# GWs from FOPTs: Recent progress on bubble expansion


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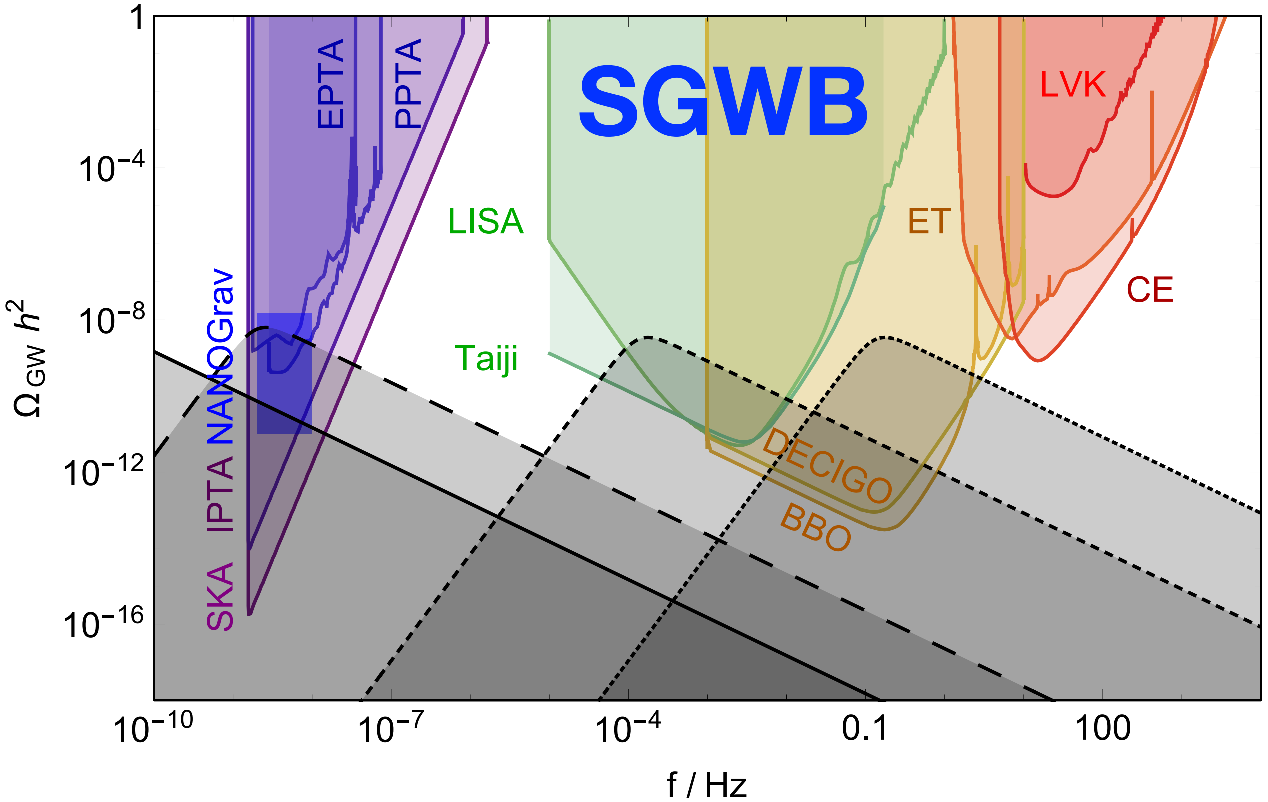
**Based on**

**2011.11451 Rong-Gen Cai, SJW, Effective picture of bubble expansion JCAP03 (2021)096**

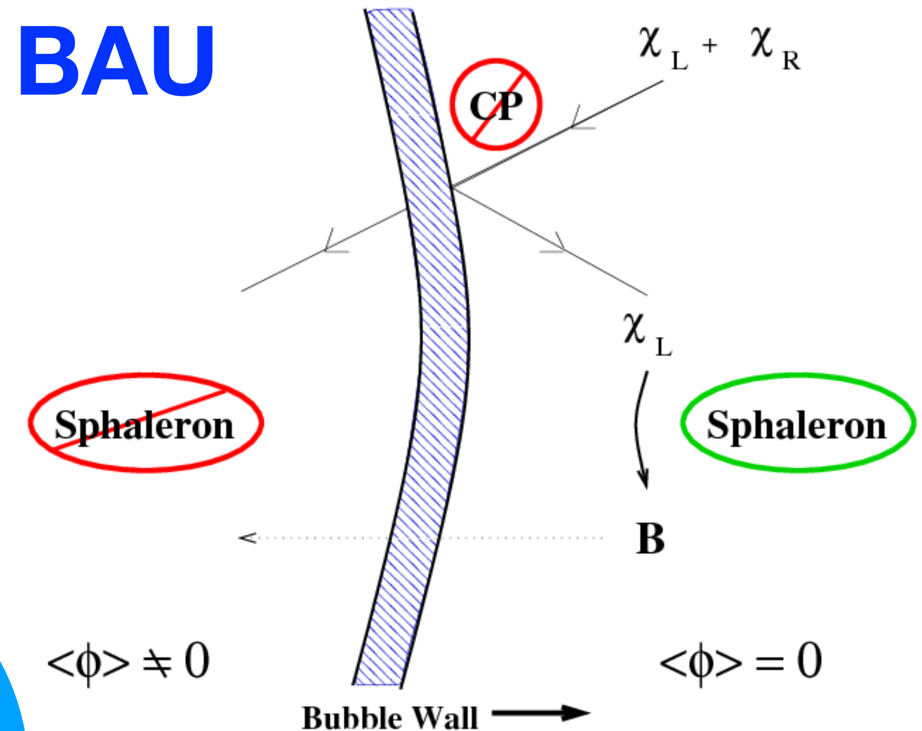
**2205.02492 SJW, Zi-Yan Yuwen Hydrodynamic backreaction force of cosmological bubble expansion**

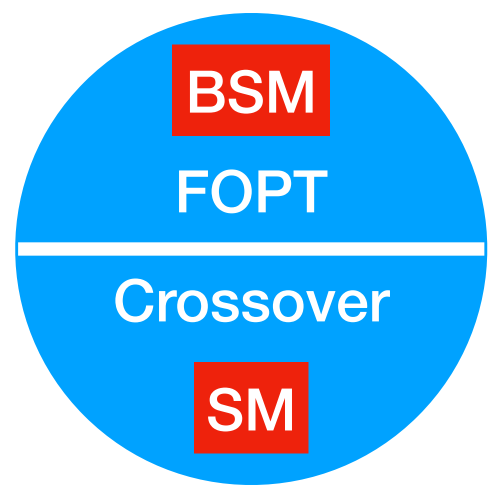
**2206.01148 SJW, Zi-Yan Yuwen The energy budget of cosmological first-order phase transitions beyond the bag equation of state**

