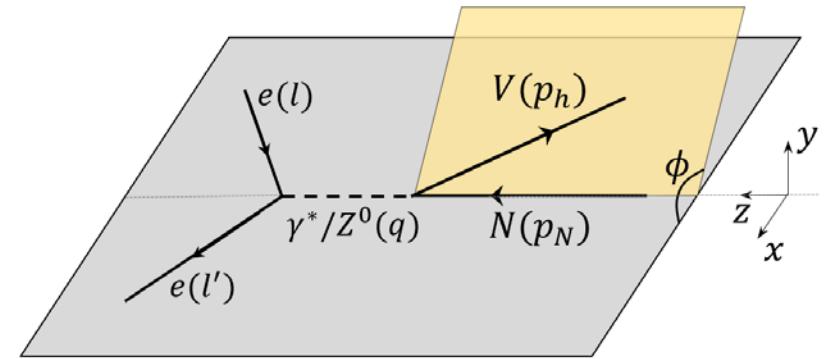
# Kinematics and structure functions for $eN \rightarrow eVX$

## ■ The process

$$e(l) + N(p_N) \rightarrow e(l') + V(p_h, S) + X$$

$V$ : Vector meson (spin 1)

$$d\sigma = \frac{2\alpha_{\text{em}}^2}{sQ^4} A_r L_r^{\mu\nu}(l, l') W_{r,\mu\nu}(q, p_N, p_h, S) \frac{d^3 l' d^3 p_h}{2E_{l'} 2E_h}, \quad (r = \gamma\gamma, ZZ, \gamma Z)$$



$$L_{\gamma\gamma}^{\mu\nu}(l, l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l \cdot l')$$

$$L_{ZZ}^{\mu\nu}(l, l') = c_1^e L_{\gamma\gamma}^{\mu\nu}(l, l') - 2ic_3^e \epsilon^{\mu\nu ll'}$$

$$L_{\gamma Z}^{\mu\nu}(l, l') = c_V^e L_{\gamma\gamma}^{\mu\nu}(l, l') - 2ic_A^e \epsilon^{\mu\nu ll'}$$

$$W_{\gamma\gamma}^{\mu\nu}(q, p_N, p_h, S) = \sum_X \delta^4(p_N + q - p_h - p_X) \langle p_N | J_{\gamma\gamma}^\mu(0) | p_h, S; X \rangle \langle p_h, S; X | J_{\gamma\gamma}^\nu(0) | p_N \rangle$$

$$W_{ZZ}^{\mu\nu}(q, p_N, p_h, S) = \sum_X \delta^4(p_N + q - p_h - p_X) \langle p_N | J_{ZZ}^\mu(0) | p_h, S; X \rangle \langle p_h, S; X | J_{ZZ}^\nu(0) | p_N \rangle$$

$$W_{Z\gamma}^{\mu\nu}(q, p_N, p_h, S) = \sum_X \delta^4(p_N + q - p_h - p_X) \langle p_N | J_{Z\gamma}^\mu(0) | p_h, S; X \rangle \langle p_h, S; X | J_{Z\gamma}^\nu(0) | p_N \rangle$$

$$A_{\gamma\gamma} = e_q^2, \quad Q^2 = -q^2,$$

$$A_{ZZ} = \frac{Q^4}{[(Q^2 + M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W} \equiv \chi, \quad x = \frac{Q^2}{2p_N \cdot q},$$

$$A_{\gamma Z} = \frac{2e_q Q^2 (Q^2 + M_Z^2)}{[(Q^2 + M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^2 2\theta_W} \equiv \chi_{\text{int}}, \quad y = \frac{p_N \cdot q}{p_N \cdot l},$$

$$z_h = \frac{p_N \cdot p_h}{p_N \cdot q},$$

$$s = (p_N + l)^2$$



# Kinematics and structure functions for $eN \rightarrow eVX$

## ■ General form of the hadronic tensor

$$W_{ZZ}^{\mu\nu}(q, p_N, p_h, S) = W_{ZZ}^{S\mu\nu}(\text{symmetric}) + iW_{ZZ}^{A\mu\nu}(\text{antisymmetric})$$

$$= \sum_{\sigma,i} W_{\sigma i}^S h_{\sigma i}^{S\mu\nu} + i \sum_{\sigma,j} W_{\sigma j}^A h_{\sigma j}^{A\mu\nu}$$

$$+ \sum_{\sigma,k} \tilde{W}_{\sigma k}^S \tilde{h}_{\sigma k}^{S\mu\nu} + i \sum_{\sigma,l} \tilde{W}_{\sigma l}^A \tilde{h}_{\sigma l}^{A\mu\nu}$$

Scalar coefficients

Basic Lorentz tensors (BLTs)

Hermiticity:

$$W_{ZZ}^{*\mu\nu} = W_{ZZ}^{\nu\mu}$$

Current conservation:

$$q_\mu W_{ZZ}^{\mu\nu} = q_\nu W_{ZZ}^{\mu\nu} = 0$$

$h$ : Parity even,  $\hat{\mathcal{P}}h^{\mu\nu} = h^{\mu\nu}$

$\tilde{h}$ : Parity odd,  $\hat{\mathcal{P}}\tilde{h}^{\mu\nu} = -\tilde{h}^{\mu\nu}$

$\sigma$ : polarizations,  $U, V, LL, LT, TT$

- **Unpolarized BLTs**

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \quad p_{Nq}^\mu p_{Nq}^\nu, \quad p_{Nq}^{\{\mu} p_{hq}^{\nu\}} \right. \\ \left. \quad p_{hq}^\mu p_{hq}^\nu \right\}$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_N p_h} p_{Nq}^{\nu\}} \right.$$

$$h_U^{A\mu\nu} = p_{Nq}^{[\mu} p_{hq}^{\nu]}$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_N}, \quad \varepsilon^{\mu\nu q p_h} \right\}$$

$$p_q^\mu \equiv p^\mu - \frac{p \cdot q}{q^2} q^\mu$$

$$q \cdot p_q = 0$$

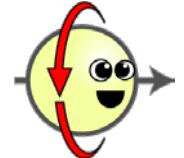


# Kinematics and structure functions for $eN \rightarrow eVX$

## Spin dependence

$S = 1/2$

$$2 \times 2 \text{ spin density matrix: } \rho = \frac{1}{2} (1 + \vec{S} \cdot \vec{\sigma})$$



$$\text{Polarization vector: } S^\mu = (0, \vec{S}_T, \lambda), \quad \vec{S}_T = (S_T^x, S_T^y)$$

