Nonuniform-temperature effects on the phase transition

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Collaborated with Jun-Hui Zheng

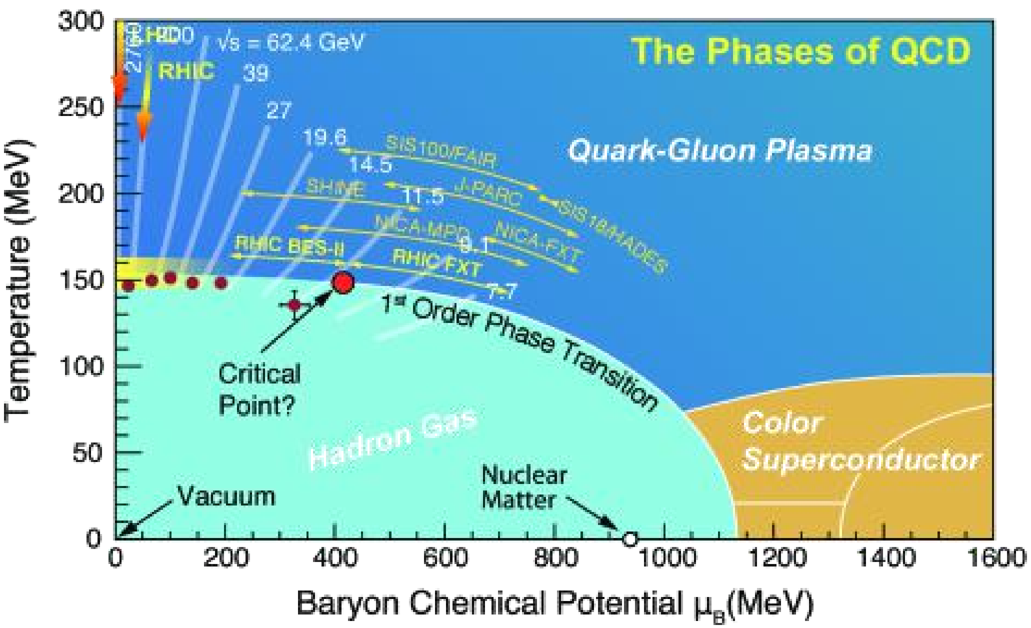
Phys. Rev. D 104.016031(2021), and work in preparation

QCD phase transition and the phase diagram

Critical Point --- the landmark of the QCD phase diagram.

- **Lattice simulation :**
  - μ=0, finite T
  - **crossover**
- **Effective theories:**
  - (P)NJL, QM, FRG, DSE, RM)
  - finite T and large μ
  - **first order**
  - **CP** is predicted.
- **Experimental facilities:**
  - RHIC (BES I & II), FAIR, NICA,HIAF

➢ **The location of CP? The 1st order phase transition? The signals?**



A. Bzdak, S.Esumi, V.Koch, J.Liao, M.Stephanov, N.XuPhys. Rept. 853 (2020)

Theoretical predictions

- **Non-interacting gas**

$$\langle \Delta n_p \Delta n_k \rangle = \left\langle (\Delta n_p)^2 \right\rangle \delta_{pk} = \langle n_p \rangle \delta_{pk}$$

- **Near the CP, 2-particle correlator**

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k} \xi^2$$

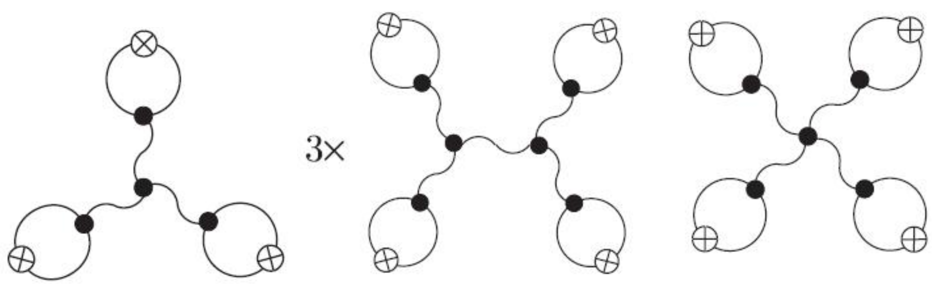
σNN coupling leads to a singular contribution as one approaches the CP.

