Origin of Neutrino Mass on Convex Cone of Positivity

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报告基于与周顺研究员合作的文章：arXiv: 2202.12907

Motivation

- 非零中微子质量 → $O^{(5)} = \overline{L}\tilde{H}\tilde{H}^T L^c$

  大气中微子，太阳中微子，反应堆中微子的振荡...　　　有效算符

- SM 不是一个完整理论而是一个有效理论

  Weinberg算符的树图完成：type-I seesaw, type-II seesaw, type-III seesaw,...

  [P. Minkowski,PLB(1977)] [W. Konetschny and W. Kummer, PLB(1977)] [R. Foot, et al, ZPC 44(1989)]

- 新物理 (UV) 非常有可能耦合到 L 和 H 二重态

- 几种Seesaws 模型在五维中是不可区分的

  对低能可观测量的模型依赖的预测出现在更高的维度中...

目标: 使用模型无关的方法在更高维的空间来对seesaw模型进行区分

➢ **凭借SMEFT的框架，近年发展起来的正定性约束的几何方法将会有帮助**

SMEFT, 正定性和锥

- **SMEFT 框架**: 通过积掉重自由度，用更高维的算符来描述“紫外完成”的红外行为,

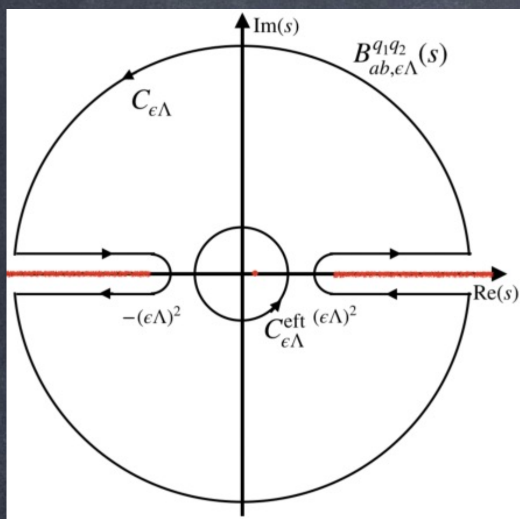
$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(7)} O_i^{(7)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

- **正定结构**: 利用量子场论的基本原理，如因果性，幺正性，洛伦兹对称性的等 -> 获得对Wilson系数的约束
  - 2对2振幅: $$\mathcal{M}_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + \cdots c_{n,m} s^n t^m$$
  - $c_2 > 0$; 或者在SMEFT中: $C^{(8)} > 0$ [A. Adams et al., JHEP 06]
  - 对于更高s依赖性的约束可见：

    近期发展: [S. Ghosh et al., 2204.07617] [L.-Y. Chiang et al., 2204.07140] [S. D. Bakshi et al., 2205.03301] [D. Chowdhury et al., 2205.13762] [G. N. Remmen, N. L. Rodd, 2206.13524]
- **几何观点:** 紫外态作为更高维算符的树图完成，在凸锥中是可分辨的

  [C.Zhang and S.-Y. Zhou, PRL(2020)]

主公式



因果性 -> 柯西积分公式

对于$ij \to kl$的2对2向前散射$(t=0)$：

[C. Zhang and S.-Y. Zhou, PRL(2020)]

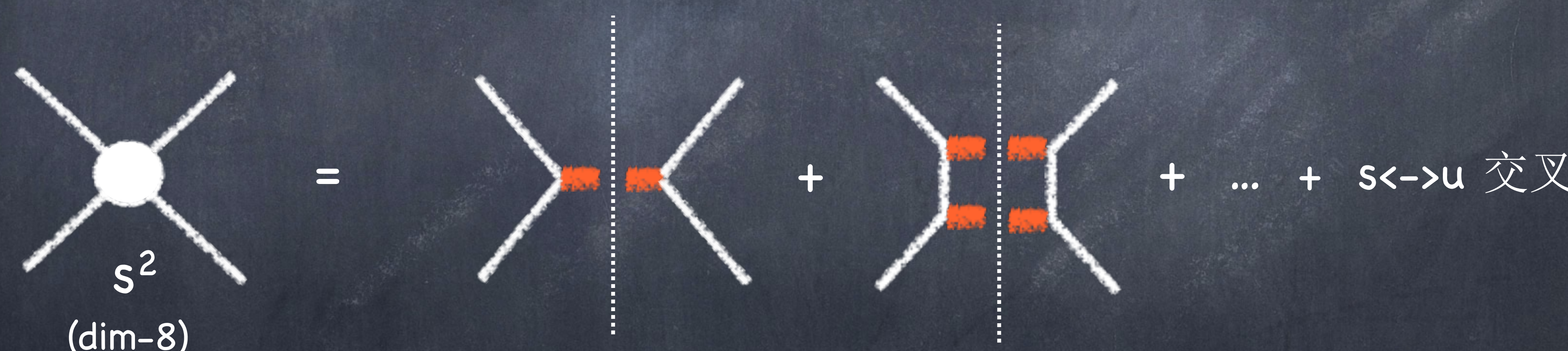
$$M^{ijkl} \equiv \frac{d^2}{ds^2}\mathcal{M}_{ij\to kl}(s,0)$$

$$= \frac{1}{2\pi i}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\,\text{Disc}\mathcal{M}_{ij\to kl}(\mu) + (j\leftrightarrow l)$$

$$+ \text{ residues at poles}$$

幺正性 -> 光学定理:

$$\text{Disc}\mathcal{M}^{ijkl}(s) = \sum_X \mathcal{M}^{ij\to X}(s)\left(\mathcal{M}^{kl\to X}(s)\right)^*$$



$$M^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X[\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X} + (j\leftrightarrow l)]$$

