Nuclear Science Computing Center at CCNU

Fluctuations of conserved charges in strong magnetic fields

Jun-Hong Liu(刘俊宏)
Central China Normal University(华中师范大学)

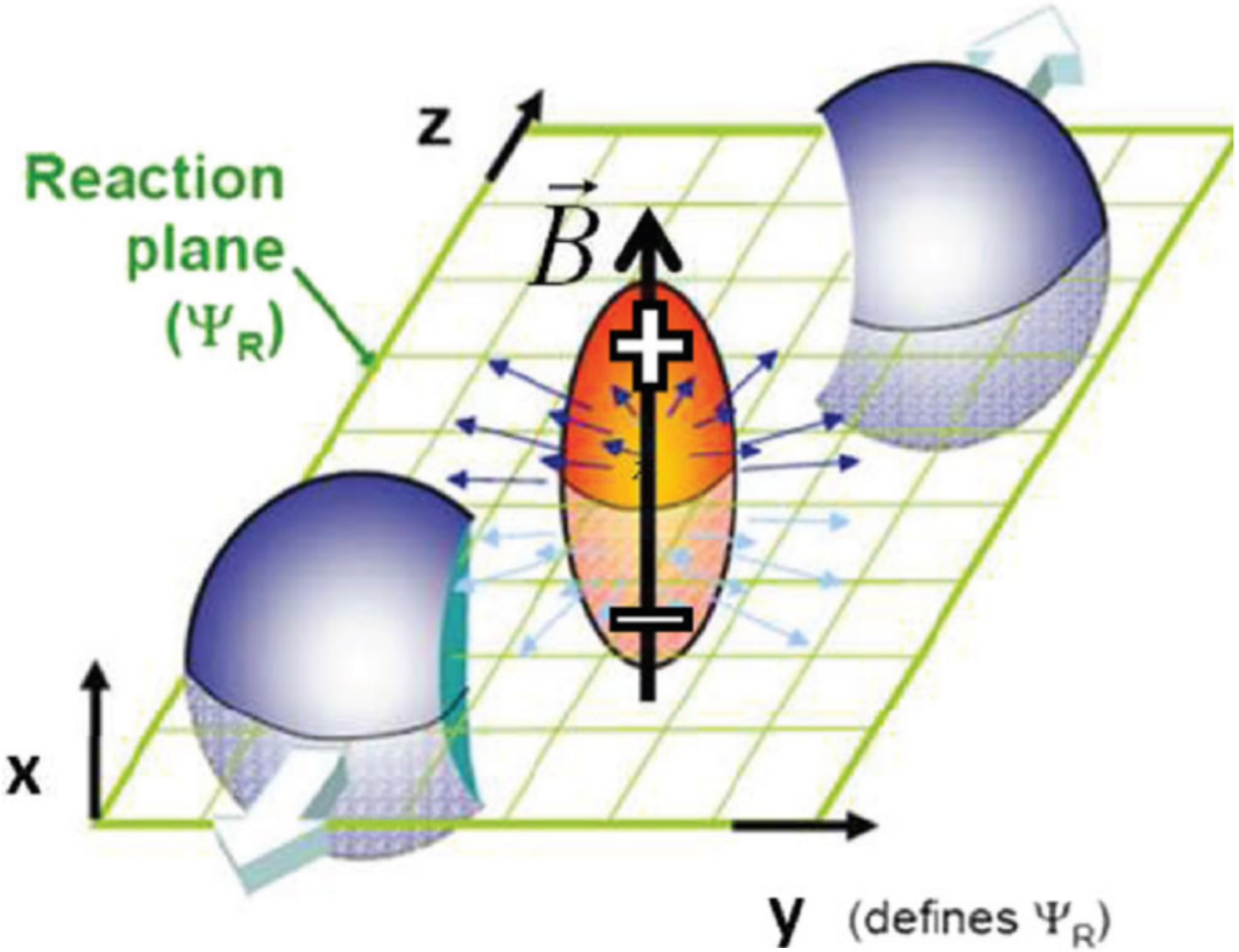
H.-T. Ding, S.-T. Li, Q. Shi, X.-D. Wang, Eur.Phys.J.A 57 (2021) 6, 202

and work in progress

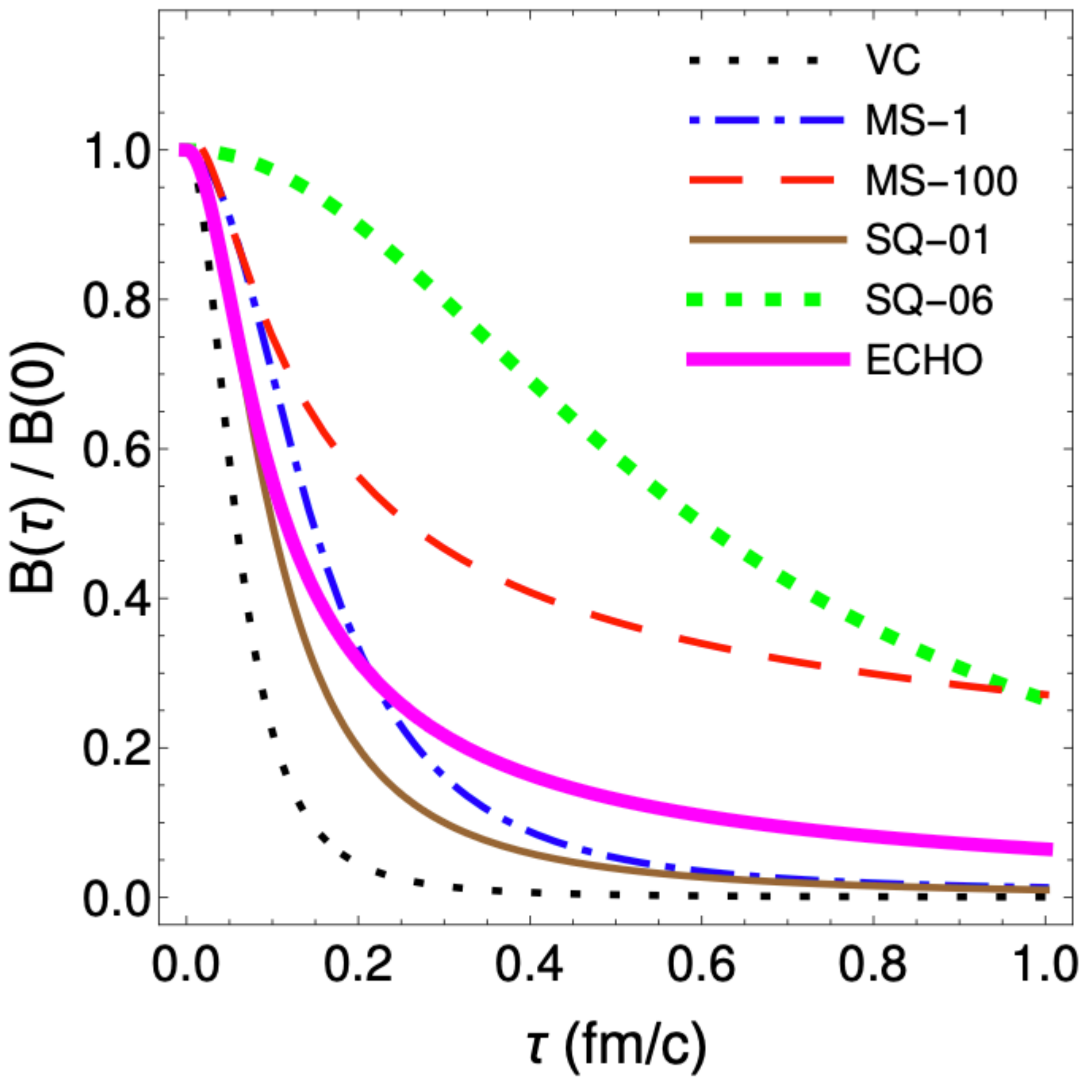
中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会

