Quantum kinetic theory and collisional contributions to shear induced polarization

林树

中山大学

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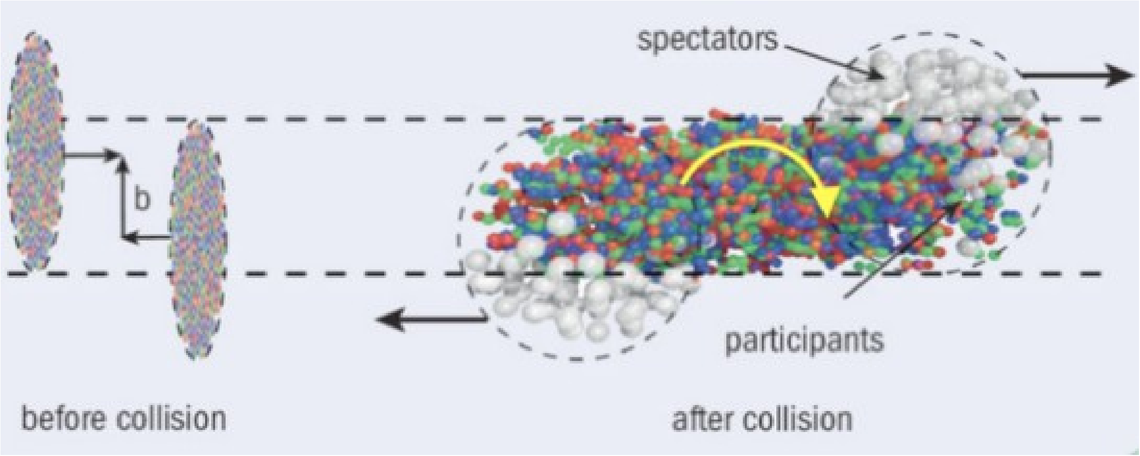
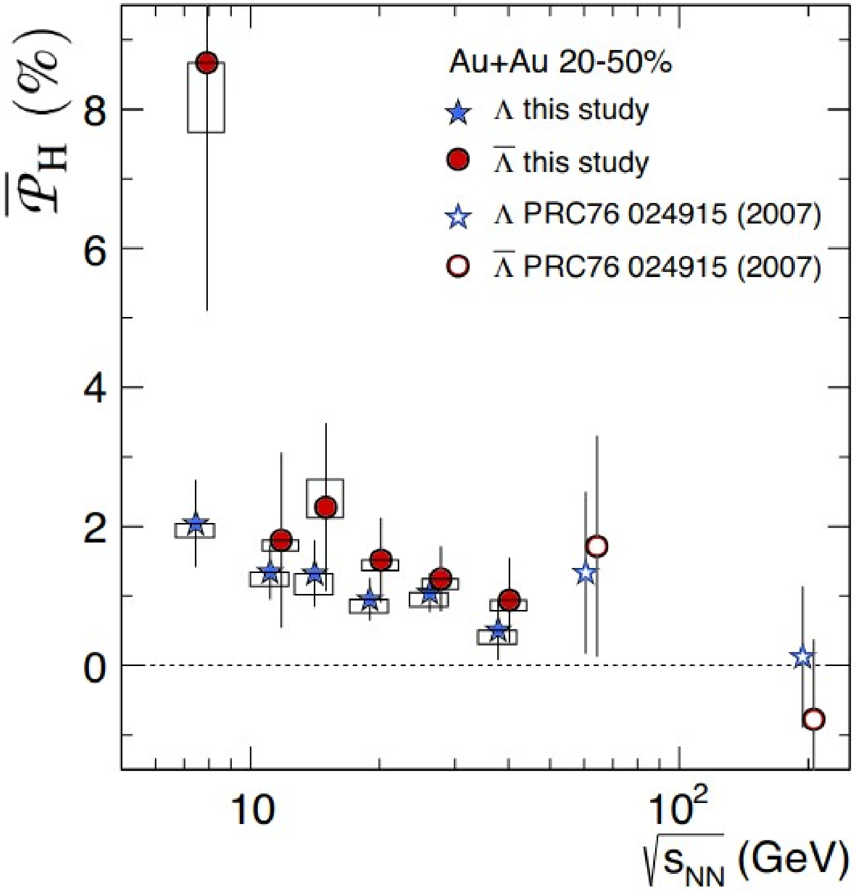
Λ Global Polarization at RHIC