凸锥

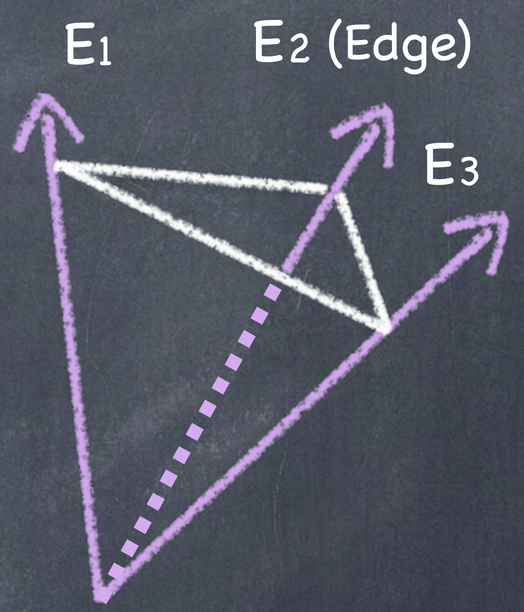
两个重点: $M^{ijkl}=\frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X[\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X}+(j\leftrightarrow l)]$ [C. Zhang and S.-Y. Zhou, PRL(2020)]

M^{ijkl} 是 $\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X}+(j\leftrightarrow l)$ 的正线性组合

1. M^{ijkl} 处于凸锥中

锥内的任何矢量总是能被写成E_i的正线性组合

$$\vec{x}:\quad \vec{x}=\sum_i w_i\vec{E}_i\ (w_i\geq 0)$$



$\{\vec{x}\}=\mathrm{cone}\left(\left\{\vec{E}_i\right\}\right)$

2. 当 X 属于不可约表示时，M^{ijkl} 可以成为棱

$$\mathbf{r}_i\otimes\mathbf{r}_j=\sum_\alpha C^{i,j}_{\mathbf{r},\alpha}\mathbf{r}$$

Clebsch-Gordan (CG)系数.

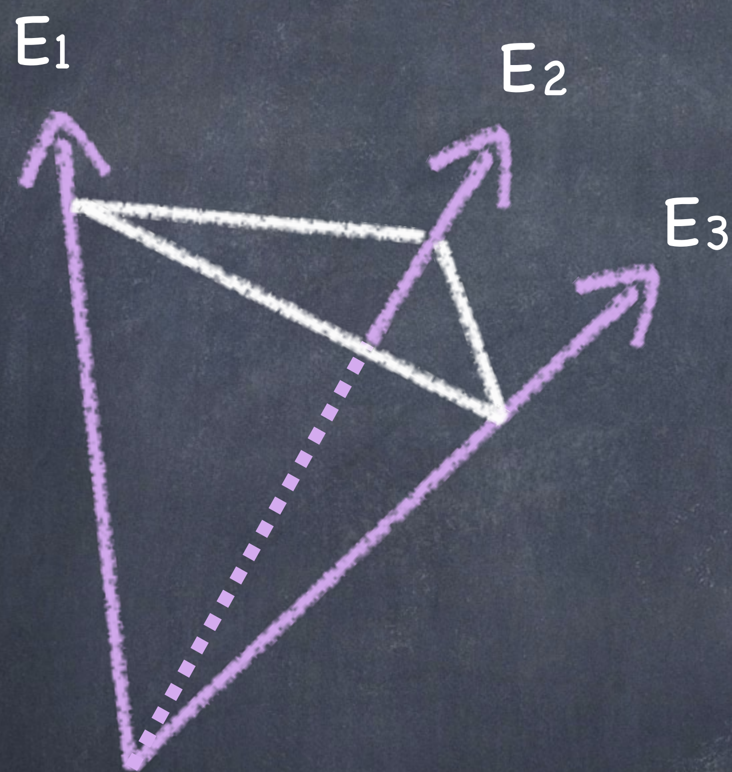
$$M^{ijkl}=\frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X|\langle X|\mathcal{M}|\mathbf{r}\rangle|^2\mathcal{G}^{ijkl}_{\mathbf{r}}$$

生成元:

$$\mathcal{G}^{ijkl}_{\mathbf{r}}\equiv\Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}(C^{k,l}_{\mathbf{r},\alpha})^*+(j\leftrightarrow l)$$

UV完成和锥

锥的棱



由生成元构造的锥

$$\mathcal{G}_{\mathbf{r}}^{ijkl} \equiv \Sigma_\alpha C_{\mathbf{r},\alpha}^{i,j} (C_{\mathbf{r},\alpha}^{k,l})^* + (j \leftrightarrow l)$$

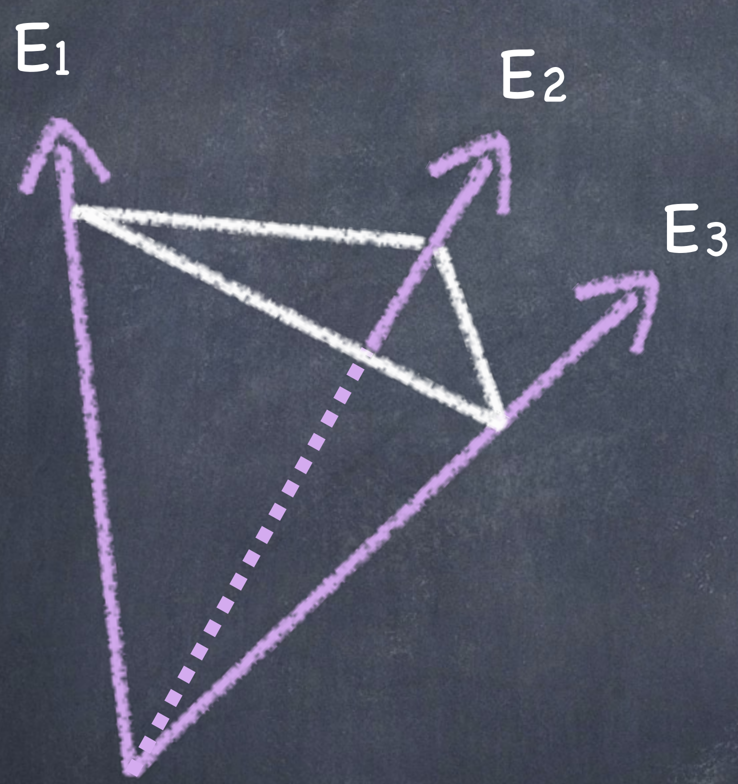
可能对应于

不可约表示中的紫外态

高维算符的树图完成

UV完成和锥

锥的棱



由生成元构造的锥

$$\mathcal{G}^{ijkl}_{\mathbf{r}} \equiv \Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}(C^{k,l}_{\mathbf{r},\alpha})^* + (j \leftrightarrow l)$$

可能对应于

seesaws能出现在八维否?