大连，辽宁师范大学，2022.8.10

Strong magnetic fields in heavy-ion collisions



Wei-Tian Deng, Xu-Guang Huang
Phys.Rev.C 85 (2012) 044907



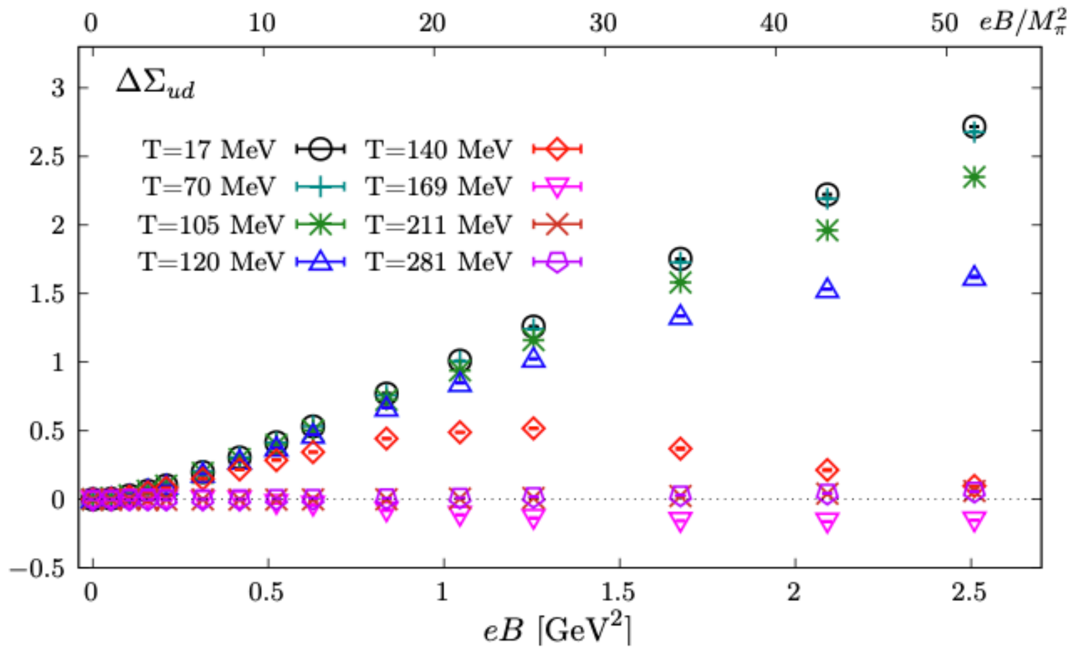
$eB_{\tau=0} \sim 3M_\pi^2$ in RHIC

$eB_{\tau=0} \sim 40M_\pi^2$ in LHC

Anping Huang et al.Phys.Lett.B 777 (2018) 177-183
A. Bzdak et al. Physics Reports 853 (2020) 1–87

