

# Deciphering the Long-distance Penguin contribution to $B \rightarrow \gamma\gamma$ decays

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QQ, Y.-L. Shen, C. Wang, Y.-M. Wang, 2207.02691



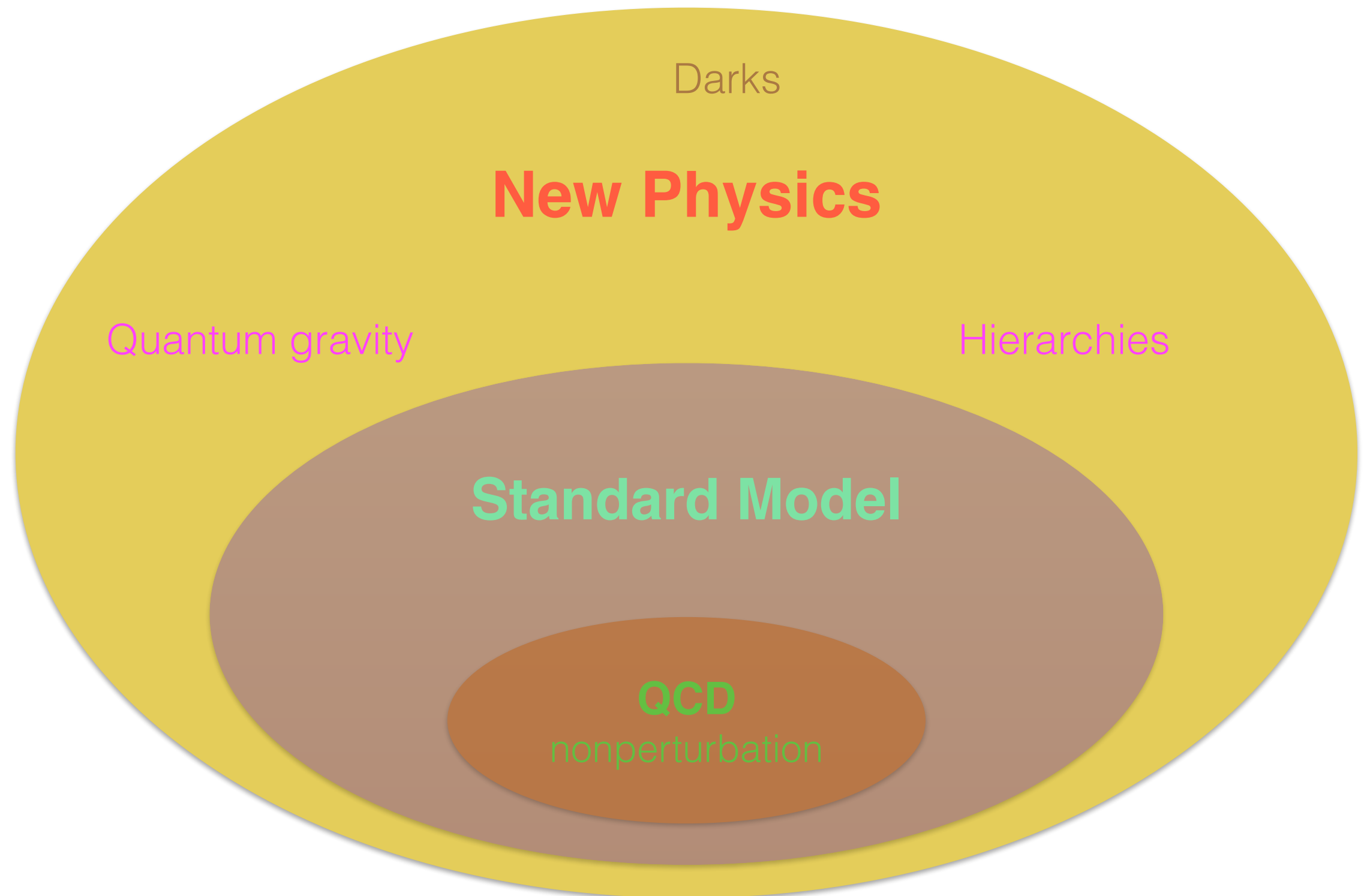
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- Why  $\bar{B} \rightarrow \gamma\gamma$ ?
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  - — a novel B-meson distribution amplitude
- Numerics
- Summary and Prospects

# B decays are important!



# Why $\bar{B} \rightarrow \gamma\gamma$ ?

- Sensitive to dynamics beyond the SM (FCNC), e.g. CP violation
- Extraction of the CKM angle  $\gamma$
- Clean environment to address the intricate strong interaction mechanism of the heavy-meson systems

— — structure of the B meson

## Belle II

## Physics Book

Process	Observable	Theory	Sys. limit (Discovery) [ab <sup>-1</sup> ]	vs LHCb	vs Belle	Anomaly	NP
● $B \rightarrow X_s l^+ l^-$	$R_{X_s}$	***	>50	***	***	**	***
● $B \rightarrow K^{(*)} e^+ e^-$	$R(K^{(*)})$	***	>50	**	***	***	***
● $B \rightarrow X_s \gamma$	$Br.$	**	1-5	***	*	*	**
● $B_{d,(s)} \rightarrow \gamma\gamma$	$Br., A_{CP}$	**	>50(5)	**	**	-	**
● $B \rightarrow K^* e^+ e^-$	$P'_5$	**	>50	***	**	***	***
● $B \rightarrow K \tau l$	$Br.$	***	>50	**	***	**	***

