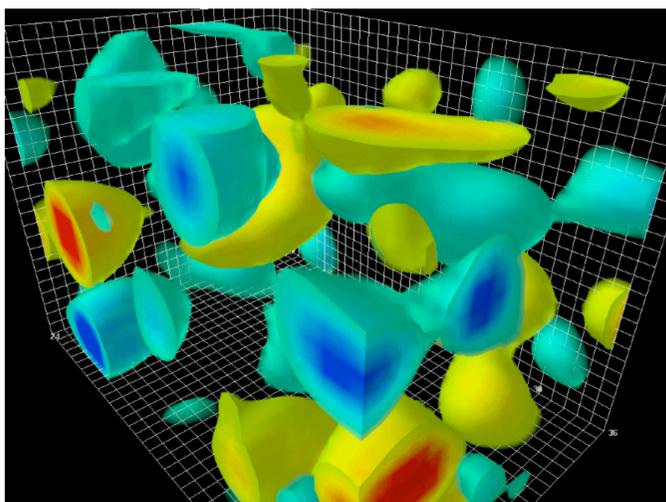


# New developments in Lattice QCD



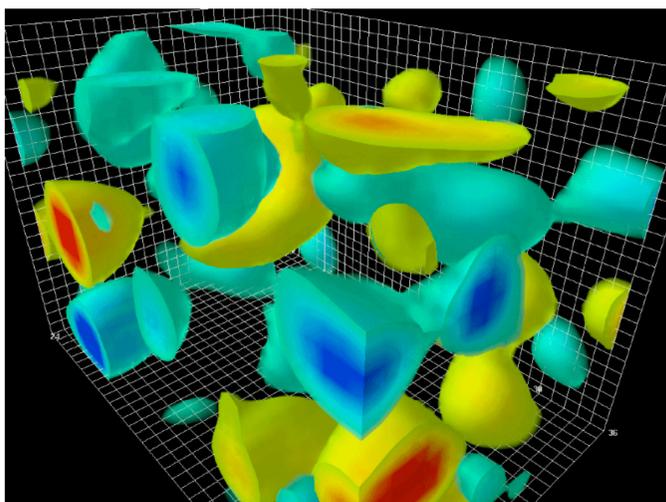
Heng-Tong Ding (丁亨通)  
Central China Normal University (华中师范大学)



中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会

大连，辽宁师范大学，2022.8.8-II

# New developments in Lattice QCD



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大连，辽宁师范大学，2022.8.8-II

# Lattice QCD talks in this conference



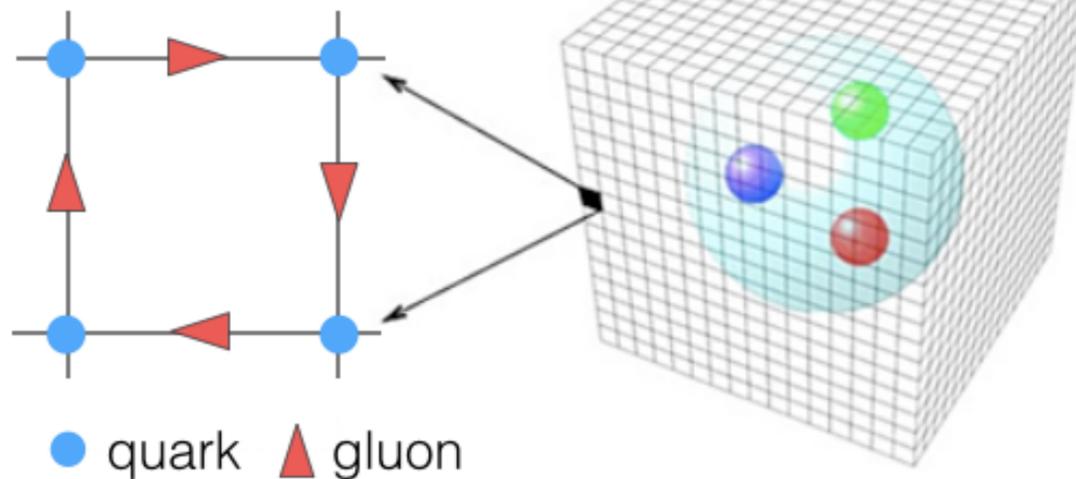
1. 孟 雨 [[Aug. 9, 15:45](#)] Lattice calculation of the  $\eta_c\eta_c$  and  $J/\psi J/\psi$  scattering length
2. 刘俊宏 [[Aug. 10, 11:15](#)] Fluctuations of conserved charges in strong magnetic fields
3. 傅 杨 [[Aug 10 , 14:20](#)], LQCD computation of two-photon exchange contribution to the muonic-hydrogen Lamb shift
4. 李胜泰 [[Aug. 10, 15:35](#)] Chiral condensates and screening masses of neutral pseudoscalar mesons
5. 黄玮平 [[Aug. 10, 15:50](#)] Criticality of QCD in correlated Dirac eigenvalues
6. 杨一玻 [[Aug. 10, 16:40](#)] Lattice QCD using Large momentum effective theory
7. 华 俊 [[Aug. 10, 17:00](#)] Pion and Kaon Distribution Amplitudes from Lattice QCD
8. 王选贺 [[Aug. 10, 19:30](#)] 质子电极化率的格点 QCD 计算
9. 马鹏翔 [[Aug. 10, 19:36](#)] 格点QCD在介子衰变辐射修正中的应用
10. 王子毓 [[Aug. 10, 19:42](#)] 双重子系统双 beta 衰变的格点计算
11. 脱心宇 [[Aug. 11, 14:00](#)] 格点计算稀有 K 介子衰变的新方案
12. 施 岐 [[Aug. 11, 14:15](#)] Lattice QCD prediction of kaon electromagnetic form factor at large  $Q^2$  up to  $10 \text{ GeV}^2$

# Supercomputing the QCD matter based on first principles

## Snowmass 2022: Theory & computational frontiers

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} [i\gamma^\mu(\partial_\mu - igA_\mu) - m_q] q$$



Discretization in  
Euclidean space

quarks: lattice sites  
gluons: lattice links

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{O} e^{-S_{lat}}$$

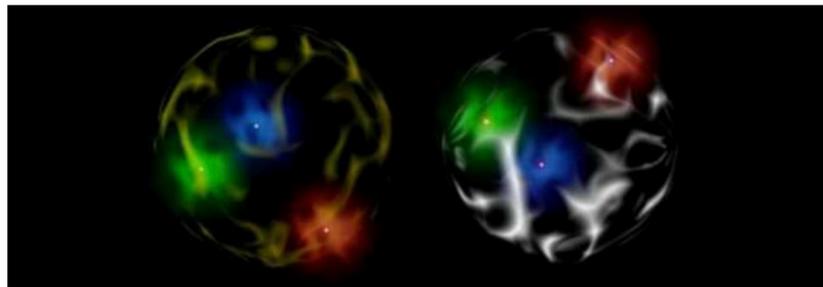
$$S_{lat} = S_g + S_f$$

$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_{lat}} = \int \mathcal{D}U e^{-S_g} \det M_f$$

Physics  
from  $\mu\text{eV}$  to  $\text{TeV}$

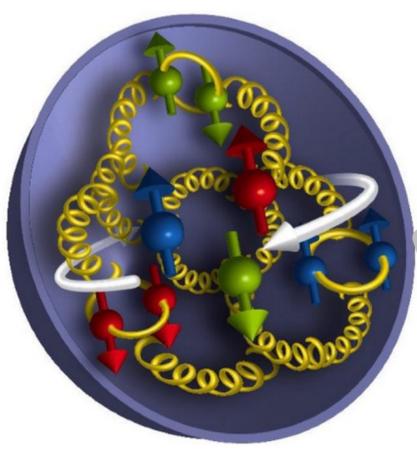
# Hadron spectroscopy & interactions

EIC,  
EicC,  
Jlab,  
...

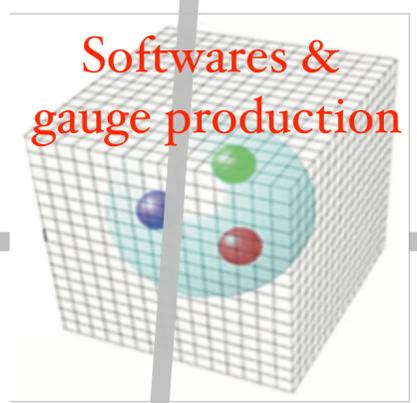


BESIII,  
LHC,  
...

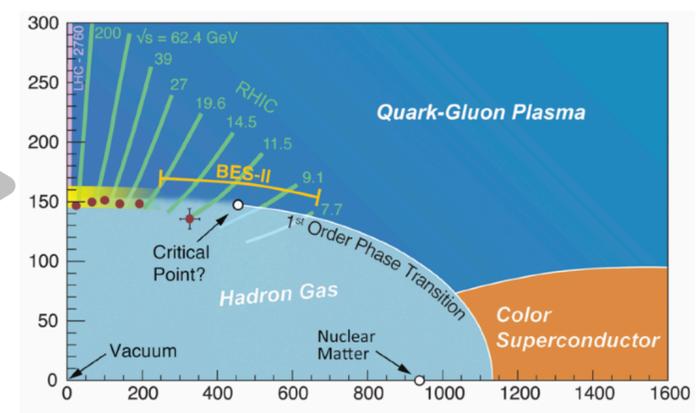
## Hadron structure



Softwares &  
gauge production

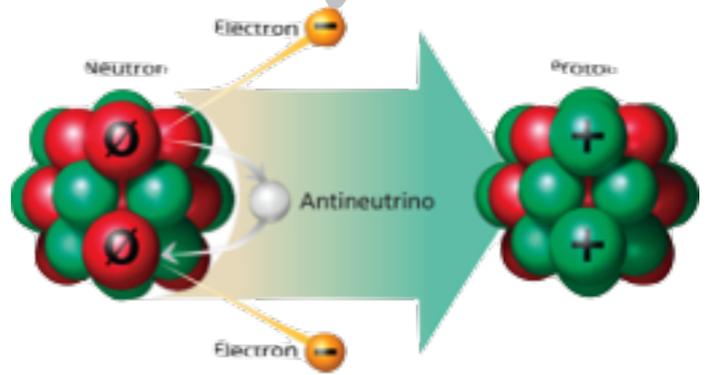


## Hot & dense QCD

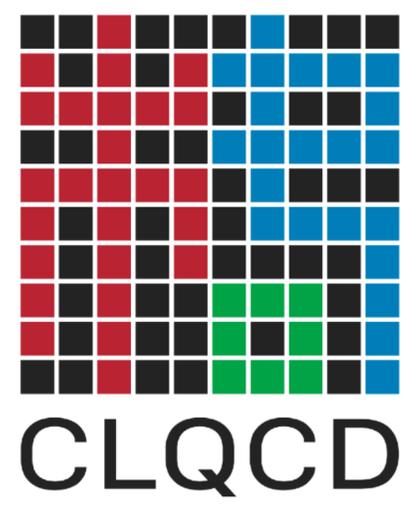


RHIC, LHC  
CEE, CBM, NICA  
...

BESIII, Belle2,  
CJPL, FNAL, LHC,  
...

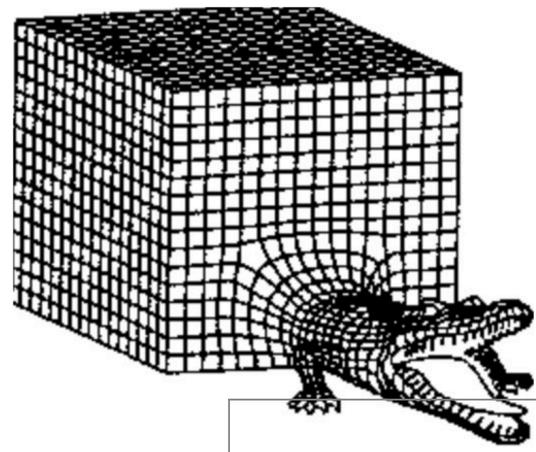


## Standard Model parameters



China Lattice QCD  
(CLQCD)  
collaboration

北大, 北航, 北师大,  
湖南师大, 华南师大,  
华中师大, 江苏大学,  
南开大学, 上海交大,  
四川大学, 西安工大,  
浙江大学, 郑州大学,  
中科院大学/高能所/  
近物所/理论所...



# Discretization schemes

Discretization schemes	Chiral symmetry	Cost	Collaborations
Wilson fermions	Explicitly broken	OK	CLQCD, ETMC, HALQCD, FlowQCD, PACS,...
Improved Staggered fermions	Partially reserved	Cheap	MILC, HotQCD, CLQCD, NPQCD, BMW,...
Domain Wall fermions	Almost reserved	Expensive	JLQCD, RBRC, UKQCD, TWQCD,...
Overlap fermions	Exactly reserved	Very expensive	chiQCD, Bielefeld-Chennai,...

To recover QCD, thermodynamic and continuum limits are required!

# Ensemble generation in CLQCD

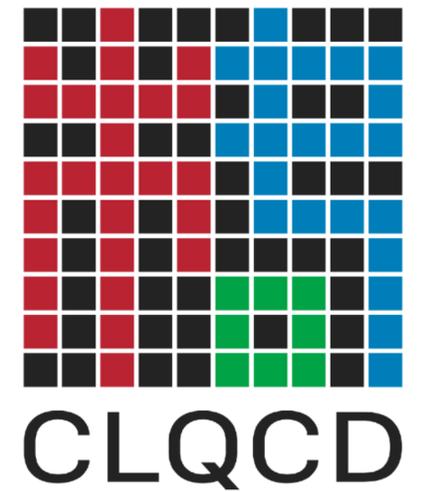
Zero temperature, Wilson-clover fermions, IHEP-IMP-ITP-SCNU



name	Volume	Lattice spacing	beta	Pion mass	Eta_s mass	L x M_pion	L	conf
C11P29S	24 <sup>3</sup> x72	0.105fm	6.20	290MeV	640MeV	3.8	2.6fm	900
C11P29M	32 <sup>3</sup> x64	0.105fm	6.20	290MeV	640MeV	5.0	3.5fm	700
C11P22M	32 <sup>3</sup> x64	0.105fm	6.20	220MeV	640MeV	3.9	3.5fm	450
C11P22L	48 <sup>3</sup> x96	0.105fm	6.20	220MeV	640MeV	5.6	5.4fm	400
C11P14L	48 <sup>3</sup> x96	0.105fm	6.20	135MeV	700MeV	3.4	5.4fm	60
C08P30S	32 <sup>3</sup> x96	0.08fm	6.41	300MeV	650MeV	3.9	2.6fm	500
C08P30M	48 <sup>3</sup> x96	0.08fm	6.41	300MeV	650MeV	5.0	3.8fm	400
C08P21S	32 <sup>3</sup> x64	0.08fm	6.41	210MeV	650MeV	4.1	2.6fm	460
C08P21M	48 <sup>3</sup> x96	0.08fm	6.41	210MeV	650MeV	4.1	3.8fm	450
C06P30S	48 <sup>3</sup> x144	0.054fm	6.72	300MeV	650MeV	4.0	2.6fm	300

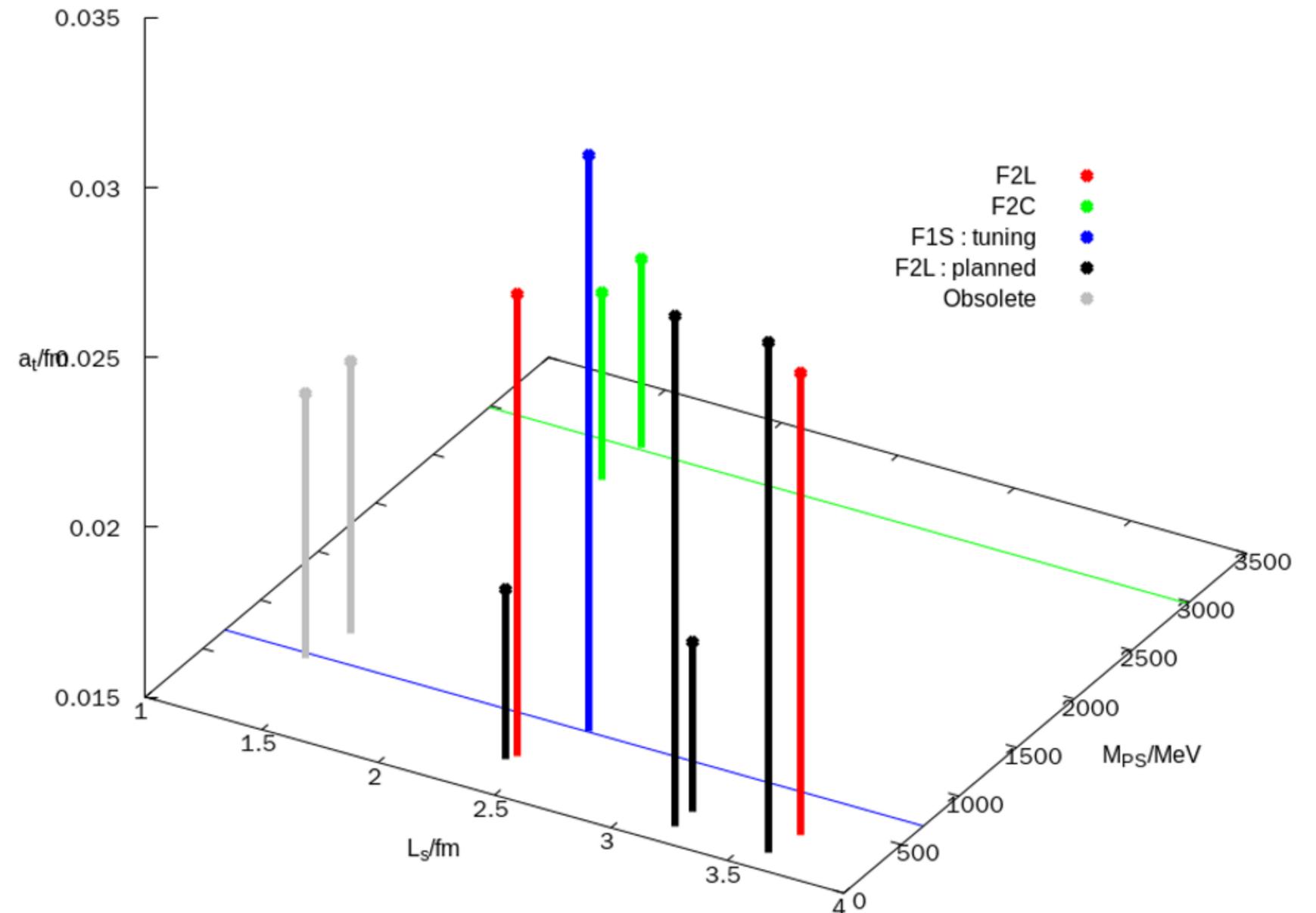
# Ensemble generation in CLQCD

Zero temperature,  
anisotropic Wilson-clover fermions, IHEP



Charmed hadrons, glueballs  
and exotic particles

陈莹 et al., arXiv:2207.04694, 2206.02724,  
arXiv:2206.06185, 2205.12541 [hep-lat],  
Chin. Phys. C46 (2022) 043102 ,  
Phys. Lett. B 827 (2022) 136960,  
Chin. Phys. C42 (2018) 093103  
... ..



# Ensemble generation in CLQCD

Nonzero & zero temperature, HISQ fermions, CCNU



## Nonzero magnetic field ensembles

$N_b$	$eB$ [GeV <sup>2</sup> ]	$N_b$	$eB$ [GeV <sup>2</sup> ]	$N_\sigma^3 \times N_\tau$	$T$ [MeV]	# conf.
0	0	16	0.836	$32^3 \times 6$	280.9	$\mathcal{O}(4000)$
1	0.052	20	1.045	$32^3 \times 8$	210.8	$\mathcal{O}(5000)$
2	0.104	24	1.255	$32^3 \times 10$	168.5	$\mathcal{O}(5000)$
3	0.157	32	1.673	$32^3 \times 12$	140.4	$\mathcal{O}(5000)$
4	0.209	40	2.09	$32^3 \times 14$	120.4	$\mathcal{O}(5000)$
6	0.314	48	2.510	$32^3 \times 16$	105.3	$\mathcal{O}(6000)$
8	0.418	-	-	$32^3 \times 18$	93.6	$\mathcal{O}(6000)$
10	0.523	-	-	$32^3 \times 24$	70.2	$\mathcal{O}(1000)$
12	0.627	-	-	$32^3 \times 96$	17.6	$\mathcal{O}(3000)$

丁亨通 et al., Phys.Rev.D 105 (2022) 034514

Phys.Rev.Lett. 126 (2021) 082001,

Phys.Rev.D 104 (2021) 014505,

Eur.Phys.J.A 57 (2021) 202,

Phys.Rev.D 102 (2020) 5, 054505,

Phys.Rev. Lett. 123 (2019) 062002,

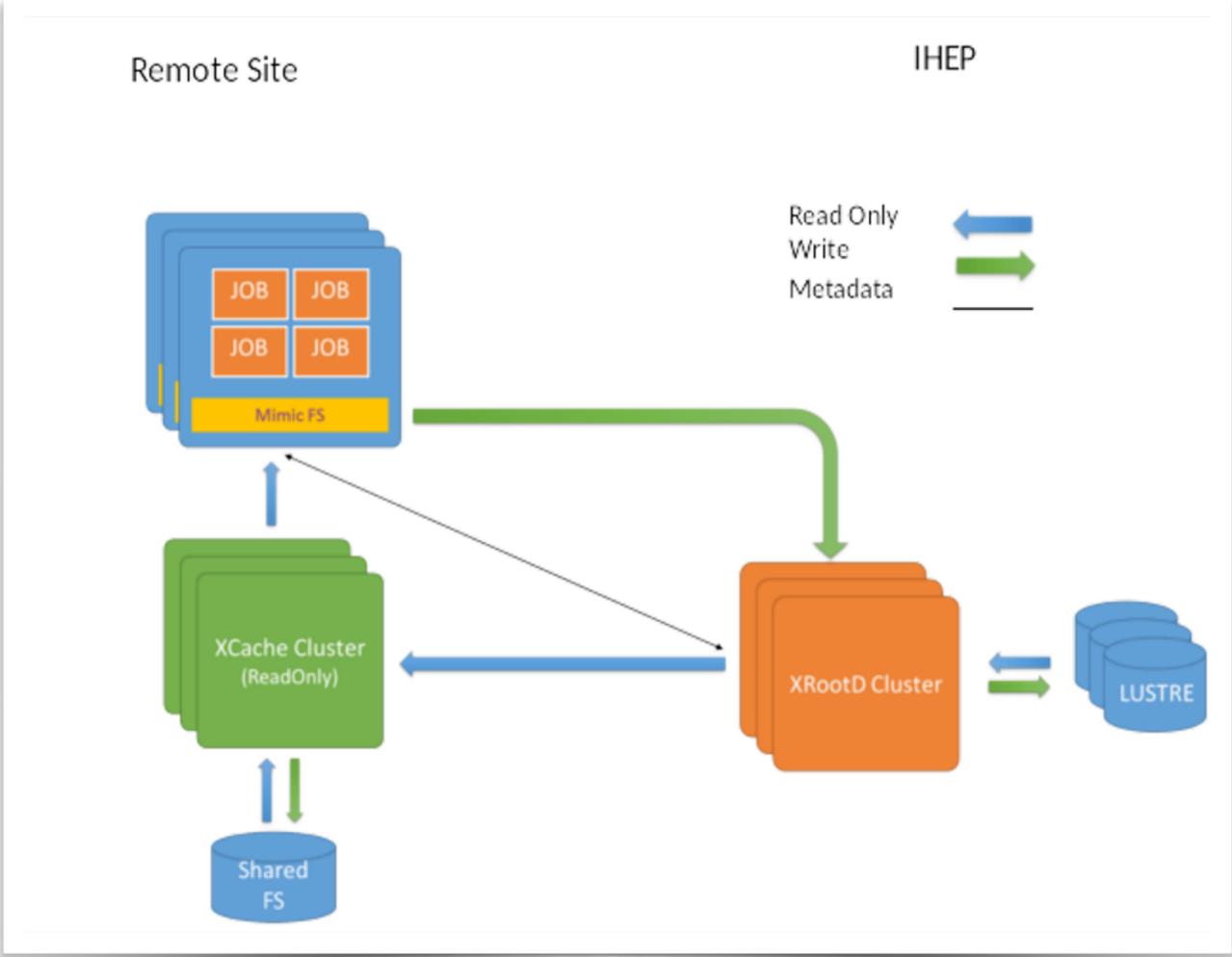
... ..

## Small quark mass ensembles

$\beta$	$am_s$	$am_l$	$m_\pi$ [MeV]	$N_\sigma^3 \times N_\tau$	#conf
6.664	0.0514				
		0.002570	160	$32^3 \times 8$	4054
		0.001904	140	$32^3 \times 8$	6707
		0.001285	110	$40^3 \times 8$	5625
		0.0006425	80	$32^3 \times 8$	3842
		0.0006425	80	$40^3 \times 8$	11863
7.078	0.034	0.0006425	80	$56^3 \times 8$	7341
		0.0006425	80	$72^3 \times 8$	5954
		0.00032125	55	$56^3 \times 8$	8473
		0.00170	160	$48^3 \times 12$	8507
		0.001259	140	$48^3 \times 12$	6575
		0.000850	110	$60^3 \times 12$	5314
7.356	0.026	0.000425	80	$48^3 \times 12$	9021
		0.000425	80	$60^3 \times 12$	6746
		0.000425	80	$72^3 \times 12$	2365
		0.0013	160	$64^3 \times 16$	3227
		0.000963	140	$64^3 \times 16$	3639
		0.000650	110	$64^3 \times 16$	3498
		0.000325	80	$64^3 \times 16$	4092
		0.000325	80	$80^3 \times 16$	5316

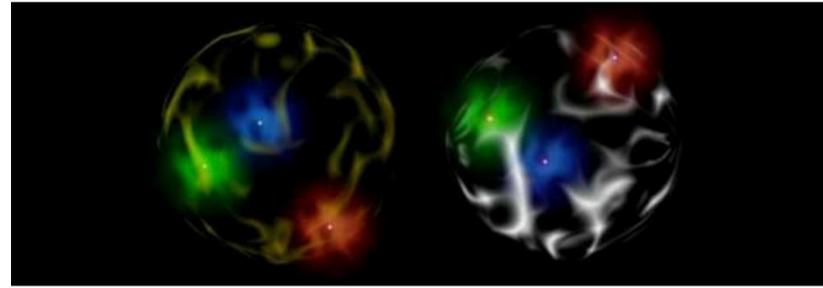
... ..

