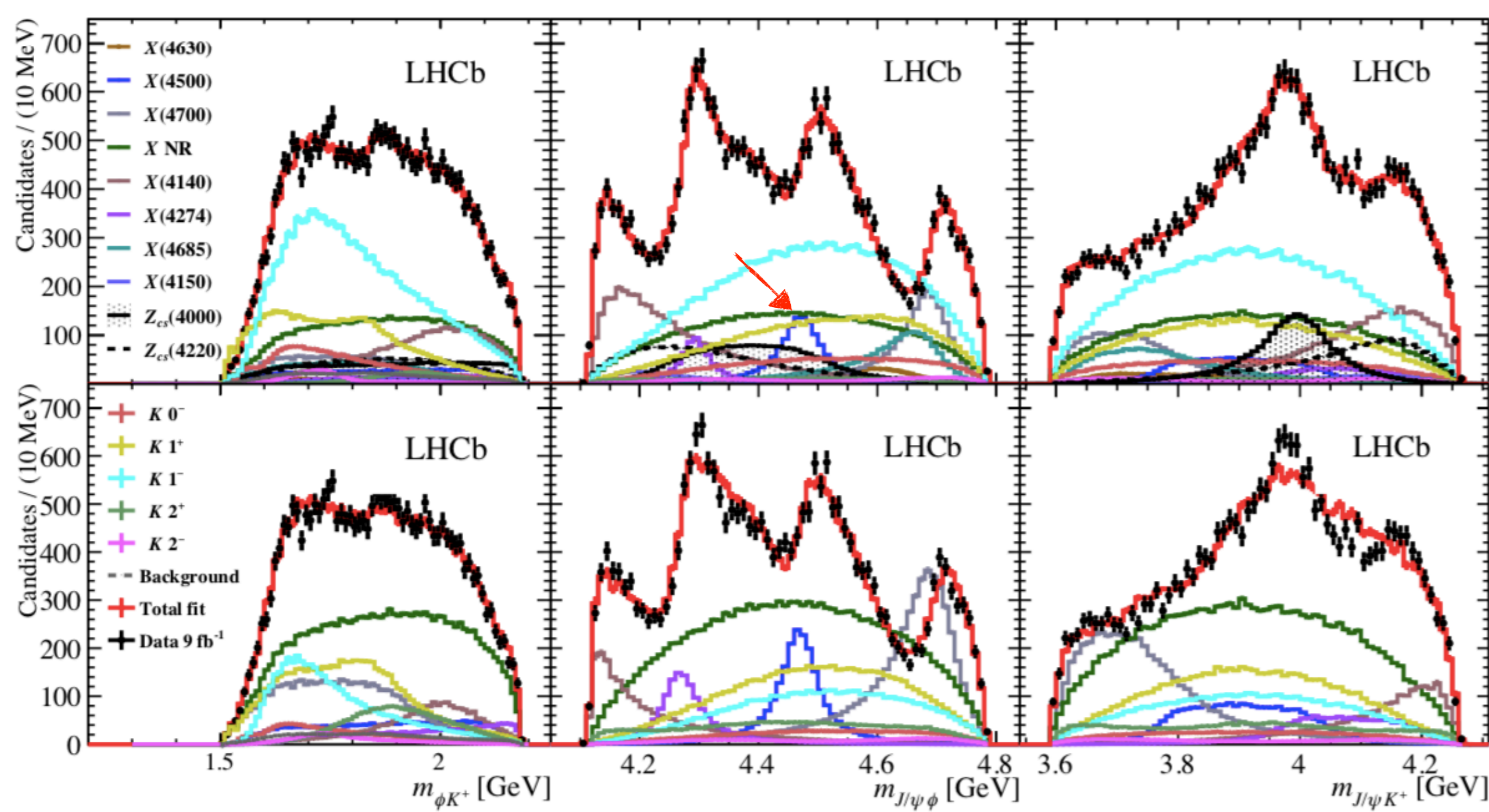


D-wave excited $c\bar{s}\bar{c}\bar{s}$ tetraquark states with $J^P = 1^{++}$ and 1^{+-}