# Why $\bar{B} \rightarrow \gamma\gamma$ ?

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- Clean environment to address the intricate strong interaction mechanism of the heavy-meson systems

— — structure of the B meson

## Belle II Physics Book

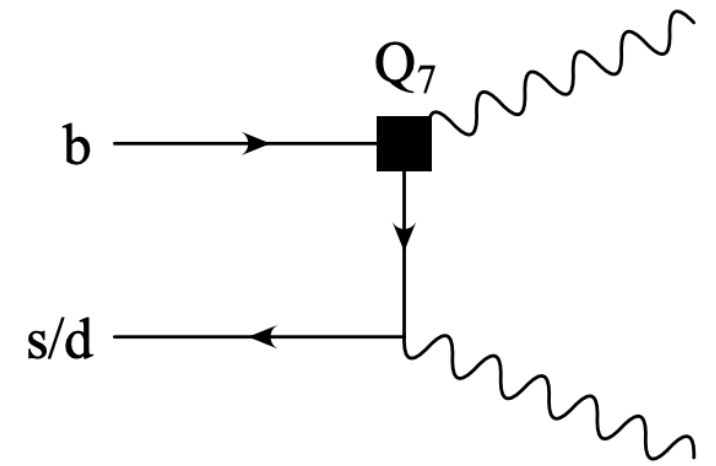
Observables	Belle 0.71 ab <sup>-1</sup> (0.12 ab <sup>-1</sup> )	Belle II 5 ab <sup>-1</sup>	Belle II 50 ab <sup>-1</sup>
Br( $B_d \rightarrow \gamma\gamma$ )	< 740%	30%	9.6%
$A_{CP}(B_d \rightarrow \gamma\gamma)$	—	78%	25%
Br( $B_s \rightarrow \gamma\gamma$ )	< 250%	23%	—

$$\mathcal{BR}(B_d \rightarrow \gamma\gamma) = (1.352_{-0.745}^{+1.242}) \times 10^{-8}, \quad \mathcal{BR}(B_s \rightarrow \gamma\gamma) = (2.964_{-1.614}^{+1.800}) \times 10^{-7}$$

[Y.-L. Shen, Y.-M. Wang, Y.-B. Wei, 2009.02723]

# History of $\bar{B} \rightarrow \gamma\gamma$

- LO + NLO



$$\bar{A}(\bar{B}_q \rightarrow \gamma\gamma) = -\frac{4 G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon^{*\alpha}(p) \epsilon^{*\beta}(q) \times \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i T_{i,\alpha\beta}^{(p)},$$

Leading power

$$T_{i,\alpha\beta}^{(p)} = i m_{B_q}^3 \left[ \left( g_{\alpha\beta}^\perp - i \epsilon_{\alpha\beta}^\perp \right) F_{i,L}^{(p)} - \left( g_{\alpha\beta}^\perp + i \epsilon_{\alpha\beta}^\perp \right) F_{i,R}^{(p)} \right],$$

Two polarizations

$F_L^{\text{LP}} \propto m_b \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu) \propto \frac{m_b}{\lambda_b}$

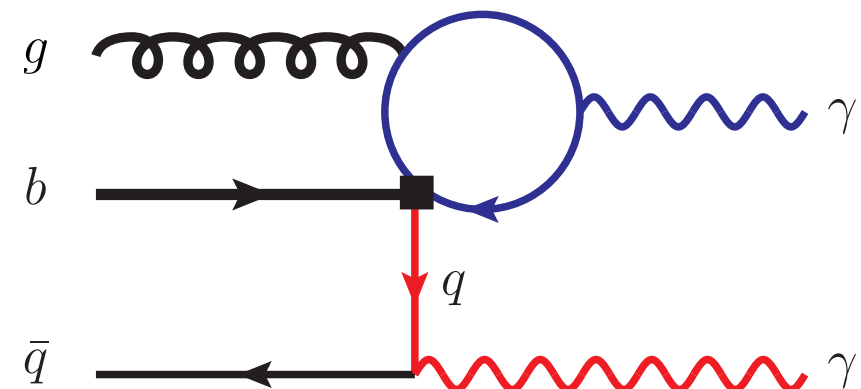
[Bosch, Buchalla, hep-ph/0208202; Descotes-Genon, Sacharajda, hep-ph/0212162]

- NLL corrections + Systematic power corrections

both  $\sim \mathcal{O}(10\%)$

[Y.-L. Shen, Y.-M. Wang, Y.-B. Wei, 2009.02723]

- One important but tough piece missing — — **long-distance penguin contribution**