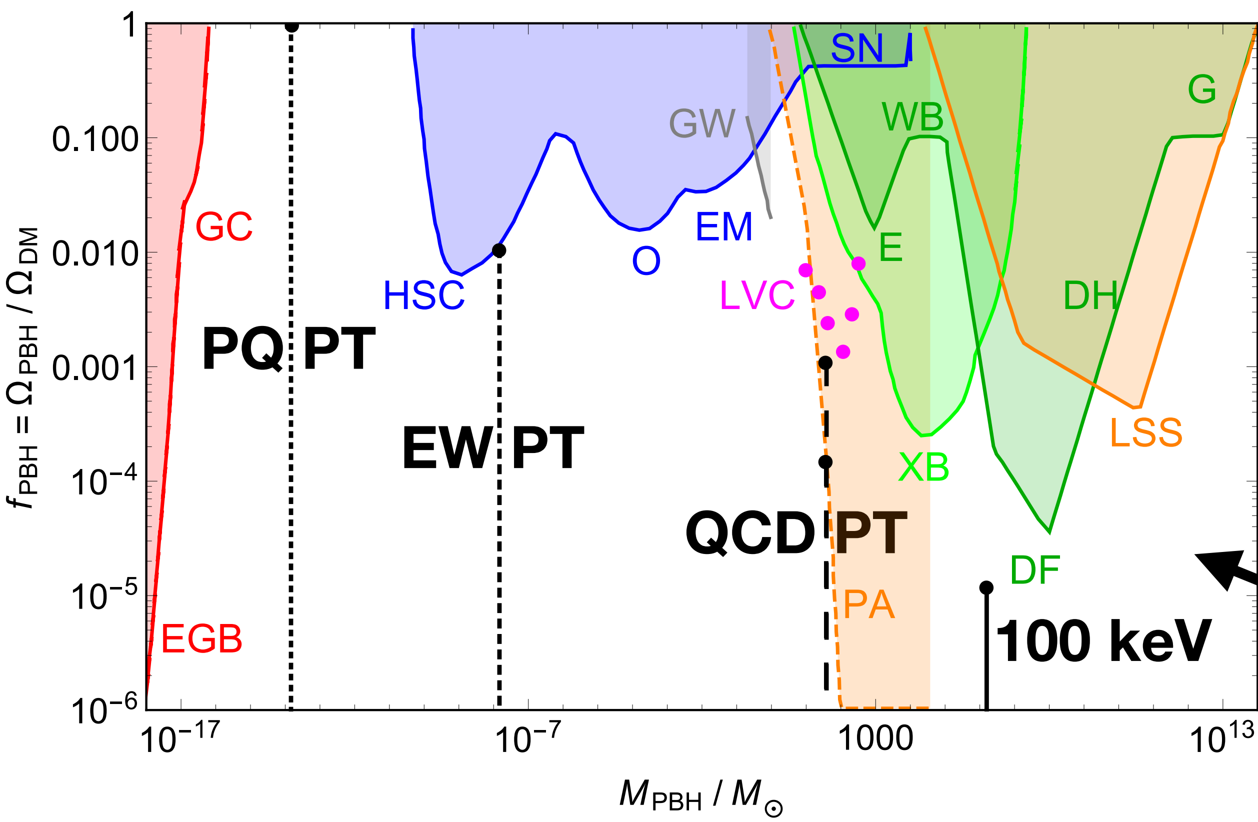
# FOPT Probe for BSM New Physics



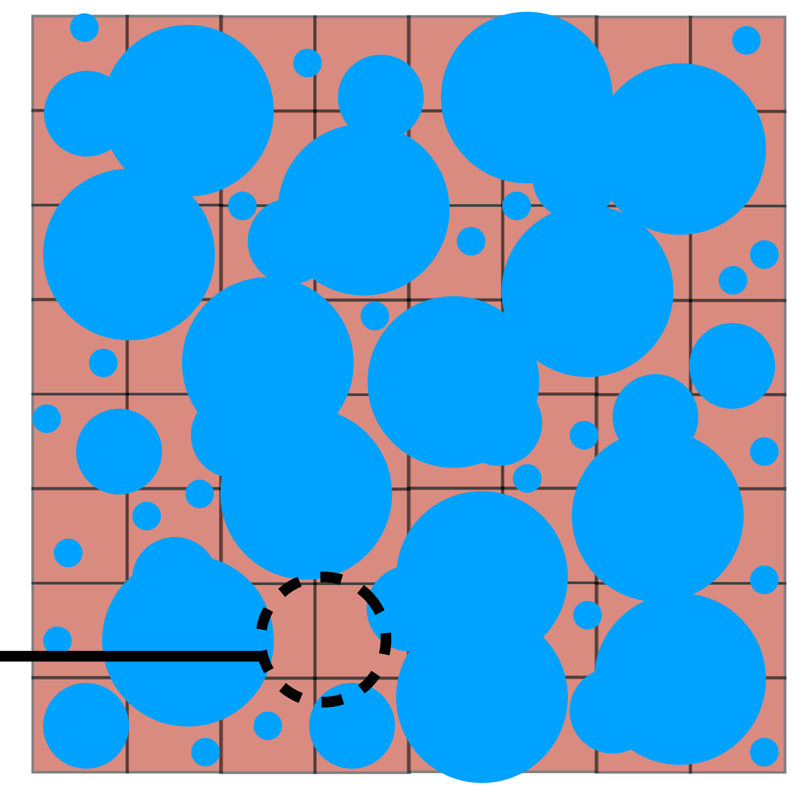

BAU



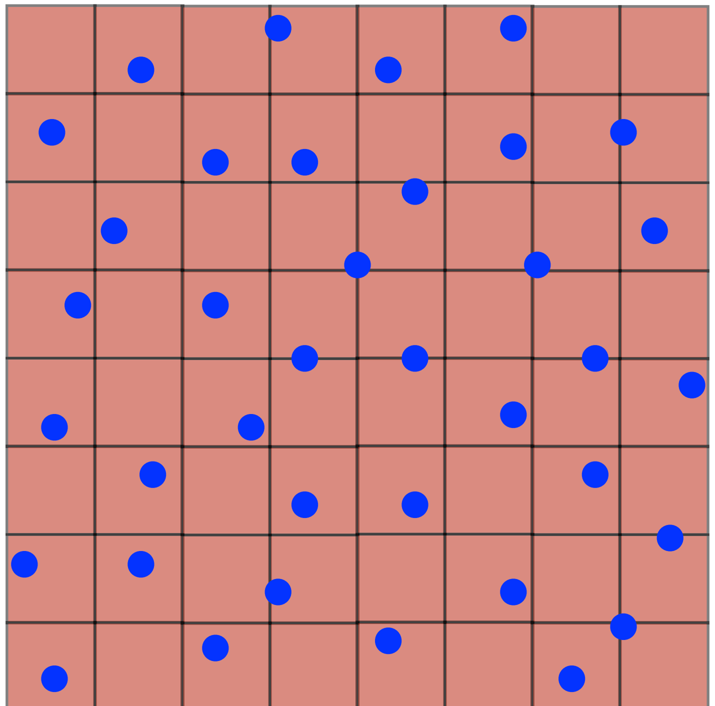




PBH



**2106.05637 Primordial black hole production during first-order phase transitions (PRD Letter)**

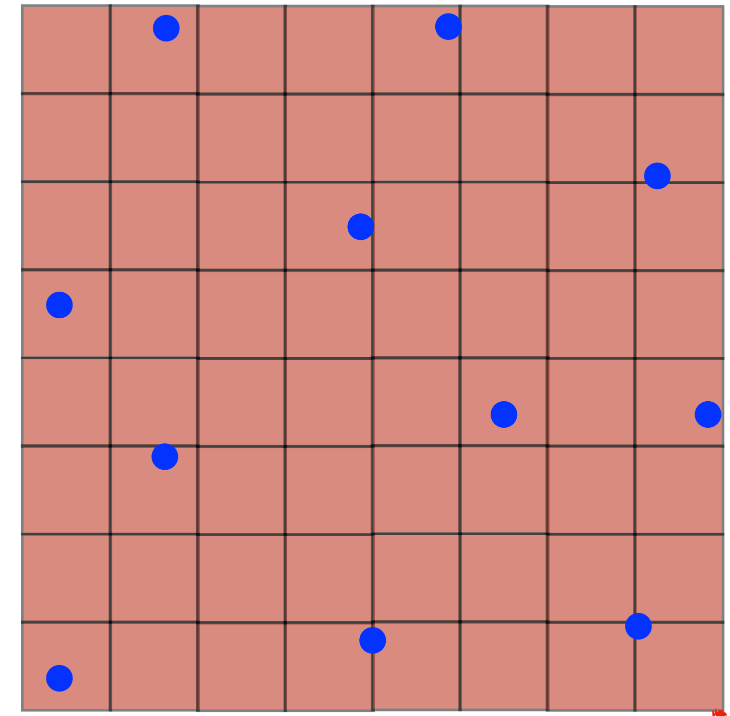
# GWs from FOPT



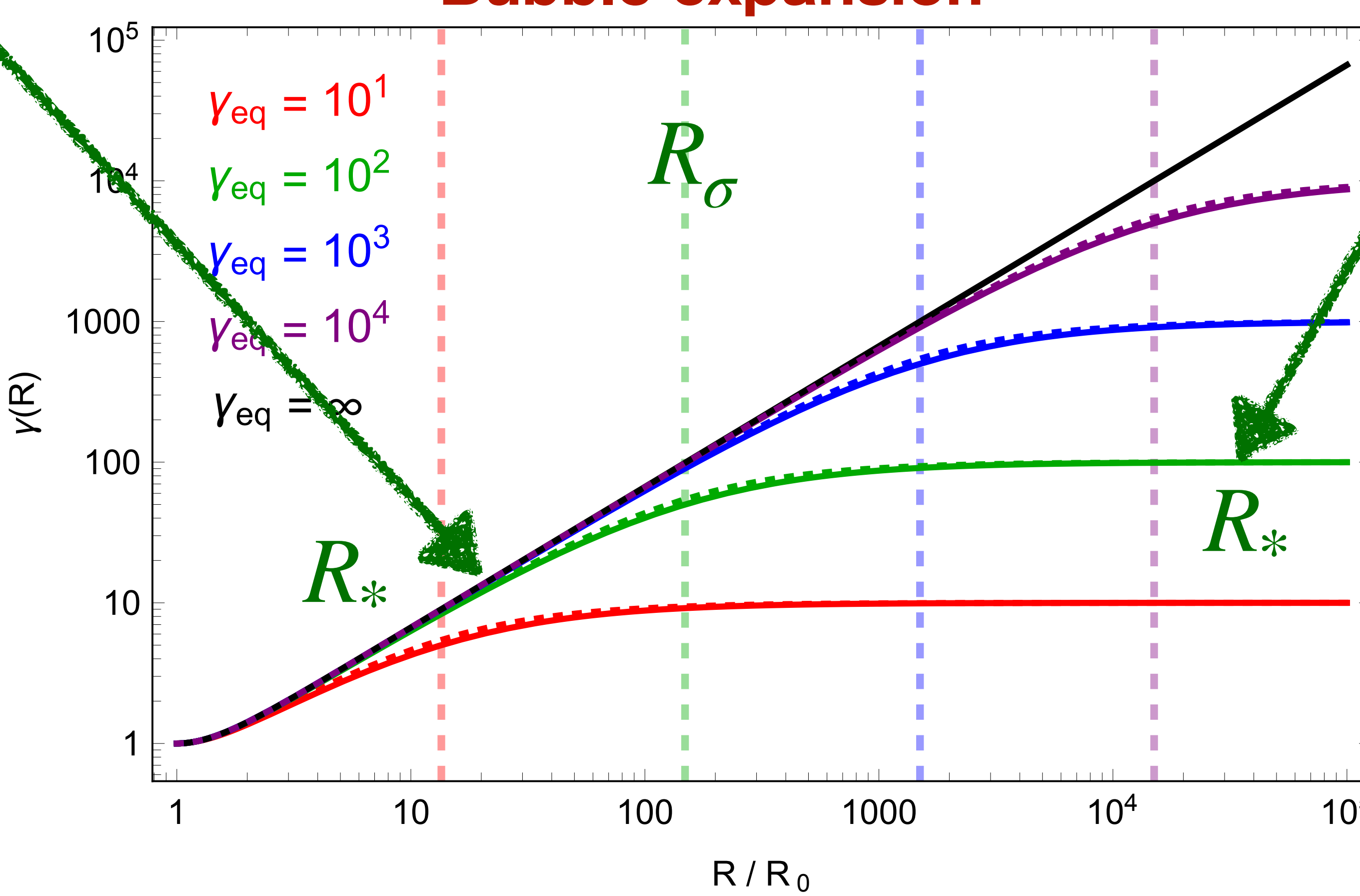
**Bubble nucleation**

$\leftarrow$ **large** $\frac{\beta}{H}$ **small** $R_*$

**large** $R_*$ **small** $\frac{\beta}{H}$ $\rightarrow$



**Bubble expansion**




**+ Small Friction**

$R_* \lesssim R_\sigma$

**GWs dominated By wall collisions**

$$\kappa_\phi = \frac{K_{\rm wall}}{\Delta V_{\rm eff}}$$

**+ Large Friction**

$R_\sigma << R_*$

**GWs dominated By sound waves**

$$\kappa_v = \frac{K_{\rm fluid}}{\Delta V_{\rm eff}}$$

2011.11451 Rong-Gen Cai, SJW,
Effective picture of bubble expansion JCAP03 (2021)096

# Effective picture of bubble expansion

Bodeker & Moore 17

Hoche et al 2007.10343

Gouttenoire et al 2112.07686

$p_{\rm dr} = \Delta V_{\rm eff}$ **vs** $\Delta p_{\rm fr} = \Delta p_{\rm LO} + h(\gamma)\Delta p_{NLO}$

**$\gamma$-independent friction force**

Bodeker & Moore 09

**$\gamma$-dependent friction force**

$$\sigma\gamma^3\ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\rm dr} - \Delta p_{\rm fr}$$

**Kinetic energy of bubble wall**

**Potential energy of bubble**

**Work done by friction force on bubble wall**

**Conservation Law** $$E = 4\pi\sigma R^2\gamma - \frac{4}{3}\pi R^3(\Delta p_{\rm dr} - \Delta p_{\rm fr})$$

$$\left(\sigma + \frac{R}{3}\frac{\mathrm{d}\Delta p_{\rm fr}}{\mathrm{d}\gamma}\right)\gamma^3\ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\rm dr} - \Delta p_{\rm fr}$$