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

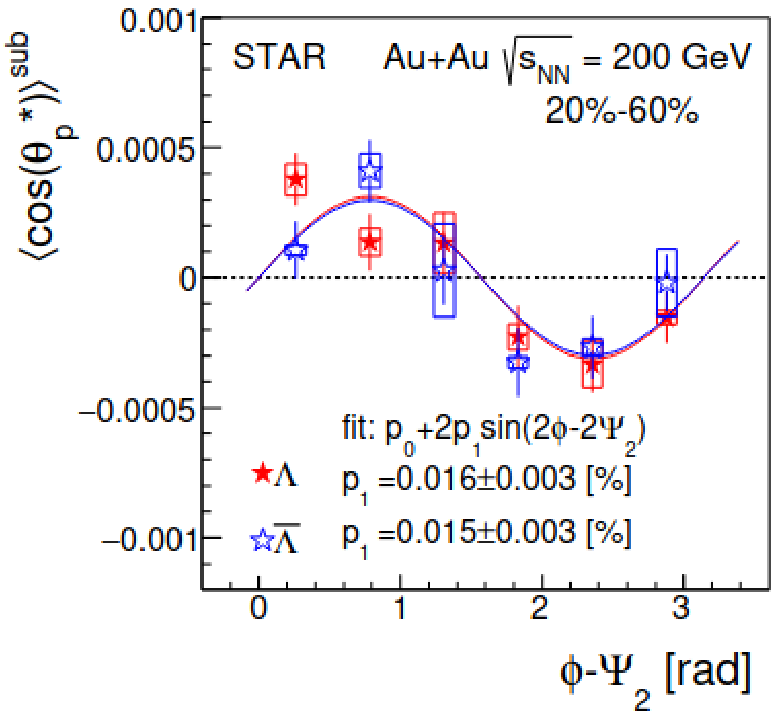
Liang, Wang, PRL 2005

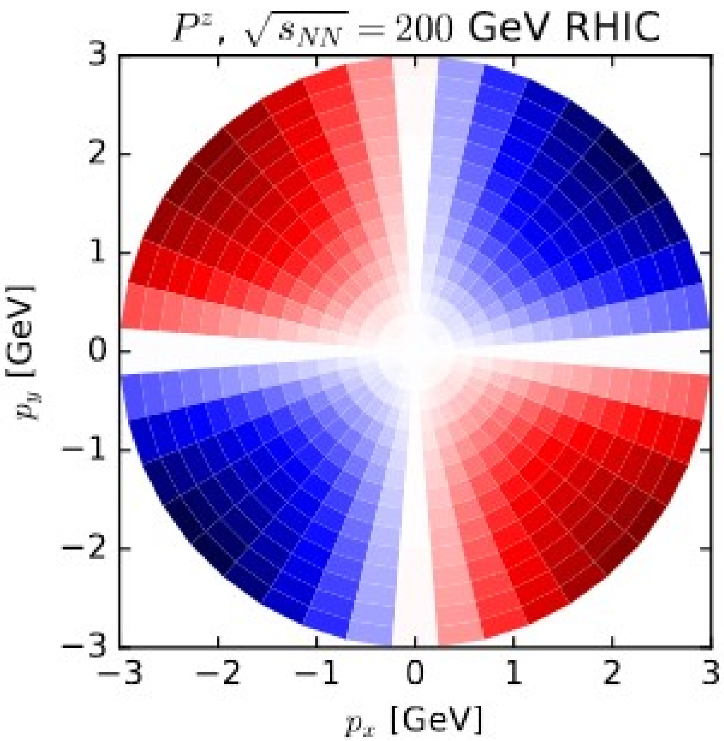


STAR collaboration, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Λ Local polarization: sign puzzle



