Light Meson Spectroscopy at BESIII

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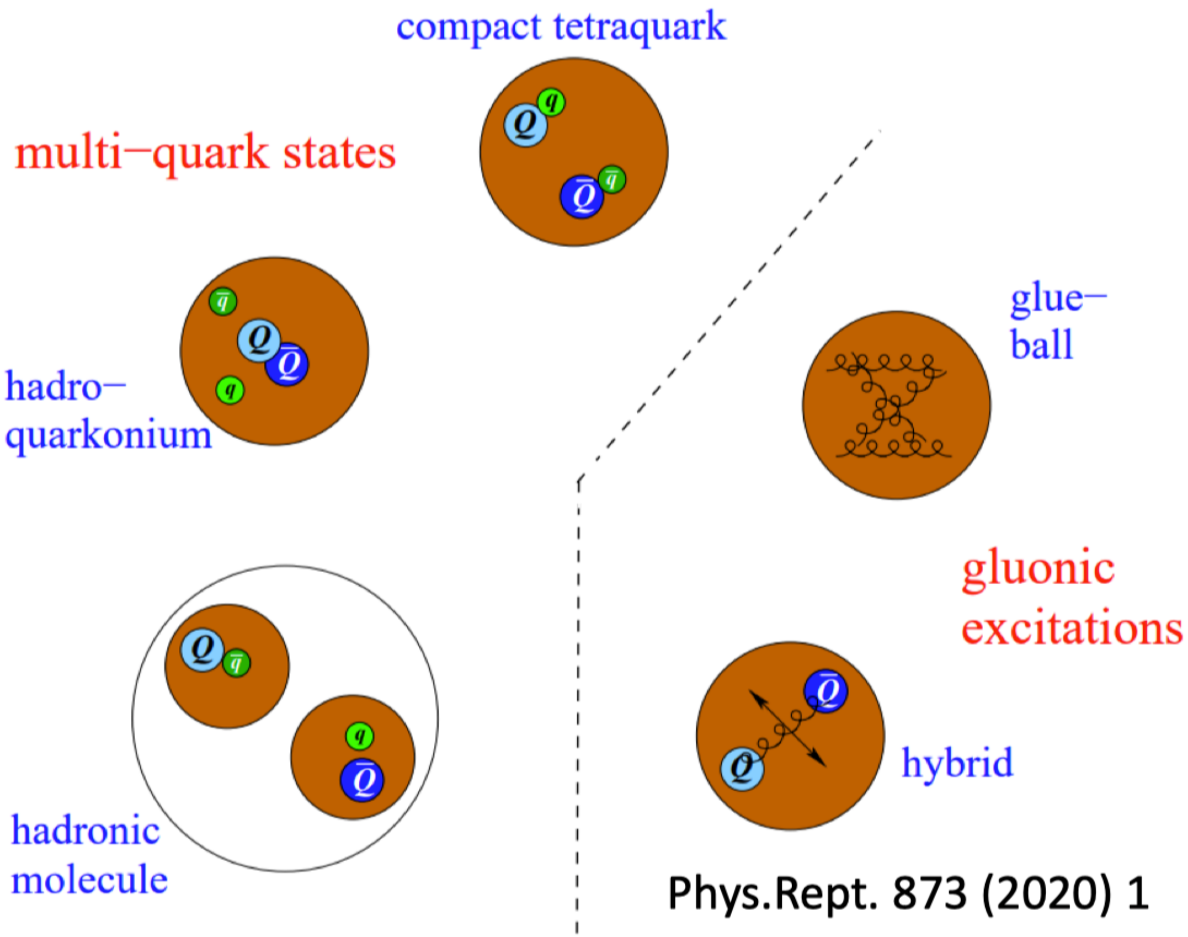
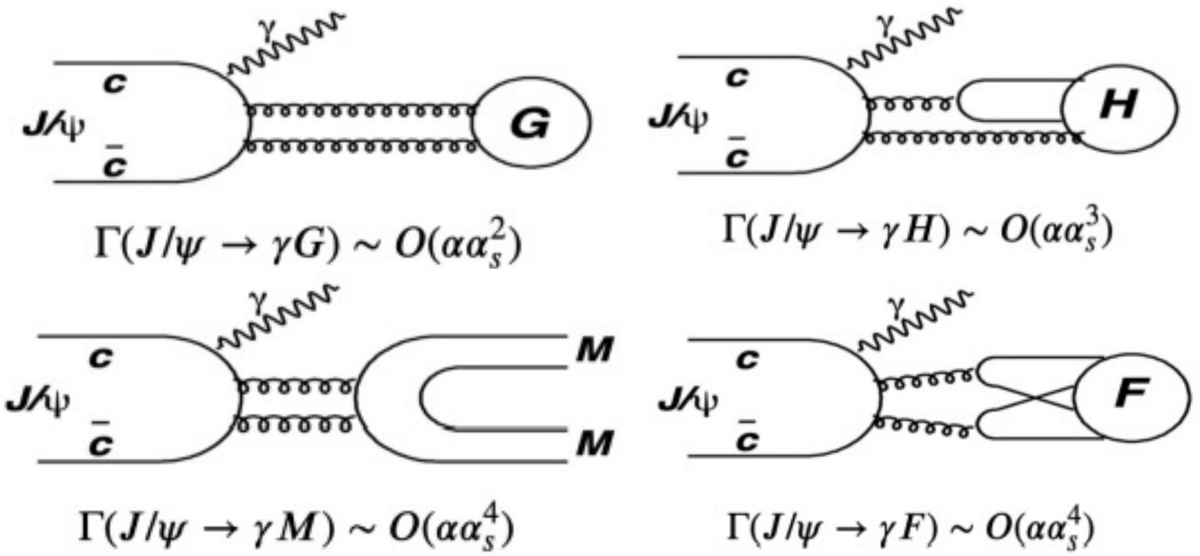
Charmonium decays provides an ideal lab for light hadron physics



Phys.Rept. 873 (2020) 1

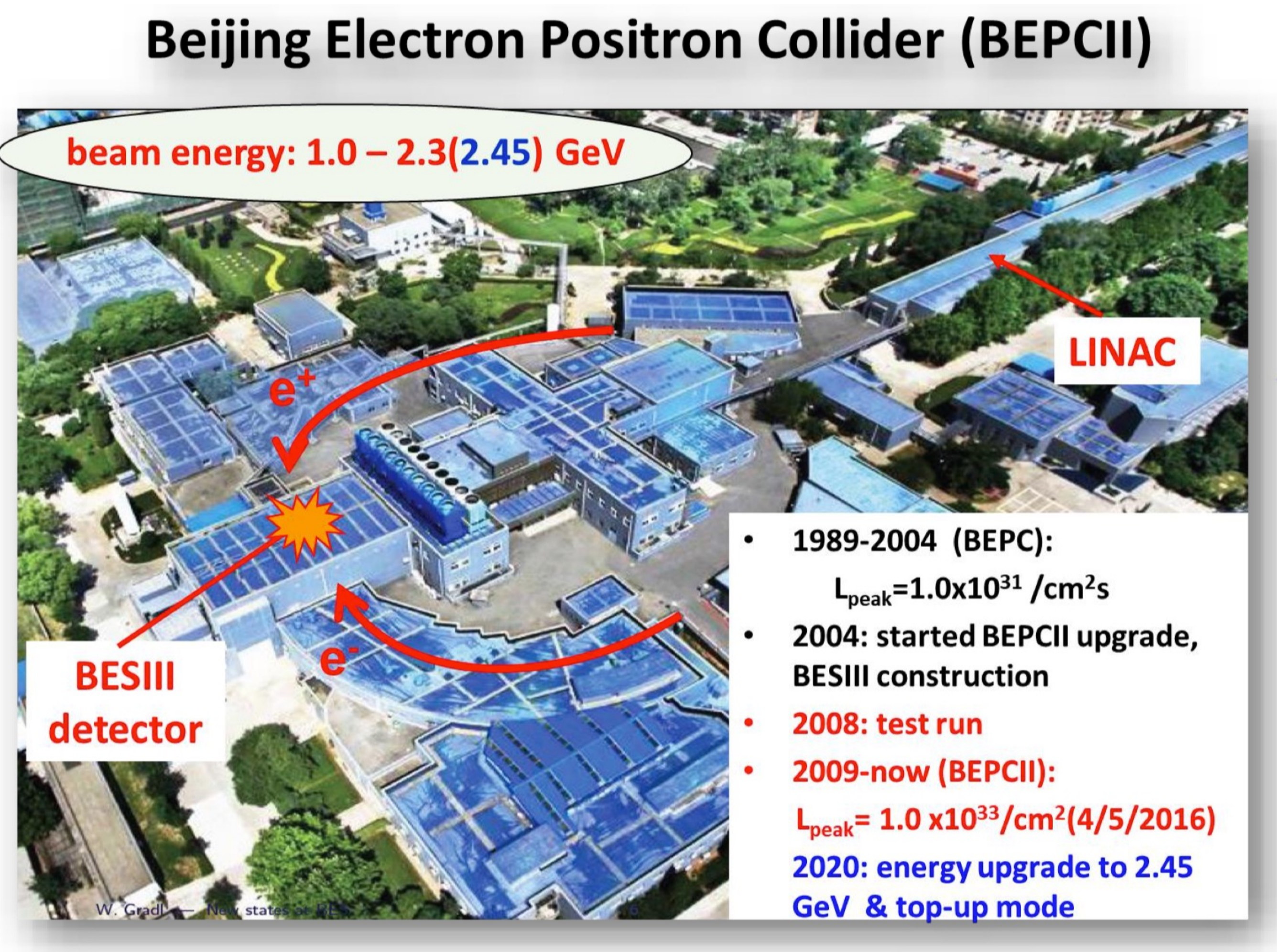
- Clean **high statistics** data samples
- Well defined initial and final states
  - Kinematic constraints
  - $I(J^{PC})$ filter
- "**Gluon-rich**" processes

