Extending Precision Perturbative QCD with Track Functions

Yibei Li (李依蓓)
Zhejiang University

in collaboration with

Hao Chen (陈豪), Max Jaarsma, Ian Moult, Wouter Waalewijn and Hua Xing Zhu (朱华星)

中国物理学会高能物理分会学术年会

2022.08

Motivation

Track-based measurements offer:

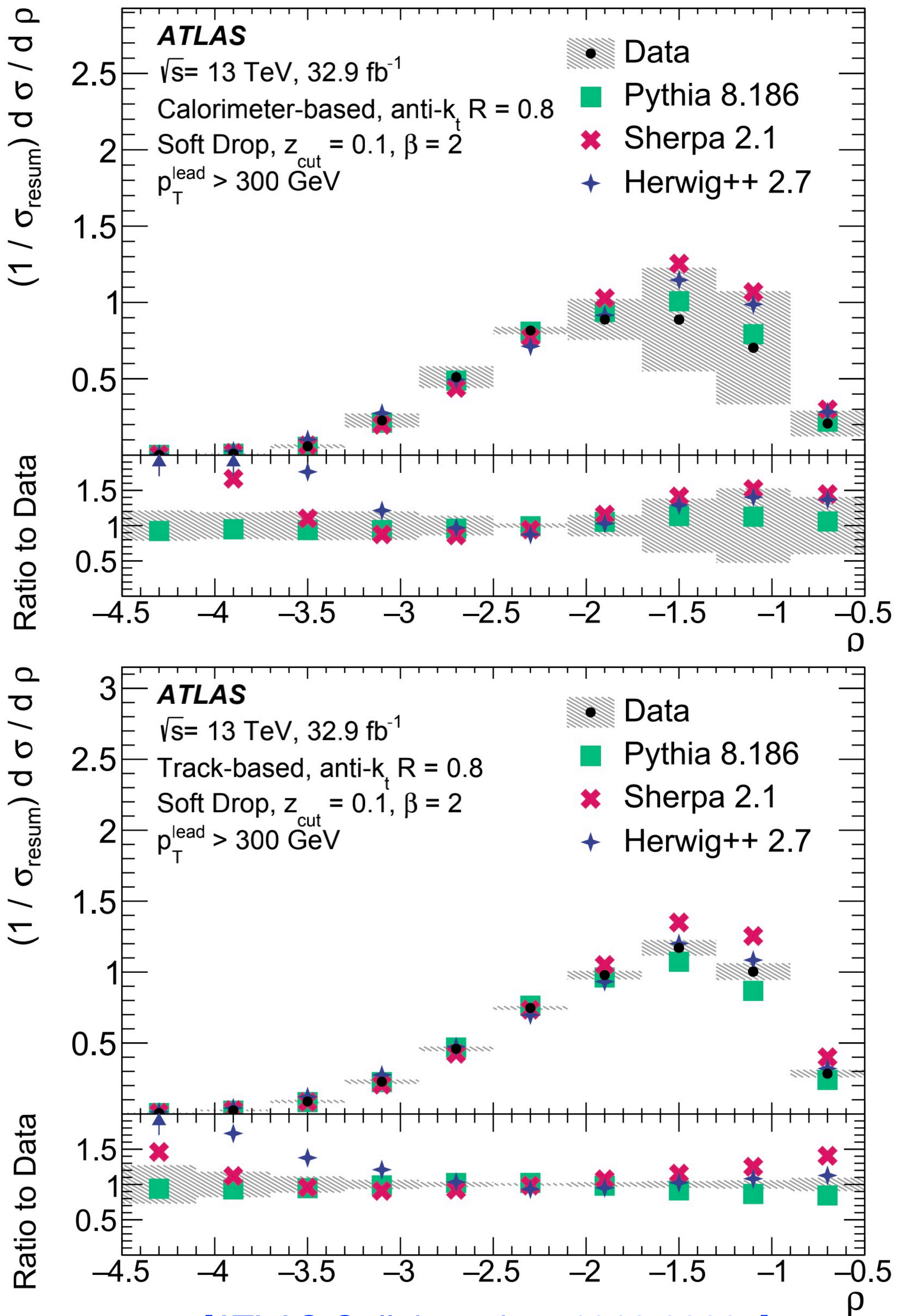
- Superior angular resolution
- Pileup mitigation
- One problem: Track-based calculations are not IR safe in perturbation theory.

Track Functions

- ▸ IR divergences are absorbed into **universal** non-perturbative functions.

calorimeter-based (all-particle)

track-based (charged-particle)



[ATLAS Collaboration, 1912.09837]

✓ Track functions introduced and studied at $\mathcal{O}(\alpha_s)$.

[H. Chang, M. Procura, J. Thaler, W. Waalewijn, 1303.6637, 1306.6630]

- Complicated:

observables. For all of these observables, the uncertainties for the track-based observables are significantly smaller than those for the calorimeter-based observables, particularly for higher values of β, where more soft radiation is included within the jet. However, since no track-based calculations exist at the present time, calorimeter-based measurements are still useful for precision QCD studies.

[ATLAS Collaboration, 1912.09837]

the selection of charged particle jets. Note that track-based observables are IRC-unsafe. In general, nonperturbative track functions can be used to directly compare track-based measurements to analytical calculations [67–69]; however, such an approach has not yet been developed for jet angularities. Two techniques are used, described in the following subsections, to apply the nonperturbative corrections.

[ALICE Collaboration, 2107.11303]

0.4 and 0.8. For each quantity, we define a variant where the observables is calculated using only the charged constituents in anti-k_T algorithm ("charged"). While observables computed with both charged and neutral constituents can be described more easily from first-principle calculations, the charged variants can be measured with a better resolution as a result of the high efficiency and precision of the tracking detector.

[CMS Collaboration, 2109.03340]

✓ This talk: Track function formalism beyond leading order.

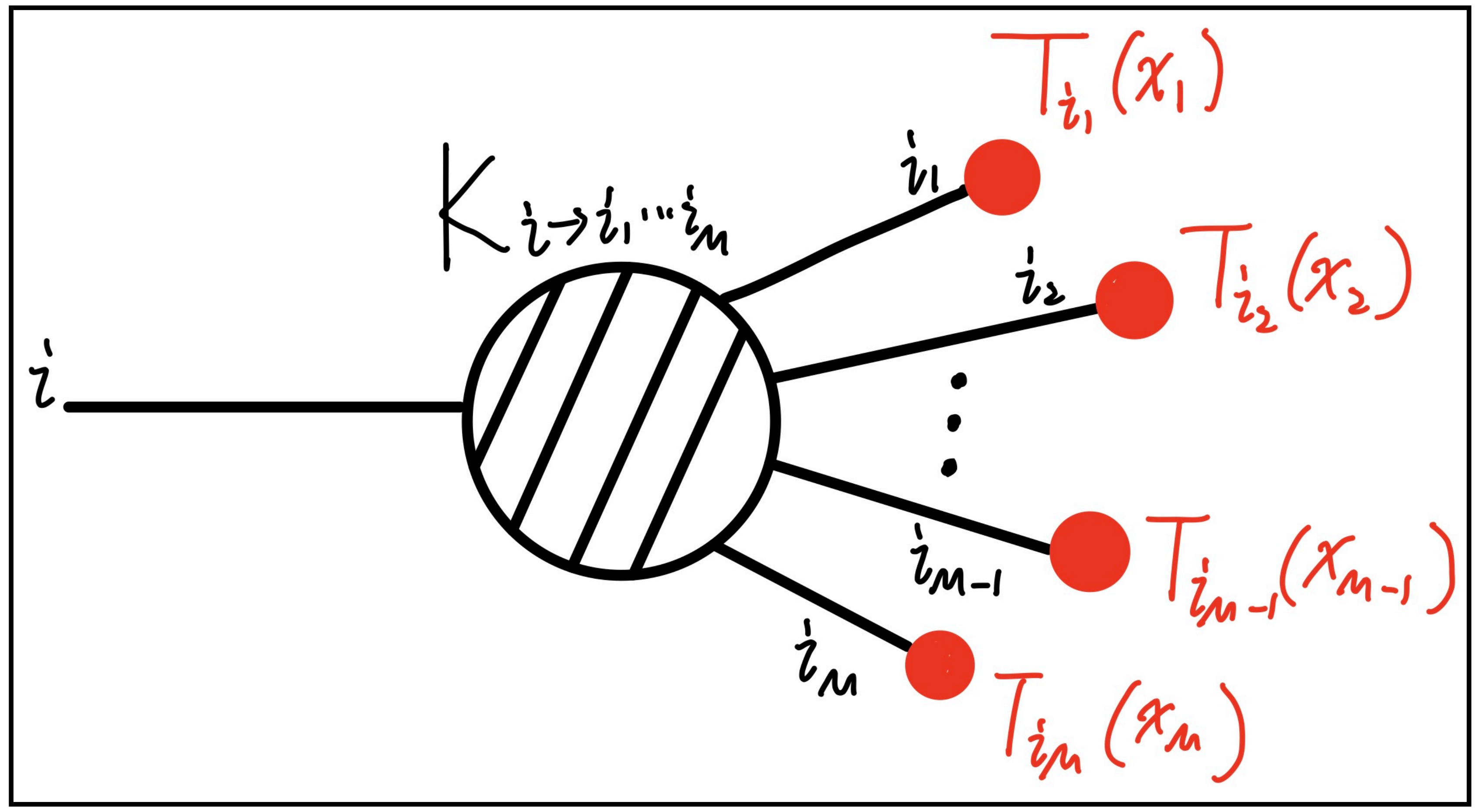
✦ New: Results for the non-linear x-space evolution at $\mathcal{O}(\alpha_s^2)$. [To appear soon]

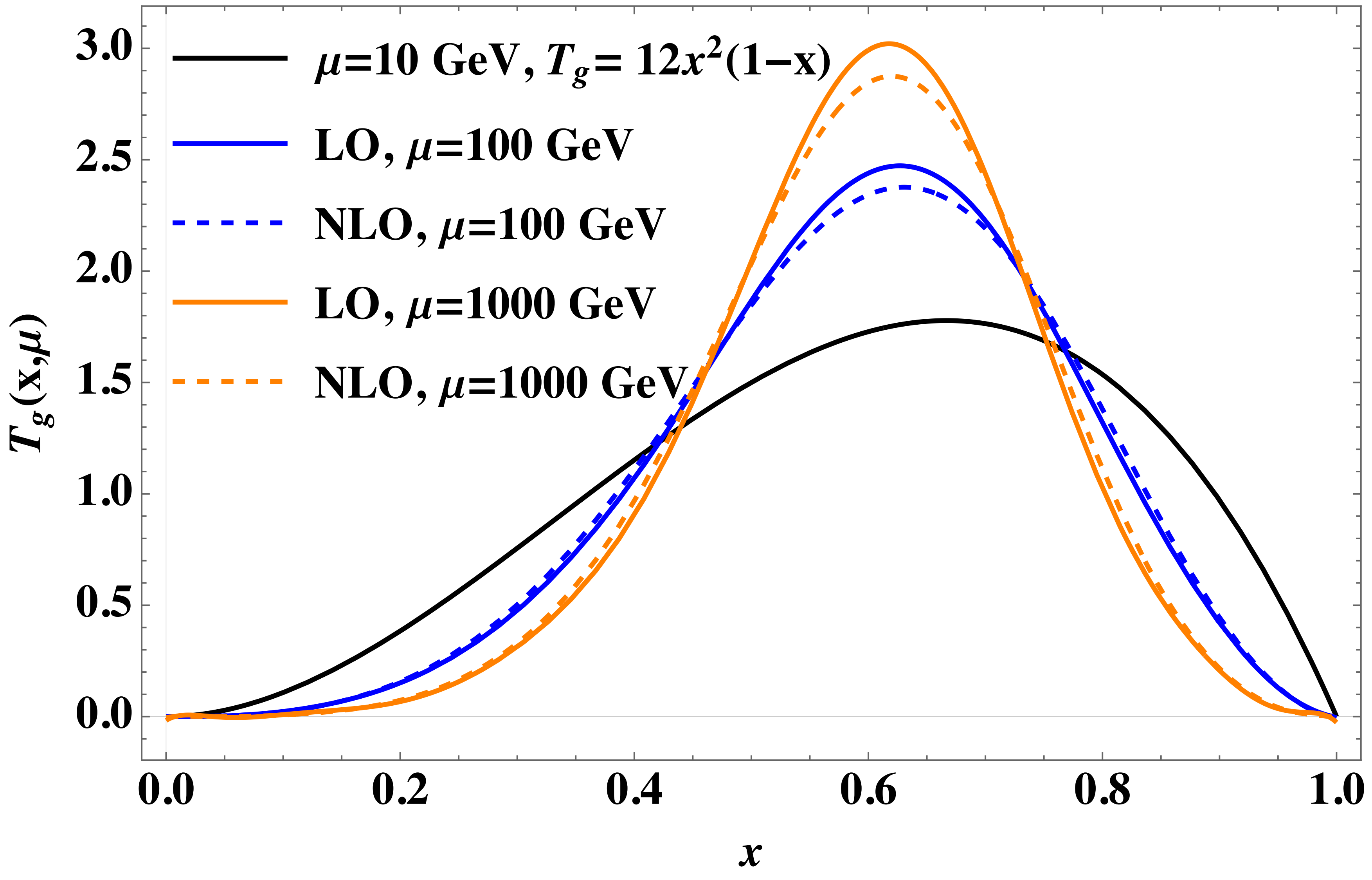
✓ Evolution of track functions in moment space and track energy correlators on tracks at $\mathcal{O}(\alpha_s^2)$.