不可约表示中的紫外态

$$O^{(5)} = \overline{L}\tilde{H}\tilde{H}^T L^c$$

Type-I	Type-II	Type-III
$\overline{L^c} \otimes H$	$\overline{L^c} \otimes L$	$\overline{L^c} \otimes H$
1	3	3 (不可约表示)
$\overline{L^c} \otimes H$	$H \otimes H$	$\overline{L^c} \otimes H$

Seesaw 模型

[L.-F. Li, PRD(1980)] [E. Ma, PRL(1998)]

[H.-L. Li, et al. PRD(2021)],
[C. W. Murphy, JHEP(2020)]

- $LLHH$:

$$\mathcal{O}_1 = (\bar{L}\gamma_\mu i\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger D^\nu H\right),$$
$$\mathcal{O}_2 = (\bar{L}\gamma_\mu\sigma^I i\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger\sigma^I D^\nu H\right)\ ;$$

- $LLLL$:

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L,H 被分配到不可约表示**2**：
则 **X** 可被分配到 **1** 或 **3**

$$\mathbf{2}\times\mathbf{2}=\mathbf{1}+\mathbf{3}$$
$$\mathbf{2}\times\bar{\mathbf{2}}=\mathbf{1}+\mathbf{3}$$

CG 系数:

$$C^{ab}_{\mathbf{1},c} = \epsilon^{ab}\ ,\quad C^{ab}_{\mathbf{3},c} = \left(\epsilon\sigma^I\right)^{ab}\ ;$$
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X is Boson, $i,j=\mathbf{2},\mathbf{r}=\mathbf{1}$ or $\mathbf{3}$

$\mathcal{M}_{ij\to\mathbf{r}} =$

	H_b	L_b	$H_b^\dagger$	$\bar{L}_b$
H_a	$C^{ab}_{\mathbf{1/3},c}$	$C^{ab}_{\mathbf{1/3},c}$	$\bar{C}^{ab}_{\mathbf{1/3},c}$	$\bar{C}^{ab}_{\mathbf{1/3},c}$
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x 是HH和LL之间耦合的相对大小

得到生成元!

$$\mathcal{G}^{ijkl}_{\mathbf{r}} \equiv \Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}\left(C^{k,l}_{\mathbf{r},\alpha}\right)^* + (j\leftrightarrow l)$$
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[C. Zhang, 2112.11665]

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[C. W. Murphy, JHEP(2020)]

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[C. Zhang, 2112.11665]

八维的树图紫外完成

特征矢量

UV State	Spin	$SU(2)_L \otimes U(1)_Y$	Interaction	Seesaw	Extremal Ray	$\vec{c}$
E	1/2	$\mathbf{1}_{-1}$	$g\bar{E}\,(H^\dagger L)$		✔	$\frac{1}{2}(-1,-1,0,0,0,0,0)$
Σ_1	1/2	$\mathbf{3}_{-1}$	$g\bar{\Sigma}_1^I\,(H^\dagger\sigma^I L)$		✗	$\frac{1}{2}(-3,1,0,0,0,0,0)$
N	1/2	$\mathbf{1}_0$	$g\bar{N}\,(H^{\mathrm{T}}\epsilon L)$	Type-I	✔	$\frac{1}{2}(-1,1,0,0,0,0,0)$
Σ	1/2	$\mathbf{3}_0$	$g\bar{\Sigma}^I\,(H^{\mathrm{T}}\epsilon\sigma^I L)$	Type-III	✗	$\frac{1}{2}(-3,-1,0,0,0,0,0)$
$\mathcal{B}_1$	1	$\mathbf{1}_1$	$g\mathcal{B}_1^\mu\left[(H^\dagger\epsilon\mathrm{i}\overleftrightarrow{D}_\mu H^*)+x(\bar{L}^c\epsilon\mathrm{i}\overleftrightarrow{D}_\mu L)\right]$		✗	$\frac{1}{2}(0,0,x^2,-x^2,16,0,-16)$
Ξ_1	0	$\mathbf{3}_1$	$g\Xi_1^I\left[M(H^\dagger\epsilon\sigma^I H^*)+x(\bar{L}^c\epsilon\sigma^I L)\right]$	Type-II	✗	$\frac{1}{2}(0,0,-3x^2,-x^2,0,16,0)$
$\mathcal{S}$	0	$\mathbf{1}_{0\mathrm{S}}$	$gM\mathcal{S}\,(H^\dagger H)$		✔	$2(0,0,0,0,0,0,1)$
$\mathcal{B}$	1	$\mathbf{1}_{0\mathrm{A}}$	$g\mathcal{B}^\mu\left[H^\dagger\mathrm{i}\overleftrightarrow{D}_\nu H+x(\bar{L}\gamma_\mu L)\right]$		✗	$\frac{1}{2}(0,0,-x^2,0,-4,4,0)$
Ξ	0	$\mathbf{3}_{0\mathrm{S}}$	$gM\Xi^I(H^\dagger\sigma^I H)$		✗	$2(0,0,0,0,2,0,-1)$
$\mathcal{W}$	1	$\mathbf{3}_{0\mathrm{A}}$	$g\mathcal{W}^{I\mu}\left[(H^\dagger\sigma^I\mathrm{i}\overleftrightarrow{D}_\mu H)+x(\bar{L}\gamma_\mu\sigma^I L)\right]$		✗	$\frac{1}{2}(0,0,0,-x^2,4,4,-8)$

