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Domain walls from spontaneous breaking of discrete symmetries

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Symmetries in fundamental physics

- Noether 1915, 对称性 → 守恒律
- 时空对称性 $R_{1,3} \rtimes O(1,3)$
- Standard Model $SU(3)_c \times SU(2)_L \times U(1)_Y \supset U(1)_{\rm em}$
- 分立对称性: C, P, T, CP, CPT
- 新物理对称性: $U(1)_{B-L}$; $U(1)_{\rm PQ}$; GUTs ($SU(5)$, $SO(10)$, Pati-Salam, etc)
- 新物理引入的分立对称性: $Z_N \subset U(1)_{\rm FN}$; $A_4, S_4 \subset SU(3)_{\rm flavour}$; Modular symm Γ; Z_2^C in GUTs; R_p, Z_3 in SUSY

新物理 { 新的粒子和相互作用 → 对撞机、亮度前沿……
新的对称性破缺效应 → 宇宙学检验 (引力波……)

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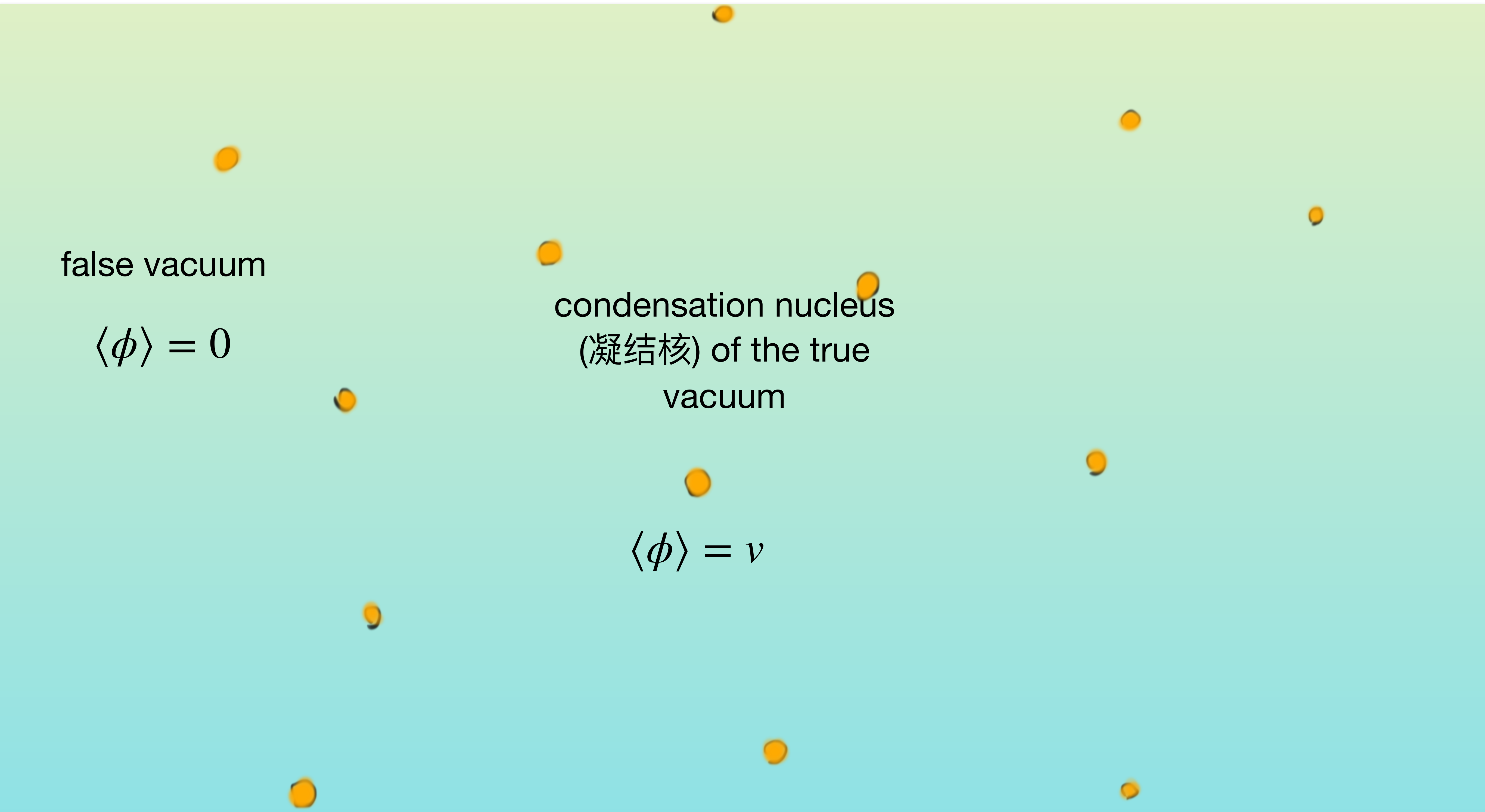
本报告 ⟹ 初步探讨Z_N畴壁(domain wall)的物理性质

吴永成、谢柯盼、周也铃，2204.04374; 2205.11529

A general picture of phase transition



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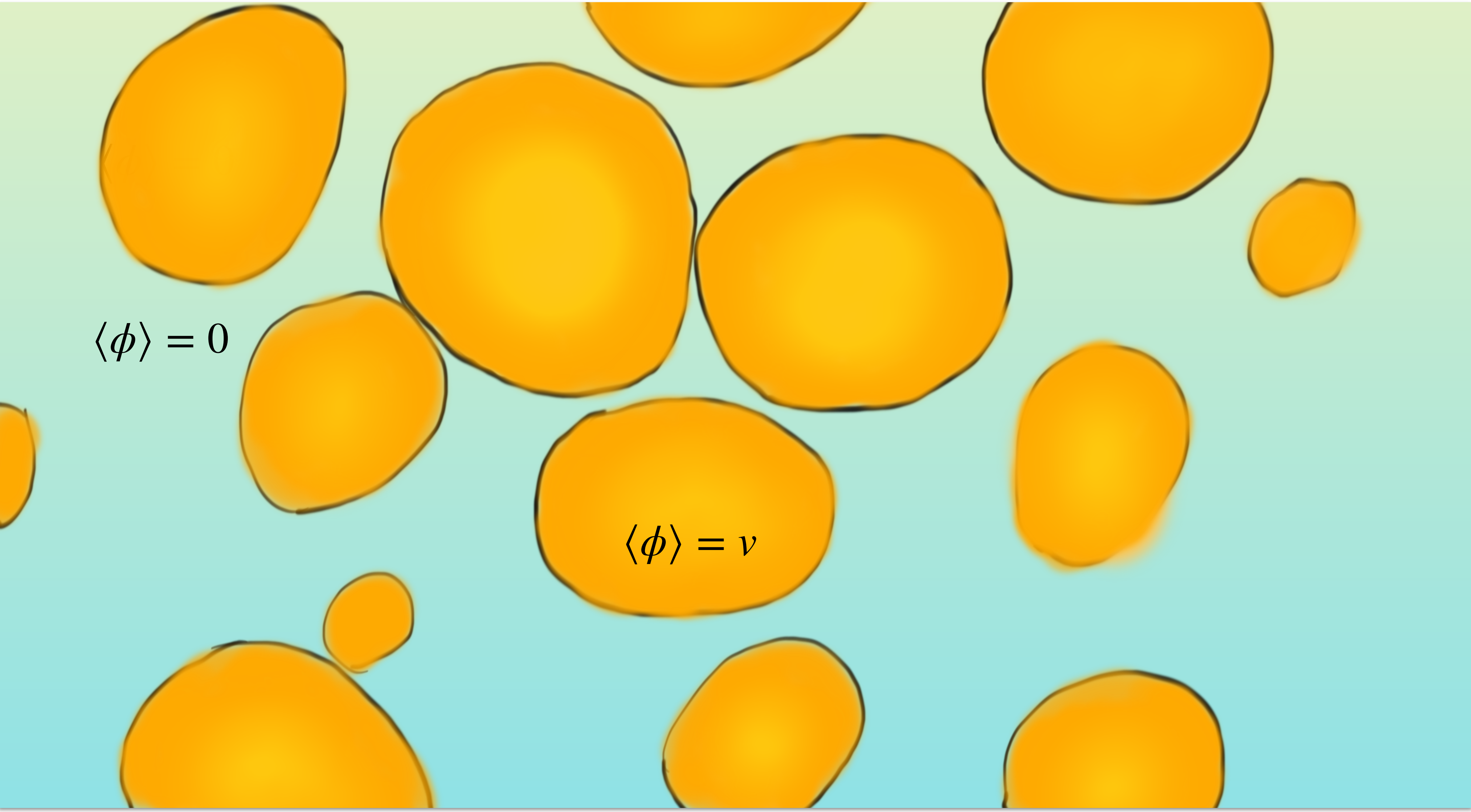