- **3, 4-particle correlators are more sensitive to ξ**

M. Stephanov, PRL 102, 032301(2009)

$$\omega_3(N_p)_\sigma \approx 6\left(\frac{\tilde{\lambda}_3}{4}\right)\left(\frac{g}{10}\right)^3\left(\frac{\xi}{1\ \mathrm{fm}}\right)^{9/2},$$

$$\omega_4(N_p)_\sigma \approx 46\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4\left(\frac{\xi}{1\ \mathrm{fm}}\right)^7.$$

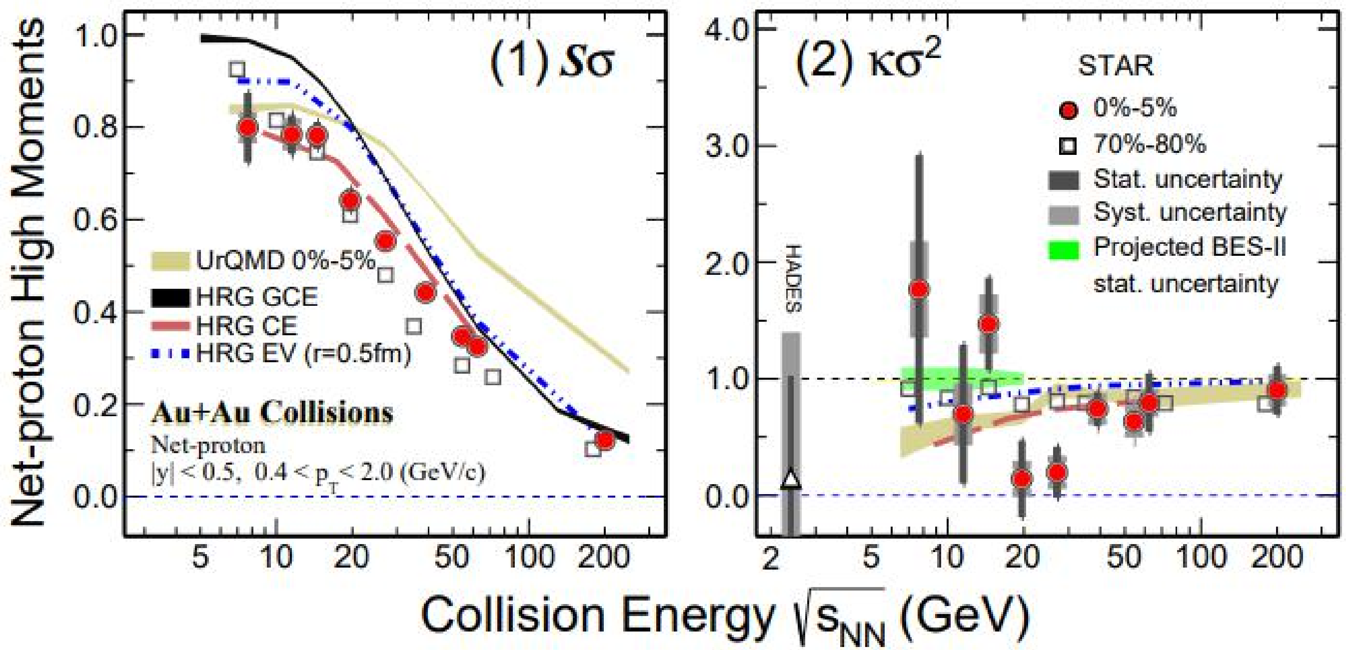


➢ Relationship with the **susceptibility ratio**:

$S\sigma \sim \chi_B^{(3)}/\chi_B^{(2)}$ $\kappa\sigma^2 \sim \chi_B^{(4)}/\chi_B^{(2)}$ with $\kappa = \frac{<(N-<N>)^4>}{\sigma^4} - 3$ $s = \frac{<(N-<N>)^3>}{\sigma^3}$

STAR BES: Cumulant ratios

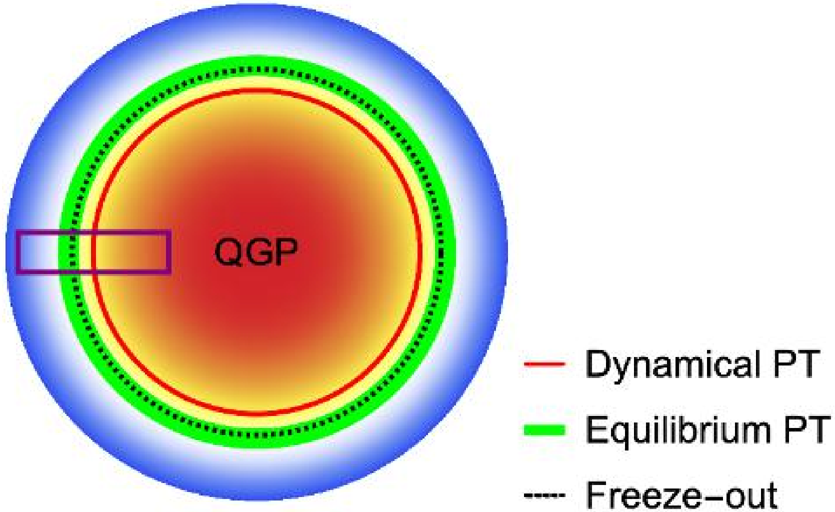
J. Adam et al. (STAR Collaboration), PRL 2021.