eB induced effects

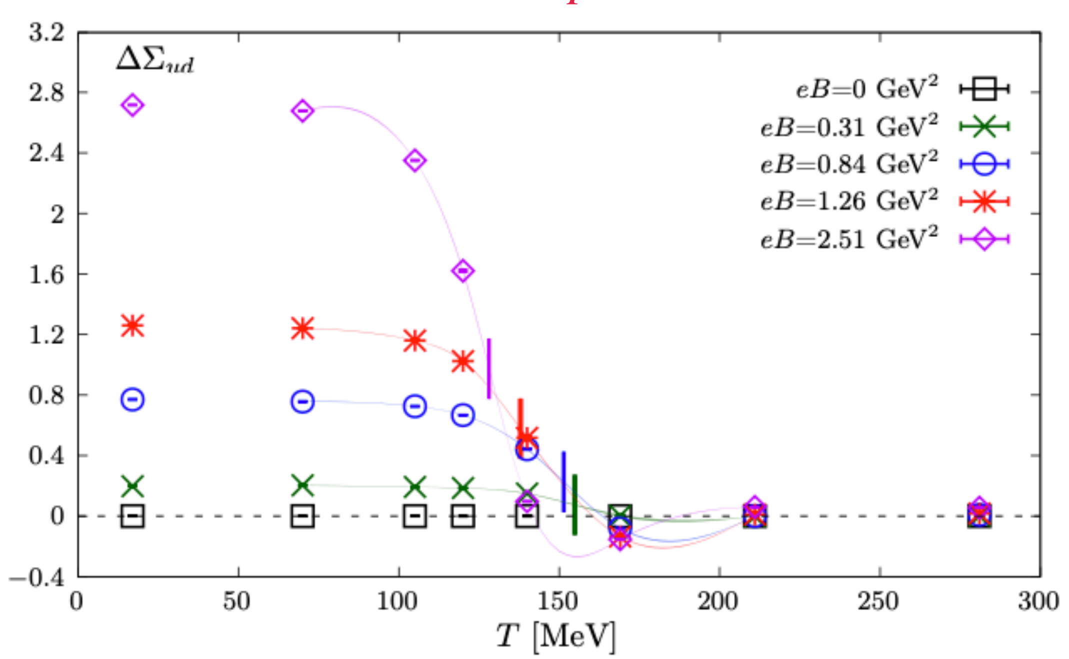
Inverse magnetic catalysis



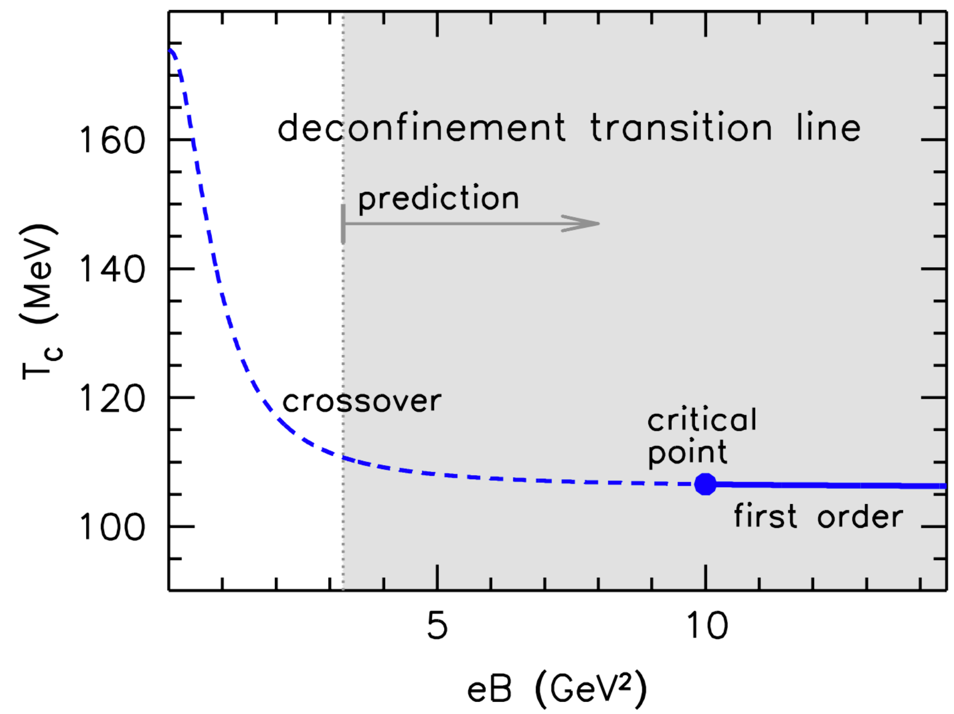
H.-T.Ding et al. ,Phys.Rev.D 105 (2022) 3, 034514