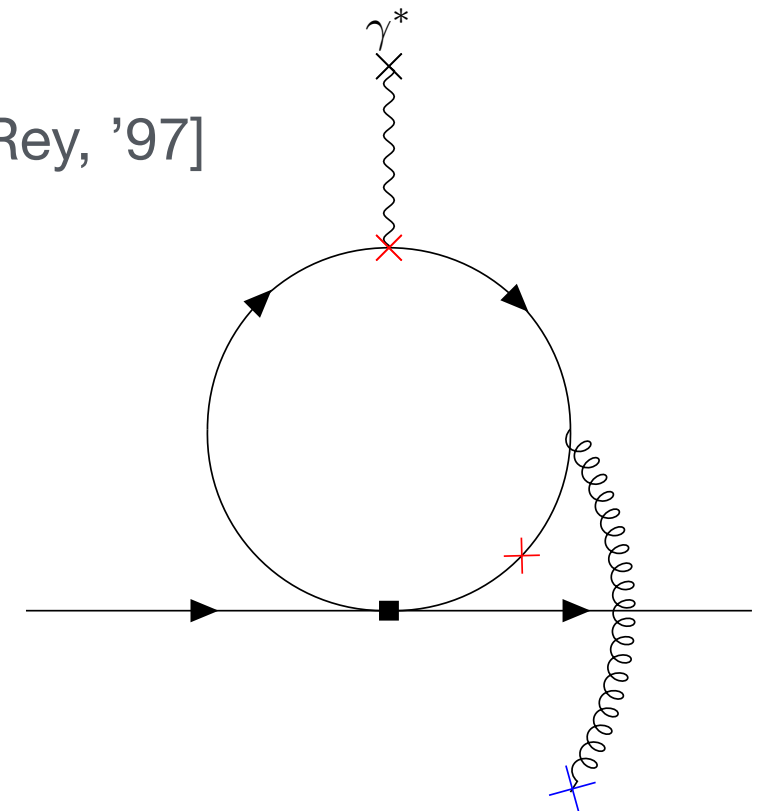
# History of Long-distance penguin contribution

In inclusive  $b \rightarrow s$  decays

- Realized in  $\bar{B} \rightarrow X_s \gamma$   
[Voloshin, '96; Ligeti, Randall, Wise, '97; Buchalla, Isidori, Rey, '97]
- Factorization in  $\bar{B} \rightarrow X_s \gamma$   
[Benzke, Lee, Neubert, Paz, 1003.5012]
- Factorization in  $\bar{B} \rightarrow X_s \ell \ell$   
[Benzke, Hurth, Turczyk, 1705.10366]

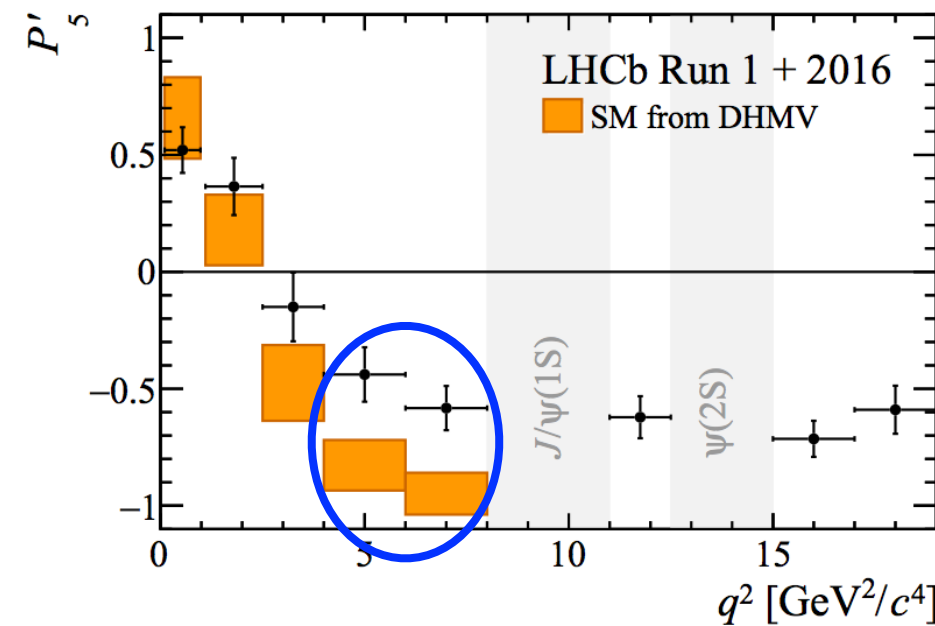
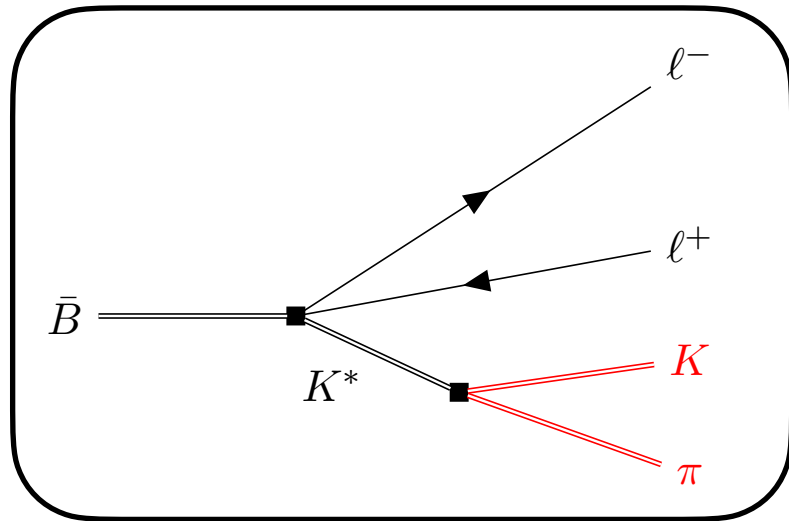
In exclusive  $b \rightarrow s$  decays