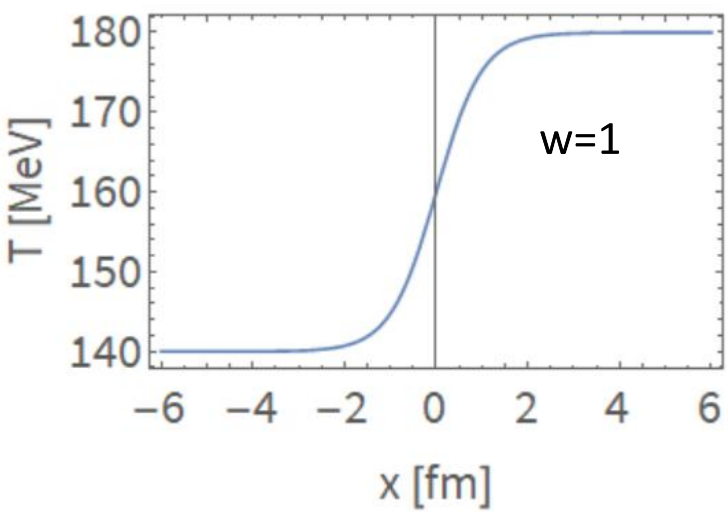


- **Nonmonotonic deviations of $\kappa\sigma^2$ at around $\sqrt{s_{NN}} \sim 20\ GeV$.**
- **Could not be explained by UrQMD, HRG, and equi/non-equi critical models.**

B. Berdnikov and K. Rajagopal, PRD 2000. M. Stephanov, PRL 2009. M. A. Stephanov, PRL, 2011. S. Mukherjee, R. Venugopalan, and Y. Yin, PRC 2015. L. Jiang, P. Li, and H. Song, PRC 2016. L. Jiang, S. Wu, and H. Song, NPA 2017. S. Wu, Z. Wu, and H. Song, PRC 2019. L. Jiang, J.-H. Zheng, and H. Stoecker, , arXiv:1711.05339. M. Stephanov and Y. Yin, PRD 2018. L. Du, U. Heinz, K. Rajagopal, and Y. Yin PRC 2020.

Assumptions





Approximations we made:

- Markov process
- Fast relaxation
- local equilibrium
- uniform chemical potential

◆ Temperature profile:

$$T(x) = T_c + \frac{\delta T}{2}\tanh\left(\frac{x}{w}\right).$$

the temperature gradiant is about 20 – 40 MeV/fm in the PT region, with $\delta T = 40$MeV, and w=1 or 0.5.

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).

E.o.m

- **The probability distribution function in continuous limit is written as**

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).

$$P[\sigma] = \exp\left\{-\int d\boldsymbol{r} \frac{(\nabla\sigma)^2/2 + V[\sigma(\boldsymbol{r})]}{T(x)}\right\},$$

- **We adopt the Ising-like effective potential**

$$V[\sigma] = a(T-T_c)(\sigma-\sigma_0)+b(\mu-\mu_c)(\sigma-\sigma_0)^2+c(\sigma-\sigma_0)^4,$$

Parameter sets: a = 0.5 fm^{-2} , b = −0.25 fm^{-1} , c = 3.6. $T_c = 160$MeV.

- **Stationary solution $\boldsymbol{\sigma_c}$ satisfies (** $\delta P[\sigma]/\delta\sigma = 0.$ **)**

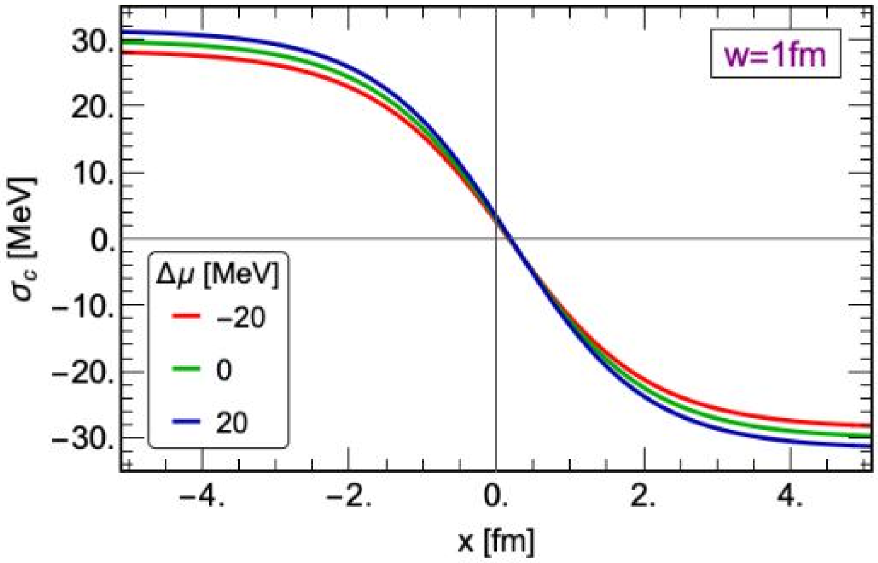
$$\nabla^2\sigma = \frac{1}{T}\nabla T \cdot \nabla\sigma + \frac{\delta V}{\delta\sigma}.$$