also see G. S. Bali et al. Phys. Rev. D86 (2012) 071502

$eB \uparrow \; T_{pc} \downarrow$

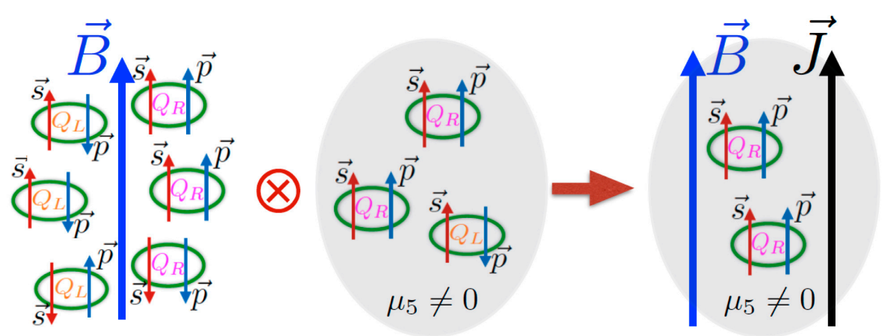


CEP in T-eB plane



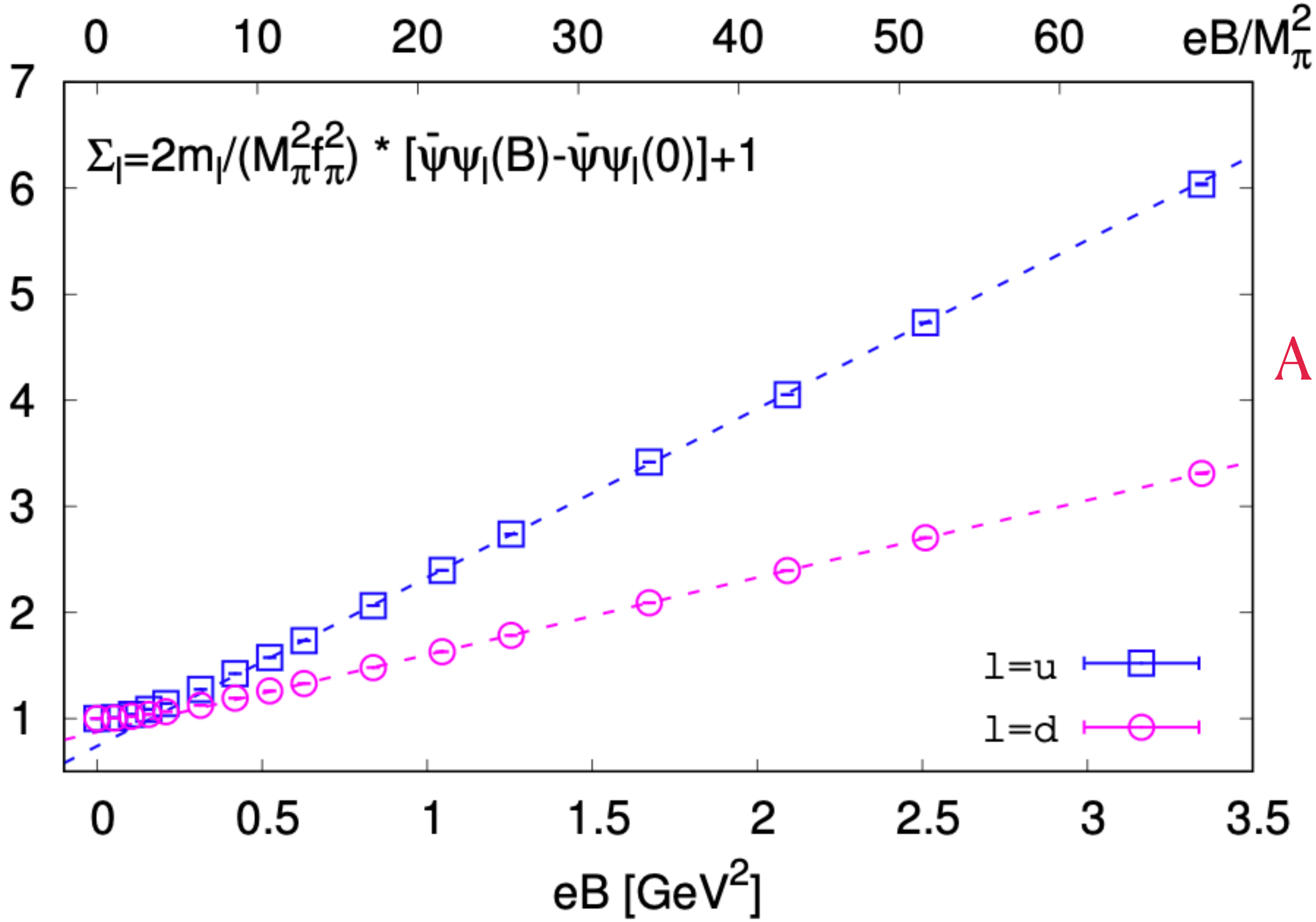
G. Endrodi, JHEP 1507(2015) 173

Chiral magnetic effects



see recent reviews e.g.
D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



A clear effect but Not accessible in HIC experiments!

H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001
See also in reviews e.g. M. D'Elia, Lect.NotesPhys.871(2013)181

Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k} \right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k} \right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$

$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$

$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}\,.$$

See recent reviews:
LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

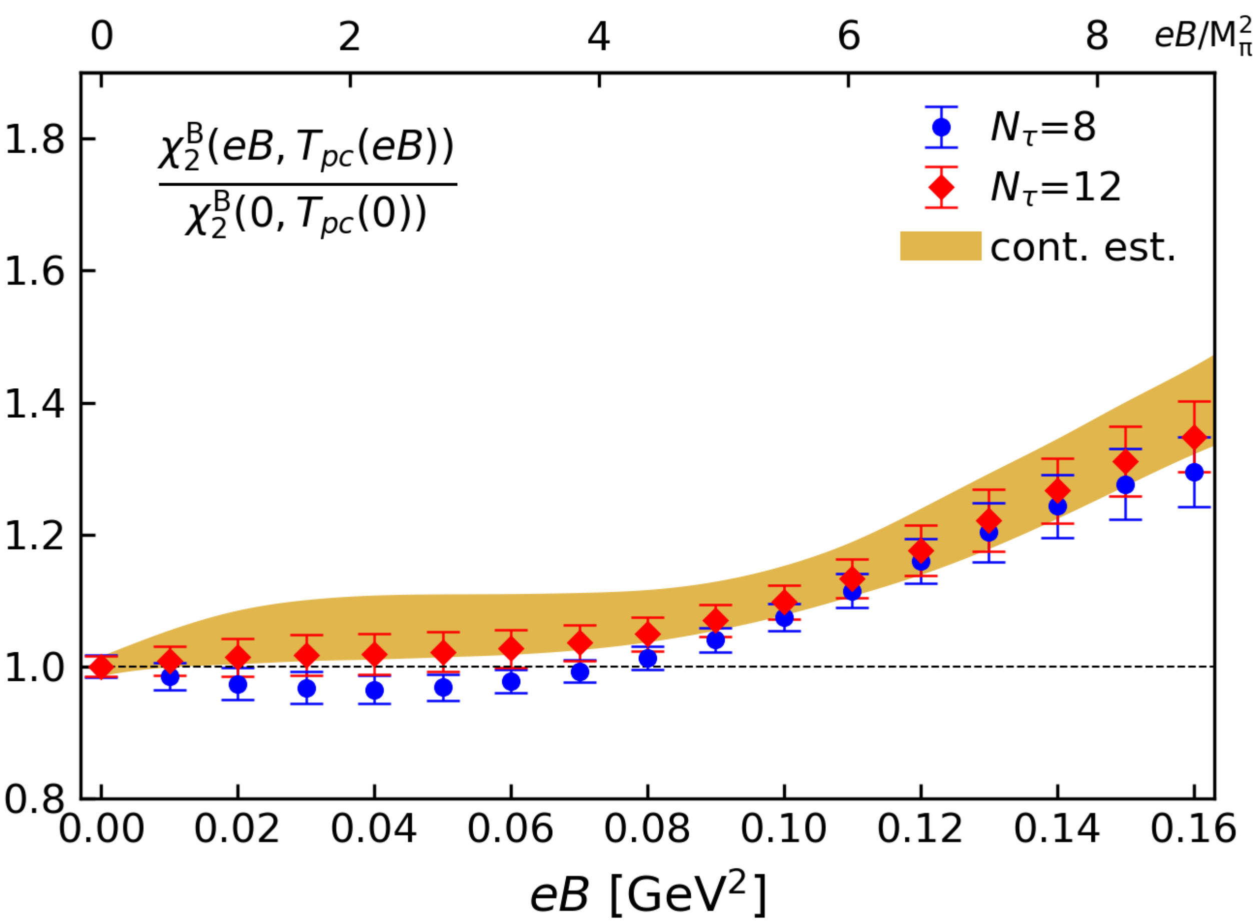
At $eB \neq 0$ a lot more need to be explored

HRG: *G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301, Bhattacharyya et al., EPL115(2016)62003*