[PRL, arXiv:2108.01674; JHEP, arXiv:2201.05166]

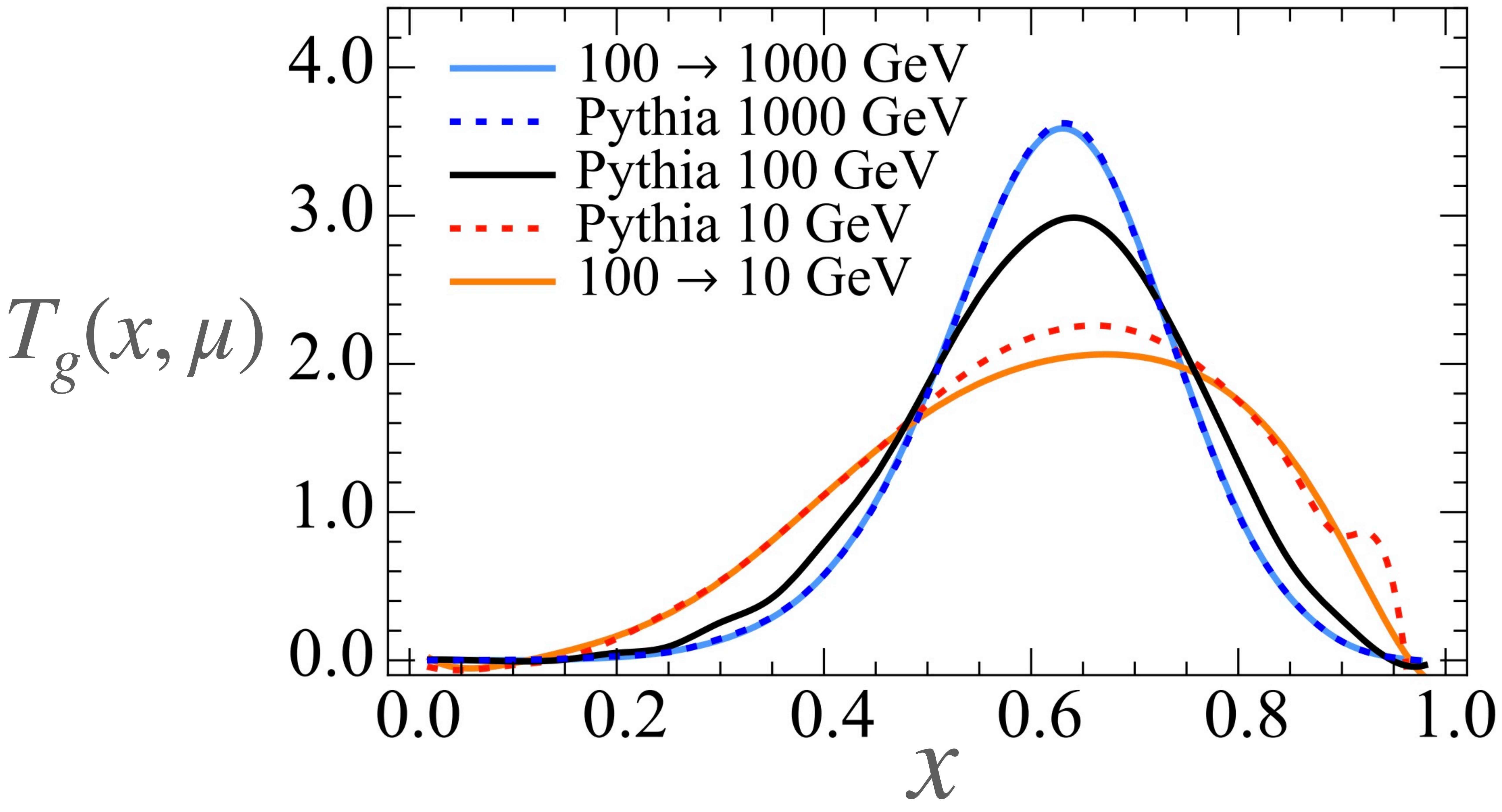
Outline

- Introduction to Track Functions
- Calculational Techniques & Results
- Reduction to DGLAP and Multi-hadron Fragmentation
- Summary





Introduction to Track Functions



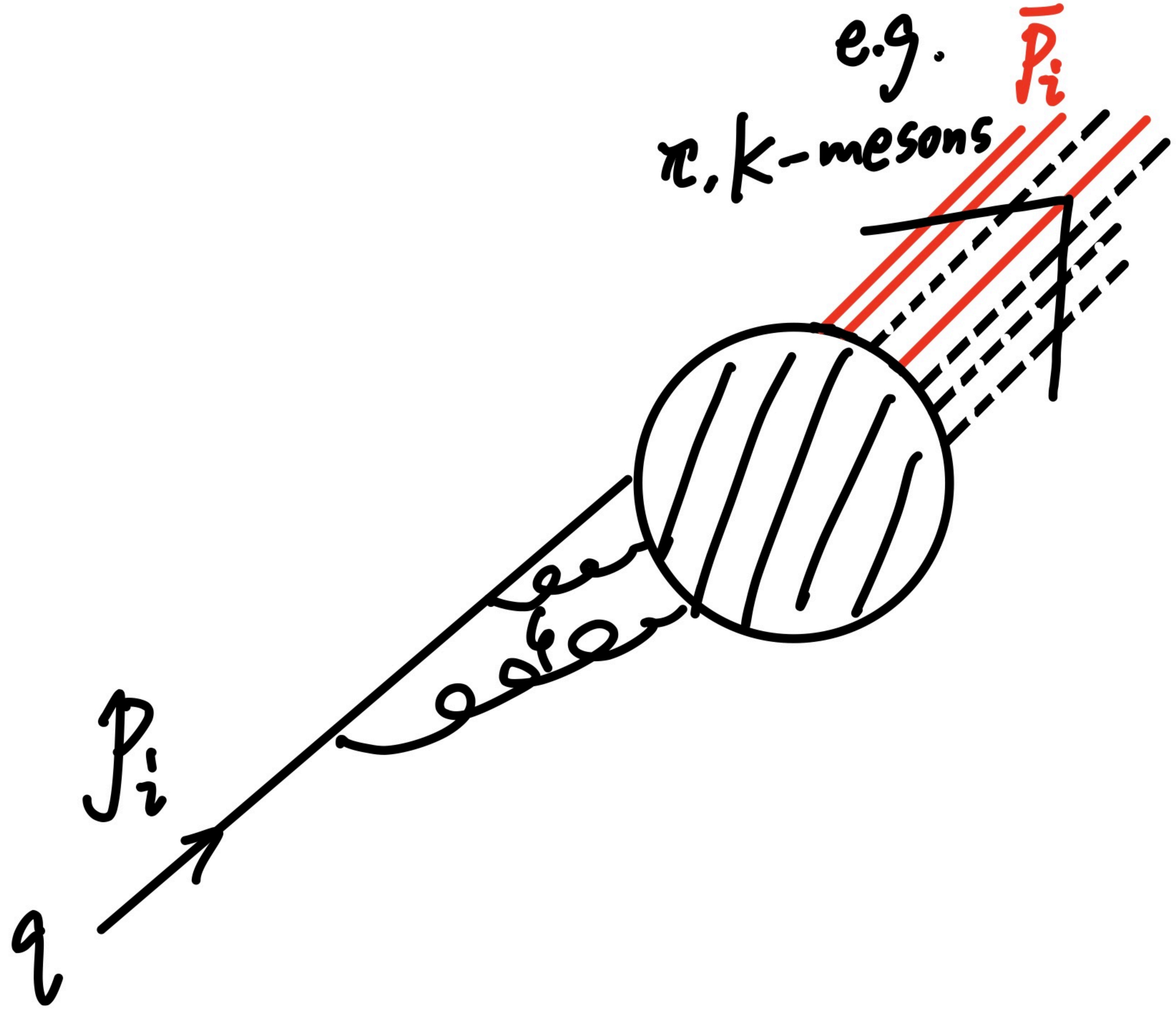
Track Functions $T_i(x, \mu)$ [H. Chang, M. Procura, J. Thaler, W. Waalewijn, 1303.6637, 1306.6630]

Definition

- The track function $T_i(x, \mu)$ describes the total momentum fraction x of **all** charged particles (tracks) in a jet initiated by a hard parton i.

