Nuclear Science Computing Center at CCNU

Criticality of QCD in correlated Dirac eigenvalues

Wei-Ping Huang

Central China Normal University

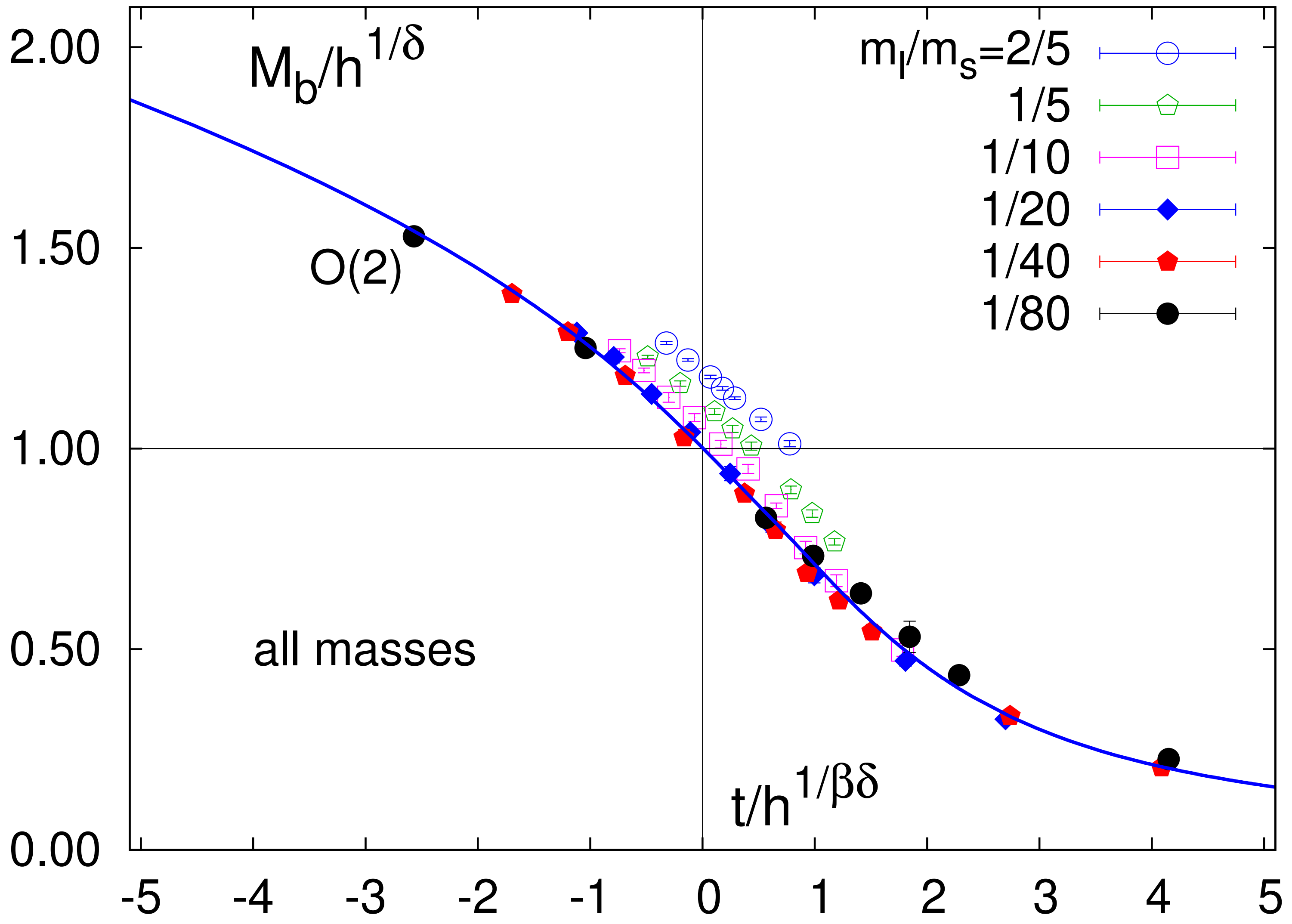
in collaboration with

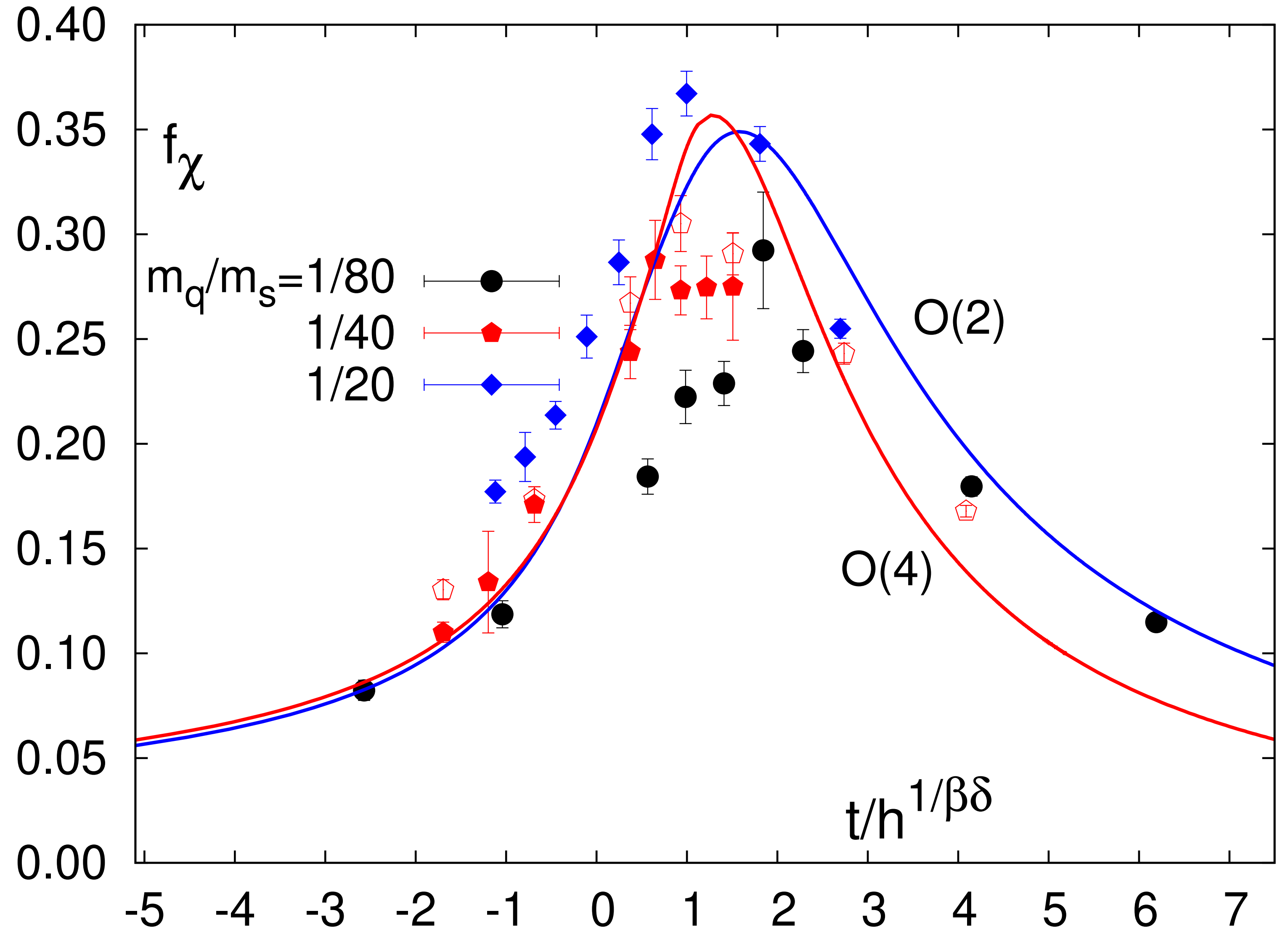
H.-T. Ding, S. Mukherjee, P. Petreczky and Y. Zhang