三种**seesaws**模型作为紫外完成的子集出现

八维的树图紫外完成

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N	1/2	$\mathbf{1}_0$	$g\bar{N}\left(H^{\mathrm{T}}\epsilon L\right)$	Type-I	✓	$\frac{1}{2}(-1,1,0,0,0,0,0)$
Σ	1/2	$\mathbf{3}_0$	$g\bar{\Sigma}^I\left(H^{\mathrm{T}}\epsilon\sigma^I L\right)$	Type-III	✗	$\frac{1}{2}(-3,-1,0,0,0,0,0)$
$\mathcal{B}_1$	1	$\mathbf{1}_1$	$g\mathcal{B}_1^\mu\left[(H^\dagger\epsilon \mathrm{i}\overleftrightarrow{D}_\mu H^*)+x(\bar{L}^c\epsilon \mathrm{i}\overleftrightarrow{D}_\mu L)\right]$		✗	$\frac{1}{2}(0,0,x^2,-x^2,16,0,-16)$
Ξ_1	0	$\mathbf{3}_1$	$g\Xi_1^I\left[M(H^\dagger\epsilon\sigma^I H^*)+x(\bar{L}^c\epsilon\sigma^I L)\right]$	Type-II	✗	$\frac{1}{2}(0,0,-3x^2,-x^2,0,16,0)$
$\mathcal{S}$	0	$\mathbf{1}_{0\mathrm{S}}$	$gM\mathcal{S}\left(H^\dagger H\right)$		✓	$2(0,0,0,0,0,0,1)$
$\mathcal{B}$	1	$\mathbf{1}_{0\mathrm{A}}$	$g\mathcal{B}^\mu\left[H^\dagger \mathrm{i}\overleftrightarrow{D}_\nu H+x(\bar{L}\gamma_\mu L)\right]$		✗	$\frac{1}{2}(0,0,-x^2,0,-4,4,0)$
Ξ	0	$\mathbf{3}_{0\mathrm{S}}$	$gM\Xi^I(H^\dagger\sigma^I H)$		✗	$2(0,0,0,0,2,0,-1)$
$\mathcal{W}$	1	$\mathbf{3}_{0\mathrm{A}}$	$g\mathcal{W}^{I\mu}\left[(H^\dagger\sigma^I \mathrm{i}\overleftrightarrow{D}_\mu H)+x(\bar{L}\gamma_\mu\sigma^I L)\right]$		✗	$\frac{1}{2}(0,0,0,-x^2,4,4,-8)$