$S = 1$

$$3 \times 3 \text{ spin density matrix: } \rho = \frac{1}{3} \left( 1 + \frac{3}{2} \vec{S} \cdot \vec{\Sigma} + 3 T^{ij} \Sigma^{ij} \right), \quad \Sigma^{ij} = \frac{1}{2} (\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{2}{3} \delta^{ij} I.$$

$$\text{Polarization vector: } S^\mu = (0, \vec{S}_T, \lambda)$$

Polarization tensor:  $\mathbf{T} = \frac{1}{2} \begin{pmatrix} -\frac{2}{3} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3} S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3} S_{LL} \end{pmatrix}.$

$S_{LL}$   
 $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$   
 $S_{TT}^{x\mu} = (0, S_{TT}^{xx}, S_{TT}^{xy}, 0)$

A. Bacchetta and P.J. Mulders, PRD62, 114004 (2000)



# Kinematics and structure functions for $eN \rightarrow eVX$

- Polarized BLTs**

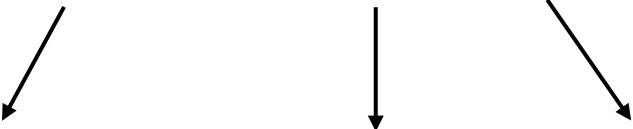
The polarized BLTs can be constructed from the unpolarized ones by multiplying with the polarization dependent scalars.

$$h_{Vi}^{S\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] \tilde{h}_{Ui}^{S\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_T} h_{Uj}^{S\mu\nu} \}$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] h_{Ui}^{S\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_T} \tilde{h}_{Uj}^{S\mu\nu} \}$$

$$h_{Vi}^{A\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] \tilde{h}_{Ui}^{A\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_T} h_U^{A\mu\nu} \}$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] h_U^{A\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_T} \tilde{h}_{Uj}^{A\mu\nu} \}$$



$$\begin{pmatrix} \text{Polarization} \\ \text{dependent BLTs} \end{pmatrix} = \begin{pmatrix} \text{Polarization} \\ \text{dependent scalars} \end{pmatrix} \times \begin{pmatrix} \text{Unpolarized} \\ \text{BLTs} \end{pmatrix}$$

**81 independent BLTs in total**

K.B. Chen, W.H. Yang, S.Y. Wei and Z.T. Liang, Phys. Rev. D94, 034003 (2016)

$$\left\{ \begin{array}{l} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LL}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{array} \right\} = S_{LL} \left\{ \begin{array}{l} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{array} \right\}$$

$$h_{LTi}^{S\mu\nu} = \{ (p_{h\perp} \cdot S_{LT}) h_{Ui}^{S\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_{LT}} \tilde{h}_{Uj}^{S\mu\nu} \}$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \{ (p_{h\perp} \cdot S_{LT}) \tilde{h}_{Ui}^{S\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_{LT}} h_{Uj}^{S\mu\nu} \}$$

$$h_{LTi}^{A\mu\nu} = \{ (p_{h\perp} \cdot S_{LT}) h_U^{A\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_{LT}} \tilde{h}_{Uj}^{A\mu\nu} \}$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \{ (p_{h\perp} \cdot S_{LT}) \tilde{h}_{Ui}^{A\mu\nu}, \quad \varepsilon_{\perp}^{p_h S_{LT}} h_U^{A\mu\nu} \}$$

$$h_{TTi}^{S\mu\nu} = \{ S_{TT}^{p_h p_h} h_{Ui}^{S\mu\nu}, \quad \tilde{S}_{TT}^{p_h p_h} \tilde{h}_{Uj}^{S\mu\nu} \}$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \{ S_{TT}^{p_h p_h} \tilde{h}_{Ui}^{S\mu\nu}, \quad \tilde{S}_{TT}^{p_h p_h} h_{Uj}^{S\mu\nu} \}$$

$$h_{TTi}^{A\mu\nu} = \{ S_{TT}^{p_h p_h} h_U^{A\mu\nu}, \quad \tilde{S}_{TT}^{p_h p_h} \tilde{h}_{Uj}^{A\mu\nu} \}$$

$$\tilde{h}_{TTi}^{A\mu\nu} = \{ S_{TT}^{p_h p_h} \tilde{h}_{Ui}^{A\mu\nu}, \quad \tilde{S}_{TT}^{p_h p_h} h_U^{A\mu\nu} \}$$



# Kinematics and structure functions for $eN \rightarrow eVX$

## ■ The cross section in terms of structure functions

$$\frac{d\sigma}{dxdydz_h d\psi d^2 p_{h\perp}} = \frac{\alpha_{\text{em}}^2}{xyQ^2} (\mathcal{W}_U + \lambda \mathcal{W}_L + |S_T| \mathcal{W}_T + S_{LL} \mathcal{W}_{LL} + |S_{LT}| \mathcal{W}_{LT} + |S_{TT}| \mathcal{W}_{TT})$$

$$\begin{aligned}\mathcal{W}_U &= A(y)W_U^T + E(y)W_U^L + B(y)(\sin \phi \tilde{W}_{U1}^{\sin \phi} + \cos \phi W_{U1}^{\cos \phi}) + E(y)(\sin 2\phi \tilde{W}_U^{\sin 2\phi} + \cos 2\phi W_U^{\cos 2\phi}) \\ &\quad + C(y)W_U + D(y)(\sin \phi \tilde{W}_{U2}^{\sin \phi} + \cos \phi W_{U2}^{\cos \phi}),\end{aligned}$$

$$\begin{aligned}\mathcal{W}_L &= A(y)\tilde{W}_L^T + E(y)\tilde{W}_L^L + B(y)(\sin \phi W_{L1}^{\sin \phi} + \cos \phi \tilde{W}_{L1}^{\cos \phi}) + E(y)(\sin 2\phi W_L^{\sin 2\phi} + \cos 2\phi \tilde{W}_L^{\cos 2\phi}) \\ &\quad + C(y)\tilde{W}_L + D(y)(\sin \phi W_{L2}^{\sin \phi} + \cos \phi \tilde{W}_{L2}^{\cos \phi}),\end{aligned}$$