PNJL: *W.-J. Fu, Phys. Rev. D 88 (2013) 014009*

Ratio for 2nd order diagonal fluctuations

N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV, with HISQ action

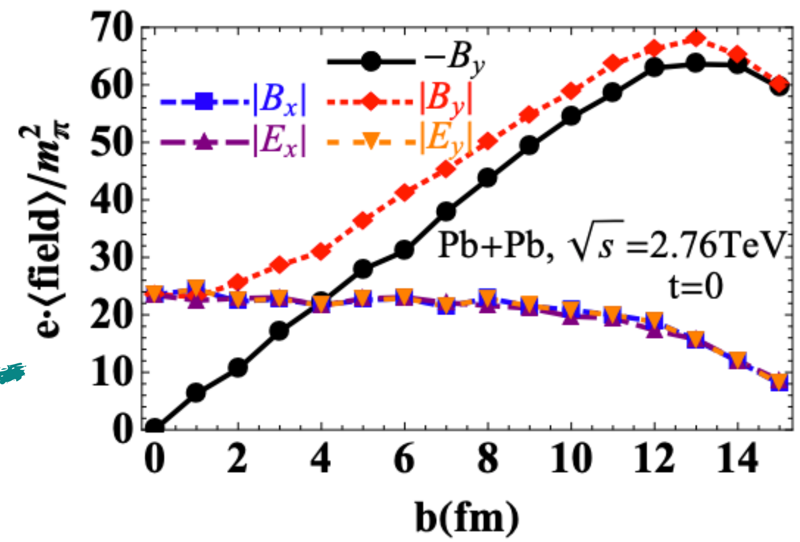


$$\frac{X\left(\mathrm{eB}, T_{pc}(\mathrm{eB})\right)}{X\left(0, T_{pc}(0)\right)}$$: R_{cp} like observable