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**Effective picture of bubble expansion**
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$$h(\gamma_{\rm eq}) \equiv \frac{\Delta p_{\rm dr} - \Delta p_{\rm LO}}{\Delta p_{NLO}}$$

$$\frac{h(\gamma) - h(1)}{h(\gamma_{\rm eq}) - h(1)} + \frac{3\gamma}{2R} = 1 + \frac{1}{2R^3}$$

**General solution**

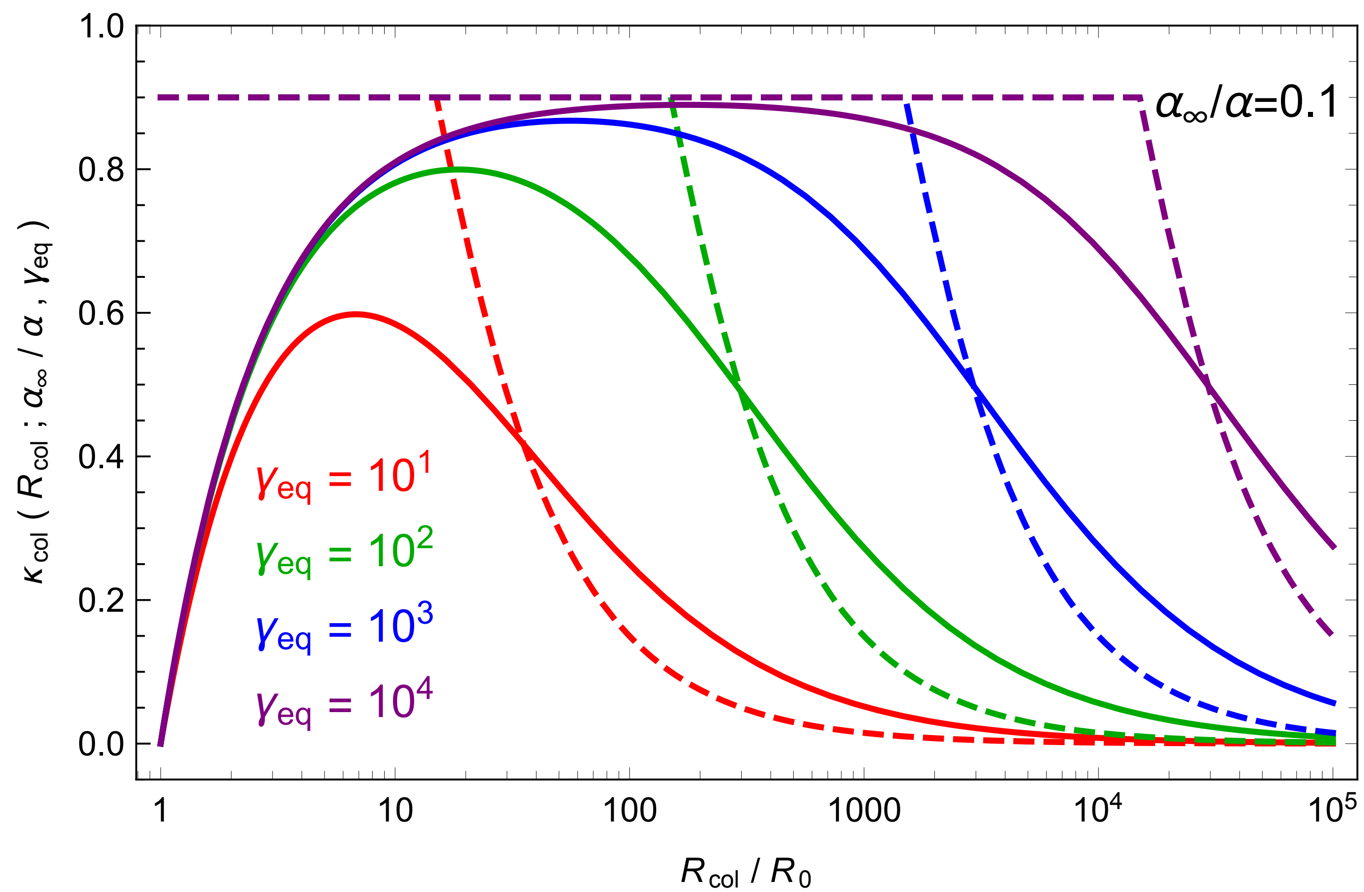
# Efficiency factor for wall collisions

**Example** $\Delta p_{\rm fr} = \Delta p_{\rm LO} + h(\gamma)\Delta p_{NLO}$

$$\alpha = \frac{\Delta V_{\rm eff}}{\rho_R}, \quad \alpha_\infty = \frac{\Delta p_{\rm LO}}{\rho_R}, \quad h(\gamma) = \gamma$$

$$\kappa_\phi = \frac{E_{\rm wall}(R_*)}{\frac{4}{3}\pi R_*^3 \Delta V_{\rm eff}} = \int_{R_0}^{R_*} \frac{{\rm d}R}{R_*} \frac{\Delta p_{\rm dr} - \Delta p_{\rm fr}}{\Delta V_{\rm eff}}$$

$$= \left(1 - \frac{\alpha_\infty}{\alpha}\right) \int_{R_0}^{R_*} \frac{{\rm d}R}{R_*} \left[1 - \frac{h(\gamma(R))}{h(\gamma_{\rm eq})}\right]$$




$$\gamma_{\rm eq} \equiv \frac{\Delta V_{\rm eff} - \Delta p_{\rm LO}}{\Delta p_{NLO}} = \left(1 - \frac{\alpha_\infty}{\alpha}\right) \Big/ \frac{\Delta p_{NLO}}{\Delta V_{\rm eff}}$$

$$= \frac{1 - \alpha_\infty/\alpha}{27\gamma_*^2\gamma_{\rm eq}(\gamma_{\rm eq} - 1)} \left[\gamma_*(3\gamma_{\rm eq} - 4)(3\gamma_{\rm eq} - 1)^2 \log\frac{3(\gamma_{\rm eq} + \gamma_* - 1)}{3\gamma_{\rm eq} - 1} - 4\gamma_* \log\frac{2}{3\gamma_*} - 2(\gamma_{\rm eq} - 1)(3\gamma_* - 2)\right]$$

$$\gamma_* \approx \frac{2R_*}{3R_0}$$

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**Effective picture of bubble expansion**
**JCAP03 (2021)096**