$$\begin{aligned}\mathcal{W}_T &= \sin \phi_S [B(y)W_{T1}^{\sin \phi_S} + D(y)W_{T2}^{\sin \phi_S}] + \sin(\phi + \phi_S)E(y)W_T^{\sin(\phi + \phi_S)} \\ &\quad + \sin(\phi - \phi_S)[A(y)W_T^{T,\sin(\phi - \phi_S)} + E(y)W_T^{L,\sin(\phi - \phi_S)} + C(y)W_T^{\sin(\phi - \phi_S)}] \\ &\quad + \sin(2\phi - \phi_S)[B(y)W_{T1}^{\sin(2\phi - \phi_S)} + D(y)W_{T2}^{\sin(2\phi - \phi_S)}] + \sin(3\phi - \phi_S)E(y)W_T^{\sin(3\phi - \phi_S)} + \cos \phi_S [B(y)\tilde{W}_{T1}^{\cos \phi_S} + D(y)\tilde{W}_{T2}^{\cos \phi_S}] \\ &\quad + \cos(\phi + \phi_S)E(y)\tilde{W}_T^{\cos(\phi + \phi_S)} + \cos(\phi - \phi_S)[A(y)\tilde{W}_T^{T,\cos(\phi - \phi_S)} + E(y)\tilde{W}_T^{L,\cos(\phi - \phi_S)} + C(y)\tilde{W}_T^{\cos(\phi - \phi_S)}] \\ &\quad + \cos(2\phi - \phi_S)[B(y)\tilde{W}_{T1}^{\cos(2\phi - \phi_S)} + D(y)\tilde{W}_{T2}^{\cos(2\phi - \phi_S)}] + \cos(3\phi - \phi_S)E(y)\tilde{W}_T^{\cos(3\phi - \phi_S)},\end{aligned}$$

$$\begin{aligned}\mathcal{W}_{LL} &= A(y)W_{LL}^T + E(y)W_{LL}^L + B(y)(\sin \phi \tilde{W}_{LL1}^{\sin \phi} + \cos \phi W_{LL1}^{\cos \phi}) + E(y)(\sin 2\phi \tilde{W}_{LL}^{\sin 2\phi} + \cos 2\phi W_{LL}^{\cos 2\phi}) \\ &\quad + C(y)W_{LL} + D(y)(\sin \phi \tilde{W}_{LL2}^{\sin \phi} + \cos \phi W_{LL2}^{\cos \phi}),\end{aligned}$$

$$A(y) = y^2 - 2y + 2,$$

$$B(y) = 2(2 - y)\sqrt{1 - y},$$

$$C(y) = y(2 - y),$$

$$D(y) = 2y\sqrt{1 - y},$$

$$E(y) = 2(1 - y).$$



# Kinematics and structure functions for $eN \rightarrow eVX$

$$\begin{aligned}\mathcal{W}_{LT} = & \sin \phi_{LT} [B(y) \tilde{W}_{LT1}^{\sin \phi_{LT}} + D(y) \tilde{W}_{LT2}^{\sin \phi_{LT}}] + \sin(\phi + \phi_{LT}) E(y) \tilde{W}_{LT}^{\sin(\phi + \phi_{LT})} \\ & + \sin(\phi - \phi_{LT}) [A(y) \tilde{W}_{LT}^{T,\sin(\phi - \phi_{LT})} + E(y) \tilde{W}_{LT}^{L,\sin(\phi - \phi_{LT})} + C(y) \tilde{W}_{LT}^{\sin(\phi - \phi_{LT})}] \\ & + \sin(2\phi - \phi_{LT}) [B(y) \tilde{W}_{LT1}^{\sin(2\phi - \phi_{LT})} + D(y) \tilde{W}_{LT2}^{\sin(2\phi - \phi_{LT})}] + \sin(3\phi - \phi_{LT}) E(y) \tilde{W}_{LT}^{\sin(3\phi - \phi_{LT})} \\ & + \cos \phi_{LT} [B(y) W_{LT1}^{\cos \phi_{LT}} + D(y) W_{LT2}^{\cos \phi_{LT}}] + \cos(\phi + \phi_{LT}) E(y) W_{LT}^{\cos(\phi + \phi_{LT})} \\ & + \cos(\phi - \phi_{LT}) [A(y) W_{LT}^{T,\cos(\phi - \phi_{LT})} + E(y) W_{LT}^{L,\cos(\phi - \phi_{LT})} + C(y) W_{LT}^{\cos(\phi - \phi_{LT})}] \\ & + \cos(2\phi - \phi_{LT}) [B(y) W_{LT1}^{\cos(2\phi - \phi_{LT})} + D(y) W_{LT2}^{\cos(2\phi - \phi_{LT})}] + \cos(3\phi - \phi_{LT}) E(y) W_{LT}^{\cos(3\phi - \phi_{LT})}\end{aligned}$$

$$\begin{aligned}\mathcal{W}_{TT} = & \sin(\phi - 2\phi_{TT}) [B(y) \tilde{W}_{TT1}^{\sin(\phi - 2\phi_{TT})} + D(y) \tilde{W}_{TT2}^{\sin(\phi - 2\phi_{TT})}] + \sin 2\phi_{TT} E(y) \tilde{W}_{TT}^{\sin 2\phi_{TT}} \\ & + \sin(2\phi - 2\phi_{TT}) [A(y) \tilde{W}_{TT}^{T,\sin(2\phi - 2\phi_{TT})} + E(y) \tilde{W}_{TT}^{L,\sin(2\phi - 2\phi_{TT})} + C(y) \tilde{W}_{TT}^{\sin(2\phi - 2\phi_{TT})}] \\ & + \sin(3\phi - 2\phi_{TT}) [B(y) \tilde{W}_{TT1}^{\sin(3\phi - 2\phi_{TT})} + D(y) \tilde{W}_{TT2}^{\sin(3\phi - 2\phi_{TT})}] + \sin(4\phi - 2\phi_{TT}) E(y) \tilde{W}_{TT}^{\sin(4\phi - 2\phi_{TT})} \\ & + \cos(\phi - 2\phi_{TT}) [B(y) W_{TT1}^{\cos(\phi - 2\phi_{TT})} + D(y) W_{TT2}^{\cos(\phi - 2\phi_{TT})}] + \cos 2\phi_{TT} E(y) W_{TT}^{\cos 2\phi_{TT}} \\ & + \cos(2\phi - 2\phi_{TT}) [A(y) W_{TT}^{T,\cos(2\phi - 2\phi_{TT})} + E(y) W_{TT}^{L,\cos(2\phi - 2\phi_{TT})} + C(y) W_{TT}^{\cos(2\phi - 2\phi_{TT})}] \\ & + \cos(3\phi - 2\phi_{TT}) [B(y) W_{TT1}^{\cos(3\phi - 2\phi_{TT})} + D(y) W_{TT2}^{\cos(3\phi - 2\phi_{TT})}] + \cos(4\phi - 2\phi_{TT}) E(y) W_{TT}^{\cos(4\phi - 2\phi_{TT})}\end{aligned}$$

$$A(y) = y^2 - 2y + 2,$$

$$B(y) = 2(2-y)\sqrt{1-y},$$

$$C(y) = y(2-y),$$

$$D(y) = 2y\sqrt{1-y},$$

$$E(y) = 2(1-y).$$

- 81 structure functions (SFs) in total
- 39 SFs are space reflection odd
- 45 tensor polarization dependent SFs