[arXiv: 2206.06051]

Introduction

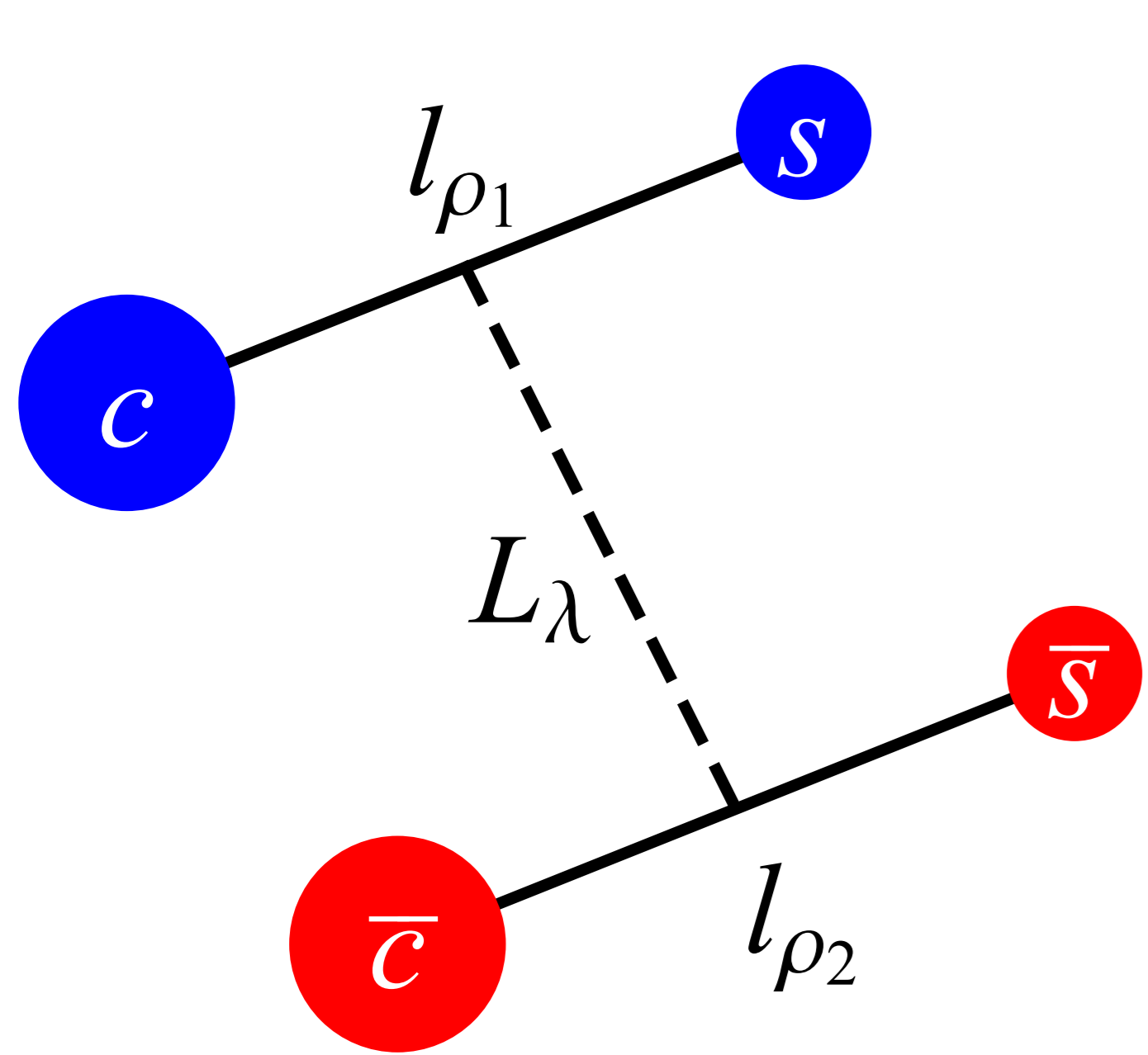


- CDF, CMS, D0, BABAR: X(4140), X(4274) in $B^+ \rightarrow J/\psi\phi K^+$ [PRL. 102,242002; Mod.Phys.Lett.A 32 (2017) 26, 1750139; PLB. 734, 261; PRD. 89, 012004; PRD. 91,012003]
- LHCb: X(4140), X(4274), X(4500), X(4700) in $B^+ \rightarrow J/\psi\phi K^+$ [PRD. 95,012002; PRL. 118, 022003]
- Inspired by these structures observed in the $J/\psi\phi$ invariant mass spectrum, X(4140), X(4274) are considered to be assigned as $c\bar{s}\bar{c}\bar{s}$ tetraquark ground states while X(4500) and X(4700) were interpreted as $c\bar{s}\bar{c}\bar{s}$ tetraquark excited states in various theoretical methods. [W.Chen, F.Stancu, H-X. Chen, D.Ebert, Q-F. Lv, J.Wu, C. Deng, X. Liu, Z-G. Wang etc.]

- X(4685) may be interpreted as D-wave $c\bar{s}\bar{c}\bar{s}$ tetraquark excited state with $J^{PC} = 1^{++}$.
- D-wave $c\bar{s}\bar{c}\bar{s}$ tetraquark excited states were studied in various method:
 - Relativistic quark model, the masses for λ -mode excitation are around 4.8 GeV [D. Ebert, etc.]
 - Color flux-tube model, the masses for λ -mode excitation are around 4.9 GeV and 5.2 GeV for antisymmetric and symmetric color structures, respectively. [C.Deng, etc.]
 - Relativistic quark model, the masses for ρ -mode excitation are around 4.6-4.7 GeV, much lower than λ -mode excitation. [Q-F. Lv, etc.]
- Other interpretation: S-wave $c\bar{s}\bar{c}\bar{s}$ tetraquark [A. Turkan, etc.], 2S radial excited $c\bar{s}\bar{c}\bar{s}$ tetraquark [Z-G. Wang]

Observation of new resonances decaying to $J/\psi\phi$
[LHCb, PRL. 127, 082001 (2021)]