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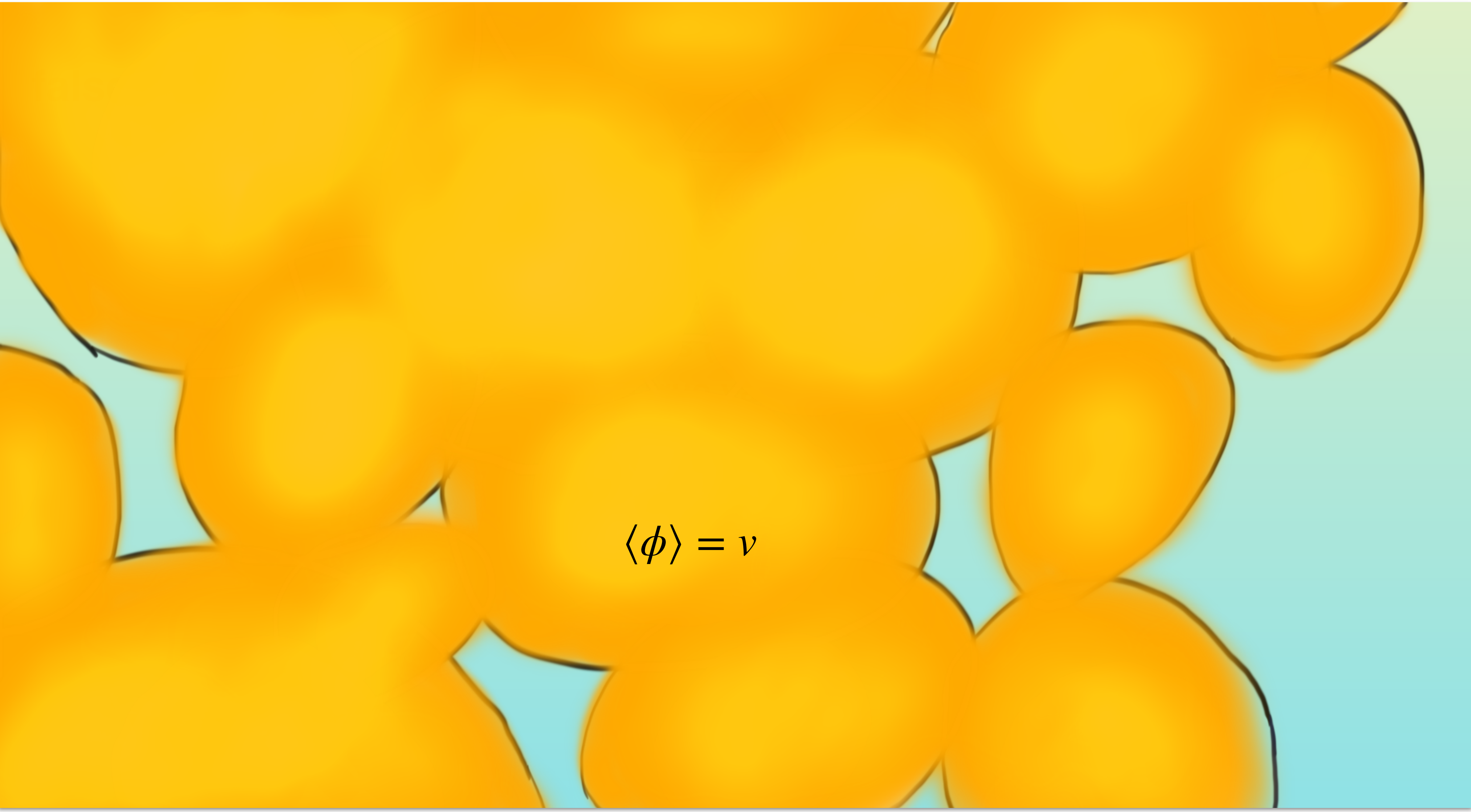
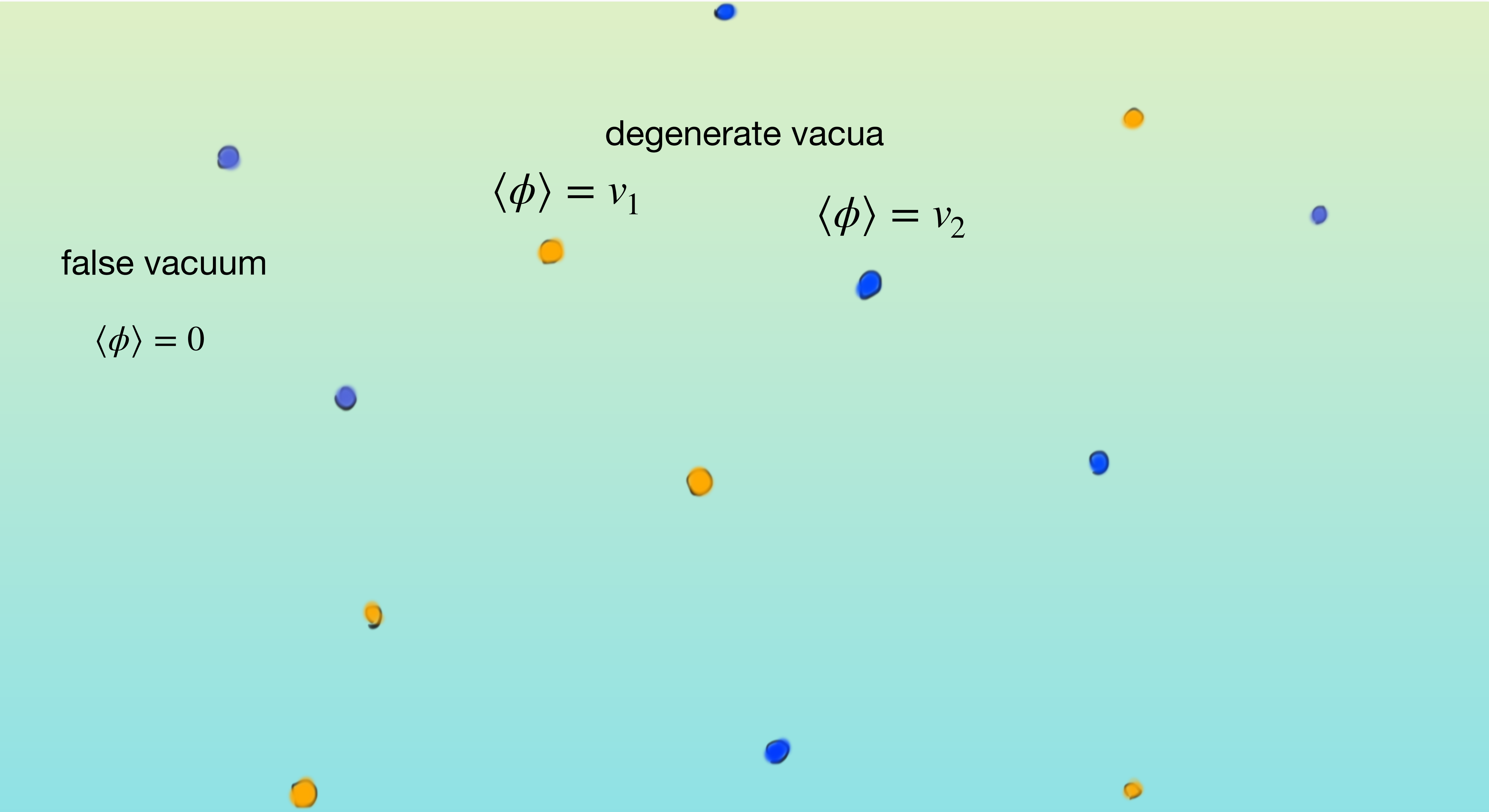
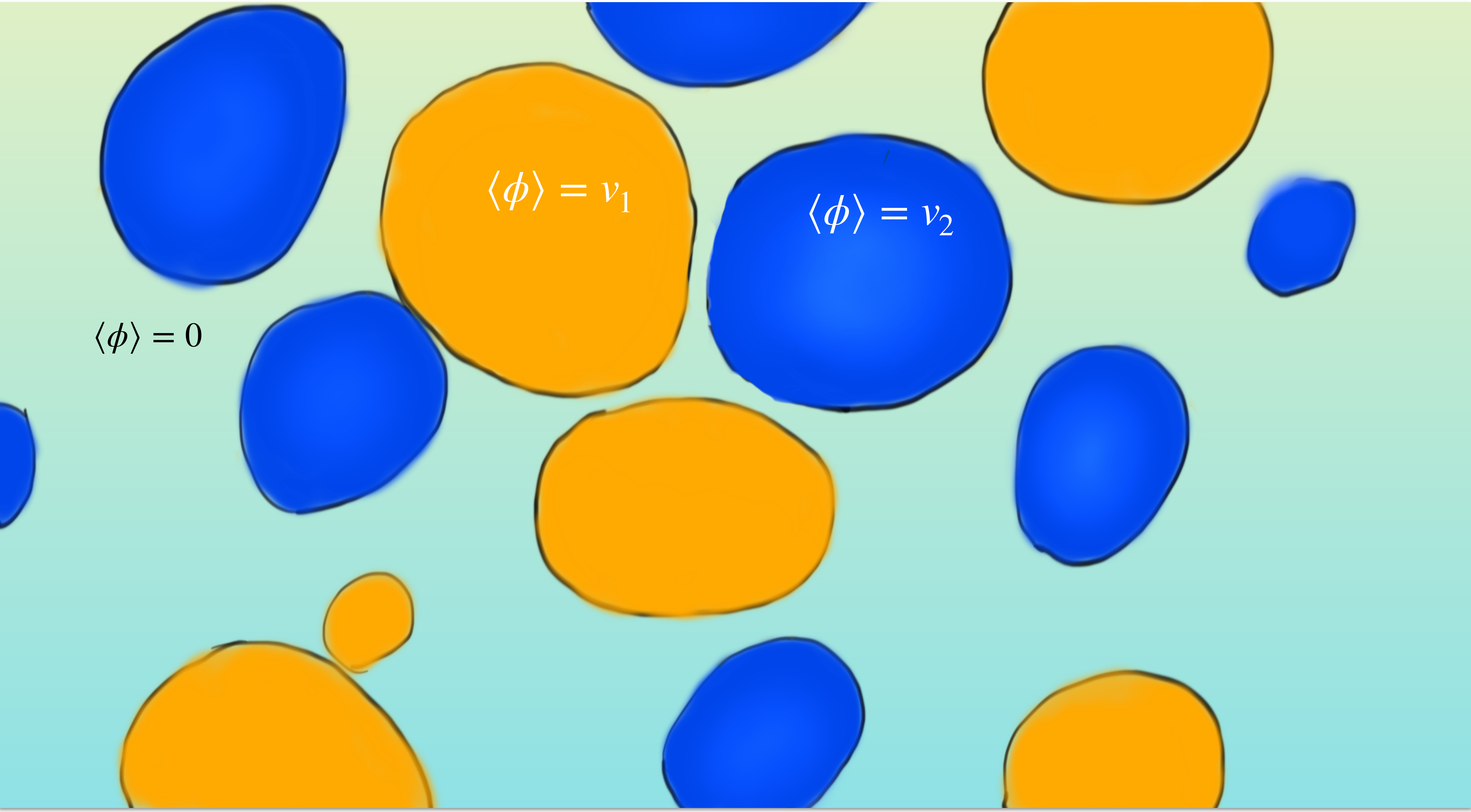