- Initiated in  $B \rightarrow K^* \gamma$   
[Khodjamirian, Ruckl, Stoll, Wyler, '97]
- Developed in  $B \rightarrow K^* \ell \ell$   
[Khodjamirian, Mannel, Pivovarov, Wang, 1006.4945]



Soft gluon from charm-loop

# History of Long-distance penguin contribution



$P'_5$ : an angular-distribution observable

[LHCb, 2003.04831]

## Charm-loop effect in $B \rightarrow K^{(*)} \ell^+ \ell^-$ and $B \rightarrow K^* \gamma$

A. Khodjamirian (Siegen U.), Th. Mannel (Siegen U.), A.A. Pivovarov (Siegen U.), Y.-M. Wang (Siegen U.)

Jun, 2010

35 pages

Published in: *JHEP* 09 (2010) 089

e-Print: [1006.4945](https://arxiv.org/abs/1006.4945) [hep-ph]

pdf

cite

414 citations



# Factorization

# Factorization

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pq}^* \left[ C_1(\nu) P_1^p(\nu) + C_2(\nu) P_2^p(\nu) + \sum_{i=3}^8 C_i(\nu) P_i(\nu) \right. \\ \left. + \sum_{i=3}^6 C_i(\nu) P_i^Q(\nu) \right] + \text{h.c.},$$

$$P_1^p = (\bar{q}_L \gamma_\mu T^a p_L) (\bar{p}_L \gamma^\mu T^a b_L),$$

$$P_2^p = (\bar{q}_L \gamma_\mu p_L) (\bar{p}_L \gamma^\mu b_L),$$

$$P_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q'),$$

$$P_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q'),$$

$$P_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q'),$$

$$P_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q'),$$

# Factorization

Integrate out the hard and hard-collinear d.o.f.

$$M = H * J * S \quad (m_b \gg m_c \sim \mathcal{O}(\sqrt{\Lambda m_b}) \gg \Lambda)$$

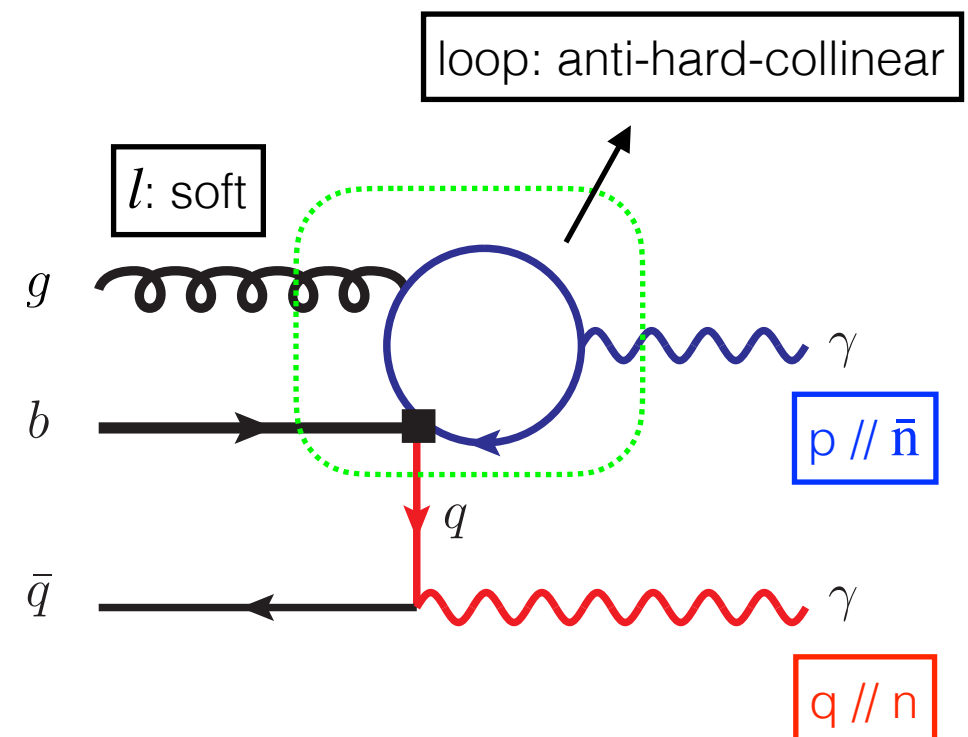
First-step match:

$$M \ni \left( C_2 - \frac{C_1}{2N_c} \right) Q_p \left[ F\left( \frac{m_p^2 - i0^+}{(p-l)^2} \right) - 1 \right] \frac{p^\alpha}{(p-l)^2} \left[ \bar{q}(\tilde{q}) \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b(v) \right]$$

$$F(x) = 4x \arctan^2 \left( \frac{1}{\sqrt{4x-1}} \right)$$

$$(p-l)^2 = -2p \cdot l = -m_b \bar{n} \cdot l$$

➡ Non-local operator!