# Contents

- Introduction
- Kinematics and structure functions for  $eN \rightarrow eVX$
- Parton model results and the spin alignment
- Summary



# Parton model results and the spin alignment

## ■ The hadronic tensor in terms of TMD PDFs and FFs

TMD factorization applies at  $P_{h\perp} \ll Q$ :

$$W_{ZZ}^{\mu\nu} = 2z_h \int d^2k_{iT} d^2k_{fT} \delta^2(\vec{k}_{iT} - \vec{k}_{fT} - \vec{p}_{h\perp}/z_h) \text{Tr}[\hat{\Phi}(x, k_{iT}) \Gamma_q^\mu \hat{\Xi}(z_h, k_{fT}, S) \Gamma_q^\nu] \quad \Gamma_q^\mu = \gamma^\mu(c_V^q - c_A^q \gamma_5)$$

The TMD parton correlators:

$$\hat{\Phi}_{ij}(x, k_{iT}) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ixp_N^+ \xi^- + ik_{iT} \cdot \xi_T} \langle p_N | \bar{\psi}_i(0) \psi_j(\xi^-, \xi_T) | p_N \rangle$$

$$\hat{\Xi}_{ij}(z_h, k_{fT}, S) = \sum_X \int \frac{d\eta^+ d^2\eta_T}{2z_h (2\pi)^3} e^{-i\eta^+ p_h^- / z_h - i\eta_T \cdot k_{fT}} \langle 0 | \psi_i(0) | p_h, S; X \rangle \langle p_h, S; X | \bar{\psi}_j(\eta^+, \eta_T) | 0 \rangle$$

For unpolarized nucleon:

$$\hat{\Phi} = \frac{1}{2} (\Phi_\alpha \gamma^\alpha - \tilde{\Phi}_\alpha \gamma_5 \gamma^\alpha + \Phi_{\alpha\beta} i\sigma^{\alpha\beta} \gamma_5) + \dots$$

$$\begin{aligned} \Phi^\alpha &= \bar{n}^\alpha f_1(x, k_{iT}) + \dots \\ \Phi^{\alpha\beta} &= -\frac{1}{M_N} \bar{n}^\alpha \tilde{k}_{iT}^\beta h_1^\perp(x, k_{iT}) + \dots \end{aligned}$$



# Parton model results and the spin alignment

For polarized vector meson production:  $\hat{\Xi} = \frac{1}{2} (\Xi_\alpha \gamma^\alpha + \tilde{\Xi}_\alpha \gamma_5 \gamma^\alpha + \Xi_{\alpha\beta} i\sigma^{\alpha\beta} \gamma_5) + \dots$

$$\Xi^\alpha = n^\alpha \left( D_1 + \frac{\tilde{k}_{fT} \cdot S_T}{M_h} D_{1T}^\perp + S_{LL} D_{1LL} + \frac{k_{fT} \cdot S_{LT}}{M_h} D_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} D_{1TT}^\perp \right) + \dots$$

$$\tilde{\Xi}^\alpha = n^\alpha \left( \lambda_h G_{1L} + \frac{k_{fT} \cdot S_T}{M_h} G_{1T}^\perp + \frac{\tilde{k}_{fT} \cdot S_{LT}}{M_h} G_{1LT}^\perp - \frac{\tilde{S}_{TT}^{k_f k_f}}{M_h^2} G_{1TT}^\perp \right) + \dots$$

$$\begin{aligned} \Xi^{\alpha\beta} = & -\frac{n^{[\alpha} \tilde{k}_{fT}^{\beta]}}{M_h} (H_1^\perp + S_{LL} H_{1LL}^\perp) + n^{[\alpha} S_T^{\beta]} H_{1T} + \frac{n^{[\alpha} k_{fT}^{\beta]}}{M_h} \left( \lambda_h H_{1L}^\perp + \frac{k_{fT} \cdot S_T}{M_h} H_{1T}^\perp \right) \\ & - n^{[\alpha} \tilde{S}_{LT}^{\beta]} H_{1LT} - \frac{n^{[\alpha} \tilde{S}_{TT}^{k_f \beta]}}{M_h} H_{1TT}^\perp - \frac{n^{[\alpha} \tilde{k}_{fT}^{\beta]}}{M_h} \left( \frac{k_{fT} \cdot S_{LT}}{M_h} H_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} H_{1TT}^\perp \right) + \dots \end{aligned}$$

Ten tensor polarization dependent TMD FFs at twist-2.



# Parton model results and the spin alignment

The hadronic tensor results:

$$W_{ZZ}^{\mu\nu} = \frac{2z_h}{x} \mathcal{C} \left\{ (c_3^q g_{\perp}^{\mu\nu} + i c_1^q \varepsilon_{\perp}^{\mu\nu}) f_1 \mathcal{G}_1 - (c_1^q g_{\perp}^{\mu\nu} + i c_3^q \varepsilon_{\perp}^{\mu\nu}) f_1 \mathcal{D}_1 + 2c_2^q h_1^{\perp} \left[ \frac{\alpha_T^{\mu\nu}(\tilde{k}_{iT}, S_T)}{M_N} H_{1T} + \frac{\alpha_T^{\mu\nu}(k_{iT}, S_{LT})}{M_N} H_{1LT} \right. \right. \\ \left. \left. + \frac{\alpha_T^{\mu\nu}(\tilde{k}_{iT}, k_{fT})}{M_N M_h} \left( \lambda_h H_{1L}^{\perp} + \frac{k_{fT} \cdot S_T}{M_h} H_{1T}^{\perp} \right) + \frac{\alpha_T^{\mu\nu}(k_{iT}, S_{TT}^{k_f})}{M_N M_h} H_{1TT}^{\perp} + \frac{\alpha_T^{\mu\nu}(k_{iT}, k_{fT})}{M_N M_h} \mathcal{H}_1 \right] \right\}.$$

$$\alpha_T^{\mu\nu}(a_T, b_T) \equiv a_T^{\{\mu} b_T^{\nu\}} - (a_T \cdot b_T) g_{\perp}^{\mu\nu}$$

$$\mathcal{C}[wfD] = x \int d^2 k_{iT} d^2 k_{fT} \delta^2(\vec{k}_{iT} - \vec{k}_{fT} - \vec{p}_{h\perp}/z_h) wf(x, k_{iT}) D(z_h, k_{fT})$$