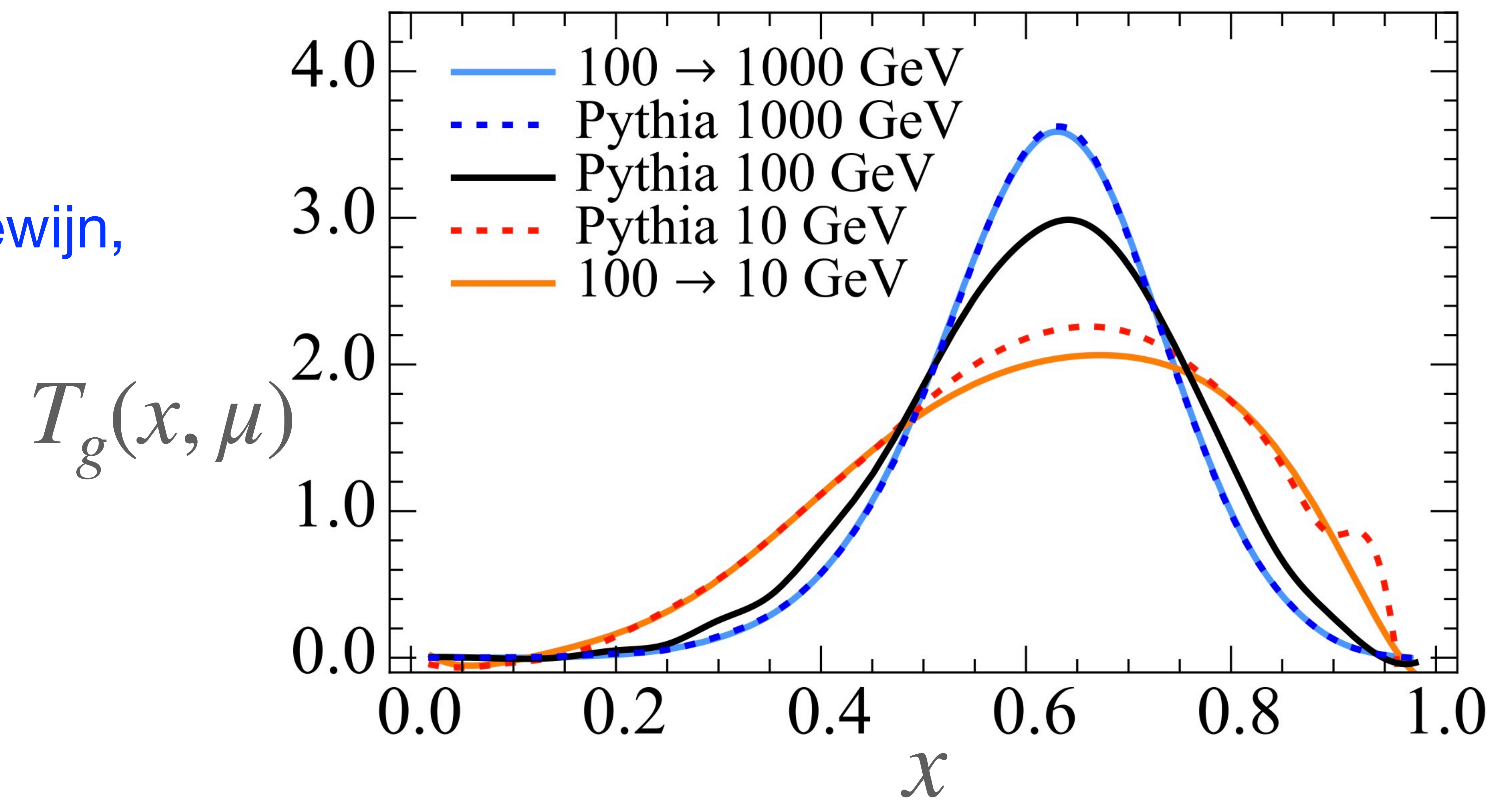
$$\bar{p}_i^\mu = x p_i^\mu + O(\Lambda_{\mathrm{QCD}}) \, , (0 \leq x \leq 1) \, .$$

- This formalism applies to other subsets of particles (positively-charged, strange, etc).



Track Functions

Features [H. Chang, M. Procura, J. Thaler, W. Waalewijn, 1303.6637, 1306.6630]

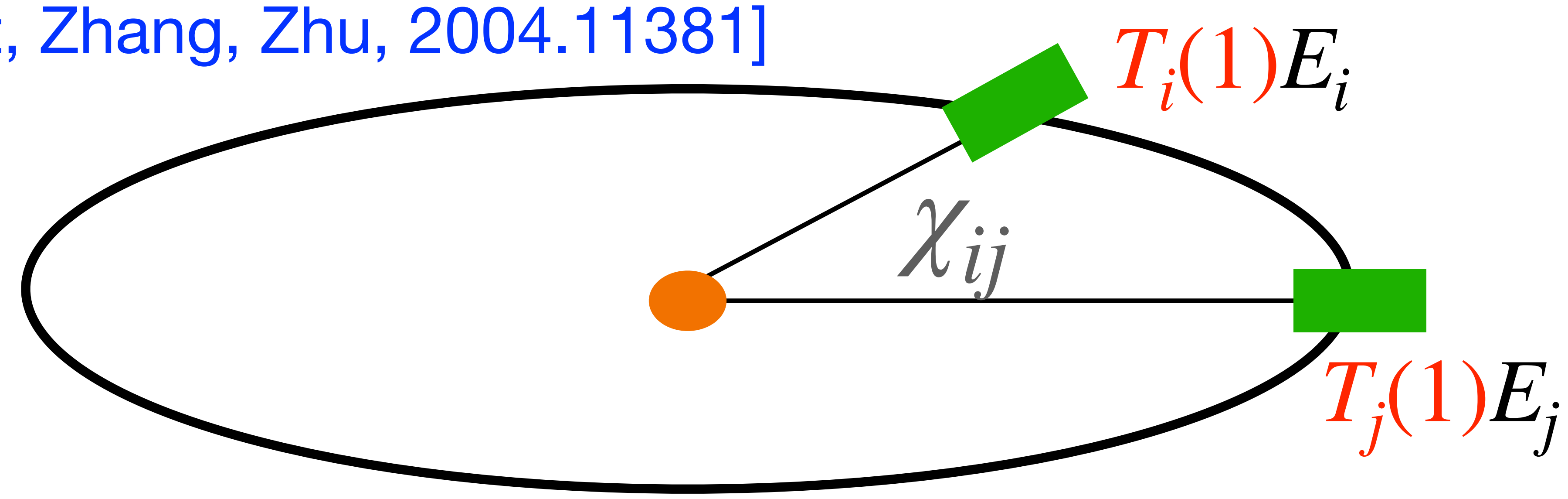


- A generalization of the fragmentation function (FF).
  - Independent of hard process.
  - Fundamentally non-perturbative, with a calculable scale (μ) dependence.
  - Incorporating correlations between final-state hadrons, like multi-hadron FFs.
  - Sum rule: $\int_0^1 dx\ T_i(x,\mu) = 1\ .$

- The single-hadron fragmentation function:
  - The probability of a parton to produce a single-hadron state considered.
  - The momentum sum rule: $\sum_h \int_0^1 dz\ z\ D_{i\to h}(z,\mu) = 1\ .$

Incorporating Tracks

[Chen, Moult, Zhang, Zhu, 2004.11381]



[1303.6637]

- For a δ-function type observable e measured using partons:

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta\left[e - \hat{e}(p_i^\mu)\right]$$

tracks

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i T_i(x_i) \delta\left[\bar{e} - \hat{e}(x_i p_i^\mu)\right]$$

full functional form of T

- Energy correlators: E.g., 2-point correlator (EEC)

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta\left(\cos\chi - \cos\chi_{ij}\right) d\sigma$$

$$E_i^n \to \int dx_i T_i(x_i) x_i^n E_i^n = T_i(n) E_i^n$$

Mellin moments

only moments of T

$$\left(\frac{d\Sigma}{d\cos\chi}\right)_{\text{tr}} = \sum_{i\neq j} T_i(1) T_j(1) \int \frac{E_i E_j}{Q^2} \delta\left(\cos\chi - \cos\chi_{ij}\right) d\bar{\sigma} + \sum_k T_k(2) \int \frac{E_k^2}{Q^2} \delta\left(\cos\chi - 1\right) d\bar{\sigma}$$

Track EEC

Track Function Evolution

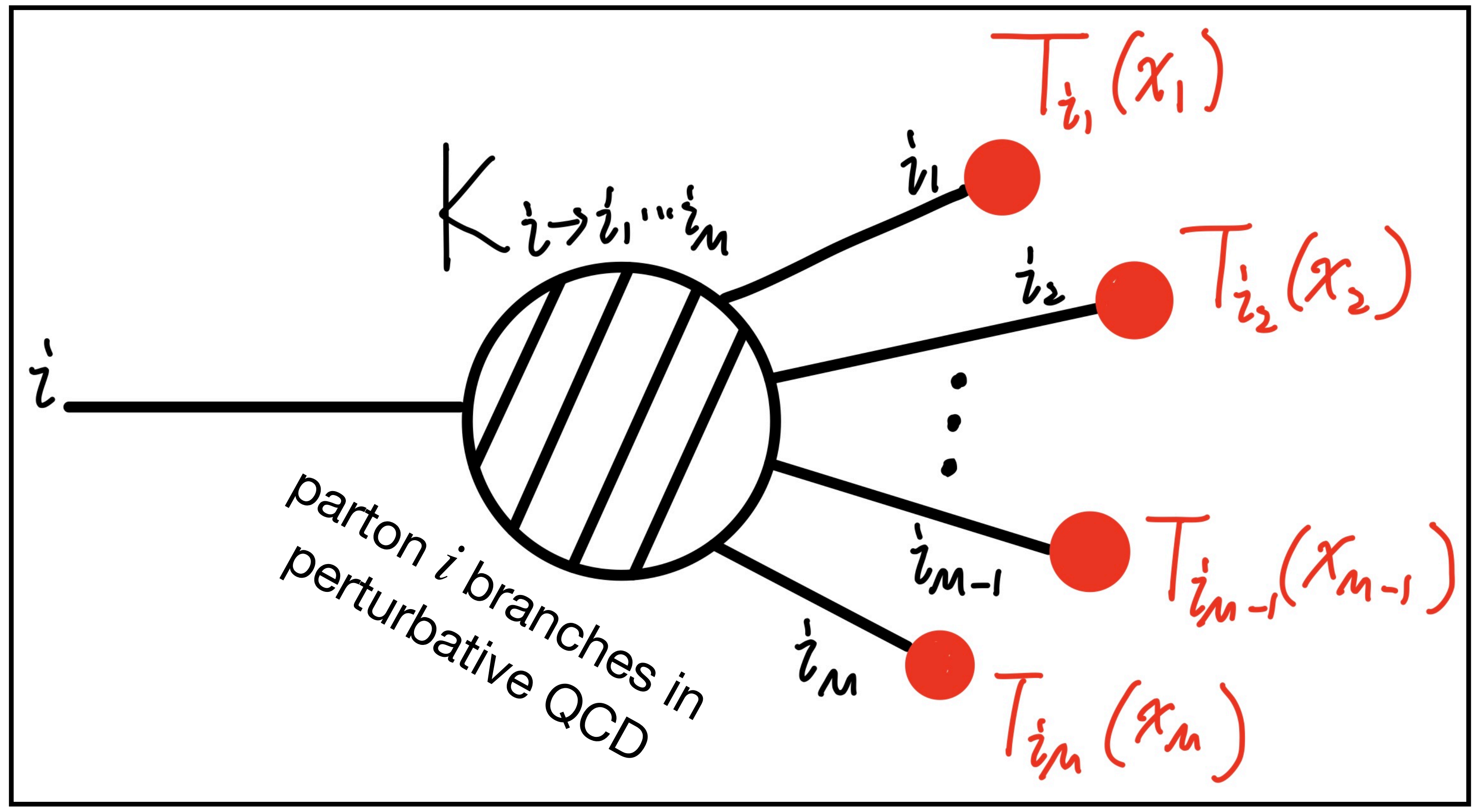
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x) = \sum_M \sum_{\{i_f\}} \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}z_m\right]\delta\Big(1-\sum_{m=1}^{M} z_m\Big)K_{i\to\{i_f\}}(\{z_f\})$$
$$\times\left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}x_m\, T_{i_m}(x_m)\right]\delta\Big(x-\sum_{m=1}^{M} z_m x_m\Big)$$

$(i, i_f = g, u, \bar{u}, d, \cdots)$

- **Nonlinear**, involving contributions from all branches of splittings.
- LO evolution:

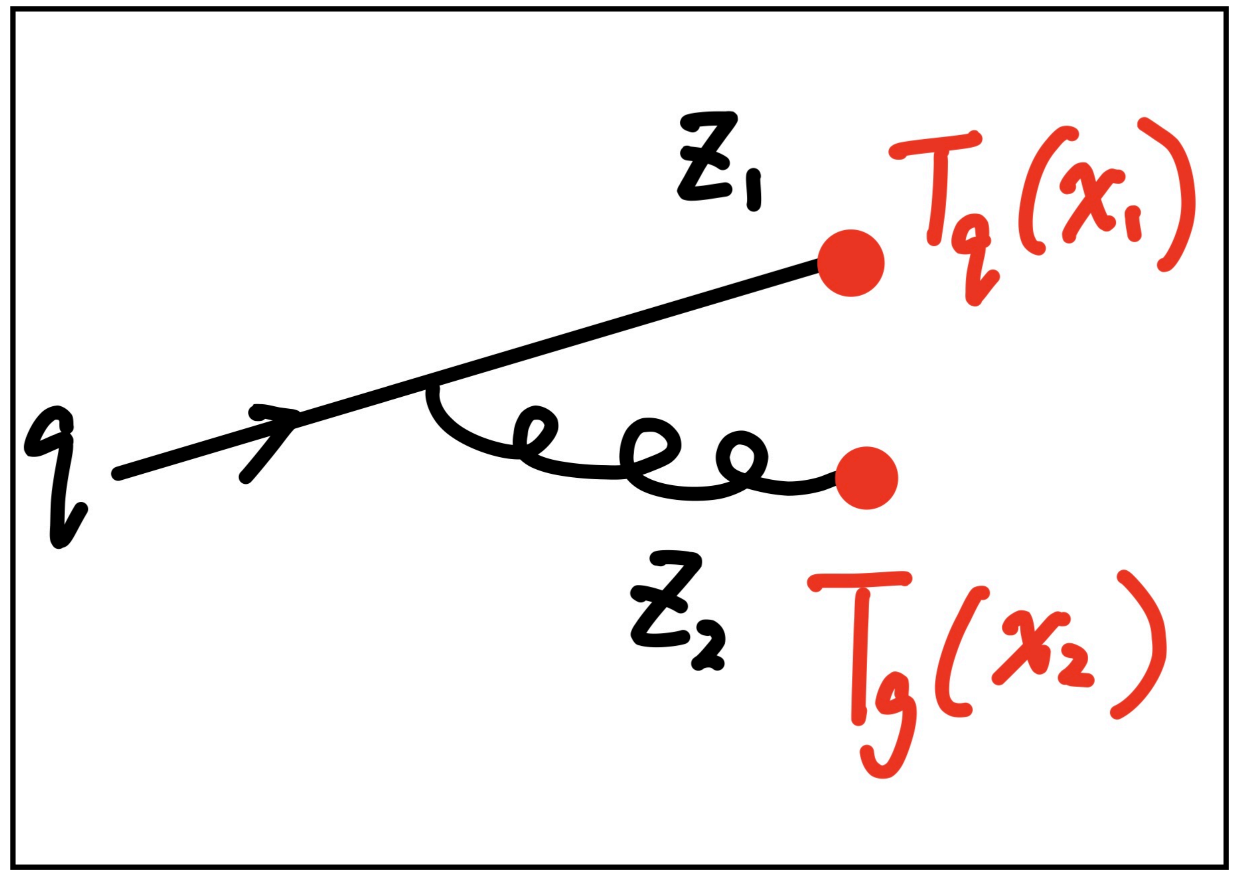
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x,\mu) = a_s(\mu)\sum_{\{jk\}}\int \mathrm{d}z_1\mathrm{d}z_2\; K^{(0)}_{i\to jk}(z_1,z_2)\delta(1-z_1-z_2)$$
$$\times\int \mathrm{d}x_1\mathrm{d}x_2 T_j(x_1,\mu)T_k(x_2,\mu)\delta[x-z_1x_1-z_2x_2]\ .$$

Involving contributions from both the branches of the splitting.

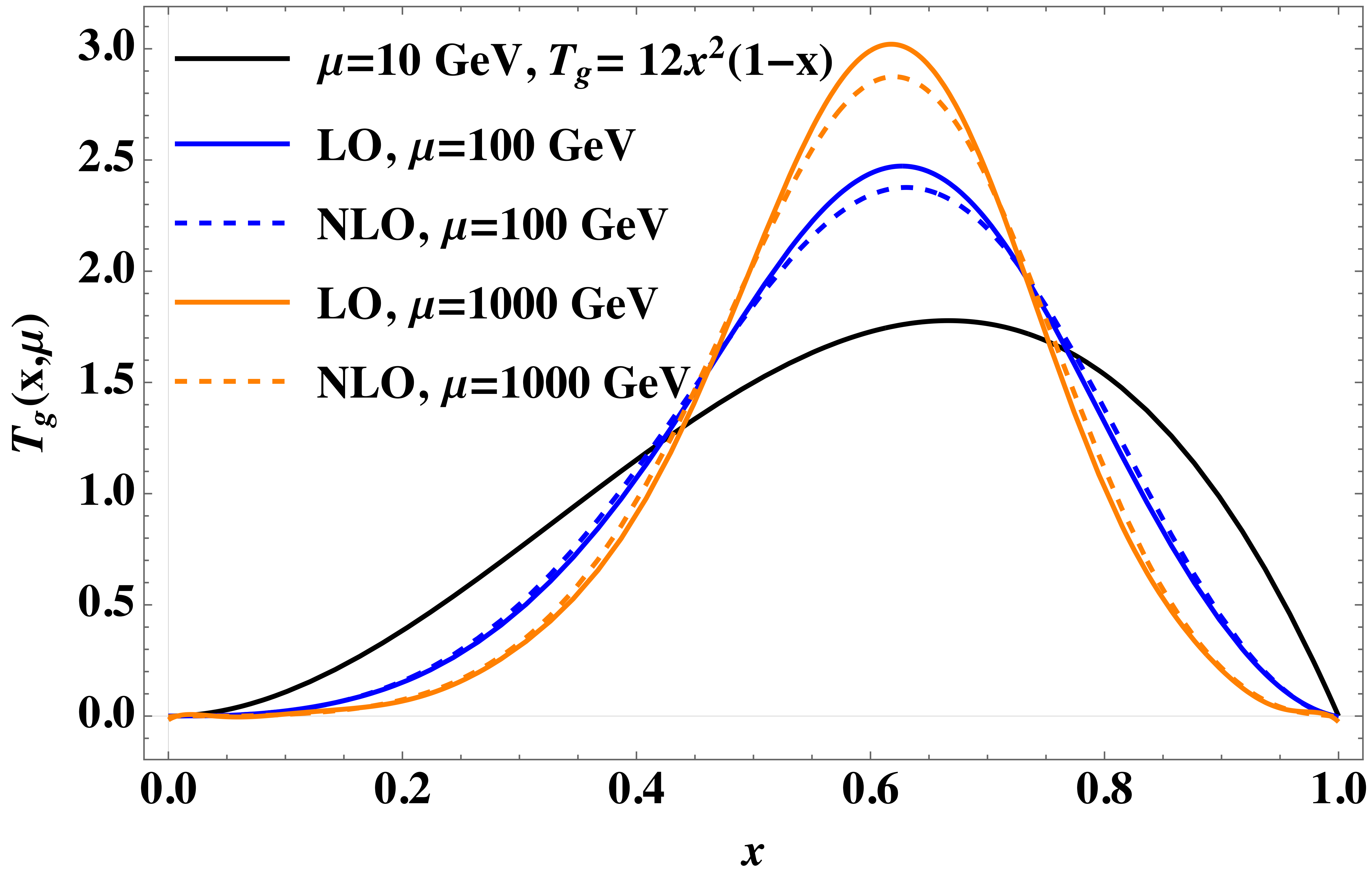


- For fragmentation functions: Only one branch observed → Linearity

$$\frac{d}{d\ln\mu^2}D_{i\to h}(x) = \sum_j D_{j\to h}\otimes P^T_{ji}(x)$$



Calculational Techniques & Results



Track Jet Functions

To calculate directly...

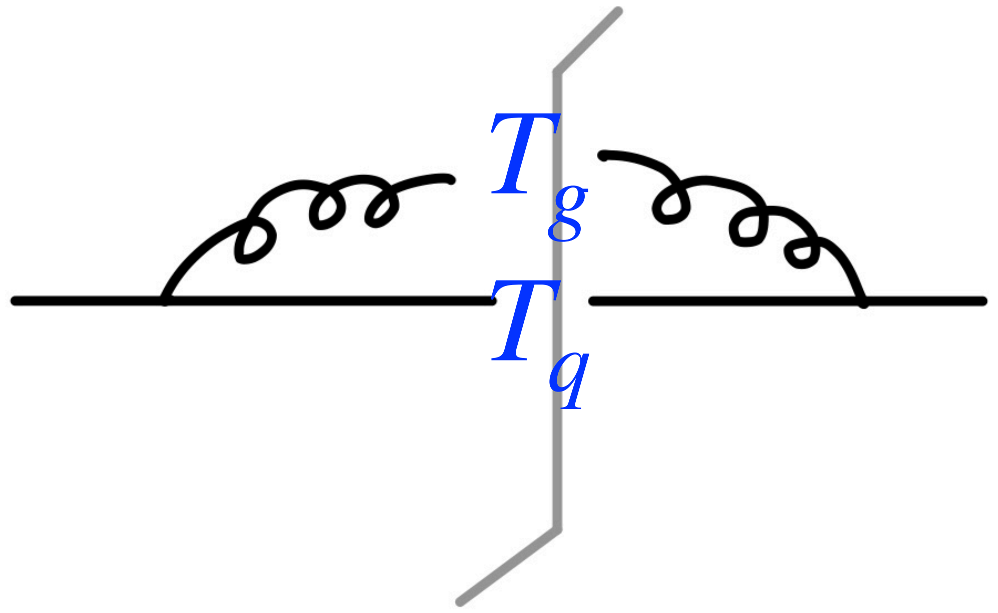
In DR: $T_i^{(0)} = T_i^{\text{bare}}$

LO track jet function:
$J_i^{(0)} = \delta(s) T_i^{(0)}$

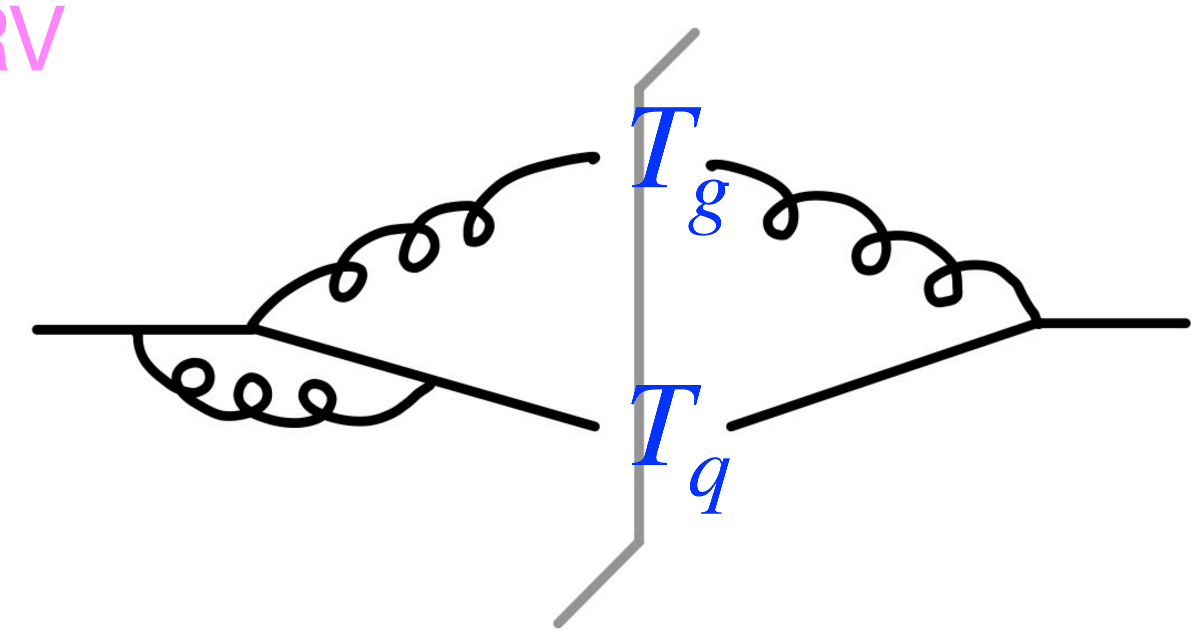
The definition for track jet functions is that