At $eB \simeq 9M_\pi^2$: ~1.3-1.4

Central Collisions

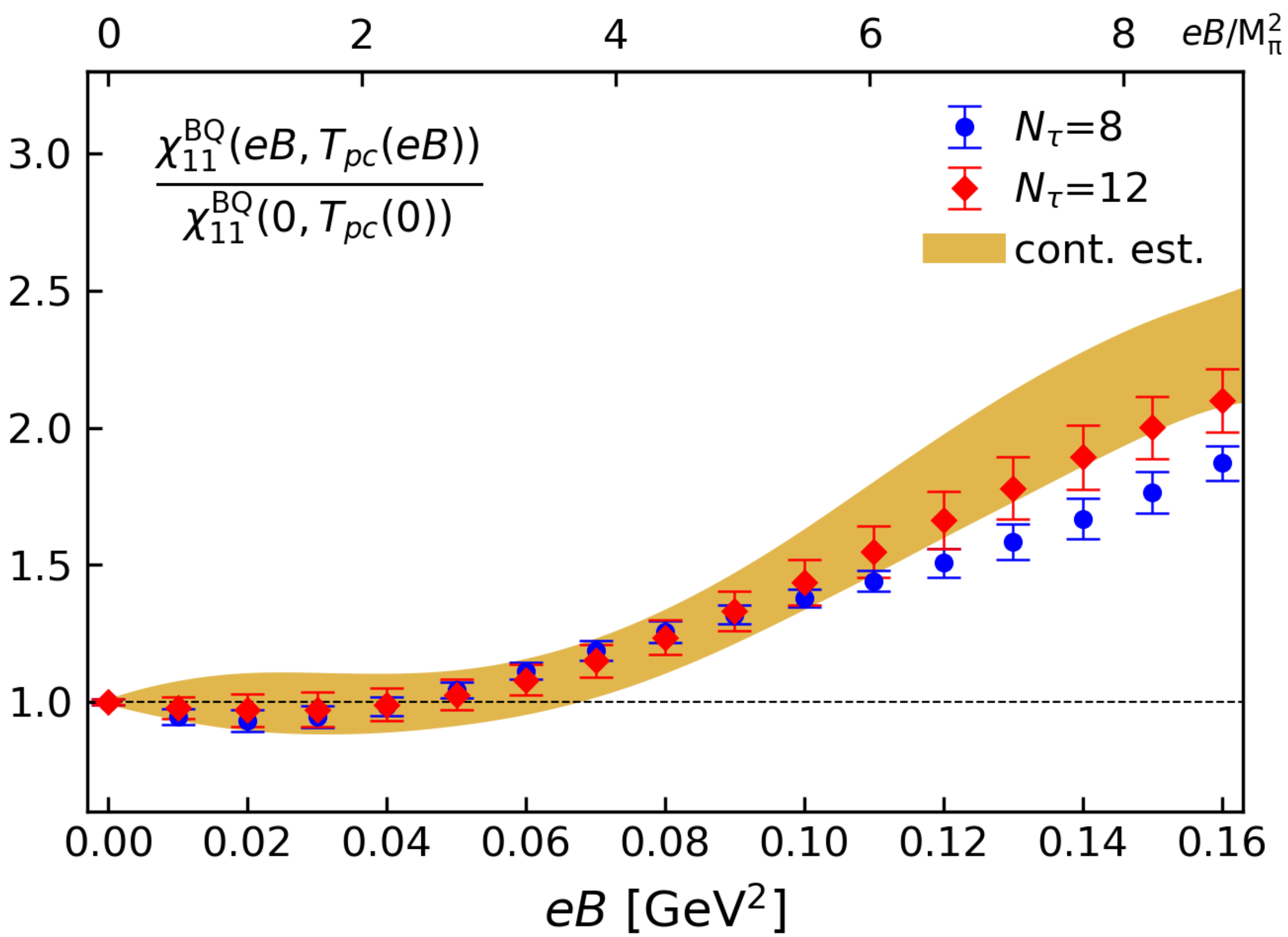
Peripheral collisions



Wei-Tian Deng, Xu-Guang Huang Phys.Rev.C 85 (2012) 044907

Ratio for 2nd order off-diagonal fluctuations

Nf=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV,with HISQ action



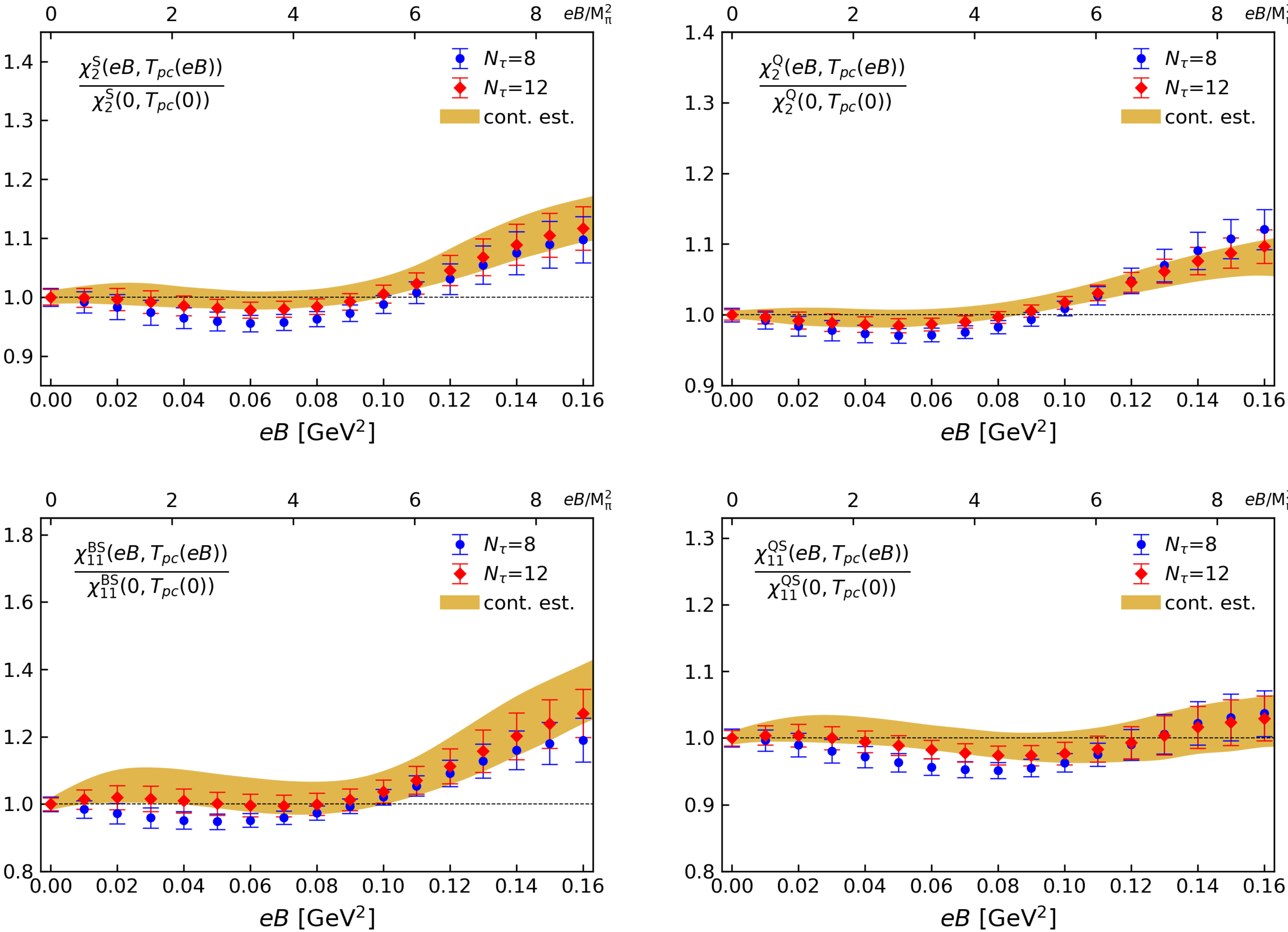
Central Collisions

Peripheral collisions

$$\frac{X\left(\mathrm{eB}, T_{pc}(\mathrm{eB})\right)}{X\left(0, T_{pc}(0)\right)}$$: R_{cp} like observable

At $eB \simeq 9M_\pi^2$: ~2-2.4

Ratio for other 2nd order fluctuations



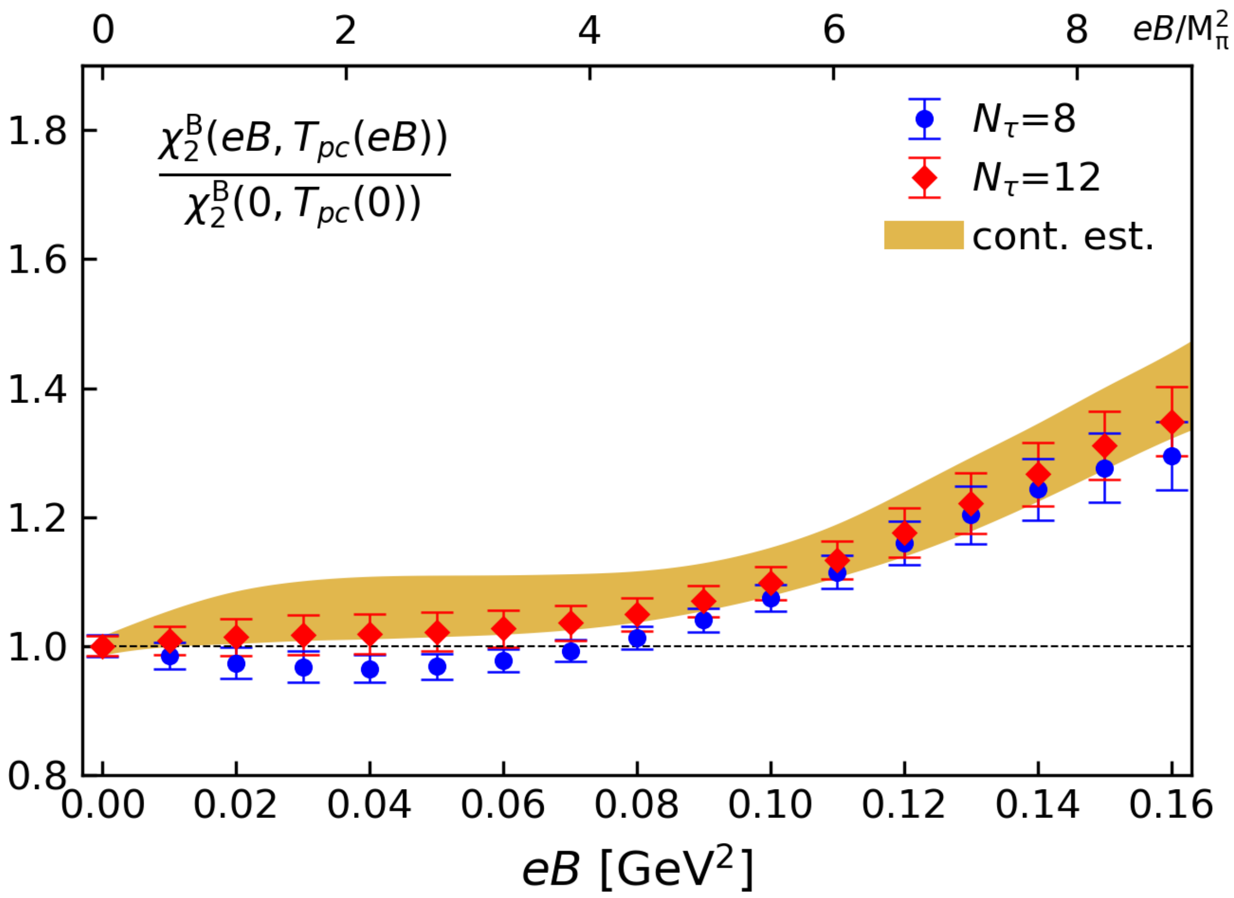
At $eB \simeq 9M_\pi^2$:

Ratio of χ_2^S ~ 1.1
Ratio of χ_2^Q ~ 1.07

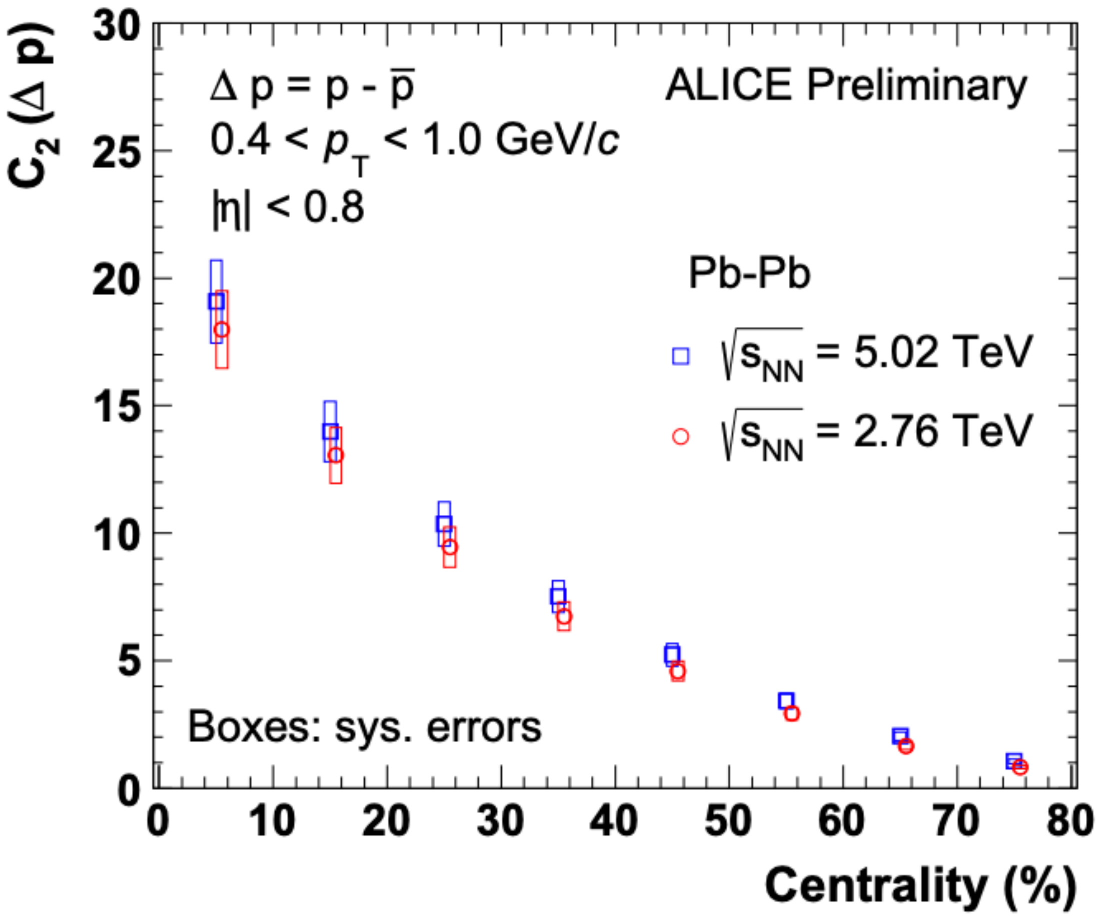
Ratio of χ_{11}^{BS} ~ 1.25
Ratio of χ_{11}^{QS} ~ 1.03

Lattice QCD meets experiment

Lattice QCD



Proxy of χ_2^B



$C_2 = VT^3\chi_2^B$

Volume!