$$\mathcal{D}_1 \equiv D_1 + \frac{\tilde{k}_{fT} \cdot S_T}{M_h} D_{1T}^{\perp} + S_{LL} D_{1LL} + \frac{k_{fT} \cdot S_{LT}}{M_h} D_{1LT}^{\perp} + \frac{S_{TT}^{k_f k_f}}{M_h^2} D_{1TT}^{\perp},$$

$$\mathcal{G}_1 \equiv \lambda_h G_{1L} + \frac{k_{fT} \cdot S_T}{M_h} G_{1T}^{\perp} + \frac{\tilde{k}_{fT} \cdot S_{LT}}{M_h} G_{1LT}^{\perp} - \frac{\tilde{S}_{TT}^{k_f k_f}}{M_h^2} G_{1TT}^{\perp},$$

$$\mathcal{H}_1 \equiv H_1^{\perp} + S_{LL} H_{1LL}^{\perp} + \frac{k_{fT} \cdot S_{LT}}{M_h} H_{1LT}^{\perp} + \frac{S_{TT}^{k_f k_f}}{M_h^2} H_{1TT}^{\perp}.$$

Parameters for weak, electromagnetic and interference contributions

|                |                     | $A_r$          | $L_r^{\mu\nu}$        | $W_r^{\mu\nu}$ |
|----------------|---------------------|----------------|-----------------------|----------------|
| ZZ             | $\chi$              | $c_1^e, c_3^e$ | $c_1^q, c_2^q, c_3^q$ |                |
| $\gamma Z$     | $\chi_{\text{int}}$ | $c_V^e, c_A^e$ | $c_V^q, c_V^q, c_A^q$ |                |
| $\gamma\gamma$ | $e_q^2$             | 1, 0           | 1, 1, 0               |                |



# Parton model results and the spin alignment

## ■ The structure functions

$$W_U^T = c_{11}^{\text{ew}} \mathcal{C}[f_1 D_1],$$

$$W_U = c_{33}^{\text{ew}} \mathcal{C}[f_1 D_1],$$

$$W_U^{\cos 2\phi} = 2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_{1LL}^\perp],$$

$$\tilde{W}_L^T = -c_{13}^{\text{ew}} \mathcal{C}[f_1 G_{1L}],$$

$$\tilde{W}_L = -c_{31}^{\text{ew}} \mathcal{C}[f_1 G_{1L}],$$

$$W_L^{\sin 2\phi} = -2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_{1L}^\perp],$$

$$W_T^{T,\sin(\phi-\phi_S)} = c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1T}^\perp],$$

$$W_T^{\sin(\phi-\phi_S)} = c_{33}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1T}^\perp],$$

$$\tilde{W}_T^{T,\cos(\phi-\phi_S)} = c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp],$$

$$\tilde{W}_T^{\cos(\phi-\phi_S)} = c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp],$$

$$W_T^{\sin(\phi+\phi_S)} = c_{12}^{\text{ew}} \mathcal{C}[w_1 h_1^\perp (\bar{w}_0 H_{1T}^\perp - 2H_{1T})],$$

$$W_T^{\sin(3\phi-\phi_S)} = c_{12}^{\text{ew}} \mathcal{C}[w_3 h_1^\perp H_{1T}^\perp],$$

$$W_{LL}^T = c_{11}^{\text{ew}} \mathcal{C}[f_1 D_{1LL}],$$

$$W_{LL} = c_{33}^{\text{ew}} \mathcal{C}[f_1 D_{1LL}],$$

$$W_{LL}^{\cos 2\phi} = 2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_{1LL}^\perp],$$

$$W_{LT}^{T,\cos(\phi-\phi_{LT})} = -c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1LT}^\perp],$$

$$W_{LT}^{\cos(\phi-\phi_{LT})} = -c_{33}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1LT}^\perp],$$

$$\tilde{W}_{LT}^{T,\sin(\phi-\phi_{LT})} = -c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1LT}^\perp],$$

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$$W_{LT}^{\cos(3\phi-\phi_{LT})} = -c_{12}^{\text{ew}} \mathcal{C}[w_3 h_1^\perp H_{1LT}^\perp],$$

$$W_{TT}^{T,\cos(2\phi-2\phi_{TT})} = c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 D_{1TT}^\perp],$$

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$$\tilde{W}_{TT}^{\sin(2\phi-2\phi_{TT})} = c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 G_{1TT}^\perp],$$

$$W_{TT}^{\cos 2\phi_{TT}} = c_{12}^{\text{ew}} \mathcal{C}[w'_0 h_1^\perp (\bar{w}_0 H_{1TT}^\perp - 2H'_{1TT})],$$

$$W_{TT}^{\cos(4\phi-2\phi_{TT})} = c_{12}^{\text{ew}} \mathcal{C}[w_4 h_1^\perp H_{1TT}^\perp].$$

$$\bar{w}_0 = -\frac{k_{fT}^2}{M_h^2}, \quad w'_0 = -\frac{k_{iT} \cdot k_{fT}}{M_N M_h},$$

$$w_1 = -\frac{\hat{h} \cdot k_{iT}}{M_N}, \quad \bar{w}_1 = -\frac{\hat{h} \cdot k_{fT}}{M_h},$$

$$w_2 = \frac{2(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}}{M_N M_h},$$

$$w'_2 = \frac{(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}}{M_N M_h},$$

$$\bar{w}_2 = \frac{2(\hat{h} \cdot k_{fT})^2 + k_{fT}^2}{M_h^2},$$

$$w_3 = \frac{-1}{M_N M_h^2} [4(\hat{h} \cdot k_{fT})^2 (\hat{h} \cdot k_{iT}) + k_{fT}^2 (\hat{h} \cdot k_{iT}) \\ + 2(\hat{h} \cdot k_{fT})(k_{iT} \cdot k_{fT})],$$

$$w_4 = \frac{1}{M_N M_h^3} \{ k_{fT}^2 [k_{iT} \cdot k_{fT} + 4(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})] \\ + 4(\hat{h} \cdot k_{fT})^2 [k_{iT} \cdot k_{fT} + 2(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})] \}.$$

$$c_{11}^{\text{ew}} = c_1^e c_1^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2,$$

$$c_{12}^{\text{ew}} = c_1^e c_2^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2,$$

$$c_{13}^{\text{ew}} = c_1^e c_3^q \chi + c_V^e c_A^q \chi_{\text{int}},$$

$$c_{31}^{\text{ew}} = c_3^e c_1^q \chi + c_A^e c_V^q \chi_{\text{int}},$$

$$c_{33}^{\text{ew}} = c_3^e c_3^q \chi + c_A^e c_A^q \chi_{\text{int}}.$$



# Parton model results and the spin alignment

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$$\tilde{W}_T^{T,\cos(\phi-\phi_S)} = c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp],$$

$$\tilde{W}_T^{\cos(\phi-\phi_S)} = c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp],$$

$$W_T^{\sin(\phi+\phi_S)} = c_{12}^{\text{ew}} \mathcal{C}[w_1 h_1^\perp (\bar{w}_0 H_{1T}^\perp - 2H_{1T})],$$