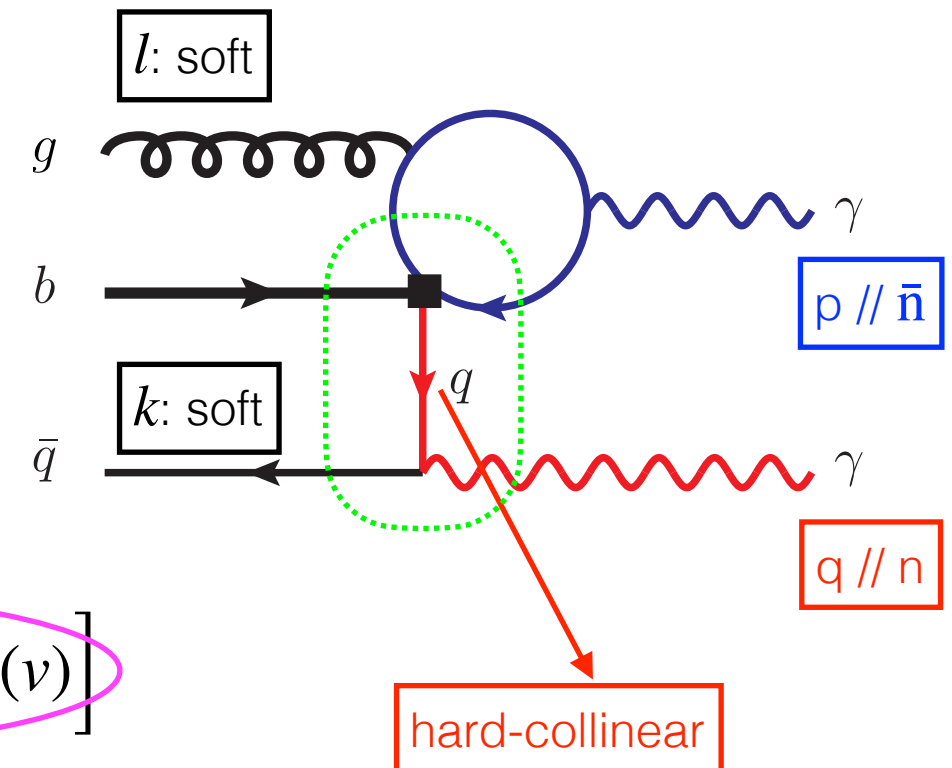
# Factorization

Second-step match:

$$\langle \gamma(p) \gamma(q) | \bar{q} \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b | g(l) b(v) \bar{q}(k) \rangle$$

$$\Rightarrow \frac{i g_{\text{em}} e_q}{(q-k)^2} \epsilon^{\mu\beta\lambda\tau} p_\lambda \epsilon_\tau^*(p) \epsilon_\rho^*(q) \times \left[ \bar{q}(k) \gamma_\perp^\rho \not{q} \gamma_\beta P_L G_{\mu\alpha}(\ell) b(v) \right]$$

$$(q-k)^2 = -2q \cdot k = -m_b \mathbf{n} \cdot \mathbf{k}$$



- The hard-kernel (jet functions) depends on 2 different light-cone components of the gluon and light quark momenta.
- It becomes evident to introduce the 3-particle B-meson distribution amplitude with 2 light-cone directions.

$$H \star J \star \bar{J} \star \Phi_G$$

# Factorization

The explicit factorization formula:

$$\sum_{i=1}^8 C_i F_{i,L}^{(p), \text{soft } 4q} = -\frac{Q_q f_{B_q}}{m_{B_q}} \int_0^\infty \left( \frac{d\omega_1}{\omega_1} \right) \int_0^\infty \left( \frac{d\omega_2}{\omega_2} \right) \left( C_2 - \frac{C_1}{2N_c} \right) Q_p \left[ F\left(-\frac{m_p^2}{m_b \omega_2}\right) - 1 \right] \times \Phi_G(\omega_1, \omega_2, \mu)$$

The light quark momentum component  $\omega_1 = n \cdot k$  ;

The soft gluon momentum component  $\omega_2 = \bar{n} \cdot l$  .

The novel B-meson DA:

$$\begin{aligned} & \langle 0 | \bar{q}_s(\tau_1 n) (g_s G_{\mu\nu})(\tau_2 \bar{n}) \bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_v(0) | \bar{B}_v \rangle \\ & = 2 \tilde{f}_B(\mu) m_B \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \exp \left[ -i(\omega_1 \tau_1 + \omega_2 \tau_2) \right] \Phi_G(\omega_1, \omega_2, \mu) \end{aligned}$$

- The quark and gluon fields are localized on 2 distinct light-cone directions.

It might opens an exciting new research subfield aiming at the multidimensional tomography of the composite bottom-meson systems.

# Factorization

The normalization conditions of  $\Phi_G$ :

Matching the conventional 3-particle B meson DAs as  $\tau_1$  or  $\tau_2 \rightarrow 0$ .