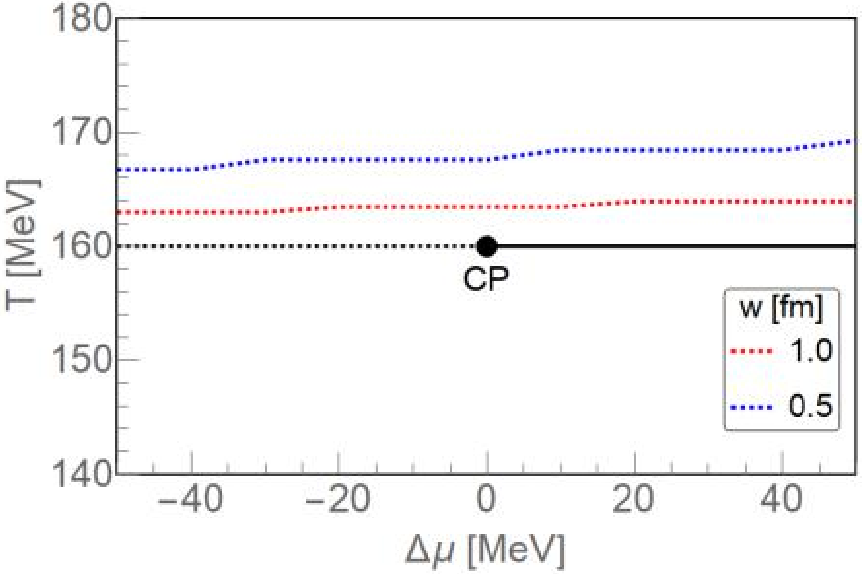
- **Boundary condition:** σ(x = −L/2) = σL, σ(x = L/2) = σR,

where σL and σR represent the global minimum at the local T.

stationary solution and the lift of Tc

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).