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$$\tilde{W}_{LT}^{\sin(\phi-\phi_{LT})} = -c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1LT}^\perp],$$

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$$W_{TT}^{\cos 2\phi_{TT}} = c_{12}^{\text{ew}} \mathcal{C}[w'_0 h_1^\perp (\bar{w}_0 H_{1TT}^\perp - 2H'_{1TT})],$$

$$W_{TT}^{\cos(4\phi-2\phi_{TT})} = c_{12}^{\text{ew}} \mathcal{C}[w_4 h_1^\perp H_{1TT}^\perp].$$

- 27 nonzero SFs at the leading twist
- 15 tensor polarization dependent SFs
- 13 new SFs exist only when considering weak interaction

$$c_{11}^{\text{ew}} = c_1^e c_1^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2,$$

$$c_{12}^{\text{ew}} = c_1^e c_2^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2,$$

$$c_{13}^{\text{ew}} = c_1^e c_3^q \chi + c_V^e c_A^q \chi_{\text{int}},$$

$$c_{31}^{\text{ew}} = c_3^e c_1^q \chi + c_A^e c_V^q \chi_{\text{int}},$$

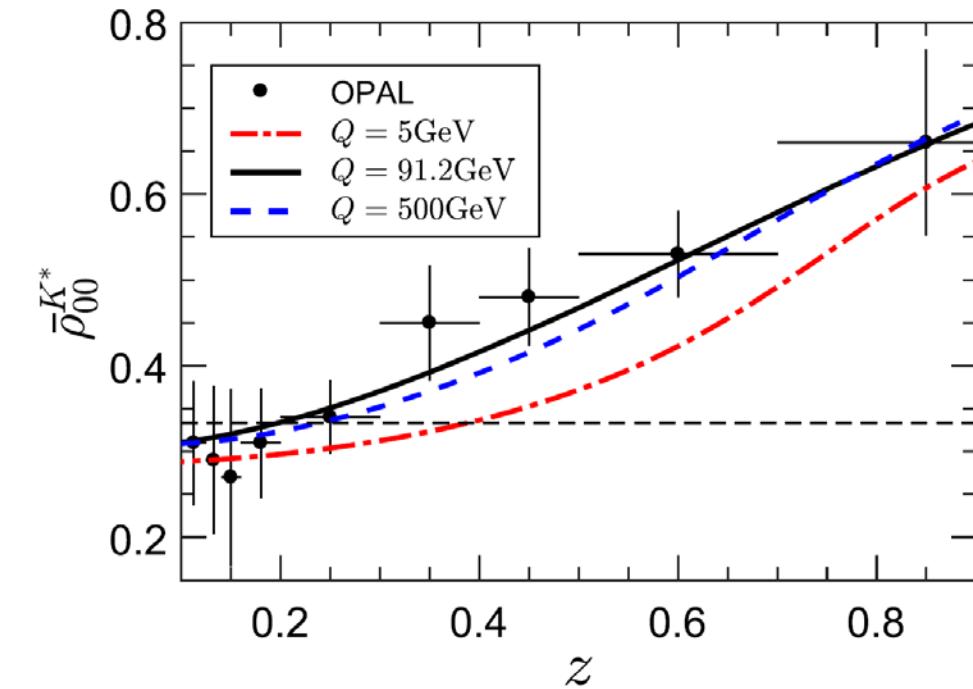
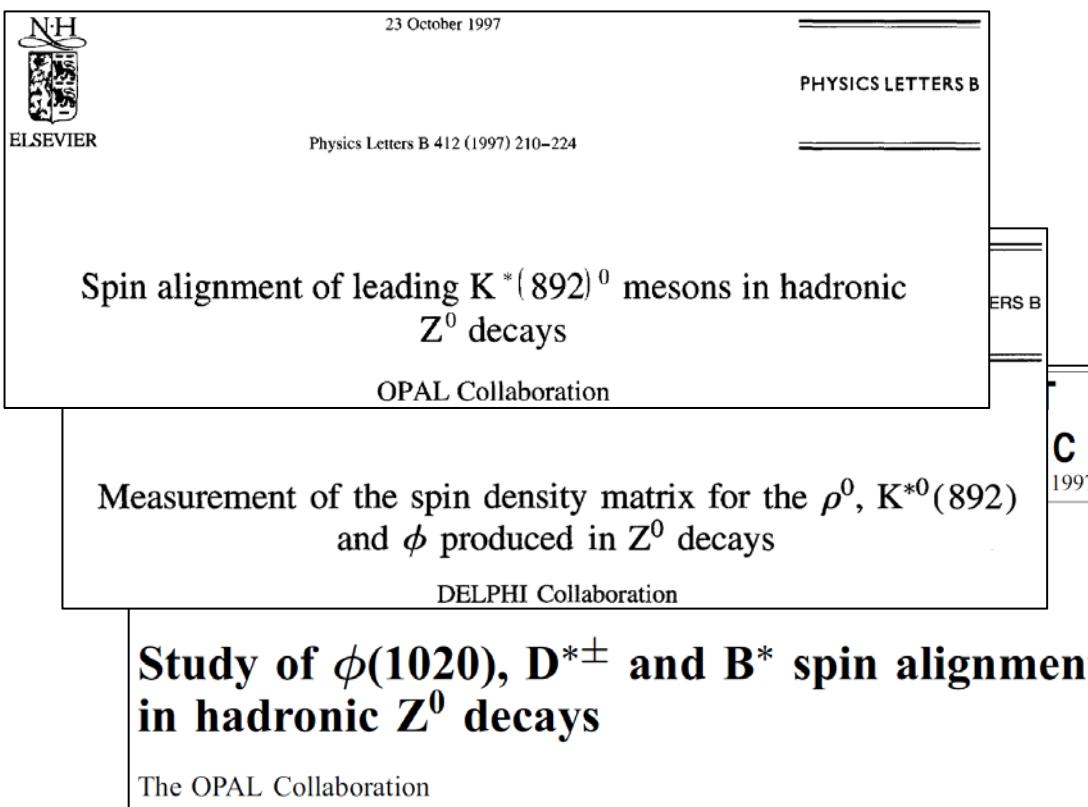
$$c_{33}^{\text{ew}} = c_3^e c_3^q \chi + c_A^e c_A^q \chi_{\text{int}}.$$