# Lattice QCD data sharing within CLQCD and beyond

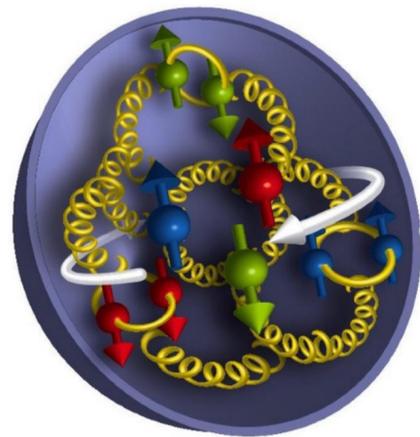


孙鹏, 宫明, 刘柳明, 杨一玻, 丁亨通, [CLQCD], Lattice 2022 talk  
<https://indico.hiskp.uni-bonn.de/event/40/contributions/853/>

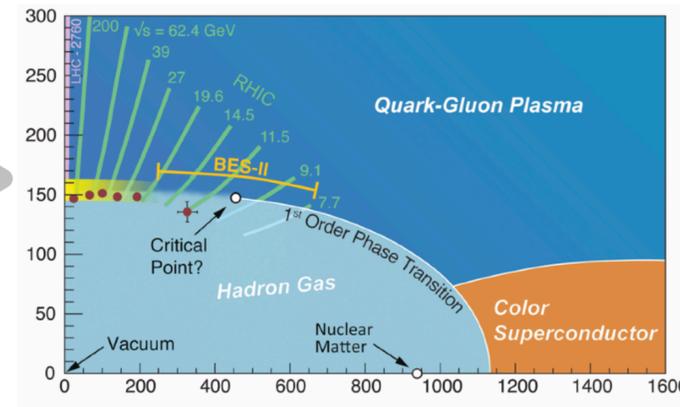
# Hadron spectroscopy & interactions



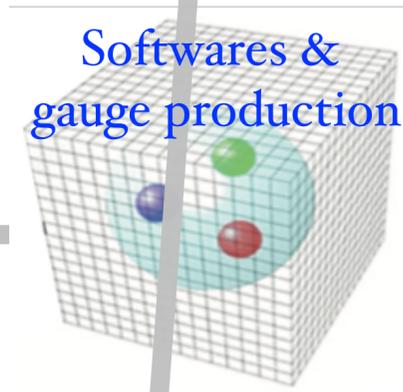
Hadron structure



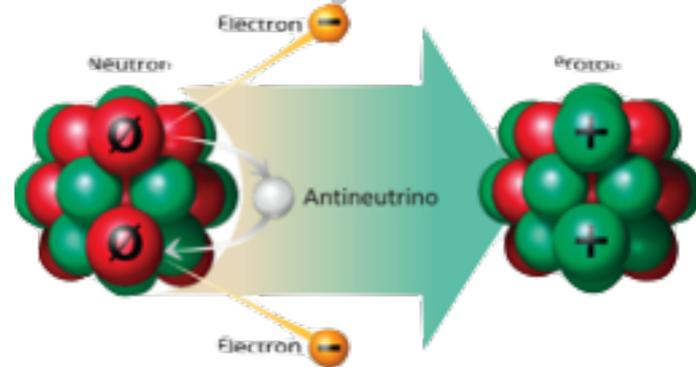
Hot & dense QCD



Softwares & gauge production



Standard Model parameters



孟雨 [Aug. 9, 15:45]

Lattice calculation of the  $\eta_c\eta_c$  and  $J/\psi J/\psi$  scattering length

...

# Lattice calculation of the $\eta_c\eta_c$ and $J/\psi J/\psi$ scattering length

孟雨

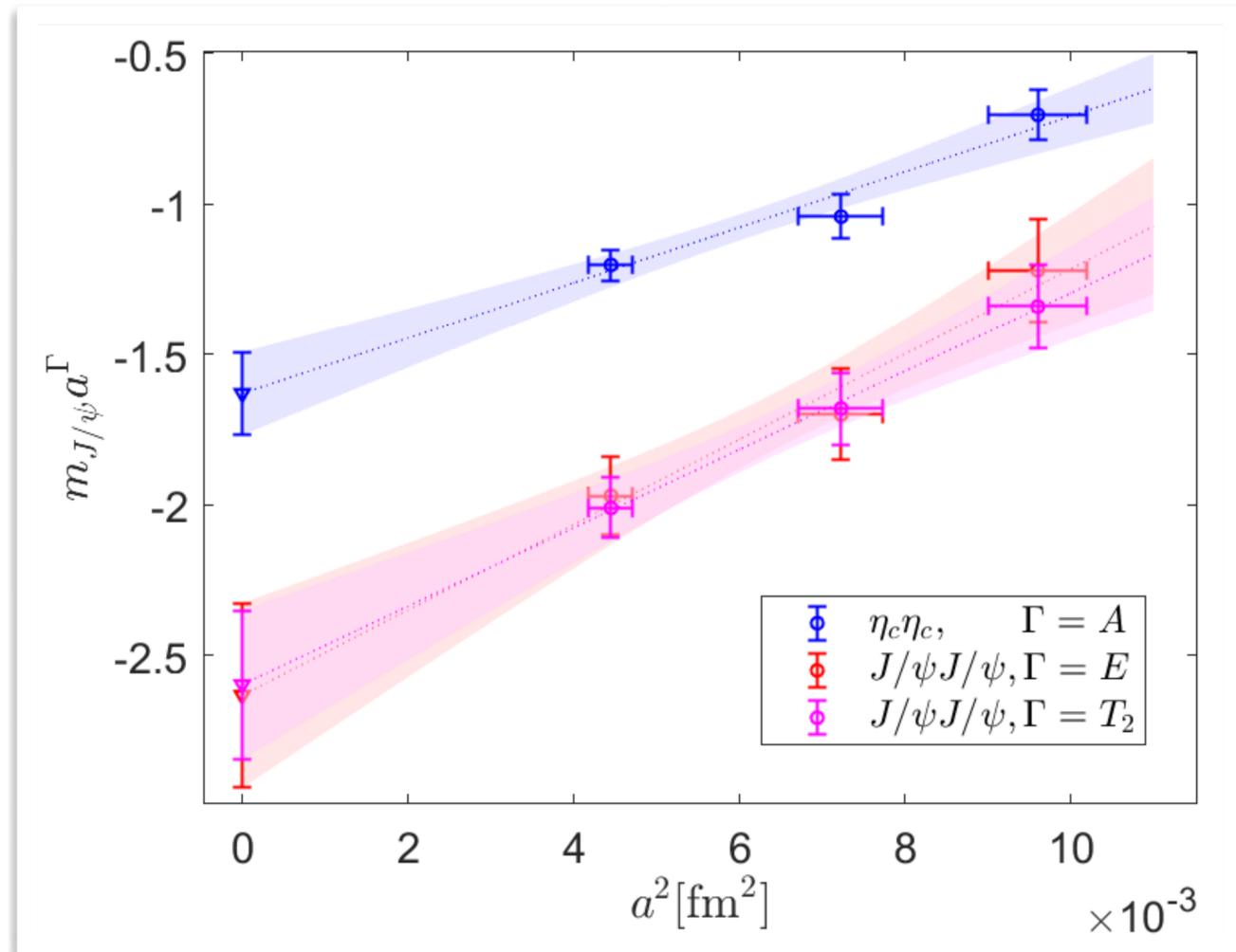
Tue Aug. 9

15:45-16:00

LHCb, CMS & ATLAS: Narrow structures in  $J/\psi J/\psi$  invariant mass spectrum

Interpretations vary from model studies

Lüscher method: 
$$\delta E = -\frac{4\pi a^\Gamma}{mL^3} \left[ 1 + c_1 \frac{a^\Gamma}{L} + c_2 \left( \frac{a^\Gamma}{L} \right)^2 + \mathcal{O}(L^{-3}) \right]$$



## S-wave scattering length

$$a_{\eta_c\eta_c}^{0+} = -0.104(09) \text{ fm}$$

$$a_{J/\psi J/\psi}^{2+} = -0.165(16) \text{ fm}$$

## Weak repulsive interaction

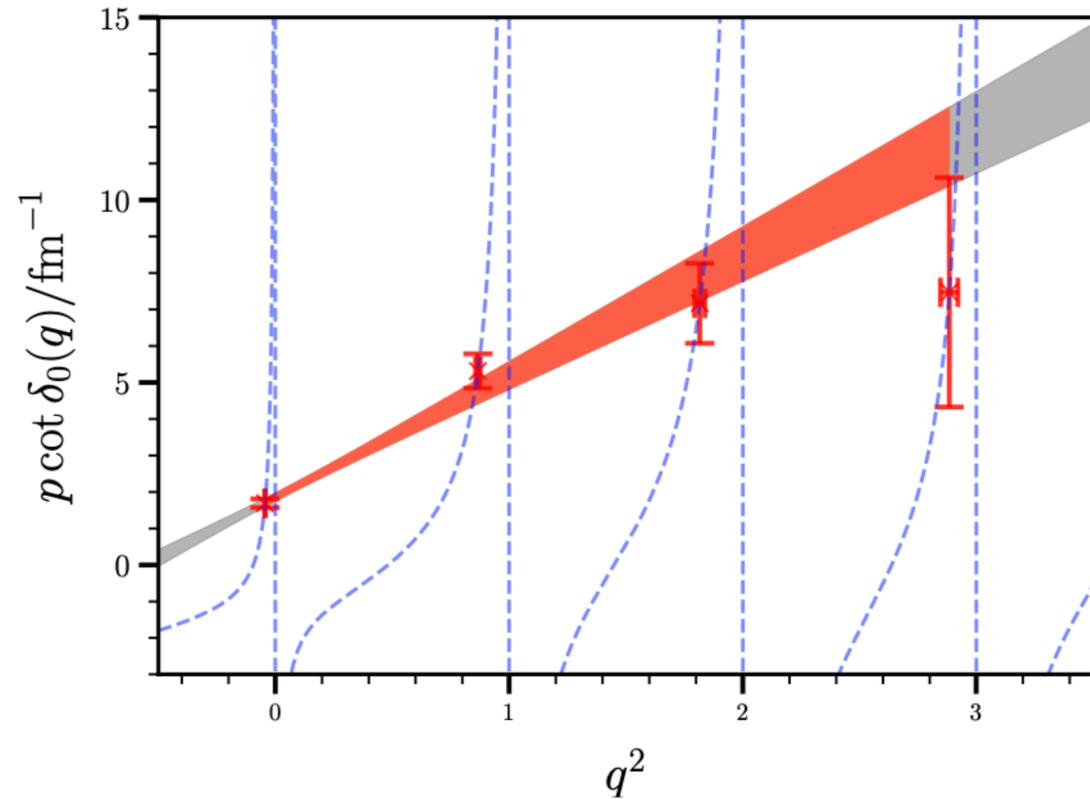
孟雨 et al., work in preparation

# $T_{cc}^+(3875)$ relevant S-wave $DD^*$ scattering from $N_f = 2$ LQCD

S. Chen et al., IHEP,  
arXiv:2206.06185

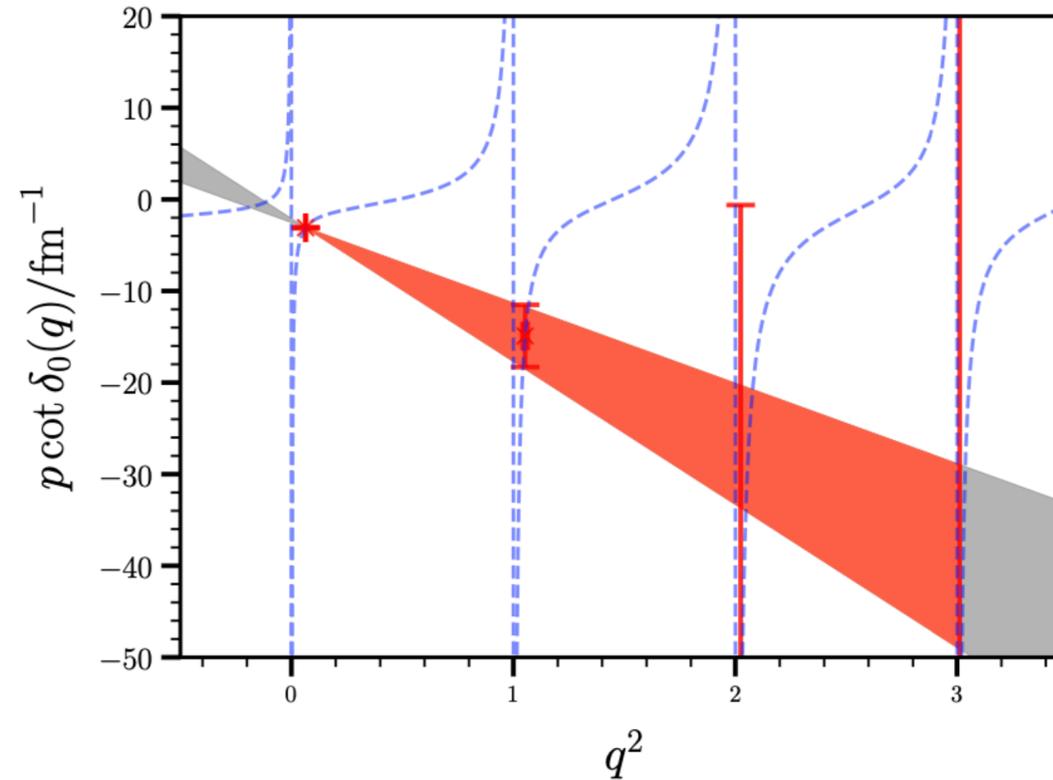
$$I = 0 : |DD^*\rangle = \frac{1}{\sqrt{2}} (|D^0 D^{*+}\rangle - |D^+ D^{*0}\rangle)$$

$$I = 1 : |DD^*\rangle = \frac{1}{\sqrt{2}} (|D^0 D^{*+}\rangle + |D^+ D^{*0}\rangle)$$



$$a_0^{(I=0)} = 0.538(33) \text{ fm}, \quad r_0^{(I=0)} = 0.99(11) \text{ fm}.$$

Attractive interaction with  
scattering in a wide  
momentum range



$$a_0^{(I=1)} = -0.433(43) \text{ fm}, \quad r_0^{(I=1)} = -3.6(1.0) \text{ fm}.$$

Repulsive interaction  
near the  $DD^*$  threshold

Conceptually in  
agreement:  
 $T_{cc}^+(3875)$  only  
found in  $D^0 D^{*+}$   
system

# Hidden-charm Hexaquarks ( $usc\bar{d}\bar{s}\bar{c}$ )

$$J^{PC} = 0^{++}, 0^{-+}, 1^{++} \text{ \& } 1^{--}$$

$$0^{++} : \mathcal{O}_1 = \epsilon^{abc} \epsilon^{def} [u_a^T C \gamma_5 s_b] [\bar{d}_d C \gamma_5 \bar{s}_e^T] \times [\bar{c}_f c_c]$$

$$0^{-+} : \mathcal{O}_2 = \epsilon^{abc} \epsilon^{def} [u_a^T C \gamma_5 s_b] [\bar{c}_d C \gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_5 c_c]$$

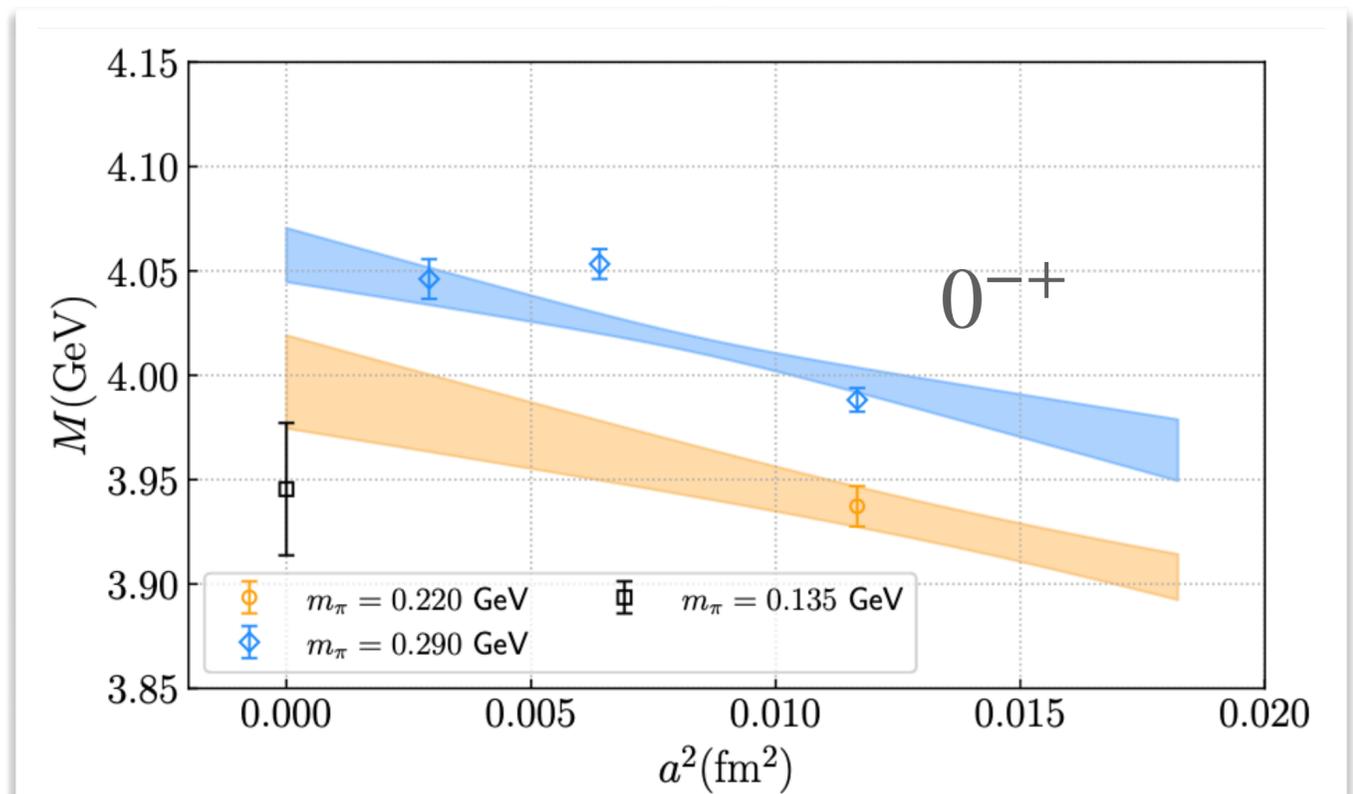
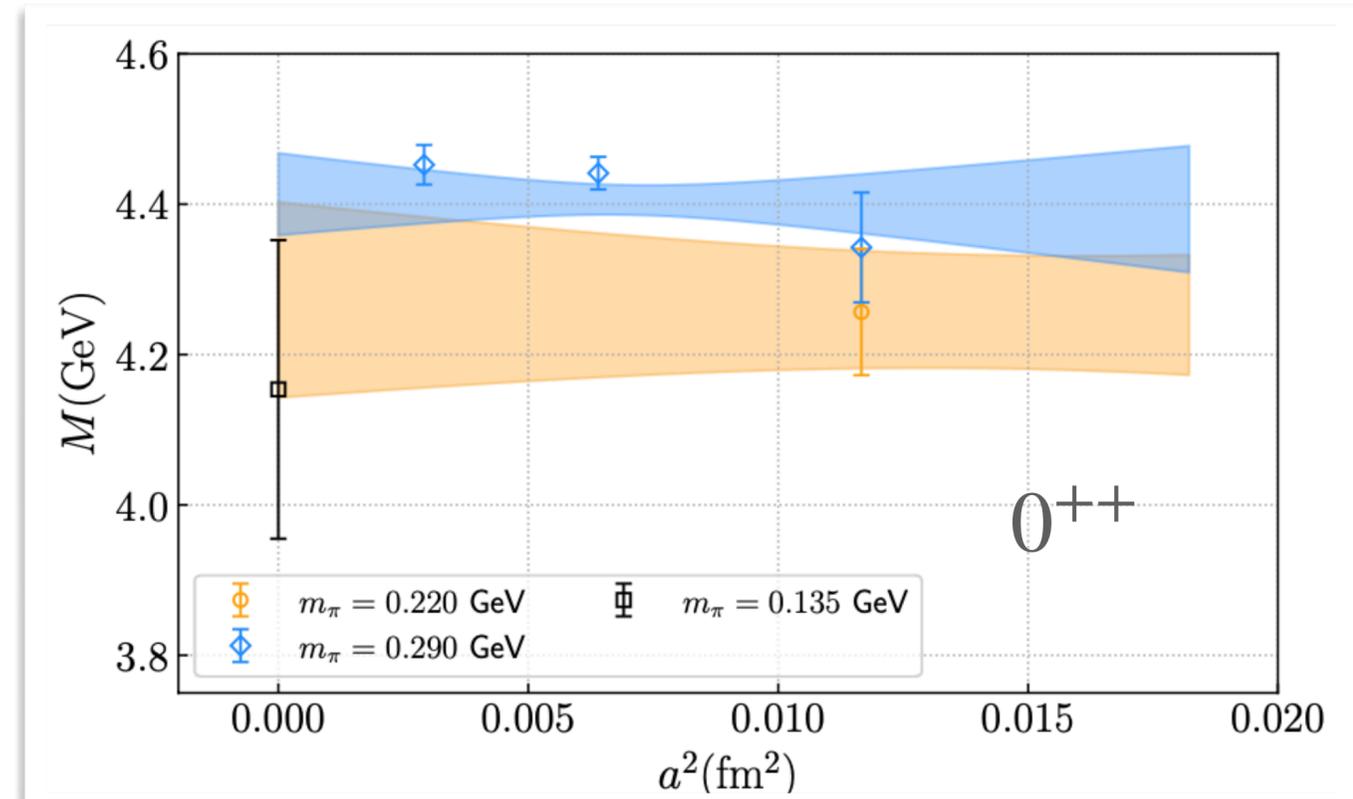
$$1^{++} : \mathcal{O}_3 = \epsilon^{abc} \epsilon^{def} [u_a^T C \gamma_5 s_b] [\bar{d}_d C \gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_i \gamma_5 c_c]$$

$$1^{--} : \mathcal{O}_4 = \epsilon^{abc} \epsilon^{def} [u_a^T C \gamma_5 s_b] [\bar{d}_d C \gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_i c_c]$$

$I(J^{PC})$	$1(0^{-+})$	$1(1^{--})$	$1(0^{++})$	$1(1^{++})$
mass(GeV)	3.945(32)	4.144(35)	4.15(20)	4.22(23)

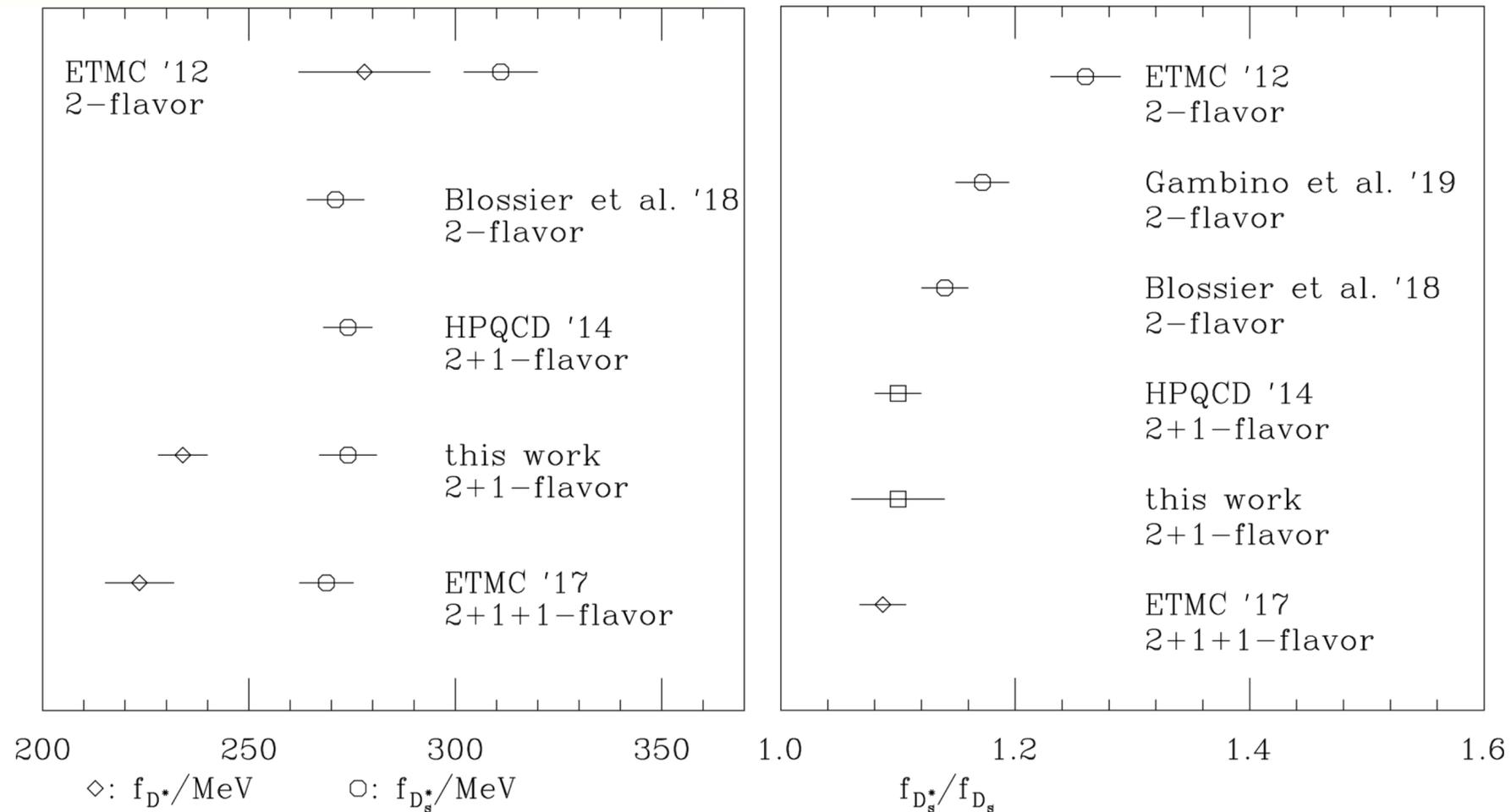
All below  $\Xi_c \bar{\Xi}_c$  threshold  
the lightest one around  $\eta_c K^+ K^-$  threshold

H. Liu et al., SJTU-IMP-ITP-BUAA, arXiv: 2207.00183



# Charmed and $\phi$ meson decay constants in $N_f=2+1$ LQCD

Domain wall fermions with physical pion mass



**Table 9.** Decay constants of  $D_{(s)}^{(*)}$  and  $\phi$  in units of MeV.  $f_V^T/f_V$  is given in the  $\overline{\text{MS}}$  scheme at the scale of 2 GeV.

	$D_s$	$D_s^*$	$D$	$D^*$	$\phi$
$f_{P/V}/\text{MeV}$	249(7)	274(7)	213(5)	234(6)	241(9)
$f_V^T/f_V$	–	0.92(4)	–	0.91(4)	–

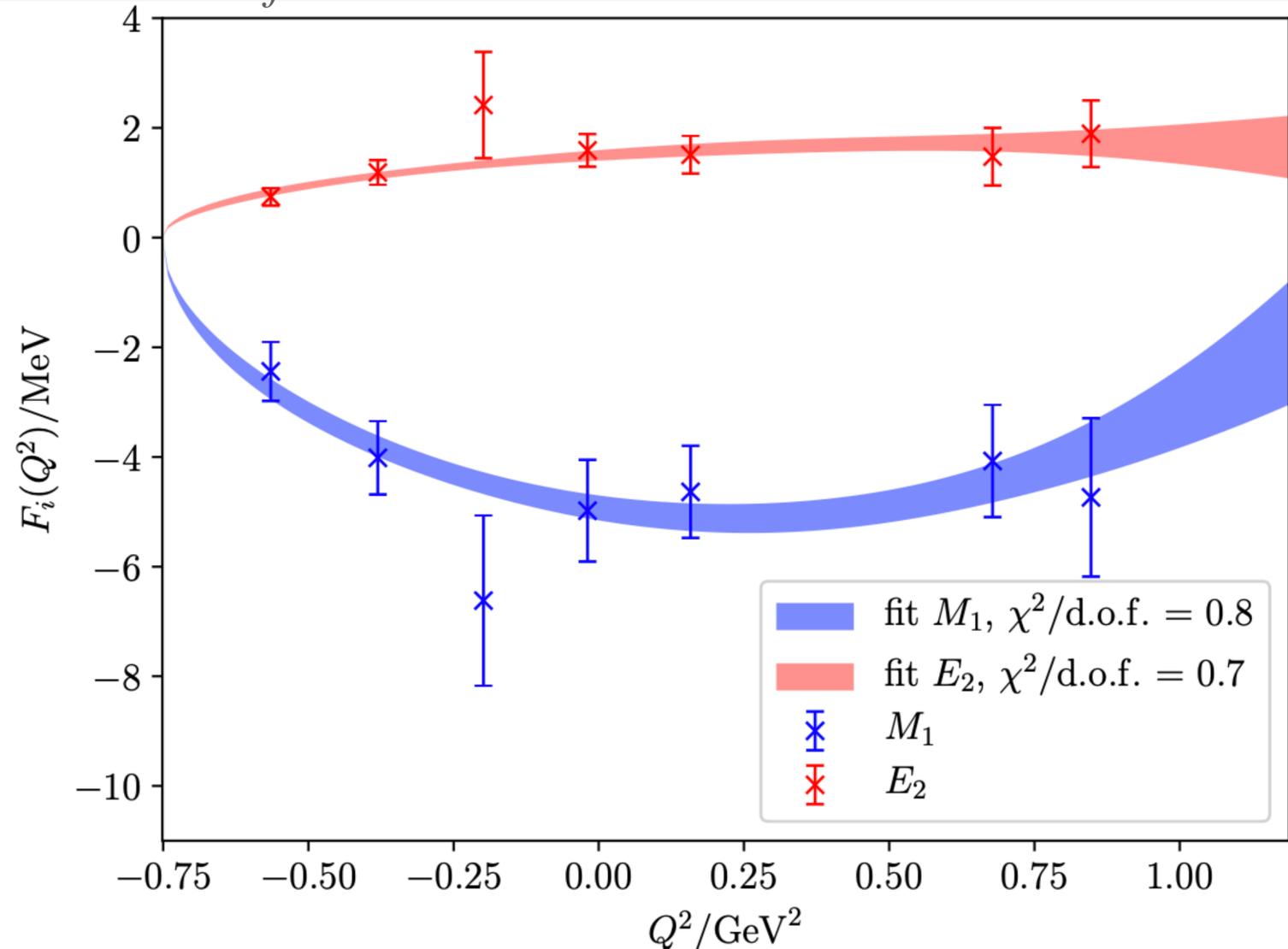
A first computation of couplings of  $D^*$  and  $D_s^*$  to the tensor current  $f_V^T$

陈莹, 邱伟峰, 宫明, 刘朝峰 & 马运恒, IHEP, [ $\chi$ QCD], Chin. Phys. C 45(2021)023109

# $1^{-+}$ Hybrid in $J/\psi$ radiative decays

Nature of  $\eta_1(1855)$ ?

