中国物理学会高能物理分会
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Non-unitary Leptonic Flavor Mixing and CP Violation in Neutrino-antineutrino Oscillations

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Based on : Y. Wang and S. Zhou, Phys. Lett. B 824 (2022) 136797.

Outline

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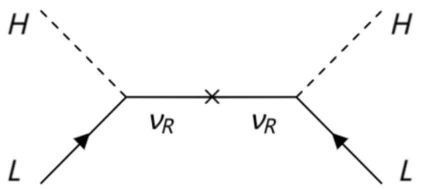
V. Summary

I. Background and Motivation

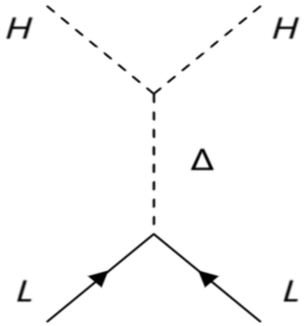
- 中微子振荡是超出标准模型新物理的确凿证据[1,2]， 它表明中微子具有**非零的绝对质量**，以及轻子部分存在**味混合**。
- 目前的中微子振荡实验结果表明轻子部分可能存在的**CP破坏**。
- CP破坏也是解释**宇宙物质-反物质不对称**的必要条件。

Seesaw model:



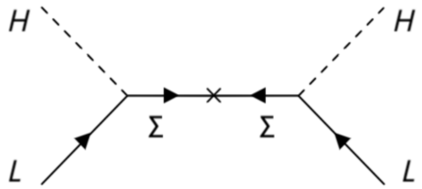
Type-I

H. Fritzsch et al., PLB 1975
P. Minkowski et al, PLB 1977
T. Yanagida et al, 1979
M. Gell-Mann et al., 1979
S.L. Glashow et al, 1980
R.N. Mohapatra et al., 1980



Type-II

W. Konetschny et al., PLB 1977
M. Magg et al., PLB 1980
J. Schechter et al., PRD 1980
T.P. Cheng et al., PRD 1980
G. Lazarides et al., NPB 1981
R.N. Mohapatra et al., PRD 1981



Type-III

R. Foot et al., ZPC 1989
E. Ma, PRL 1998

[1]P. A. Zyla et al, PTEP 2020, no.8, 083C01 (2020).
[2] Z. z. Xing, Phys. Rept. 854, 1-147 (2020).

I. Background and Motivation

- Type-I seesaw :

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i}\not{\partial} N_{\rm R} - \left[\frac{1}{2} \overline{N_{\rm R}^{c}} M_{\rm R} N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.} \right] , \quad \tilde{H} \equiv {\rm i}\sigma_2 H^*$$

$$\boxed{M_\nu \equiv V \widehat{M}_\nu V^{\rm T} \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^{\rm T}}$$

规范对称性破缺后，中微子的**质量项**为：

$$\mathcal{L}_\nu = -\frac{1}{2} \overline{\left(\nu_{\rm L} \ N_{\rm R}^{\rm C} \right)} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu_{\rm L}^{\rm C} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.} \, , \qquad \nu_{\rm L}^{\rm C} \equiv {\rm C} \overline{\nu_{\rm L}}^{\rm T} \quad N_{\rm R}^{\rm C} \equiv {\rm C} \overline{N_{\rm R}}^{\rm T}$$