# Parton model results and the spin alignment

## ■ The spin alignment of the vector meson

$\rho_{00}$  gives the relative intensity of mesons in helicity 0 state

- In  $e^+e^-$  annihilation:



### General features:

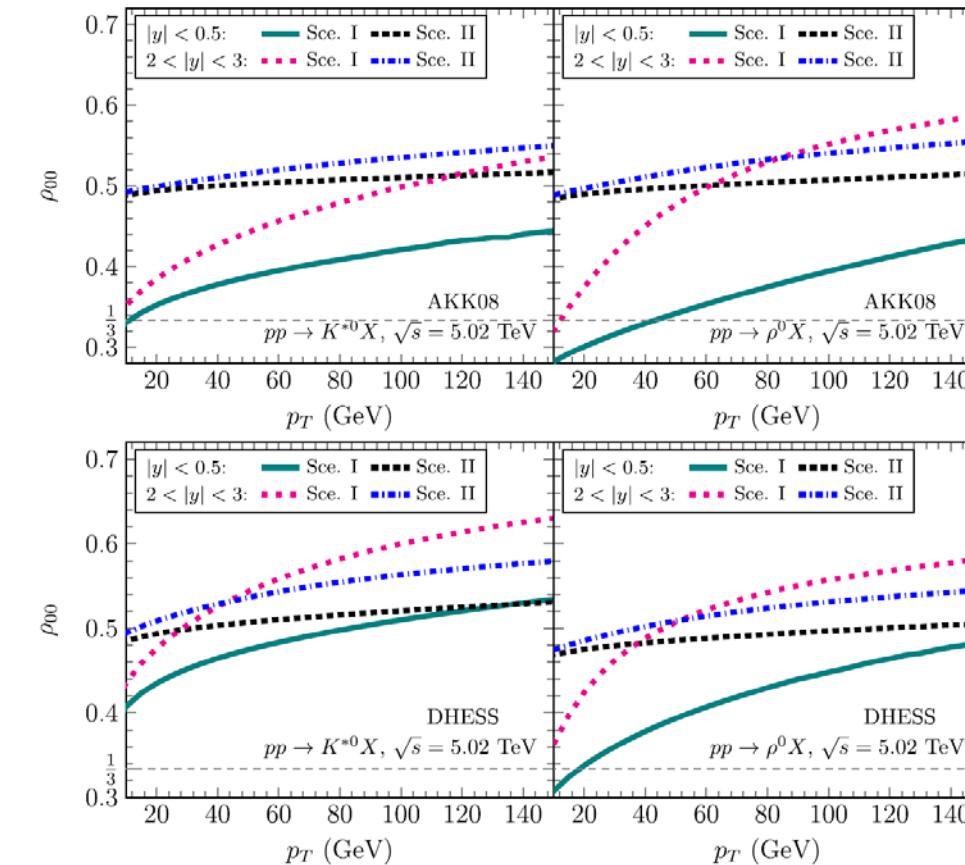
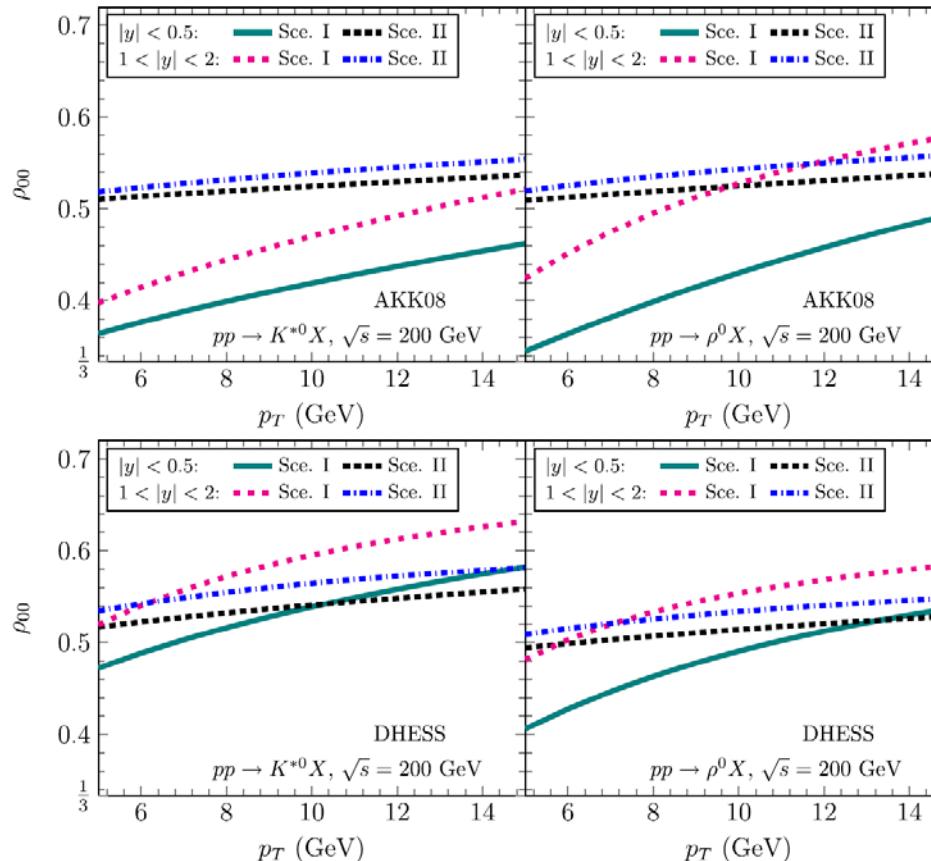
- Independent of the polarization of the fragmenting quark
- Weak energy dependence

K.B. Chen, W.H. Yang, Y.J. Zhou and Z.T. Liang, Phys. Rev. D95, 034009 (2017)

# Parton model results and the spin alignment

## ■ The spin alignment of the vector meson

- In  $pp$  collision:



K.B. Chen, Z.T. Liang, Y.K. Song, S.Y. Wei, Phys. Rev. D102, 034001 (2020)



# Parton model results and the spin alignment

## ■ The spin alignment of the vector meson

- In SIDIS:

$$\rho_{00}^V = \frac{1}{3} - \frac{\mathcal{W}_{LL}}{3\mathcal{W}_U} = \frac{1}{3} \left\{ 1 - \frac{[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] \mathcal{C}[f_1 D_{1LL}] + 2c_{12}^{\text{ew}} E(y) \mathcal{C}[w_2 h_1^\perp H_{1LL}^\perp] \cos 2\phi}{[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] \mathcal{C}[f_1 D_1] + 2c_{12}^{\text{ew}} E(y) \mathcal{C}[w_2 h_1^\perp H_1^\perp] \cos 2\phi} \right\}.$$