**2206.01148 SJW, Zi-Yan Yuwen The energy budget of cosmological first-order phase transitions beyond the bag equation of state**

# Efficiency factor for sound waves

**Bag equation of state**

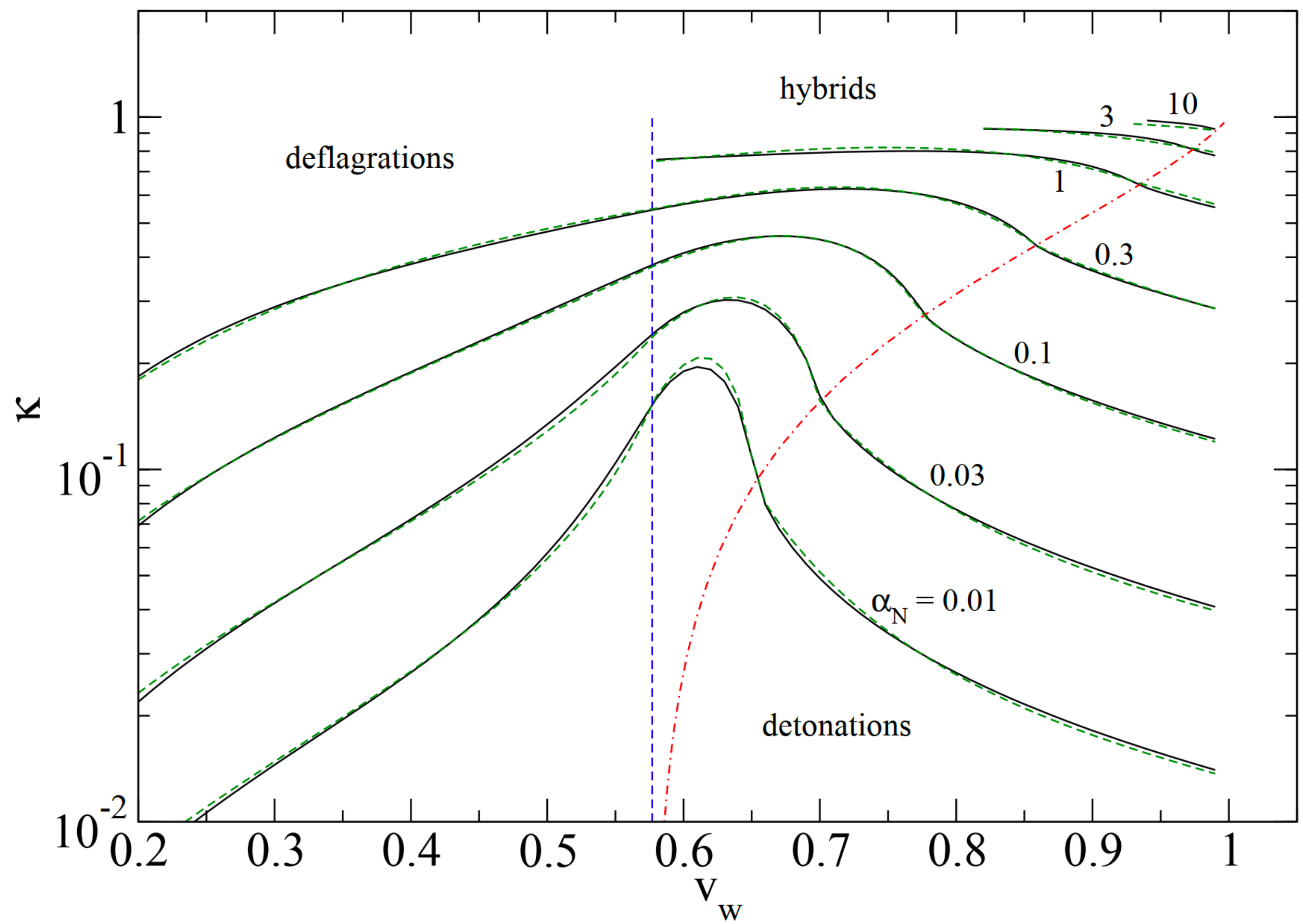
$$p_\pm = c_s^2 a_\pm T^4 - \epsilon_\pm$$

$$\rho_\pm = a_\pm T^4 + \epsilon_\pm$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3}, \quad \epsilon = V_0(\phi_\pm)$$

$$a = \frac{\pi^2}{30}\left(\sum_{\text{light B}} g_i + \frac{7}{8}\sum_{\text{light F}} g_j\right)$$

Espinosa et al. 1004.4187




**Beyond bag equation of state — constant sound velocity : $\nu$ − model**

$$p_\pm = c_{s,\pm}^2 a_\pm T^{\nu_\pm} - \epsilon_\pm$$

$$\rho_\pm = a_\pm T^{\nu_\pm} + \epsilon_\pm$$

$$c_{s,\pm}^2 = \frac{\partial p_\pm}{\partial \rho_\pm} = \frac{1}{\nu_\pm - 1}$$

Leitao & Megevand 1410.3875

Giese et al. 2004.06995, 2010.09744

Wang et al. 2010.13770, 2112.14650

# Beyond constant sound velocity

**Effective potential**

$$V_T(\phi,T) = \sum_{i=\mathrm{B,F}} \pm g_i T \int \frac{d^3\vec{k}}{(2\pi)^3} \ln\left[1 \mp e^{-\frac{\sqrt{\vec{k}^2+m_i^2}}{T}}\right] \equiv \frac{T^4}{2\pi^2} \sum_{i=\mathrm{B,F}} g_i J_i\left(\frac{m_i^2}{T^2}\right)$$

**Heavy particles**

$$J_{\mathrm{B/F}}\left(\frac{m_i^2}{T^2}\right) = -\left(\frac{m_i}{2\pi T}\right)^{\frac{3}{2}} e^{-m_i/T}\left[1 + \mathcal{O}\left(\frac{T}{m_i}\right)\right]$$

**Light particles**

$$J_{\mathrm{B}}\left(\frac{m_i^2}{T^2}\right) = -\frac{\pi^2}{90} + \frac{1}{24}\left(\frac{m_i}{T}\right)^2 - \frac{1}{12\pi}\left(\frac{m_i}{T}\right)^3 - \frac{1}{32\pi^2}\left(\frac{m_i}{T}\right)^4 \ln\frac{m_i e^{\gamma_E - 3/4}}{4\pi T} - \frac{1}{16\pi^{5/2}}\left(\frac{m_i}{T}\right)^4 \sum_{l=1}^{\infty}(-1)^l \frac{\zeta(2l+1)}{(l+1)!}\Gamma\left(l+\frac{1}{2}\right)\left(\frac{m_i^2}{4\pi^2 T^2}\right)^l$$

$$J_{\mathrm{F}}\left(\frac{m_i^2}{T^2}\right) = -\frac{7}{8}\frac{\pi^2}{90} + \frac{1}{48}\left(\frac{m_i}{T}\right)^2 + \frac{1}{32\pi^2}\left(\frac{m_i}{T}\right)^4 \ln\frac{m_i e^{\gamma_E - 3/4}}{\pi T} + \frac{1}{8\pi^{5/2}}\left(\frac{m_i}{T}\right)^4 \sum_{l=1}^{\infty}(-1)^l \frac{\zeta(2l+1)}{(l+1)!}\left(1 - \frac{1}{2^{2l+1}}\right)\Gamma\left(l+\frac{1}{2}\right)\left(\frac{m_i^2}{\pi^2 T^2}\right)^l$$

**EoS**

$$p = -\mathscr{F} = -V_{\mathrm{eff}} = -V_0(\phi) + \frac{1}{3}aT^4 - bT^2 + cT$$

$$\rho = T\partial_T p - p = V_{\mathrm{eff}} - T\partial_T V_{\mathrm{eff}} = V_0(\phi) + aT^4 - bT^2$$

$$c_s^2(T) = \frac{1}{3}\left(1 - \frac{4bT - 3c}{12aT^3 - 6bT}\right)$$

$$a = \sum_{i=\mathrm{B}} g_i + \frac{7}{8}\sum_{i=\mathrm{F}} g_i$$

$$b = \frac{1}{24}\left(\sum_{i=\mathrm{B}} g_i m_i^2 + \frac{1}{2}\sum_{i=\mathrm{F}} g_i m_i^2\right)$$

$$c = \frac{1}{12\pi}\sum_{i=\mathrm{B}} g_i m_i^3$$

# Solving fluid EoM iteratively

**Fluid equations of motion**

$$2\frac{v}{\xi} = \gamma(v)^2(1-\xi v)\left(\frac{\mu(\xi,v)^2}{c_s^2(T(\xi))} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

$$\frac{\mathrm{d}w}{\mathrm{d}\xi} = w\gamma(v)^2\mu(\xi,v)\left(\frac{1}{c_s^2(T(\xi))} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

$$T(\xi) = T(\xi_0)\exp\left[\int_{v(\xi_0)}^{v(\xi)}\gamma(v)^2\mu(\xi,v)\mathrm{d}v\right]$$