新引入的重自由度会引起**幺正性破坏**：

Non-unitary ⟵ V

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm R} \end{pmatrix} ,$$

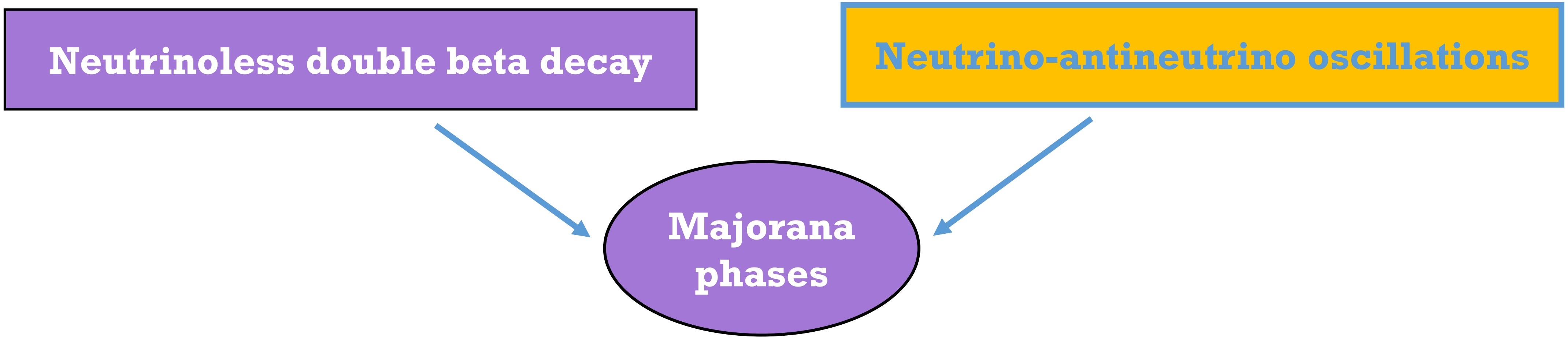
幺正性条件：

$$V V^\dagger + R R^\dagger = \mathbf{1}$$
$$V S^\dagger + R U^\dagger = \mathbf{0}.$$

- 由于轻重自由度的混合，V, R, S, U 本身都不再是幺正矩阵。

I. Background and Motivation

➢ 若中微子具有**Majorana**属性，则一定存在中微子-反中微子振荡[1]和无中微子双贝塔衰变$(0\nu\beta\beta)$[2]这种**轻子数破坏**的相互作用。



Questions:

- **非幺正**混合矩阵所引起的CP破坏与**幺正**情况下有什么不同?
- 根据最新对非幺正参数的实验限制，这种差别会达到多少?

[1] B. Pontecorvo, Sov. Phys. JETP 6, 429 (1957). [2] W. H. Furry, Phys.Rev. 56 (1939) 1184-1193.

II. Non-unitary flavor mixing

- **第一次**利用QR分解定理**严格地**给出了非幺正矩阵的厄米参数化[1]和下三角参数化[2]形式的关系。

Hermitian × unitary

$$V = \begin{pmatrix} 1-\eta_{ee} & -\eta_{e\mu} & -\eta_{e\tau} \\ -\eta^*_{e\mu} & 1-\eta_{\mu\mu} & -\eta_{\mu\tau} \\ -\eta^*_{e\tau} & -\eta^*_{\mu\tau} & 1-\eta_{\tau\tau} \end{pmatrix} \cdot V'$$

QR Factorization

lower-triangular × unitary

$$V = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \cdot \widetilde{V}$$

complex (α_{21}, α_{31}, α_{32})

$$\mathbf{1} - \eta \approx \begin{pmatrix} 1-\eta_{ee} & 0 & 0 \\ -2\eta^*_{e\mu} & 1-\eta_{\mu\mu} & 0 \\ -2\eta^*_{e\tau} & -2\eta^*_{\mu\tau} & 1-\eta_{\tau\tau} \end{pmatrix} \cdot \begin{pmatrix} 1 & -\eta_{e\mu} & -\eta_{e\tau} \\ +\eta^*_{e\mu} & 1 & -\eta_{\mu\tau} \\ +\eta^*_{e\tau} & +\eta^*_{\mu\tau} & 1 \end{pmatrix} .$$