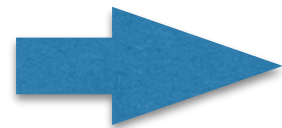
$$\langle 0 | \bar{q}(z_1)(g_s G_{\mu\nu})(z_2) \bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_\nu(0) | \bar{B}_\nu \rangle = 2 \tilde{f}_B(\mu) \Phi_4(z_1, z_2, \mu)$$

Twist 4

$$\langle 0 | \bar{q}(z_1)(g_s G_{\mu\nu})(z_2) n^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_\nu(0) | \bar{B}_\nu \rangle = 2 \tilde{f}_B(\mu) \Phi_5(z_1, z_2, \mu)$$

Twist 5

[Braun, Ji, Manashov, 1703.02446]



$$\begin{aligned} \int_0^\infty d\omega_1 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_1 \Phi_4(\omega_1, \omega_2, \mu), \\ \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_2 \Phi_5(\omega_1, \omega_2, \mu), \\ \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \frac{\lambda_E^2 + \lambda_H^2}{3}, \end{aligned}$$

The asymptotic behaviors of  $\Phi_G$ :

$$\Phi_G(\omega_1, \omega_2, \mu) \sim \omega_1 \omega_2^2 \text{ at } \omega_1, \omega_2 \rightarrow 0$$

# Factorization

The explicit factorization formula:

$$\sum_{i=1}^8 C_i F_{i,L}^{(p), \text{soft } 4q} = -\frac{Q_q f_{B_q}}{m_{B_q}} \int_0^\infty \left( \frac{d\omega_1}{\omega_1} \right) \int_0^\infty \left( \frac{d\omega_2}{\omega_2} \right) \left( C_2 - \frac{C_1}{2N_c} \right) Q_p \left[ F\left(-\frac{m_p^2}{m_b \omega_2}\right) - 1 \right] \times \Phi_G(\omega_1, \omega_2, \mu)$$

$$\Phi_G(\omega_1, \omega_2, \mu) \sim \omega_1 \omega_2^2 \text{ at } \omega_1, \omega_2 \rightarrow 0$$

➡ The convolution integral converges.

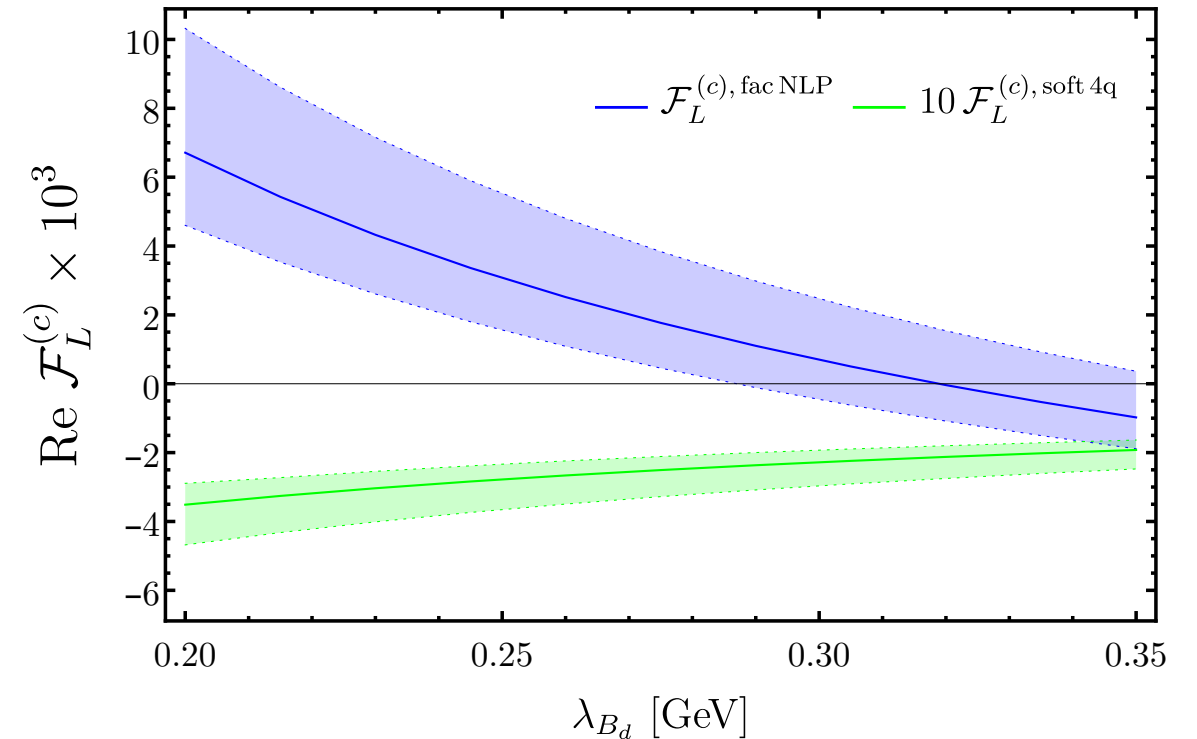
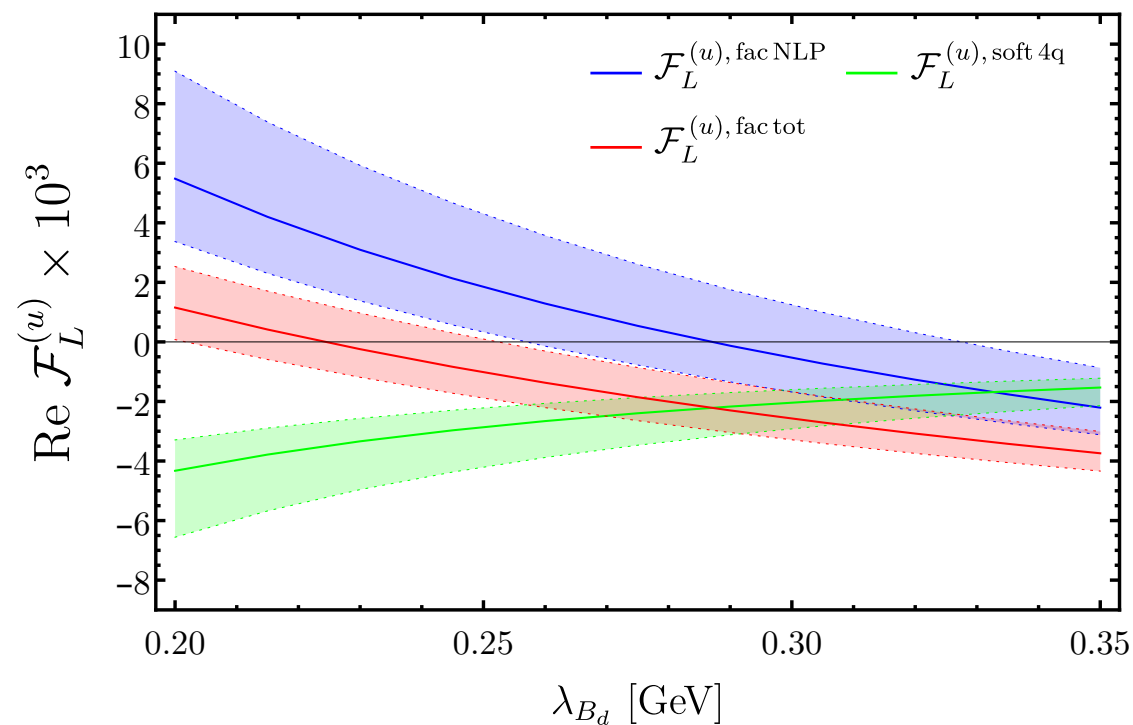
$$\begin{aligned} \int_0^\infty d\omega_1 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_1 \Phi_4(\omega_1, \omega_2, \mu), \\ \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \int_0^\infty d\omega_2 \Phi_5(\omega_1, \omega_2, \mu), \\ \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) &= \frac{\lambda_E^2 + \lambda_H^2}{3}, \end{aligned}$$