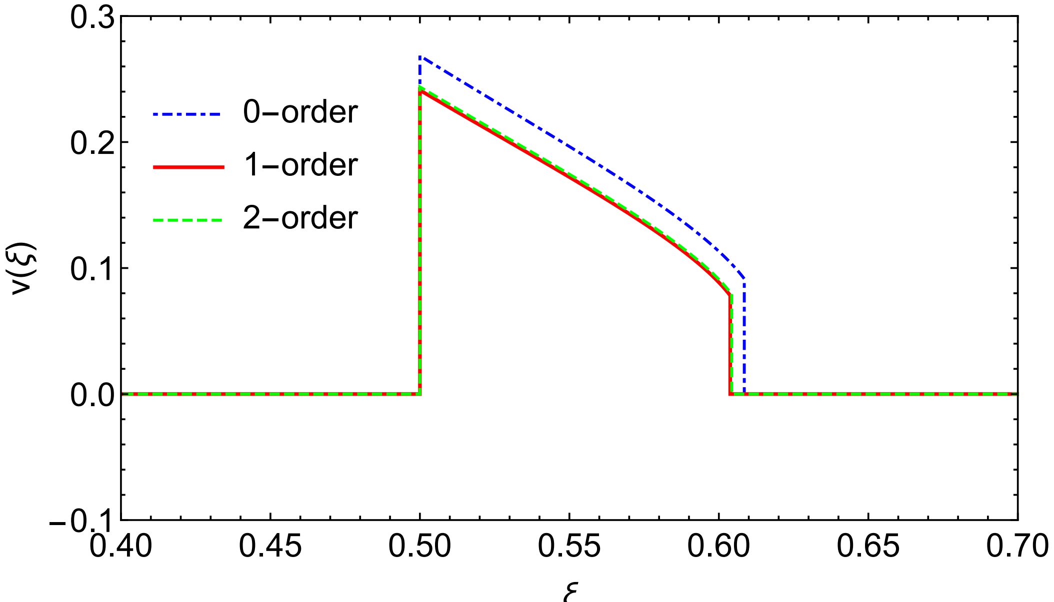
**Iteration method**

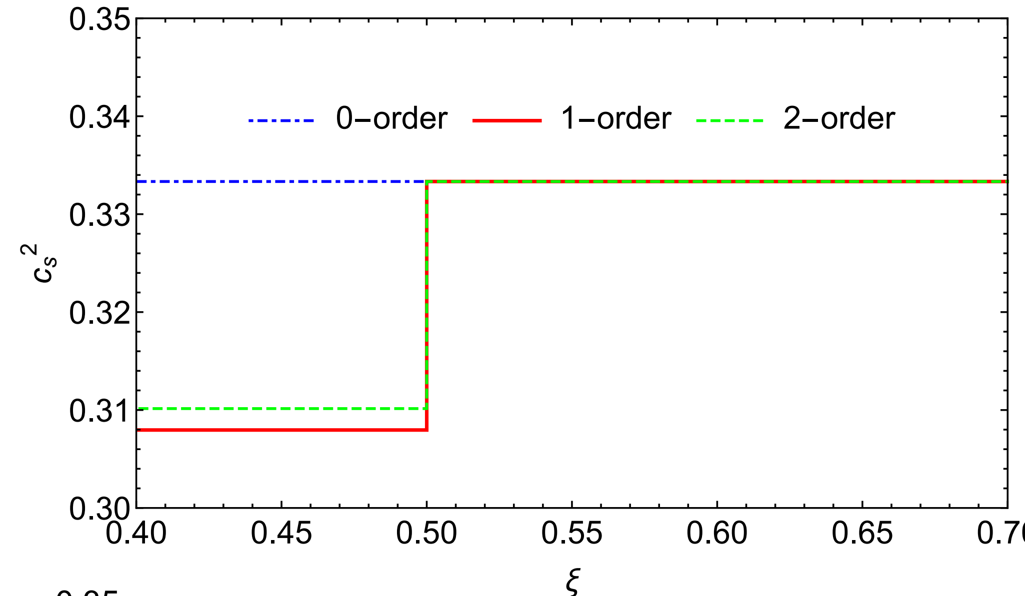
$$c_s^{(0)}(\xi) = 1/3 \Rightarrow v^{(0)}(\xi),\quad w^{(0)}(\xi),\quad T^{(0)}(\xi)$$

$$c_s^{(1)}(\xi) = c_s(T^{(0)}(\xi)) \Rightarrow v^{(1)}(\xi),\quad w^{(1)}(\xi),\quad T^{(1)}(\xi)$$

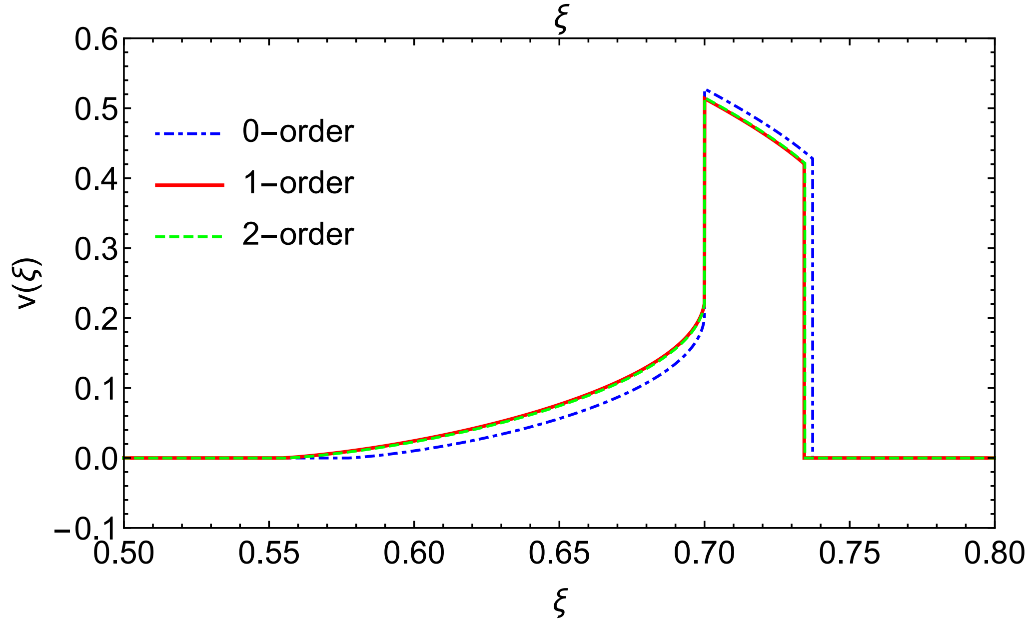
$$c_s^{(2)}(\xi) = c_s(T^{(1)}(\xi)) \Rightarrow v^{(2)}(\xi),\quad w^{(2)}(\xi),\quad T^{(2)}(\xi)$$