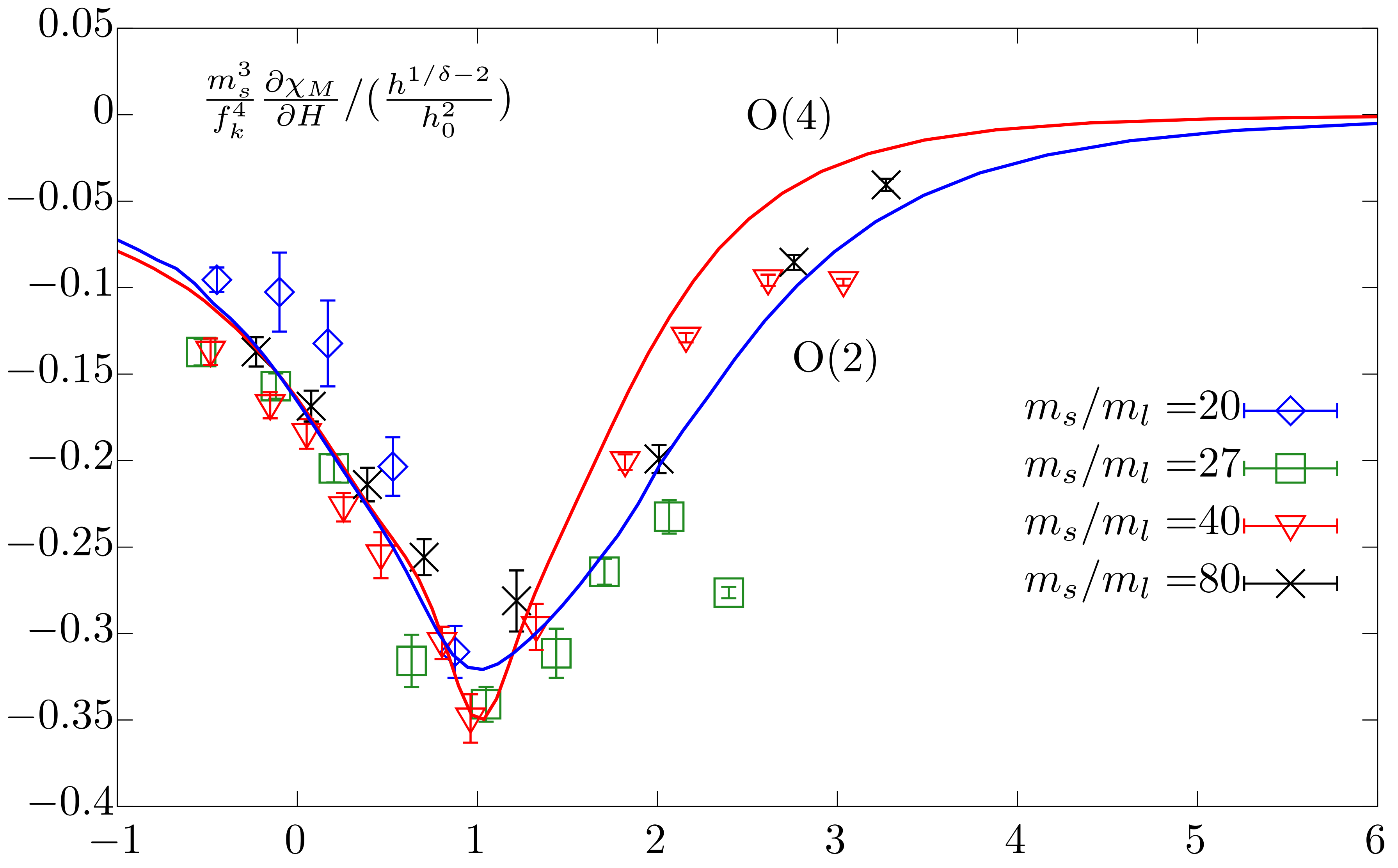
中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会

Critical behavior and Magnetic Equation of State

$$M(t,h) = h^{1/\delta} f_G(z) + f_{reg}(T,H) \qquad \chi_M(t,h) = \frac{\partial M}{\partial H} = h_0^{-1} h^{1/\delta-1} f_\chi(z) + f'_{reg} \qquad \frac{\partial \chi_M(t,h)}{\partial H} = h_0^{-2} h^{1/\delta-2} f_{pp}(z) + f''_{reg}$$







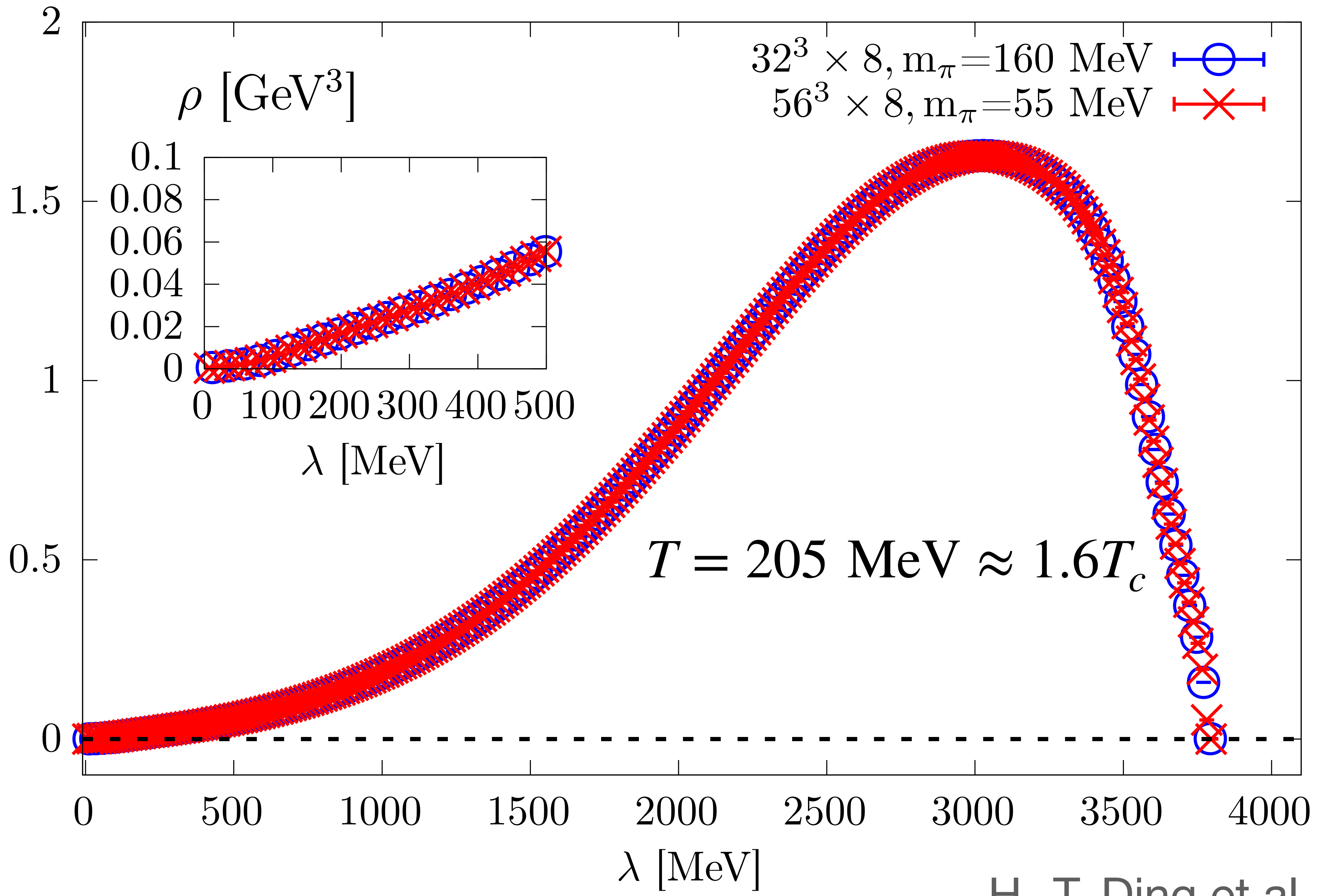
S. Ejiri et al., Phys. Rev. D 80, 094505 (2009)

$$\langle\bar\psi\psi\rangle_l = \int_0^\infty \frac{4m_l \cdot \rho(\lambda, m_l)}{\lambda^2 + m_l^2}\, d\lambda \qquad \chi_{disc} = \int_0^\infty d\lambda \frac{4m_l \cdot \partial\rho/\partial m_l}{\lambda^2 + m_l^2} \qquad \chi_2 \equiv \int_0^\infty d\lambda \frac{4m_l \cdot \partial^2\rho/\partial m_l^2}{\lambda^2 + m_l^2}$$