$N_f=2$  QCD with  $m_\pi \approx 350$  MeV



$$\Gamma(J/\psi \rightarrow \gamma\eta_1) = \frac{4\alpha}{27} \frac{|\vec{p}_\gamma|}{m_{J/\psi}^2} (|M_1(0)|^2 + |E_2(0)|^2)$$

$$\Gamma(J/\psi \rightarrow \gamma\eta_1) = 2.29(77) \text{ eV}$$

Consider  $\eta_1(1855)$  is an  $1^{-+}$  hybrid

$$\text{Br}(J/\psi \rightarrow \gamma\eta_1(1855)) = 6.2(2.2) \times 10^{-5}$$

$$\text{Br}(\eta_1(1855) \rightarrow \eta\eta') \sim 4.3 \%$$

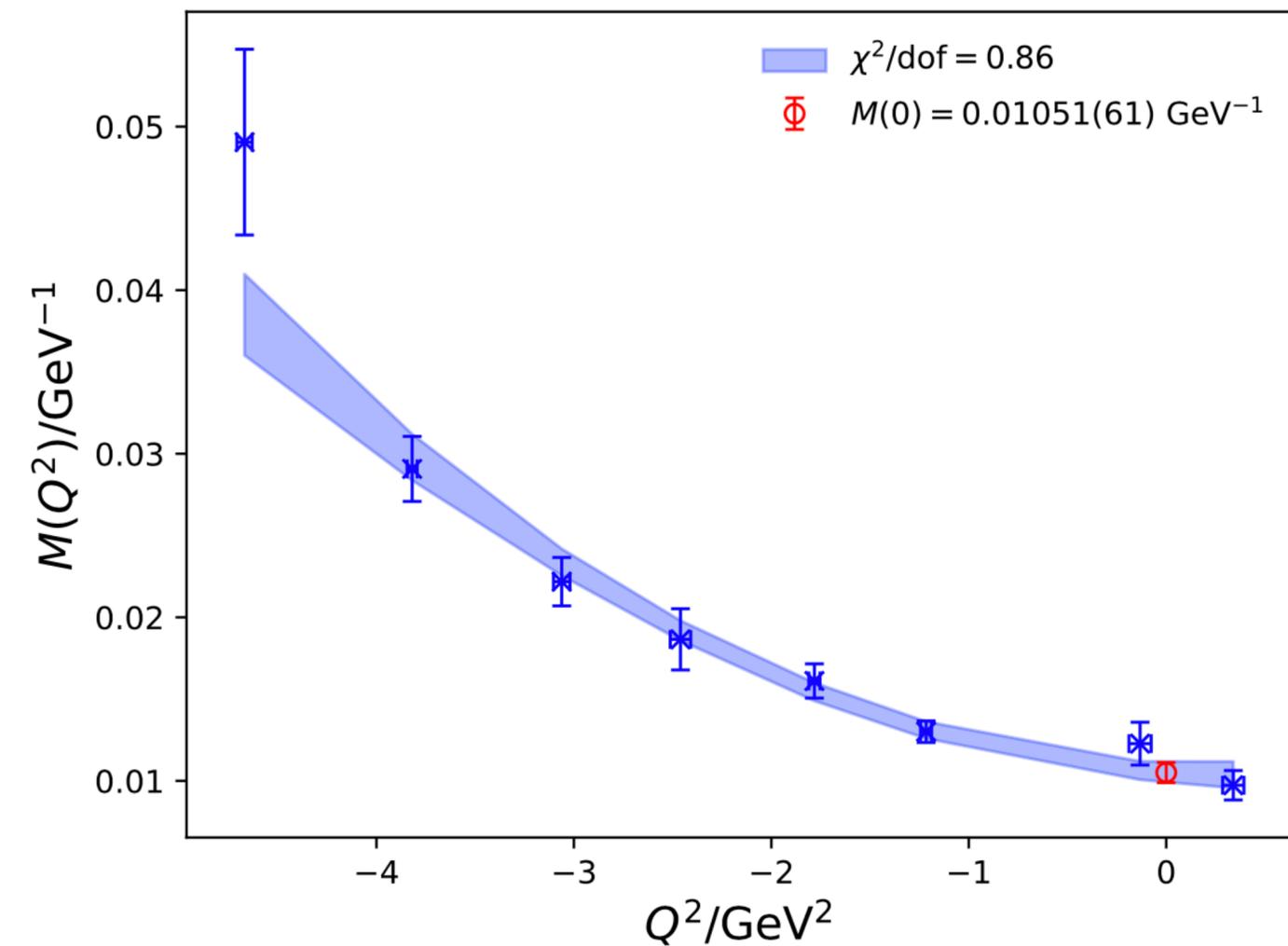
F. Chen et al., IHEP, arXiv:2207.04694

# A first LQCD computation on radiative decay width of $J/\psi \rightarrow \gamma\eta$

## Effect of axial U<sub>1</sub> anomaly

X. Jiang et al., IHEP,  
arXiv:2206.02724

$$\Gamma(J/\psi \rightarrow \gamma\eta) = \frac{4\alpha}{27} |\vec{q}_\gamma|^3 M^2(0)$$



📌  $N_f = 2$  QCD with  $m_\pi \approx 350$  MeV

$$\Gamma(J/\psi \rightarrow \gamma\eta) = 0.385(45) \text{ keV}$$

$$\text{Br}(J/\psi \rightarrow \gamma\eta) = 4.16(49) \times 10^{-3}$$

➤ Extrapolating to  $N_f = 3$  considering  $\eta - \eta'$  mixing

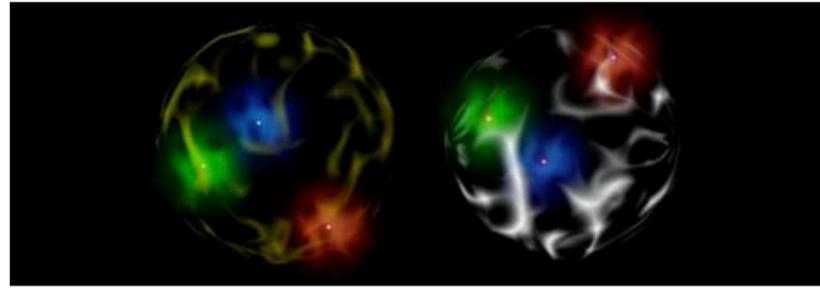
$$\text{Br}(J/\psi \rightarrow \gamma\eta) = 1.15(14) \times 10^{-3}$$

$$\text{Br}(J/\psi \rightarrow \gamma\eta') = 4.49(53) \times 10^{-3}$$

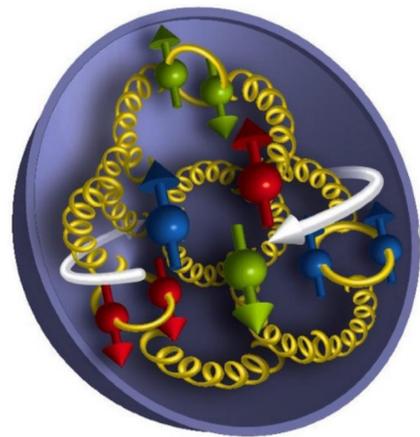
In good agreement with PDG 2020 values

$$\text{Br}(J/\psi \rightarrow \gamma\eta) = 1.11(3) \times 10^{-3}, \text{Br}(J/\psi \rightarrow \gamma\eta') = 5.25(7) \times 10^{-3}$$

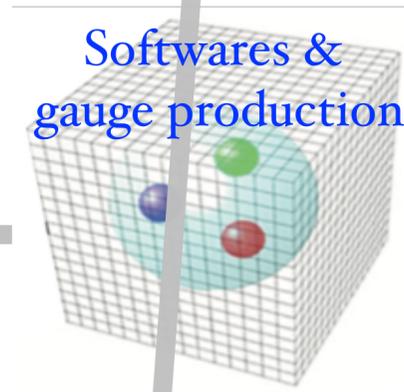
# Hadron spectroscopy & interactions



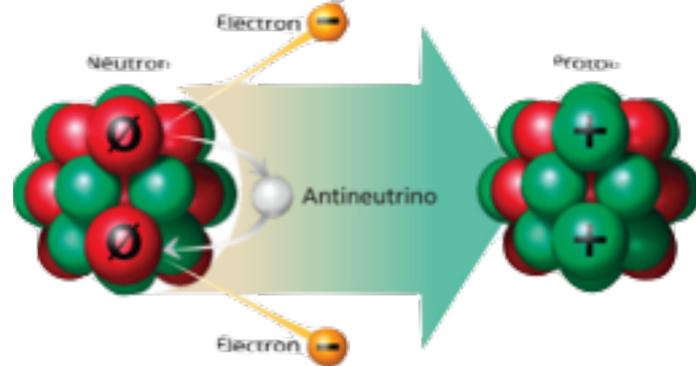
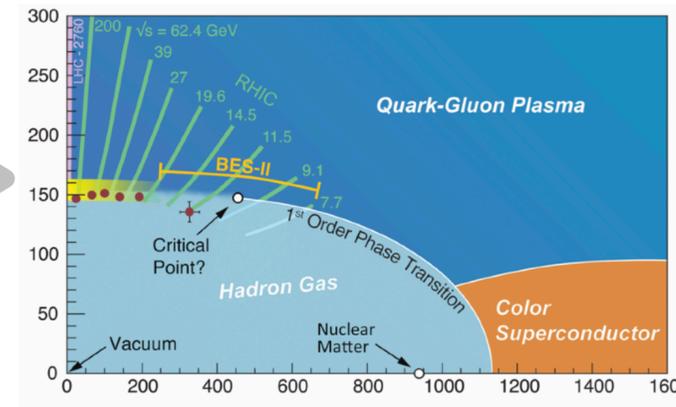
Hadron structure



Softwares & gauge production



Hot & dense QCD

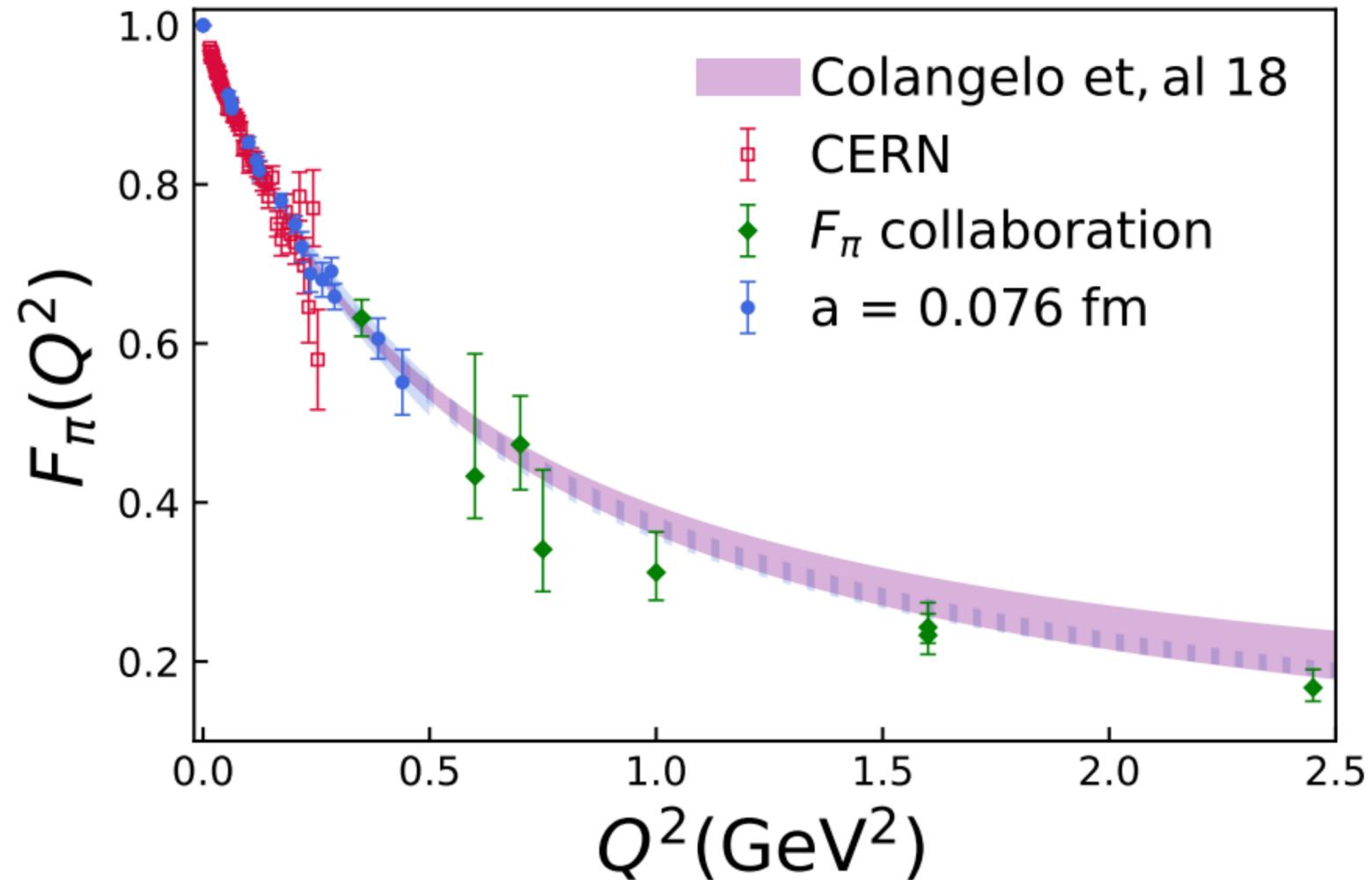


Standard Model parameters

- 杨一玻 [Aug. 10, 16:40]  
Lattice QCD using Large momentum effective theory
- 华俊 [Aug. 10, 17:00]  
Pion and Kaon Distribution Amplitudes from Lattice QCD
- 施岐 [Aug. 11, 14:15]  
Lattice QCD prediction of kaon electromagnetic form factor at large  $Q^2$  up to 10  $\text{GeV}^2$
- ...

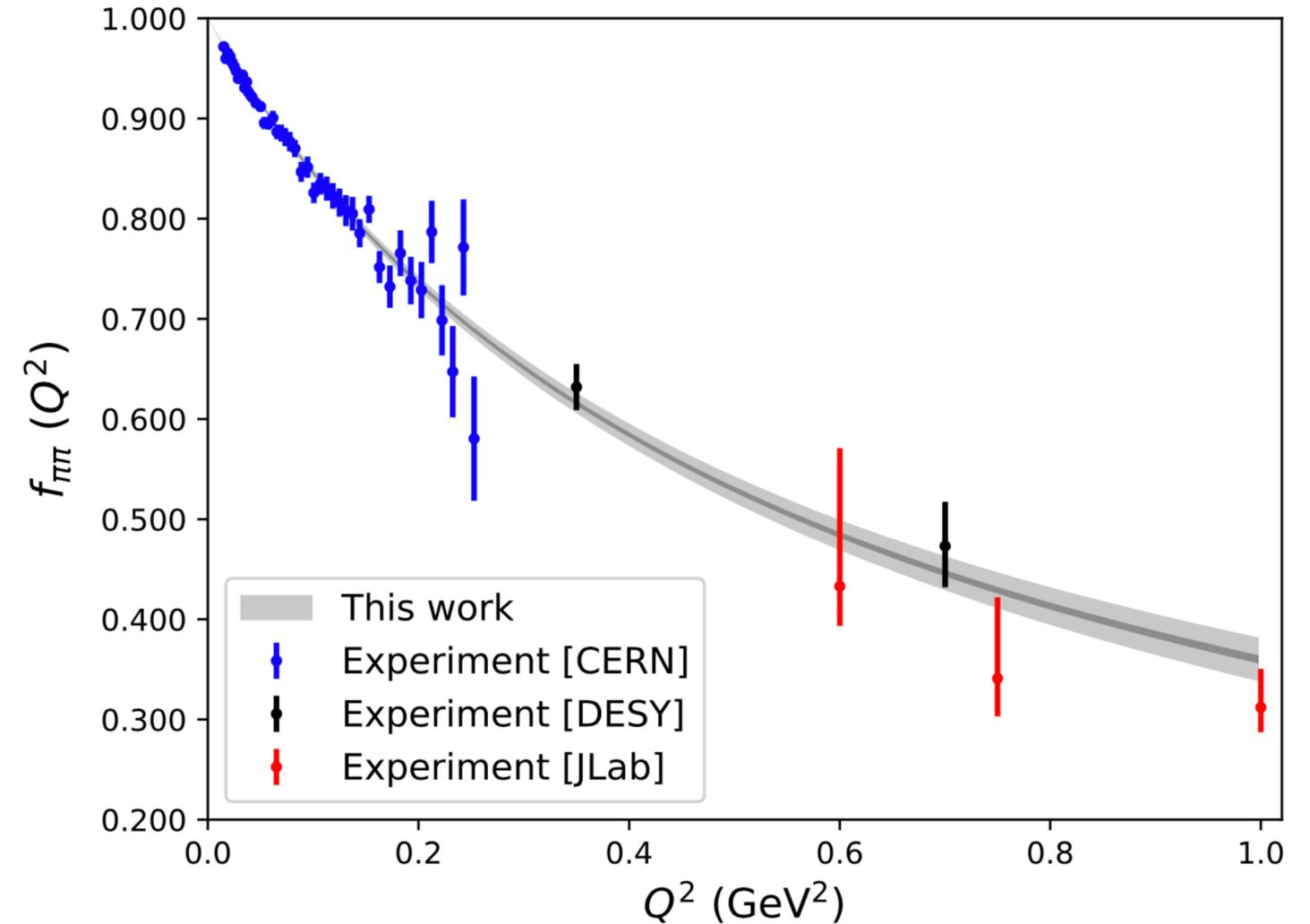
# Pion electromagnetic form factor

Wilson-Clover fermion  
on HISQ ensembles



高翔 et al., Tsinghua-BNL-ANL,  
Phys. Rev. D 104 (2021) 114515

Overlap fermions  
on Domain Wall ensembles

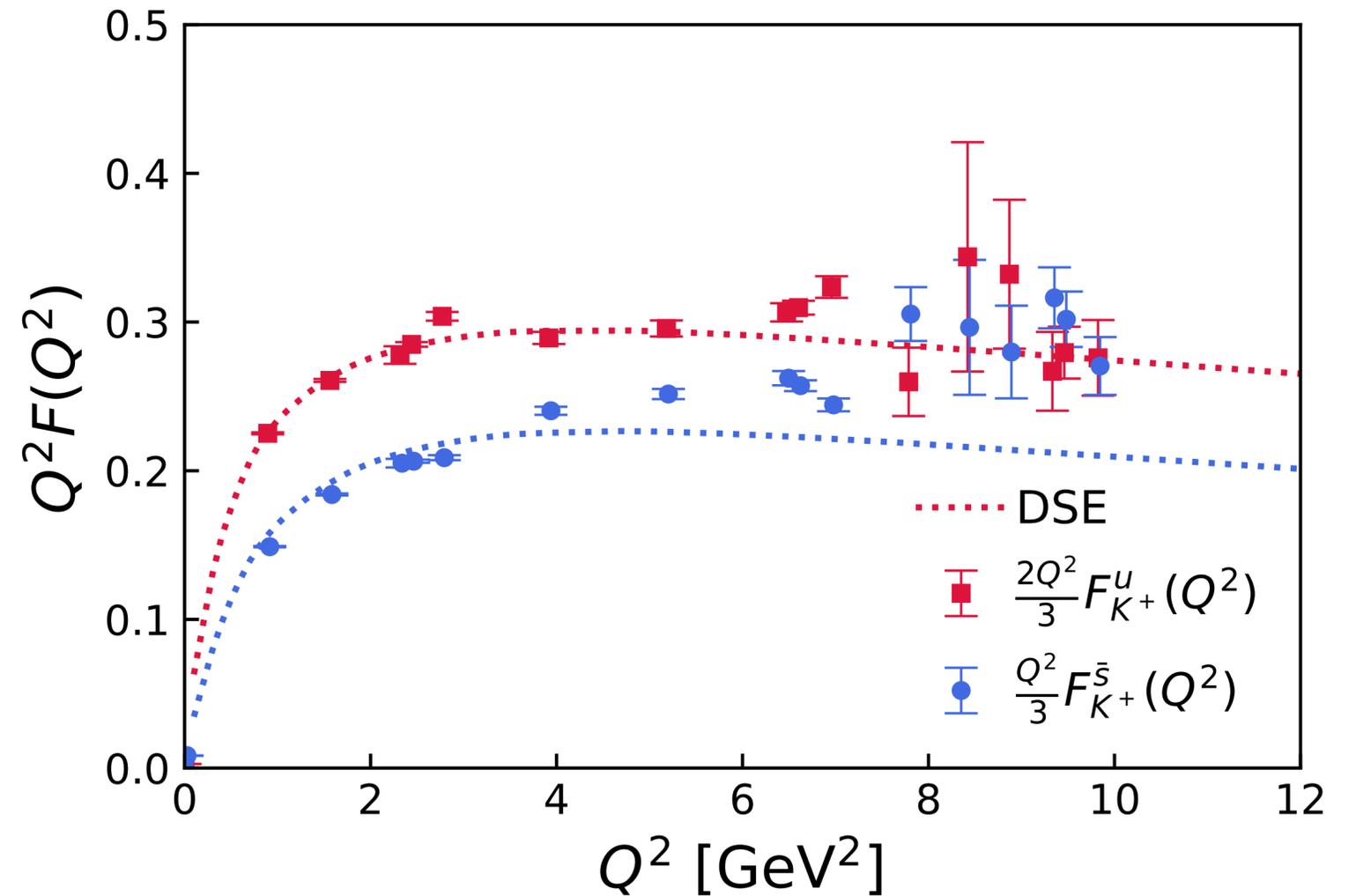
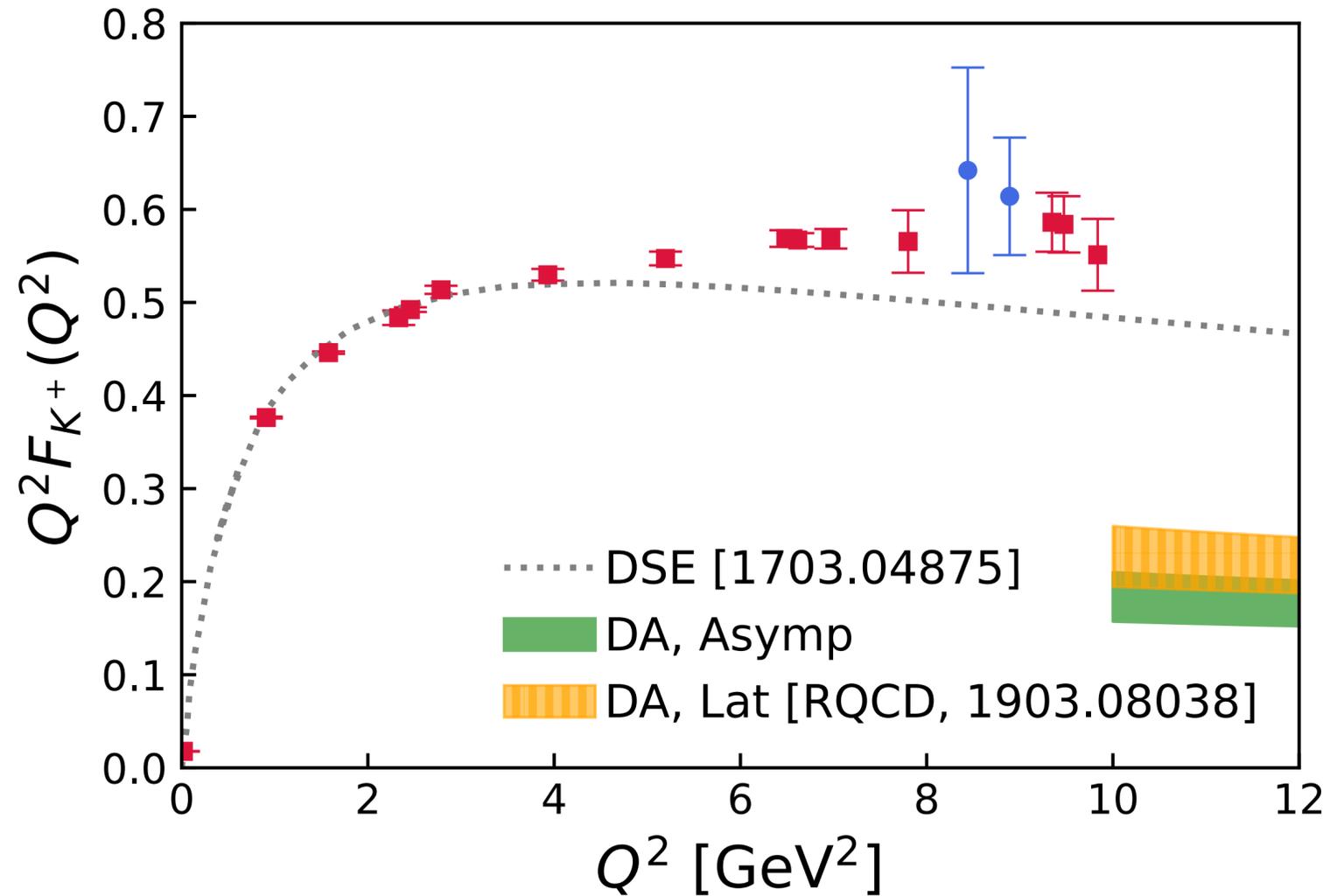