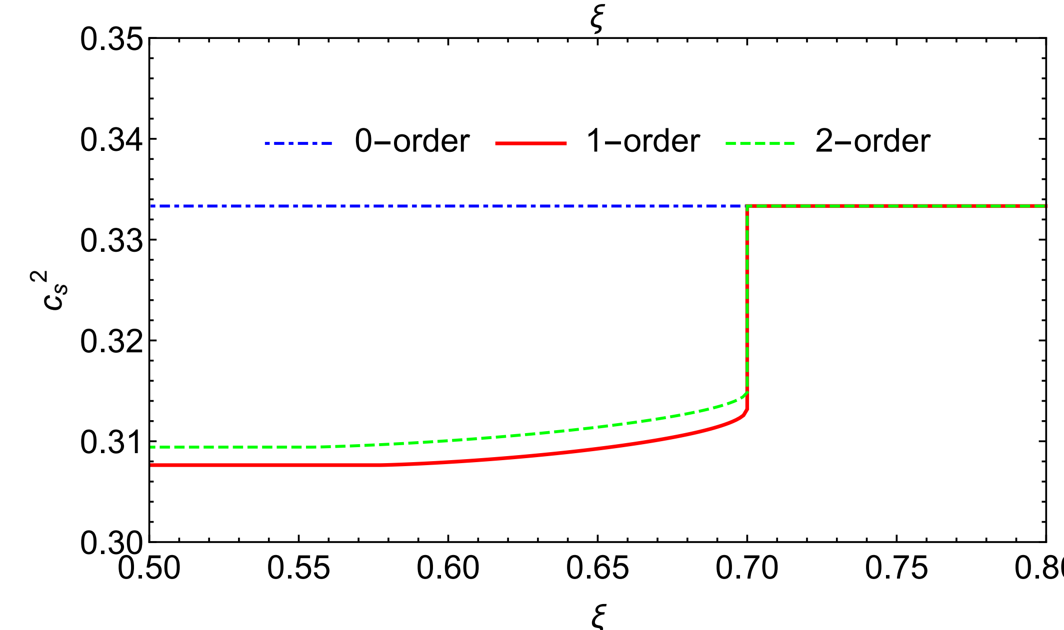
# Iterative solutions



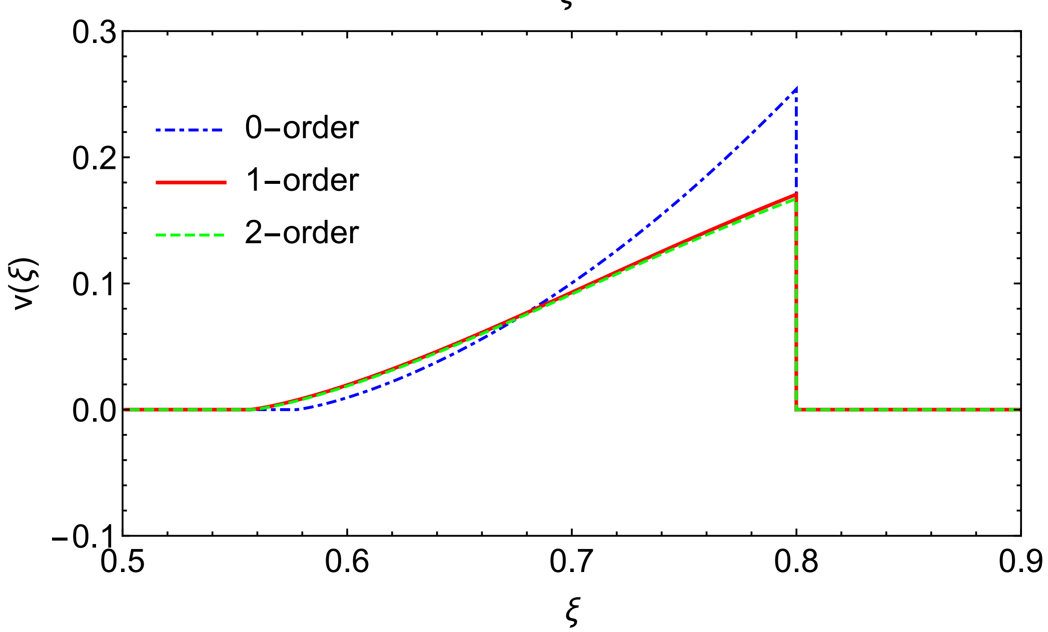



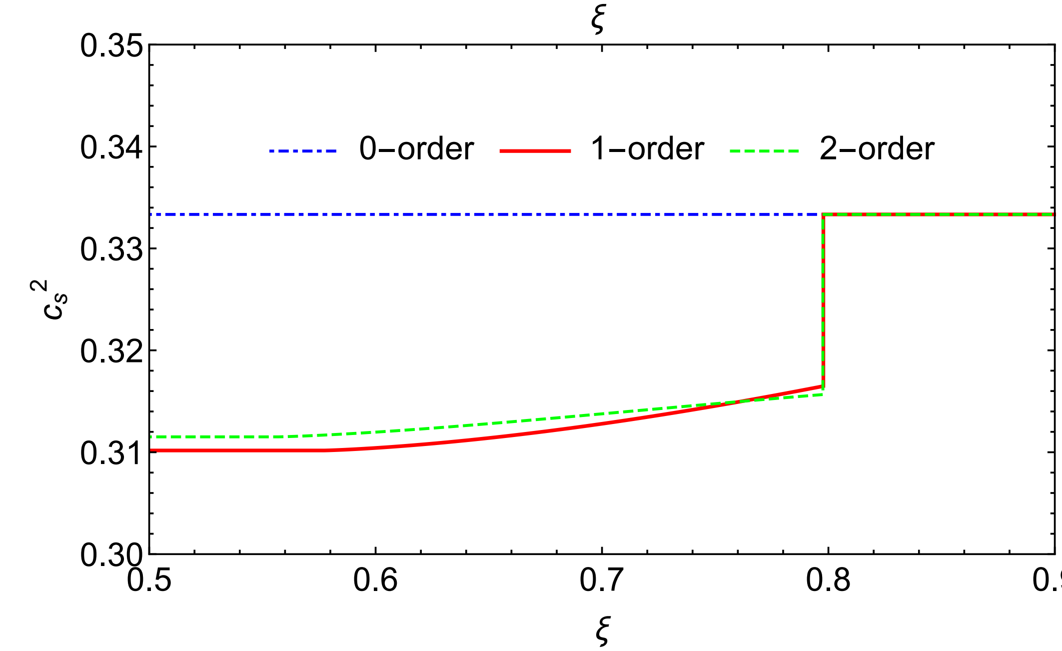

**Deflagration**



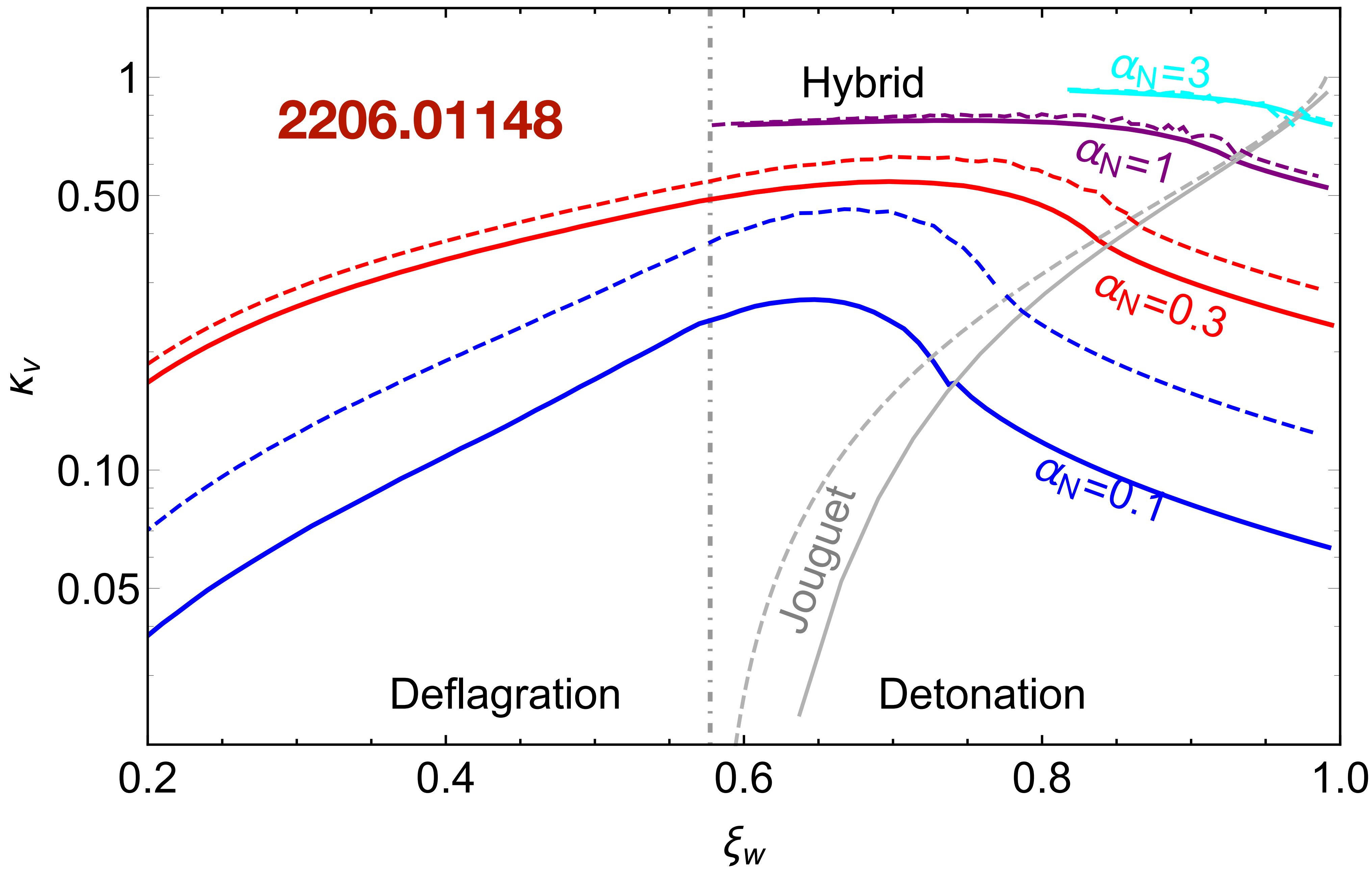

**Hybrid**




**Detonation**

# Efficiency factor for sound waves




1. The efficiency factor beyond the constant sound velocity is suppressed with respect to the case with a bag EoS

2. The suppression effect is less announced for a strong first-order phase transition with a large strength factor alpha_N