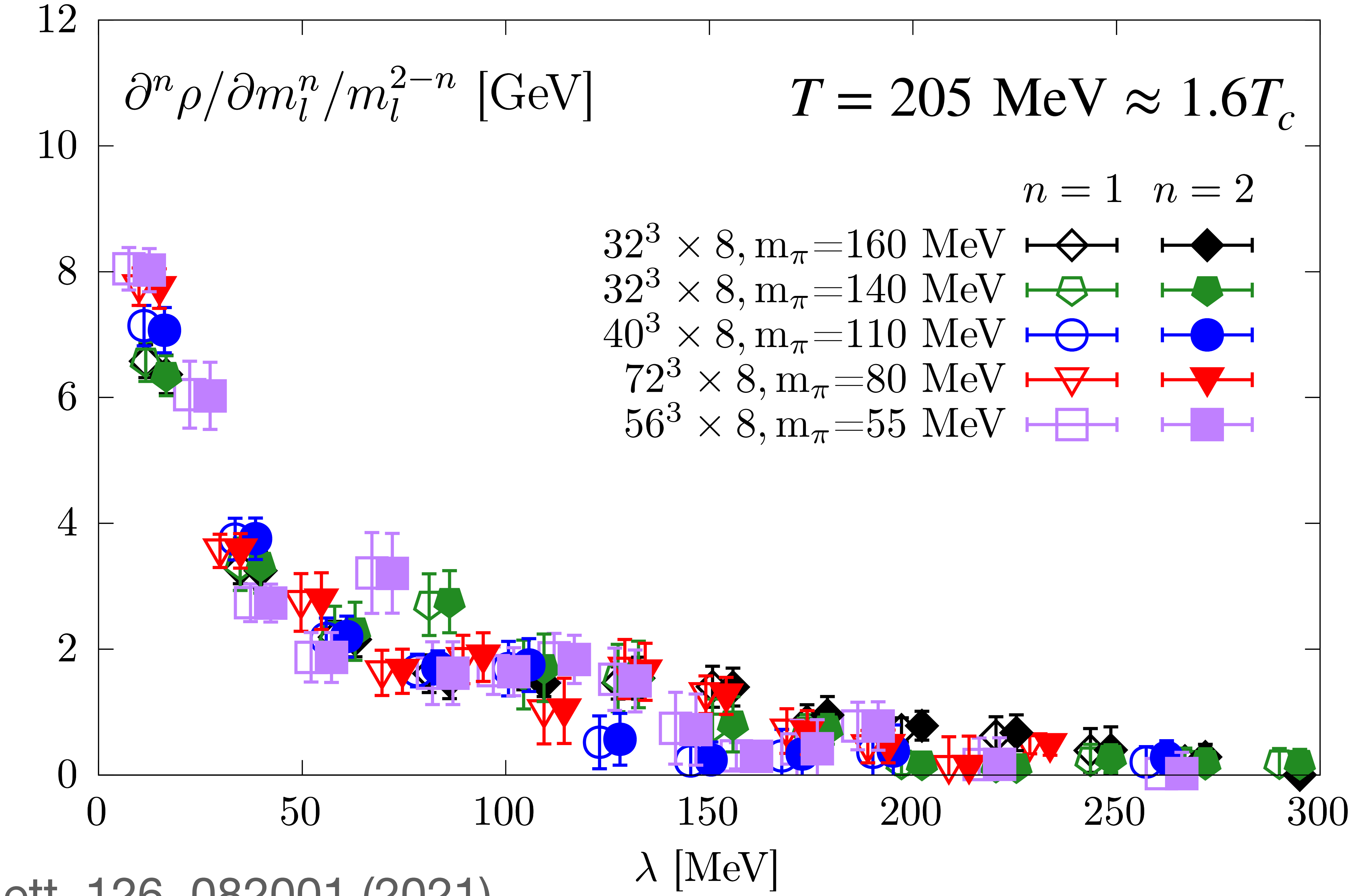
How critical behavior is manifested in ρ, $\partial^n\rho/\partial m_l^n$ around T_c ?

Dirac eigenvalue spectra and their correlations

Chebyshev filtering technique



A novel relation between $\partial^n\rho/\partial m^n$ and C_{n+1}



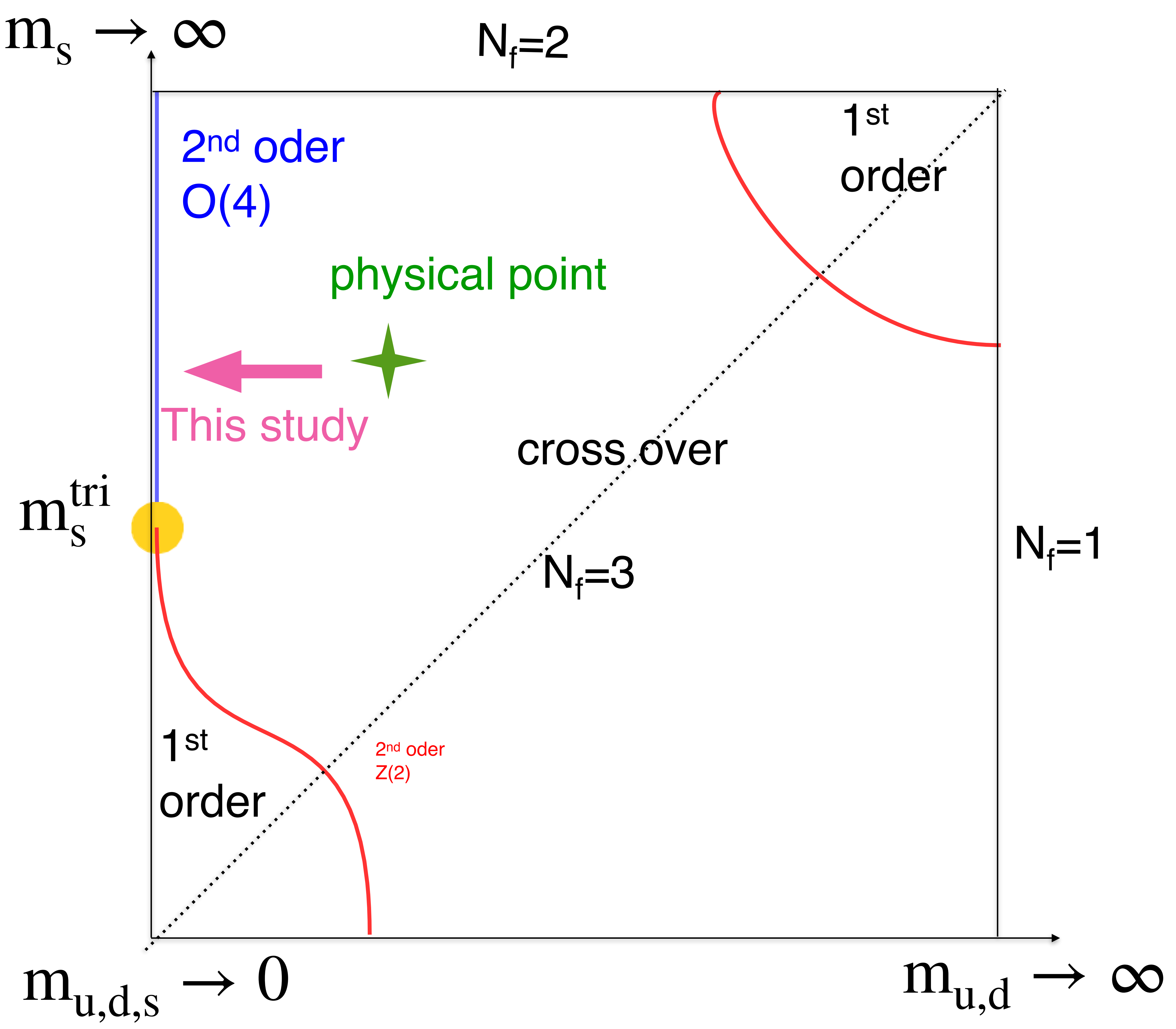
H.-T. Ding et al., Phys. Rev. Lett. 126, 082001 (2021)

At $1.6T_c$, $m_l^{-1}\partial\rho/\partial m_l \approx \partial^2\rho/\partial m_l^2$ & $\partial^3\rho/\partial m_l^3 \approx 0 \implies \rho(\lambda \to 0, m_l) \propto m_l^2\delta(\lambda)$

How about the case at $T \sim T_c$?

