Lattice calculation of the $\eta_c\eta_c$ and $J/\psi J/\psi$ scattering length

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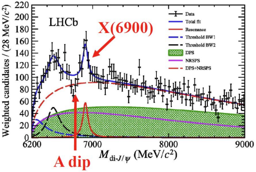
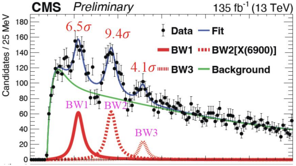
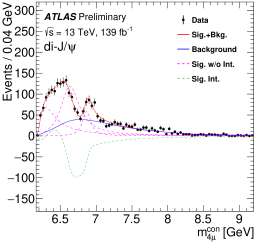
J/ψ-pair mass spectrum from experiments







- In 2020, LHCb first observed a narrow structure X(6900) in the di-J/ψ spectrum Sci.Bull.65 (2020) 23
- In 2022, CMS found 3 significxant structures, named as X(6600),X(6900) and X(7200) ICHEP(2022)
- In 2022, ATLAS observed X(6900) in di-J/ψ channnel and X(6900),X(7200) in $J/\psi + \psi(2S)$ channel ICHEP(2022)

Phenomenological studies

Is there compact bound state below di-heavy-quarkonium threshold ?

- No $\cdots$
- Yes $\cdots$

What is the nature of $X(6900)$?

- Tetraquark
- Gluonic tetracharm
- Coupled channel effect

What is the less certain? relatively certain? very certain?

Role of lattice QCD

- $X(6900)$: very challenging for current lattice QCD   less certain
    - Unknown internal structure
    - Very dense energy levels
    - Coupled-channel and many-body effect

- Basic information of di-charmonium scattering(Lattice QCD)
  $\rightarrow$ inputs for phenomenological studies
    - Constrain various phenomenological models
    - Improve the predictive power of phenomenology

  less certain $\rightarrow$ relatively certain

  ...

- Experiments$+$phenomenology$+$lattice QCD $\rightarrow$ very certain

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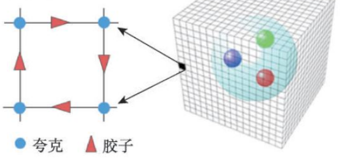
- Experiments+phenomenology+lattice QCD $\rightarrow$ very certain

Target : 0^+ $\eta_c\eta_c$ and 2^+ $J/\psi J/\psi$ scattering lengths

Lattice QCD

- K.G.Wilson, PRD 10, 2445(1974)
- Idea: put QCD on 4-d lattice
  - Quark field $\rightarrow$ site
  - Gauge filed $\rightarrow$ link

$$\mathcal{L}=-\frac{1}{4}\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu}+\sum_{q=u,d,s,c,b,t}\bar{q}\left[\mathrm{i}\gamma^{\mu}(\partial_{\mu}-\mathrm{i}g\mathbf{A}_{\mu})-m_{q}\right]q$$



格点量子色动力学在中国 [J]. 现代物理知识,2020,32(01):36-44.

- “格点场论既是世界观 (非微扰的定义) 又是方法论 (非微扰的计算)”

— 刘川,《格点量子色动力学导论》

  - 世界观 $\Rightarrow$ Non-perturbative definition of QCD,
    natural ultraviolet and infrared truncation
  - 方法论 $\Rightarrow$ Non-perturbative calculation of QCD,
    Monte-Carlo simulation

Lattice method

- Method: Lüscher finite volume formula

$$\delta E^{\Gamma} = -\frac{4\pi a^{\Gamma}}{mL^3}\left[1 + c_1\frac{a^{\Gamma}}{L} + c_2\left(\frac{a^{\Gamma}}{L}\right)^2 + \mathcal{O}(L^{-3})\right], \Gamma = A_1, E, T_2$$

- Single-particle operator: $\mathcal{P}(t) = \bar{c}\gamma_5 c(t), \mathcal{V}_i(t) = \bar{c}\gamma_i c(t)$
- Two-particle operator

$$\begin{aligned}
\mathcal{O}^{A_1}(t) &= \mathcal{P}(t)\mathcal{P}(t) \\
\mathcal{O}^{E}(t) &= \left\{\frac{1}{\sqrt{2}}\left[\mathcal{V}_1(t)\mathcal{V}_1(t) - \mathcal{V}_2(t)\mathcal{V}_2(t)\right],\right. \\
&\quad \left.\frac{1}{\sqrt{2}}\left[\mathcal{V}_2(t)\mathcal{V}_2(t) - \mathcal{V}_3(t)\mathcal{V}_3(t)\right]\right\} \\
\mathcal{O}^{T_2}(t) &= \left\{\mathcal{V}_2(t)\mathcal{V}_3(t), \mathcal{V}_3(t)\mathcal{V}_1(t), \mathcal{V}_1(t)\mathcal{V}_2(t)\right\}
\end{aligned}$$