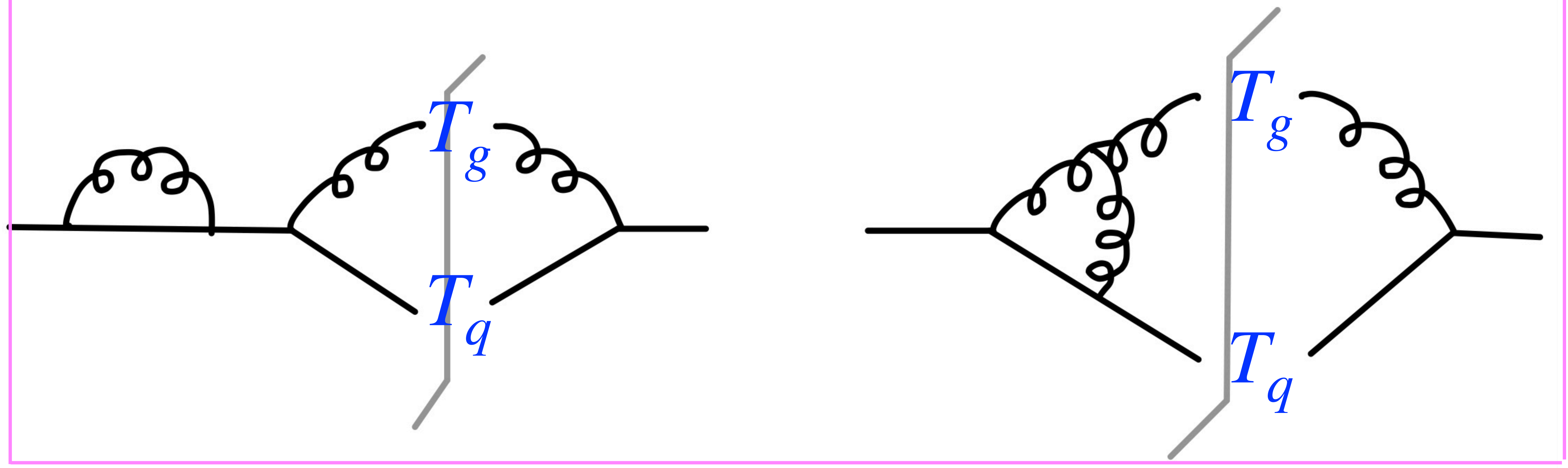
$$J_{\text{tr},i}^{\text{bare}}(s,x) = \sum_N \sum_{\{i_f\}} \int \mathrm{d}\Phi_N^c \delta(s-s') \sigma_{i\to\{i_f\}}^c(\{i_f\},\{s_{ff'}\},s') \int \left[\prod_{m=1}^N \mathrm{d}x_m T_{i_m}^{(0)}(x_m)\right] \delta\left(x - \sum_{m=1}^N x_m z_m\right)$$

E.g., the quark case:



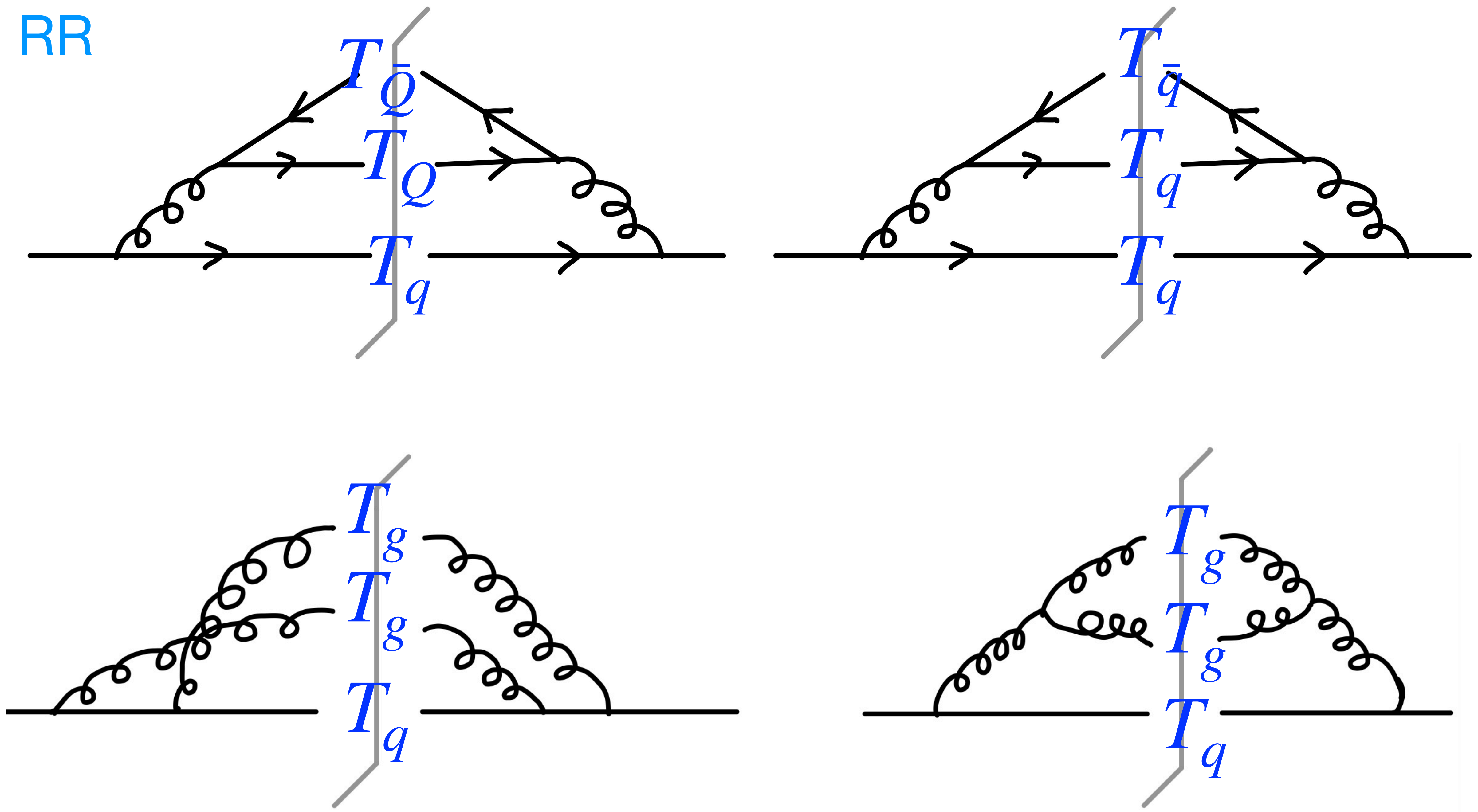
RV





[G. Sborlini, D. Florian, G. Rodrigo: arXiv:1310.6841]

RR



[S. Catani, M. Grazzini: arXiv:hep-ph/9908523]

Calculation of Track Jet Functions

After integration over angular variables,

$$J_{\mathrm{tr},i}(s,x) \supset \int \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \int_0^1 \mathrm{d}z_1\mathrm{d}z_2\mathrm{d}z_3\delta(1-z_1-z_2-z_3)P_{i\to i_1i_2i_3}(z_1,z_2,z_3)$$

have not been expanded in ϵ

$$\times\, T_{i_1}^{(0)}(x_1)T_{i_2}^{(0)}(x_2)T_{i_3}^{(0)}(x_3)\delta(x-z_1x_1-z_2x_2-z_3x_3)$$

- For $z_{i_1} < z_{i_2} < z_{i_3}$ $(i_1, i_2, i_3 = 1,2,3)$, do the coordinate transformation

$$z_{i_1} \to \frac{zt}{1+z+zt}, z_{i_2} \to \frac{z}{1+z+zt}, z_{i_3} \to \frac{1}{1+z+zt}$$

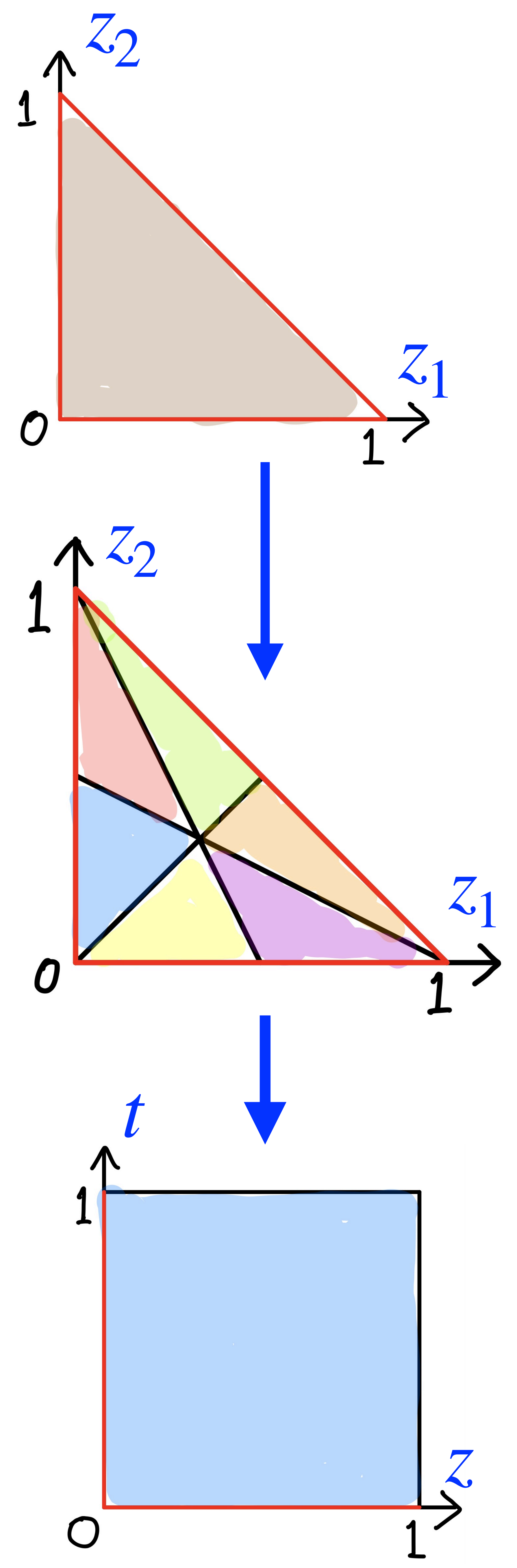
[Sector decomposition (Heinrich, arXiv:0803.4177)]

- For $1 \to n+1$ splitting $P_{1\to n+1}(z_1, z_2, \cdots, z_{n+1})$, we can set

$$z_{i_1} < z_{i_2} < \cdots < z_{i_n} < z_{i_{n+1}} \quad \text{and} \quad t_1 \to \frac{z_{i_1}}{z_{i_2}}, t_2 \to \frac{z_{i_2}}{z_{i_3}}, \cdots, t_n \to \frac{z_{i_n}}{z_{i_{n+1}}}$$

  to divide the integration region and then separate the singularities.

▸ For $1 \to 2$ splittings, $z_{i_1} \to \frac{z}{1+z}$, $z_{i_2} \to \frac{1}{1+z}$ for $z_{i_1} < z_{i_2}$.