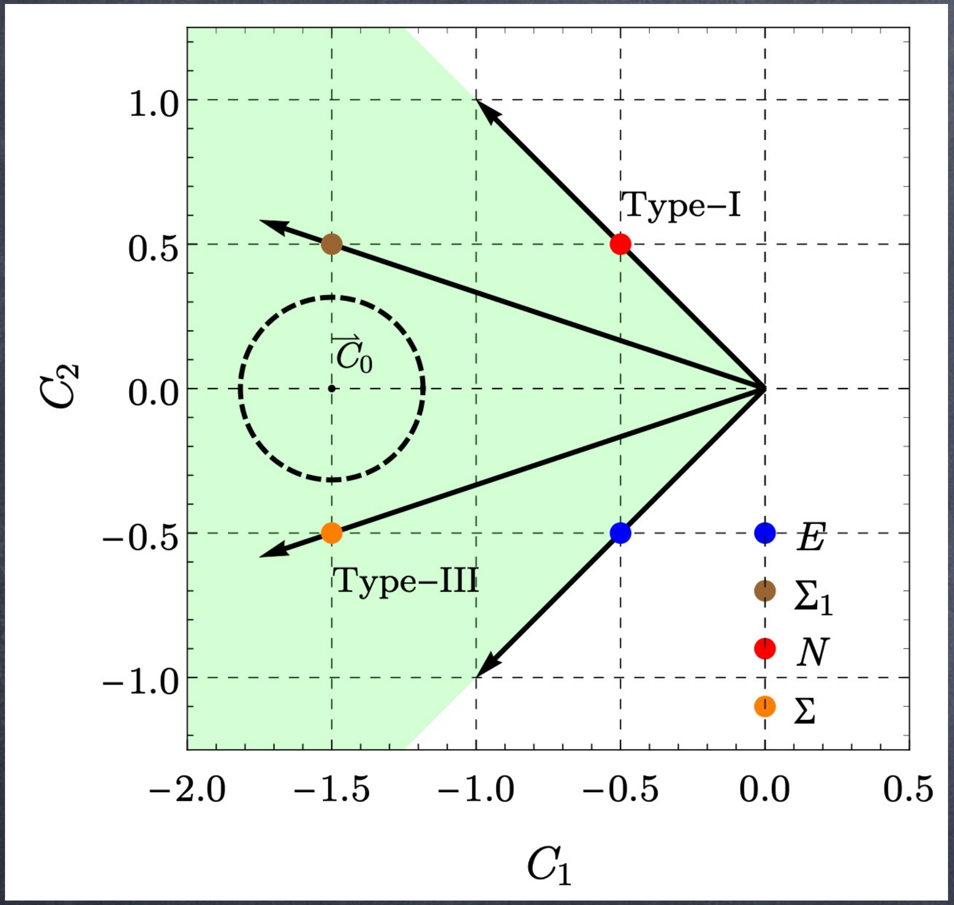
费米子

玻色子

玻色子类型和费米子类型的紫外态分别处于七维空间中两个不同的子空间

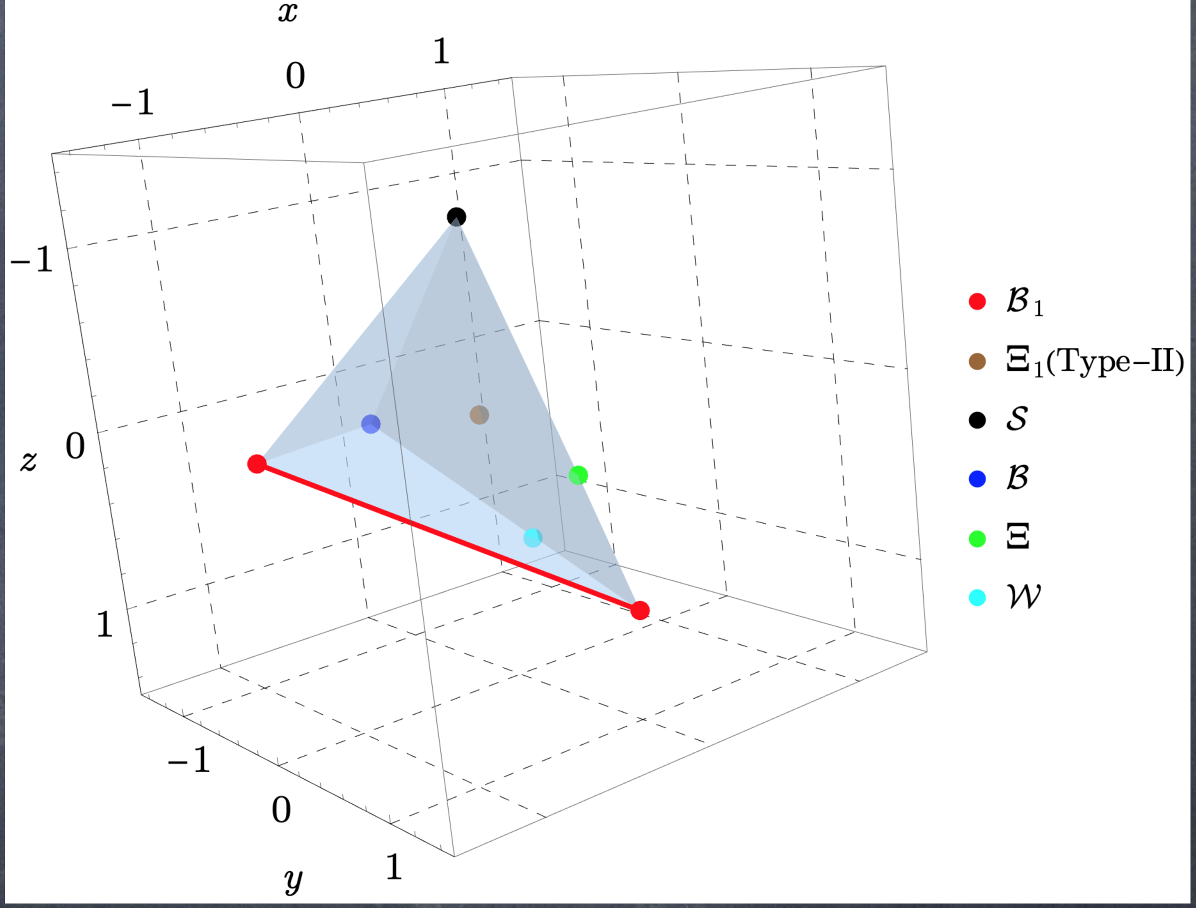
锥的子空间截面

紫外态是费米子



$$\mathcal{O}_1 = (\bar{L}\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger D^\nu H\right),$$
$$\mathcal{O}_2 = (\bar{L}\gamma_\mu \sigma^I \mathrm{i}\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger \sigma^I D^\nu H\right)$$