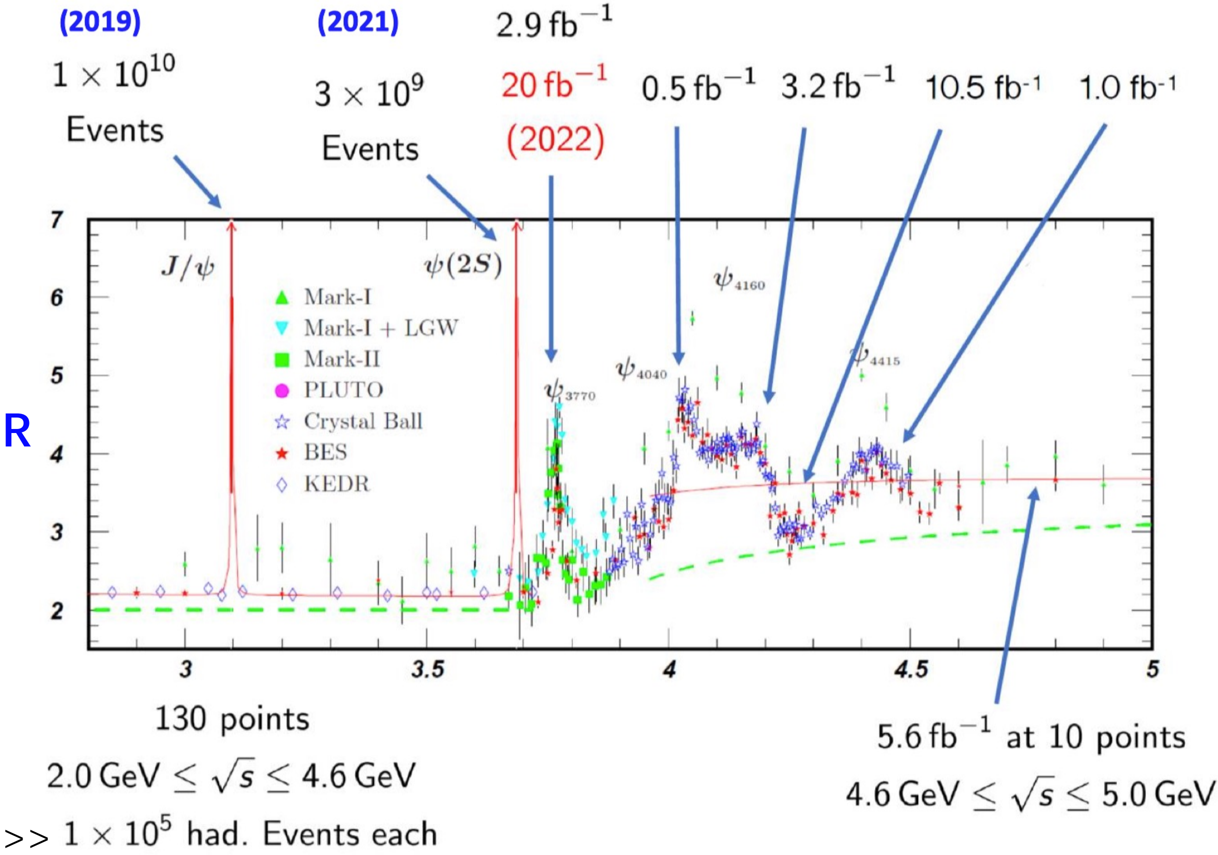
$$\Gamma(J/\psi \to \gamma G) > \Gamma(J/\psi \to \gamma H) > \Gamma(J/\psi \to \gamma M) > \Gamma(J/\psi \to \gamma F)$$



What's the role of gluonic excitation and how does it connect to the confinement?

World's Largest τ-charm Data Sets in e^+e^- Annihilation

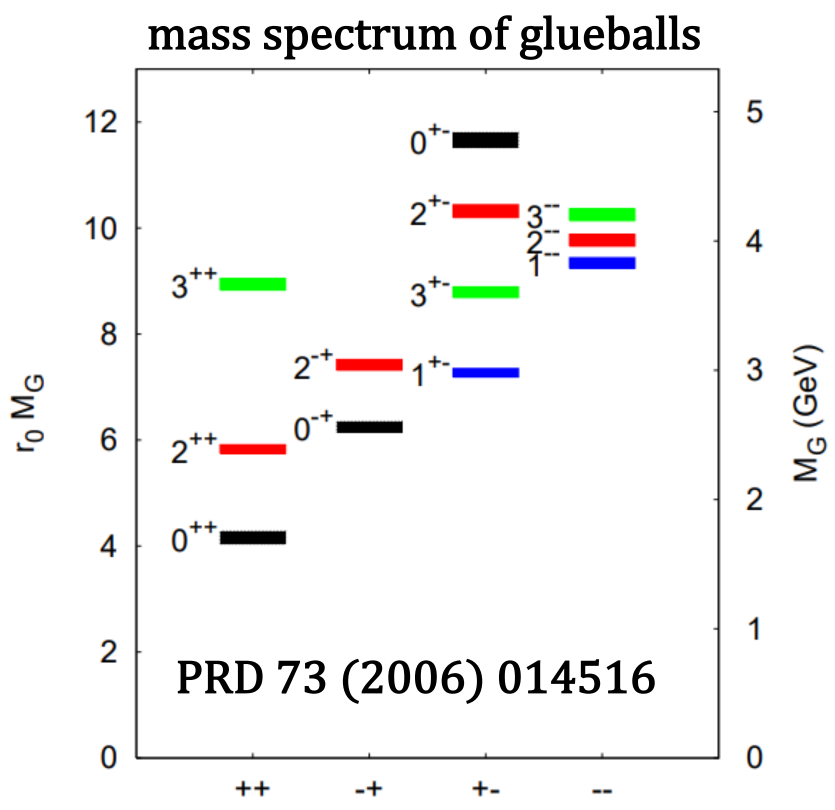




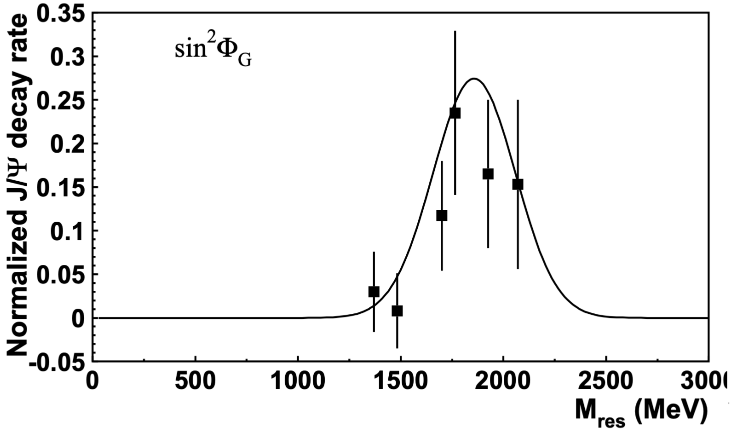
Glueballs

- States composed of only gluons
- **Low-lying glueballs** with ordinary J^{PC} → **mixing** with $q\bar{q}$ meson
- **LQCD predictions**
  - ➢ **0^{++} glueball**
    - lightest mass : 1.5~1.7 GeV/c^2
    - $B(J/\psi \to \gamma G_{0^{++}}) = 3.8(9)\times 10^{-3}$
  - ➢ **2^{++} glueball**
    - lightest mass : 2.3~2.4 GeV/c^2
    - $B(J/\psi \to \gamma G_{2^{++}}) = 1.1(2)(1)\times 10^{-2}$

LQCD prediction of glueball spectrum



P.L.B 826, 136906 (2022)

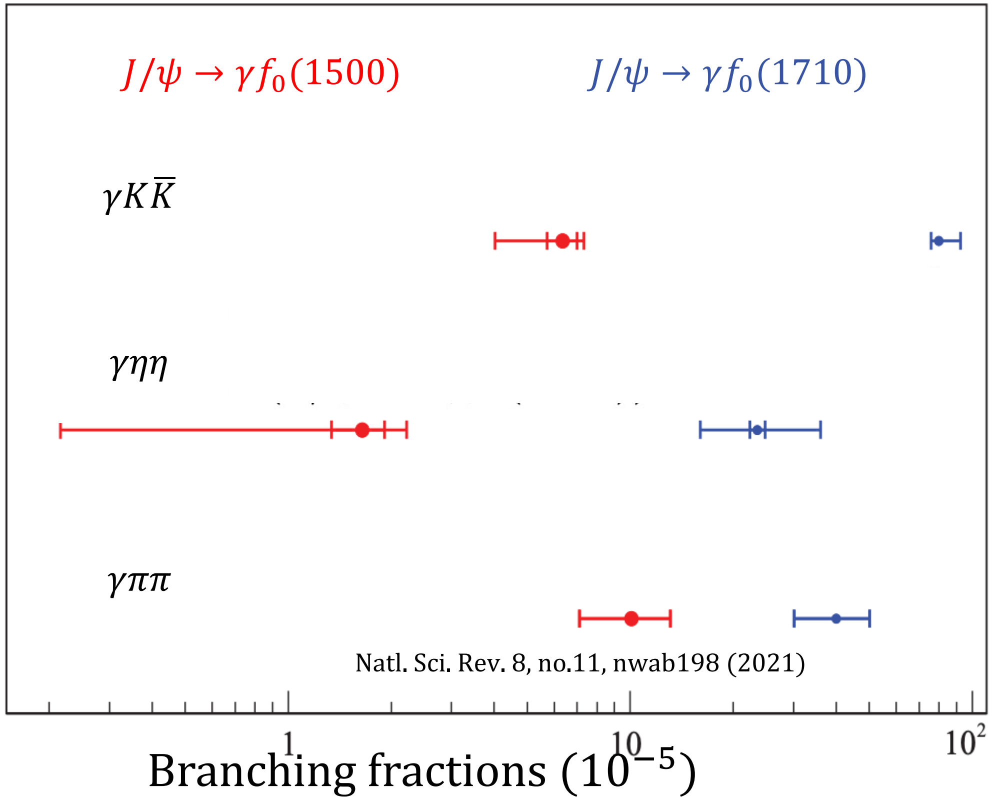


Glueballs

$$\mathcal{B}(J/\psi \to \gamma f_0(1500)) \sim 0.29 \times 10^{-3},$$
$$\mathcal{B}(J/\psi \to \gamma f_0(1710)) \sim 2.2 \times 10^{-3},$$

- **Production properties :**
  - $B(J/\psi \to \gamma f_0(1710))$ is compatible with LQCD predictions for a scalar glueball
  - Observed $B(J/\psi \to \gamma f_0(1710))$ is x10 larger than $f_0(1500)$
  - $f_0(1710)$ largely overlapped with scalar glueball
- **Decay properties : $G \to \eta\eta'$ decay** is expected to be **suppressed**
  - $SU(3)_f$ symmetry for a pure glueball

    $\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$
  - $B(G \to \eta\eta')/B(G \to \pi\pi) < 0.04$, predicted by Ref. [1]



$J/\psi \to \gamma\eta\eta'$ provides important information

[1]P.R.D 92 12,121902 (2015)

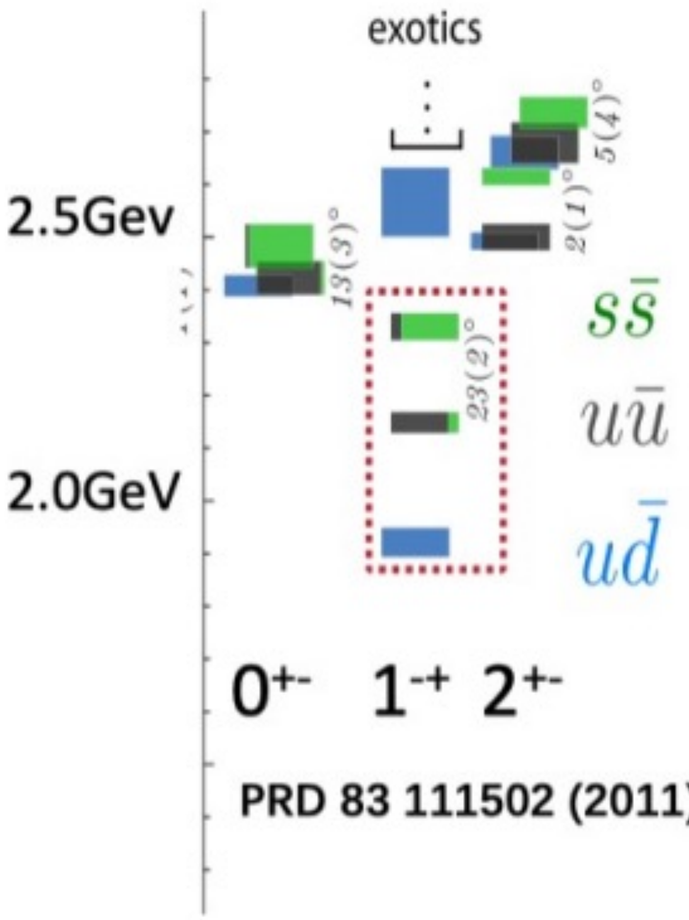
Hybrids

- Formed by quarks, anti-quarks, and excitated gluon field

- Low-lying hybrids can have **exotic quantum numbers** $0^{+-}, 1^{-+}, 2^{+-}$, which is **forbidden by $q\bar{q}$** configuration