D-wave $c\bar{s}\bar{c}\bar{s}$ excited tetraquark



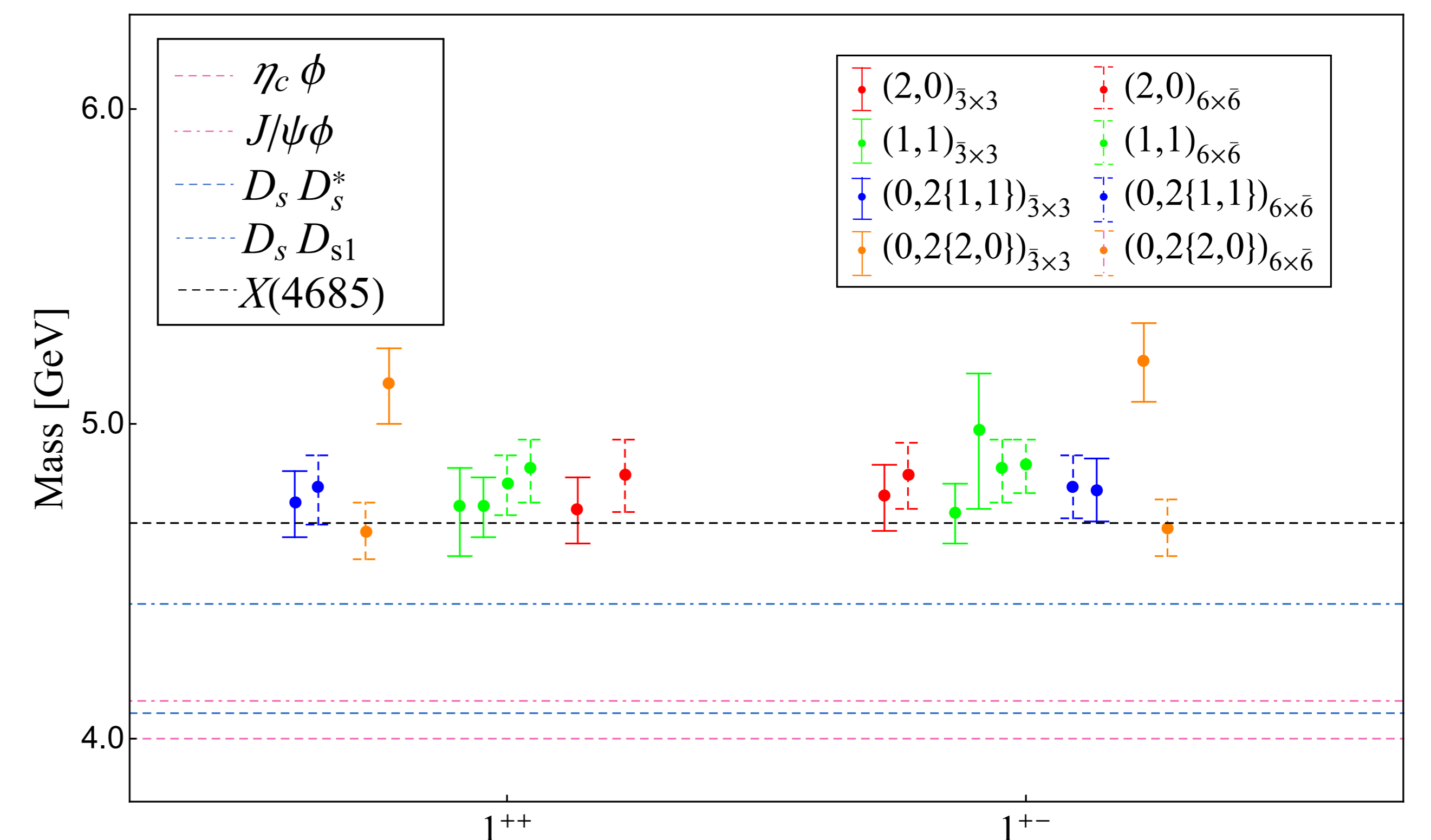