Y. Wang, S. Zhou, Phys. Lett. B 824 (2022) 136797.

- 这种非幺正性的影响也会体现在**中微子-反中微子振荡**的过程中，带来**额外的**CP破坏来源。

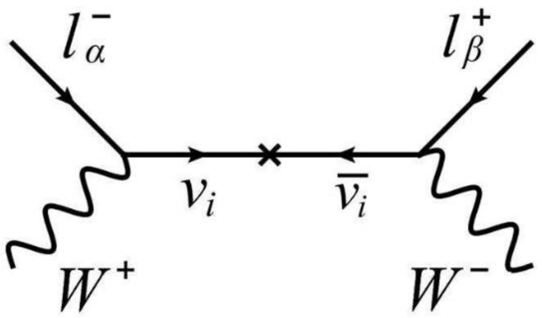
[1] S. Antusch, et al., JHEP 10, 084 (2006). [2] Z. z. Xing, Phys. Rev. D 85, 013008 (2012).

III. CP asymmetries

Flavor eigenstates ← $|\nu_\alpha\rangle = \frac{1}{\sqrt{(VV^\dagger)_{\alpha\alpha}}} \sum_i V^*_{\alpha i} |\nu_i\rangle$, → **Mass eigenstates**

➢ 质量基下，轻、重中微子的带电流相互作用：

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_L} V\gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu + \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_L} R\gamma^\mu \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_L W^-_\mu\right] + \text{h.c.}\,,$$



$$A(\nu_\alpha \to \overline{\nu}_\beta) = \frac{K}{\sqrt{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}} \sum_{i=1}^{3} V^*_{\alpha i} V^*_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i}\frac{m_i^2 L}{2E}\right)\,,$$

$$A(\overline{\nu}_\alpha \to \nu_\beta) = \frac{\overline{K}}{\sqrt{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}} \sum_{i=1}^{3} V_{\alpha i} V_{\beta i} \frac{m_i}{E} \exp\left(-\mathrm{i}\frac{m_i^2 L}{2E}\right)\,,$$

Z. z. Xing, PRD 87, no.5, 053019 (2013).
A. de Gouvea, et al., PRD 67, 053004 (2003).

Y. Wang, S. Zhou, Phys. Lett. B 824 (2022) 136797.

III. CP asymmetries

➤ 振荡概率：

$$\langle m\rangle_{\alpha\beta} \equiv V_{\alpha 1}V_{\beta 1}m_1 + V_{\alpha 2}V_{\beta 2}m_2 + V_{\alpha 3}V_{\beta 3}m_3 \ (\text{for } \alpha, \beta = e, \mu, \tau)$$

$$P\left(\nu_\alpha \to \overline{\nu}_\beta\right) = \frac{|K|^2}{\left(VV^\dagger\right)_{\alpha\alpha}\left(VV^\dagger\right)_{\beta\beta}}\left[\frac{\left|\langle m\rangle_{\alpha\beta}\right|^2}{E^2} - 4\sum_{i<j}\frac{m_i m_j}{E^2}\mathcal{C}^{ij}_{\alpha\beta}\sin^2\frac{F_{ji}}{2} + 2\sum_{i<j}^{3}\frac{m_i m_j}{E^2}\mathcal{V}^{ij}_{\alpha\beta}\sin F_{ji}\right]$$

➤ CP不对称度：

$$\mathcal{A}_{\alpha\beta} \equiv \frac{P\left(\nu_\alpha \to \overline{\nu}_\beta\right) - P\left(\overline{\nu}_\alpha \to \nu_\beta\right)}{P\left(\nu_\alpha \to \overline{\nu}_\beta\right) + P\left(\overline{\nu}_\alpha \to \nu_\beta\right)} = \frac{2\sum_{i<j} m_i m_j \mathcal{V}^{ij}_{\alpha\beta}\sin F_{ji}}{\left|\langle m\rangle_{\alpha\beta}\right|^2 - 4\sum_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta}\sin^2\frac{F_{ji}}{2}},$$

$$F_{ji} \equiv \Delta m^2_{ji}L/(2E), \quad \mathcal{C}^{ij}_{\alpha\beta} \equiv \text{Re}\left[V_{\alpha i}V_{\beta i}V^*_{\alpha j}V^*_{\beta j}\right], \quad \mathcal{V}^{ij}_{\alpha\beta} \equiv \text{Im}\left[V_{\alpha i}V_{\beta i}V^*_{\alpha j}V^*_{\beta j}\right].$$