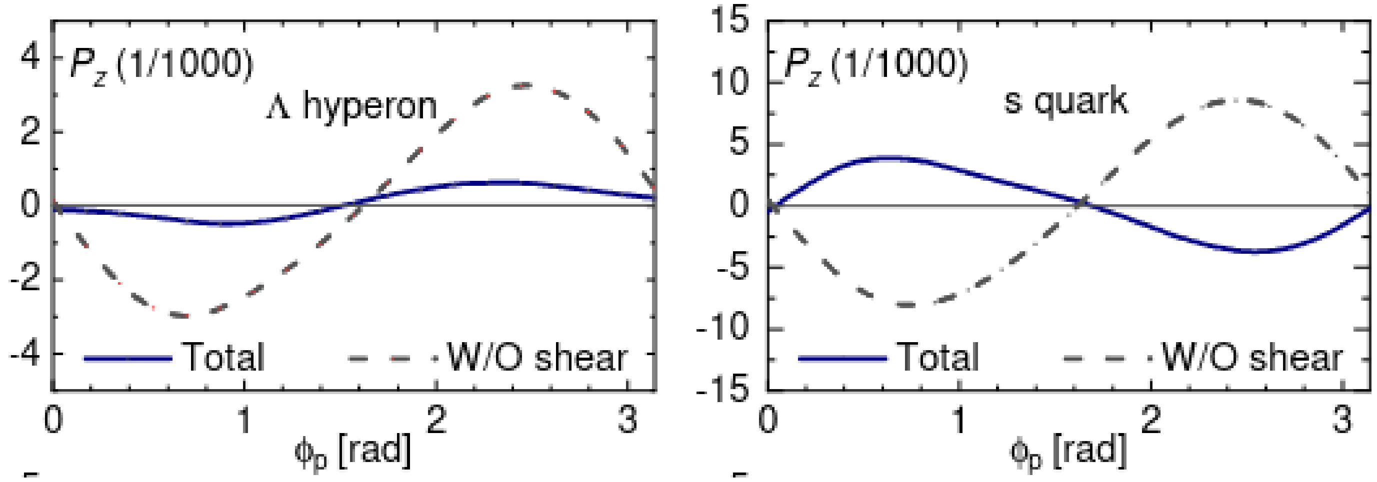


STAR collaboration, PRL 2019

Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

$$e^{-\beta(H_0-\mathbf{S}\cdot\omega)}$$

Shear induced polarization



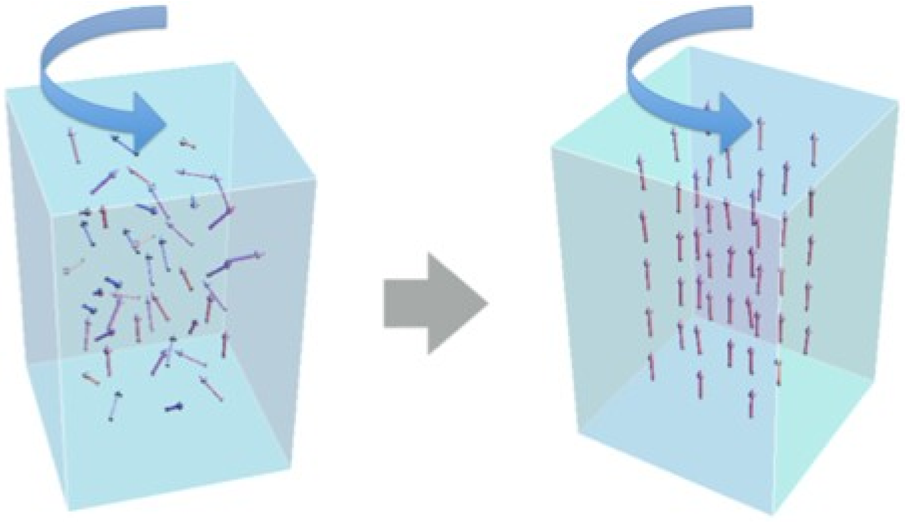
Liu, Yin JHEP 2021
Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PLB 2021, PRL 2021
Yi, Pu, Yang, PRC 2021

vorticity $\frac{1}{2}(\partial_x u_y - \partial_y u_x)$

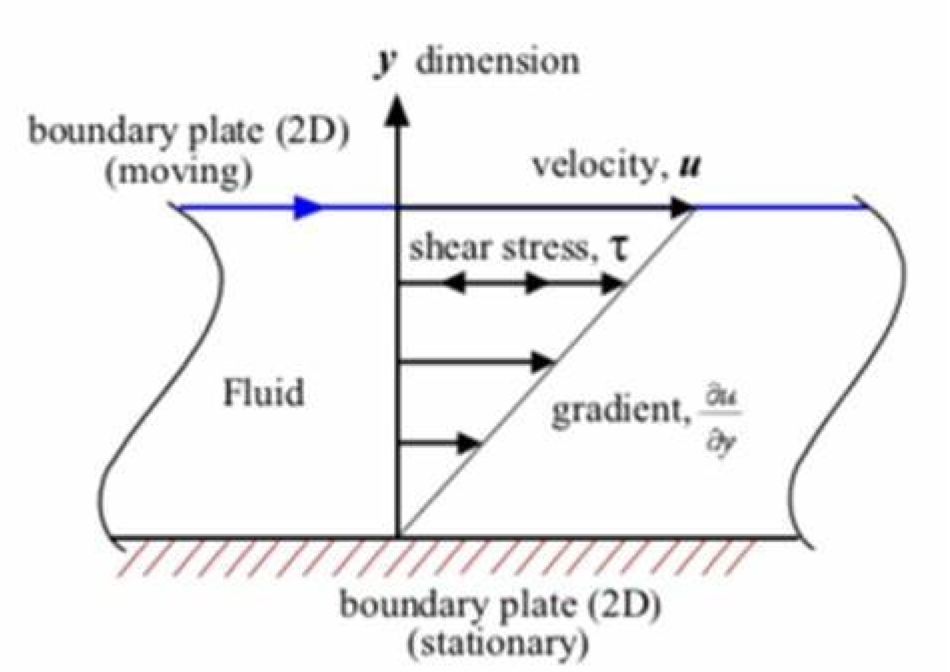
shear $\frac{1}{2}(\partial_x u_y + \partial_y u_x)$