Phase transition with degenerate vacua



Phase transition with degenerate vacua



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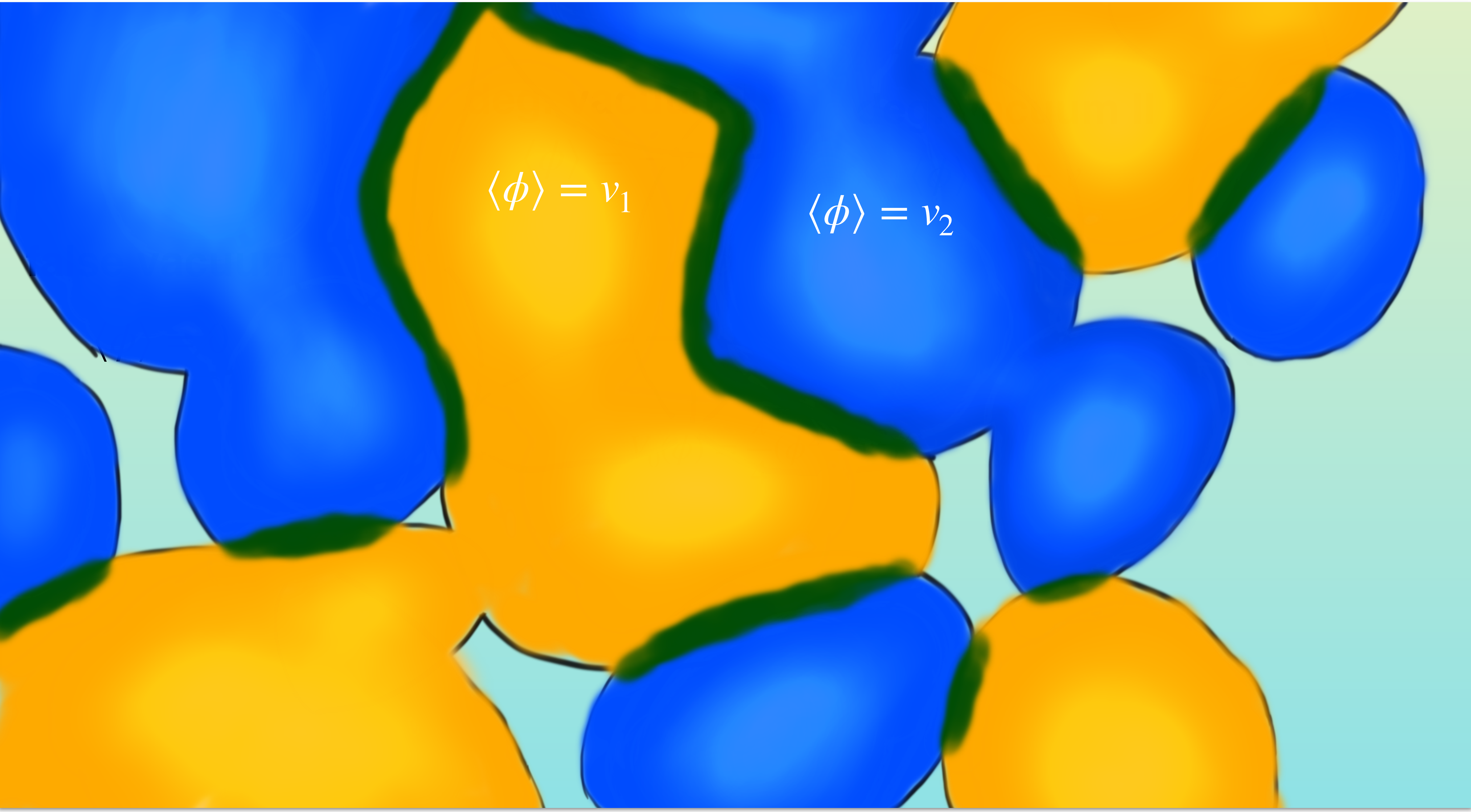

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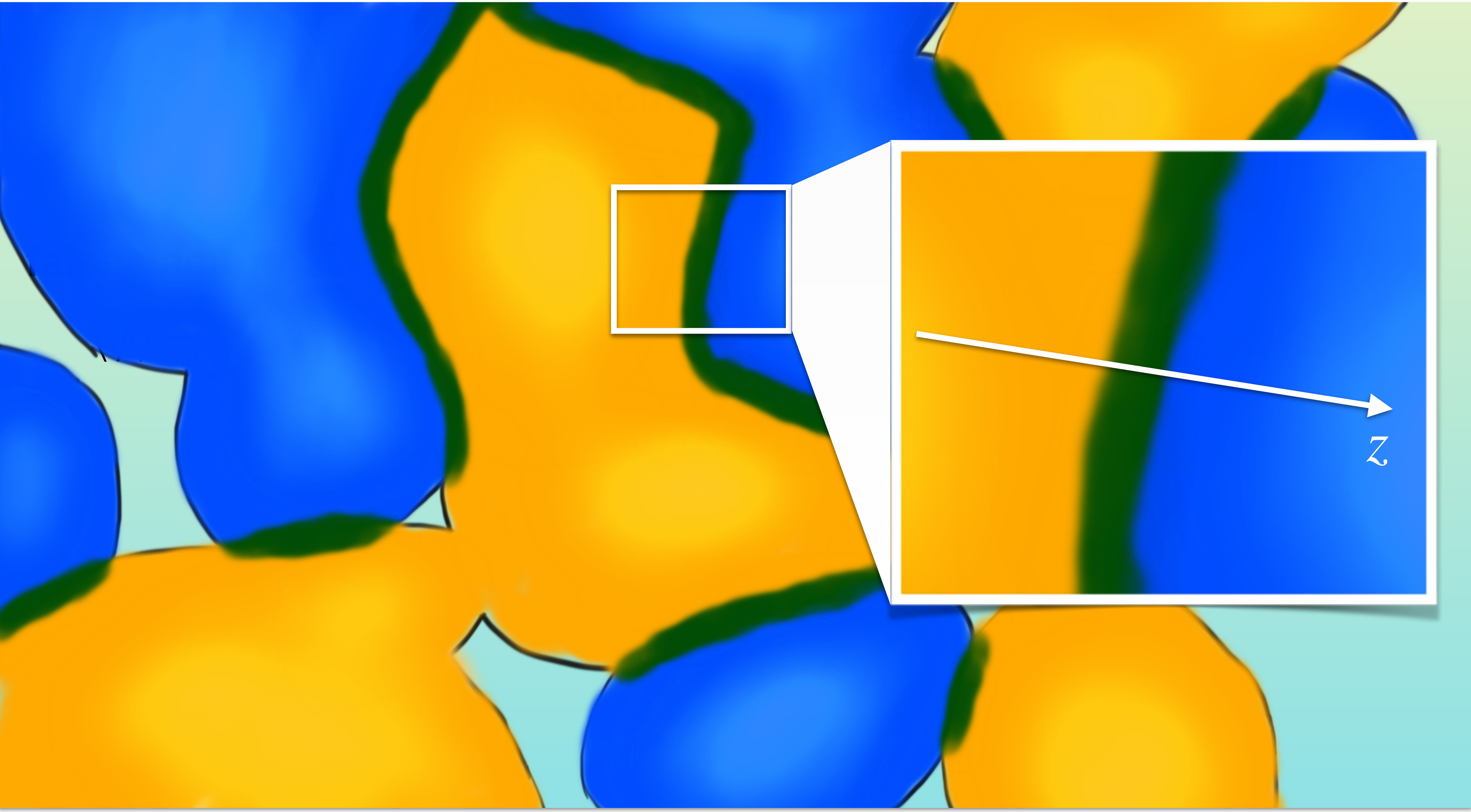
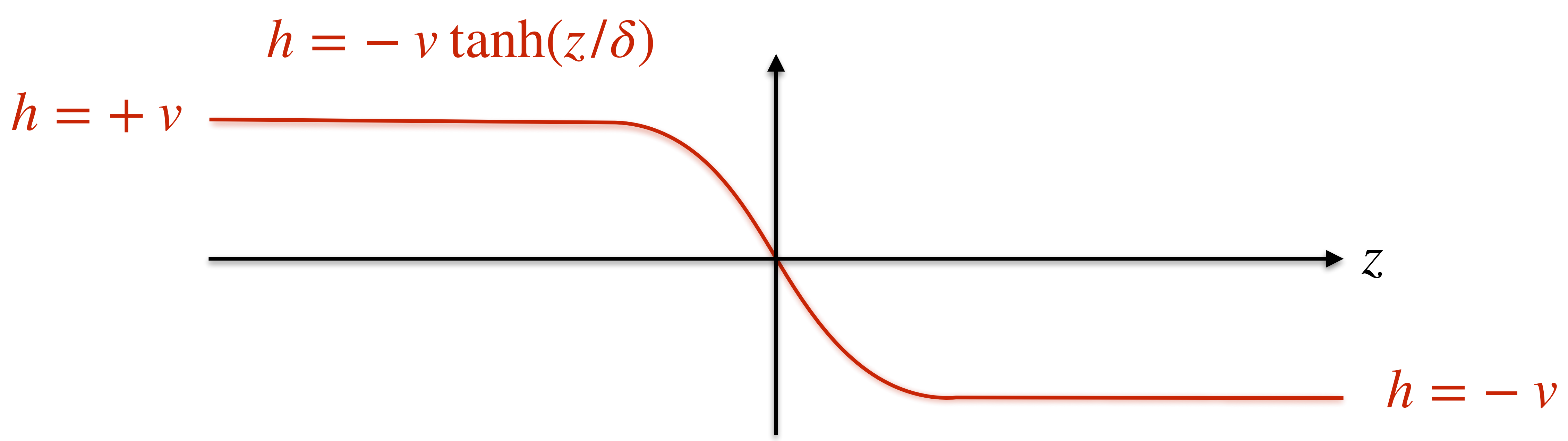