2205.02492 SJW, Zi-Yan Yuwen

Hydrodynamic backreaction force of cosmological bubble expansion

# Boltzmann equations of motion

$$T^{\mu\nu}_{\phi} = \nabla^{\mu}\phi\,\nabla^{\nu}\phi + g^{\mu\nu}\left[-\frac{1}{2}(\nabla\phi)^2 - V_0(\phi)\right]$$

$$T^{\mu\nu}_{f} = \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{k^{\mu}k^{\nu}}{k^0}\bigg|_{k^0=E_i(\mathbf{k})} f_i(\mathbf{x},\mathbf{k})$$

**Scalar EoM**

$$\nabla_{\mu}T^{\mu\nu}_{\phi} \equiv [\nabla_{\mu}\nabla^{\mu}\phi - V_0'(\phi)]\,\nabla^{\nu}\phi = +f^{\nu}$$

**Plasma EoM**

$$\nabla_{\mu}T^{\mu\nu}_{f} \equiv \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{k^{\mu}k^{\nu}}{E_i(\mathbf{k})}\nabla_{\mu}f_i = -f^{\nu}$$

$$f^{\nu} = \nabla^{\nu}\phi \sum_{i=\mathrm{B,F}} g_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f_i}{2E_i}$$

$$\frac{\partial V_T^{1-\mathrm{loop}}}{\partial\phi} = \sum_{i=\mathrm{B,F}} g_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f_i^{\mathrm{eq}}}{2E_i}$$

$$f_i = f_i^{\mathrm{eq}} + \Delta f_i^{\mathrm{eq}} + \delta f_i \qquad f_i^{\mathrm{eq}} = \frac{1}{e^{E_i/T} \mp 1}$$

$$f^{\nu} = \nabla^{\nu}\phi\left(\frac{\partial V_T^{1-\mathrm{loop}}}{\partial\phi} + \frac{\partial \Delta V_T}{\partial\phi} - \frac{\partial p_{\delta f}}{\partial\phi}\right)$$

**Out-of-equilibrium effect**

$$\nabla_{\mu}\nabla^{\mu}\phi - \frac{\partial V_{\mathrm{eff}}}{\partial\phi} = -\frac{\partial p_{\delta f}}{\partial\phi} \qquad \nabla_{\mu}T^{\mu\nu}_{f} + \nabla^{\nu}\phi\frac{\partial V_T}{\partial\phi} = \nabla^{\nu}\phi\frac{\partial p_{\delta f}}{\partial\phi}$$

# New picture of bubble expansion

**Scalar EoM**

$$\int \mathrm{d}\xi \frac{\mathrm{d}\phi}{\mathrm{d}\xi}\left(\nabla^2\phi - \frac{\partial V_{\rm eff}}{\partial \phi} + \frac{\partial p_{\delta f}}{\partial \phi}\right) = 0$$

$$\phi'(\xi \neq \xi_w)^2 = 0 \qquad \frac{\partial V_{\rm eff}}{\partial \phi}\frac{\mathrm{d}\phi}{\mathrm{d}\xi} = \frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}\xi} - \frac{\partial V_{\rm eff}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}\xi}$$

$$\frac{F_{\rm driving}}{A} \equiv \Delta V_{\rm eff} = \int \mathrm{d}\xi \frac{\mathrm{d}T}{\mathrm{d}\xi}\frac{\partial V_{\rm eff}}{\partial T} + \int \mathrm{d}\xi \frac{\mathrm{d}\phi}{\mathrm{d}\xi}\frac{\partial p_{\delta f}}{\partial \phi} \equiv \frac{F_{\rm thermal}}{A} + \frac{F_{\rm friction}}{A} \equiv \frac{F_{\rm backreaction}}{A}$$

**Driving force from vacuum difference** = **Thermal force from Temperature gradience** + **Friction force from Out-of-equilibrium**

**Plasma EoM**

$$u_\nu \nabla_\mu (w u^\mu u^\nu + p_f \eta^{\mu\nu}) + u_\nu \nabla^\nu \phi \frac{\partial V_T}{\partial \phi} = u_\nu \nabla^\nu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

$$u_\nu u^\nu = -1, \quad u_\nu \nabla_\mu u^\nu = 0 \qquad \nabla_\mu p_f = -\nabla_\mu V_T = -\nabla_\mu T \frac{\partial V_T}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi} = -\nabla_\mu T \frac{\partial V_{\rm eff}}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi}$$

**Entropy flow** $$T\nabla_\mu (s u^\mu) = -u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

**Enthalpy flow** $$-\nabla_\mu (w u^\mu) = u^\mu \nabla_\mu T \frac{\partial V_{\rm eff}}{\partial T} + u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

# Hydrodynamic evaluation

**Driving force** $$\frac{F_{\rm drive}}{A} \equiv \Delta\left(-\frac{1}{3}aT^4 + V_0\right) = -\frac{1}{4}\Delta w + \frac{3}{4}\alpha_N w_N$$

**Thermal force** $$\frac{F_{\rm themo}}{A} \equiv \int \mathrm{d}\xi \frac{\mathrm{d}T}{\mathrm{d}\xi}\frac{\partial V_{\rm eff}}{\partial T} = \int \mathrm{d}\xi \frac{\mathrm{d}T}{\mathrm{d}\xi}\left(-\frac{4}{3}aT^3\right)$$

→ $s(v)$ **unknown from junction condition**

**Friction force** $$\frac{F^{\rm hydro}_{\rm fric}}{A} = \int \mathrm{d}\xi \frac{\mathrm{d}\phi}{\mathrm{d}\xi}\frac{\partial p_{\delta f}}{\partial \phi} = \int \mathrm{d}r \frac{T\,\nabla_\mu (su^\mu)}{\gamma(\xi - v)} = \int_0^1 \mathrm{d}\xi \left(-T\frac{\mathrm{d}s}{\mathrm{d}\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu}\frac{\mathrm{d}v}{\mathrm{d}\xi}\right)$$

**Backreaction force** $$\frac{F^{\rm hydro}_{\rm back}}{A} = \int \mathrm{d}r \frac{\nabla_\mu (wu^\mu)}{\gamma(\xi - v)} = \int_0^1 \mathrm{d}\xi \left(-\frac{\mathrm{d}w}{\mathrm{d}\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu}\frac{\mathrm{d}v}{\mathrm{d}\xi}\right)$$