Lattice Setup

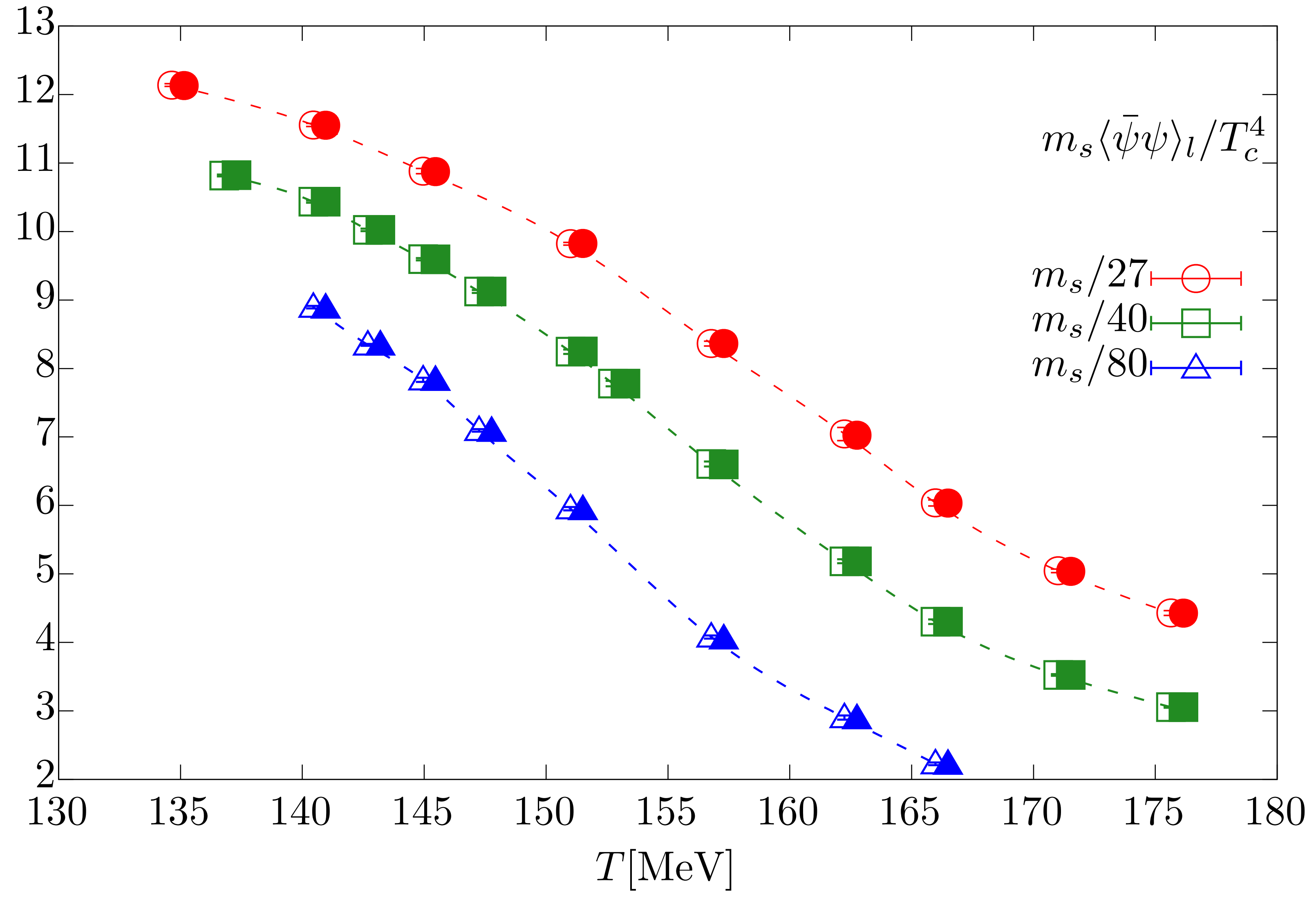
- Actions: Highly improved staggered quarks and tree-level Symanzik gauge action
- Lattices size: $N_\tau = 8, \;\; N_\sigma = 32, \; 40, \; 56$
- Quark mass: $m_s^{\mathrm{phy}}/m_l = 27, \; 40, \; 80$
  $(m_\pi \approx 140, \; 110, \; 80 \;\mathrm{MeV})$
- Temperatures: $T \in (134{,}176)$ MeV
- Measurements carried out on $\mathrm{NSC^3}$ at CCNU



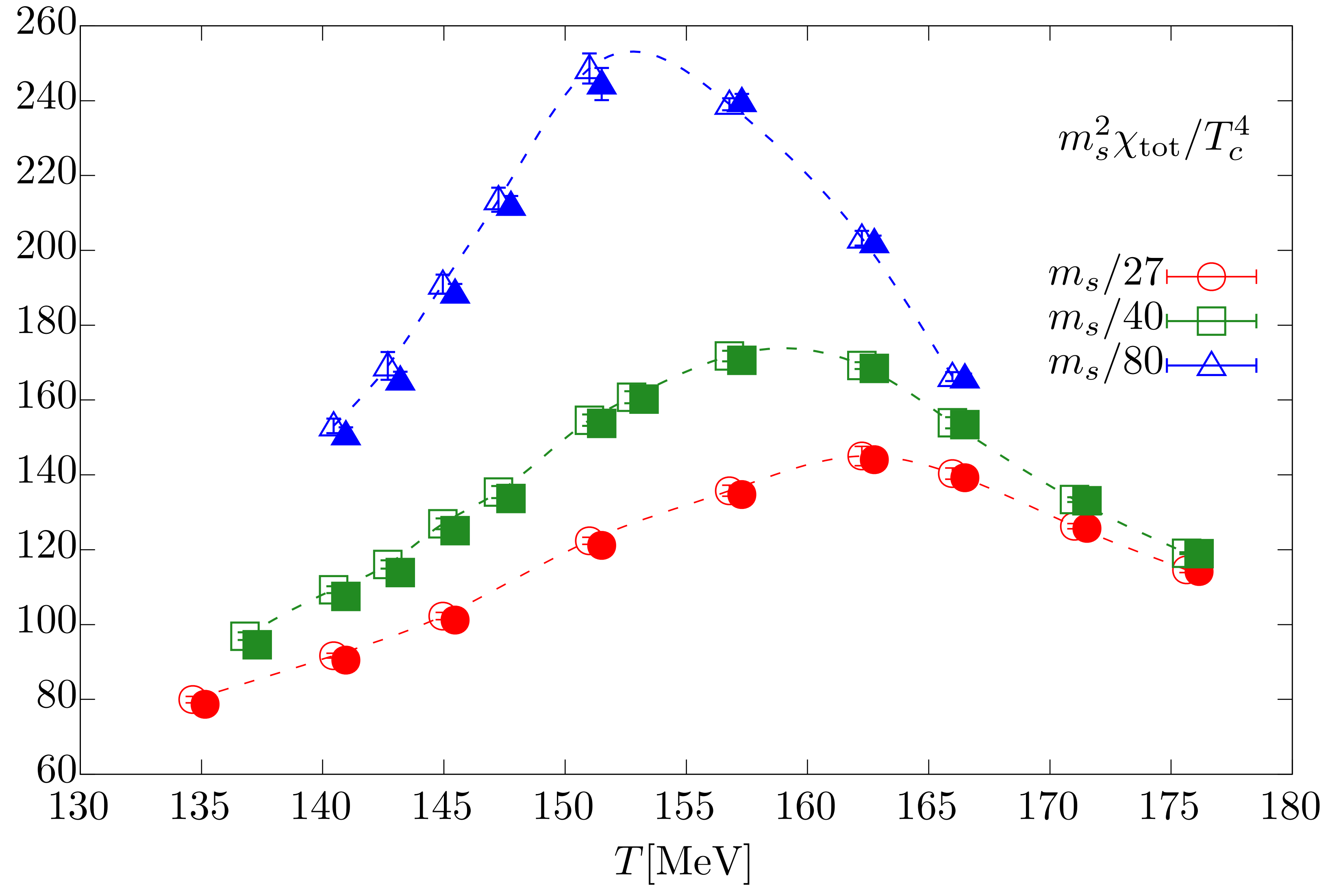
Nuclear Science
Computing Center at CCNU