$\phi$  integrated  $\rightarrow$

$$\langle \rho_{00}^V \rangle = \frac{1}{3} - \frac{(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1(x, k_{iT}) D_{1LL}(z_h, k_{fT})]}{3(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1(x, k_{iT}) D_1(z_h, k_{fT})]}$$

Take Gaussian form for TMDs:

$$f_1(x, k_{iT}) = f_{1q}(x) \frac{1}{\pi \Delta_f^2} e^{-\vec{k}_{iT}^2 / \Delta_f^2},$$

$$D_1(z_h, k_{fT}) = D_{1q}^{K^{*0}}(z_h) \frac{1}{\pi \Delta_D^2} e^{-\vec{k}_{fT}^2 / \Delta_D^2},$$

$$D_{1LL}(z_h, k_{fT}) = D_{1LLq}^{K^{*0}}(z_h) \frac{1}{\pi \Delta_{LL}^2} e^{-\vec{k}_{fT}^2 / \Delta_{LL}^2}.$$

$p_{h\perp}$  integrated  $\rightarrow$

$$\langle \bar{\rho}_{00}^{K^{*0}} \rangle = \frac{1}{3} - \frac{[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] f_{1q}(x) D_{1LLq}^{K^{*0}}(z_h)}{3[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] f_{1q}(x) D_{1q}^{K^{*0}}(z_h)}$$

Only depends on the **collinear part** of the TMDs

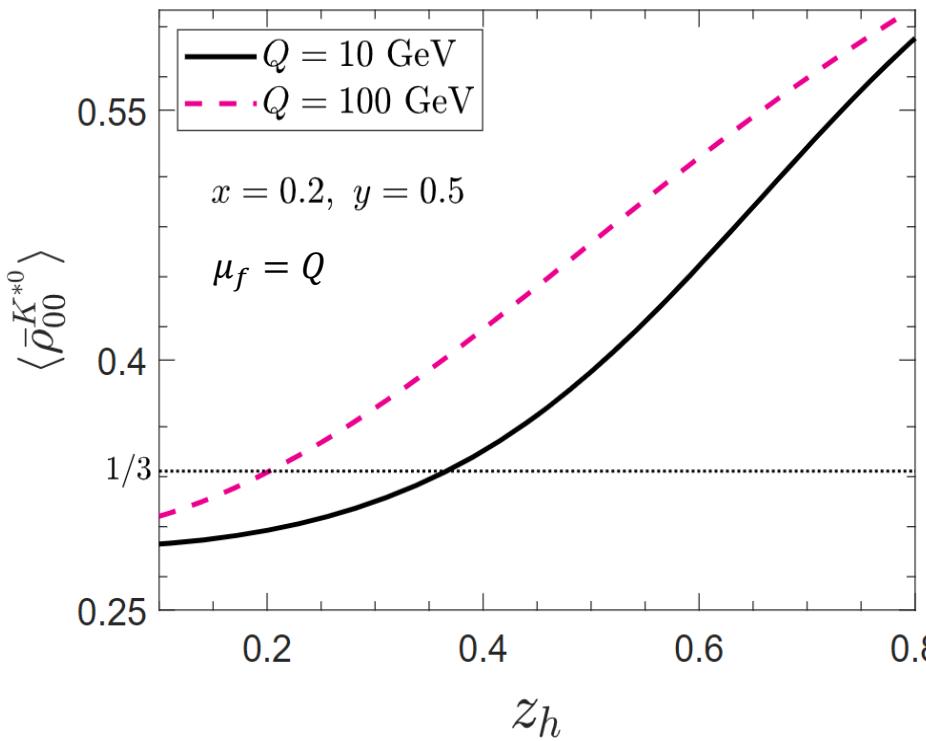
$f_{1q}(x)$ : CTEQ14 NLO

$D_{1q}^{K^{*0}}(z_h), D_{1LLq}^{K^{*0}}(z_h)$ : K.B. Chen et al., PRD102, 034001 (2020)

# Parton model results and the spin alignment

## ■ The spin alignment of the vector meson

- In SIDIS:

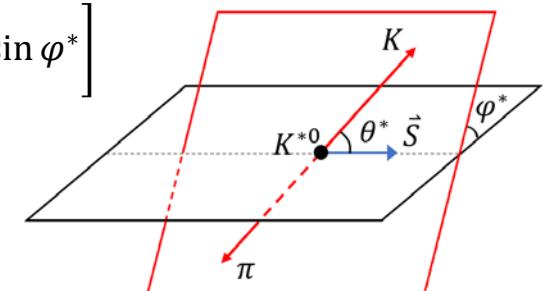


A rough numerical estimate of the spin alignment for  $K^{*0}$  production

- $\langle \bar{\rho}_0^{K^{*0}} \rangle$  deviates from  $1/3$  at both low and high  $Q$
- $\langle \bar{\rho}_0^{K^{*0}} \rangle$  increases monotonically with  $z_h$
- Scale dependence is relatively small

$$W(\cos \theta^*, \varphi^*) = \frac{3}{4\pi} \left[ \frac{1 - \rho_{00}}{2} + \frac{3\rho_{00} - 1}{2} \cos^2 \theta^* - \text{Re} \rho_{1-1} \sin^2 \theta^* \cos 2\varphi^* \right. \\ \left. - \frac{1}{\sqrt{2}} \text{Re}(\rho_{10} - \rho_{0-1}) \sin 2\theta^* \cos \varphi^* + \text{Im} \rho_{1-1} \sin^2 \theta^* \sin 2\varphi^* \right. \\ \left. + \frac{1}{\sqrt{2}} \text{Im}(\rho_{10} - \rho_{0-1}) \sin 2\theta^* \sin \varphi^* \right]$$
$$W(\cos \theta^*) = \frac{3}{4} [1 - \rho_{00} + (3\rho_{00} - 1) \cos^2 \theta^*]$$

K. Ackerstaff et al., Z. Phys. C 74, 437 (1997)



Possible measurements at EicC, EIC, JLab ...



# Summary

- We make a kinematic analysis for polarized vector meson production SIDIS. Considering the parity violating effects, the cross section is expressed by 81 structure functions.
- We give a parton model calculation in TMD factorization. There are 27 nonzero SFs at the leading twist; 15 tensor polarization dependent SFs; 13 new SFs generated by weak interaction.
- A rough numerical estimate of the spin alignment for  $K^{*0}$  production is given.

Thank you for your attention!