- LQCD predicts the mass of **lightest** exotic $J^{PC} = \mathbf{1^{-+}}$ **nonet of hybrids** is 1.7~2.1 GeV/c^2

Lattice QCD Predictions:



PRD 83 111502 (2011)

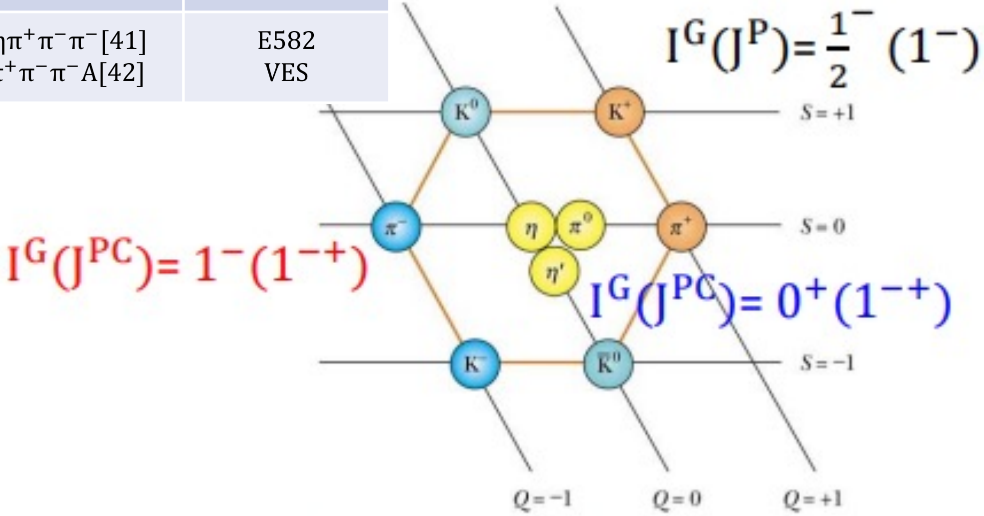
Hybrids(1^{-+})

- **Only isovector 1^{-+}** candidates observed : $\pi_1(1400), \pi_1(1600), \pi_1(2015)$

	π_1 decay mode	decay channel	Collaboration
$\pi_1(1400)$	$\eta\pi$	$\pi^- p \to \pi^- \eta p$[28] $\pi^- p \to \pi^0 \eta n$[27] $\pi^- p \to \pi^- \eta p$[29] $\pi^- p \to \pi^0 \eta n$[30] $\bar{p}n \to \pi^- \pi^0 \eta$[31] $\bar{p}p \to \pi^0 \pi^0 \eta$[32]	GAMS KEK E852 E852 CBAR CBAR
	$\rho\pi$	$\bar{p}p \to 2\pi^+ 2\pi^-$[33]	Obelix
$\pi_1(2015)$	$f_1\pi$	$\pi^- p \to \omega\pi^-\pi^0 p$[38]	E582
	$b_1\pi$	$\pi^- p \to \rho\eta\pi^+\pi^-\pi^-$[41]	

	π_1 decay mode	decay channel	Collaboration
$\pi_1(1600)$	$\eta'\pi$	$\pi^- Be \to \eta'\pi^-\pi^0 Be$[34] $\pi^- p \to \pi^-\eta' p$[35] $\chi_{c1} \to \eta'\pi^+\pi^-$[36]	VES E852 CLEO-c
	$b_1\pi$	$\pi^- Be \to \omega\pi^-\pi^0 Be$[34] $\bar{p}p \to \omega\pi^+\pi^-\pi^0$[37] $\pi^- p \to \omega\pi^-\pi^0 p$[38]	VES CBAR E582
	$\rho\pi$	$\pi^- Pb \to \pi^+\pi^-\pi^- X$[39] $\pi^- p \to \pi^+\pi^-\pi^- p$[40]	COMPASS E582
	$f_1\pi$	$\pi^- p \to \rho\eta\pi^+\pi^-\pi^-$[41] $\pi^- A \to \eta\pi^+\pi^-\pi^- A$[42]	E582 VES

- **Isoscalar 1^{-+}** is critical to establish the **hybrid nonet**
  - Can be produced in the gluon-rich J/ψ radiative decays
  - Can decays to $\eta\eta'$ in P-wave [2][3][4]



➢ **Search for Isoscalar 1^{-+} in $J/\psi \to \gamma\eta\eta'$**

[2] PRD 83,014021 (2011)
[3] PRD 83,014006 (2011)
[4] Eur.Phys.J.Plus 135, 945(2020)

Observation of An Exotic Isoscalar State $\eta_1(1855)$ (1^{-+}) in $J/\psi \to \gamma\eta\eta'$

10 billion J/ψ

arXiv:2202.00621

arXiv:2202.00623

- The η' is reconstructed from $\gamma\pi^+\pi^-$ & $\eta\pi^+\pi^-$, η from $\gamma\gamma$

- Partial wave analysis of $J/\psi \to \gamma\eta\eta'$

  Quasi two-body decay amplitudes in the sequential decay processes $J/\psi \to \gamma X, X \to \eta\eta'$ and $J/\psi \to \eta X, X \to \gamma\eta'$ and $J/\psi \to \eta' X, X \to \gamma\eta$ are constructed using the covariant tensor formalism[5]

- All kinematically allowed known resonances with 0^{++}, 2^{++}, 4^{++} $(\eta\eta')$ and 1^{+-}, $1^{--}$$(\gamma\eta^{(\prime)})$ are considered

  1^{-+} in $\eta\eta'$ is also considered (η/η' not identical particle)

Decay mode	Resonance	M (MeV/c^2)	Γ (MeV)	B.F. ($\times 10^{-5}$)	Sig.
$J/\psi \to \gamma X \to \gamma\eta\eta'$	$f_0(1500)$	1506	112	$1.81\pm0.11^{+0.19}_{-0.13}$	$\gg30\sigma$
	$f_0(1810)$	1795	95	$0.11\pm0.01^{+0.04}_{-0.03}$	11.1σ
	$f_0(2020)$	$2010\pm6^{+6}_{-4}$	$203\pm9^{+13}_{-11}$	$2.28\pm0.12^{+0.29}_{-0.20}$	24.6σ
	$f_0(2330)$	$2312\pm7^{+7}_{-3}$	$65\pm10^{+3}_{-12}$	$0.10\pm0.02^{+0.01}_{-0.02}$	13.2σ
	$\eta_1(1855)$	$1855\pm9^{+6}_{-1}$	$188\pm18^{+3}_{-8}$	$0.27\pm0.04^{+0.02}_{-0.04}$	21.4σ
	$f_2(1565)$	1542	122	$0.32\pm0.05^{+0.12}_{-0.02}$	8.7σ
	$f_2(2010)$	$2062\pm6^{+10}_{-7}$	$165\pm17^{+10}_{-5}$	$0.71\pm0.06^{+0.10}_{-0.06}$	13.4σ
	$f_4(2050)$	2018	237	$0.06\pm0.01^{+0.03}_{-0.01}$	4.6σ
	0^{++} PHSP	-	-	$1.44\pm0.15^{+0.10}_{-0.20}$	15.7σ
$J/\psi \to \eta' X \to \gamma\eta\eta'$	$h_1(1415)$	1416	90	$0.08\pm0.01^{+0.01}_{-0.02}$	10.2σ
	$h_1(1595)$	1584	384	$0.16\pm0.02^{+0.03}_{-0.01}$	9.9σ

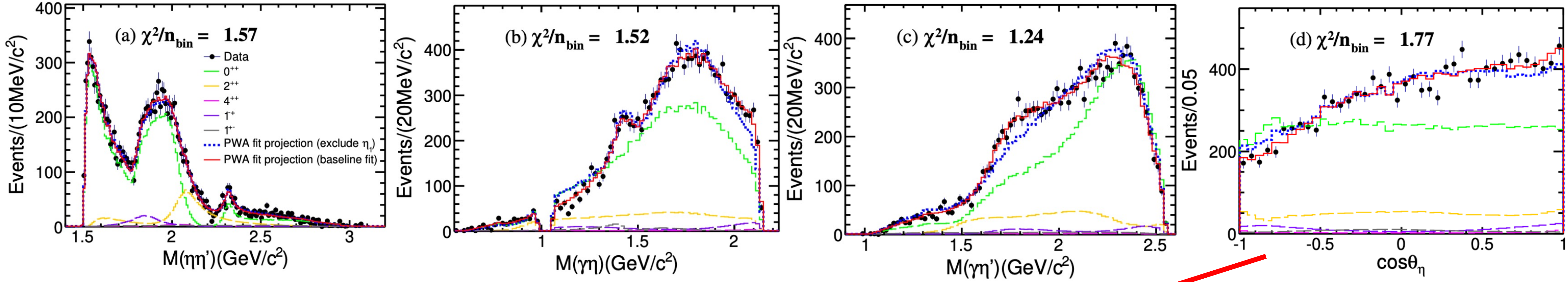
significance of all other additional resonances are less than 3 σ

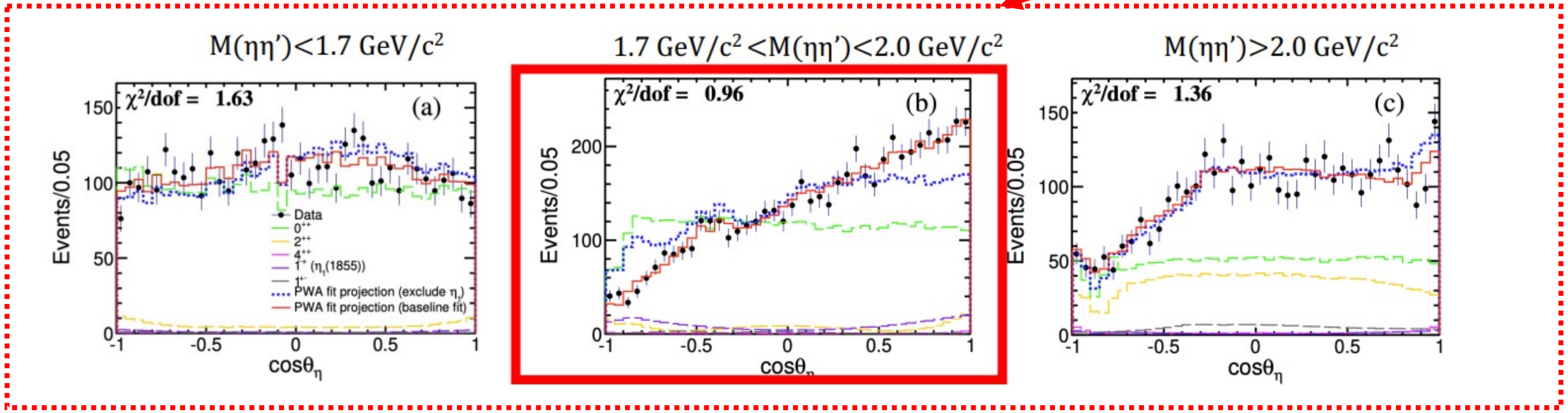
✓ An isoscalar resonance with exotic $J^{PC} = 1^{-+}$