Reproduction of chiral observables via ρ and $\partial^n\rho/\partial m_l^n$

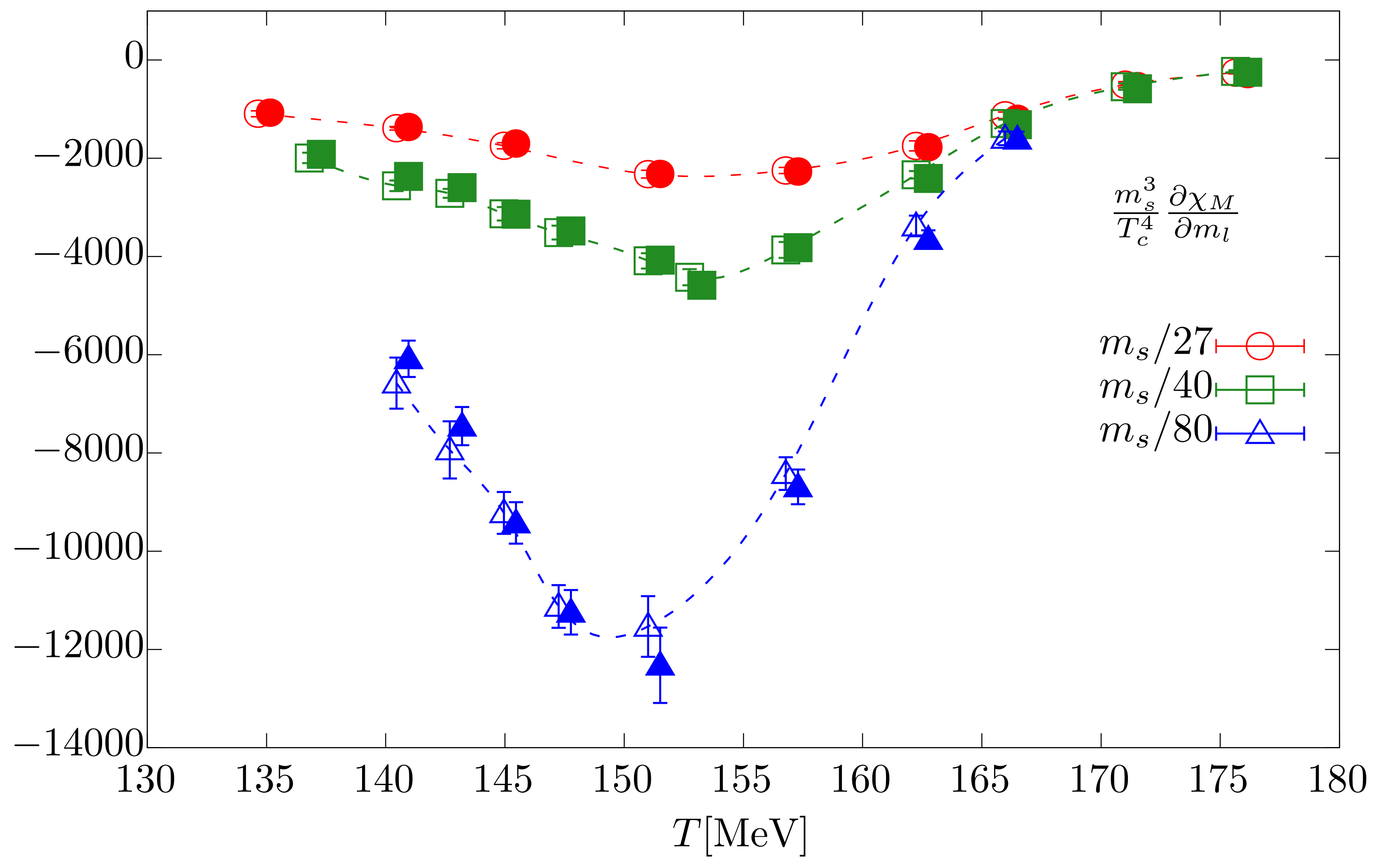
$$\langle\bar\psi\psi\rangle_l = \int_0^\infty \frac{4m_l\cdot\rho(\lambda,m_l)}{\lambda^2+m_l^2}\,\mathrm{d}\lambda$$



$$\chi_{\mathrm{tot}} = \int_0^\infty \mathrm{d}\lambda\frac{4m_l\cdot\partial\rho/\partial m_l}{\lambda^2+m_l^2} + \int_0^\infty \mathrm{d}\lambda\frac{4\left(\lambda^2-m_l^2\right)\rho}{\left(\lambda^2+m_l^2\right)^2}$$



$$\frac{\partial\chi_{\mathrm{tot}}}{\partial m_l} = \int_0^\infty \mathrm{d}\lambda\frac{4m_l\cdot\partial^2\rho/\partial m_l^2}{\lambda^2+m_l^2} + 2\cdot\int_0^\infty \mathrm{d}\lambda\frac{4(\lambda^2-m_l^2)\partial\rho/\partial m_l}{(\lambda^2+m_l^2)^2} + \int_0^\infty \mathrm{d}\lambda\,\rho\left[\frac{-8m_l}{(\lambda^2+m_l^2)^2} - \frac{16m_l(\lambda^2-m_l^2)}{(\lambda^2+m_l^2)^3}\right]$$

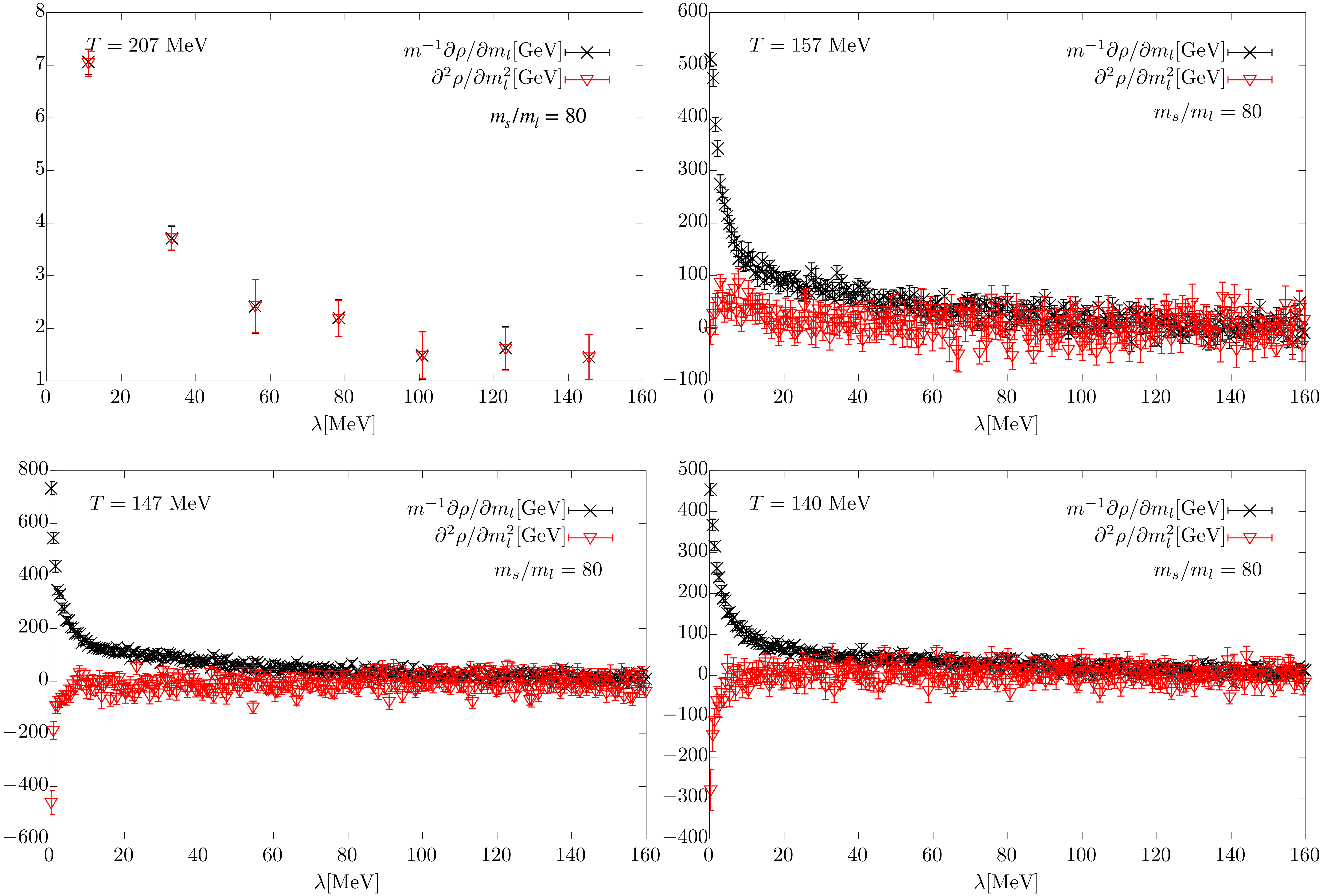


Open symbols: computation via ρ and $\partial^n\rho/\partial m_l^n$

Filled symbols: computation via inversions of the fermion matrix

Quantities related to ρ and $\partial^n\rho/\partial m_l^n$ can successfully reproduce their corresponding directly-computed results