This talk: more contribution from the shear

A fundamental difference between vorticity & shear



spin-vorticity coupling only (Barnett effect)



spin-shear coupling + particle redistribution

Equilbrium: collision vanishes
by detailed balance

Nonequilibrium:
Collision nonvanishing

Particle redistribution from spin-averaged kinetic theory

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$: distributions of quarks and transverse gluons

$C_s^{2\leftrightarrow 2}[f]$: elastic collisions

$C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$: inelastic collisions

Arnold, Moore and Yaffe, early 00s

shear induced particle redistribution → shear viscosity

$$\delta f \sim \partial f^{\text{leq}}(p \cdot u)\,\tau \qquad \tau \sim \frac{1}{g^4 T}$$

Spin information averaged out

Quantum kinetic theory (QKT)

- QKT in collisionless limit

  sufficient for vorticity induced polarization

Hattori, Hidaka, Yang, PRD 2019
Weickgenannt, Sheng, Wang, Rishcke, PRD 2019
Gao, Liang, PRD 2019
Liu, Mameda, Huang, CPC 2020
Guo, CPC 2020

- Collisionful QKT

  needed for shear induced polarization

Yang, Hattori, Hidaka JHEP 2020
Hattori, Hidaka, Yamamoto, Yang JHEP 2021
Weickgnnant et al, PRL 2021
Sheng et al, PRD 2021
Wang, Guo, Zhuang, EPJC 2021
Shi, Gale, Jeon, PRC 2021
SL, PRD 2022
Fang, Pu, Yang, PRD 2022
Wang, 2205.09334

QKT for QED: spin-averaged part

$$\frac{i}{2}\partial\!\!\!/ S^< + \frac{P\!\!\!/ - m}{\hbar} S^< = \frac{i}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right) - \frac{\hbar}{4}\left(\{\Sigma^>, S^<\}_{\rm PB} - \{\Sigma^<, S^>\}_{\rm PB}\right)$$

equation for photon not shown

$$S^<(X,P) = S^{<(0)}(X,P) + \cdots$$

$$S^{<(0)}(X,P) = -2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)(P\!\!\!/ + m)f(X,P)$$

SL, PRD 2022

$$\left(\partial_t + \hat{\boldsymbol{p}}\cdot\boldsymbol{\nabla}_{\boldsymbol{x}}\right) f_s(\boldsymbol{x},\boldsymbol{p},t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

$$f = f_{(0)} + f_{(1)} + \cdots \qquad f_{(1)} \sim \hbar\partial_X f_{(0)}\tau$$

particle redistribution

Boltzmann equation not as classical as we thought

QKT for QED: spin polarized part

$$S^{<}(X,P) = S^{<(0)}(X,P) + S^{<(1)}(X,P) + \cdots$$

$$S^{<(1)}(X,P) = \gamma^5 \gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu, \gamma_\nu]}{4} \mathcal{S}^{\mu\nu}$$

$$\mathcal{A}^\mu = -2\pi\hbar \left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \mathcal{D}_\nu f}{2(P \cdot u + m)} \right] \delta(P^2 - m^2)$$ ~ spin polarization

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^{>} - \Sigma_\nu^{<} \frac{1-f}{f}$$

to be compared with $f_{(1)} \sim \hbar \partial_X f_{(0)} \tau$

- hbar not independent from gradient in counting
- simialr contribution in spin averaged/polarized parts

SL, PRD 2022

Composition of spin polarization

- Spin polarization ~ $\mathcal{A}^\mu = -2\pi\hbar\left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \mathcal{D}_\nu f}{2(P\cdot u + m)}\right]\delta(P^2 - m^2)$

dynamical non-dynamical

a^μ, dynamical spin vector

f_A parity violating distribution

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f}{f}$$

green: derivative contribution (in literature) same for QED/QCD

blue: self-energy contribution collision dependent

Solving for particle redistribution: QED example

$$(\partial_t + \hat{p}\cdot\nabla_x) f_p = -\frac{1}{2}\int_{p',k',k} (2\pi)^4\delta^4(P+K-P'-K')\frac{1}{16p_0k_0p'_0k'_0}\times$$
$$\Big[|\mathcal{M}|^2_{\mathrm{Coul},f} \left(f_p f_k(1-f_{p'})(1-f_{k'}) - f_{p'}f_{k'}(1-f_p)(1-f_k)\right)$$
$$+ |\mathcal{M}|^2_{\mathrm{Comp},f} \left(f_p \tilde{f}_k(1+\tilde{f}_{p'})(1-f_{k'}) - \tilde{f}_{p'}f_{k'}(1-f_p)(1+\tilde{f}_k)\right)$$
$$+ |\mathcal{M}|^2_{\mathrm{anni},f} \left(f_p f_k(1+\tilde{f}_{p'})(1+\tilde{f}_{k'}) - \tilde{f}_{p'}\tilde{f}_{k'}(1-f_p)(1-f_k)\right)\Big]$$