Results in $\mathcal{N} = 4$ SYM

a: t' Hooft coupling constant

$$\frac{d}{d\ln\mu^2}T(x) = a^2 \Bigg\{ K^{(1)}_{1\to 1}\, T(x) + \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}z\; K^{(1)}_{1\to 2}(z)\; T(x_1)T(x_2)\; \delta\left(x - x_1\frac{1}{1+z} - x_2\frac{z}{1+z}\right)$$

$$+ \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}x_3 \int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}t\; K^{(1)}_{1\to 3}(z,t)\; T(x_1)T(x_2)T(x_3)$$

$$\times\, \delta\left(x - x_1\frac{1}{1+z+zt} - x_2\frac{z}{1+z+zt} - x_3\frac{zt}{1+z+zt}\right)\Bigg\}$$

where

$$K^{(1)}_{1\to 1} = -25\zeta_3 \qquad K^{(1)}_{1\to 2}(z) = \frac{8}{3}\pi^2\left[\frac{1}{z}\right]_+ + \frac{32\ln^2(z+1)}{z} - \frac{16\ln(z)\ln(z+1)}{z}$$

$$K^{(1)}_{1\to 3}(z,t) = 8\Bigg\{\frac{4\ln(1+z)}{z}\left[\frac{1}{t}\right]_+ + \left[\frac{1}{z}\right]_+\left(4\left[\frac{\ln t}{t}\right]_+ - \frac{\ln t}{1+t} - \frac{7\ln(1+t)}{t}\right)$$

$$+\frac{2\left[\ln(1+tz) - \ln(1+z+tz)\right]}{(1+t)(1+z)(1+tz)} + \frac{10\left[\ln(1+z+tz) - \ln(1+z)\right]}{tz} + \frac{\ln(1+tz)}{(1+t)z(1+z)}$$

$$-\frac{7\ln(1+tz)}{tz} + \frac{\ln(1+t) - \ln t}{(1+t)(1+tz)} + \frac{\ln(1+z) + \ln(1+t)}{(1+t)(1+z)} - \frac{\ln(1+z)}{(1+t)z} - \frac{z\ln(1+z)}{(1+z)(1+tz)}\Bigg\}$$

Results in QCD

E.g. Gluon case:

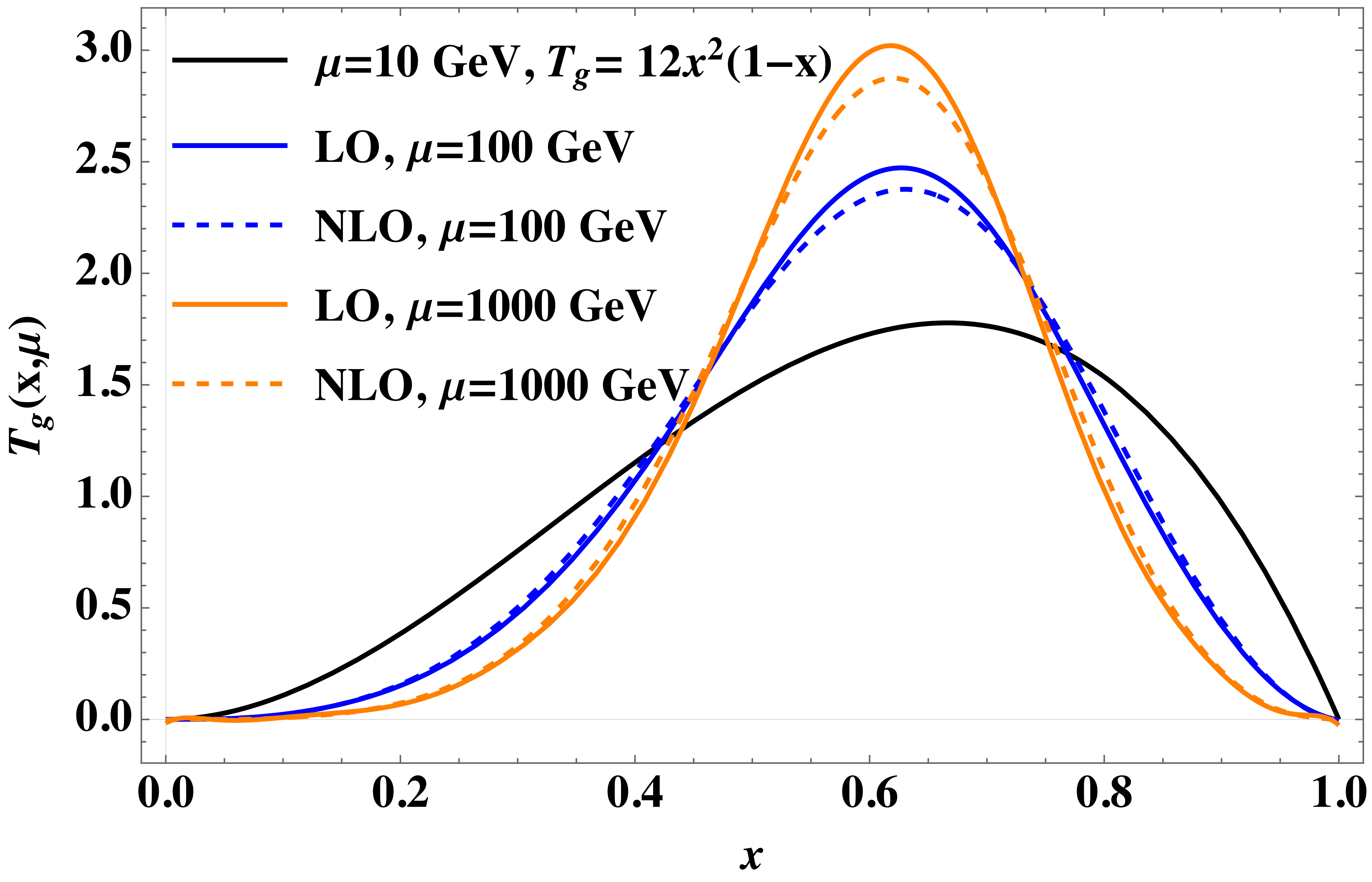
For brevity, $a_s^2 = [\alpha_s(\mu)/(4\pi)]^2$ is suppressed.

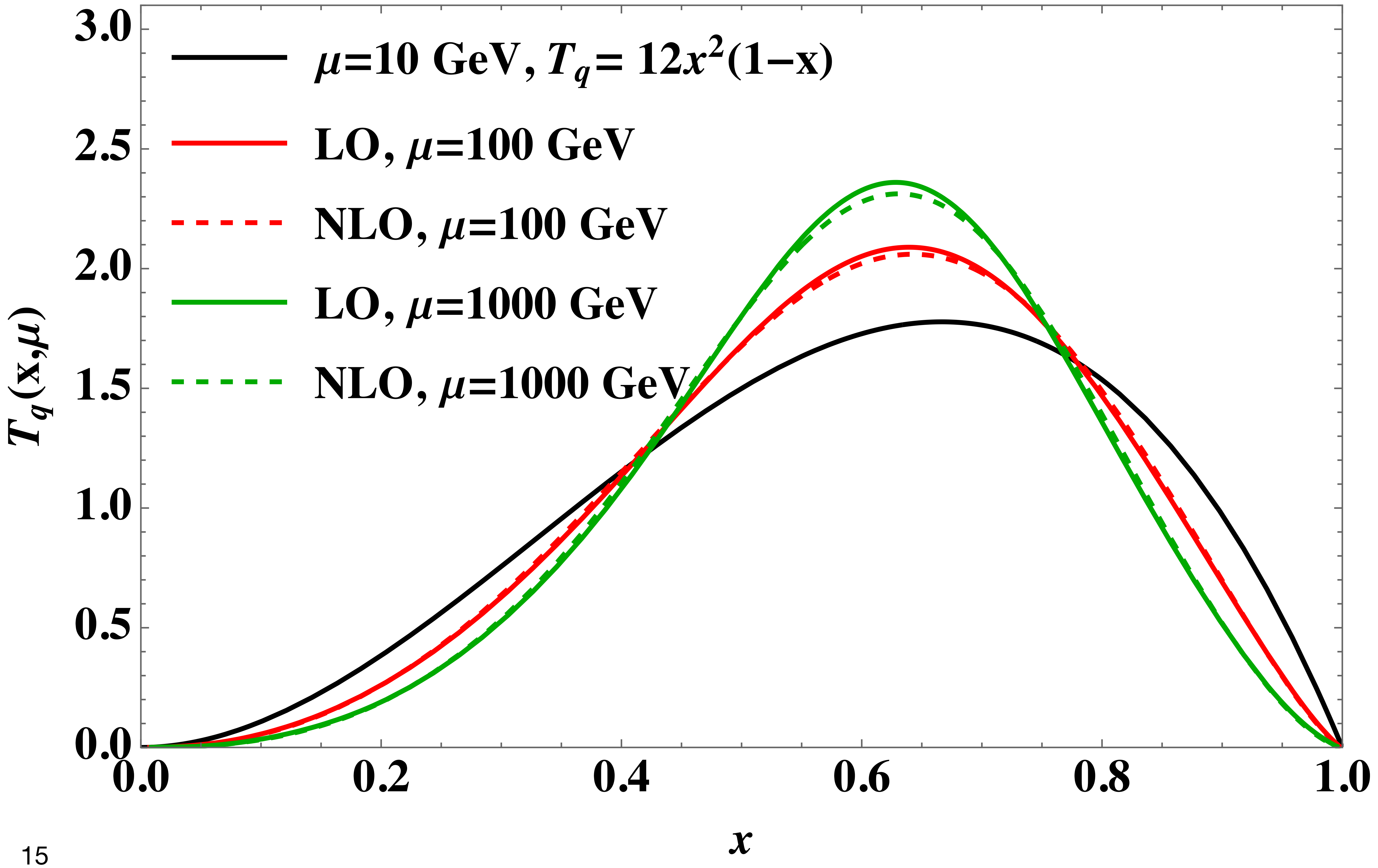
$$
\begin{aligned}
\frac{d}{d\ln\mu^2} T_g(x) =& T_g(x)\ K_g^{(1)} \\
&+ \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}z \delta\left(x - x_1 \frac{1}{1+z} - x_2 \frac{z}{1+z}\right) \Big[T_g(x_1) T_g(x_2)\ K_{gg,1}^{(1)}(z) \\
&+ \sum_q \left(T_q(x_1) T_{\bar{q}}(x_2) + T_q(x_2) T_{\bar{q}}(x_1)\right)\ K_{q\bar{q},1}^{(1)}(z) \Big] \\
&+ \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}x_3 \int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}t\ \delta\left(x - x_1 \frac{1}{1+z+zt} - x_2 \frac{z}{1+z+zt} - x_3 \frac{zt}{1+z+zt}\right) \\
&\times \Big\{ 6\ T_g(x_1) T_g(x_2) T_g(x_3)\ K_{ggg,1}^{(1)}(z,t) \\
&\quad + \sum_q \Big[T_g(x_3)\left(T_q(x_2) T_{\bar{q}}(x_1) + T_q(x_1) T_{\bar{q}}(x_2)\right)\ K_{gq\bar{q},1}^{(1)}(z,t) \\
&\quad + T_g(x_2)\left(T_q(x_3) T_{\bar{q}}(x_1) + T_q(x_1) T_{\bar{q}}(x_3)\right)\ K_{gq\bar{q},2}^{(1)}(z,t) \\
&\quad + T_g(x_1)\left(T_q(x_3) T_{\bar{q}}(x_2) + T_q(x_2) T_{\bar{q}}(x_3)\right)\ K_{gq\bar{q},3}^{(1)}(z,t) \Big] \Big\}
\end{aligned}
$$