➢ consistent with LQCD calculation for the 1^{-+} hybrid (1.7~2.1 GeV/c^2)

[5] Eur. Phys. J. A 16, 537 (2003)

Observation of An Exotic Isoscalar State $\eta_1(1855)$ (1^{-+}) in $J/\psi \to \gamma\eta\eta'$





✓ A clear asymmetry largely due to $\eta_1(1855)$ signal

Further Checks on the 1^{-+} State $\eta_1(1855)$

Angular distribution as a function of M(ηη′) can be expressed **model-independently** in terms of Legendre polynomial moments

$$\langle Y_0^0 \rangle \equiv \sum_{i=1}^{N_k} W_i Y_l^0(cos\theta_\eta^i)$$

- **Neglecting** resonance contributions in the **γη and γη′** subsystems, the moments are related to the spin-0(S), spin-1(P), spin-2(D) amplitudes in ηη′ by:

$$\sqrt{4\pi}\langle Y_0^0 \rangle = S^2 + P^2 + D^2$$

$$\sqrt{4\pi}\langle Y_1^0 \rangle = 2SPcos\phi_P + 4PDcos(\phi_P - \phi_D)$$

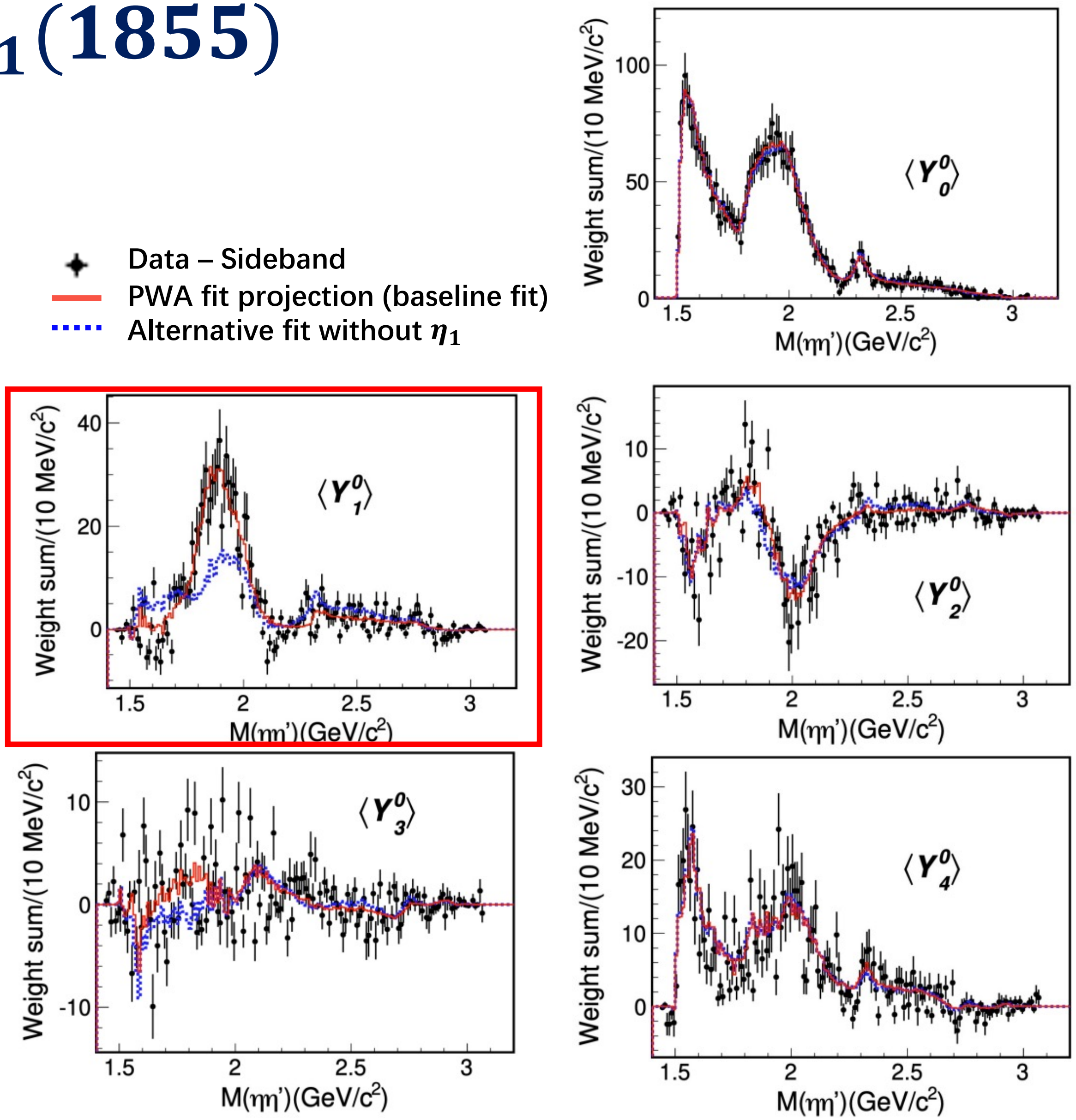
$\langle Y_1^0 \rangle = 0$ without P-wave contribution

$$\sqrt{4\pi}\langle Y_2^0 \rangle = \frac{2}{\sqrt{5}}P^2 + \frac{2\sqrt{5}}{7}D^2 + 2SDcos\phi_D$$

$$\sqrt{4\pi}\langle Y_3^0 \rangle = \frac{6}{5}\sqrt{\frac{15}{7}}PDcos(\phi_P - \phi_D)$$

$$\sqrt{4\pi}\langle Y_4^0 \rangle = \frac{6}{7}D^2$$

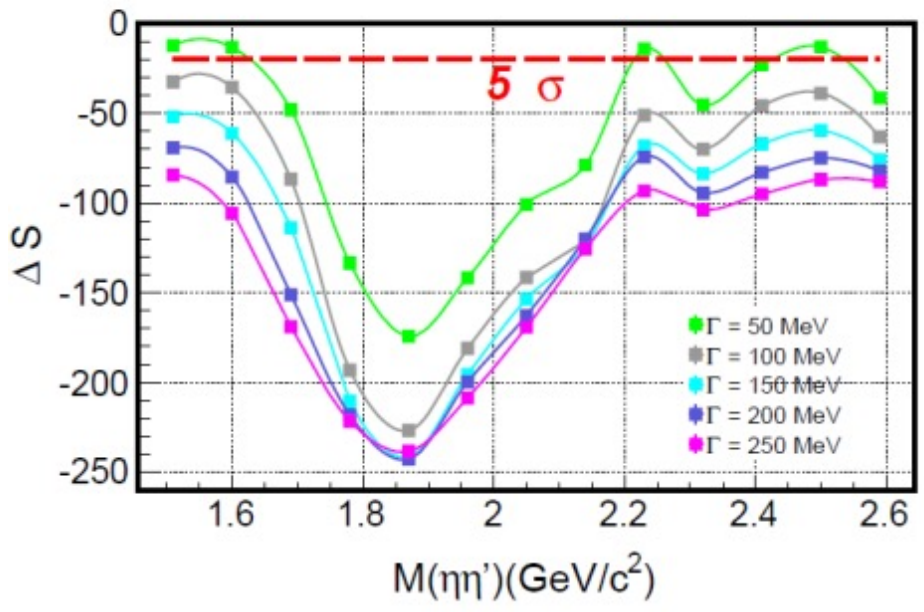
- **Narrow structure** in $\langle Y_1^0 \rangle$
  - ➢ **Cannot be described by resonances in $\gamma\eta(\eta')$**
  - **$\eta_1(1855) \rightarrow \eta\eta'$ needed**



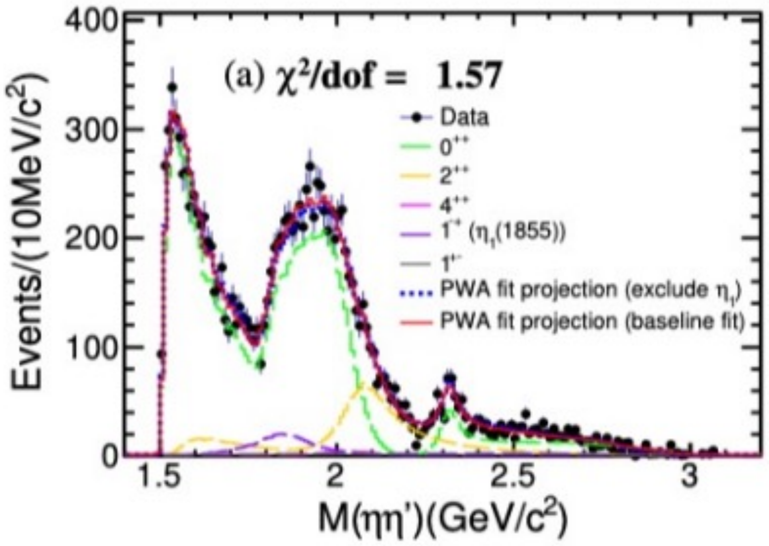
Further Checks on the 1^{-+} State $\eta_1(1855)$

- Change J^{PC} of $\eta_1(1855)$: log-likelihood ↓235
  - ➢ J^{PC} prefer 1^{-+}
- Remove BW phase motion of $\eta_1(1855)$: log-likelihood ↓43
  - ➢ Resonance structure needed
- Assuming $\eta_1(1855)$ as additional resonance, evaluate its significance with various masses and widths
  - ➢ Significant 1^{-+} contribution around $1.8\ \mathrm{GeV}/c^2$ needed
- Systematic uncertainties are studied, and significance of $\eta_1(1855)$ remains larger than 19σ in all cases

significance of $\eta_1(1855)$ with various masses and widths



Discussions about $f_0(1500)$ & $f_0(1710)$



- Significant $f_0(1500)$

$$\frac{B(f_0(1500) \to \eta\eta')}{B(f_0(1500) \to \pi\pi)} = (8.96^{+2.95}_{-2.87})\times 10^{-2}$$

consistent with PDG

- Absence of $f_0(1710)$

$$\frac{B(f_0(1710) \to \eta\eta')}{B(f_0(1710) \to \pi\pi)} < 1.61\times 10^{-3}\ @90\%\ \mathrm{C.L.}$$

➢ Supports to the hypothesis that $f_0(1710)$ overlaps with the ground state scalar (0^{++}) glueball
  - Scalar glueball expected to be suppressed in $\eta\eta'$: $B(G \to \eta\eta')/B(G \to \pi\pi) < 0.04$