王根 et al., Kentucky-SCNU-ITP, [ $\chi$ QCD],  
Phys. Rev. D 104 (2021) 074502

# LQCD prediction of Kaon electromagnetic form factor at $Q^2 \lesssim 10 \text{ GeV}^2$

HISQ ensembles with  $a=0.076 \text{ fm}$

施岐  
Thu Aug. 11  
14:15-14:30

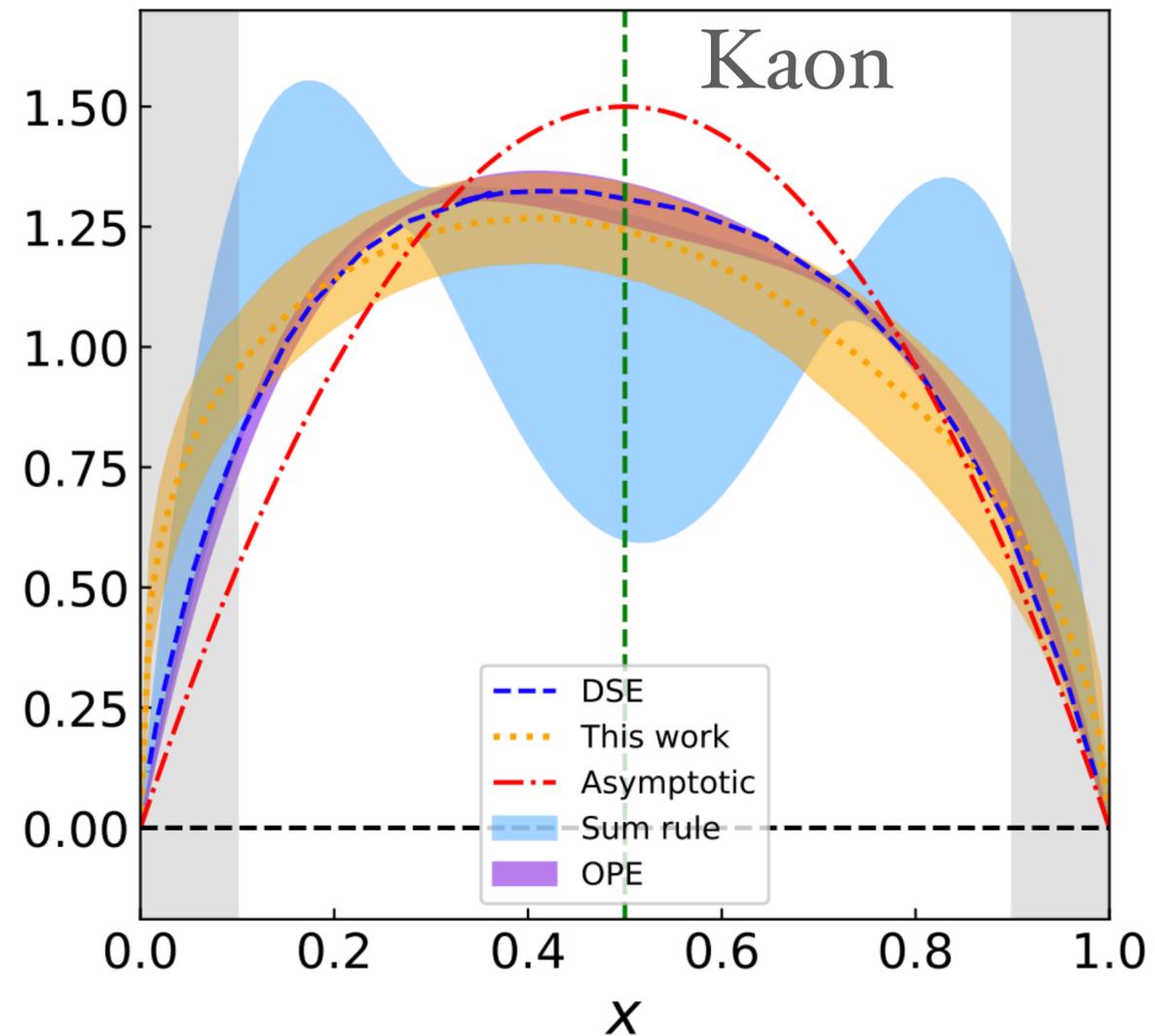
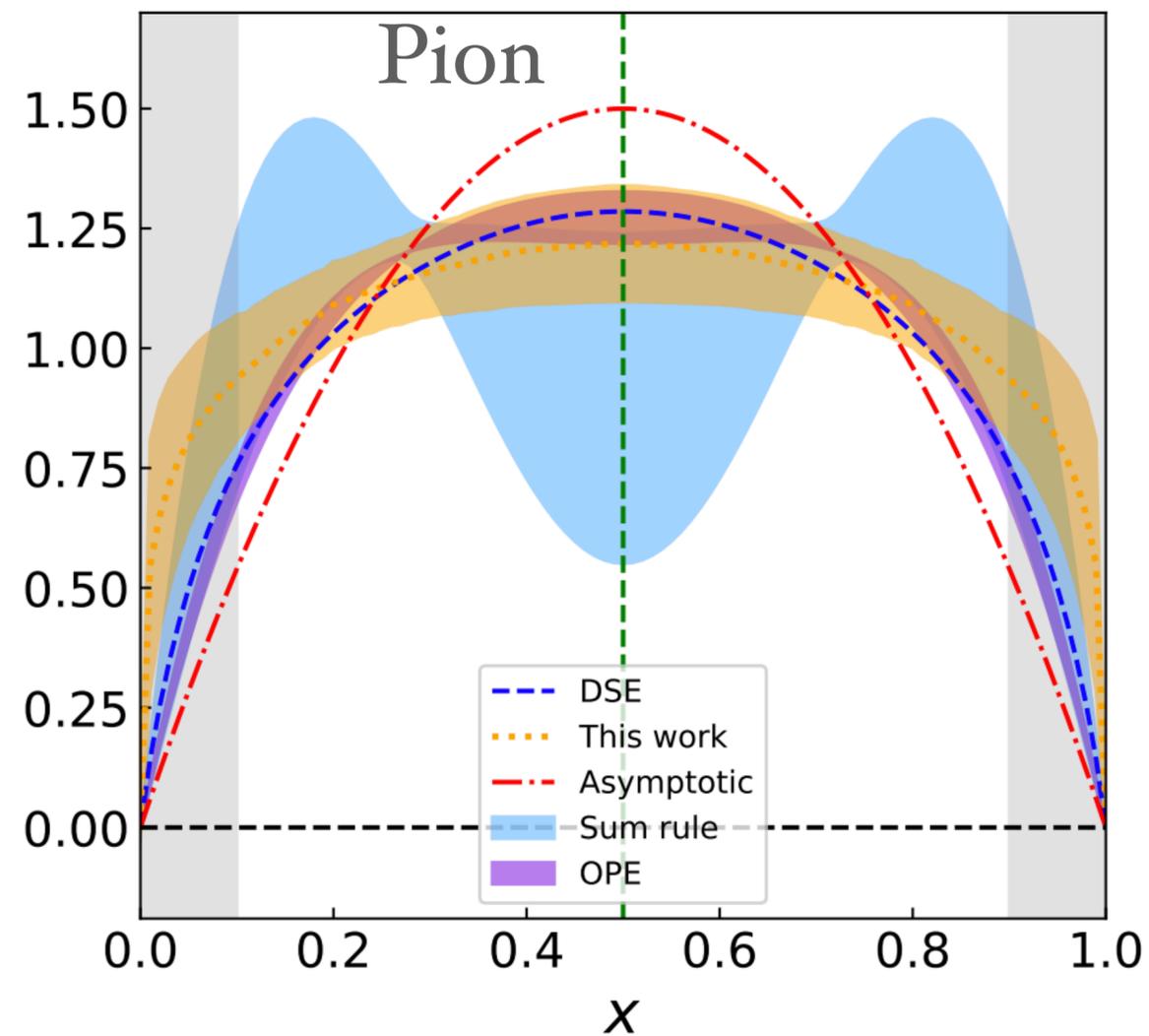


施岐 et al., CCNU-BNL-ANL, in preparation

# Distribution Amplitudes of Pion & Kaon

Hybrid renormalization scheme + continuum extrapolation

华俊  
Wed. Aug. 10  
17:00-17:15

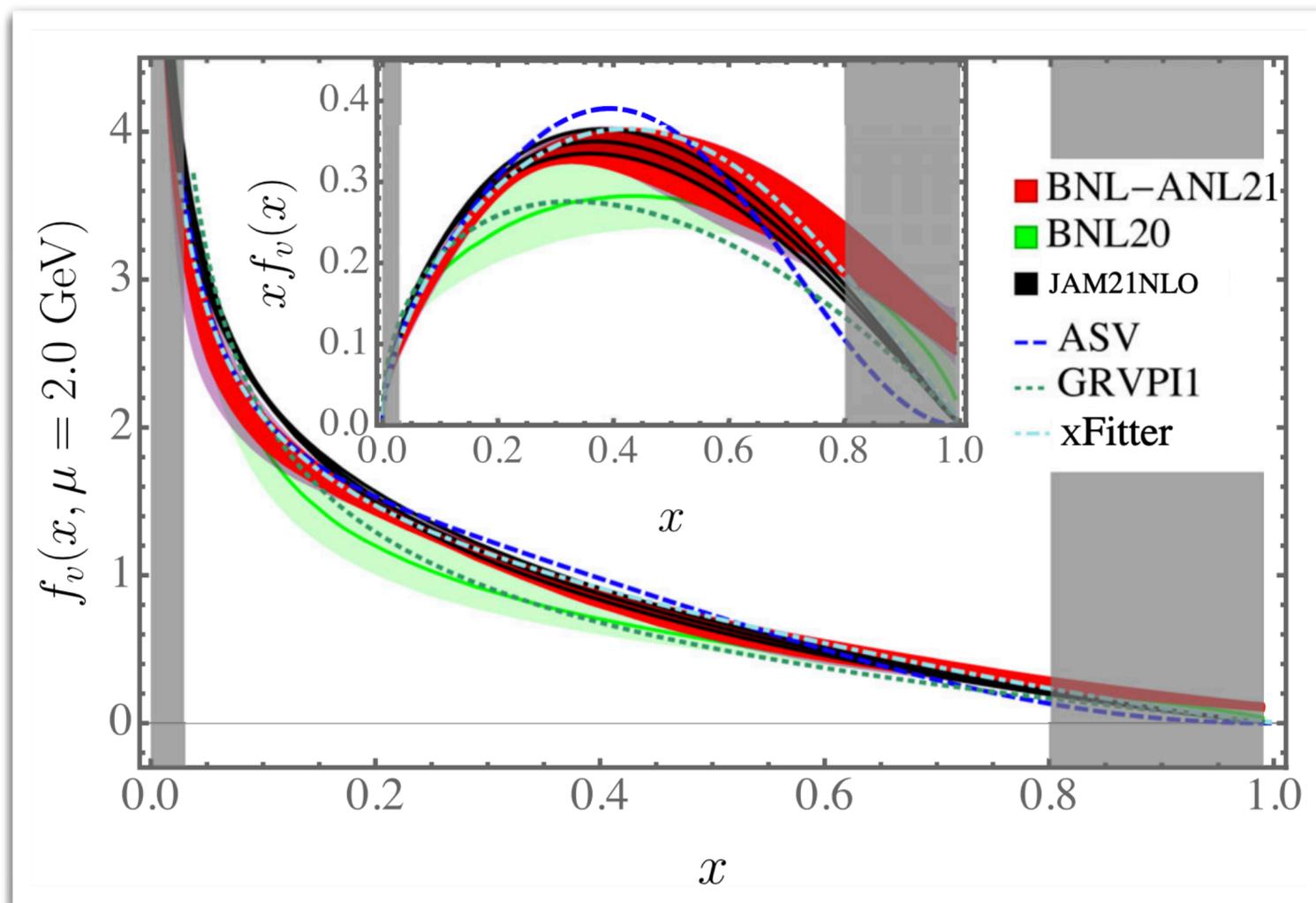


华俊 et al., SJTU-NJNU-ZU-IMP-BUAA, [LPC collaboration],  
Distribution of  $K^*$  and  $\phi$ , Phys. Rev. Lett. 127 (2021) 062002

华俊 et al., SCNU-SJTU-Maryland-Regensburg-ZU-ITP-BNU-BUAA [LPC collaboration], arXiv:2201.09173

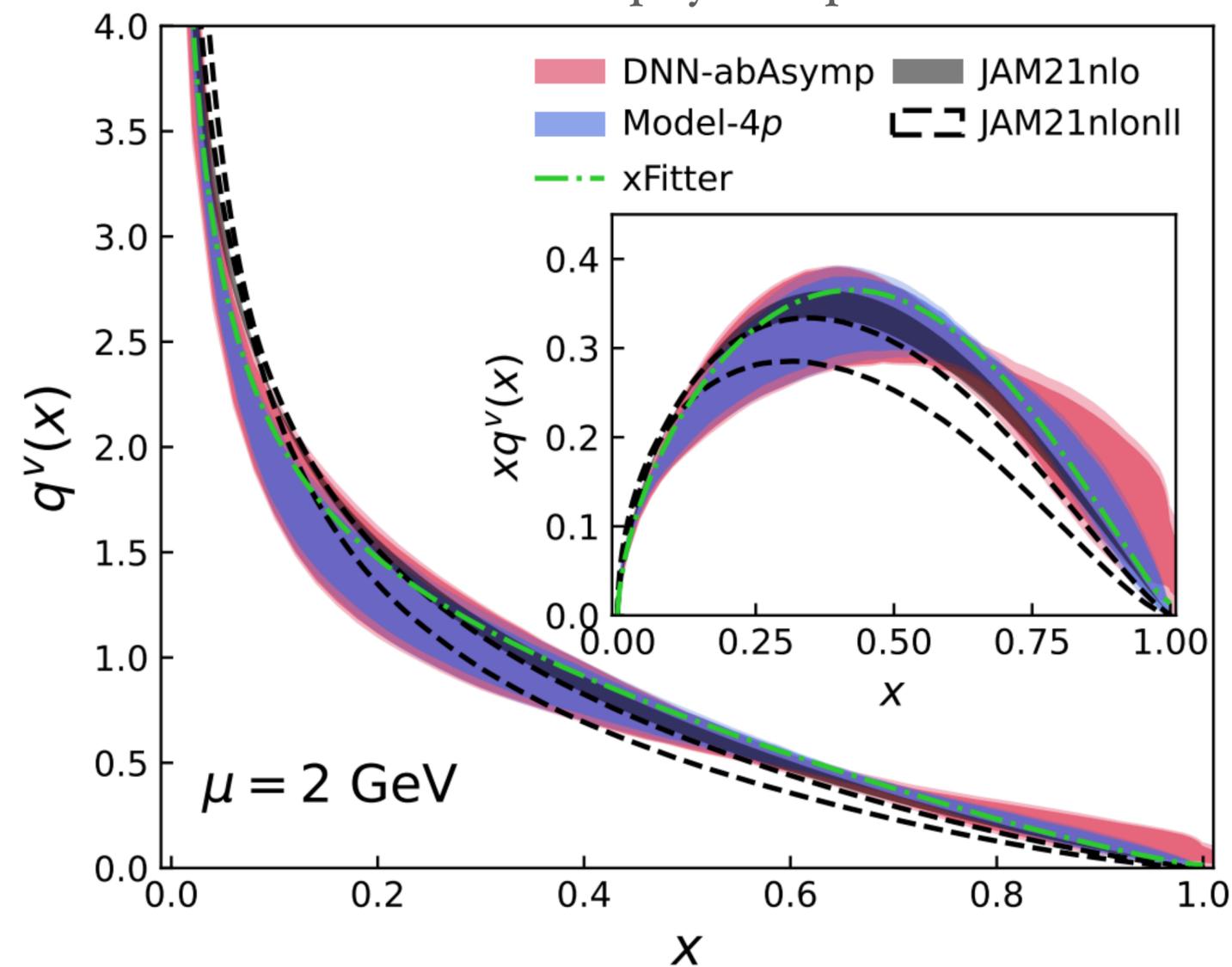
# Bjorken-x dependence of pion valence Parton Distribution Functions at NNLO

Nf=2+1 QCD using HISQ,  
 $a=0.04$  and  $0.06$  fm &  $m_{\pi}^{valence} = 300$  MeV



高翔 et al., CCNU-BNL-SBU-ANL,  
 Phys. Rev. Lett. 128 (2022) 142003

Continuum extrapolated results  
 at the physical point



高翔 et al., ANL-BNL-SBU-Frankfurt,  
 arXiv:2208.02297

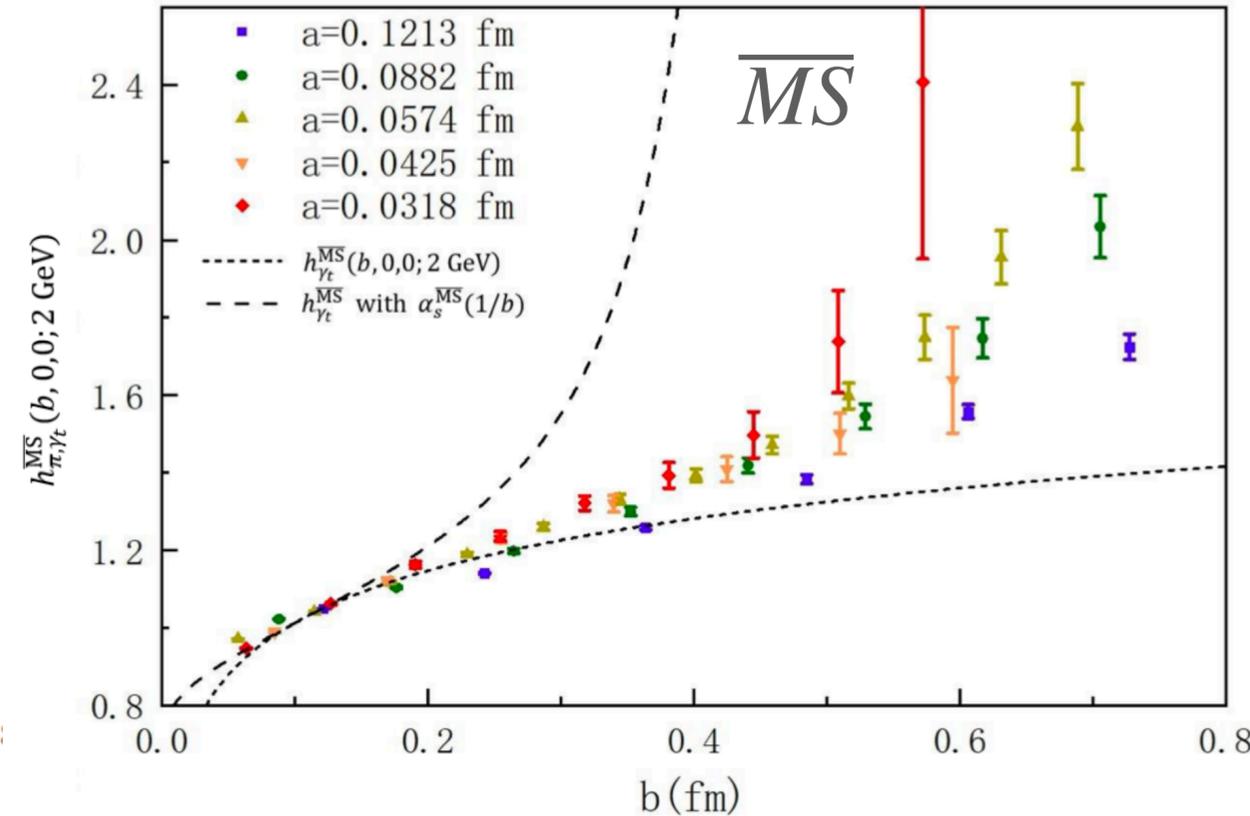
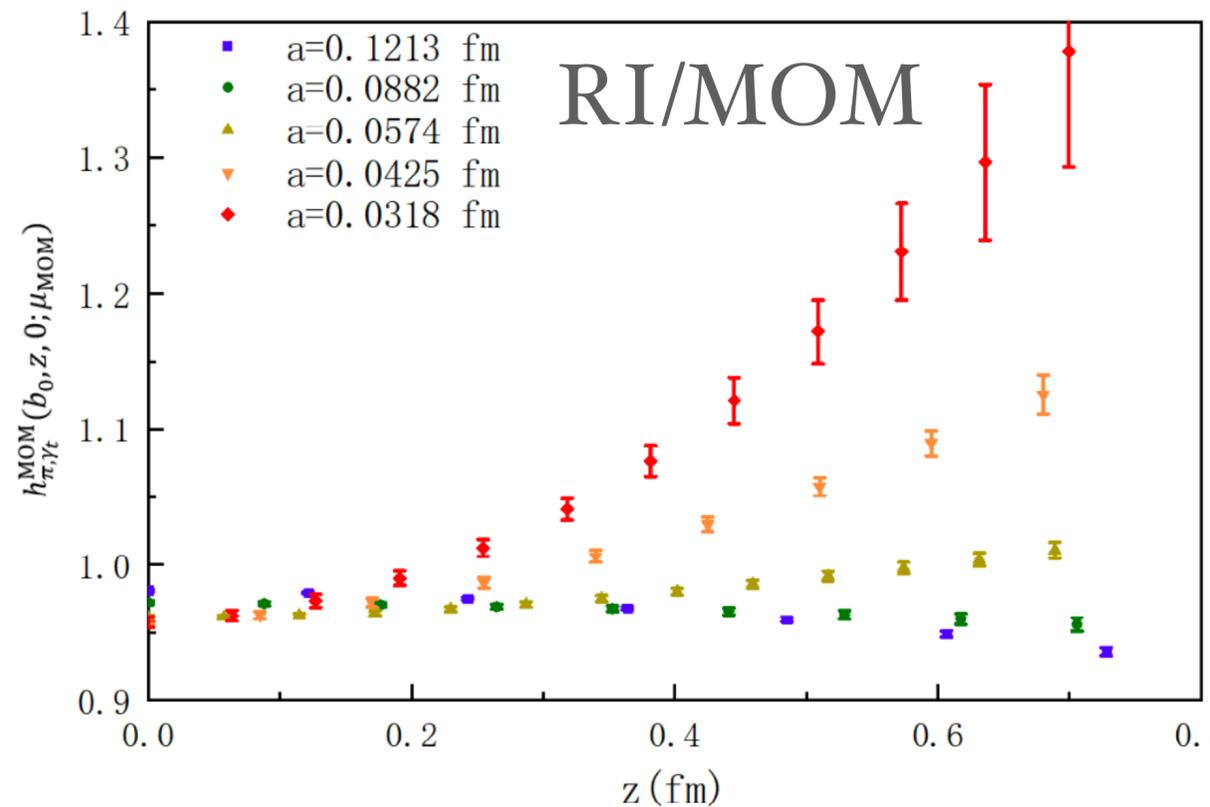
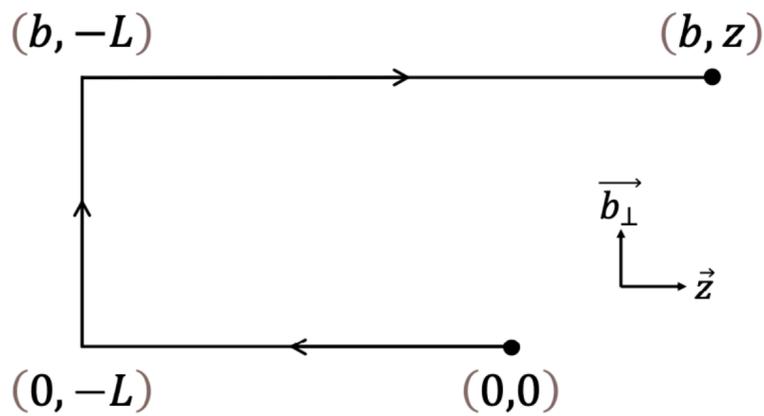
# Renormalization of transverse-momentum-dependent parton distribution (TMDPDF) in LaMET on the lattice

杨一玻  
Wed. Aug. 10  
16:40-17:00

## Pion matrix element of the TMDPDF operator

$$M_{\pi}^{HISQ,sea} \sim 310 \text{ MeV}, \quad M_{\pi}^{valence} \sim 280 - 320 \text{ MeV}$$

Wilson line needed in the quasi-TMDPDF operator

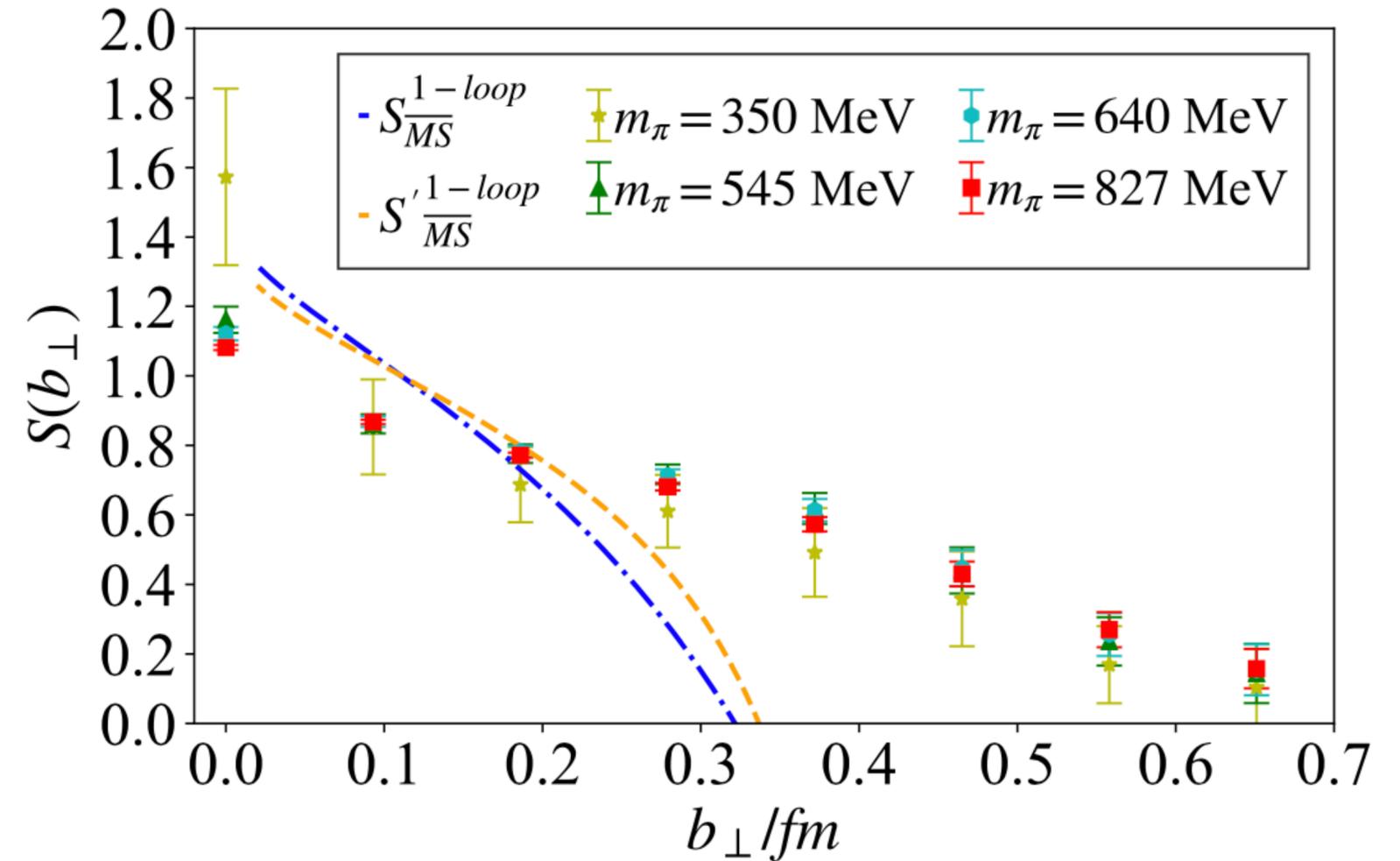
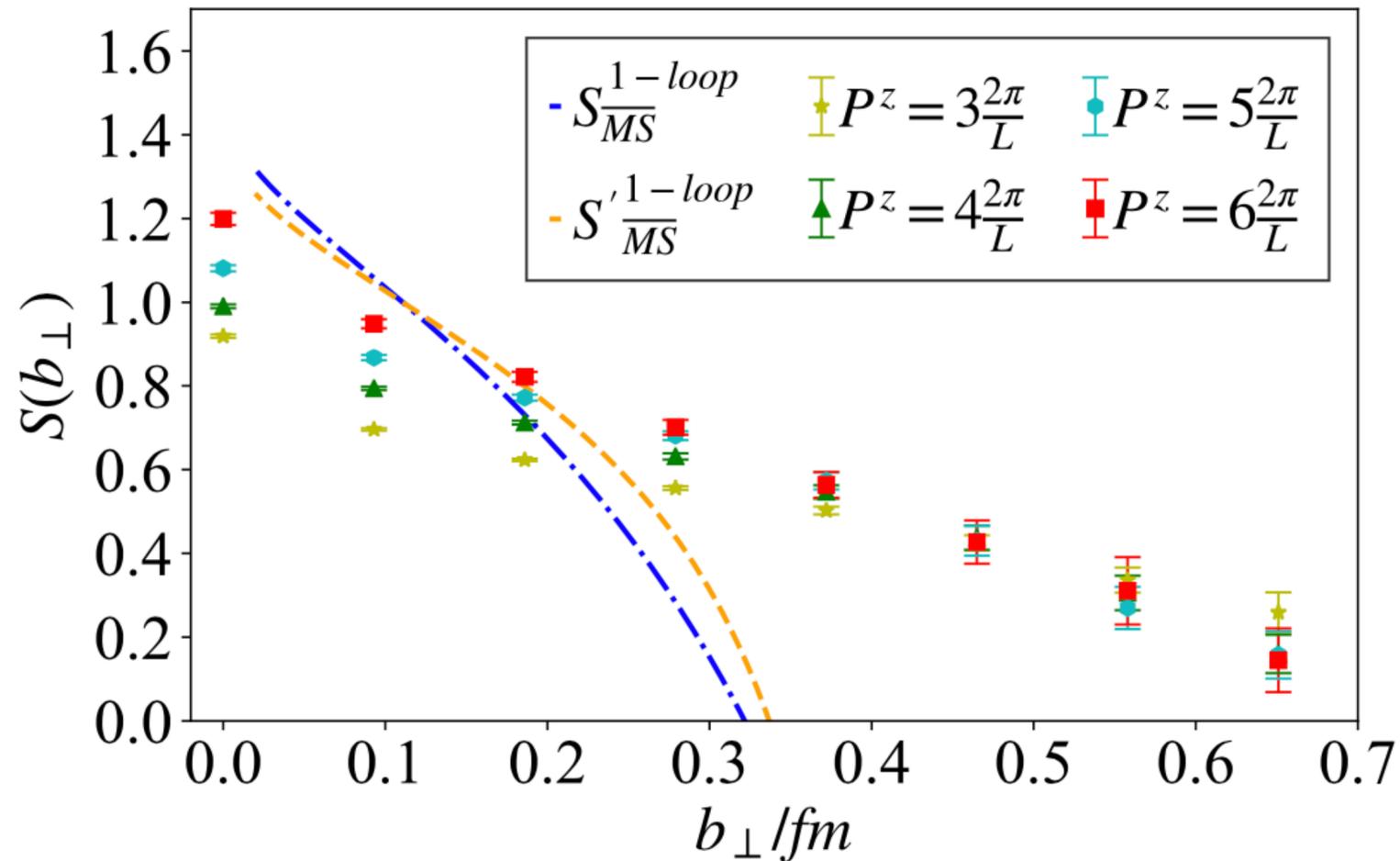