Z_2 wall —— the simplest wall

Given a toy potential for a real scalar in Z_2 $\qquad V = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$

EOM $\qquad h''(z) = \lambda h(h^2 - v^2) \qquad v = \sqrt{\mu^2/\lambda}$

Soliton solution: scalar solution along z direction



Vilenkin, Phys. Rept.121 (1985) 263

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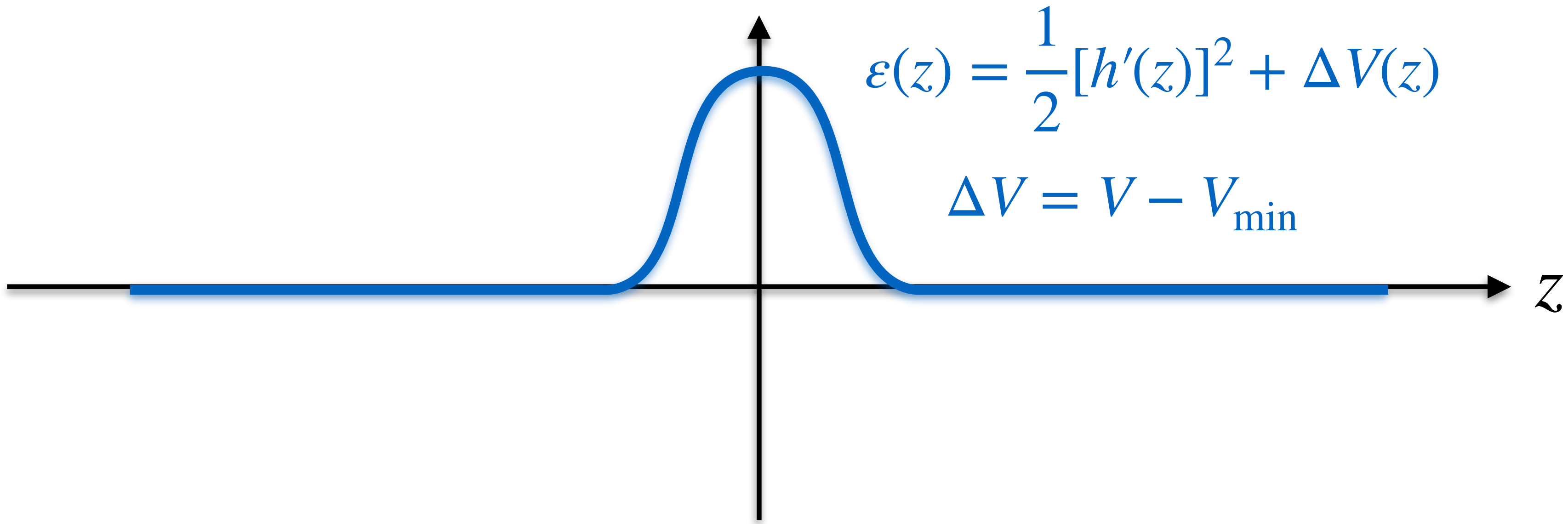
Tension (即表面能量密度) $\quad \sigma = \frac{4}{3}\sqrt{\frac{\lambda}{2}}v_0^3 \qquad \sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz$

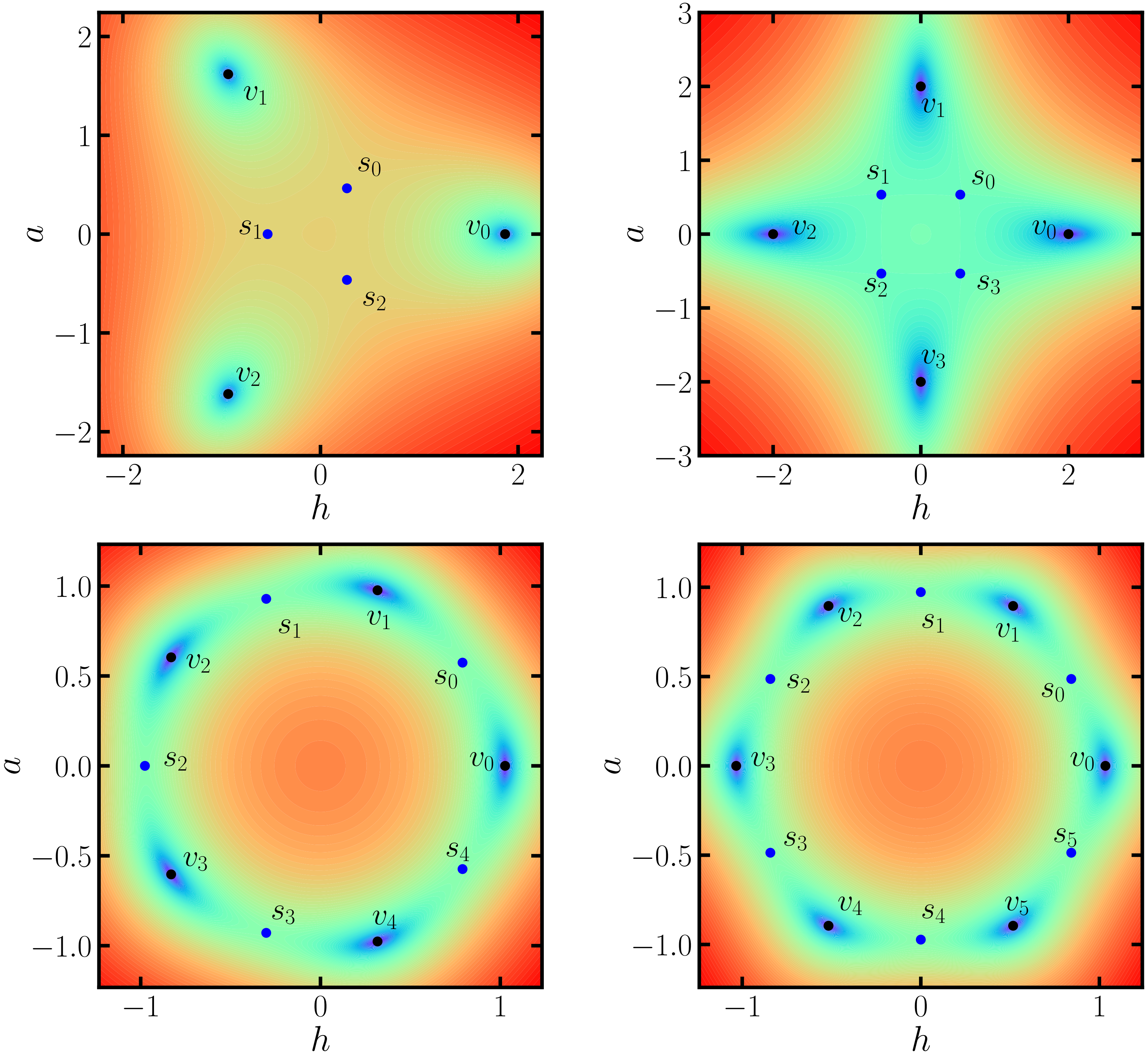
Thickness (即畴壁的厚度) $\quad \delta = \sqrt{\frac{2}{\lambda v_0^2}}$



Vilenkin, Phys. Rept.121 (1985) 263

How about walls beyond Z_2?

Z_N-invariant potential $\quad V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu^{4-N}(\phi^N + \phi^{*N})$



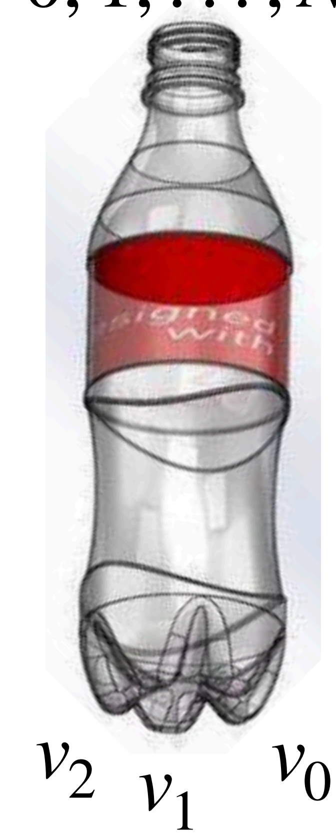
$$\phi = \frac{1}{\sqrt{2}}(h + ia)$$

(assuming CP conservation, simplest form)