Decay mode	Resonance	M (MeV/c^2)	Γ (MeV)	M_{PDG} (MeV/c^2)	Γ_{PDG} (MeV)	B.F. ($\times 10^{-5}$)	Sig.
$J/\psi \to \gamma X \to \gamma\eta\eta'$	$f_0(1500)$	1506	112	1506	112	$1.81\pm0.11^{+0.19}_{-0.13}$	$\gg 30\sigma$
	$f_0(1810)$	1795	95	1795	95	$0.11\pm0.01^{+0.04}_{-0.03}$	11.1σ
	$f_0(2020)$	$2010\pm6^{+6}_{-4}$	$203\pm9^{+13}_{-11}$	1992	442	$2.28\pm0.12^{+0.29}_{-0.20}$	24.6σ
	$f_0(2330)$	$2312\pm7^{+7}_{-3}$	$65\pm10^{+3}_{-12}$	2314	144	$0.10\pm0.02^{+0.01}_{-0.02}$	13.2σ
	$\eta_1(1855)$	$1855\pm9^{+6}_{-1}$	$188\pm18^{+3}_{-8}$	-	-	$0.27\pm0.04^{+0.02}_{-0.04}$	21.4σ
	$f_2(1565)$	1542	122	1542	122	$0.32\pm0.05^{+0.12}_{-0.02}$	8.7σ
	$f_2(2010)$	$2062\pm6^{+10}_{-7}$	$165\pm17^{+10}_{-5}$	2011	202	$0.71\pm0.06^{+0.10}_{-0.06}$	13.4σ
	$f_4(2050)$	2018	237	2018	237	$0.06\pm0.01^{+0.03}_{-0.01}$	4.6σ
	0^{++} PHSP	-	-	-	-	$1.44\pm0.15^{+0.10}_{-0.20}$	15.7σ
$J/\psi \to \eta' X \to \gamma\eta\eta'$	$h_1(1415)$	1416	90	1416	90	$0.08\pm0.01^{+0.01}_{-0.02}$	10.2σ
	$h_1(1595)$	1584	384	1584	384	$0.16\pm0.02^{+0.03}_{-0.01}$	9.9σ

Partial Wave Analysis of $J/\psi \to \gamma\eta'\eta'$

- Observation of the $f_0(2480)$, $f_0(2020)$, $f_0(2330)$ and $f_2(2340)$ decays to $\eta'\eta'$
  - ➢ **A new 0^{++} state $f_0(2480)$**

- After considering the phase-space factor :

$$\frac{\Gamma(f_0(2020)\to\eta\eta')}{\Gamma(f_0(2020)\to\eta'\eta')} = 0.0148$$

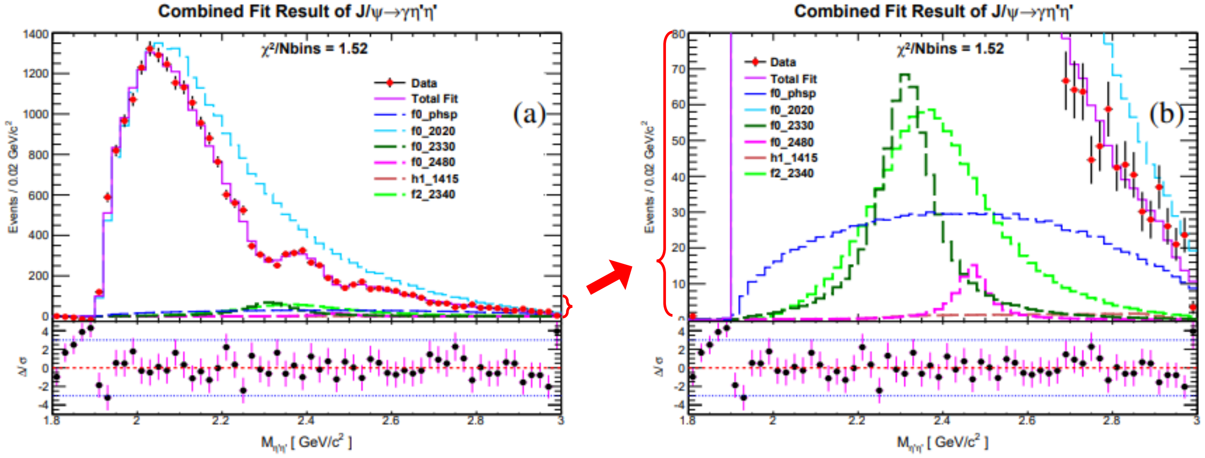
  - ➢ Indicates that $f_0(2020)$ is a flavor singlet[5]

- $B(J/\psi \to \gamma f_2(2340)) \sim 3.0\times10^{-4}$

  (LQCD : $B(J/\psi \to \gamma G_{2^{++}}) = 1.1(2)(1)\times10^{-2}$)
  - ➢ Need more measurment

10 billion J/ψ
PRD **105**,072002 (2022)

Resonance	$M(\mathrm{MeV}/c^2)$	$\Gamma(\mathrm{MeV})$	B.F.	Significance (σ)
$f_0(2020)$	$1982 \pm 3^{+54}_{-0}$	$436 \pm 4^{+46}_{-49}$	$(2.63 \pm 0.06^{+0.31}_{-0.46}) \times 10^{-4}$	$\gg 25$
$f_0(2330)$	$2312 \pm 2^{+10}_{-0}$	$134 \pm 5^{+30}_{-9}$	$(6.09 \pm 0.64^{+4.00}_{-1.68}) \times 10^{-6}$	16.3
$f_0(2480)$	$2470 \pm 4^{+4}_{-6}$	$75 \pm 9^{+11}_{-8}$	$(8.18 \pm 1.77^{+3.73}_{-2.23}) \times 10^{-7}$	5.2
$h_1(1415)$	$1384 \pm 6^{+9}_{-0}$	$66 \pm 10^{+12}_{-10}$	$(4.69 \pm 0.80^{+0.74}_{-1.82}) \times 10^{-7}$	5.3
$f_2(2340)$	$2346 \pm 8^{+22}_{-6}$	$332 \pm 14^{+26}_{-12}$	$(8.67 \pm 0.70^{+0.61}_{-1.67}) \times 10^{-6}$	16.1
0^{++} PHSP	...	...	$(1.17 \pm 0.23^{+4.09}_{-0.70}) \times 10^{-5}$	15.7

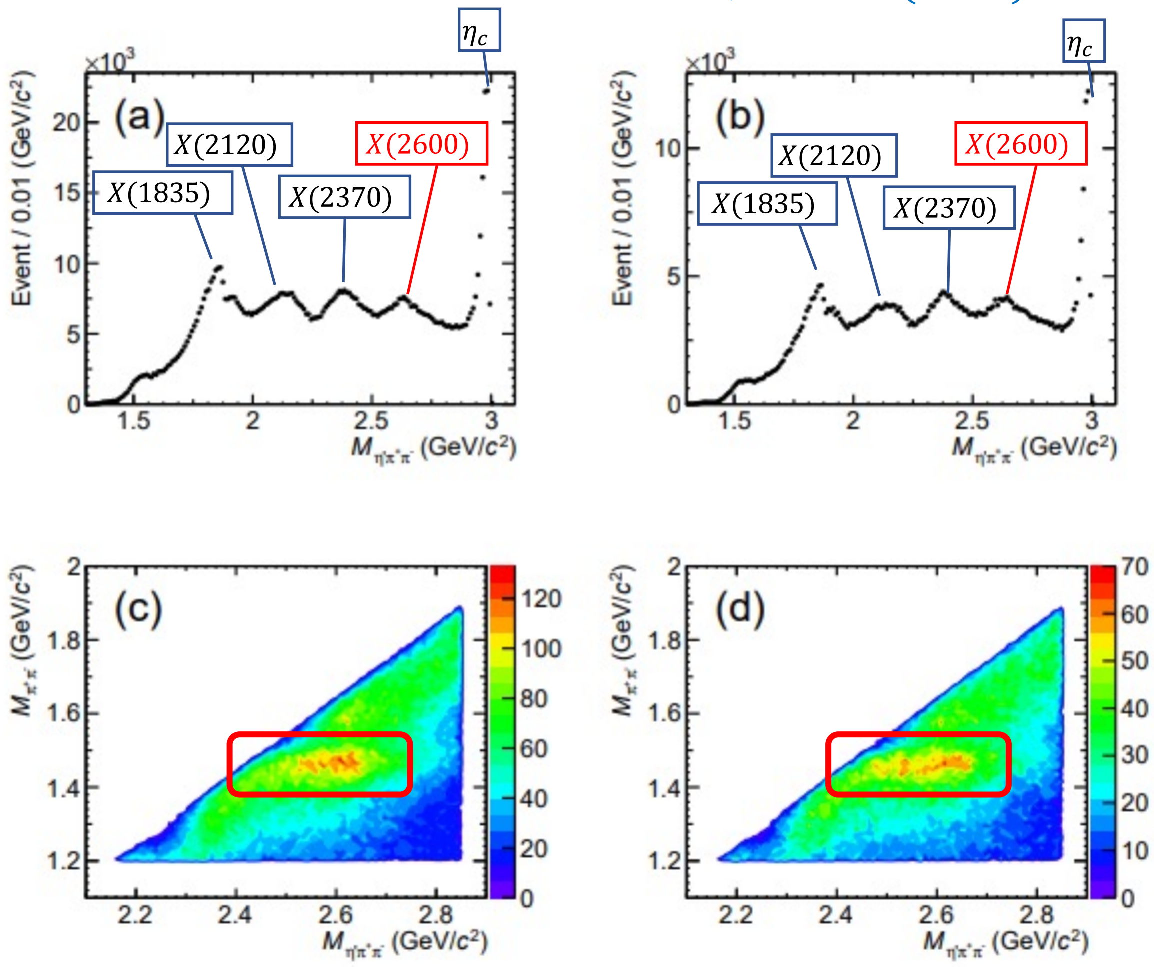


[5] Phys. Lett. B 826, 136906 (2022)

A New State X(2600) Observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$

PRL 129, 042001 (2022)

- X(1835) was first observed and confirmed in $J/\psi \to \gamma\pi^+\pi^-\eta'$[6][7], with $J^{PC} = 0^{-+}$[8], and an anomalous line shape at $p\bar{p}$ threshold[9]
- X(2120), X(2370) also observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$[7]
- With the 10 billon J/ψ events, a **new state X(2600)** in $M(\eta'\pi^+\pi^-)$ is observed, which is correlated to a structure @1.5 GeV/c^2 in $M(\pi^+\pi^-)$



reconstruct η' from $\gamma\pi^+\pi^-$(left) & $\eta(\to\gamma\gamma)\pi^+\pi^-$ (right)

[6] PRL 95, 262001 (2005)
[7] PRL 106, 072002 (2011)
[8] PRL 115, 091803 (2015)
[9] PRL 117, 042002 (2016)