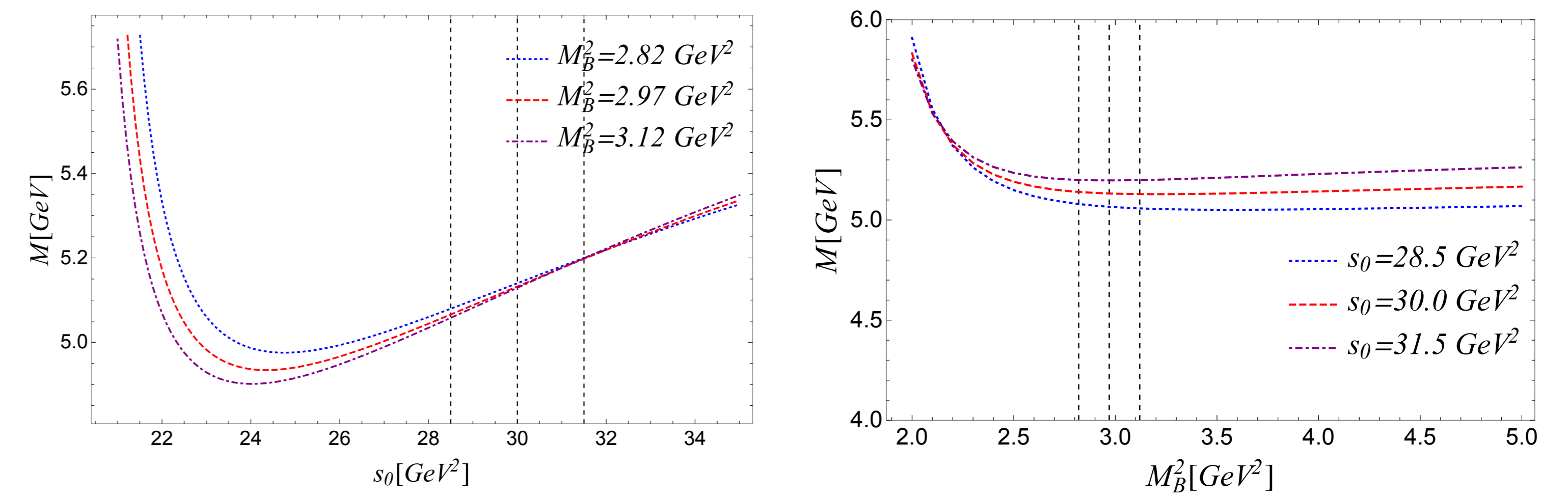
$$(L_\lambda, L_\rho \{l_{\rho_1}, l_{\rho_2}\}) =$$