fermion

$$(\partial_t + \hat{p}\cdot\nabla_x) \tilde{f}_p = -\frac{1}{2}\int_{p',k',k} (2\pi)^4\delta^4(P+K-P'-K')\frac{1}{16p_0k_0p'_0k'_0}\times$$
$$\Big[|\mathcal{M}|^2_{\mathrm{Comp},\gamma} \left(\tilde{f}_p f_k(1-f_{p'})(1+\tilde{f}_{k'}) - f_{p'}\tilde{f}_{k'}(1+\tilde{f}_p)(1-f_k)\right)$$
$$+ 2N_f|\mathcal{M}|^2_{\mathrm{anni},\gamma} \left(\tilde{f}_p \tilde{f}_k(1-\tilde{f}_{p'})(1-\tilde{f}_{k'}) - f_{p'}f_{k'}(1+\tilde{f}_p)(1+\tilde{f}_k)\right)\Big]$$

photon

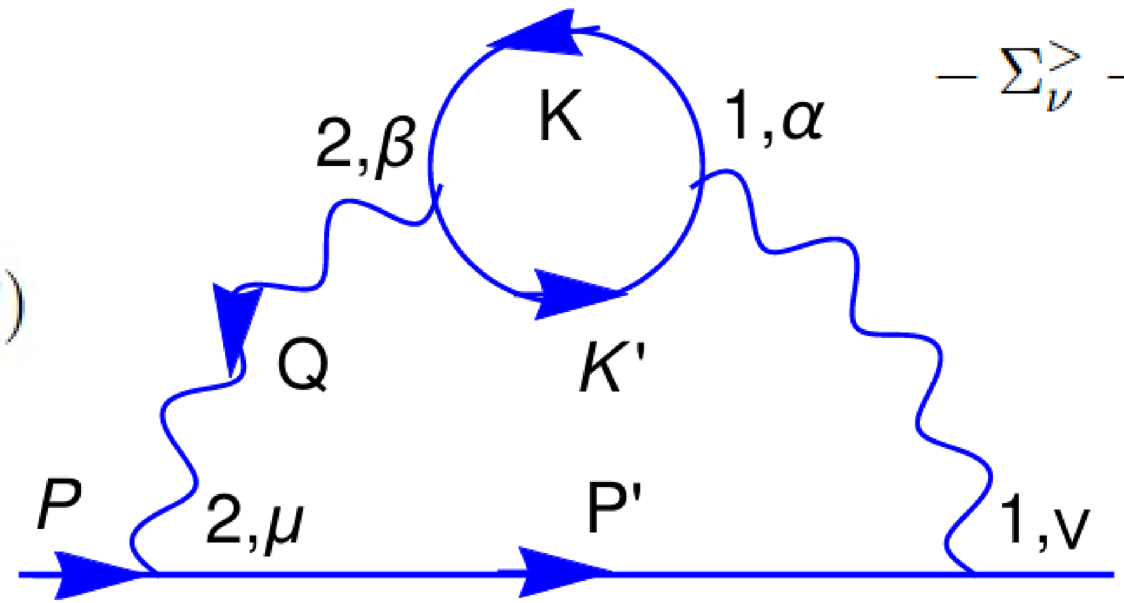
$$f_{(1)} \sim \frac{\partial_X f_{(0)}}{e^4 \ln e^{-1}}$$

Probe fermion in QED plasma with shear

shear induced medium
fermion redistribution

SL, Wang, 2206.12573

$\Sigma^{>}(P)$



$$-\Sigma^{>}_{\nu}-\Sigma^{<}_{\nu}\frac{1-f}{f}\sim e^4\ln e^{-1} f_{(1)}\sim\frac{f_{(1)}}{\tau}$$

$$f_{(1)}\sim\frac{\partial_X f_{(0)}}{e^4\ln e^{-1}}\sim\partial_X f_{(0)}\tau$$

$$-\Sigma^{>}_{\nu}-\Sigma^{<}_{\nu}\frac{1-f}{f}\sim\partial_X f_{(0)}$$

coupling constant cancels!