➡ The power counting:  $F_L^{\text{soft}, 4q} / F_L^{\text{LP}} \sim \lambda_B / m_b$

# Numerics

The  $\Phi_G$  parametrization:

$$\Phi_G(\omega_1, \omega_2, \mu_0) = \frac{\lambda_E^2 + \lambda_H^2}{6} \frac{\omega_1 \omega_2^2}{\omega_0^5} \exp\left(-\frac{\omega_1 + \omega_2}{\omega_0}\right) \frac{\Gamma(\beta + 2)}{\Gamma(\alpha + 2)} U\left(\beta - \alpha, 4 - \alpha, \frac{\omega_1 + \omega_2}{\omega_0}\right)$$



- The up-loop contribution dominates; the charm-loop is 1-order smaller.
- The new power correction accidentally cancels the previous ones.



Clean channel to determine  $\lambda_B$  and to probe new physics.



# Numerics

The  $B_d$  results:

	Central Value	Total Error	$\lambda_{B_d}$	$\{\hat{\sigma}_{B_d}^{(1)}, \hat{\sigma}_{B_d}^{(2)}\}$	$\mu$	$\nu$	$\mu_h$	$\bar{\Lambda}$	$m_c^{\text{PS}}$
$10^8 \times \mathcal{BR}$	1.929 [1.900]	+1.096 -1.012	+0.680 -0.439	+0.736 -0.779	+0.083 -0.299	+0.278 -0.287	+0.246 -0.066	+0.212 -0.200	+0.043 -0.043
$f_{\parallel}$	0.408 [0.407]	+0.044 -0.046	+0.015 -0.015	+0.016 -0.033	+0.002 -0.009	+0.037 -0.026	+0.007 -0.002	+0.005 -0.006	+0.002 -0.002
$f_{\perp}$	0.592 [0.593]	+0.046 -0.044	+0.015 -0.015	+0.033 -0.016	+0.009 -0.002	+0.026 -0.037	+0.002 -0.007	+0.006 -0.005	+0.002 -0.002
$\mathcal{A}_{\text{CP}}^{\text{dir}, \parallel}$	0.126 [0.129]	+0.043 -0.027	+0.007 -0.004	+0.017 -0.010	+0.013 -0.008	+0.027 -0.018	+0.024 -0.012	+0.007 -0.007	+0.004 -0.004
$\mathcal{A}_{\text{CP}}^{\text{mix}, \parallel}$	-0.197 [-0.154]	+0.053 -0.084	+0.019 -0.036	+0.001 -0.002	+0.021 -0.047	+0.026 -0.040	+0.015 -0.029	+0.011 -0.013	+0.008 -0.009
$\mathcal{A}_{\Delta\Gamma}^{\parallel}$	-0.972 [-0.980]	+0.024 -0.013	+0.009 -0.004	+0.003 -0.002	+0.013 -0.005	+0.013 -0.007	+0.010 -0.004	+0.004 -0.003	+0.002 -0.002
$\mathcal{A}_{\text{CP}}^{\text{dir}, \perp}$	0.330 [0.326]	+0.078 -0.053	+0.015 -0.012	+0.060 -0.035	+0.035 -0.014	+0.012 -0.024	+0.014 -0.010	+0.018 -0.016	+0.018 -0.017
$\mathcal{A}_{\text{CP}}^{\text{mix}, \perp}$	0.136 [0.101]	+0.087 -0.066	+0.043 -0.028	+0.015 -0.035	+0.025 -0.014	+0.060 -0.038	+0.026 -0.012	+0.003 -0.003	+0.009 -0.008
$\mathcal{A}_{\Delta\Gamma}^{\perp}$	0.934 [0.940]	+0.017 -0.030	+0.000 -0.003	+0.009 -0.019	+0.007 -0.017	+0.001 -0.002	+0.005 -0.009	+0.006 -0.007	+0.007 -0.008