N degenerate vacua:

$$v_k = ve^{i2\pi k/N}$$

$$k = 0, 1, \ldots, N-1$$

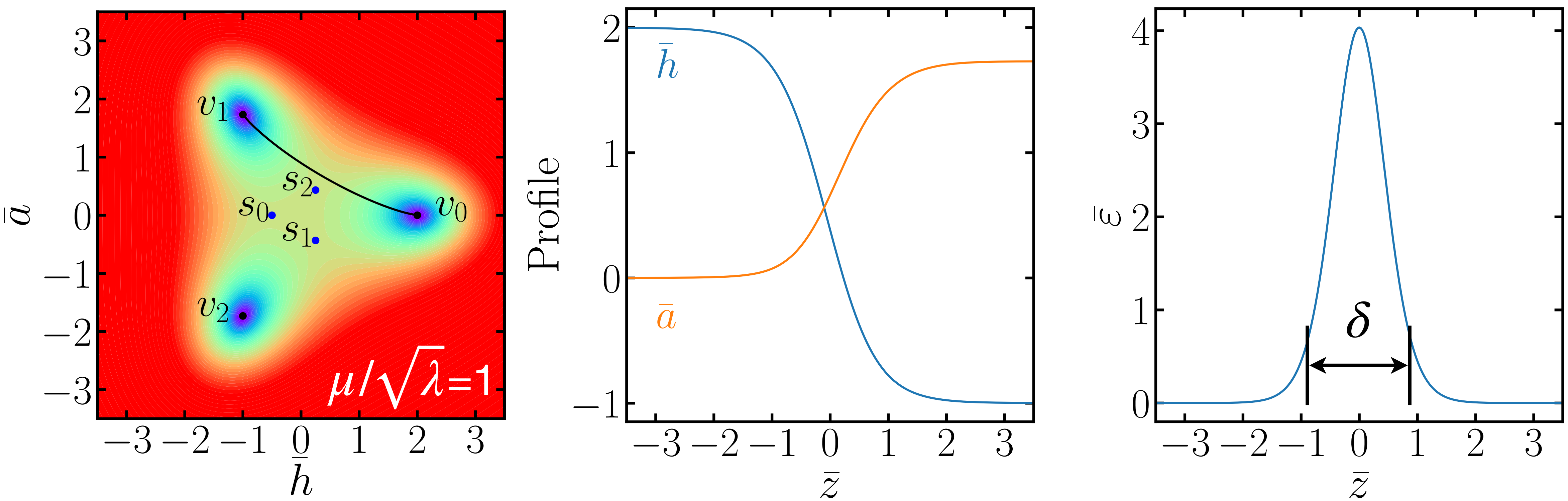


Z_3 wall

Z_3-invariant potential $\quad V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu(\phi^3 + \phi^{*3})$

3 vacua $\quad v_k = \frac{\mu}{\sqrt{2\lambda_1}}(\beta + \sqrt{1+\beta^2})\, e^{i2\pi k/3} \quad \beta = 3\lambda_2/\sqrt{8\lambda_1} > 0$

$\beta = 3/4$



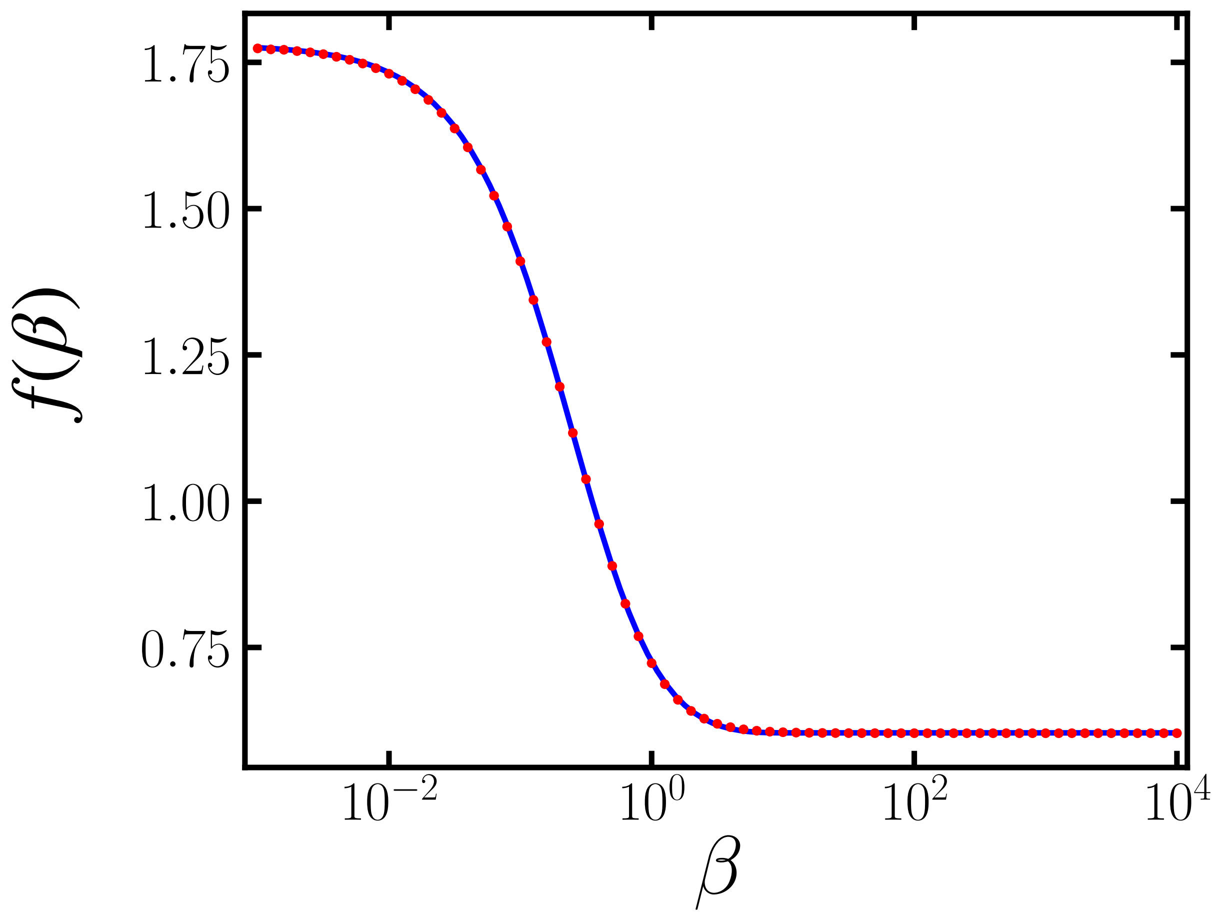
$$\varepsilon(z) = \frac{1}{2}\{[h'(z)]^2 + [a'(z)]^2\} + \Delta V(z)$$

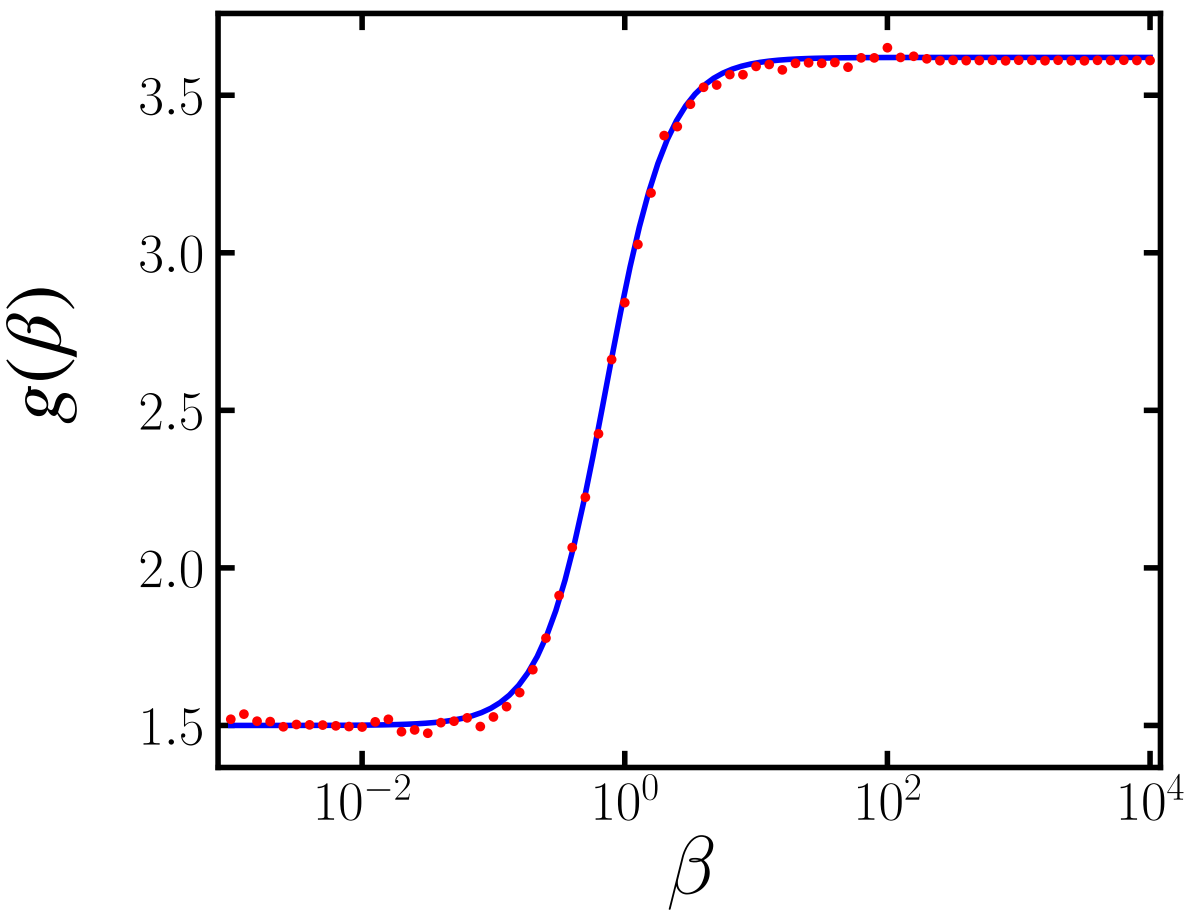
$$\Longrightarrow \quad \sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz \qquad \int_{-\delta/2}^{\delta/2} dz\varepsilon(z) = 64\% \times \sigma\,.$$