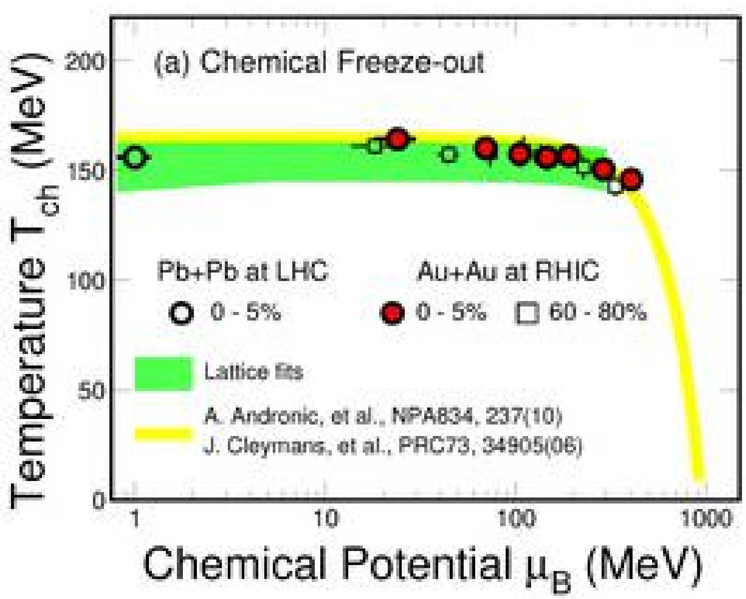




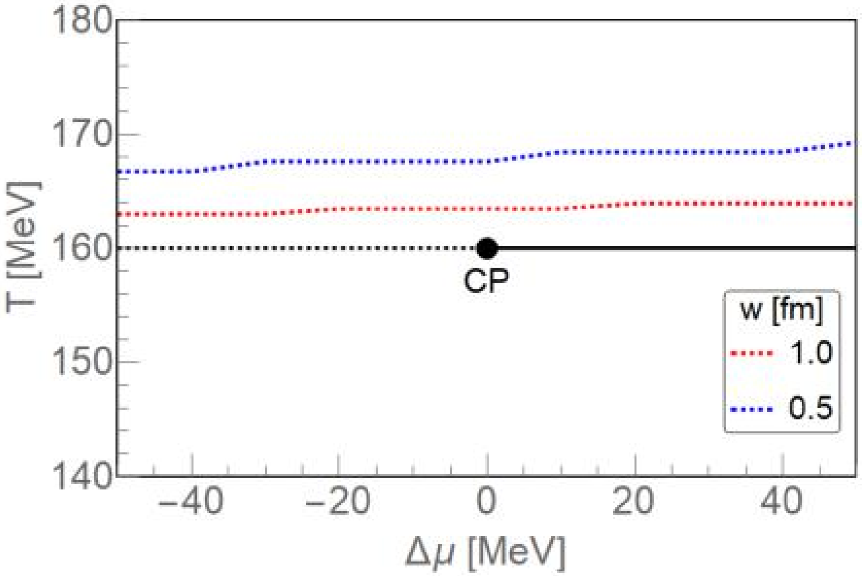
- For different order of phase transition, $\sigma_c(x)$ changes its sign at $x > 0$.
- By identifying the point of sign change of σ_c as the PT point, the PT temperature are lifted about 3 Mev and 5 MeV from Tc.

lift of Tc

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).

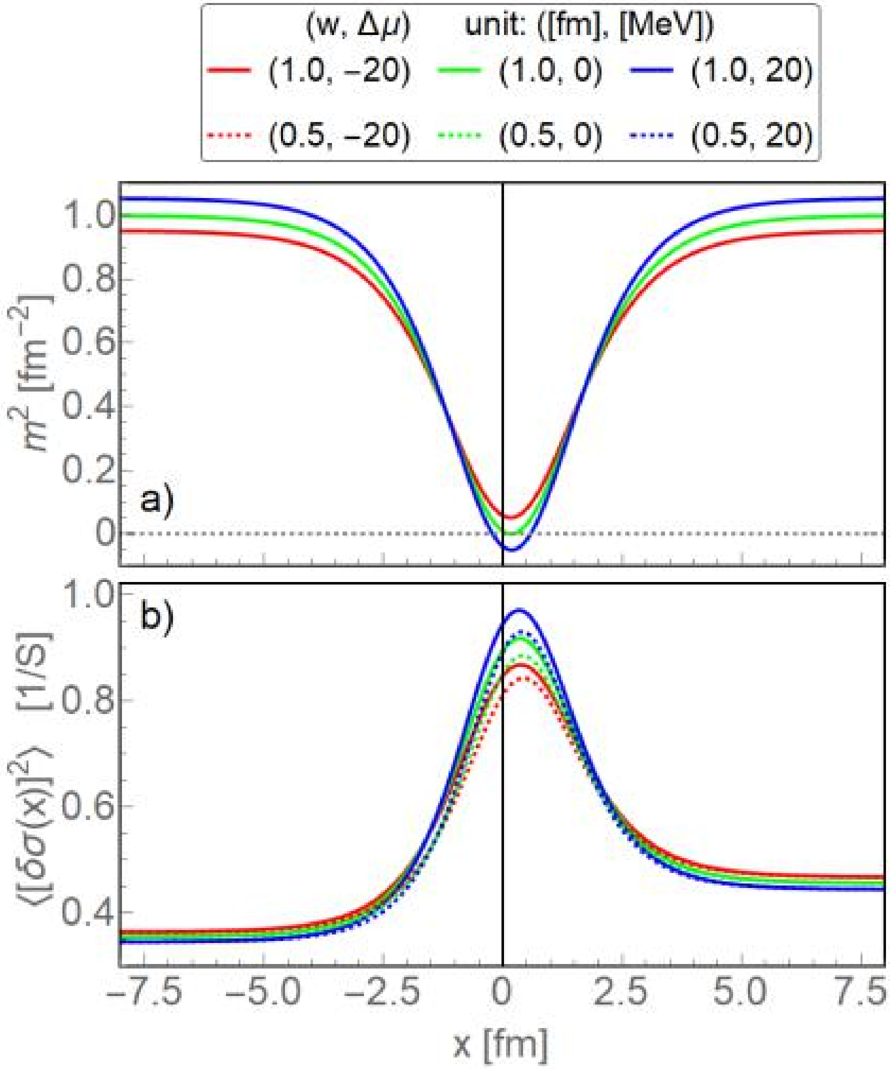


X. Luo, S. Shi, N. Xu, and Y. Zhang, Particles 3, 278 (2020).



Hints that the QCD phase transition may happen at a higher temperature than the equilibrium lattice Tc.

mass and variance



J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).