紫外态是玻色子

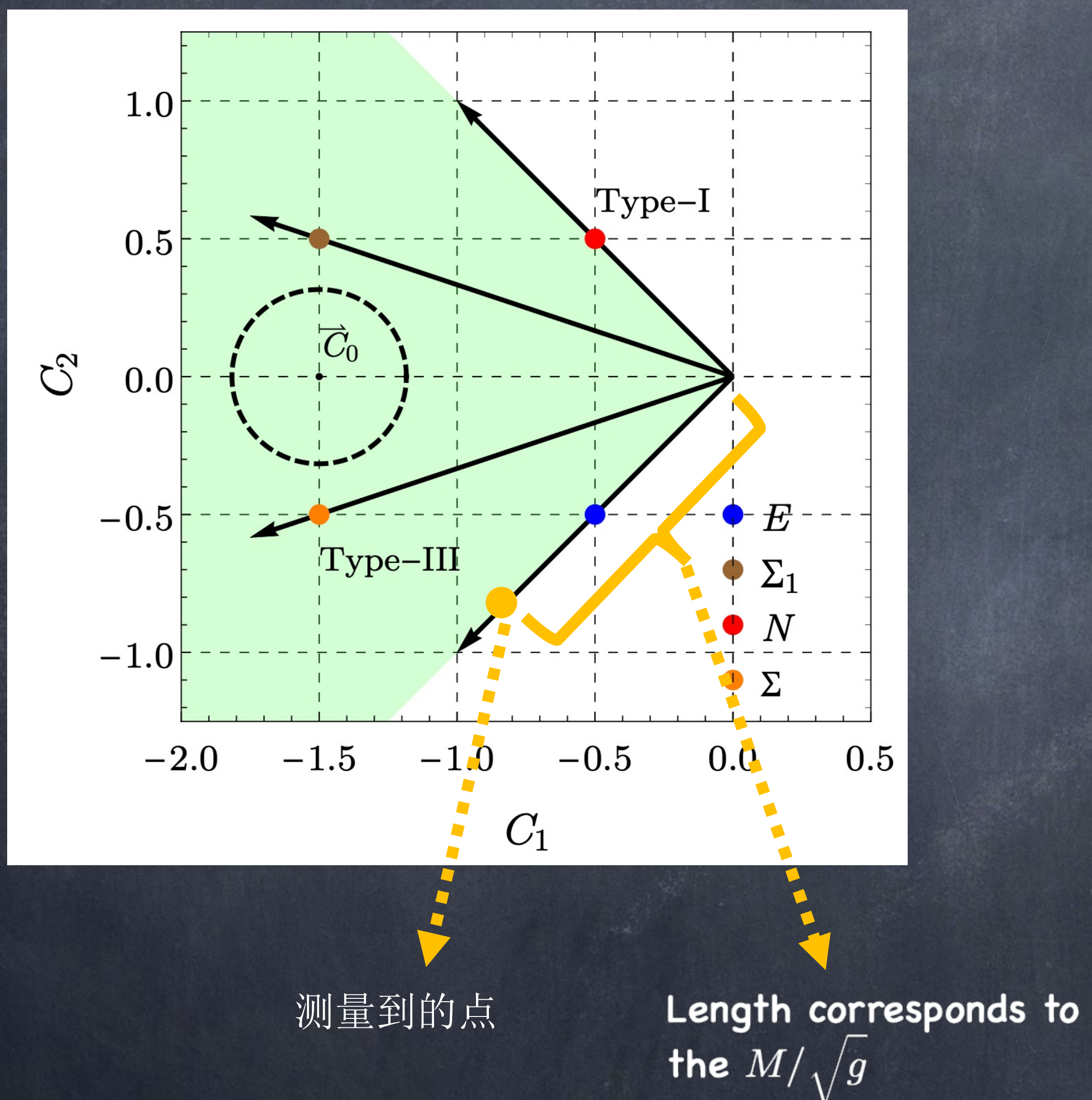


$$(x, y, z) = \frac{A(C_i)}{C_3 + 2C_4 - 2C_5 - 3C_6 - C_7}$$

$$A(C_i) = (-0.7C_3 + 0.69C_4 + 0.11C_5 + 0.13C_6 + 0.057C_7\,, 0.031C_3 + 0.85C_5 - 0.5C_6 - 0.16C_7, -0.019C_3 + 0.31C_6 - 0.95C_7)$$

凸优化

紫外态是费米子



反向提取紫外态能标的下限：[C. Zhang, 2112.11665]

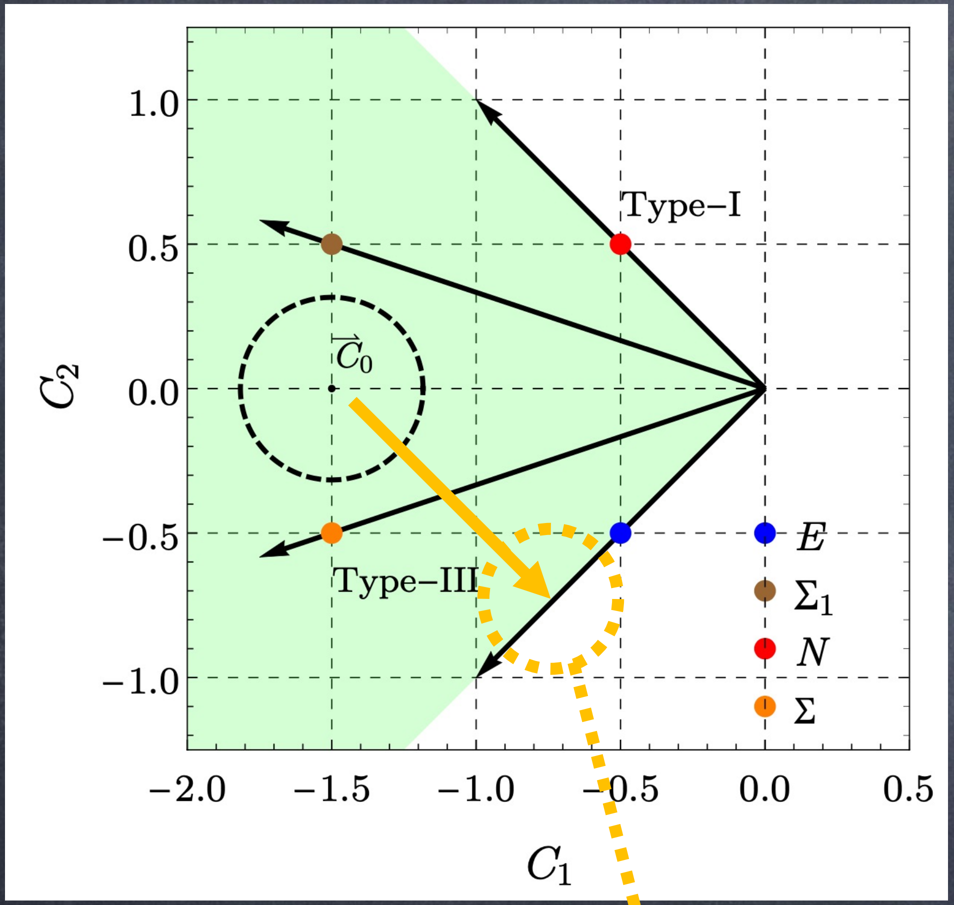
$$\vec{C}(\lambda) \equiv \vec{C}_0 - \lambda \vec{c}_i = \sum_{j\neq i} \omega_j \vec{c}_j + (\omega_i - \lambda)\vec{c}_i \ .$$

凸优化：

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \vec{C} - \lambda \vec{c}_i \subset \mathcal{C} \\ & (\vec{C} - \vec{C}_0) \cdot A \cdot (\vec{C} - \vec{C}_0) \leq \Delta \end{aligned}$$

凸优化

UV states are Fermions



反向提取紫外态能标的下限：[C. Zhang, 2112.11665]

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凸优化：

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$\vec{C}_0$	E	Σ_1	N	Σ
$(-1/2, 1/2)$	∞	∞	≥ 1.0	∞
$(-3/2, 0)$	≥ 0.9	≥ 1.07	≥ 0.9	≥ 1.07
$(-3/2, 0)$ with $\Delta = 0.1$	≥ 0.85	≥ 1.0	≥ 0.85	≥ 1.0
$(0, 0)$ with $\Delta = 0.1$	≥ 1.22	≥ 1.5	≥ 1.22	≥ 1.5

总结

- EFT 系数空间在八维会出现正定结构, 这来自于量子场论的基本原理.
- 为了实现Weinberg算符，新物理非常可能耦合到L和H，所以构建出包含LLHH的八维空间的凸结构，非常有利于我们探测该新物理。
  - 对称性的不可约表示可以形成凸锥， seesaw模型作为$SU(2)$的不可约表示，会很自然地出现在锥中。
- 反向探测紫外态的能标下限能够由完善的凸优化程序实现。