张宽 et al, ITP-Maryland-BNU, [LPC], arXiv:2205.13402, to appear in PRL

# Transverse-Momentum-Dependent (TMD) Soft function

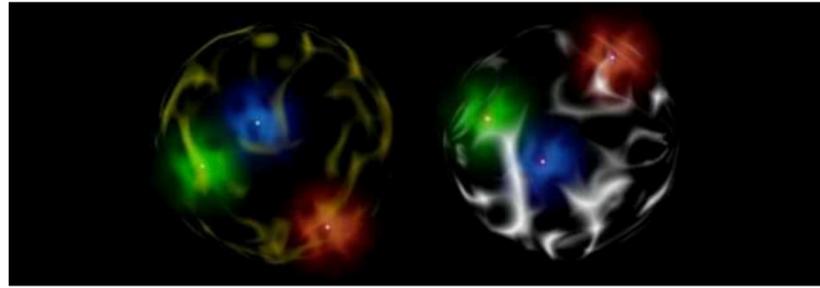
$$F_{\Gamma}(b_{\perp}, P^z) = \langle \pi(-P^z) | \bar{u} \Gamma u(b_{\perp}) \bar{d} \Gamma d(0) | \pi(P^z) \rangle$$

$$F_{\Gamma}(b_{\perp}, P^z) \stackrel{P^z \rightarrow \infty}{\underset{\text{LO kernel}}{=}} S(b_{\perp}, \mu) H_{\Gamma}^0 | \phi(z=0, b_{\perp}, l=\infty, P^z) |^2$$

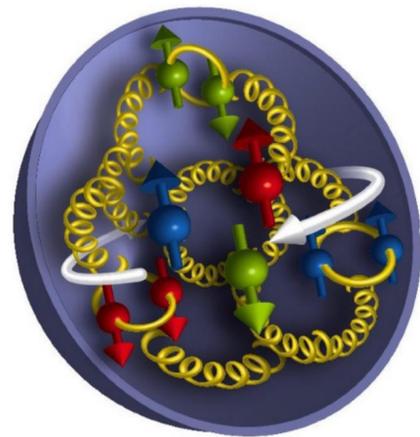


Yuan Li et al., PKU-ETMC, PRL 128 (2022)062002

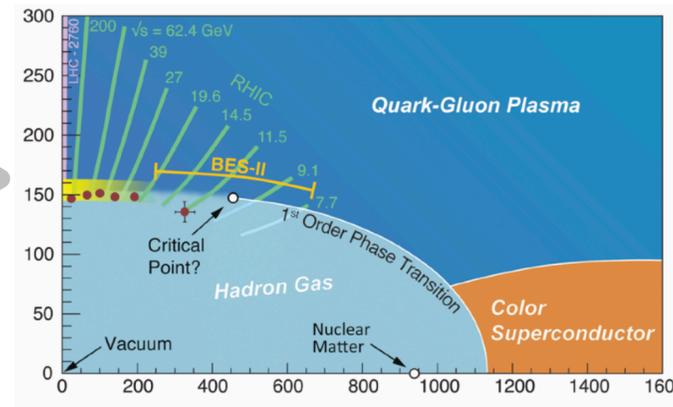
# Hadron spectroscopy & interactions



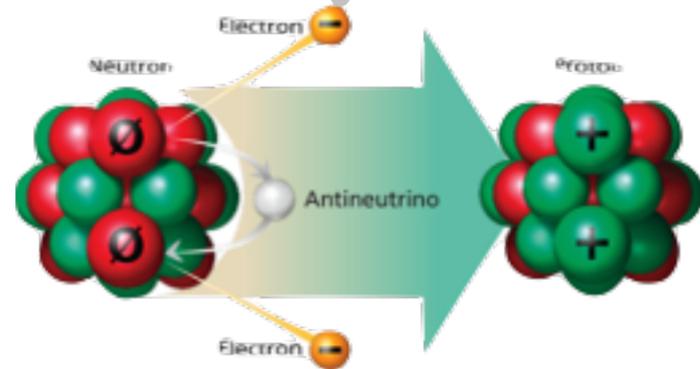
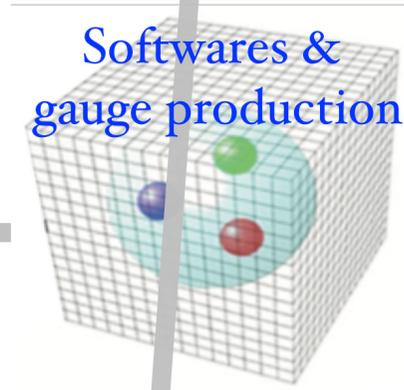
Hadron structure



Hot & dense QCD



Softwares & gauge production



Standard Model parameters

傅 杨 [Aug 10, 14:20], [晨光杯] Lattice QCD computation of two-photon exchange contribution to the muonic-hydrogen Lamb shift

王选贺 [Aug. 10, 19:30] 质子电极化率的格点 QCD 计算

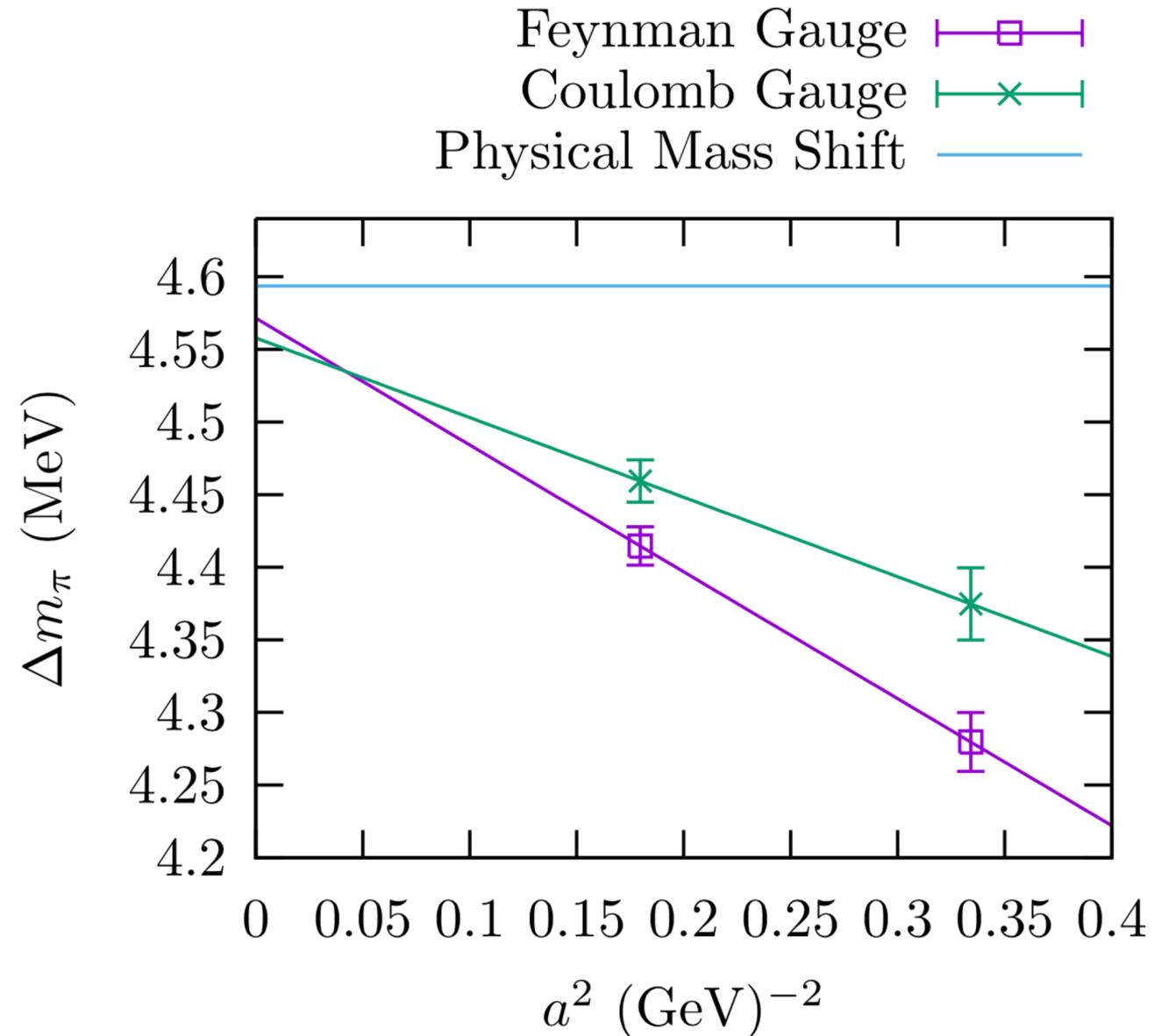
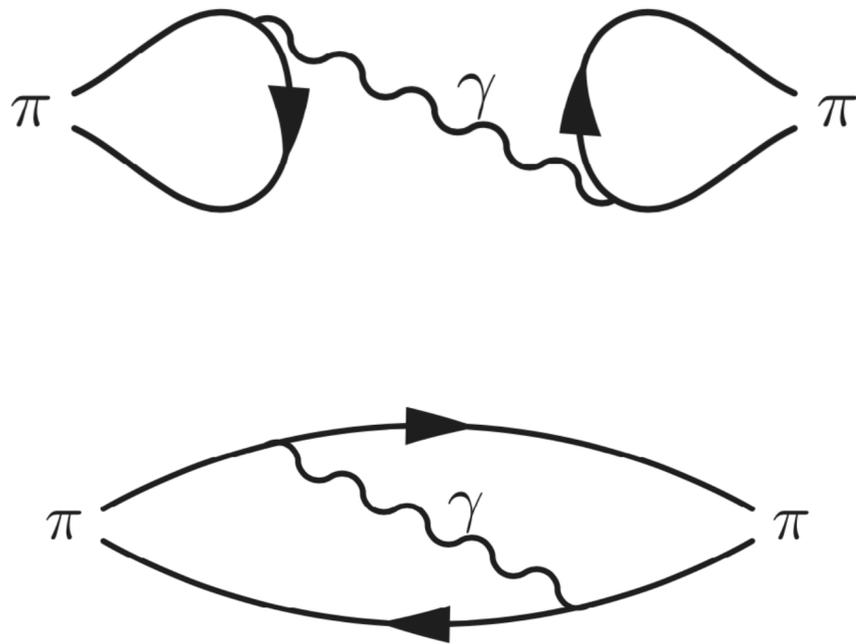
马鹏翔 [Aug. 10, 19:36] 格点QCD在介子衰变辐射修正中的应用

王子毓 [Aug. 10, 19:42] 双重子系统双 beta 衰变的格点计算

脱心宇 [Aug. 11, 14:00] 格点计算稀有 K 介子衰变的新方案

...

# Neutral and charged pion mass splitting



LQCD:  
 $\Delta m_\pi = 4.534(42)_{\text{stat}}(43)_{\text{sys}} \text{ MeV}$

PDG: 4.5936(5) MeV

# Two-photon exchange (TPE) contribution to the muonic-hydrogen Lamb shift

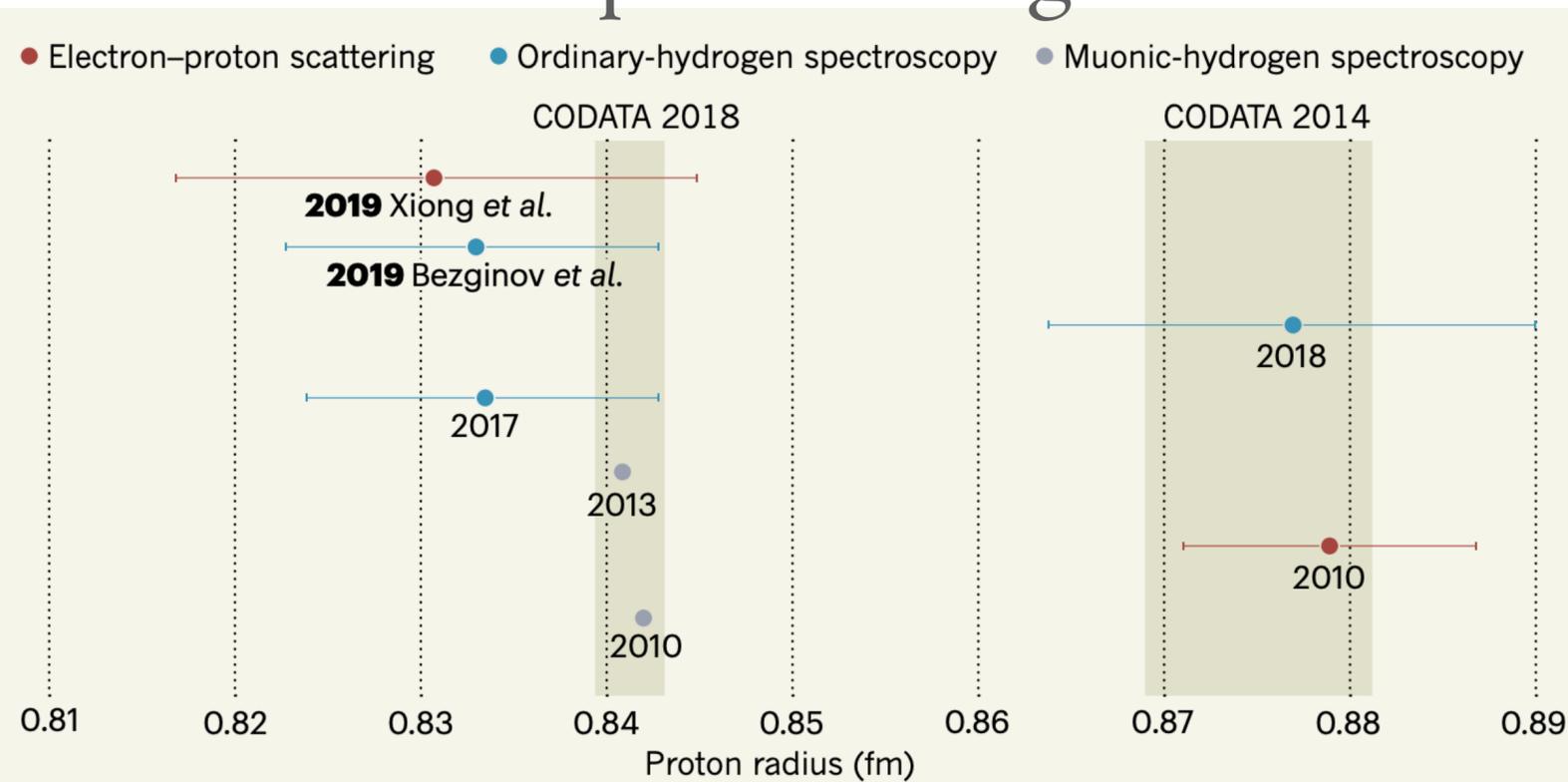
晨光杯报告

傅杨

Wed Aug. 10

14:20-14:40

## Puzzle on proton charge radius



Karr & Marchand, Nature 575 (2019) 61

$$\Delta E_{\text{LS}}^{\text{theory}} = 206033.6(1.5) - 5227.5(1.0)\langle r_p^2 \rangle + \Delta E_{\text{TPE}}$$

$$\Delta E = \frac{2m\alpha^2}{\pi M} |\phi_n(0)|^2 \left\{ \sum_{i=1,2} \left[ \int_{-t_s}^{t_s} d^4x \omega_i(\mathbf{x}, t) H_i(\mathbf{x}, t) + \int d^3\mathbf{x} L_i(\mathbf{x}, t_s) H_i(\mathbf{x}, t_s) - 2M \int_{\epsilon} \frac{dQ^2}{Q^4} \int d\theta \alpha_2(Q) (K_2 - 4\pi T_2^{pt}) \right] \right\}$$

$$\Delta E_{\text{TPE}} = 37.4(4.9) \mu\text{eV} : \text{consistent with previous theoretical computations } (20,50)\mu\text{eV}$$

Further improvements can be done with finer lattices

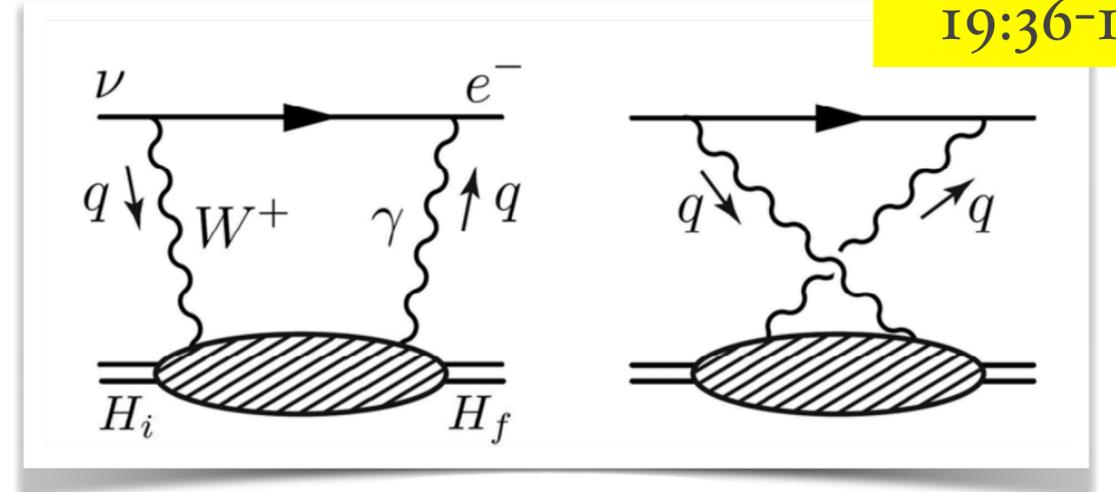
傅杨 et al., PKU-Connecticut, Phys.Rev.Lett. 128, 172002 (2022)

# Axial $\gamma W$ -box correction for the pion & kaon semileptonic decay

马鹏翔  
Wed Aug. 10  
19:36-19:42

$\pi^- \rightarrow \pi^0 \ell^- \nu$ :  $\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + \delta) I_\pi$

Sirlin's formule:  $\delta = \frac{\alpha_e}{2\pi} \left[ \bar{g} + 3 \ln \frac{m_Z}{m_p} + \ln \frac{m_Z}{m_W} + \tilde{a}_g \right] + \delta_{\text{HO}}^{\text{QED}} + 2\Box_{\gamma W}^{\text{VA}}$



$H_i \rightarrow H_f e \bar{\nu}_e$

$\Box_{\gamma W}^{\text{VA}}|_H = \frac{1}{F_+^H} \frac{\alpha_e}{\pi} \int_0^\infty dQ^2 \frac{m_W^2}{m_W^2 + Q^2} \times \int_{-\sqrt{Q^2}}^{\sqrt{Q^2}} \frac{dQ_0}{\pi} \frac{(Q^2 - Q_0^2)^{3/2}}{(Q^2)^2} T_3(Q_0, Q^2)$  Hadronic function by LQCD

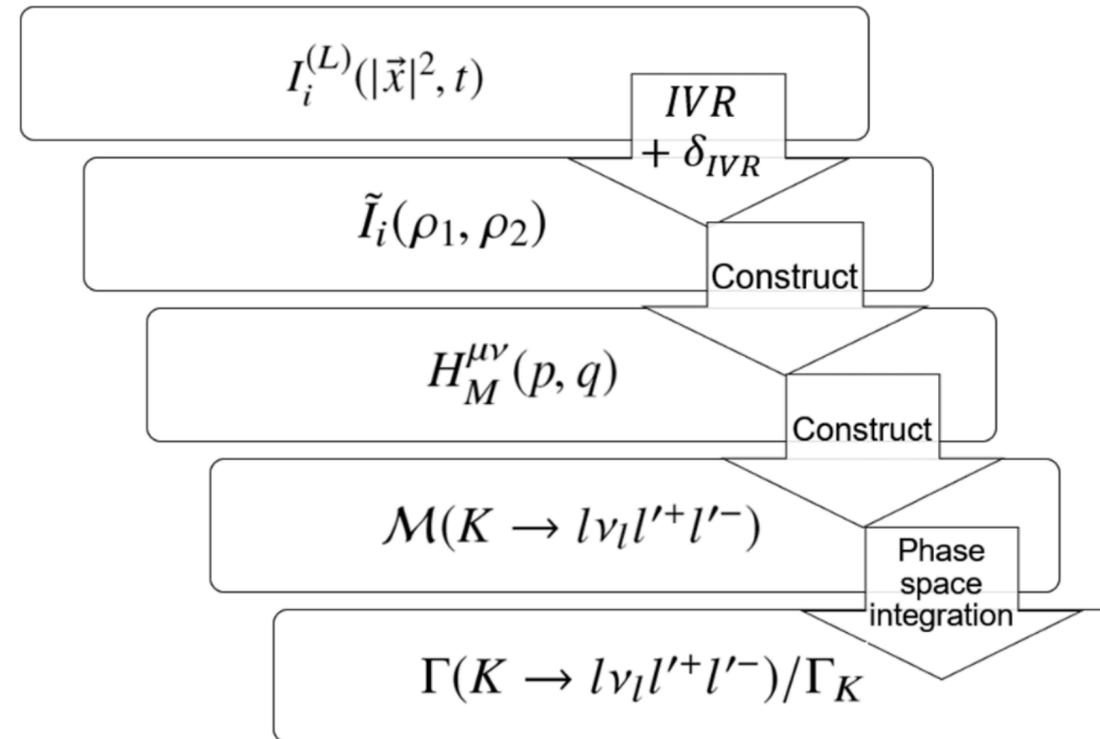
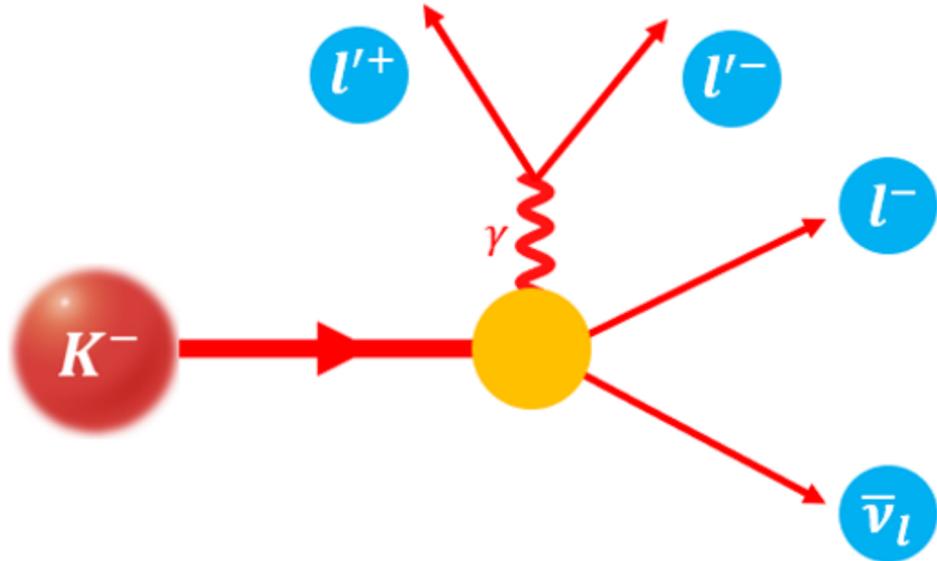
Pion:  $\delta = 0.0332(1)_{\gamma W} (3)_{\text{HO}}$  冯旭 et al., PKU-Mainz-Connecticut-Bonn, Phys.Rev.Lett., 124 (2020) 192002

$K \rightarrow \pi \ell \nu$ :  $\Gamma_{K\ell 3} = \frac{G_F^2 m_K^5}{192\pi^3} S_{\text{EW}} (1 + \delta_K^\ell + \delta_{\text{SU}2}) C^2 |V_{us}|^2 f_+^2(0) I_K^\ell$

Kaon:  $\Box_{\gamma W}^{\text{VA}}|_{K^0, \text{SU}(3)} + \text{Low energy constants} \implies \Box_{\gamma W}^{\text{VA}}|_{K^0}$  马鹏翔 et al., Phys.Rev. D 103(2021)114503

# New methodology in computing the Kaon decay on Lattice

脱心宇  
Thu Aug. 11  
14:00-14:15



Scalar function + Infinite Volume Reduction method

Channels	$m_{ee}$ cuts	Lattice ( $m_\pi = 352$ MeV)	ChPT [5]	Experiments
$\text{Br}[K \rightarrow e\nu_e e^+ e^-]$	140 MeV	$1.77(16) \times 10^{-8}$	$3.39 \times 10^{-8}$	$2.91(23) \times 10^{-8}$ [3]
$\text{Br}[K \rightarrow \mu\nu_\mu e^+ e^-]$	140 MeV	$10.59(33) \times 10^{-8}$	$8.51 \times 10^{-8}$	$7.93(33) \times 10^{-8}$ [3]
$\text{Br}[K \rightarrow e\nu_e \mu^+ \mu^-]$	...	$0.72(5) \times 10^{-8}$	$1.12 \times 10^{-8}$	$1.72(45) \times 10^{-8}$ [4]
$\text{Br}[K \rightarrow \mu\nu_\mu \mu^+ \mu^-]$	...	$1.45(6) \times 10^{-8}$	$1.35 \times 10^{-8}$	...