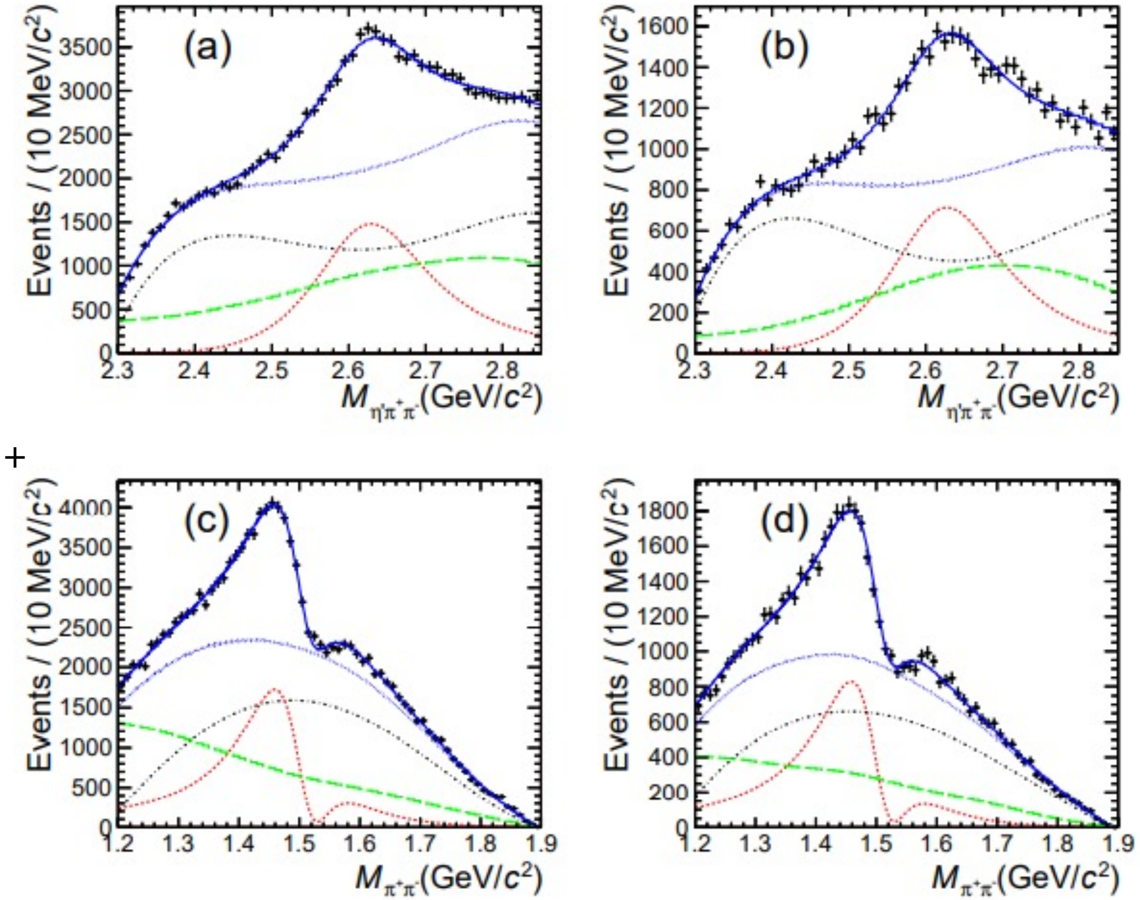
A New State X(2600) Observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$

- To study **X(2600) parameters**, a simultaneous fit to $\eta'\pi^+\pi^-$ and $\pi^+\pi^-$ is performed
- The **structure in $M(\pi^+\pi^-)$** well described with the interference between $f_0(1500)$ and X(1540)

@ > 20σ	Mass (MeV/c^2)	Width (MeV)
$f_0(1500)$	$1492.5 \pm 3.6^{+2.4}_{-20.5}$	$107 \pm 9^{+21}_{-7}$
$X(1540)$	$1540.2 \pm 7.0^{+36.3}_{-6.1}$	$157 \pm 19^{+11}_{-77}$
$X(2600)$	$2618.3 \pm 2.0^{+16.3}_{-1.4}$	$195 \pm 5^{+26}_{-17}$

X(2600)
$J^{PC} = 0^{-+}$ or 2^{-+}

Case	$f_0(1500)$	$X(1540)$
Events	24585 ± 1689	21203 ± 1456
BF ($\times 10^{-5}$)	$3.09 \pm 0.21^{+1.14}_{-0.77}$	$2.69 \pm 0.19^{+0.38}_{-1.21}$

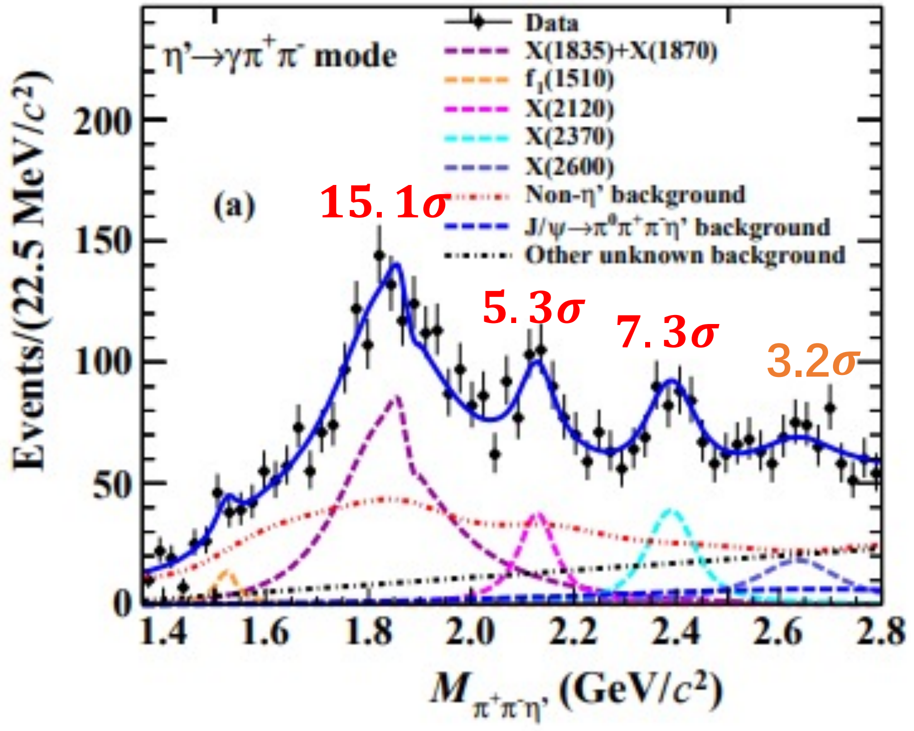


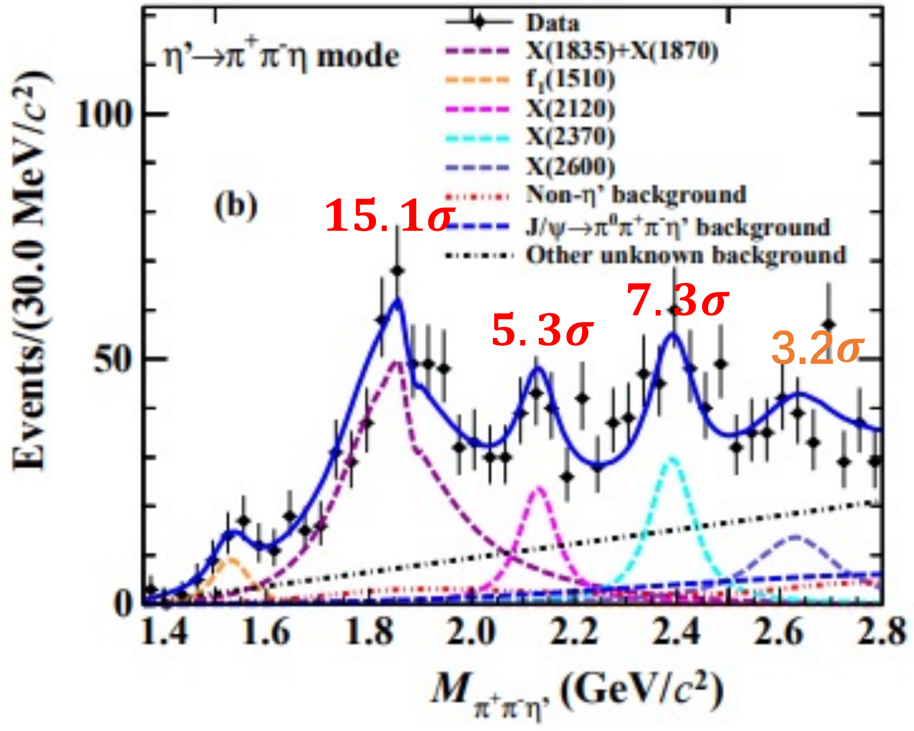
reconstruct η' from $\gamma\pi^+\pi^-$(left) & $\eta(\to\gamma\gamma)\pi^+\pi^-$ (right)

Observation of X(1835), X(2120) and X(2370) in J/ψ EM Dalitz Decays

$J/\psi \to e^+e^-\pi^+\pi^-\eta'$

- **Confirmation** of **X(1835), X(2120), X(2370)** previously observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$



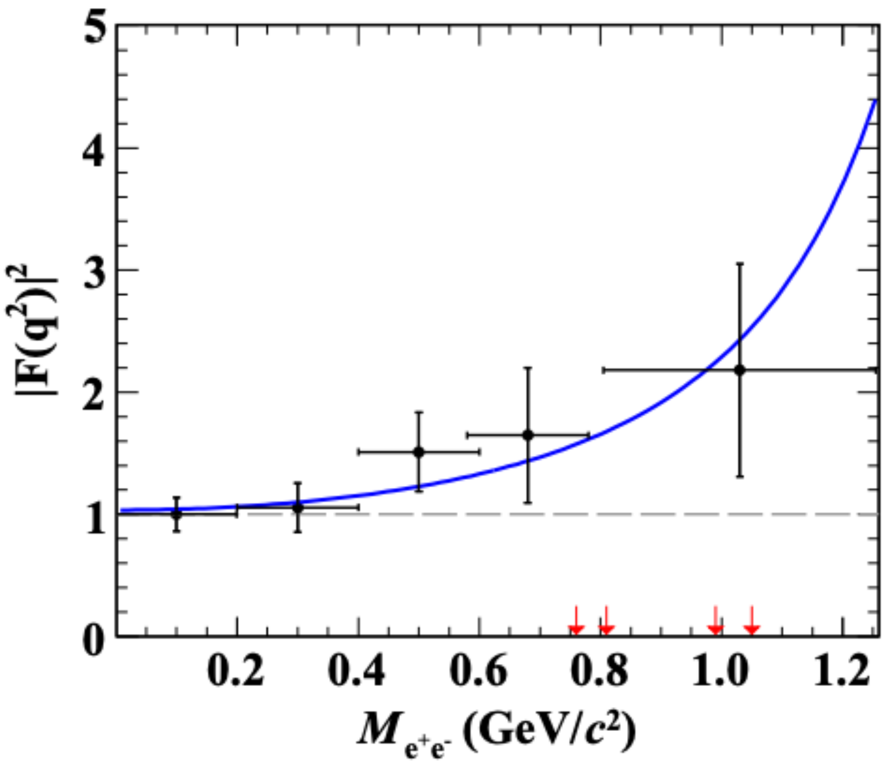


10 billion J/ψ
PRL 129 (2022) 2, 022002

Observation of X(1835), X(2120) and X(2370) in J/ψ EM Dalitz Decays

$J/\psi \to e^+e^-\pi^+\pi^-\eta'$

- Measurement of the Transition Form Factor of $J/\psi \to e^+e^-X(1835)$
  - Gives additional information of the internal structure of X(1835)



$$\frac{d\Gamma\left(J/\psi \to X(1835)e^+e^-\right)}{dq^2\Gamma(J/\psi \to X(1835)\gamma)} = \left|F\left(q^2\right)\right|^2 \times \left[\mathrm{QED}\left(q^2\right)\right]$$