Solving the RGEs Numerically

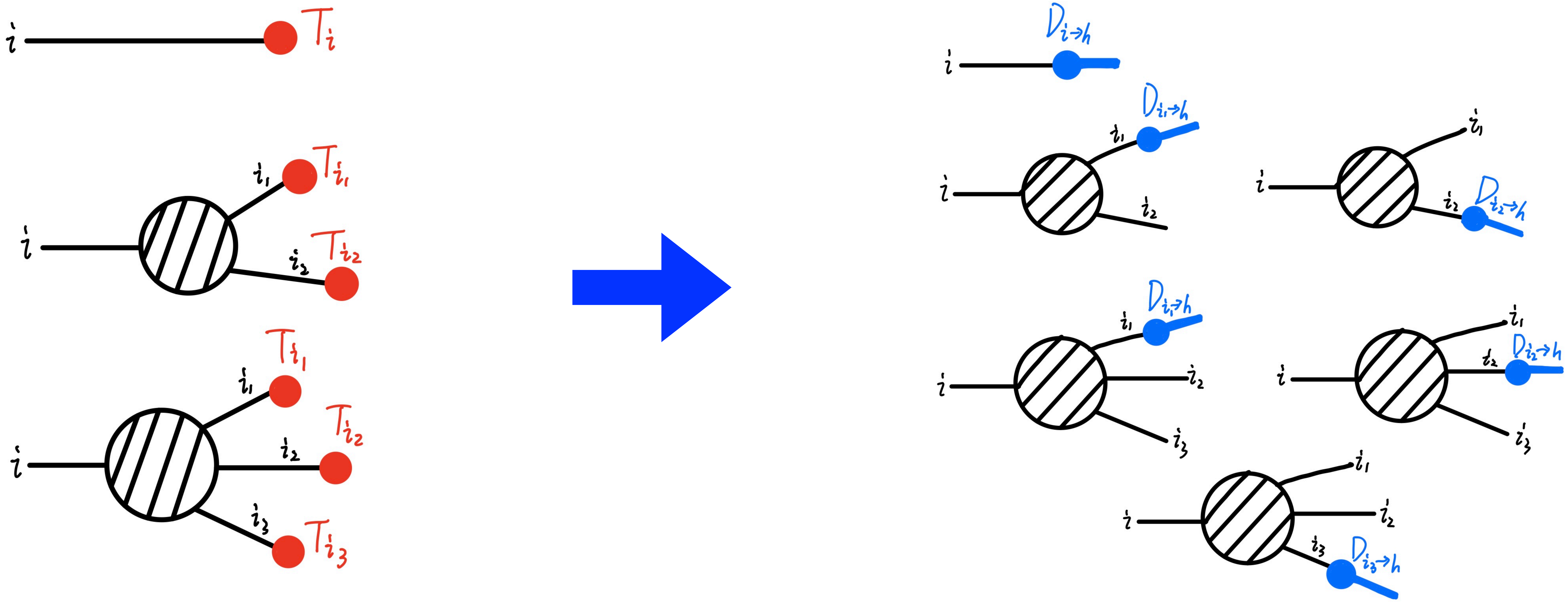
- A toy model: at $\mu_0 = 10$ GeV, $T_i(x) = 12x^2(1-x)$ $(i = q, g)$ of which the first moment is 0.6 ~ that in real world QCD.

- Suppose that the track function at any scale, $T(x, \mu_j)$, can be well described by a polynomial of some degree. $T(x, \mu_j)$ can be restored from a finite number of its moments.





Reduction to DGLAP and Multi-hadron Fragmentation



Fragmentation Functions

Single- and Multi-hadron cases

[U. P. Sukhatme and K. E. Lassila, *Phys.Rev.D* 22 (1980) 1184]
[de Florian, Vanni: arXiv:0310196]

- **The single-hadron fragmentation function** $D_{i\to h}(y)$ gives the probability of finding in a jet a single hadron h with momentum fraction y of that possessed by the jet-initiating parton i (a quark, antiquark or gluon).

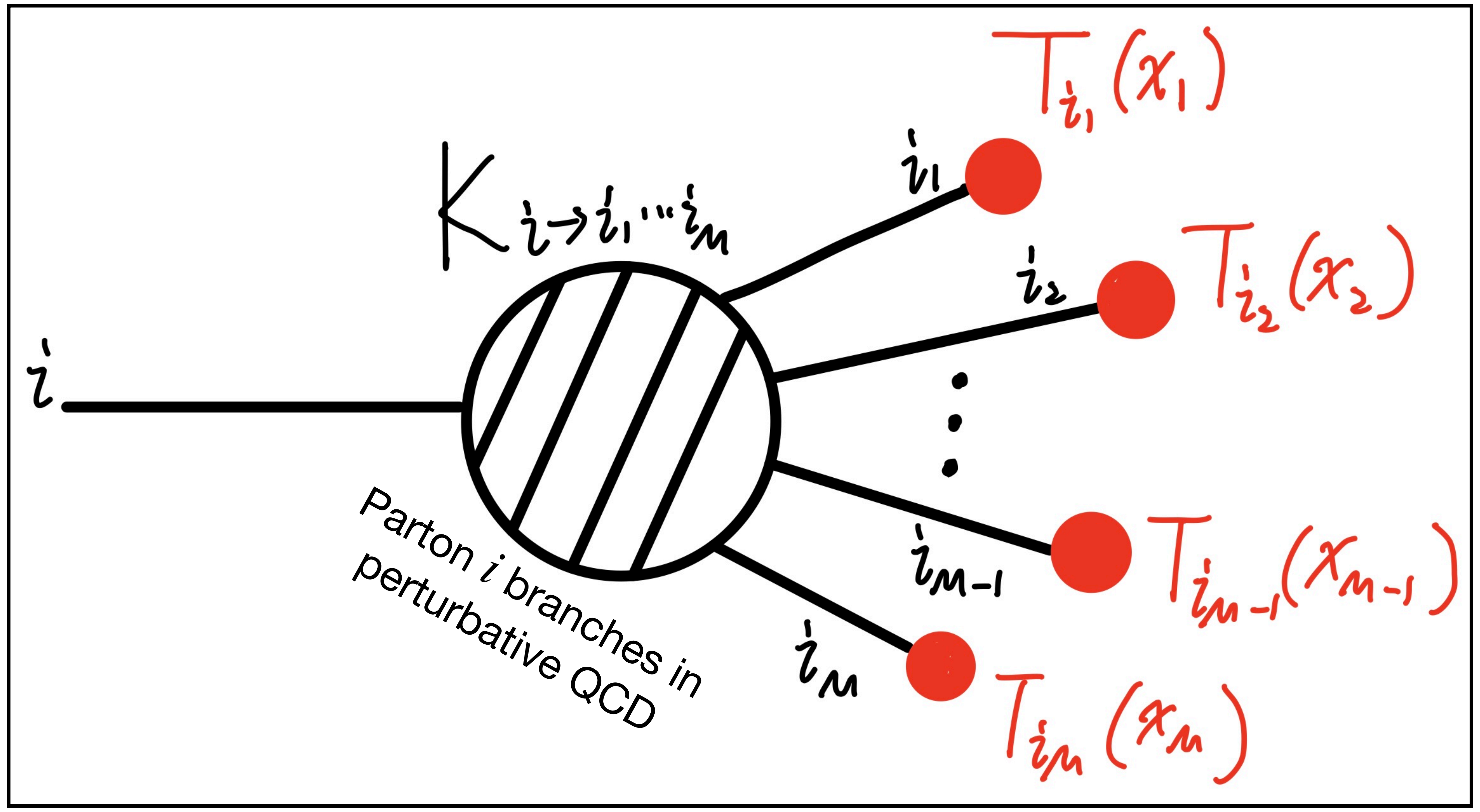
- **The** N**-hadron fragmentation function** $D_{i\to h_1h_2\cdots h_N}(y_1, y_2, \cdots, y_N)$ for fragmentation of parton i into N hadrons which carry fractions $y_1, y_2, \cdots, y_N$ of the momentum carried by the initial parton.

- $N = 2$: **Di-hadron fragmentation function** $D_{i\to h_1h_2}(y_1, y_2)$.

Notation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x) = \sum_M \sum_{\{i_f\}} \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}z_m\right]\delta\left(1-\sum_{m=1}^{M} z_m\right)K_{i\to\{i_f\}}(\{z_f\})$$

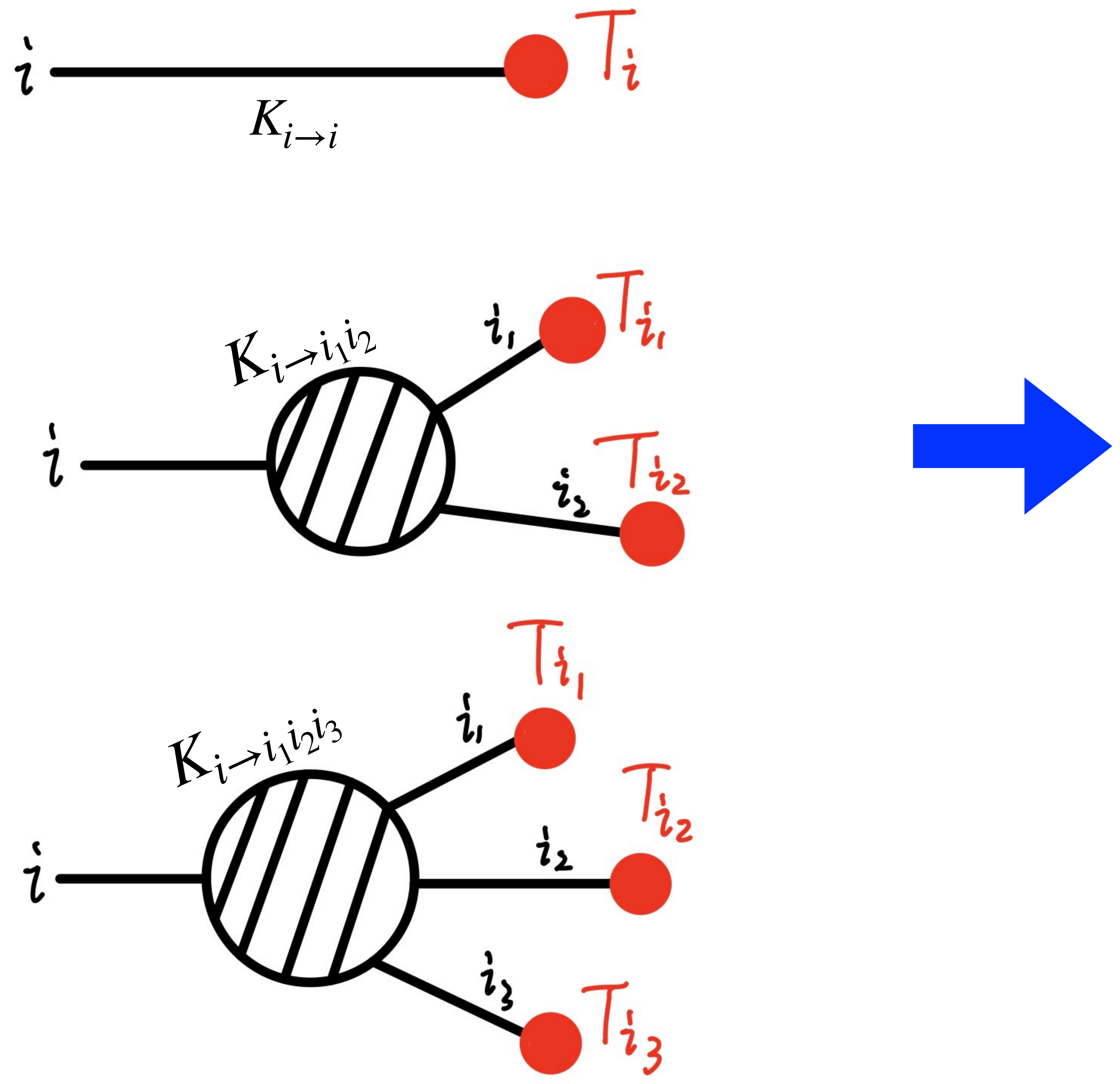
$$\times \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}x_m\, T_{i_m}(x_m)\right]\delta\left(x-\sum_{m=1}^{M} z_m x_m\right)$$