**e.g. Detonation:**

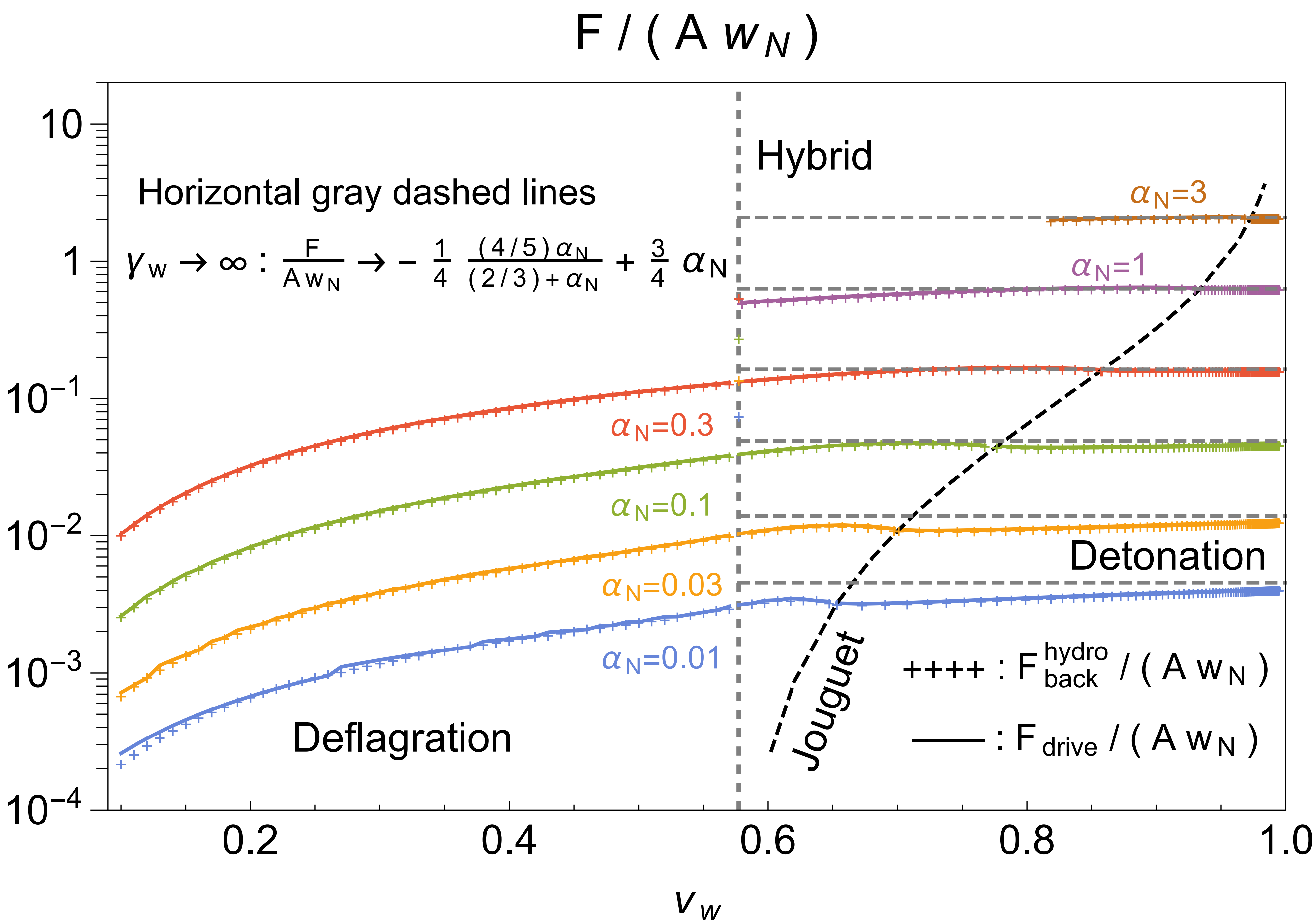
**Junction condition** ↓

$$\frac{F^{\rm hydro}_{\rm back}}{A} = -\frac{1}{4}[w(\xi = v_w) - w(\xi = c_s)] + \left.\frac{F^{\rm hydro}_{\rm back}}{A}\right|_{\rm wall}$$

$$w(v) = w_+ \frac{\bar{\gamma}_+^2 \bar{v}_+}{\bar{\gamma}^2 \bar{v}} = w_N \frac{v_w}{1 - v_w^2}\frac{1 - \mu(v_w, v)^2}{\mu(v_w, v)}$$

$$\left.\frac{F^{\rm hydro}_{\rm back}}{A}\right|_{\rm wall} = -(w_+ - w_-) + \int_{v_-}^0 \mathrm{d}v \frac{w(v)\gamma(v)^2}{\mu(v_w, v)} = -(w_+ - w_-) + \frac{v_-}{v_- - v_w} w_+$$

# Total back-reaction force




$$\frac{F_{\rm drive}}{Aw_N} = \frac{F^{\rm hydro}_{\rm back}}{Aw_N} \to -\frac{1}{4}\frac{(4/5)\alpha_N}{(2/3)+\alpha_N} + \frac{3}{4}\alpha_N$$

# Backreactoin force on the wall

**Barroso Mancha et al.**,"**Field theoretic derivation of bubble wall force,**" **2005.10875**

$$\Delta_{\rm wall}(p) \equiv p_+ - p_- = w_-\bar{\gamma}_-^2\bar{v}_-^2 - w_+\bar{\gamma}_+^2\bar{v}_+^2 \equiv -\Delta_{\rm wall}(w\bar{\gamma}^2\bar{v}^2)$$

$$\Delta_{\rm wall}(V_{\rm eff}) = \Delta_{\rm wall}(w\bar{\gamma}^2\bar{v}^2) \equiv \frac{F_{\rm back}^{\rm wall}}{A}$$

**There is a misleading deduction that this evaluation implies $\gamma_w^2$ – scaling friction force**

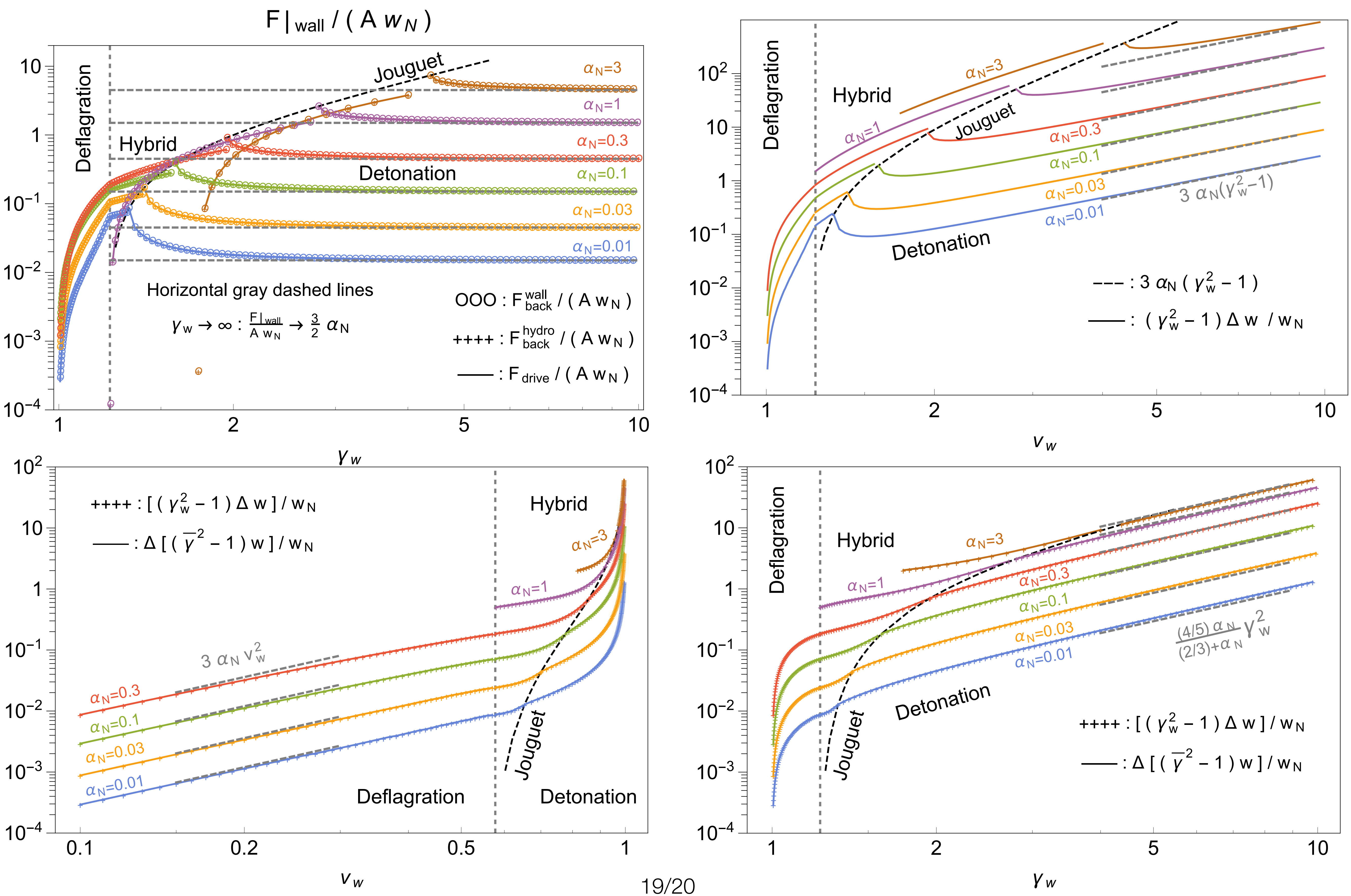
$$\Delta_{\rm wall}(w\bar{\gamma}^2\bar{v}^2) \equiv \Delta_{\rm wall}[(\bar{\gamma}^2-1)w] = (\gamma_w^2-1)\Delta_{\rm wall}(w) = (\gamma_w^2-1)T\Delta_{\rm wall}(s) \quad ✘$$

**However, we can analytically prove that** $\left(w_+\bar{\gamma}_+^2\bar{v}_+^2 - w_-\bar{\gamma}_-^2\bar{v}_-^2\right) \to \frac{3}{2}\alpha_N w_N, \quad \gamma_w \to \infty$

**In fact, we can numerically show that**

$$\Delta_{\rm wall}(V_{\rm eff}) = \Delta_{\rm wall}[(\bar{\gamma}^2-1)w] \neq \begin{cases} (\gamma_w^2-1)\Delta w = \Delta[(\bar{\gamma}^2-1)w] \to \frac{(4/5)\alpha_N w_N}{(2/3)+\alpha_N}\gamma_w^2, & \gamma_w \to \infty \\ (\gamma_w^2-1)\Delta_{\rm wall} w \to 3\alpha_N w_N(\gamma_w^2-1), & \gamma_w \to \infty \end{cases}$$

# Backreaction force on the wall



# Conclusions and discussions

**We have proposed an analytical evaluation method for the efficiency factor of bubble wall collisions**

**We have proposed an iteration method for the efficiency factor of bulk fluid motions beyond a constant sound velocity**

**We have proposed a hydrodynamical evaluation for the total backreaction force along with its wall contribution**

**We want to propose a hybrid method from hydrodynamics and microphysics to compute the friction force evolution**

*Thank you*