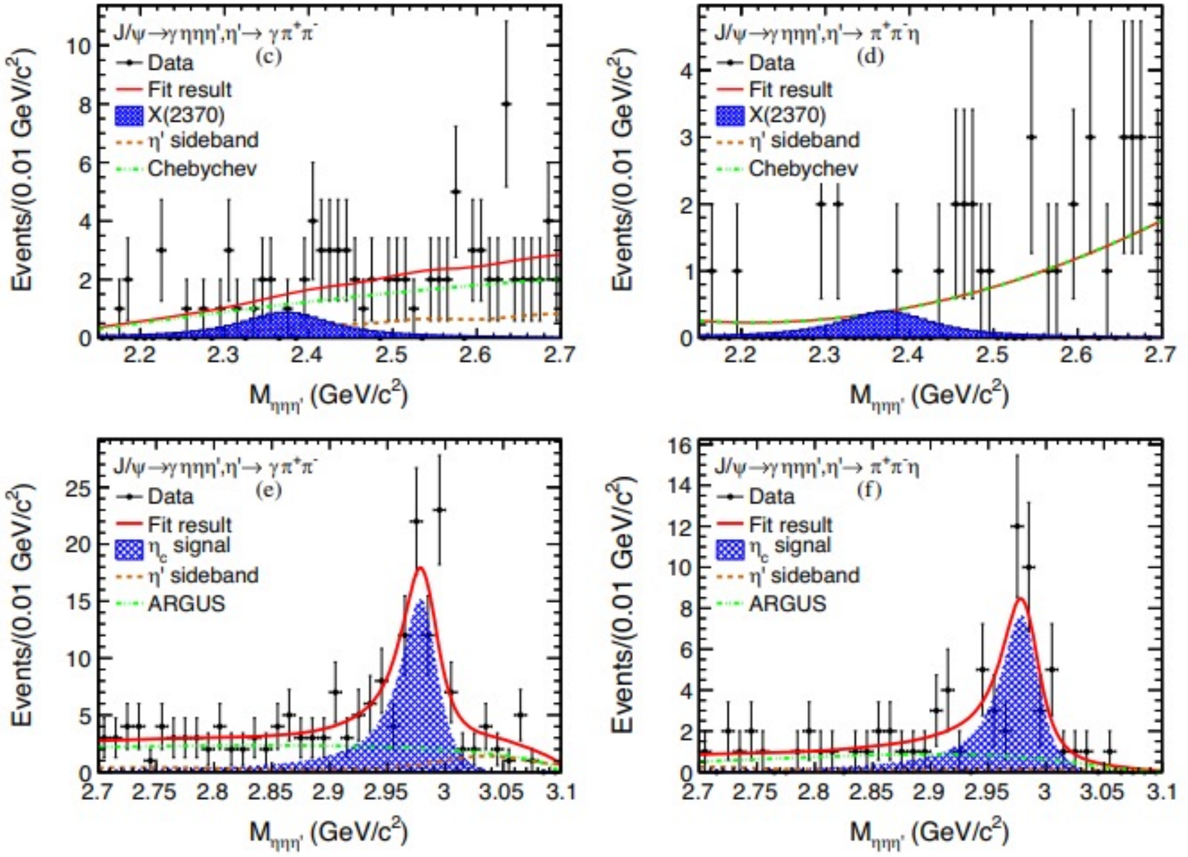
$$F(q^2) = \frac{1}{1-q^2/\Lambda^2}$$

$$\Lambda = 1.75 \pm 0.29 \pm 0.05\ \mathrm{GeV}/c^2$$

Search for X(2370) in $J/\psi \to \gamma\eta\eta'$

1.3 billion J/ψ

Phys. Rev. D 103, 012009(2021)



reconstruct η' from $\gamma\pi^+\pi^-$(left) & $\eta(\to\gamma\gamma)\pi^+\pi^-$(right)

X(2370) is previously observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$ [10] and $J/\psi \to \gamma K\bar{K}\eta'$ [11], and possibly a **pseudoscalar glueball candidate**

- **No evident signal of X(2370)** in $J/\psi \to \gamma\eta\eta'$

  $B(J/\psi \to \gamma X(2370) \to \gamma\eta\eta') < 9.2\times10^{-6}$ (@ 90% C.L.)

➢ No contradiction with prediction of the branching ratio for pseudoscalar glueball [12]

- Observation of $\eta_c \to \eta\eta'$

  $B(J/\psi \to \gamma\eta_c \to \gamma\eta\eta') = 4.86 \pm 0.62(\text{stat.}) \pm 0.45(\text{sys.})$

[10] PRL 106, 072002 (2011)
[11] Eur. Phys.J.C 80,746 (2020)
[12] PRD 87, 054036 (2013)

Summary

- $J/\psi \to \gamma\eta\eta'$
  - Observation of **exotic isoscalar 1^{-+} $\eta_1(1855)$**
    - Hybrid? Molecule? Tetraquark? … needs **further study**
  - Support $f_0(1710)$ overlap with **scalar glueball**
- $J/\psi \to \gamma\eta'\eta'$
  - $f_0(2020), f_0(2330), f_2(2340)$ and a **new state $f_0(2480)$** observed
  - $f_0(2020)$ observed to be a **flavor singlet**

- **Other partners** in hybrid nonet:
  $\pi_1(b_1\pi, f_1\pi, \dots)$ and $K_1(K_1(1270)\pi, \dots)$
- **Production & decay** of $\eta_1(1855)$
  - $J/\psi(\psi') \to VX, \dots$
  - $X \to a_1\pi, K_1K, f_1\eta, \dots$

- **New $X(2600)$** observed in $J/\psi \to \gamma\pi^+\pi^-\eta'$ in addition to $X(1835), X(2120), X(2370)$
- **Confirmation of $X(1835), X(2120), X(2370)$** in $J/\psi \to e^+e^-\pi^+\pi^-\eta'$ and measurement of **Transition form factor** of $J/\psi \to e^+e^-X(1835)$
- **Upper limit for $X(2370)$ in $J/\psi \to \gamma\eta\eta\eta'$** and observation of $\eta_c \to \eta\eta\eta'$

➢ With the world's largest charmonium data sets, BESIII provides great opportunities to map out light meson spectroscopy and study QCD exotics.

Thank you for your attention!

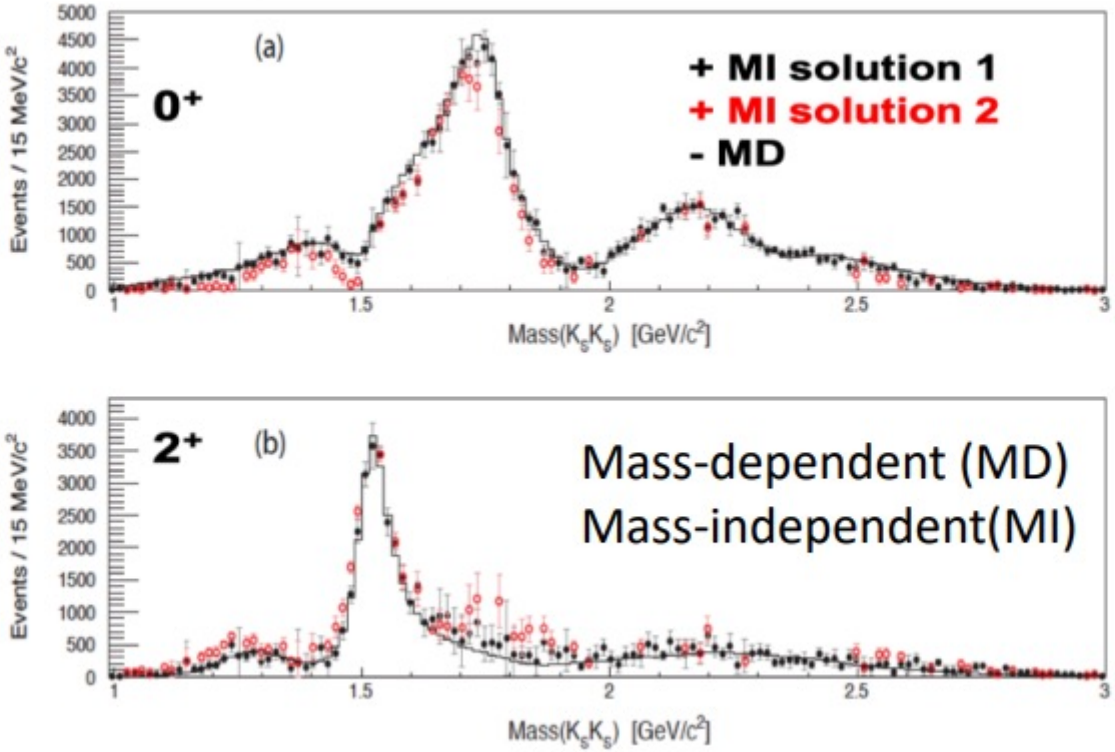
Backup slide

Amplitude analysis of $J/\psi \to \gamma K_S K_S$

1.3 billion J/ψ

Phys. Rev. D 98, 072003(2018)

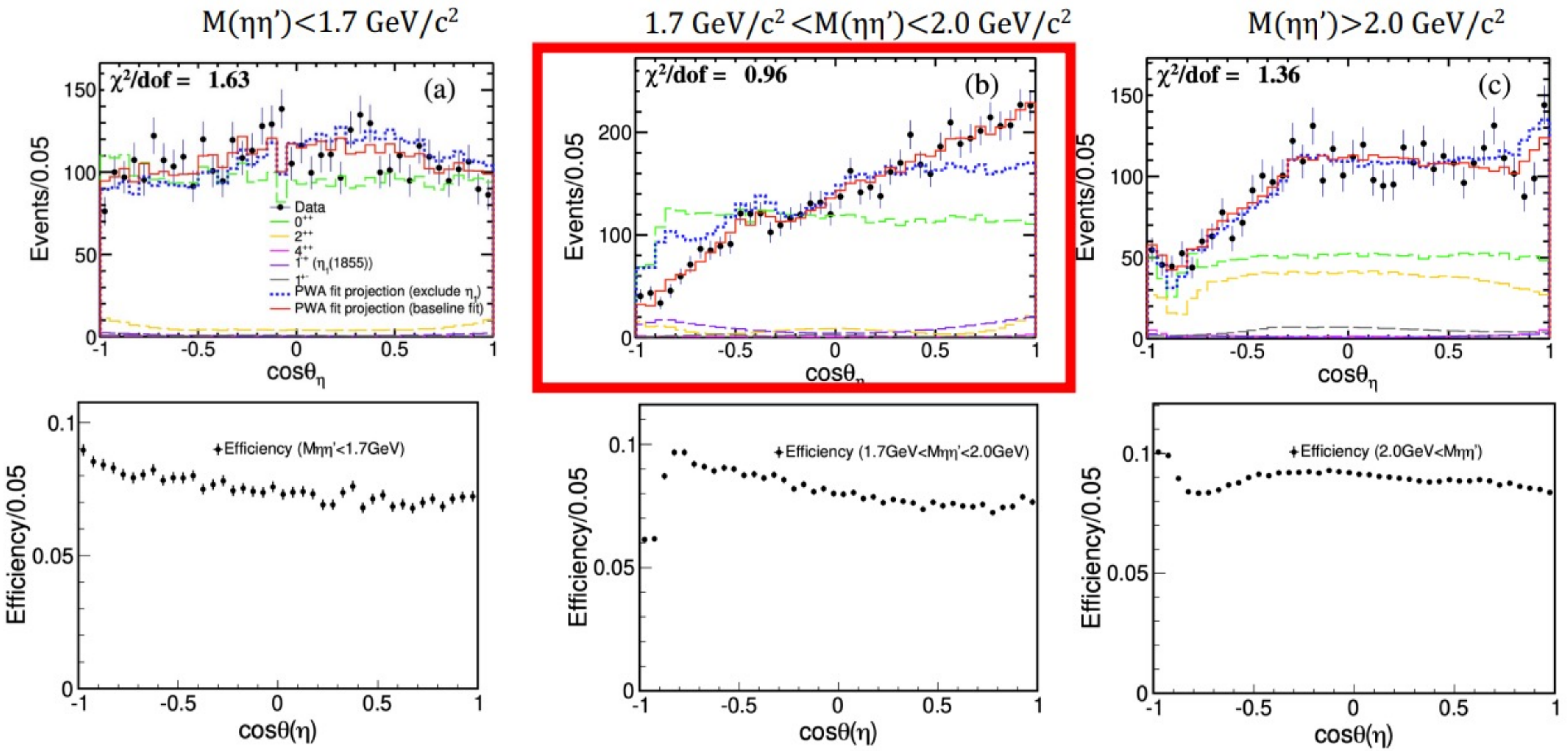
MD analysis is well consist with MI analysis