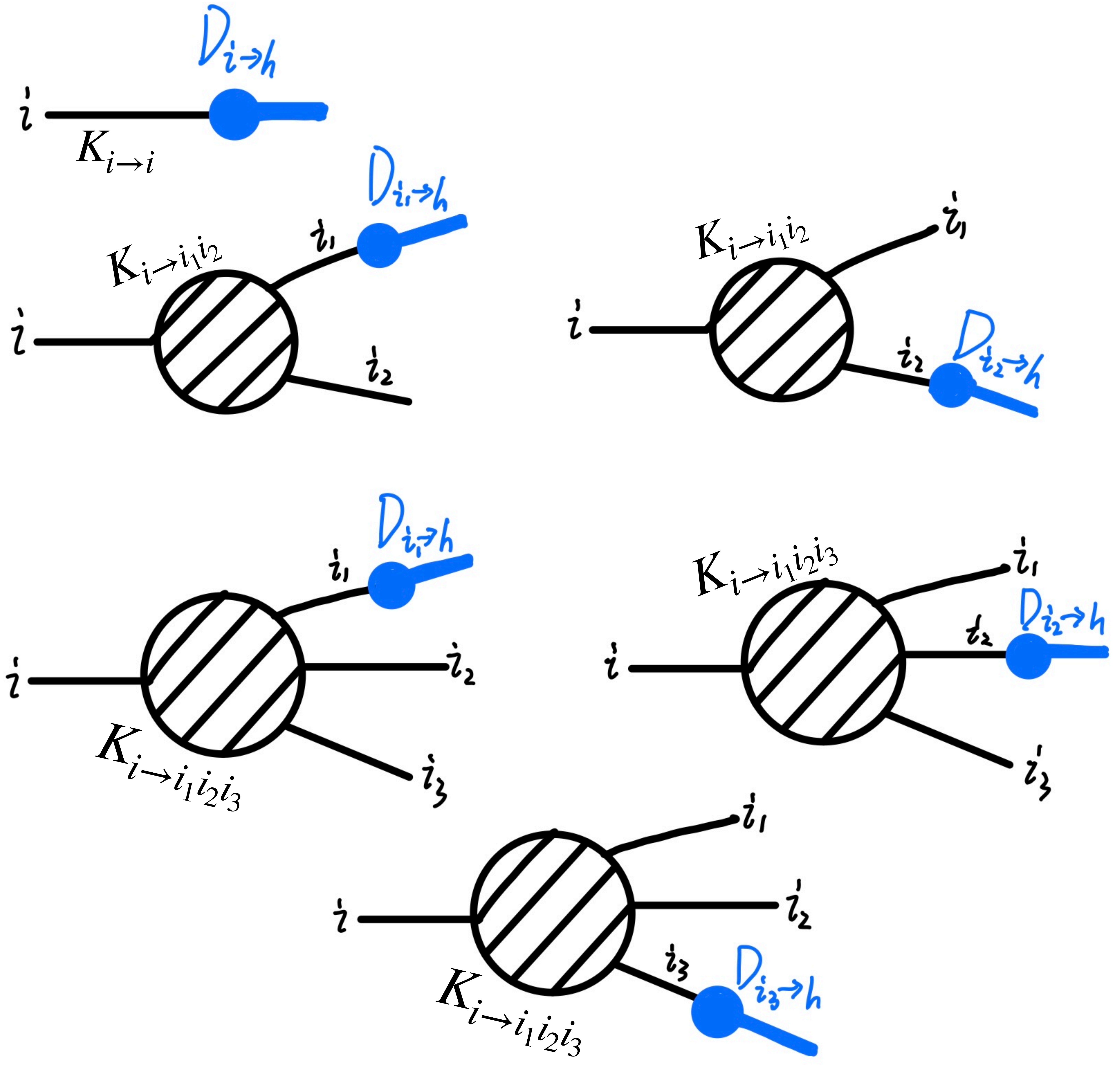


$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x) = \sum_M \sum_{\{i_f\}} K_{i\to i_1 i_2\cdots i_M}\otimes T_{i_1}T_{i_2}\cdots T_{i_M}(x)$$

Reduction to DGLAP

- For notational simplicity, set $M \leq 3$.





Reduction to DGLAP

- The NLO evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x)$$
$$= K^{(1)}_{i\to i}T_i(x)$$
$$+\sum_{\{i_f\}}K^{(1)}_{i\to i_1 i_2}\otimes T_{i_1}(x_1)T_{i_2}(x_2)$$
$$+\sum_{\{i_f\}}K^{(1)}_{i\to i_1 i_2 i_3}\otimes T_{i_1}(x_1)T_{i_2}(x_2)T_{i_3}(x_3)$$

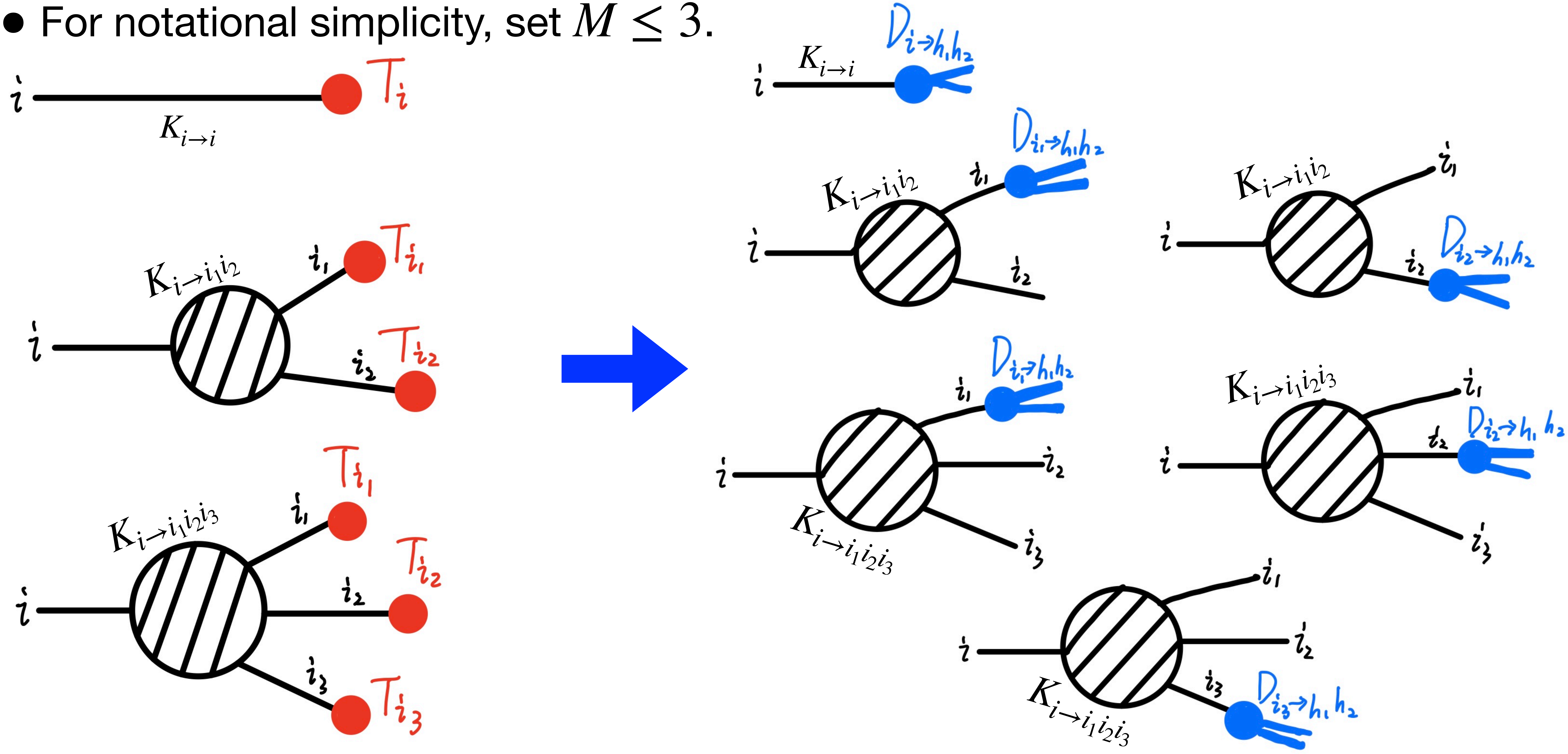
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}D_{i\to h}(x)$$
$$= K^{(1)}_{i\to i}D_{i\to h}(x)$$
$$+\sum_{\{i_f\}}K^{(1)}_{i\to i_1 i_2}\otimes [D_{i_1\to h}(x_1)+D_{i_2\to h}(x_2)]$$
$$+\sum_{\{i_f\}}K^{(1)}_{i\to i_1 i_2 i_3}\otimes [D_{i_1\to h}(x_1)+D_{i_2\to h}(x_2)$$
$$+D_{i_3\to h}(x_3)]$$

equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}D_{i\to h}(x)=\sum_j D_{j\to h}\otimes P^{T,(1)}_{ji}(x)$$

Reduction to Di-hadron Fragmentation

- For notational simplicity, set $M \leq 3$.



Reduction to Di-hadron Fragmentation

- For notational simplicity, set $M \leq 3$.

Reduction to Di-hadron Fragmentation

- The NLO evolution:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} D_{i\to h_1h_2}(y_1,y_2) = \Bigg\{ K^{(1)}_{i\to i} D_{i\to h_1h_2}(y_1,y_2) + \sum_{\{i_f\}} K^{(1)}_{i\to i_1i_2} \otimes [D_{i_1\to h_1h_2} + D_{i_2\to h_1h_2}]$$

$$+ \sum_{\{i_f\}} K^{(1)}_{i\to i_1i_2i_3} \otimes [D_{i_1\to h_1h_2} + D_{i_2\to h_1h_2} + D_{i_3\to h_1h_2}] \Bigg\}$$

$$+ \Bigg\{ \sum_{\{i_f\}} K^{(1)}_{i\to i_1i_2} \otimes [D_{i_1\to h_1} D_{i_2\to h_2} + D_{i_1\to h_2} D_{i_2\to h_1}]$$

$$+ \sum_{\{i_f\}} K^{(1)}_{i\to i_1i_2i_3} \otimes [D_{i_1\to h_1} D_{i_2\to h_2} + D_{i_1\to h_2} D_{i_2\to h_1}$$

$$+ D_{i_1\to h_1} D_{i_3\to h_2} + D_{i_1\to h_2} D_{i_3\to h_1}$$

$$+ D_{i_2\to h_1} D_{i_3\to h_2} + D_{i_2\to h_2} D_{i_3\to h_1}] \Bigg\}$$

Summary & Outlook

- Track functions offer a QFT approach to calculating track-based observables. This formalism allows IR-safe observables to be computed on any subset of final-state hadrons specified by some particular quantum numbers.
- The full result of the nonlinear x-space evolution at $\mathcal{O}(\alpha_s^2)$.
    - Numerical implementation.
- The equation can be thought of as the most general equation for collinear evolution at NLO. From that, one can derive the NLO corrections to single- and multi-hadron fragmentation functions.
- Precision phenomenology with tracks using track functions; application of the multi-hadron fragmentation functions.

Thank you for listening!