III. CP asymmetries

- 虽然中微子-反中微子振荡过程的概率被中微子**质量**比**束流能量**的**平方**压低[1-5] $\sim 10^{-14}$，但其**CP不对称度**依赖于**Majorana相位**，对揭示中微子的Majorana属性有重要作用。

- 此外，长寿命重中微子的向前传播也可以导致**重中微子-反中微子振荡**，随着质量的提升，振荡概率也会变大[6]。若引入的两个重中微子质量近乎简并，CP不对称度也会被共振放大，从而可以在LHC上被观测[7]。

[1] J. N. Bahcall and H. Primako, Phys. Rev. D 18, 3463-3466 (1978).
[2] J. Schechter and J. W. F. Valle, Phys. Rev. D 23, 1666 (1981).
[3] J. D. Vergados, Phys. Rept. 133, 1 (1986).
[4] A. de Gouvea, B. Kayser and R. N. Mohapatra, Phys. Rev. D 67, 053004 (2003).
[5] Z. z. Xing, Phys. Rev. D 87, no.5, 053019 (2013).
[6] S. Antusch and J. Rosskopp, JHEP 03, 170 (2021) .
[7] S. Bray, J. S. Lee and A. Pilaftsis, Nucl. Phys. B 786, 95-118 (2007).

IV. Numerical Results

- 以Minimal seesaw为例，在取引入重中微子进行全局拟合的参数输入[1]以及最大振荡相位下，**非幺正性**的影响在CP相位 $\delta = 195°$，$\sigma = 0°$ 的 $\boldsymbol{\mu}$−$\boldsymbol{\tau}$ 味的振荡过程中会变得非常显著。

Normal Ordering	$\delta = 195°,\ \sigma = 0°$		$\delta = 195°,\ \sigma = 45°$	
	$\varepsilon^{\rm U}_{\alpha\beta}$	$\varepsilon^{\rm L}_{\alpha\beta}$	$\varepsilon^{\rm U}_{\alpha\beta}$	$\varepsilon^{\rm L}_{\alpha\beta}$
$\alpha,\ \beta = e,\ e$	0%	0%	0%	0%
$\alpha,\ \beta = e,\ \mu$	−0.008974%	+0.008974%	−0.001717%	+0.001717%
$\alpha,\ \beta = e,\ \tau$	+1.946%	−1.948%	+0.2681%	−0.2698%
$\alpha,\ \beta = \mu,\ \mu$	+0.09932%	−0.09932%	+0.005116%	−0.005116%
$\alpha,\ \beta = \mu,\ \tau$	−206.8%	+206.8%	−0.4555%	+0.4564%
$\alpha,\ \beta = \tau,\ \tau$	−19.39%	+19.40%	+0.9050%	−0.9000%

[1] E. Fernandez-Martinez, et al., JHEP 08, 033 (2016).

$$\varepsilon_{\alpha\beta} \equiv \frac{\mathcal{A}_{\alpha\beta} - \widetilde{\mathcal{A}}_{\alpha\beta}}{\widetilde{\mathcal{A}}_{\alpha\beta}} \times 100\%\,,$$

Unitary CPA

$$\mathcal{V}^{23}_{\mu\tau} = c^2_{13} c_{23} s_{23} \left[-c^2_{12} c_{23} s_{23} \sin 2\sigma - c_{12} s_{12} s_{13} \left(c^2_{23} - s^2_{23} \right) \sin\left(\delta + 2\sigma\right) + s^2_{12} s^2_{13} c_{23} s_{23} \sin 2\left(\delta + \sigma\right) \right]$$