$\partial\rho/\partial m_l$ v.s. $\partial^2\rho/\partial m_l^2$



- High Temperature ($1.6T_c$) :

$$m_l^{-1}\partial\rho/\partial m_l \approx \partial^2\rho/\partial m_l^2$$

$$\rho \propto m_l^2$$

consistent with dilute instanton gas approximation

- Temperature approaches to T_c :

$$m_l^{-1}\partial\rho/\partial m_l \neq \partial^2\rho/\partial m_l^2$$

infrared region of $\partial^2\rho/\partial m_l^2 < 0$

Dilute instanton gas picture is not valid as getting closer to T_c

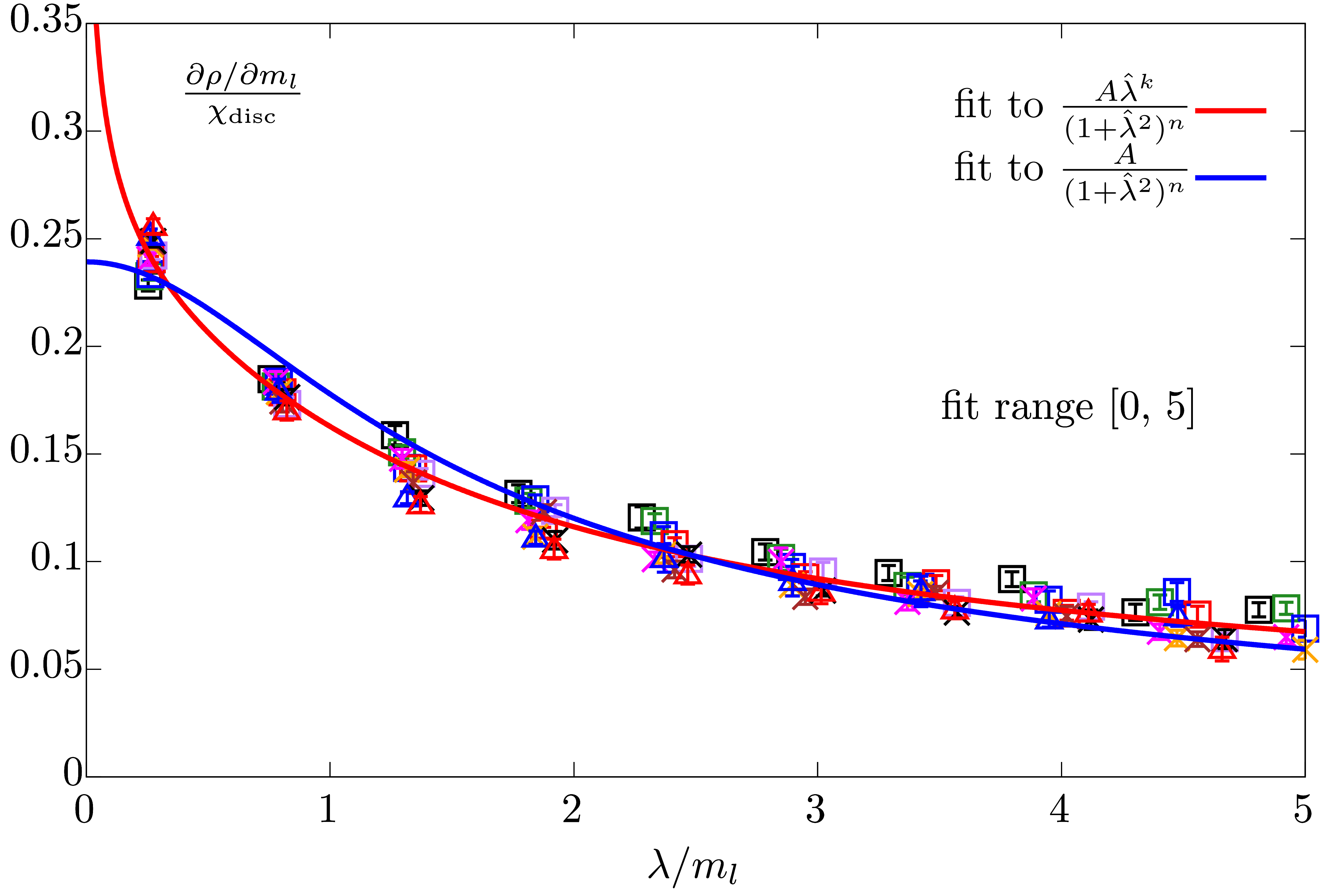
χ_{disc}-Rescaled $\partial\rho/\partial m_l$

$\frac{\partial\rho/\partial m_l(\lambda/m_l)}{\chi_{\text{disc}}}$: quark mass and temperature independent at about $T \in (140, 151)$ MeV

χ_2-Rescaled $\partial^2\rho/\partial m_l^2$

$\frac{\partial^2\rho/\partial m_l^2(\lambda/m_l)}{\chi_2}$ **: quark mass and temperature independent in the same T window**

Fit to $\partial\rho/\partial m_l/\chi_{\rm disc}$



- T & m -independent $\partial\rho/\partial m_l/\chi_{\rm disc}$ → factorize → $\frac{\partial\rho}{\partial m_l} = f(\frac{\lambda}{m_l})\cdot\chi_{\rm disc}$