脱心宇, X. Feng, L. Jin, T. Wang, PKU-Conneticut, PRD105 (2022) 054518

# Lattice study of Dibaryon nuclear $\beta\beta$ decay

Domain Wall fermions + Iwasaki gauge action

$$pp \rightarrow d \text{ fusion: } \text{NME}/g_A = 1.0(4)$$

$$2\nu 2\beta: \text{NME}/g_A^2 = 1.02(6)$$

$$0\nu 2\beta: \text{NME}/(g_A^2/2m_\nu V) = 13(4)$$

Consistent with NPLQCD Savage et al., PRL 17

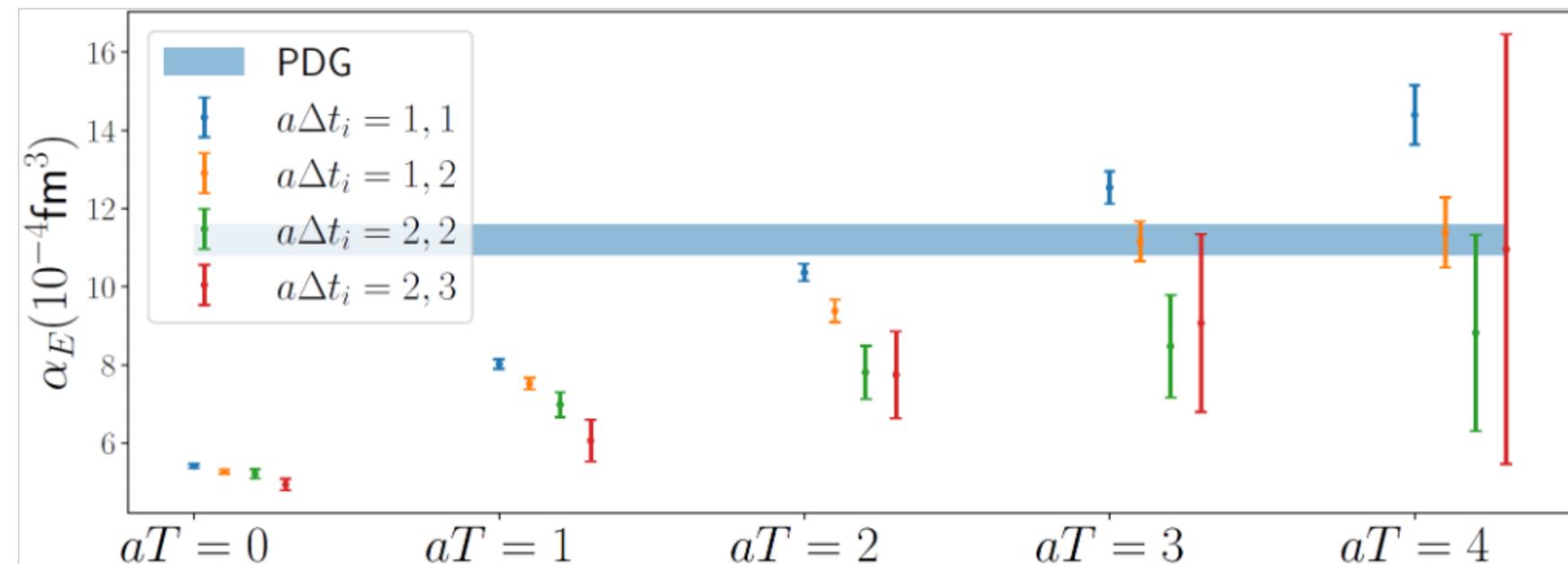
王子毓, PKU, PoS(Lattice 2021)629, work in progress

王子毓  
Wed Aug. 10  
19:42-19:48

## Electric polarizability of Proton extracted from Compton scattering

4-point correlation func. of  
proton:

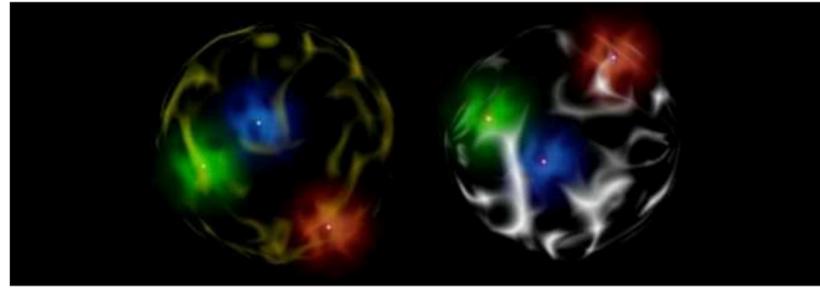
$$\langle N(t + \Delta t_1) J^\mu(t) J^\nu(0) \bar{N}(-\Delta t_2) \rangle$$



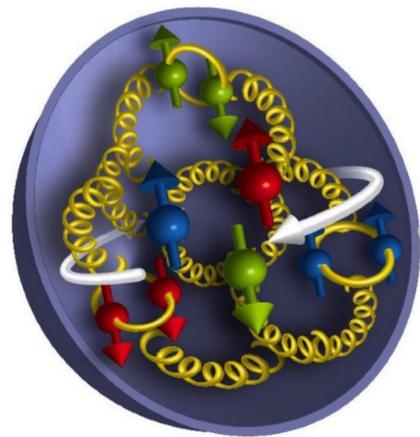
王选贺 et al., PKU-Connecticut, work in progress

王选贺  
Wed Aug. 10  
19:30-19:36

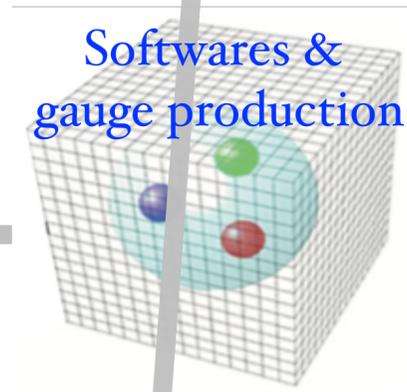
# Hadron spectroscopy & interactions



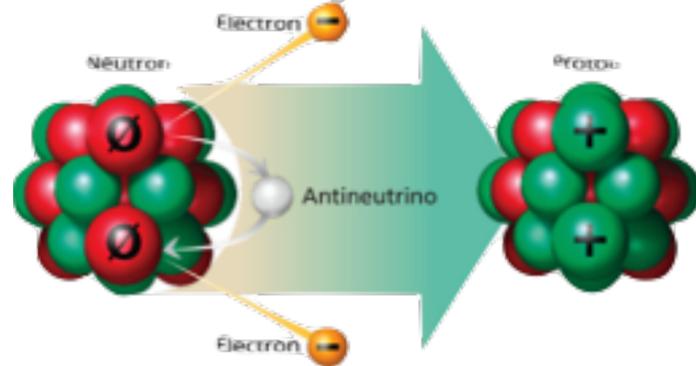
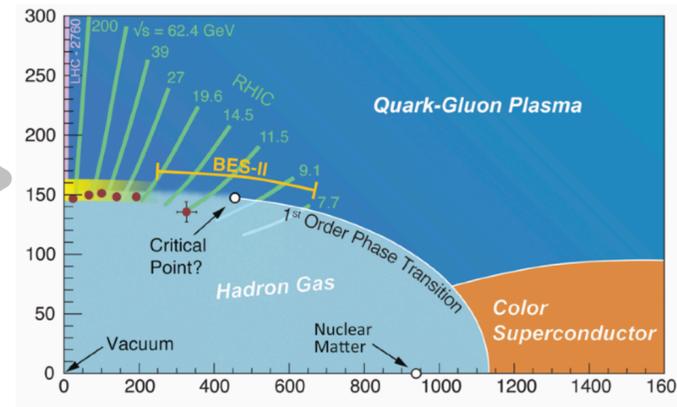
Hadron structure



Softwares & gauge production



Hot & dense QCD



Standard Model parameters

- 李胜泰 [Aug. 10, 15:35]  
Chiral condensates and screening masses of neutral pseudoscalar mesons
- 黄玮平 [Aug. 10, 15:50]  
Criticality of QCD in correlated Dirac eigenvalues
- 刘俊宏 [Aug. 10, 11:15]  
Fluctuations of conserved charges in strong magnetic fields
- ...

# HotQCD collaboration

- Brookhaven National Laboratory, USA
- Bielefeld University, Germany
- Central China Normal University, China
- Michigan State University, USA
- Tsukuba University, Japan
- Indian Institute of Science, India
- Institute of Mathematical Sciences, India

arXiv:1504.05274v1 [hep-lat] 21 Apr 2015

## THERMODYNAMICS OF STRONG-INTERACTION MATTER FROM LATTICE QCD

Heng-Tong Ding

*Key Laboratory of Quark & Lepton Physics (MOE), Institute of Particle Physics,  
Central China Normal University, Wuhan, 430079, China*

Frithjof Karsch

*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA  
and  
Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*

Swagato Mukherjee

*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

We review results from lattice QCD calculations on the thermodynamics of strong-interaction matter with emphasis on input these calculations can provide to the exploration of the phase diagram and properties of hot and dense matter created in heavy ion experiments. This review is organized as follows:

- 1) Introduction
- 2) QCD thermodynamics on the lattice
- 3) QCD phase diagram at high temperature
- 4) Bulk thermodynamics
- 5) Fluctuations of conserved charges
- 6) Transport properties
- 7) Open heavy flavors and heavy quarkonia
- 8) QCD in external magnetic fields
- 9) Summary

Review article: Int. J. Mod. Phys. E24(2015)1530007,  
cited by 298 records

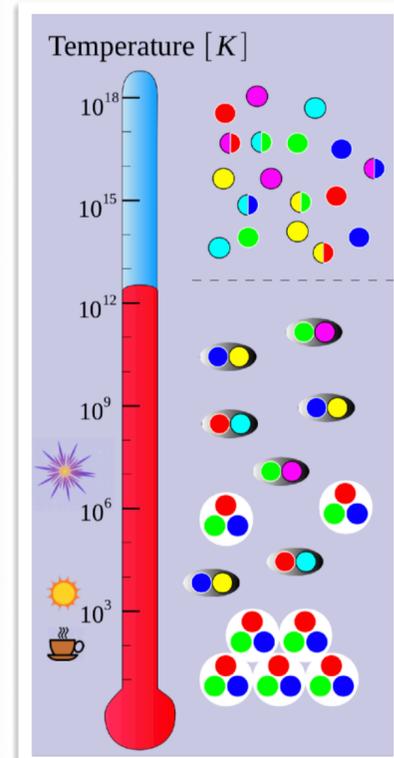
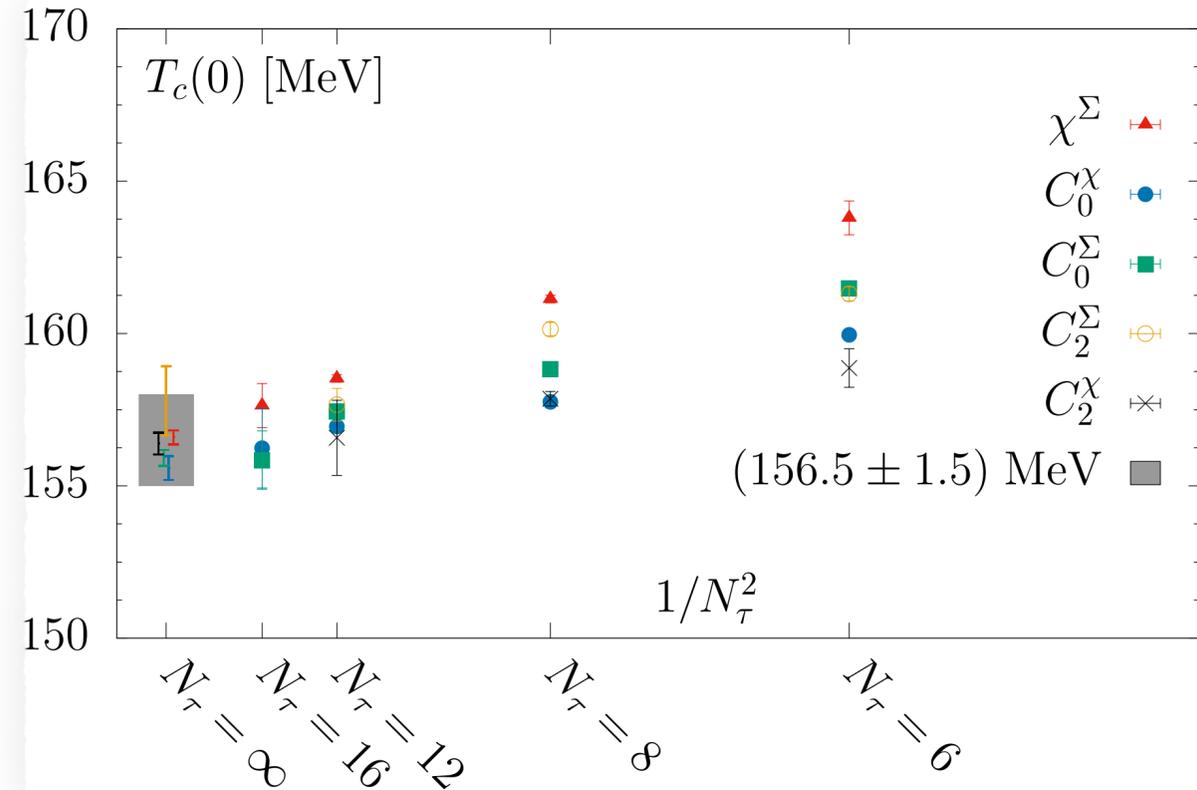
# Chiral crossover/phase transition temperature at physical point and $m_q \rightarrow 0$

Made in

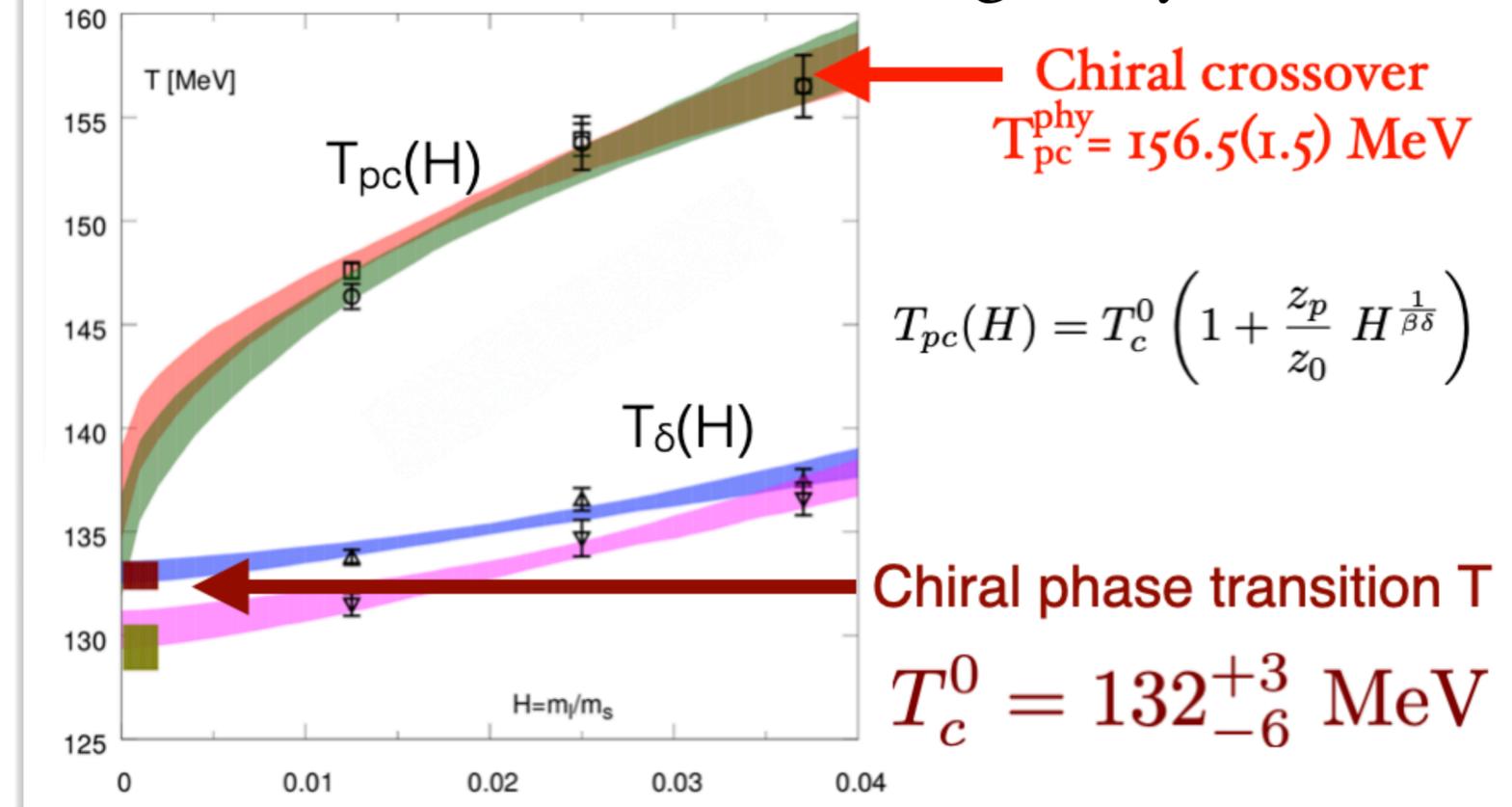
华中师大  
核科学计算中心



Rigorous definition from  $O(4)$  universality class



Based on  $O(4)$  scaling analyses



A. Bazavov, 丁亨通, P. Hegde et al. [HotQCD],  
Phys. Lett. B795 (2019) 15, 被引360次

丁亨通, P. Hegde, O. Kaczmarek et al. [HotQCD],  
Phys. Rev. Lett. 123 (2019) 062002, 被引147次

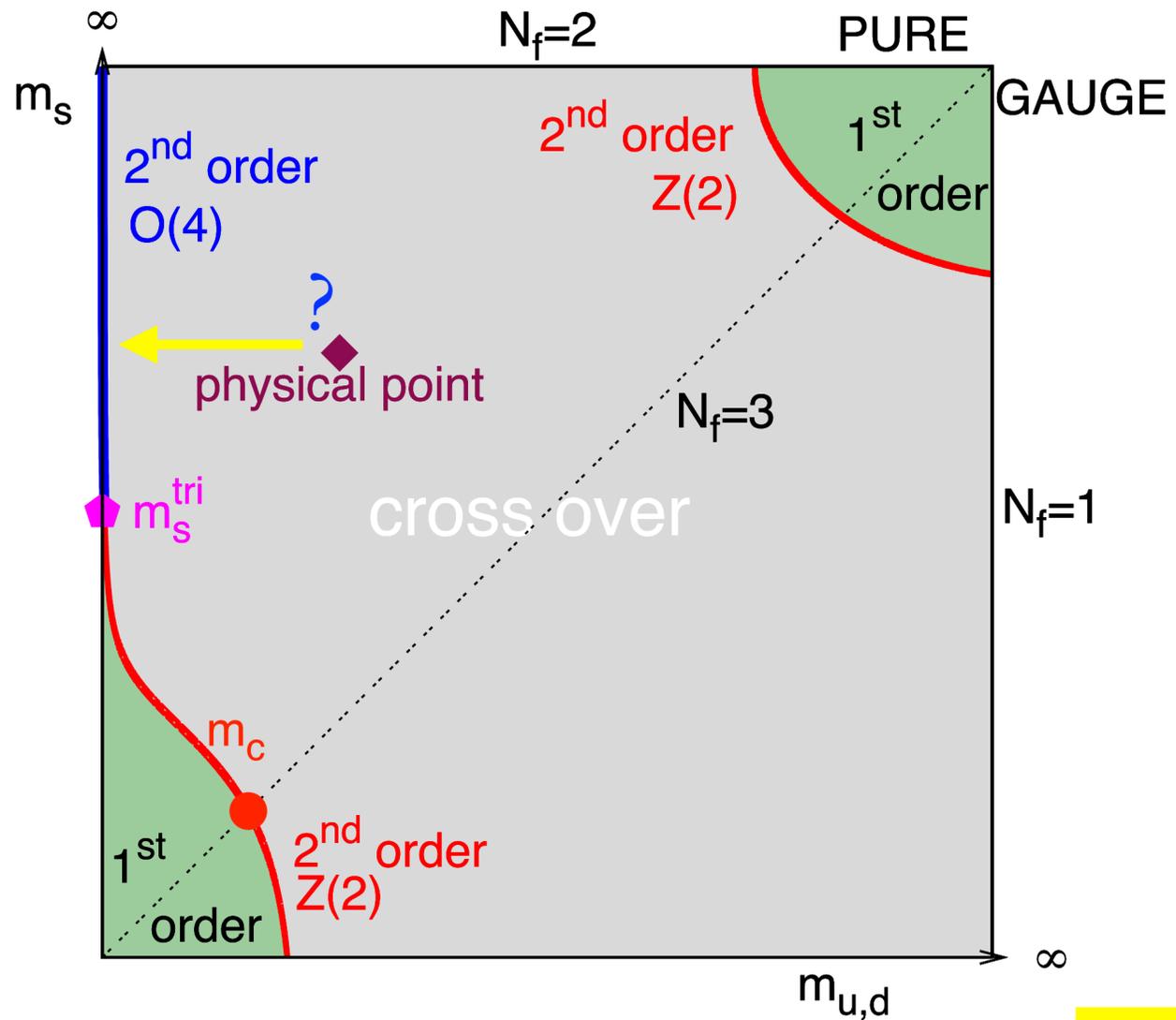
Chiral crossover transition

$T = 156.5(1.5)$  MeV  $\sim 1.8 \times 10^{12}$  K

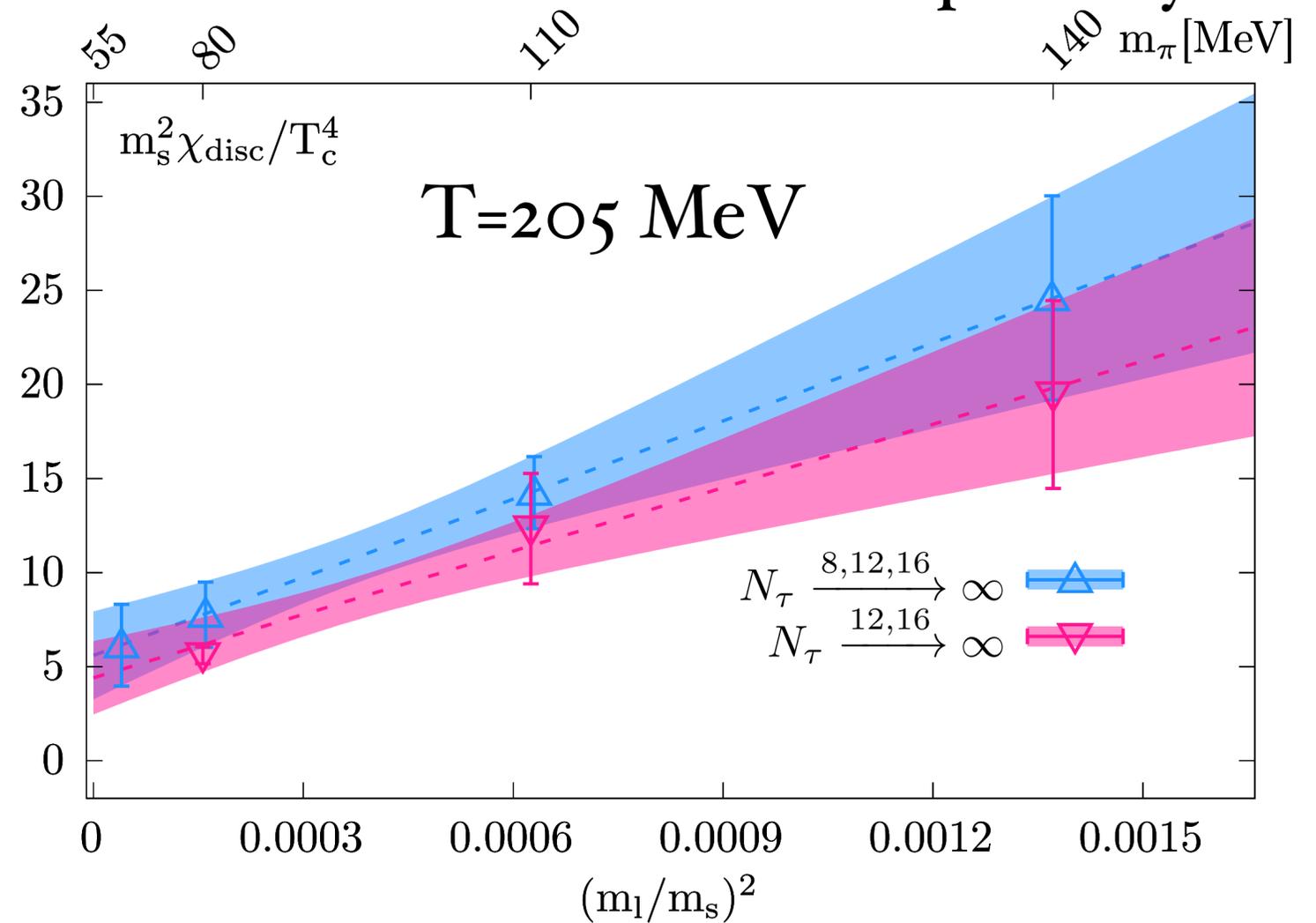
Consistent results from Wuppertal-Budapest,  
PRL125 (2020) 052001

Chiral phase transition  $T$  is a possible upper bound of  $T_{CEP}$

# Fate of the Axial U1 anomaly at nonzero T



## disconnected chiral susceptibility



丁亨通, 李胜泰, 复谷昭夫,  
S. Mukherjee, 汪晓丹, 张瑜,  
CCNU-IMP-BNL,  
Phys.Rev.Lett. 126 (2021) 082001

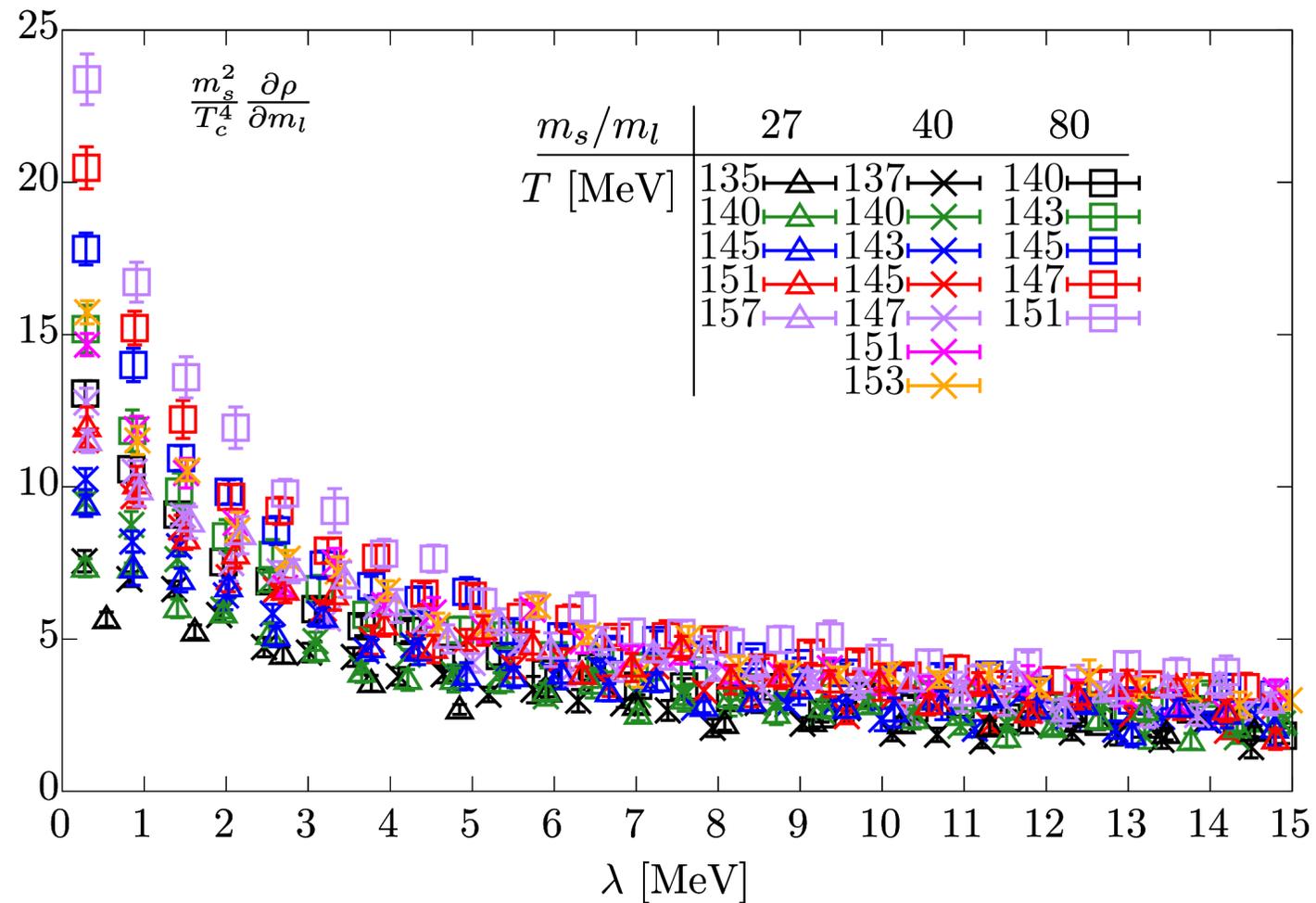
**Axial anomaly remains manifested at a 2-3 $\sigma$  level**