# Summary and prospects

- ◉ We have factorized the long-distance penguin contribution to  $\bar{B} \rightarrow \gamma\gamma$  decay, for the first time in an exclusive decay.
- ◉ A novel B-meson DA is defined, with quark and gluon fields localized on two different light-cone directions. It will open a new subfield about the inner structure of the B meson.
- ◉ The new contribution cancels the known factorizable power corrections, making  $\bar{B} \rightarrow \gamma\gamma$  a clean channel to determine  $\lambda_B$  and to probe the non-standard four-fermion interactions.
- ◉ The developed formalism has a broad field of applications to the entire spectrum of the exclusive FCNC B-meson decays, including flagship modes, e.g.  $B \rightarrow K^*\gamma$ ,  $B \rightarrow K^*\mu\mu$ .

**Thank you!**

**Backup**

	$B_d$	$B_s$
$\mathcal{A}^{\text{LP,NLL}} [10^{-4}]$	$3.4 + 1.9 i$	$-20 - 0.37 i$
$\mathcal{A}^{\text{fac,NLP}} [10^{-4}]$	$-0.15 - 0.53 i$	$0.92 + 2.6 i$
$\mathcal{A}_R^{\text{fac,NLP}} [10^{-4}]$	$0.25 - 0.36 i$	$-1.6 + 2.6 i$
$\mathcal{A}^{\text{had},\gamma} [10^{-4}]$	$-0.30 - 0.17 i$	$1.4 - 0.0021 i$
$\mathcal{A}^{\text{soft}, 4\text{q}} [10^{-4}]$	$(-0.0079 + 0.078 i)$	$-0.11 + 0.016 i$
$(F_u^{\text{LP,NLL}}, F_c^{\text{LP,NLL}})$	$(-0.056 - 0.0092i, -0.048 - 0.0019i)$	$(-0.057 - 0.0094i, -0.049 - 0.0020i)$
$(F_u^{\text{had},\gamma}, F_c^{\text{had},\gamma})$	$(0.0051 + 0.00092i, 0.0043 + 0.00019i)$	$(0.0094 + 0.0016i, 0.0034 + 0.00016i)$
$(F_u^{\text{soft},4\text{q}}, F_c^{\text{soft},4\text{q}})$	$(-0.0024, -0.00025)$	$(-0.0021, -0.00025)$
$(F_u^{\text{HC}}, F_c^{\text{HC}})$	$(0.0055, 0.0055)$	$(0.0067, 0.0067)$
$(F_u^{\text{mq}}, F_c^{\text{mq}})$	$(0.000049, 0.000049)$	$(0.00078, 0.00078) [0.00079]$
$(F_u^{\text{A}_2}, F_c^{\text{A}_2})$	$(-0.0010, -0.0010)$	$(-0.0011, -0.0011)$
$(F_u^{\text{HT}}, F_c^{\text{HT}})$	$(0.0046, 0.0046) [0.0047]$	$(0.0048, 0.0048) [0.0050]$
$(F_u^{\text{Q}_b}, F_c^{\text{Q}_b})$	$(-0.0036, -0.0036)$	$(-0.0043, -0.0043)$
$(F_u^{\text{WA}}, F_c^{\text{WA}})$	$(-0.0049 + 0.000092i, -0.0037 + 0.0056i)$	$(-0.0059 + 0.00011i, -0.0045 + 0.0065i)$
$(F_u^{\text{fac,NLP}}, F_c^{\text{fac,NLP}})$	$(0.00054 + 0.000092i, 0.0018 + 0.0056i)$	$(0.00098 + 0.00011i, 0.0023 + 0.0065i)$
	$[(0.00063 + 0.000092i, 0.0019 + 0.0056i)]$	$(0.0011 + 0.00011i, 0.0024 + 0.0065i)$
$(F_{R,u}^{\text{fac,NLP}}, F_{R,c}^{\text{fac,NLP}})$	$(-0.0046 + 0.000092i, -0.0033 + 0.0056i)$	$(-0.0054 + 0.00011i, -0.0041 + 0.0065i)$

$$A = V_{uq}^* V_{ub} F_u + V_{cq}^* V_{cb} F_c \quad (q = d, s)$$