Expand the probability distribution function around the stationary solution $\boldsymbol{\sigma_c}$

$$\sigma(\boldsymbol{r}) = \sigma_c(x) + \delta\sigma(\boldsymbol{r}).$$

We can obtain the **mass**

$$m^2(x) = 2b\Delta\mu + 12c\sigma_c^2$$

Numerically, the probability distribution function is

$$P[\sigma] \propto \exp\left\{-\frac{S}{2}\sum_{i,j}\delta\sigma_i M_{ij}\delta\sigma_j\right\},$$

with the nonzero elements of the matrix M are

$$M_{ii} = \frac{1}{\Delta x}\left[\frac{1}{T_{i-1/2}} + \frac{1}{T_{i+1/2}}\right] + \frac{m_i^2\Delta x}{T_i},$$

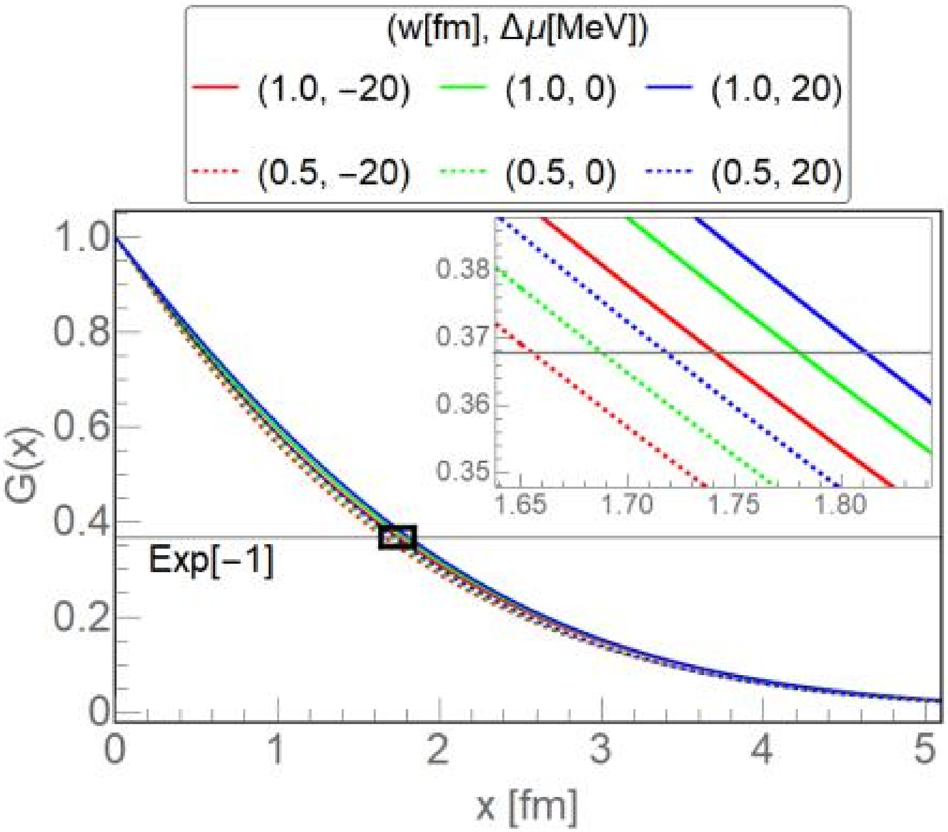
$$M_{i,i+1} = M_{i+1,i} = -\frac{1}{T_{i+1/2}\Delta x}.$$

The local variance $\langle[\delta\sigma_i]^2\rangle = \dfrac{[M^{-1}]_{ii}}{S}.$

The variances are enhanced in the PT region, and monotonically increase as the increase of μ.

Spatial correlation

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).



The nonlocal correlation

$$G(x) = \frac{\langle \delta\sigma(x_p + x/2)\delta\sigma(x_p - x/2)\rangle}{\langle \delta\sigma(x_p)\delta\sigma(x_p)\rangle}$$

Where x_p denotes the spatial location of the maximum point of the variance.

Numerically,

$$G(2j\Delta x) = [M^{-1}]_{i_0-j,i_0+j}/[M^{-1}]_{i_0,i_0}.$$

The correlation length ξ is determined by requiring

$$G(\xi) = \exp(-1).$$

The correlation length ξ again smoothly increases as the increase of μ, and is about 1.65fm – 1.9fm in the central part of the brick cell.