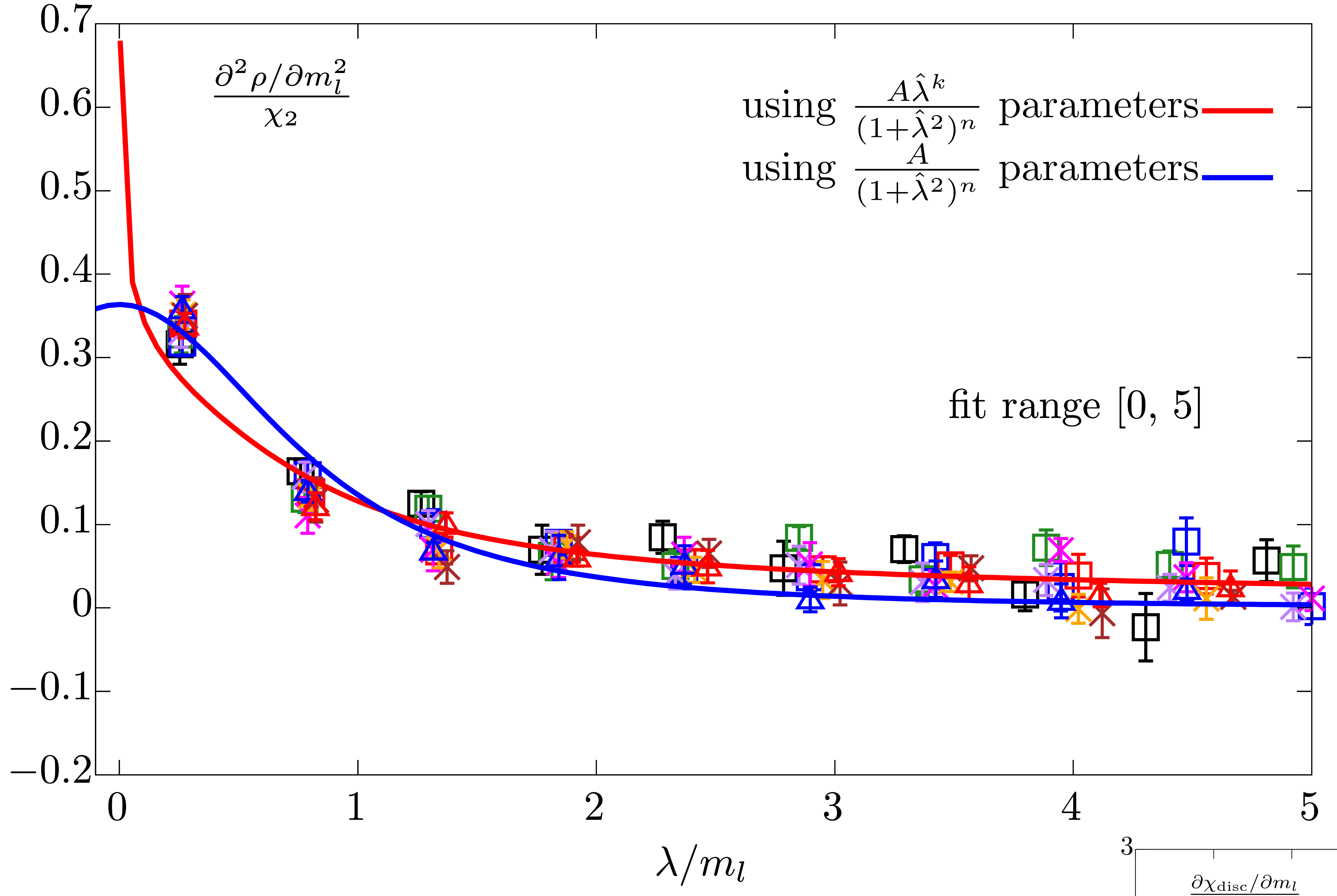
- Assuming $f(\hat{\lambda} \equiv \frac{\lambda}{m_l}) = \frac{A\hat{\lambda}^k}{(1+\hat{\lambda}^2)^n}$ → fit to → $\partial\rho/\partial m_l/\chi_{\rm disc}$

- Banks-Casher-like relations → constrain → $k = 0$

$$\lim_{m_l\to0}\lim_{V\to\infty}\chi_{\rm disc} = \lim_{m_l\to0}\lim_{V\to\infty} 2\pi\cdot\partial\rho/\partial m_l(0,m_l)$$

$$= 2\pi \lim_{m_l\to0}\lim_{V\to\infty}\chi_{\rm disc}\cdot\lim_{m_l\to0}\lim_{V\to\infty}\lim_{\lambda\to0}\frac{Am_l^{2n-k}\lambda^k}{(m_l^2+\lambda^2)^n}$$

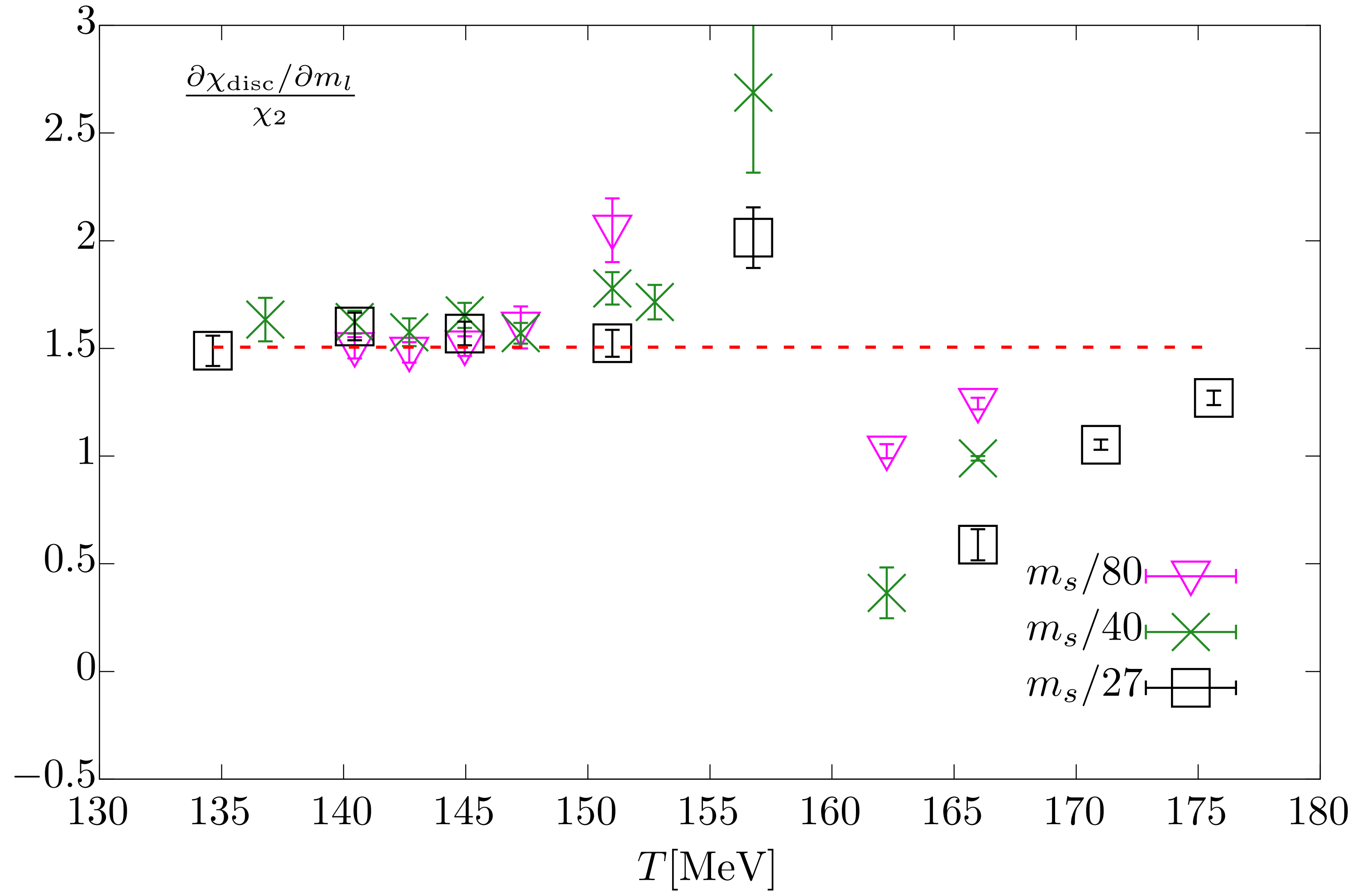
Reconstruction of $\partial^2\rho/\partial m_l^2/\chi_2$