F.R.López, A.Rusetsky and C.Urbach,PRD**98**,014503(2018)

Energy shift

- Two-point function

$$C^{(2)}_{\eta_c}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{P}(t+t_s)\mathcal{P}^\dagger(t_s)\rangle$$

$$C^{(2)}_{J/\psi}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{V}_i(t+t_s)\mathcal{V}_i^\dagger(t_s)\rangle$$

- Four-point function

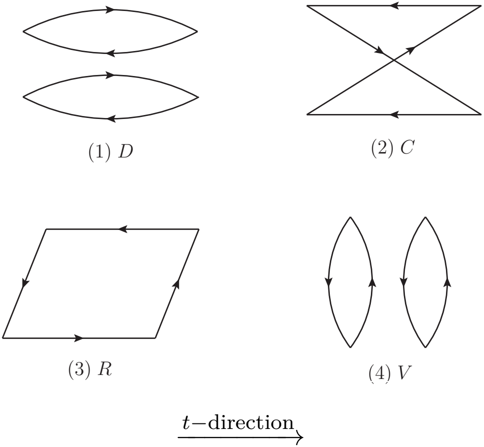
$$C^{(4)}_{\Gamma}(t) = \frac{1}{T}\sum_{t_s}\langle \mathcal{O}^\Gamma(t+t_s)\left(\mathcal{O}^\Gamma(t_s)\right)^\dagger\rangle$$

- Ratio

$$R^\Gamma(t) = \frac{C^{(4)}_\Gamma(t) - C^{(4)}_\Gamma(t+1)}{(C^{(2)}_h(t))^2 - (C^{(2)}_h(t+1))^2}$$

$$\to A_R[\cosh(\delta E^\Gamma t') + \sinh(\delta E^\Gamma t')\coth(2m_h t')], \quad t' = t + 1/2 - T/2$$

Wick contraction



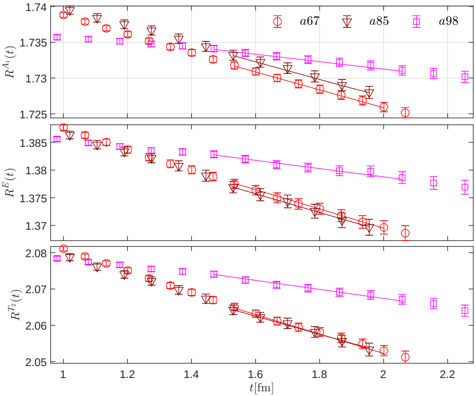
- Type-R and type-V are supposed to be highly suppressed

Ensembles

Ens	a(fm)	V/a^4	$a\mu_{sea}$	$N_{\mathrm{conf}} \times T_s$	m_π(MeV)
a98	0.098(3)	$24^3 \times 48$	0.0060	236×48	365
a85	0.085(2)	$24^3 \times 48$	0.0040	200×48	315
a67	0.0667(20)	$32^3 \times 64$	0.0030	200×64	300

- $N_f = 2$ twisted-mass guage configuration
- Dimensionless quantity $m_{J/\psi}a^{\Gamma}$ in the continuous limit
- Smeared stochastic Z_4-noise for the propagator
- Charm quark mass is tuned by physical J/ψ mass

Ratio

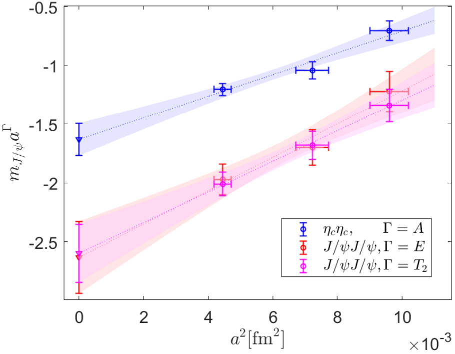


- Asymptotic behavior: $T \gg t,\ \delta E/2m_{\bar{c}c} \ll 1$

$$R^{\Gamma}(t) \quad \to \quad e^{-\delta E^{\Gamma}\cdot t}$$

- It indicates $\delta E > 0$

Preliminary results



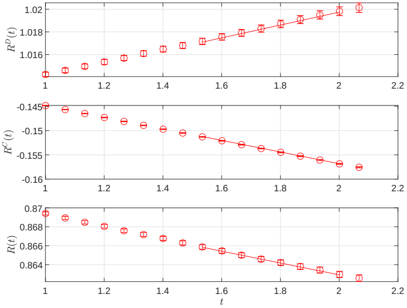
- S-wave scattering length

$$a^{0^+}_{\eta_c\eta_c} = -0.104(09)\ \mathrm{fm},\ \ a^{2^+}_{J/\psi J/\psi} = -0.165(16)\ \mathrm{fm}$$