Z_3 wall

畴壁的表面能 $\sigma = m_a v_0^2 f(\beta)$

畴壁的厚度 $\delta = m_a^{-1} g(\beta)$





$$f(\beta) = 0.604 + \frac{0.234}{e^{0.826\beta} + 0.435\beta^2 - 0.801}$$

$$g(\beta) = 3.62 - \frac{2.12}{1 + 1.85\beta^{1.81}}$$

m_a 是 PGB 的质量

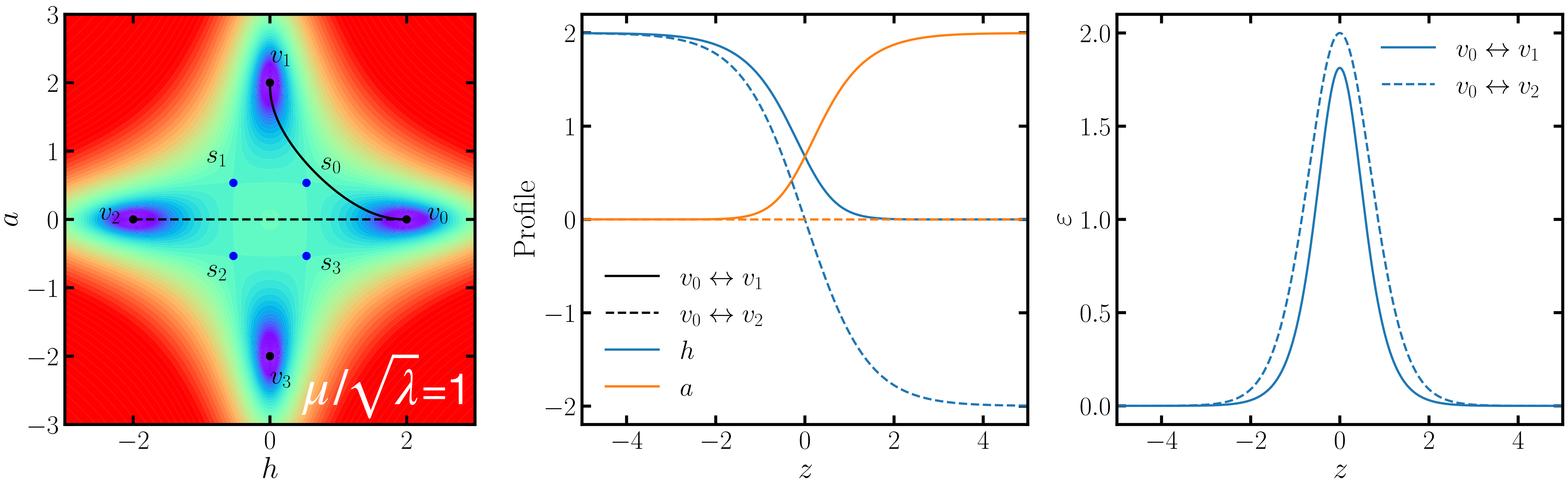
误差：<2% for $10^{-3} < \beta < 10^4$

Z_4 wall

Z_4-invariant potential $\quad V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2(\phi^4 + \phi^{*4})$

$$v_k = \frac{\mu}{\sqrt{2\lambda_1(1-\beta)}} e^{i\frac{2\pi}{4}k} \qquad \beta \equiv 2\lambda_2/\lambda_1$$