**Chiral phase transition:  
2nd order in  $O(4)$  universality class**

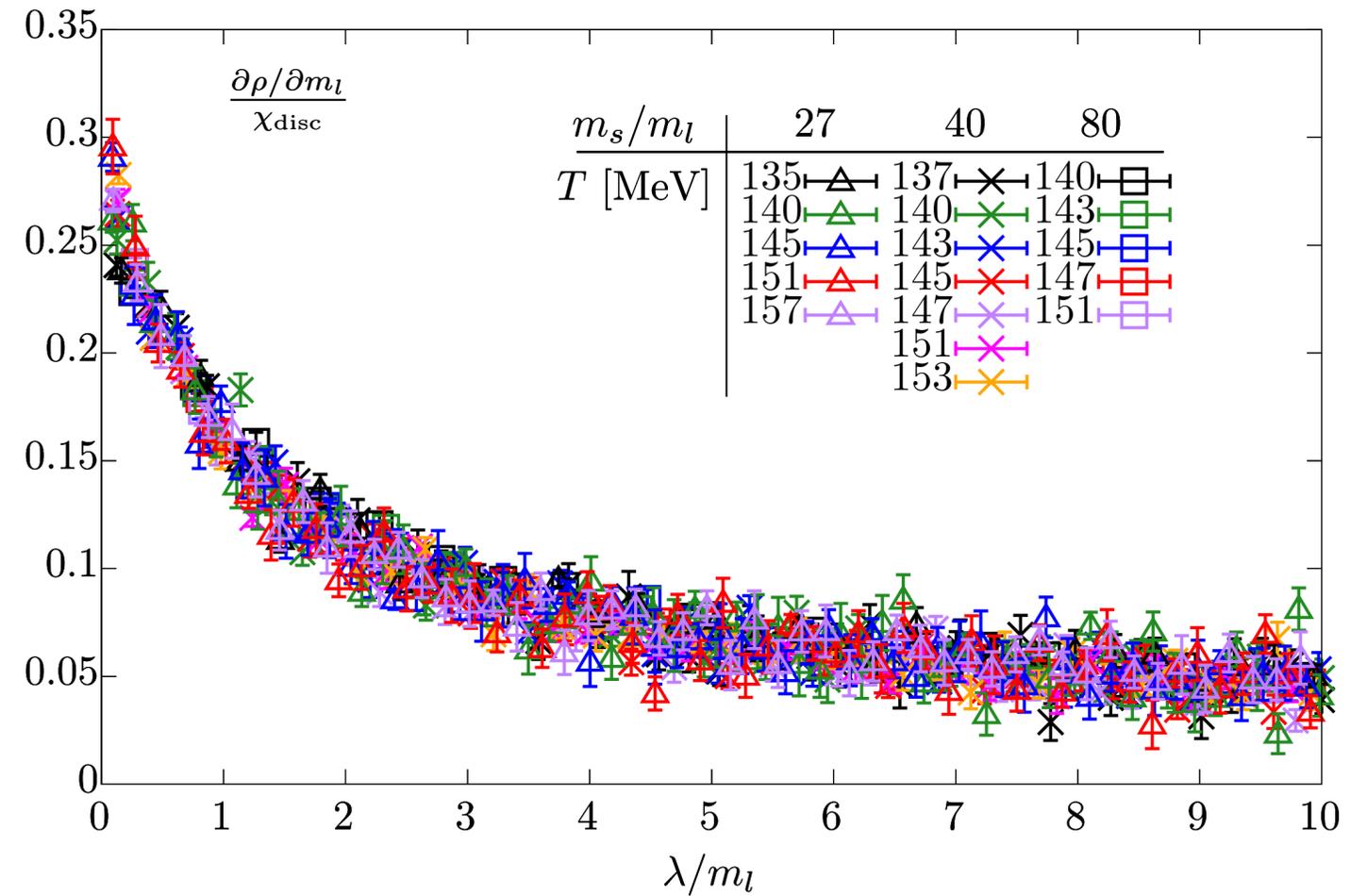
# Microscopic manifestation of chiral phase transition

黄玮平  
Wed Aug.10  
15:50-16:05

Quark mass derivative of Dirac Eigenvalue spectrum  $\partial\rho/\partial m$



Rescaled  $\partial\rho/\partial m$  by disconnected chiral susceptibility





# Lattice QCD in strong magnetic fields

No sign problem

$B$  pointing to the  $z$  direction & Gauge link multiplied by a  $U(1)$  factor

$$u_x(n_x, n_y, n_z, n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x, n_y, n_z, n_\tau) = \exp[iqa^2Bn_x],$$

$$u_z(n_x, n_y, n_z, n_\tau) = u_t(n_x, n_y, n_z, n_\tau) = 1.$$

Quantization of the magnetic field

$$qB = \frac{2\pi N_b}{N_x N_y} a^{-2} \quad \xrightarrow{q_u=2/3e, q_d=-1/3e, q_s=-1/3e} \quad eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

CCNU LQCD group has developed GPU/C++ codes for LQCD simulations at  $eB \neq 0$

contributors: Akio Tomiya, 李胜泰, 汪晓丹, 李韶荣, 张瑜

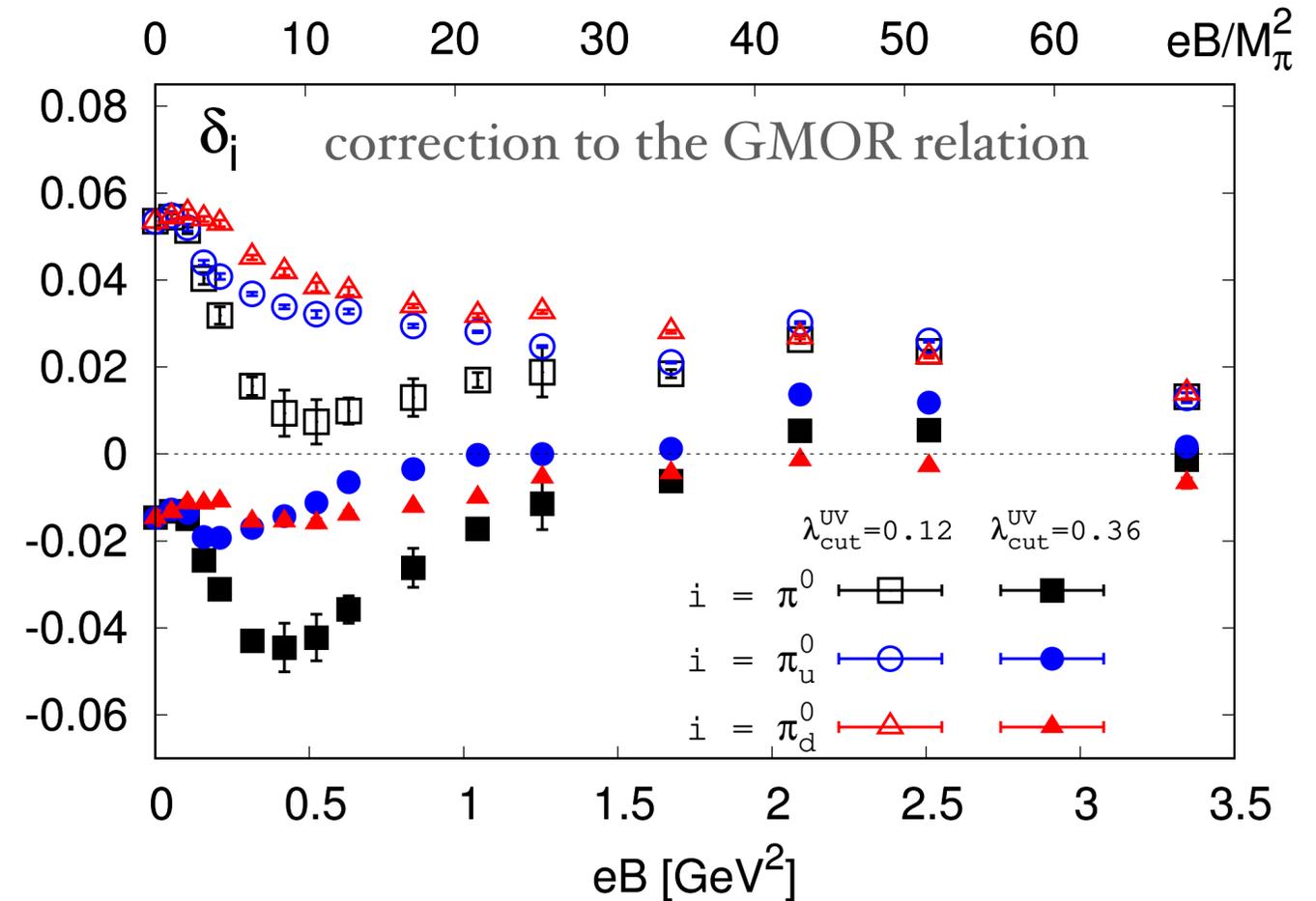
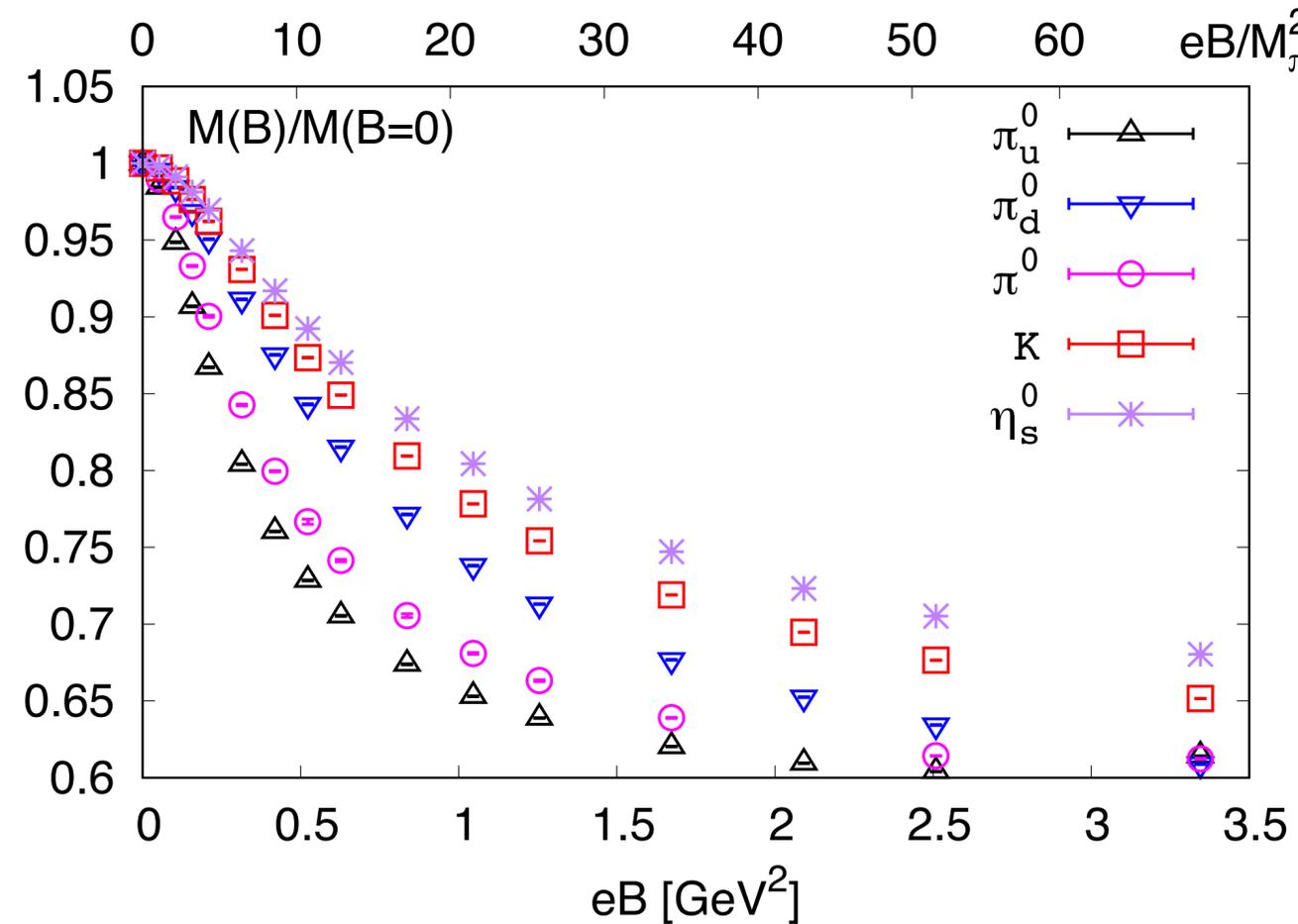
*EPJ Web Conf.* 175 (2018) 07041, *PoS LATTICE2018* (2019) 163 ,  
*PoS LATTICE2019* (2020) 250, *Phys.Rev.D* 102 (2020) 5, *Eur.Phys.J.A* 57 (2021) 6, 202,  
*Phys.Rev.D* 104 (2021) 1, *Acta Phys.Polon.Supp.* 14 (2021) 403, *Phys.Rev.D* 105 (2022) 3, 034514

... ..



# Gell-Mann-Oakes-Renner relation at $eB \neq 0$ and $T=0$

$$(m_u + m_d) (\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d) = 2f_\pi^2 M_\pi^2 (1 - \delta_\pi)$$



$N_{f=2+1}$  QCD,  $M_\pi(eB=0) \approx 220$  MeV, on  $32^3 \times 96$  lattices with  $a^{-1} \approx 1.7$  GeV<sup>-1</sup> and HISQ action at  $T=0$

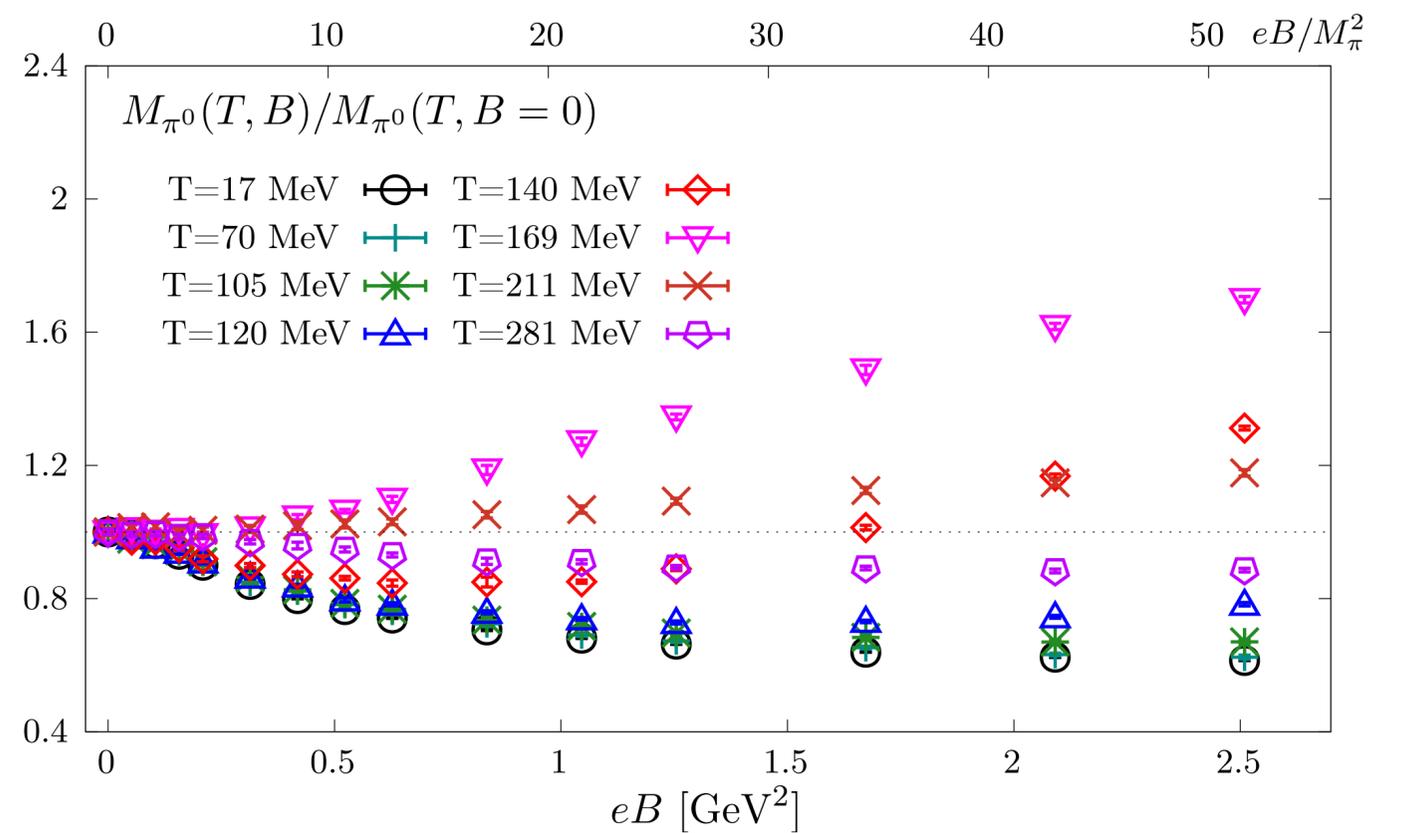
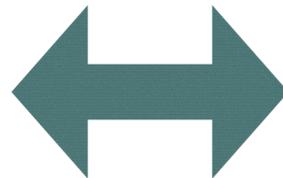
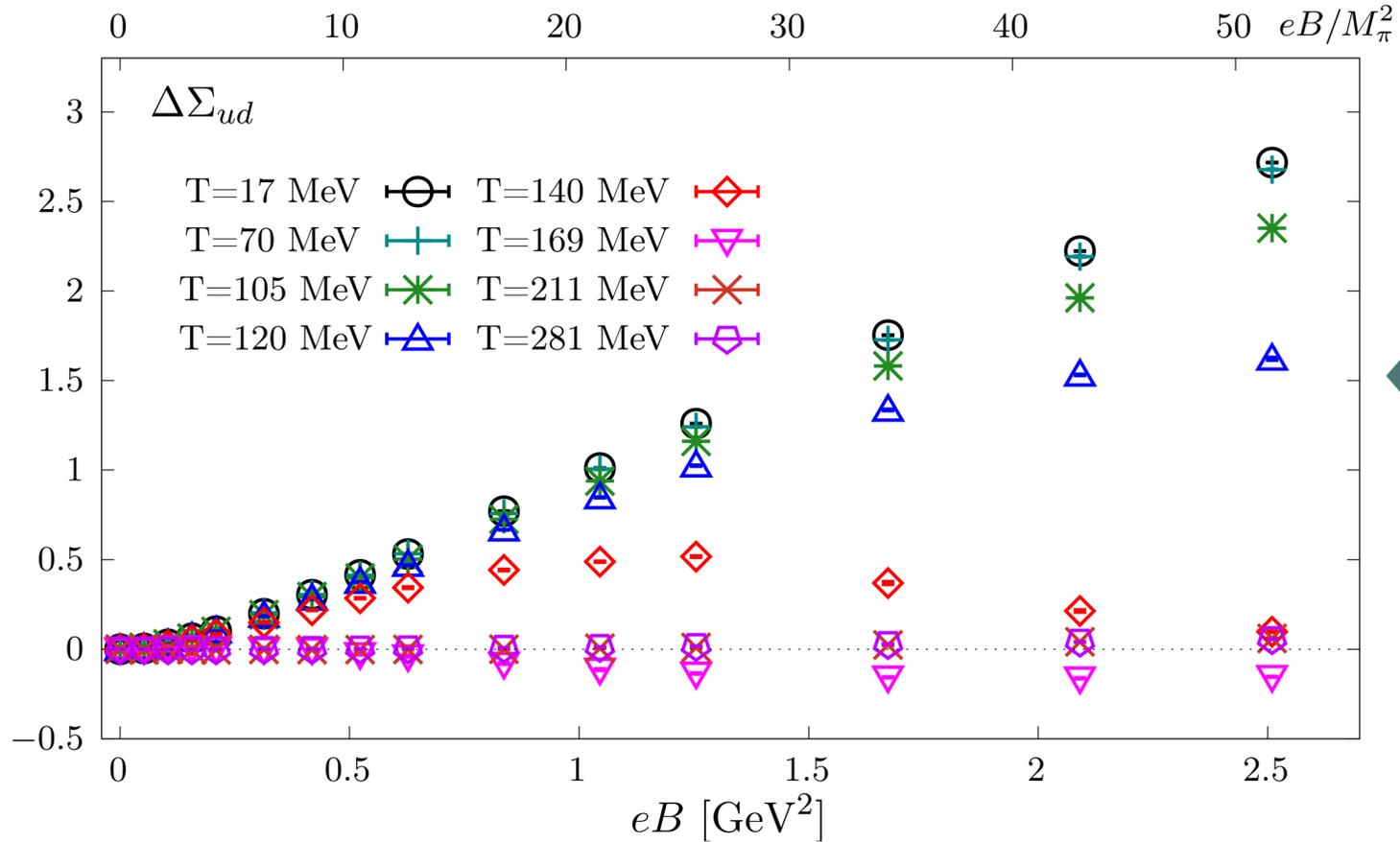
HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

# (inverse) magnetic catalysis $\iff$ hadron screening masses

李胜泰  
Wed Aug. 10  
15:35-15:50

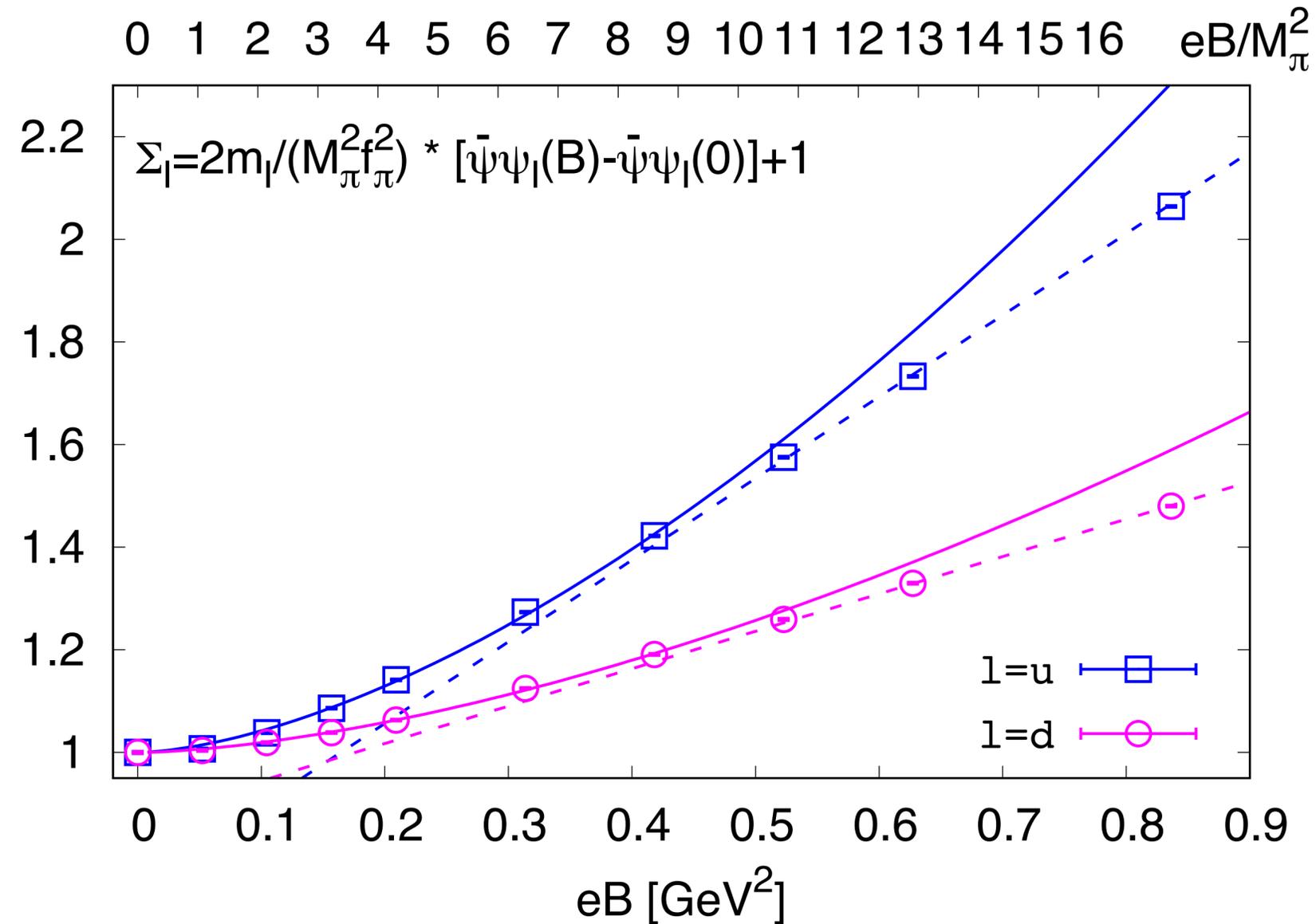
Ward Identity:  $\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d = (m_u + m_d) \chi_{\pi^0}$

$\chi_{\pi^0} = \int dz G_{\pi^0}(z), \quad \lim_{z \rightarrow \infty} G_{\pi^0}(z) = A_{\pi^0} e^{-M_{\pi^0} z}$   $M_{\pi^0}$ : Screening mass of  $\pi^0$