Backups

To formulate this approach, **symmetries of the system help**
(will also discuss cases without symmetries)

- Make use of symmetries of the problem (SM symmetries, helicities)
- Dispersion relation: $$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$
- Becomes: $$M^{ijkl} = \sum_{X\in r}{}' \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \frac{|<X|M|r>|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_r^{i(j|k|l)}$$

Dynamics

Symmetry

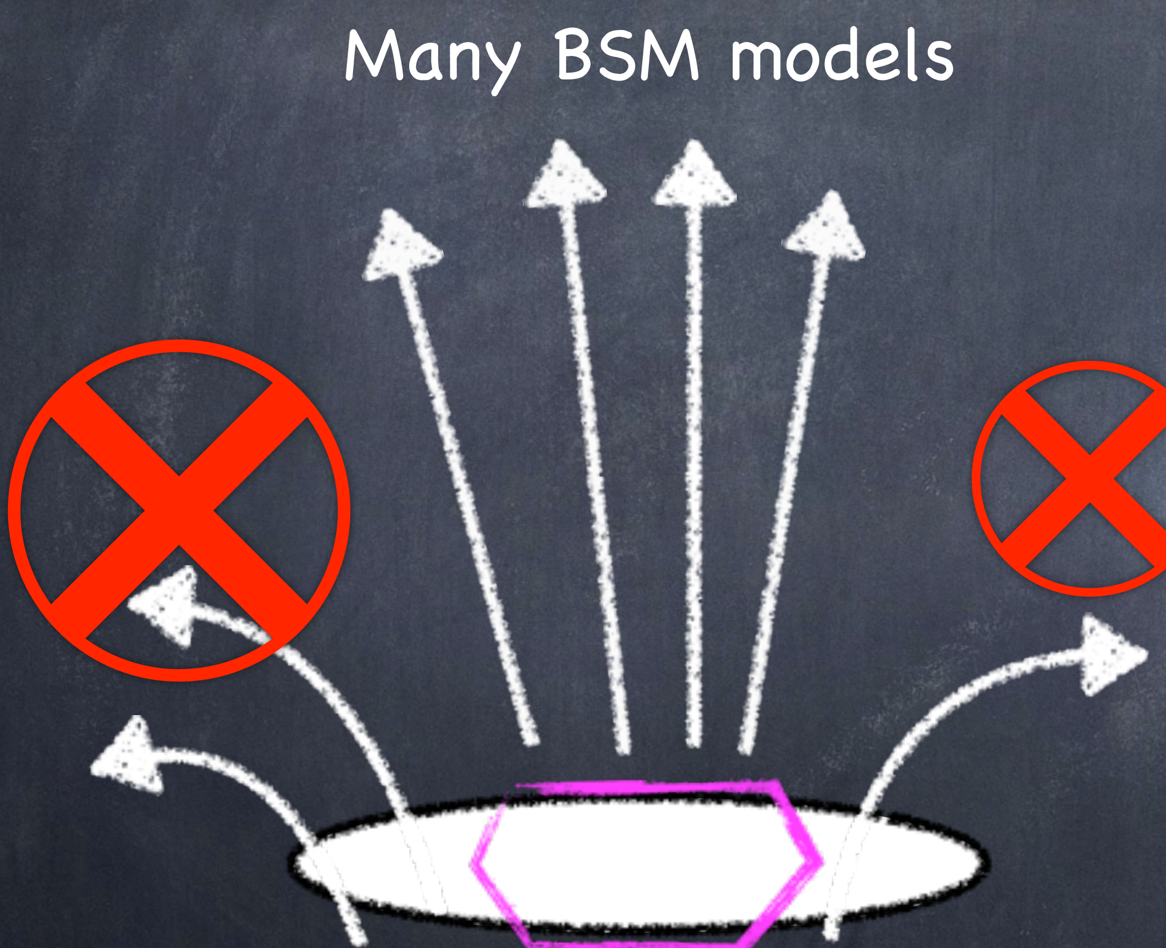
i(j|k|l): j,l symmetrized

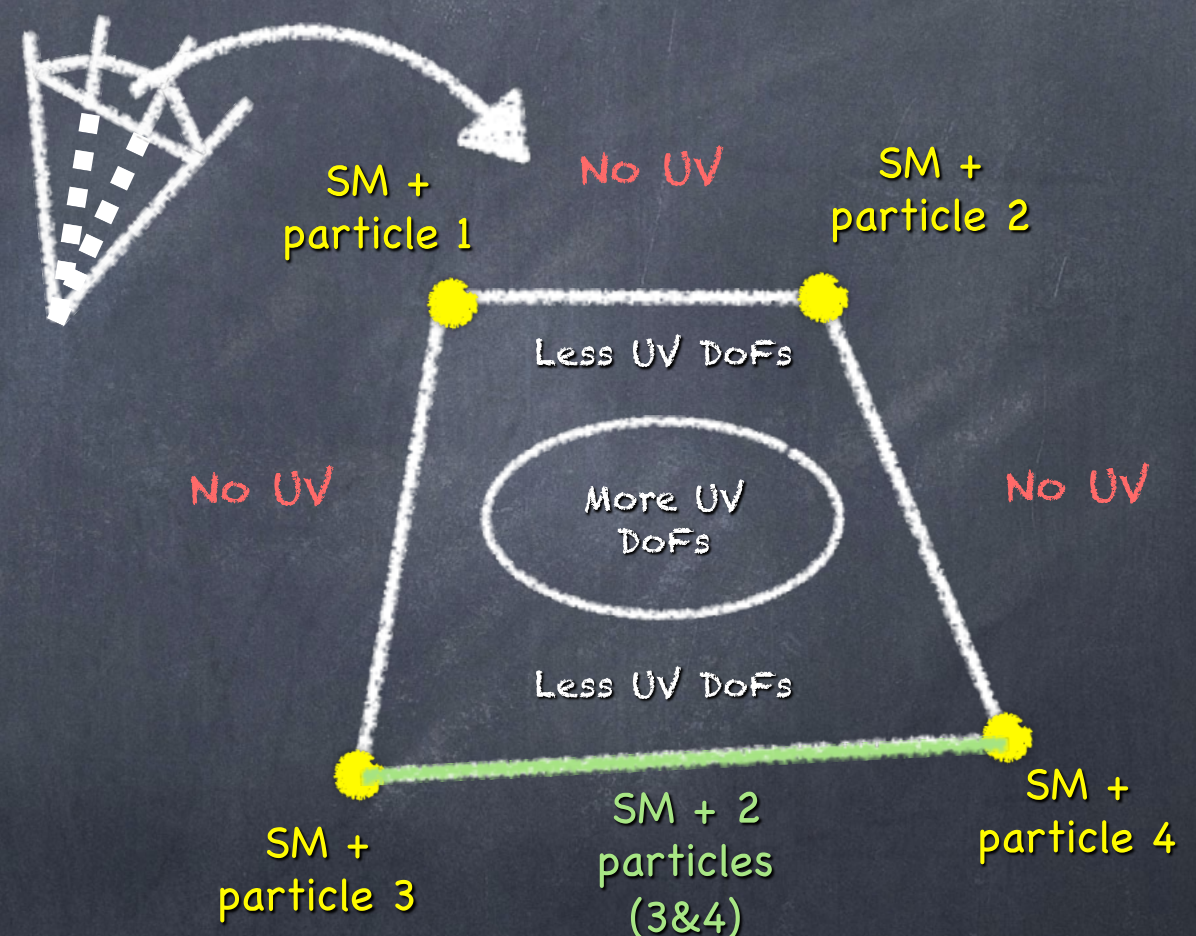
- P_r^{ijkl} is the projective operator of an irrep r, obtained by CG coefficients.
$$P_r^{ijkl} = \sum_\alpha C_{i,j}^{r,\alpha} \left(C_{k,l}^{r,\alpha}\right)^*$$
- The generators are simply (subset of) $P_r^{i(j|k|l)}$