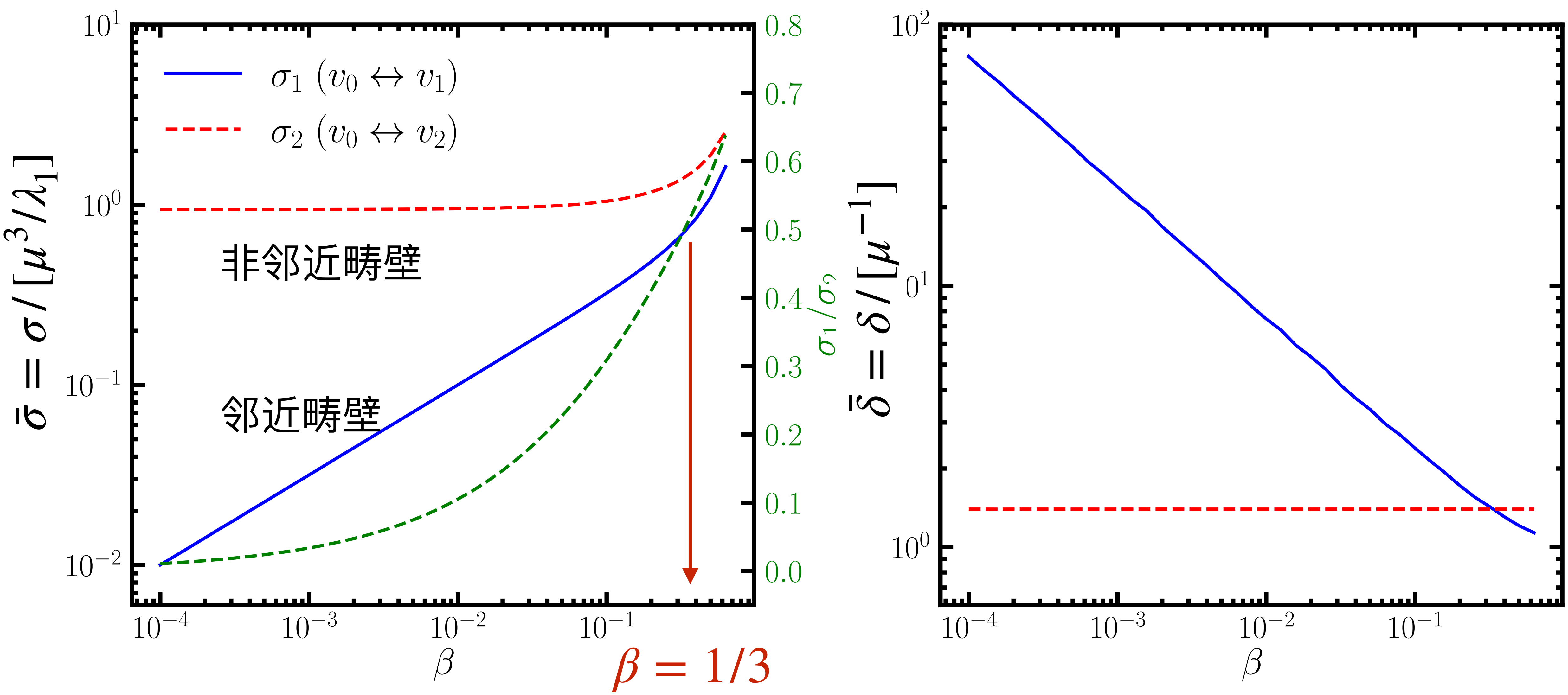
$\beta = 3/4$



Adjacent walls (邻近畴壁): 分隔邻近真空(比如 v_0 和 v_1)的畴壁

Non-adjacent walls (非邻近畴壁)：分隔非邻近真空(比如 v_0 和 v_2)的畴壁

Z_4 wall



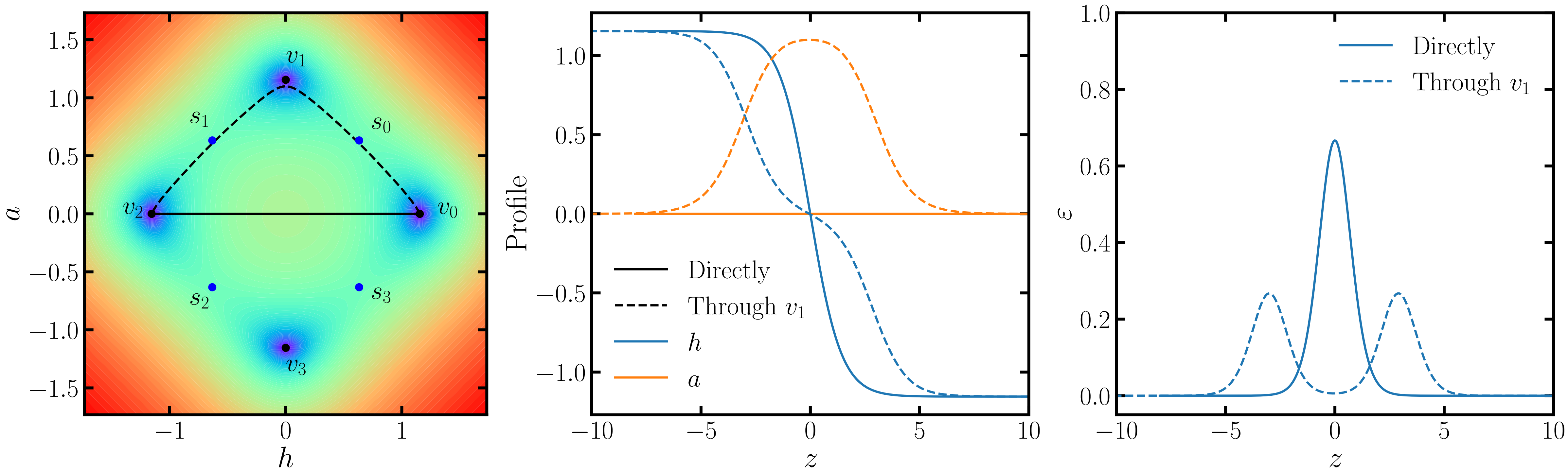
$\beta = 1/3$

$\Downarrow$

$\bar{\sigma}_{\text{non-adj}} = 2\,\bar{\sigma}_{\text{adj}} = \sqrt{2}$

Z_4 wall

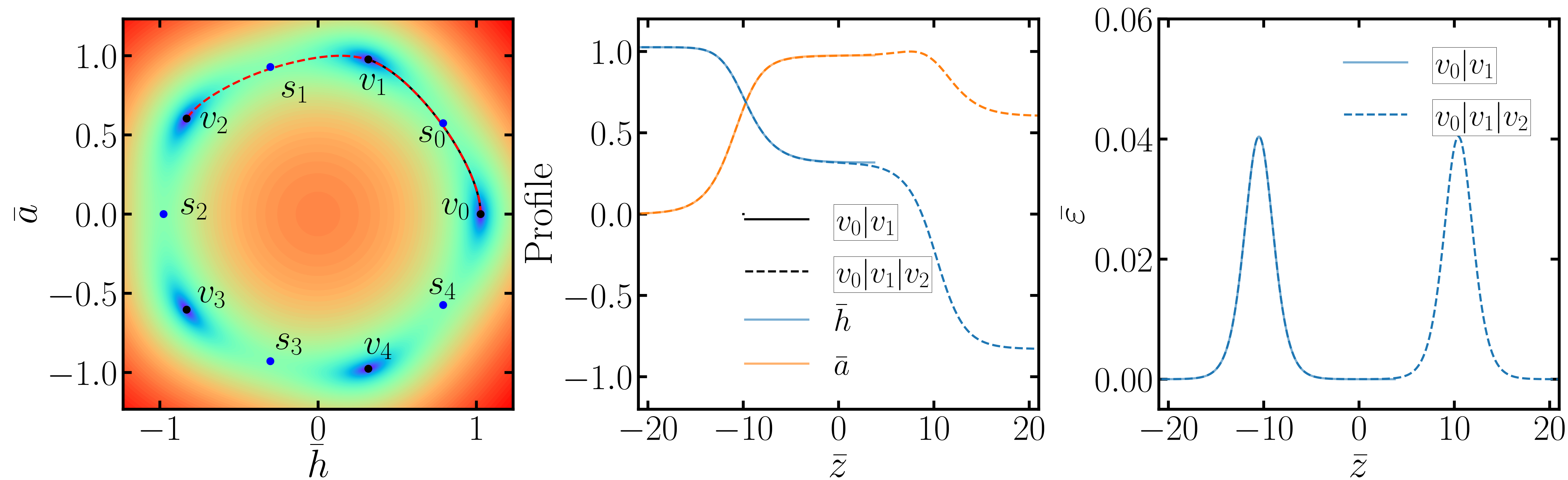
$\beta = 1/4$



当 $\beta < 1/3$ 时，$\sigma_2 > 2\sigma_1$，非邻近畴壁不稳定，会衰变为两个邻近畴壁

$$\boxed{v_0|v_2} \to \boxed{v_0|v_1|v_2} \to \boxed{v_0|v_1} + \boxed{v_1|v_2}$$

Z_N walls with small Z_N effects



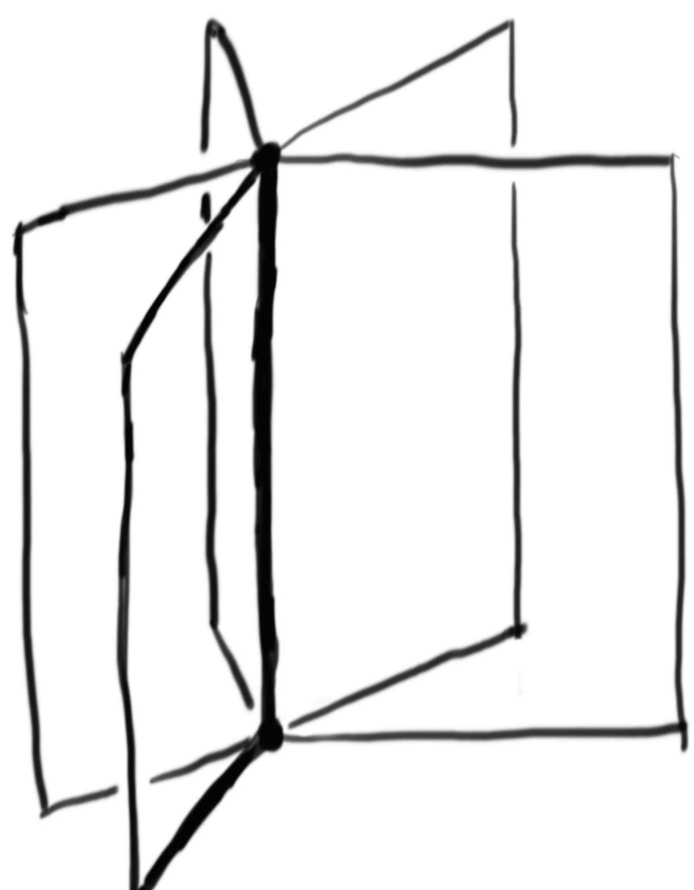
Approx U(1) $\phi = |\phi| e^{i\theta}$

$\downarrow \langle |\phi| \rangle \approx v$

Z_N

$\downarrow \langle \theta \rangle = 2\pi k / N$

1



String bounded by walls

其性质与类轴子模型中出现的畴壁相似。动力学演化可以直接参考后者的数值模拟结果，比如
Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 1207.3166; 1412.0789

Z_N walls with multiscalars

e.g., Z_6-invariant potential with two scalars

$$V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \frac{1}{2}\mu_\xi^2\xi^2 + \frac{1}{4}\lambda_\xi\xi^4 - \lambda_{\phi\xi}(\phi^3 + \phi^{*3})\xi$$