$$(2,0\{0,0\}), (1,1\{1,0\}),$$

$$(1,1\{0,1\}), (0,2\{1,1\}),$$

$$(0,2\{2,0\}), (0,2\{0,2\}).$$

Numerical Analysis



Discussion and conclusion

- Our results support to interpret the recent observed X(4685) resonance as a D-wave $c\bar{s}\bar{c}\bar{s}$ tetraquark state with $J^{PC} = 1^{++}$ in the $(0,2\{2,0\})$ excitation mode, although some other possible excitation structures cannot be excluded exhaustively within theoretical errors.
- By using the color antisymmetric current, the mass of the $c\bar{s}\bar{c}\bar{s}$ tetraquark with $J^{PC} = 1^{++}$ and 1^{+-} in the $(2,0\{0,0\})$ excitation mode is predicted to be around 4.73 GeV and 4.77 GeV respectively, which is consistent with the calculations of the relativistic quark model [1, 2].
- in Ref. [2]. Their masses were predicted about 4.7 GeV ($P - \bar{P}$) and 4.6 GeV ($D - \bar{S}$), which are in agreement with our calculations.
- Our results provide a mass relation $6_{\rho\rho} < 3_{\lambda\lambda} < 3_{\lambda\rho} < 3_{\rho\rho}$ and $6_{\rho\rho} < 3_{\lambda\lambda} < 6_{\lambda\lambda} < 3_{\rho\rho}$ for the C-positive and negative D-wave $c\bar{s}\bar{c}\bar{s}$ tetraquarks, respectively.

Reference:

- [1] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C 58, 399 (2008)
- [2] Q.-F. Lu and Y.-B. Dong, Phys. Rev. D 94, 074007 (2016)

QCD Sum Rule

The two-point correlation function for the tensor currents:

$$\Pi_{\mu\nu,\rho\sigma}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_{\mu\nu}(x) J_{\rho\sigma}^\dagger(0)] | 0 \rangle$$

Hadron Level

$$\text{Dispersion relation: } \Pi(q^2) = \frac{1}{\pi} \int_{s_c}^{\infty} ds \frac{\text{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

The imaginary part of the correlation function is defined as the spectral function,

with narrow resonance approximation: $\rho(s) = \frac{1}{\pi} \text{Im}\Pi(s) = f_X^2 \delta(q^2 - m_X^2) + \text{continuum}$