probe massive fermion
m >> eT, Coulomb dominates, $\ln e^{-1}$ enhanced

cancellation generic

Self-energy contribution to spin polarization

$$\mathcal{A}^i \simeq -\frac{1}{p_0+m}(I_2+I_3)\frac{\epsilon^{iml}p_n p_l S_{mn}}{p^5}\delta(P^2-m^2)C_f,$$

$$S_{ij} = \frac{1}{2}(\partial_i\beta_j+\partial_j\beta_i)-\frac{1}{3}\delta_{ij}\partial\cdot\beta$$ shear tensor

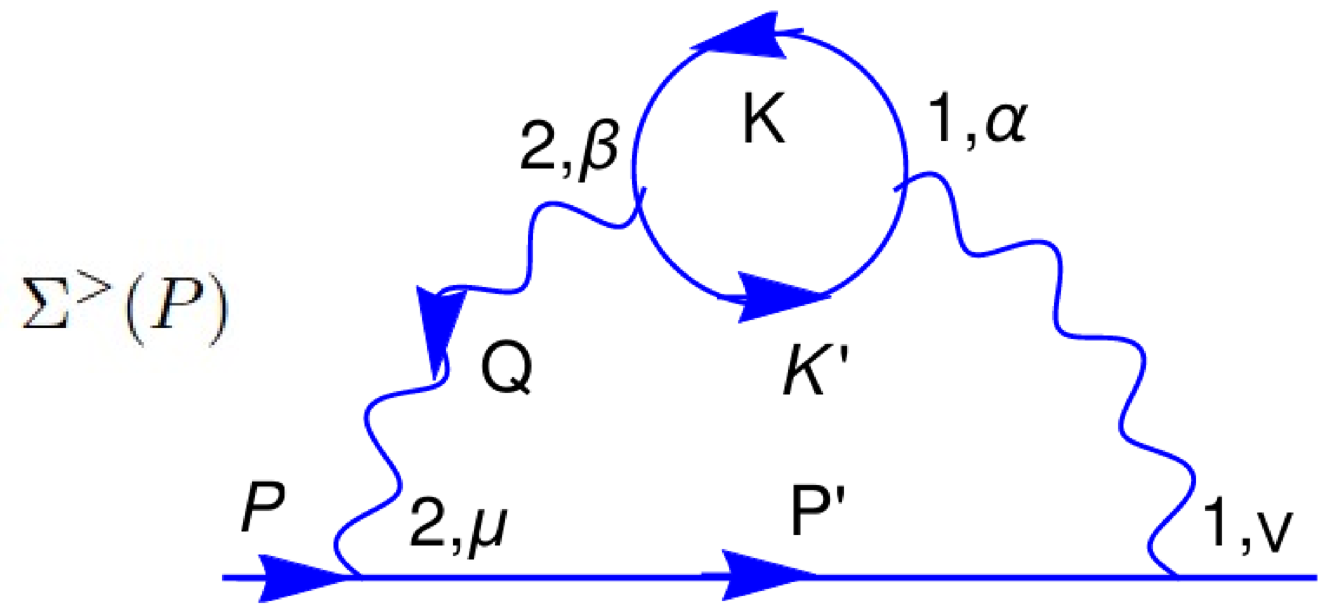
$$C_f = \frac{3N_f(1+2N_f)}{4\pi^2 N_f^2}$$ particle content dependent constant

I_2, I_3 functions of p, T

Not suppressed by coupling constant

SL, Wang, 2206.12573

Self-energy contribution gauge dependent!



Explicit results in Feynman and Coulomb gauges show difference

Self-energy gauge dependent, but spin polarization should not be!