	Complex	Real
Scalar	ϕ	ξ
Z_6 charge	1	3

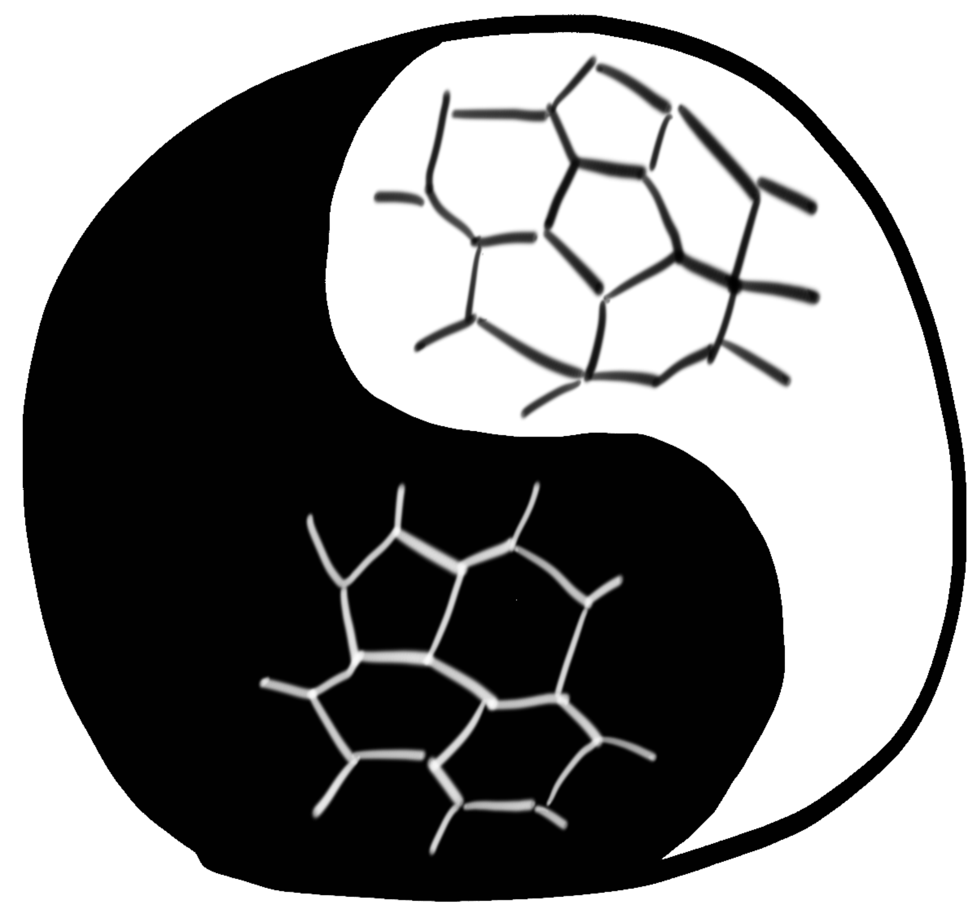
Z_6

↓ $\langle\xi\rangle = \pm\sqrt{\mu_\xi^2/2\lambda_\xi}$

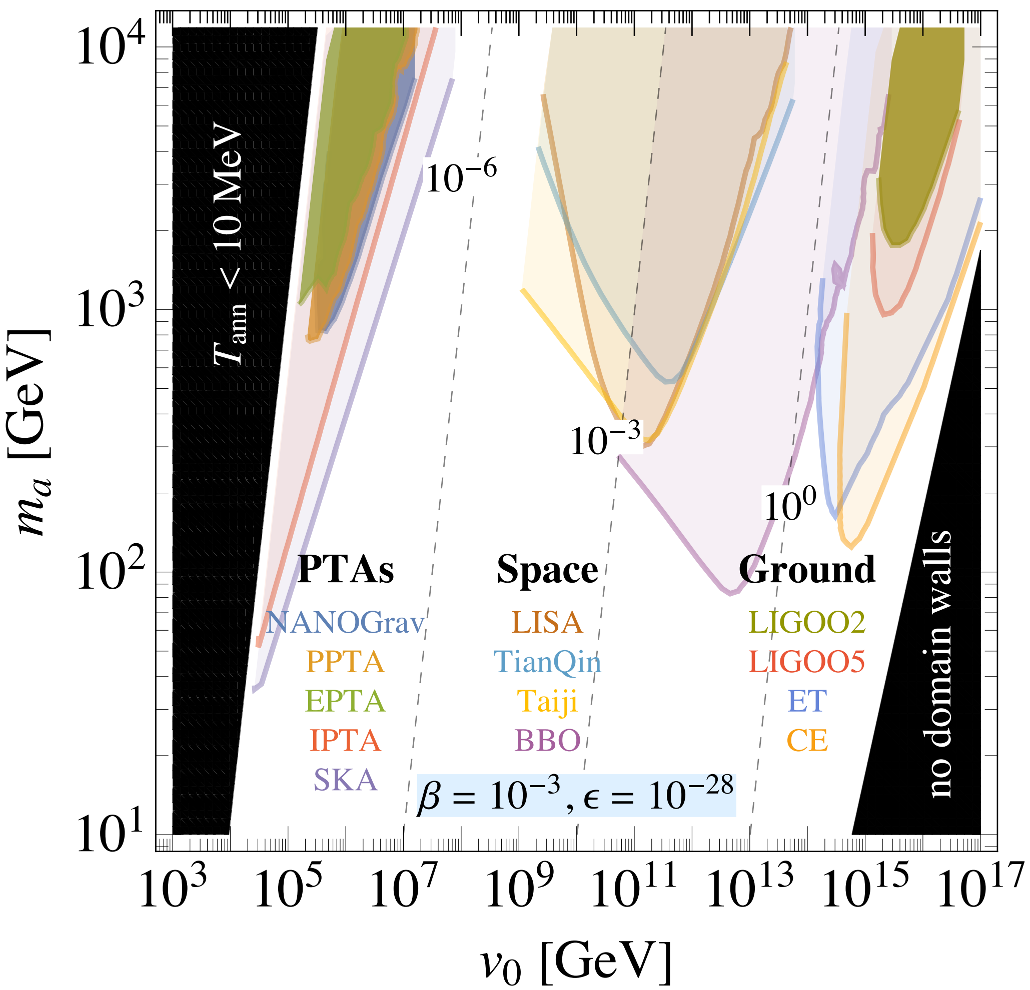
Z_3

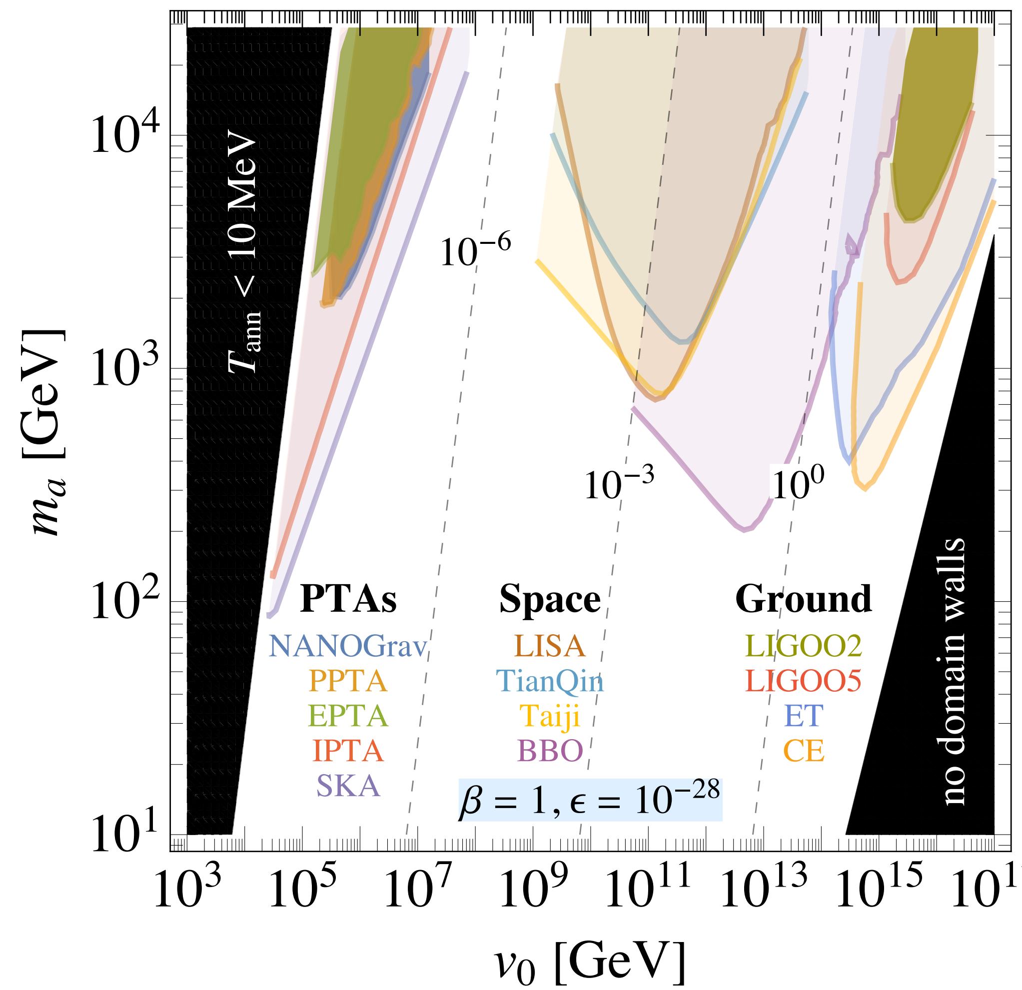
↓ $\langle\phi\rangle = \frac{\mu(\beta + \sqrt{1+\beta^2})}{\sqrt{2\lambda_1}}\, e^{\pm i2\pi k/3}$

1



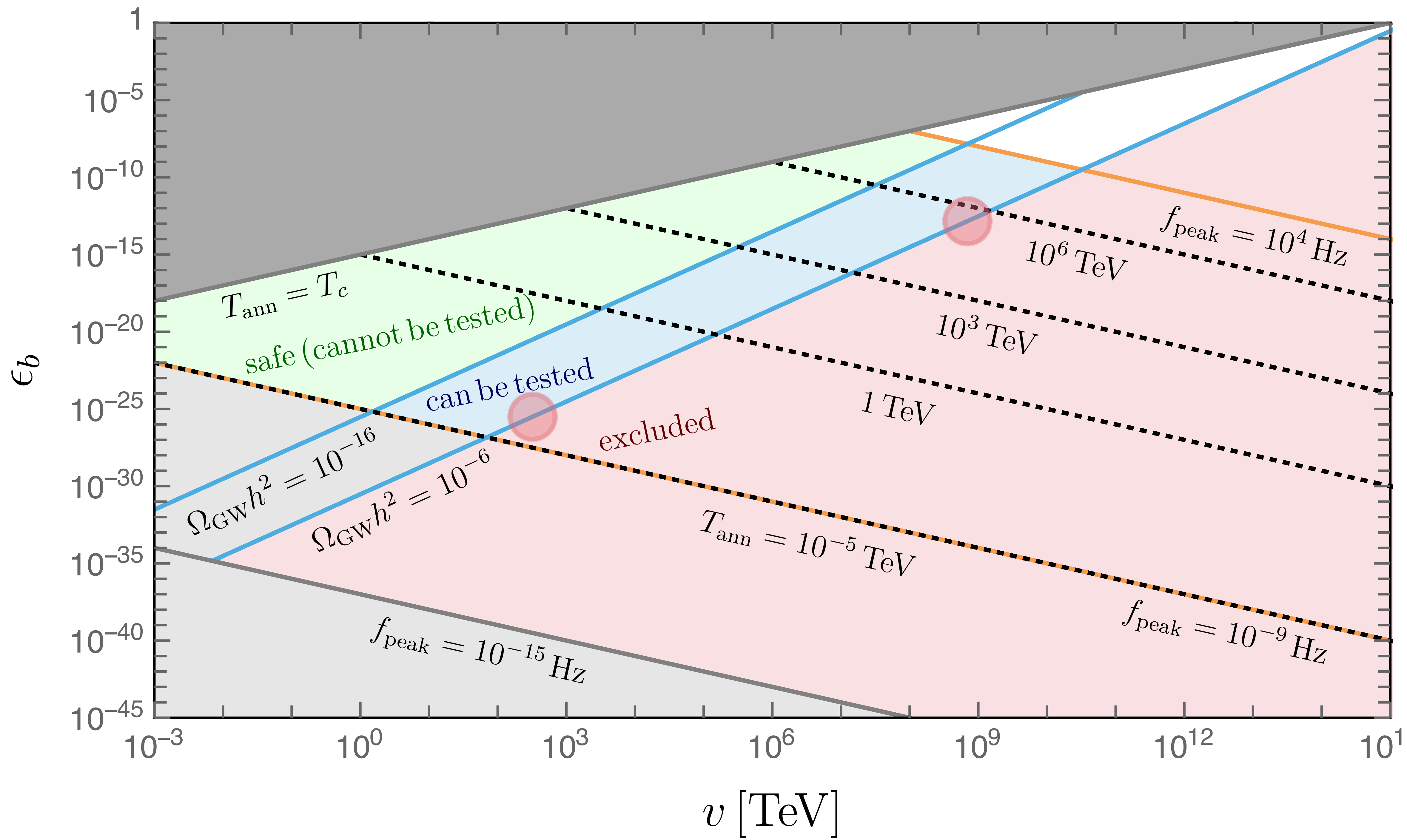
引力波观测对“Z_3对称性自发破缺”的检验能力





- 定量方面，Z_3导致的畴壁引力波与Z_2引力波不同。
- $\beta \ll 1$时，等效于类轴子模型中的畴壁引力波的结果；但$\beta \gtrsim 1$时不等效。
- 上述结论还很粗略，忽略了畴壁的动力学演化。

引力波观测对“非Abel分立对称性”的检验



畴壁引力波对A_4味对称性的检验：Gelmini, Pascoli, Vitagliano, YLZ, 2009.01903

感谢倾听！