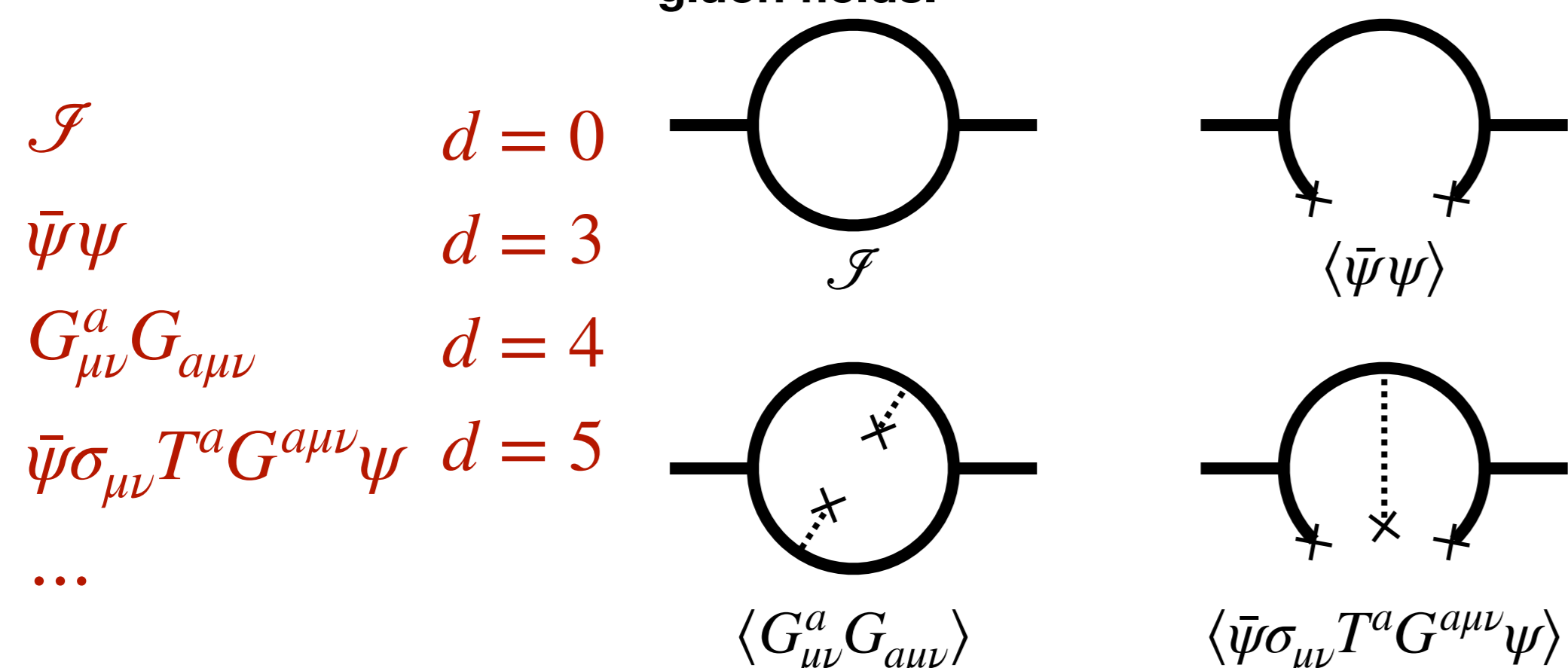
$$\mathcal{L}_k(s_0, M_B^2) = f_X^2 m_X^{2k} e^{-m_X^2/M_B^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M_B^2} \rho(s) s^k$$

$$m_X(s_0, M_B^2) = \sqrt{\frac{\mathcal{L}_1(s_0, M_B^2)}{\mathcal{L}_0(s_0, M_B^2)}}$$

Quark-Gluon Level

Operation Production Expansion(OPE): $\Pi(q^2) = C_0 \mathcal{F} + \sum_n C_n(q) \mathcal{O}_n$

$C_n(q)$ are Wilson coefficient, which can be calculated perturbatively, and \mathcal{O}_n are local gauge invariant operator constructed from quark and gluon fields.



OPE: $\Pi(s) = \Pi^{pert}(s) + \Pi^{(q\bar{q})}(s) + \Pi^{(G^2)}(s) + \dots$