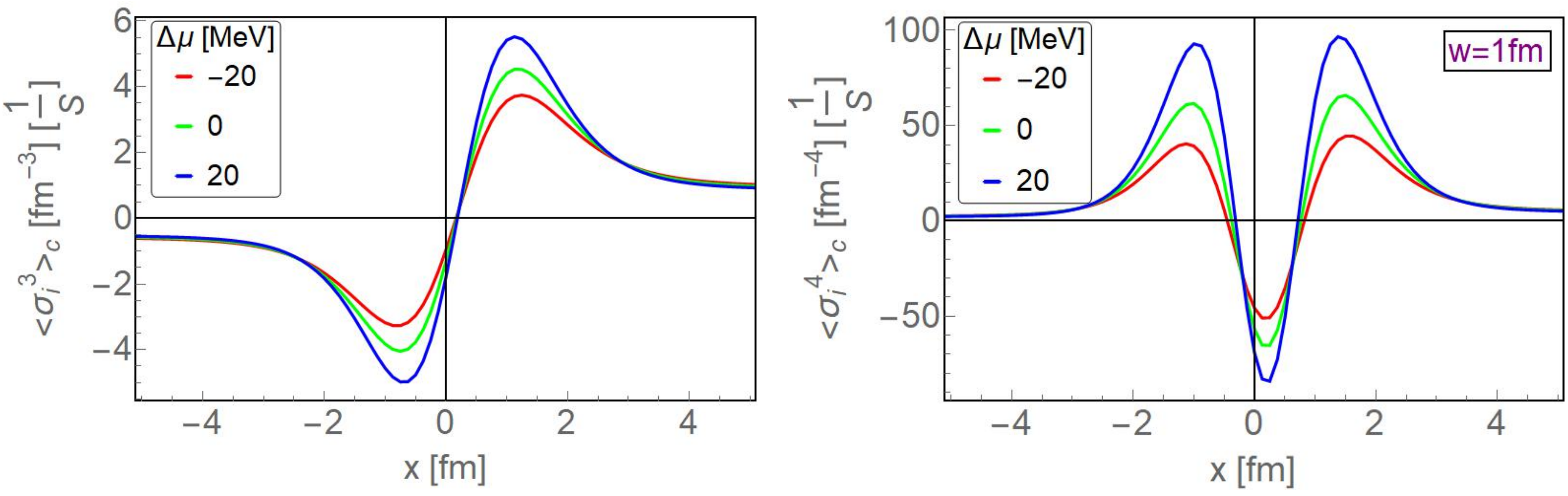
3, 4 point correlation in the space

The normalized nonlocal correlation, with $h(r) = \dfrac{12c\sigma_c\Delta r}{T(r_n)}, g(r) = \dfrac{4c\Delta r}{T(r_n)}$

in preparation

$$\langle \tilde{\sigma}_i \tilde{\sigma}_j \tilde{\sigma}_k \rangle = -\frac{1}{S}\sum_n 2h(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{kn}$$

$$\langle \tilde{\sigma}_i \tilde{\sigma}_j \tilde{\sigma}_k \tilde{\sigma}_p \rangle_c = \frac{1}{S}\sum_n \left(\sum_m 12h^2(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{mn}\left(M^{-1}\right)_{km}\left(M^{-1}\right)_{pm} - 6g(r)\left(M^{-1}\right)_{in}\left(M^{-1}\right)_{jn}\left(M^{-1}\right)_{kn}\left(M^{-1}\right)_{pn}\right)$$



Typical structure for the high order fluctuations in the nonuniform T space.

Summary

- **we studied the nonuniform-temperature effects on the stablest order parameter profile, the fluctuations and the correlation length.**

  - lift the PT temperature, which hints the QCD phase transition may happen at a higher temperature than the equilibrium lattice Tc.

  - The fluctuations are enhanced in the PT region.

  - No divergence, the uniqueness of the CP behaviors are wiped off.

- **Future work:**

  - more realistic setup of the nonuniform-temperature system.
  - combination of nonuniform-temperature effects and dynamical effects.
  -
  - …

Thank you!