- No evidence of $\eta_c\eta_c$ and $J/\psi J/\psi$ with mass below the noninteracting thresholds in the 0^+ and 2^+ channels $[\delta E > 0]$

Individual Wick contraction: A_1, a67

$$R(t) = A[\cosh(\delta E \cdot t') + \sinh(\delta E \cdot t') \coth(2mt')], \quad t' = t + 1/2 - T/2$$



- $A_D = 1.0203(4)$
  $\delta E_D = -1.10(7)\text{MeV}$
- $A_C = -0.1581(1)$
  $\delta E_C = -15.4(1)\text{MeV}$
- $A = 0.8623(3)$
  $\delta E = 1.42(7)\text{MeV}$

- $\delta E < 0$ for both Wick contractions, $\delta E > 0$ for the sum
- No physical interpretation for individual Wick contraction

Discussion: $\bar{b}\bar{b}bb$ & $\Omega_{ccc}\Omega_{ccc}$

- $\bar{b}\bar{b}bb$ C. Hughes,E. Eichten and C.T.H. Davies,PRD**97**,054505(2018)
  (i) 2+1+1 flavors from MILC collaboration,$a \sim [0.06, 0.12]\mathrm{fm}$
  (ii) No any state below the threshold in any channel$(0^{++}, 1^{+-}, 2^{++})$
  (iii) Mass below the threshold for individual Wick contraction, above after the sum $\rightarrow$ individuals misinterpreted $\cdots$

- $\Omega_{ccc}\Omega_{ccc}$ PKU&HALQCD,PRL**127**,072003(2021)
  (i) 2+1 flavor $O(a)$-improved Wilson action, $a \sim 0.085\mathrm{fm}$
  (ii) $V(r)$ replusive at short range and attractive at midrange$(^1S_0)$
  (iii) It supports a loosely bound state $[\delta E < 0]$

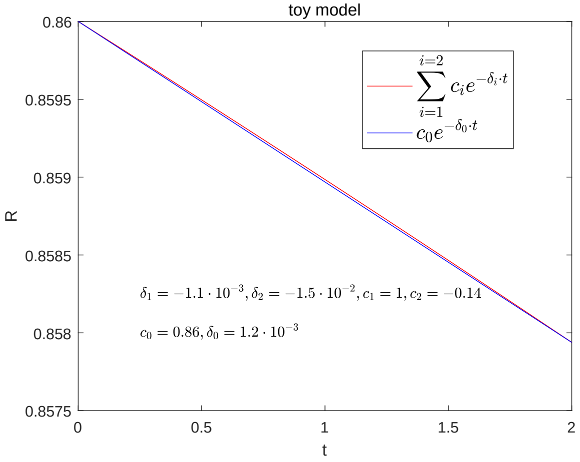
Conclusion

- We present first-principle calculation on the scattering length of $\eta_c\eta_c$ and $J/\psi J/\psi$ in 0^+ and 2^+ channels
- No evidence of $\eta_c\eta_c$ and $J/\psi J/\psi$ with mass below the noninteracting thresholds in both channels
- We observe sizeable discretization effect, weak repulsive interaction in 0^+ $\eta_c\eta_c$ and 2^+ $J/\psi J/\psi$ systems
- The scattering lengths are obtained as

$$a^{0^+}_{\eta_c\eta_c} = -0.104(09)\ \mathrm{fm},\ a^{2^+}_{J/\psi J/\psi} = -0.165(16)\ \mathrm{fm}$$

谢谢

A toy model


toy model
0.86
0.8595
0.859
0.8585
0.858
0.8575
R
0
0.5
1
1.5
2
t
$\sum_{i=1}^{i=2} c_i e^{-\delta_i \cdot t}$
$c_0 e^{-\delta_0 \cdot t}$
$\delta_1 = -1.1 \cdot 10^{-3}, \delta_2 = -1.5 \cdot 10^{-2}, c_1 = 1, c_2 = -0.14$
$c_0 = 0.86, \delta_0 = 1.2 \cdot 10^{-3}$