Resonance	M (MeV/c^2)	$M_{\rm PDG}$ (MeV/c^2)	Γ (MeV/c^2)	$\Gamma_{\rm PDG}$ (MeV/c^2)	Branching fraction	Significance
$K^*(892)$	896	895.81±0.19	48	47.4±0.6	$(6.28^{+0.16+0.59}_{-0.17-0.52})\times10^{-6}$	35σ
$K_1(1270)$	1272	1272±7	90	90±20	$(8.54^{+1.07+2.35}_{-1.20-2.13})\times10^{-7}$	16σ
$f_0(1370)$	$1350\pm9^{+12}_{-2}$	1200 to 1500	$231\pm21^{+28}_{-48}$	200 to 500	$(1.07^{+0.08+0.36}_{-0.07-0.34})\times10^{-5}$	25σ
$f_0(1500)$	1505	1504±6	109	109±7	$(1.59^{+0.16+0.18}_{-0.16-0.56})\times10^{-5}$	23σ
$f_0(1710)$	$1765\pm2^{+1}_{-1}$	1723^{+6}_{-5}	$146\pm3^{+7}_{-1}$	139±8	$(2.00^{+0.03+0.31}_{-0.02-0.10})\times10^{-4}$	$\gg35\sigma$
$f_0(1790)$	$1870\pm7^{+2}_{-3}$	-	$146\pm14^{+7}_{-15}$	-	$(1.11^{+0.06+0.19}_{-0.06-0.32})\times10^{-5}$	24σ
$f_0(2200)$	$2184\pm5^{+4}_{-2}$	2189±13	$364\pm9^{+4}_{-7}$	238±50	$(2.72^{+0.08+0.17}_{-0.06-0.47})\times10^{-4}$	$\gg35\sigma$
$f_0(2330)$	2411±10±7	-	$349\pm18^{+23}_{-1}$	-	$(4.95^{+0.21+0.66}_{-0.21-0.72})\times10^{-5}$	35σ
$f_2(1270)$	1275	1275.5±0.8	185	$186.7^{+2.2}_{-2.5}$	$(2.58^{+0.08+0.59}_{-0.09-0.20})\times10^{-5}$	33σ
$f_2'(1525)$	1516±1	1525±5	75±1±1	73^{+6}_{-5}	$(7.99^{+0.03+0.69}_{-0.04-0.50})\times10^{-5}$	$\gg35\sigma$
$f_2(2340)$	$2233\pm34^{+9}_{-25}$	2345^{+50}_{-40}	$507\pm37^{+18}_{-21}$	322^{+70}_{-60}	$(5.54^{+0.34+3.82}_{-0.40-1.49})\times10^{-5}$	26σ
0^{++} PHSP	-	-	-	-	$(1.85^{+0.05+0.68}_{-0.05-0.26})\times10^{-5}$	26σ
2^{++} PHSP	-	-	-	-	$(5.73^{+0.99+4.18}_{-1.00-3.74})\times10^{-5}$	13σ

Further Checks on the 1^{-+} State $\eta_1(1855)$

Angular distribution in different $M(\eta\eta')$ region

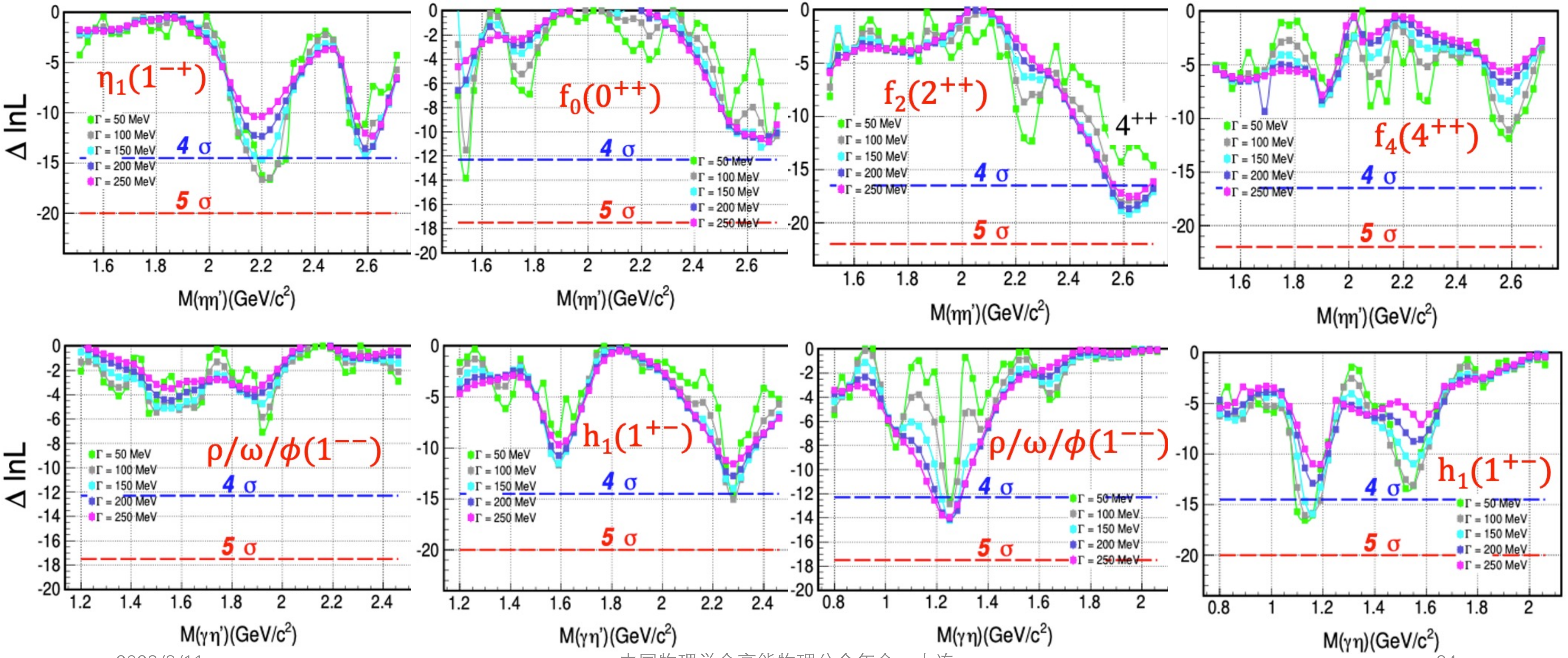


✓ A clear asymmetry largely due to $\eta_1(1870)$ signal

Further Checks on the 1^{-+} State $\eta_1(1855)$

Scan of additional states with different J^{PC}, masses and widths

- No significant contributions from additional resonances



Significance of additional resonances

Decay mode	Resonance	J^{PC}	$\Delta\ln\mathcal{L}$	Δdof	Sig.
$J/\psi \to \gamma X \to \gamma\eta\eta'$	$f_2(1525)$	2^{++}	6.3	6	1.9σ
	$f_2(1810)$	2^{++}	2.7	6	0.7σ
	$f_0(1710)$	0^{++}	3.4	2	2.1σ
	$f_2(1910)$	2^{++}	3.9	6	1.1σ
	$f_2(1950)$	2^{++}	2.6	6	0.6σ
	$f_0(2100)$	0^{++}	1.1	2	1.1σ
	$f_2(2150)$	2^{++}	2.3	6	0.5σ
	$f_0(2200)$	0^{++}	0.4	2	0.4σ
	$f_2(2220)$	2^{++}	8.6	6	2.6σ
	$f_2(2300)$	2^{++}	7.2	6	2.2σ
	$f_4(2300)$	4^{++}	2.3	6	0.5σ
	$f_0(2330)$	0^{++}	1.5	2	1.2σ
	$f_2(2340)$	2^{++}	6.3	6	1.9σ
	$f_0(2102)$[61]	0^{++}	0.1	2	0.2σ
	$f_2(2240)$[61]	2^{++}	2.9	6	0.7σ
	$f_2(2293)$[61]	2^{++}	4.1	6	1.2σ
	$f_4(2283)$[61]	4^{++}	0.9	6	0.1σ

Decay mode	Resonance	J^{PC}	$\Delta\ln\mathcal{L}$	Δdof	Sig.
$J/\psi \to \eta' X \to \gamma\eta\eta'$	$\rho(1450)$	1^{--}	3.4	2	2.1σ
	$\rho(1700)$	1^{--}	0.8	2	0.7σ
	$\rho(1900)$	1^{--}	0.0	2	0σ
	$\omega(1420)$	1^{--}	5.3	2	2.8σ
	$\omega(1650)$	1^{--}	2.6	2	1.7σ
	$\phi(1680)$	1^{--}	4.3	2	2.5σ
	$\phi(2170)$	1^{--}	0.4	2	0.4σ
$J/\psi \to \eta X \to \gamma\eta\eta'$	$h_1(1415)$	1^{+-}	1.3	4	0.5σ
	$h_1(1595)$	1^{+-}	8.1	4	2.9σ
	$\rho(1450)$	1^{--}	1.3	2	1.1σ
	$\rho(1700)$	1^{--}	3.1	2	2.0σ
	$\rho(1900)$	1^{--}	6.1	2	3.0σ
	$\omega(1420)$	1^{--}	2.5	2	1.7σ
	$\omega(1650)$	1^{--}	0.8	2	0.7σ
	$\phi(1680)$	1^{--}	2.1	2	1.5σ
	$\phi(2170)$	1^{--}	0.1	2	0.1σ