SL, Wang, 2206.12573

Gauge invariant propagator in SK contour

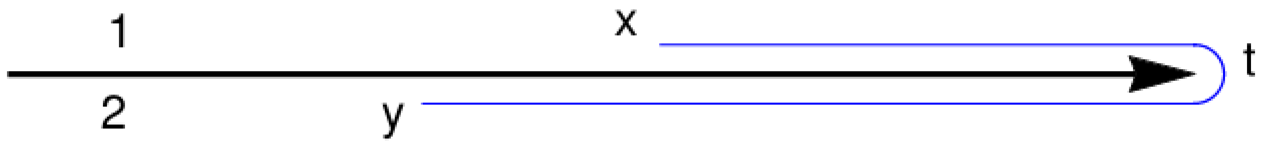
gauge transformation of propagator

$$S^<(x,y) \to e^{-ie\alpha_2(y)} S^<(x,y) e^{ie\alpha_1(x)}$$

gauge invariant propagator generalized to Schwinger-Keldysh contour

$$\bar{S}^<(x,y) = \psi_1(x)\bar{\psi}_2(y)U_2(y,\infty)U_1(\infty,x)$$

$$U_i(y,x) = \exp\left(-ie\int_y^x dw \cdot A_i(w)\right)$$



SL, Wang, 2206.12573

Path of gauge links

straight path connecting x&y

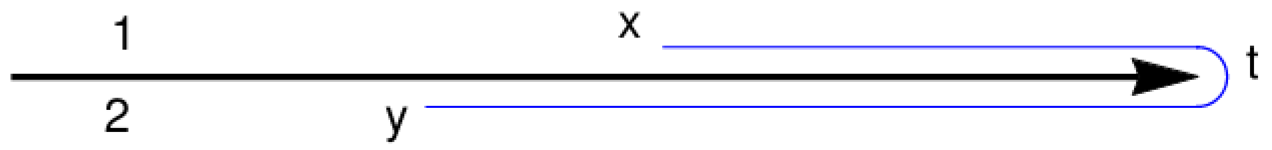
$$U(y,x) = \exp(-ie \int_y^x dw \cdot A(w))$$

Vasak, Gyulassy, Elze, Ann.Phys 1987

A(w) can be general quantum fields, but hard to incorporate collisions

Instead propose extending straight path to SK contour

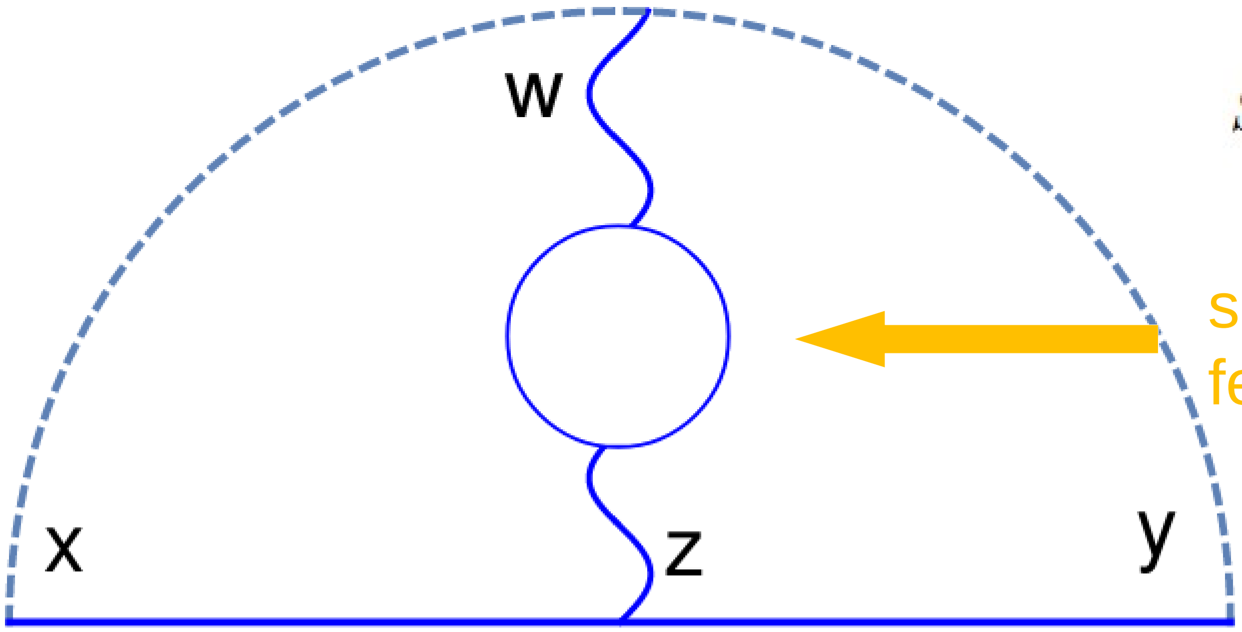
$$U_i(y,x) = \exp\left(-ie \int_y^x dw \cdot A_i(w)\right)$$



reduce to simple straight line for background A(w)

SL, Wang, 2206.12573

Gauge link contribution to spin polarization



$\bar{S}^{<}(x,y)$

shear induced medium fermion redistribution

gauge fields fluctuation A(z) from interaction, A(w) from gauge link

similar mechanism leads to cancellation of coupling constant

Gauge link contribution to spin polarization

$$\mathcal{A}^i = \frac{1}{(2\pi)} C_f \frac{9\zeta(3)}{2\beta^4} (J_1 + J_2 + J_3 + J_4) \frac{\epsilon^{iml} p_n p_l S_{mn}}{2p^5} f_p(1 - f_p)\delta(P^2 - m^2)$$

$$S_{ij} = \frac{1}{2}(\partial_i \beta_j + \partial_j \beta_i) - \frac{1}{3}\delta_{ij}\partial \cdot \beta$$