IV. Numerical Results

- 在关掉所有Dirac-和Majorana-CP相位的情况下，**非幺正性的相位**依然会提供较大的额外CP破坏。

$$\mathcal{V}^{12}_{ee} \approx \mathcal{V}^{13}_{ee} \approx \mathcal{V}^{23}_{ee} \approx \mathcal{V}^{13}_{e\mu} \approx \mathcal{V}^{23}_{e\mu} \approx 0 \, .$$

$$\mathcal{V}^{12}_{\mu\mu} \approx -2\,|\alpha_{21}|\,\alpha^3_{22} s_{12} c_{12} c^3_{23} \sin\phi_{21} \, ,$$

$$\mathcal{V}^{13}_{\mu\mu} \approx -\mathcal{V}^{23}_{\mu\mu} \approx -2\,|\alpha_{21}|\,\alpha^3_{22} s_{12} c_{12} s^2_{23} c_{23} \sin\phi_{21}$$

$$\mathcal{V}^{12}_{\tau\tau} \approx +2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s^3_{23} \sin\phi_{31} \, ,$$

$$\mathcal{V}^{13}_{\tau\tau} \approx +2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} - 2\,|\alpha_{32}|\,\alpha^3_{33} s^2_{12} s_{23} c_{23} \sin\phi_{32} \, ,$$

$$\mathcal{V}^{23}_{\tau\tau} \approx -2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} - 2\,|\alpha_{32}|\,\alpha^3_{33} c^2_{12} s_{23} c_{23} \sin\phi_{32} \, ,$$

$$\mathcal{V}^{12}_{e\mu} \approx +\alpha^2_{11}\,|\alpha_{21}|\,\alpha_{22} s_{12} c_{12} c_{23} \sin\phi_{21} \, ,$$

$$\mathcal{V}^{12}_{e\tau} \approx -\alpha^2_{11}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} \sin\phi_{31} \, ,$$

$$\mathcal{V}^{13}_{e\tau} \approx +\alpha^2_{11} \alpha_{33} c_{12} s_{13} \left(|\alpha_{31}|\, c_{12} c_{23} \sin\phi_{31} - |\alpha_{32}|\, s_{12} \sin\phi_{32}\right)$$

$$\mathcal{V}^{23}_{e\tau} \approx +\alpha^2_{11} \alpha_{33} s_{12} s_{13} \left(|\alpha_{31}|\, s_{12} c_{23} \sin\phi_{31} + |\alpha_{32}|\, c_{12} \sin\phi_{32}\right)$$

$$\mathcal{V}^{12}_{\mu\tau} \approx +\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} \, ,$$

$$\mathcal{V}^{13}_{\mu\tau} \approx -\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} + \alpha^2_{22}\,|\alpha_{32}|\,\alpha_{33} s^2_{12} s_{23} c_{23} \sin\phi_{32} \, ,$$