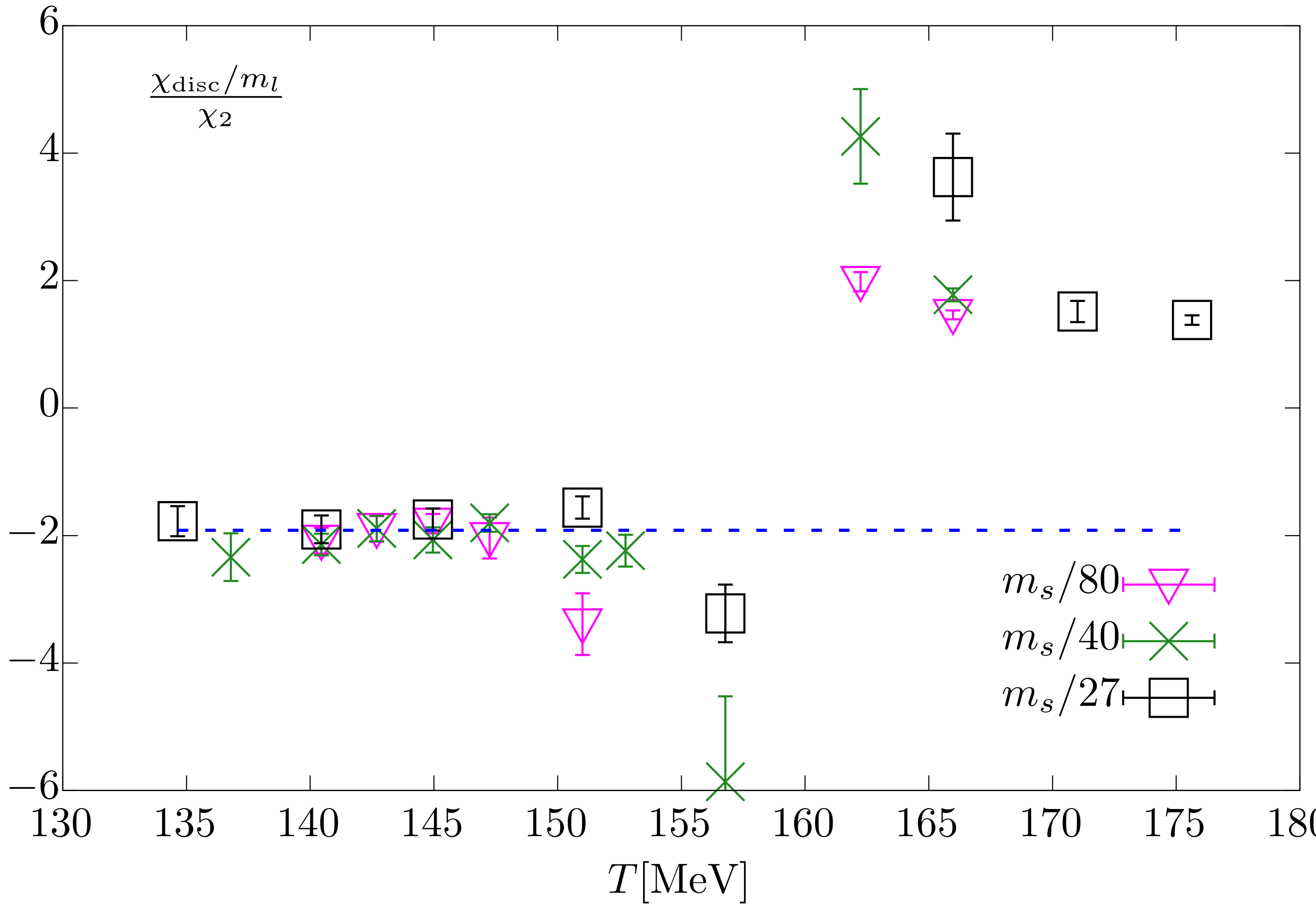


$$\frac{\partial\rho}{\partial m_l} = f(\frac{\lambda}{m_l}) \cdot \chi_{\rm disc} \qquad f(\hat{\lambda} \equiv \frac{\lambda}{m_l}) = \frac{A\hat{\lambda}^k}{(1+\hat{\lambda}^2)^n}$$

mass derivative

$$\frac{\partial^2\rho/\partial m_l^2}{\chi_2} = f(\hat{\lambda})\left[\frac{\partial\chi_{\rm disc}/\partial m_l}{\chi_2} + \frac{\chi_{\rm disc}/m_l}{\chi_2}\frac{[2n\hat{\lambda}^2 - k(1+\hat{\lambda}^2)]}{(1+\hat{\lambda}^2)}\right]$$





Summary

Based on HISQ configurations $(32^3, 40^3, 56^3) \times 8$ lattices with $m_\pi \approx (140,\ 110,\ 80)$ MeV,

- ☑ Chiral observables can be reproduced via ρ, $\partial\rho/\partial m_l$ and $\partial^2\rho/\partial m_l^2$
- ☑ As T approaches to T_c, the m_l^2 behavior in ρ does not exist any more
- ☑ $\partial\rho/\partial m_l/\chi_{\rm disc}$ and $\partial^2\rho/\partial m_l^2/\chi_2$ are temperature and quark mass independent around T_c

Outlook

- ☐ Check the scaling behavior in the Dirac eigenvalue correlations v.s. MEOS

Backup

Calculation of eigenvalue spectrum

- Commonly used method: Lanczos algorithm to calculate the individual low-lying eigenvalues

- Here we utilized the Chebyshev filtering technique combined with a stochastic estimate of the mode number

Mode number : $n_{[s,t]} \approx \frac{1}{n_v} \sum_{k=1}^{n_v} \left[\sum_{j=0}^{p} g_j^p \gamma_j v_k^T T_j(A) v_k \right]$

T_j : Chebyshev polynomial

γ_j & g_j^p : expansion coefficients

n_v : number of random vectors

p : number of polynomial orders

eigenvalue spectrum : $\rho_U(\lambda) = \frac{1}{4} \frac{n_{[\lambda-\delta/2,\lambda+\delta/2]}}{2\delta\lambda}$

$1/4$: Staggered Fermion Discretization Scheme

$1/2$: positive and negative eigenvalue pairs

$\delta\lambda$:bin-size

H.-T. Ding et al., Phys. Rev. Lett. 126, 082001 (2021)

Yu Zhang, Lattice 19', arXiv: 2001.05217

Cossu et al., arXiv: 1601.00744

$\partial^n \rho / \partial m_l^n$ and Dirac eigenvalue correlations

Eigenvalue spectrum for (2+1)-flavor QCD:

$$\rho\left(\lambda, m_l\right) = \frac{T}{VZ[U]} \int D[U] e^{-S_G[U]} \det\left[\not{D}[U] + m_s\right] \times \left(\det\left[\not{D}[U] + m_l\right]\right)^2 \rho_U(\lambda)$$

Eigenvalue spectrum for a given configuration:

$$\rho_U(\lambda) = \sum_j \delta\left(\lambda - \lambda_j\right)$$

Partition function:

$$Z[U] = \int D[U] e^{-S_G[U]} \det\left[\not{D}[U] + m_s\right] \times \left(\det\left[\not{D}[U] + m_l\right]\right)^2$$

Trace of fermion matrix:

$$\det\left[\not{D}[U] + m_l\right] = \prod_j \left(+\mathrm{i}\lambda_j + m_l\right)\left(-\mathrm{i}\lambda_j + m_l\right) = \exp\left(\int_0^\infty \mathrm{d}\lambda \rho_U(\lambda) \mathrm{In}\left[\lambda^2 + m_l^2\right]\right)$$

Mass derivative of $\rho(\lambda, m_l)$:

$$\frac{V}{T}\frac{\partial \rho}{m_l} = \int_0^\infty \mathrm{d}\lambda_2 \frac{4 m_l C_2}{\lambda_2^2 + m_l^2} \qquad C_2\left(\lambda, \lambda_2\right) = \left\langle \rho_U(\lambda) \rho_U\left(\lambda_2\right)\right\rangle - \left\langle \rho_U(\lambda)\right\rangle \left\langle \rho_U\left(\lambda_2\right)\right\rangle$$

H.-T. Ding et al., Phys. Rev. Lett. 126, 082001 (2021)

MEOS and Scaling functions

Magnetic Equation of State (regular term up to H^3):

$$z = t/h^{1/\delta\beta}, \text{ where } \; t = \frac{T - T_c}{t_0 T_c}, \; h = \frac{H}{h_0}$$

$$M(t,h) = h^{1/\delta} f_G(z) + [a_0 + a_1 \frac{T - T_c}{T_c} + a_2 (\frac{T - T_c}{T_c})^2] H + [c_0 + c_1 \frac{T - T_c}{T_c} + c_2 (\frac{T - T_c}{T_c})^2] H^3$$

$$\chi_M(t,h) = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) + [a_0 + a_1 \frac{T - T_c}{T_c} + a_2 (\frac{T - T_c}{T_c})^2] + 3[c_0 + c_1 \frac{T - T_c}{T_c} + c_2 (\frac{T - T_c}{T_c})^2] H^2$$

$$\frac{\partial \chi_M}{\partial H} = \frac{1}{h_0^2} h^{1/\delta - 2} f_{\rm pp}(z) + 6[c_0 + c_1 \frac{T - T_c}{T_c} + c_2 (\frac{T - T_c}{T_c})^2] H$$

$$f_{\rm pp}(z) \equiv \frac{1}{\delta}[(1/\delta - 1) f_G(z) + (\frac{z}{\beta} + \frac{z}{\beta^2 \delta} - 2 \cdot \frac{z}{\beta\delta}) f'_G(z) + \frac{z^2}{\beta^2 \delta} f''_G(z)]$$

Temperature dependence of $\partial^2\rho/\partial m_l^2$