shear tensor

$$C_f = \frac{3N_f(1+2N_f)}{4\pi^2 N_f^2}$$

particle content
dependent constant

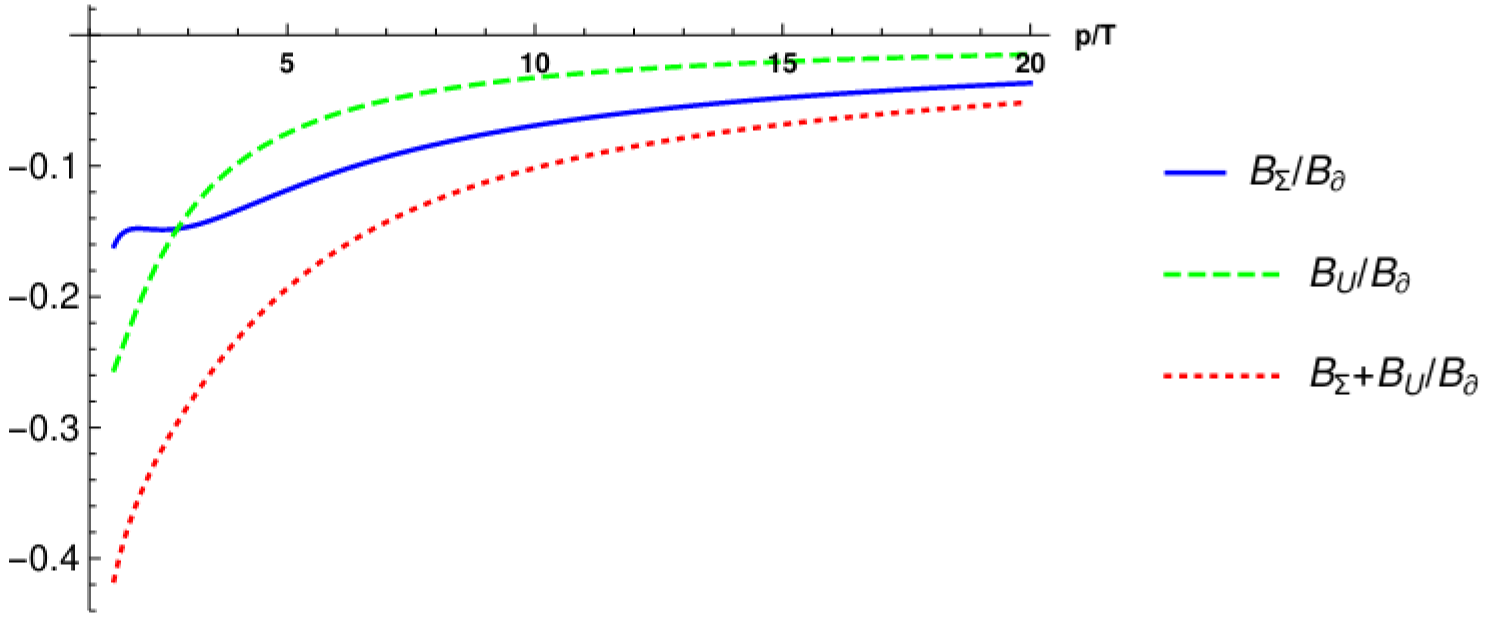
$J_1,\ J_2,\ J_3,\ J_4$ functions of p, T

Not suppressed by coupling constant

SL, Wang, 2206.12573

Suppression of spin polarization

$$\mathcal{A}^i_M = B_M \epsilon^{iml} p_n p_l S_{mn} \qquad M = \partial, \Sigma, U$$



Self-energy and gauge link contributions lead to modest suppression of derivative contribution to spin polarization

Determine dynamical contribution: massless case

$$\mathcal{A}^\mu = -2\pi\hbar\left[a^\mu f_A + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f}{2(P\cdot u+m)}\right]\delta(P^2-m^2)$$

fix dynamical contribution using frame independence

Chen, Son, Stephanov, PRL 2015

$m \to 0$ $\quad a^\mu \to p^\mu$

vorticity tensor $\Omega_{\mu\nu}$

$$f_A = \frac{\epsilon^{\mu\nu\rho\sigma}\Omega_{\mu\nu}p_\rho n_\sigma}{4p\cdot n}$$

Gao, Pang, Wang, PRD 2019

shear tensor $S_{\mu\lambda}$

$$f_A \propto -\frac{\epsilon^{\mu\nu\rho\lambda}u_\nu p_\rho n_\sigma p^\lambda S_{\mu\lambda}}{(p\cdot n)(p\cdot u)}$$

collision dependent

work in progress

n: arbitrary frame vector $\quad f_A(u) = 0$

Summary

- Derived QKT for QED allows study of spin polarization with collisional effect
- Self-energy contribution+Gauge link contribution parametrically the same, lead to suppression of derivative contribution

Outlook

- Dynamical contribution to spin polarization
- Gauge invariance of spin polarization
- Generalization to QKT for QCD

Thank you!