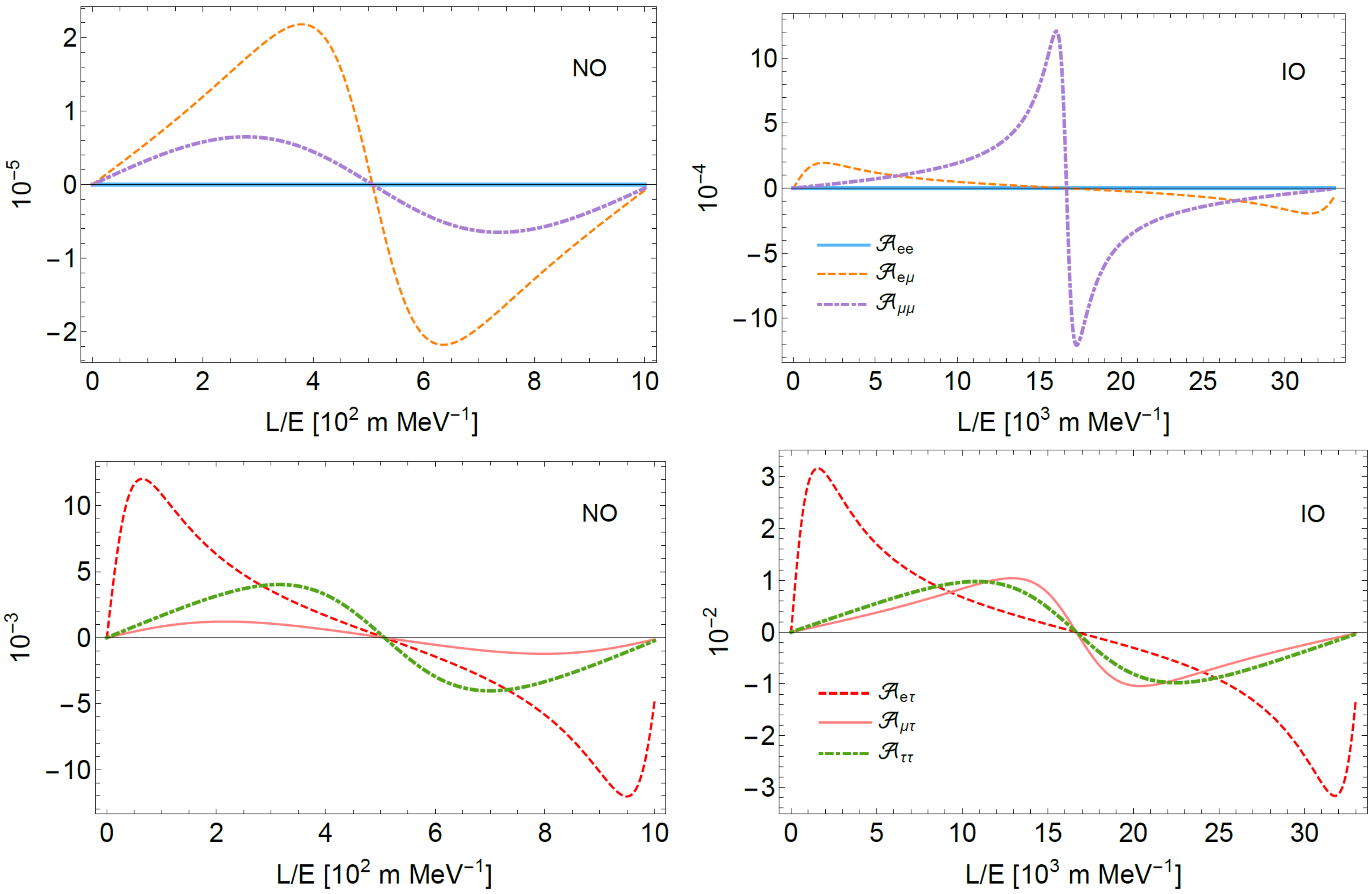
$$\mathcal{V}^{23}_{\mu\tau} \approx +\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} + \alpha^2_{22}\,|\alpha_{32}|\,\alpha_{33} c^2_{12} s_{23} c_{23} \sin\phi_{32} \, .$$

Non-unitary phases

$\delta = \sigma = 0°$

IV. Numerical Results

$\delta = \sigma = 0°$



IV. Summary

- 非幺正的轻子味混合是 type-I seesaw 模型的**自然预言**。
- 探测中微子-反中微子振荡过程是**证实**和**理解**中微子 Majorana 属性，**确定** Majorana 相位的重要过程。
- 利用QR分解定理，第一次**严格地**给出了非幺正矩阵的厄米参数化和下三角参数化形式的关系。
- 计算了在非幺正轻子味混合下，中微子-反中微子振荡过程中的CP破坏。在特定情况下，非幺正的影响会非常显著。
- 即使没有Dirac和Majorana相位的贡献，非幺正的相位依然会带来**额外的**CP破坏。

Thanks for your attention!