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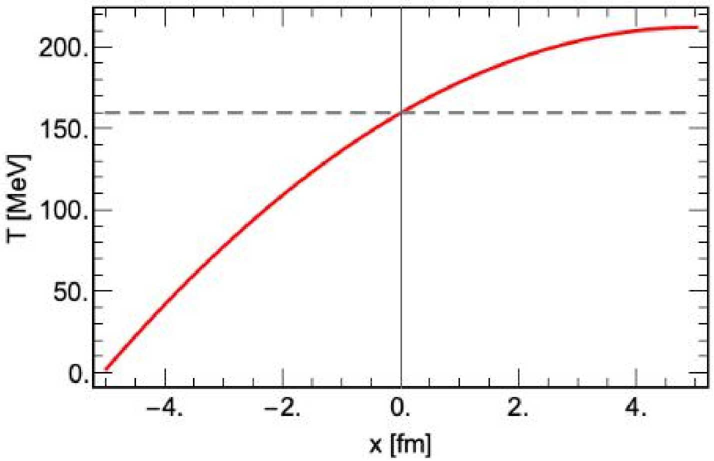
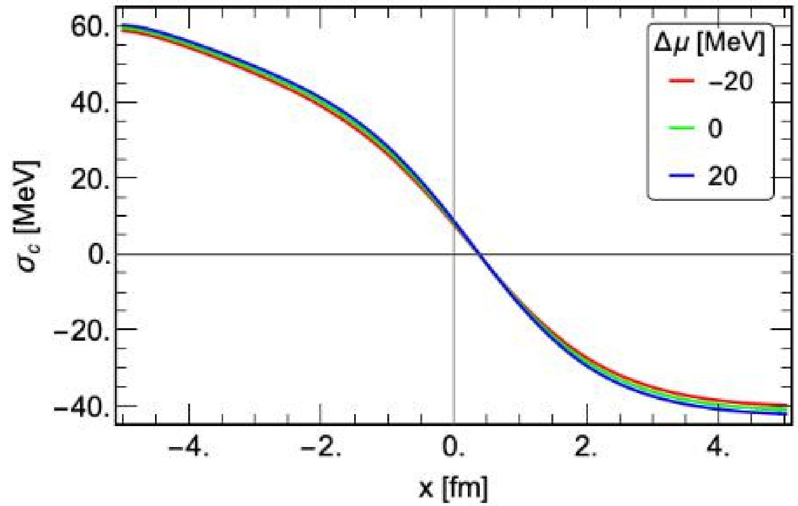
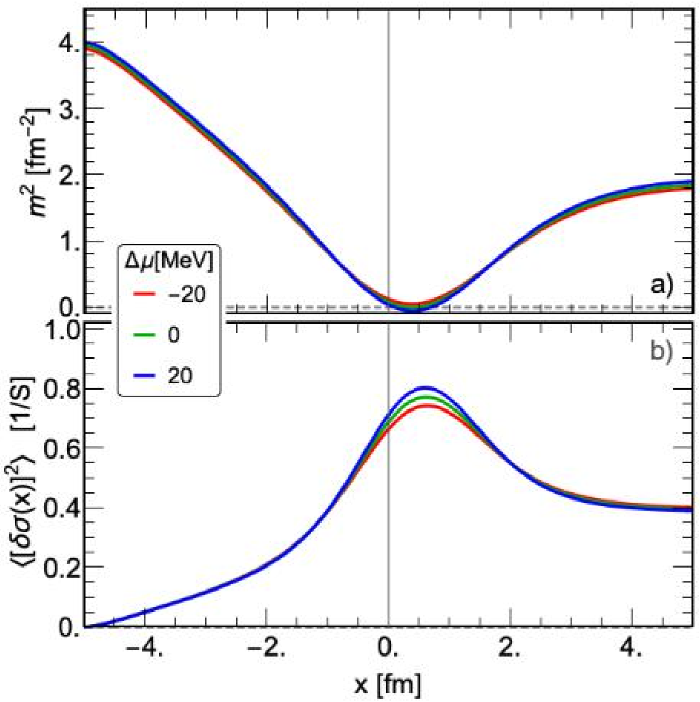
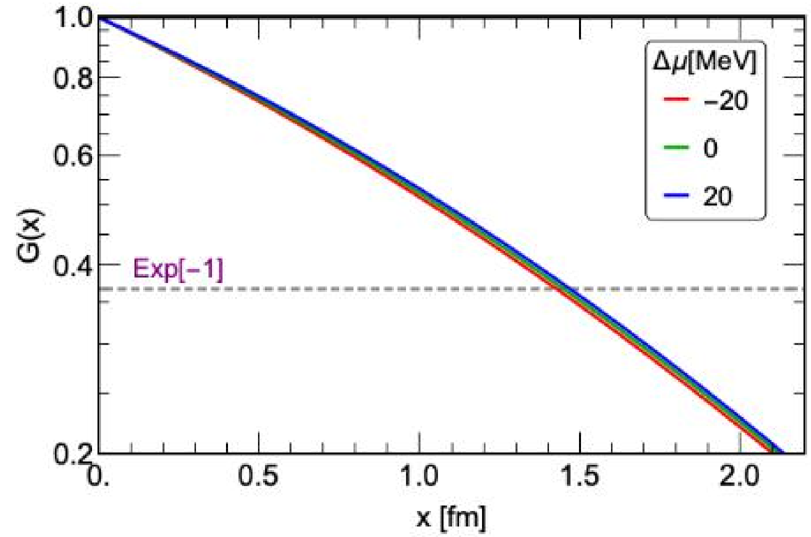
Results with a more realistic temperature profile

J-H. Zheng, L. J, Phys. Rev. D 104.016031(2021).









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Results with a more realistic temperature profile --- high order flucs

in preparation