丁亨通, 李胜泰, 刘俊宏, 汪晓丹, Phys. Rev. D105(2022)034514

# Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



Not accessible in Heavy Ion Collision experiments

$eB$  related physics in HIC:  
See next talk by Xu-Guang Huang

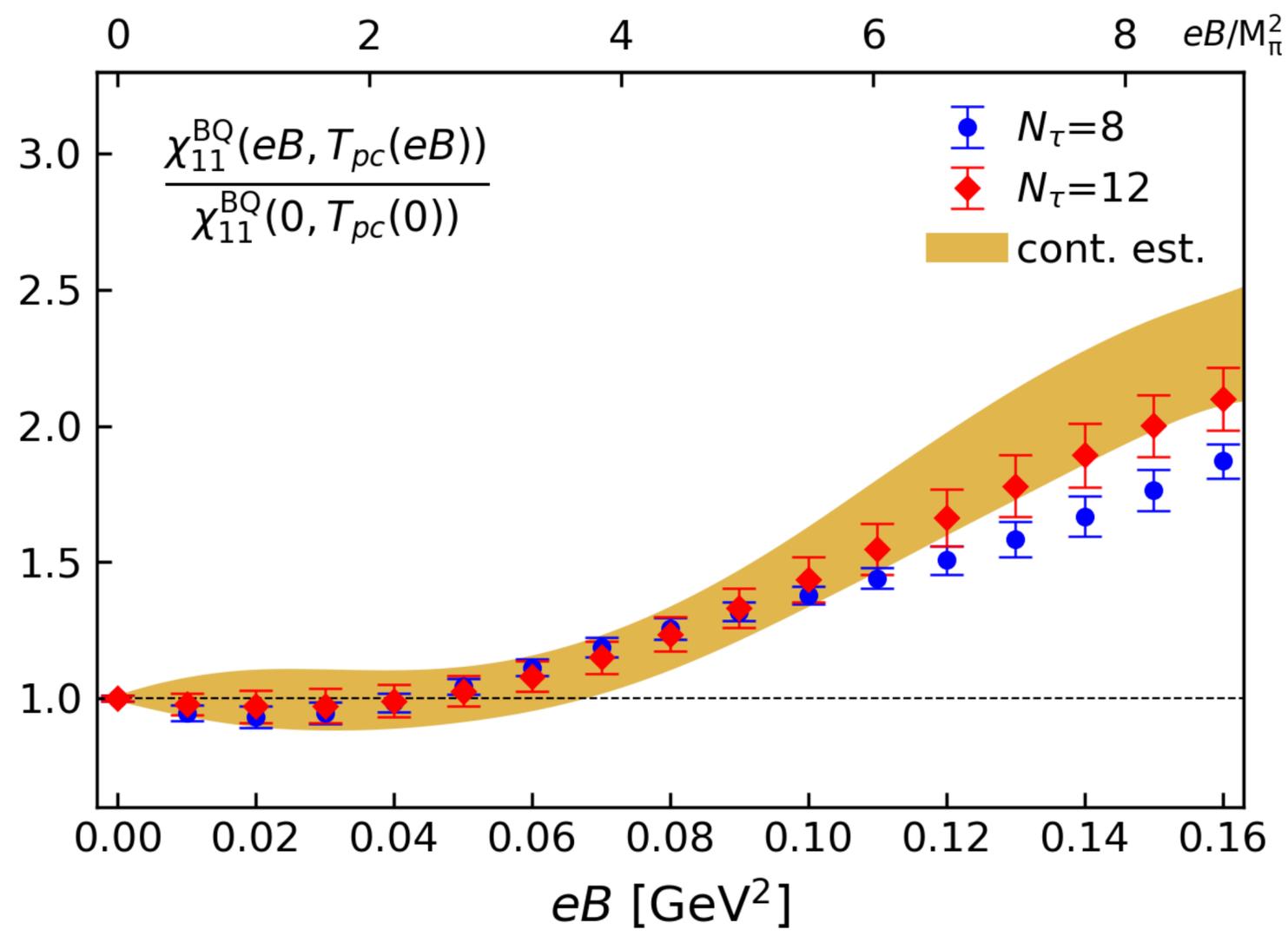
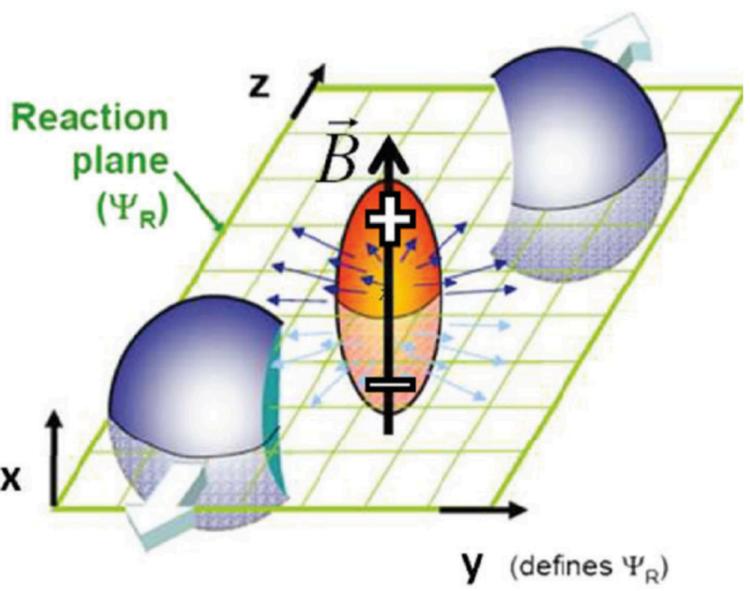
HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

See also in reviews e.g. M. D'Elia, Lect.NotesPhys.871(2013)181

# Ratio $X(eB)/X(eB=0)$ for 2nd order off-diagonal fluctuations

刘俊宏  
Wed, Aug. 10  
11:15-11:30

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 135$  MeV,  $T_{pc}(eB=0) \approx 157$  MeV,  
 $32^3 \times 8$  and  $48^3 \times 12$  lattices with HISQ action



$X(eB)/X(eB=0)$  :  
Rcp like observable

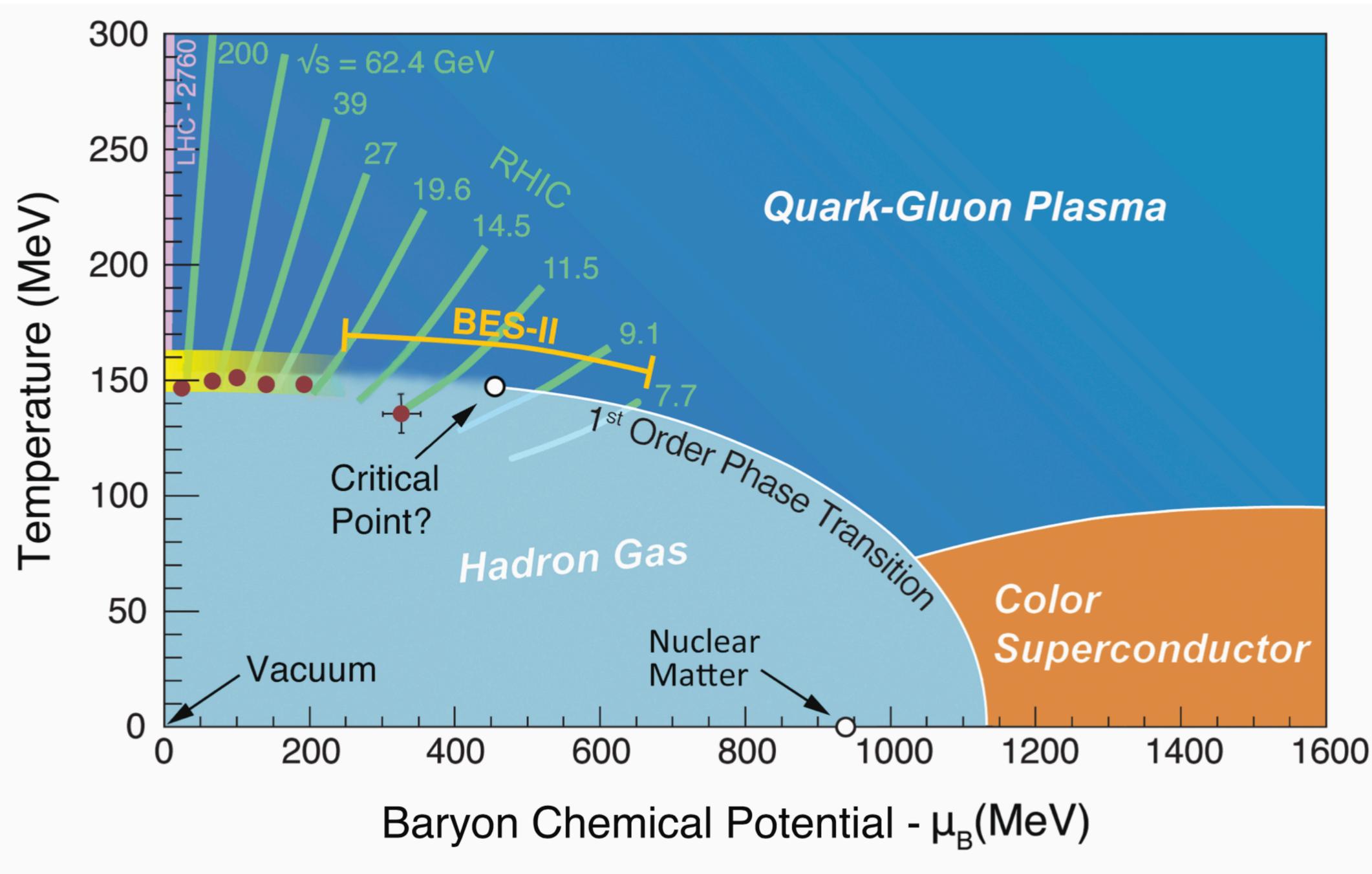
At  $eB \simeq 8M_\pi^2$ :  $\sim 2!$

刘俊宏 et al., CCNU, QM 2021 talk  
& work in progress



# QCD phase structure at nonzero baryon density

## QCD critical endpoint



- RHIC/BESII
- NA6I/SHINE
- HADES
- CEE
- CBM
- NICA

See recent reviews:

罗晓峰, 许怒, Nucl.Sci.Tech. 28(2017)112

丁亨通, Nucl.Phys.A1005(2021)21940

付伟杰, arXiv:2205.00468

# Taylor expansion of pressure in $\mu_B$

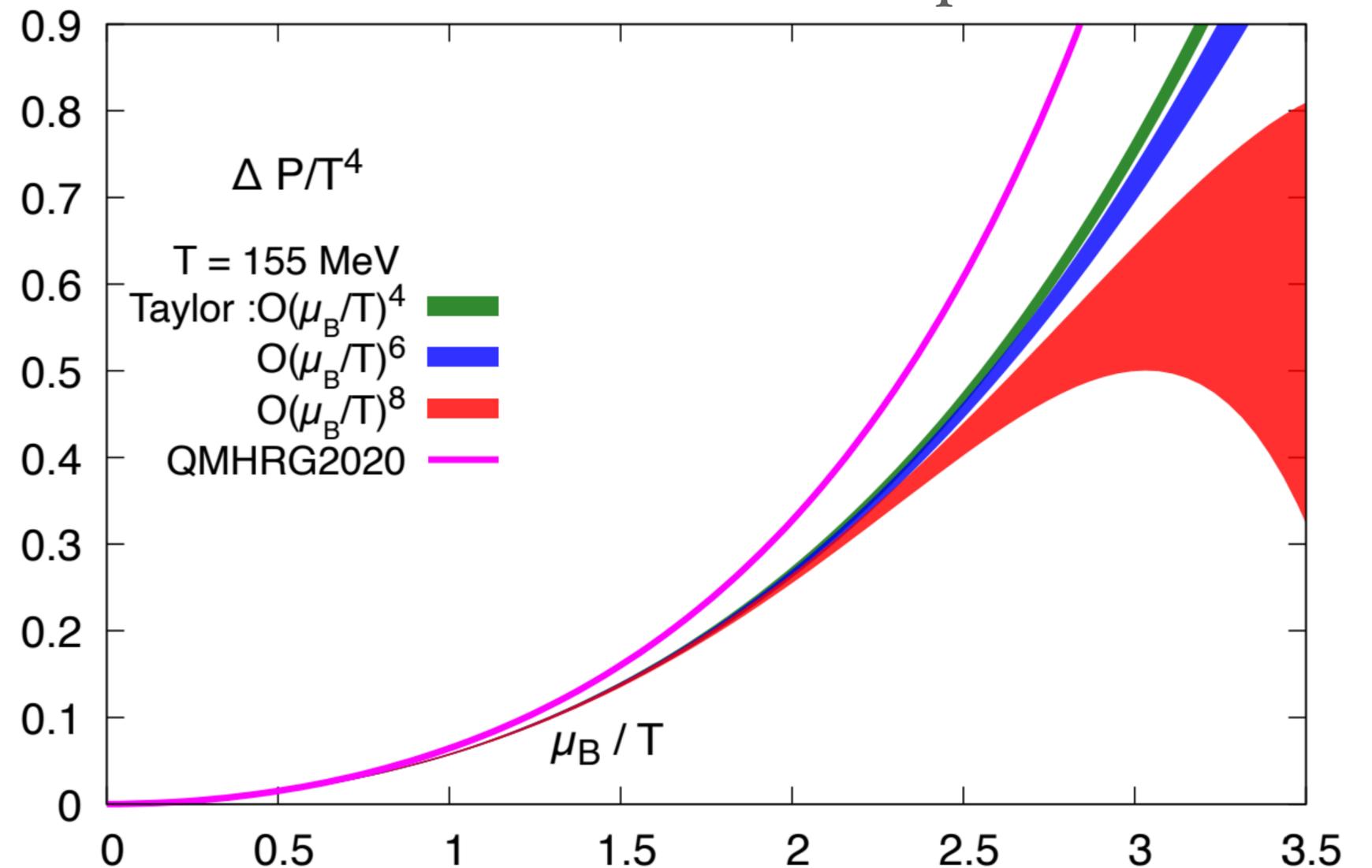
Taylor expansion in  $\mu_B$

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu) = \sum_{n=0}^{\infty} \frac{\chi_n^B}{n!} \left( \frac{\mu_B}{T} \right)^n$$

$$\chi_n^B(T) = \left. \frac{\partial^n P(T, \mu_B)/T^4}{\partial \hat{\mu}_B^n} \right|_{\hat{\mu}_B=0}$$

$$\Delta P = P(T, \mu_B) - P(T, \mu_B = 0)$$

State-of-the-art results on pressure



D. Bollweg et al., [HotQCD], Phys. Rev. D 105(2022)074511

# Resummed Taylor expansion: all orders in $\mu_B$

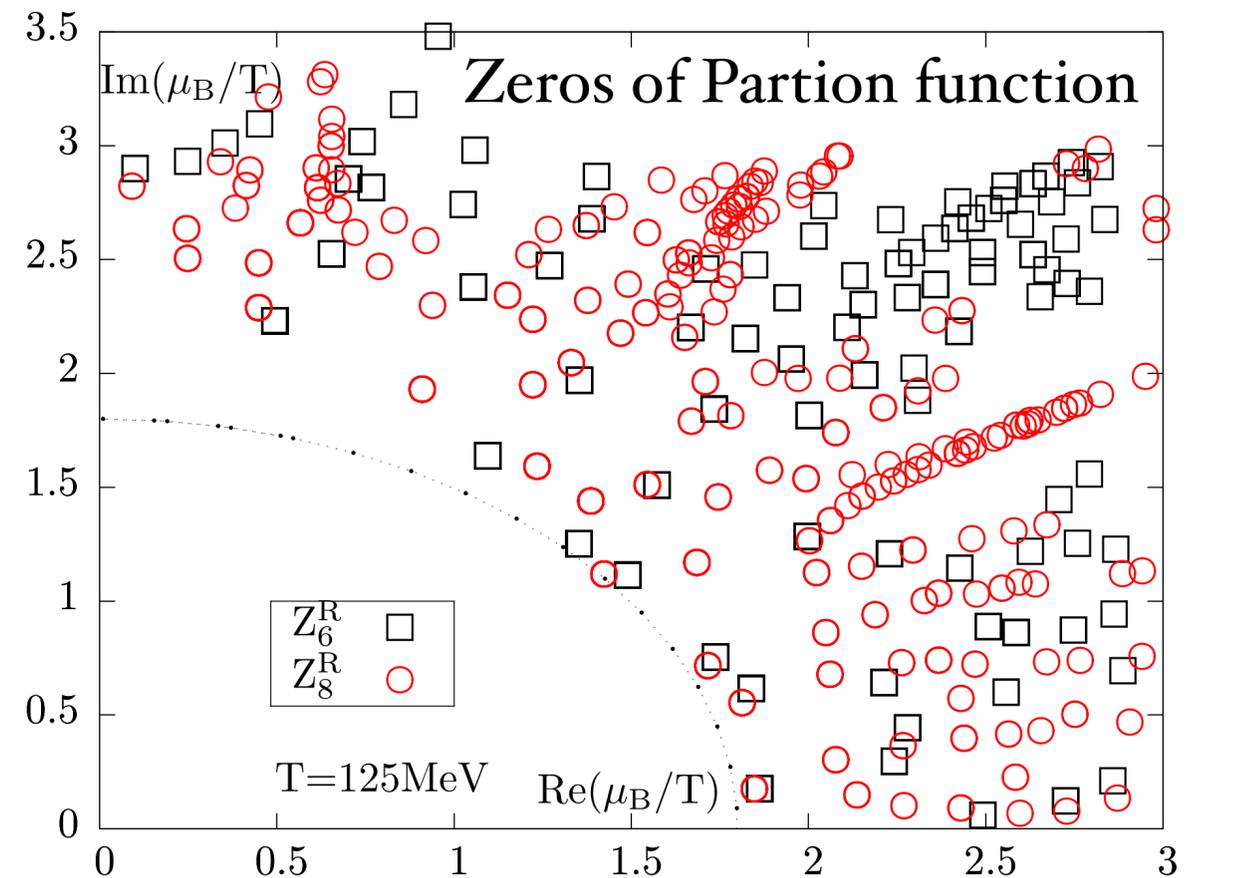
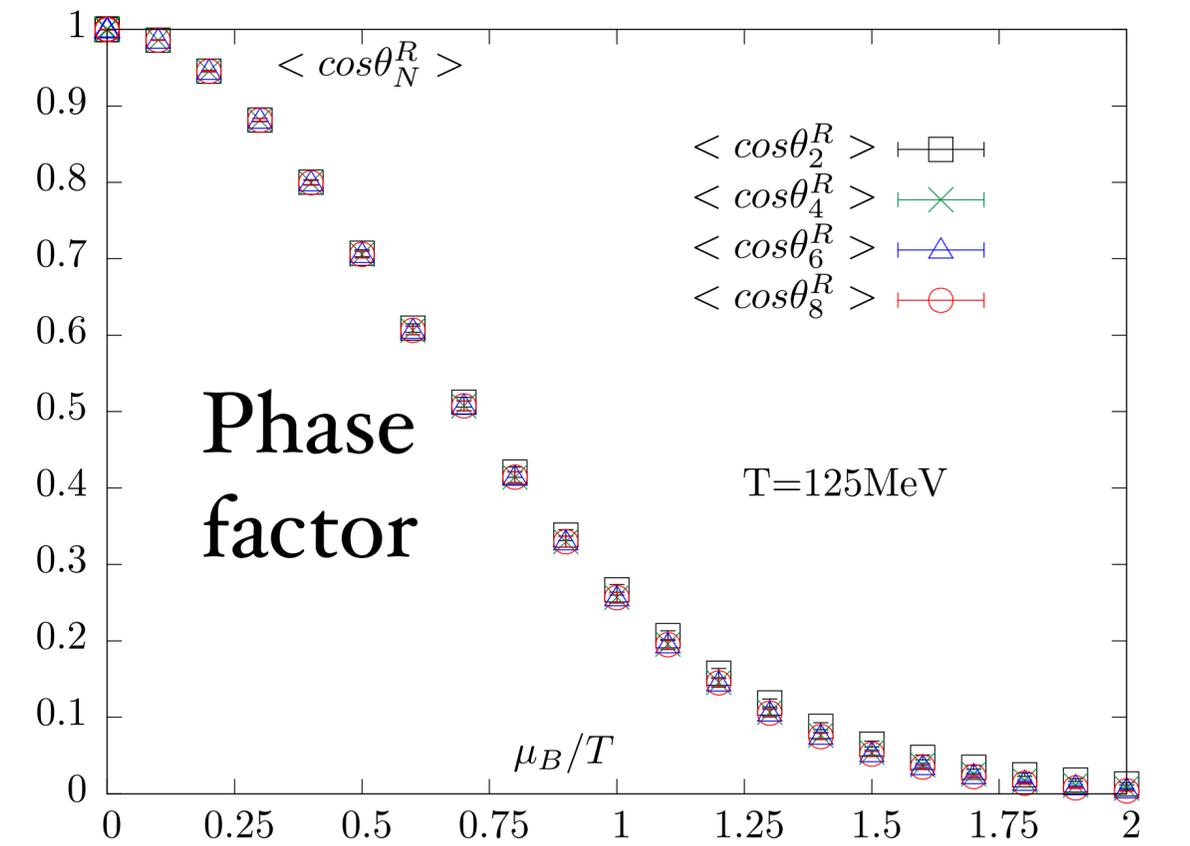
$$\bar{D}_n = \frac{1}{n!} \frac{\partial^n \ln \det[M(T, \mu_B)]}{\partial (\mu_B/T)^n} \Bigg|_{\mu_B=0}$$

$$\frac{\Delta P_N^R}{T^4} = \frac{1}{VT^3} \ln \left( \frac{Z_N^R(T, \mu_B)}{Z(T, 0)} \right) = \frac{1}{VT^3} \ln \left\langle \exp \left[ \sum_{n=1}^N \bar{D}_n \left( \frac{\mu_B}{T} \right)^n \right] \right\rangle$$

$$\langle \cos \Theta_N^R \rangle = \left\langle \cos \left( \sum_{n=1}^{N/2} \text{Im}[\bar{D}_{2n-1}] \left( \frac{\mu_B}{T} \right)^{2n-1} \right) \right\rangle$$

S. Mondal, S. Mukherjee & P. Hegde, PRL 128 (2022) 022001

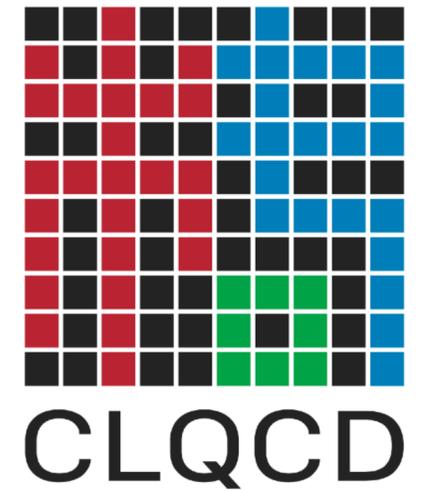
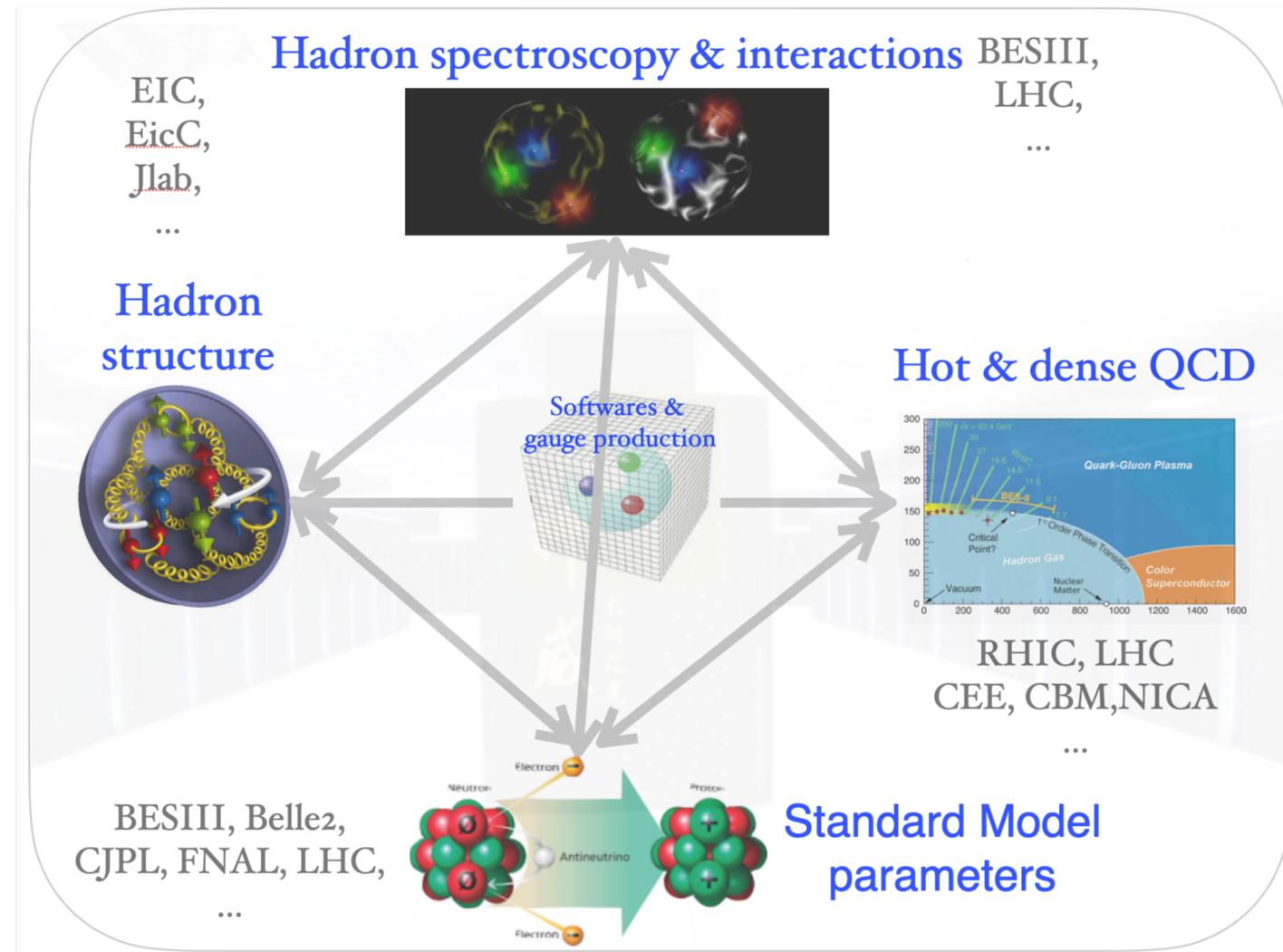
李文汐 et al., CCNU-BNL, work in preparation



# Summary & Outlook



刘川, 2006,  
中国物理学会高能物理分会  
第七届学术年会  
大会报告



2021-2022: 7 PRL + ...

Lattice 2022 plenary talks:

刘柳明(近物所)

晨光杯:

2014 桂龙成(高能所)

2021 李胜泰(华中师大)

2022 傅杨(北大, 候选人)

⇒ Lattice QCD applications on Exa-scale supercomputers in China

# 第二届中国格点QCD研讨会

时间：2022年10月7-10日

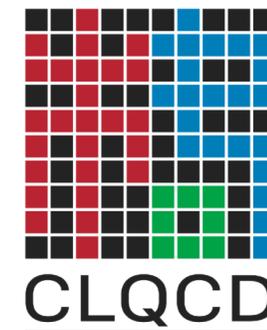
地点：李政道研究所

交流格点QCD研究最新进展

谋划E级超算上的格点应用

欢迎各位莅临交流指导！

<https://indico-tdli.sjtu.edu.cn/event/1198/overview>



第二届中国格点

量子色动力学研讨会

2022年10月7日-10月10日

李政道研究所 承办

“第二届中国格点量子色动力学研讨会”将于2022年10月7日-10月10日举办，由李政道研究所承办。会议注册已开放。  
<https://indico-tdli.sjtu.edu.cn/event/1198/overview>

强相互作用的非微扰性质是标准模型中尚待解决的重大理论疑难，也是其精细检验中需面对的挑战。从第一性原理出发，格点QCD能够利用超大规模数值模拟，精确计算上述非微扰性质，并与目前和未来的实验结果相互印证。

中国于20世纪八十年代初开始格点QCD研究，并于2005年成立了合作组CLQCD。经过多年发展和近期一大批优秀年轻研究人员的加盟，中国的格点QCD研究已经具备了利用最顶尖的超级计算机，探索强子能谱、核子结构、QCD相结构和标准模型精细检验等方向的国际前沿问题的能力。

高性能计算资源在格点QCD研究中起着至关重要的作用，除了依托本地的计算机群外，CLQCD合作组各单位亦在积极谋划依托国家即将部署的E级超算从事深入的格点研究。

研讨会诚邀相关领域专家共同探讨粒子物理、核物理中的重要前沿问题，寻求格点QCD与相关领域的合作和协同发展。