Talk by Cen Zhang (HEP 2021)

- Infer UV model from EFT measurements

Inverse problem: Given the measured values of the operator coefficients around the electroweak scale, to what extend can we possibly determine the nature of the new physics beyond the SM? [Gu, Wang, 2008.07551]
see also [S. Dawson et al. 2007.01296] [N. Arkani-Hamed et al. hep-ph/0512190]





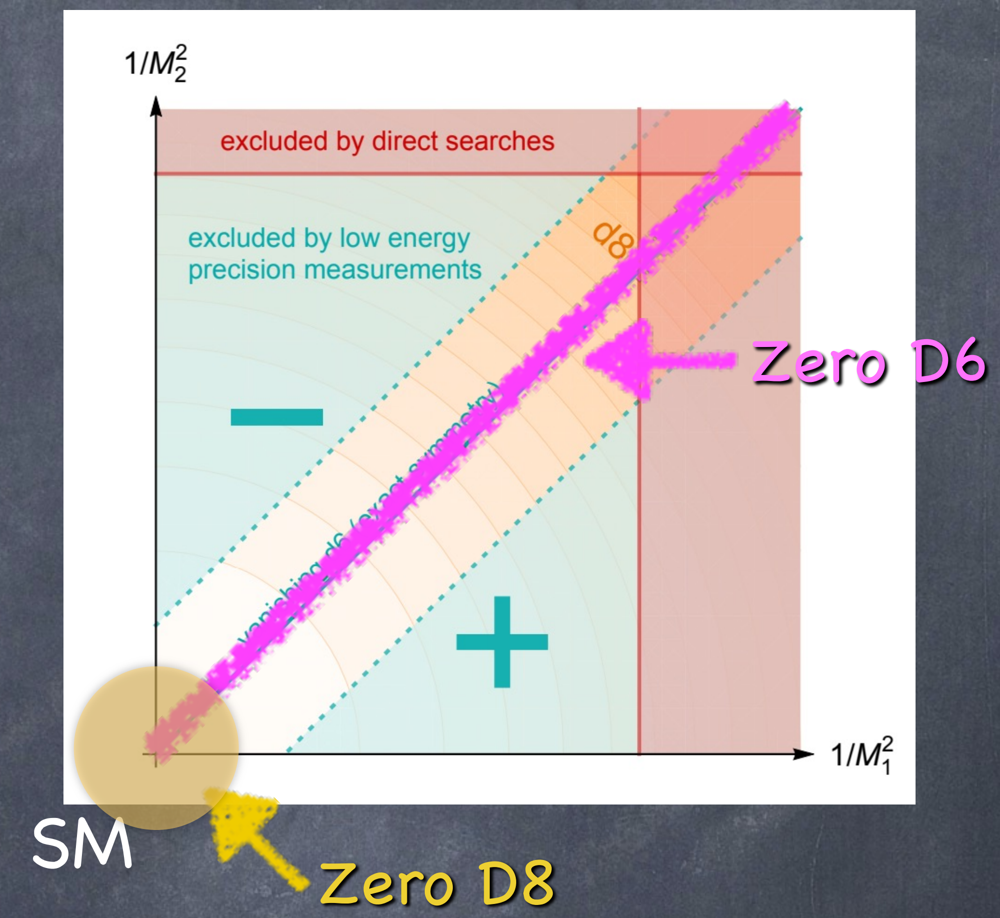
[CZ and S.-Y. Zhou 2005.03047]
[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]

Talk by Cen Zhang (HEP 2021)

- Testing and confirming the SM: Null result of measurements at dim-6 does not exclude all BSM, but does at dim-8 by using positivity bounds

- Dim-6: no positivity, different states may cancel each other's effects.
  - E.g., scalar and vector generate 4-fermion operators with opposite signs.
  - No UV particle can be absolutely excluded.

[Gu, Wang, 2008.07551]



- Dim-8: with positivity, different states are not allowed to cancel.
  - All states can be exclude to some absolute scale. (by using posi. bound)
  - Unlike dim-6 cannot lift this limit by adding more and more BSM states.
  - A robust confirmation of the SM.

[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]

Talk by Cen Zhang (HEP 2021)