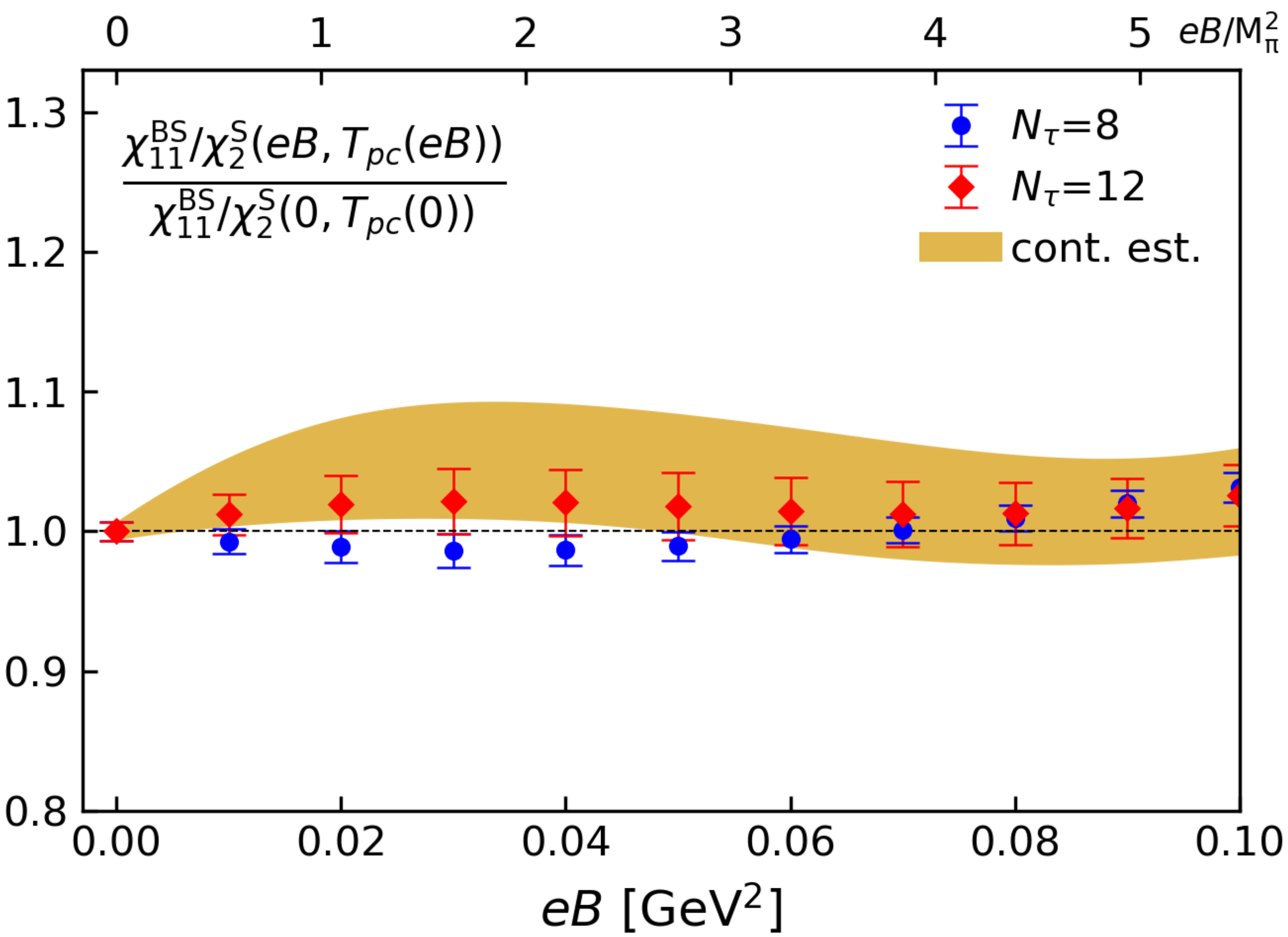
Smaller eB ⟷ Larger eB

Smaller eB ⟷ Larger eB

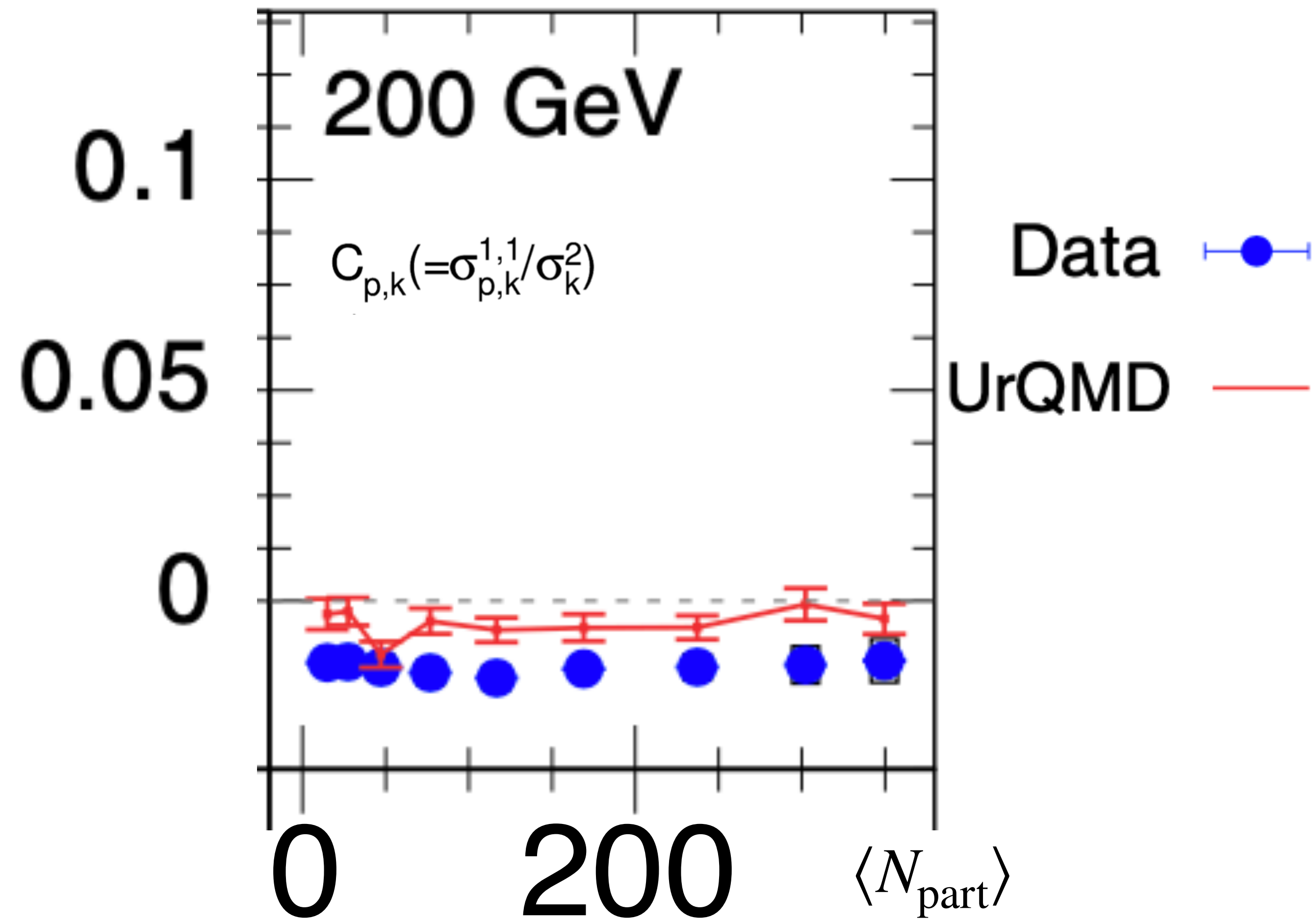
ALICE: Nucl.Phys.A 982 (2019) 851

Lattice QCD meets experiment

Lattice QCD



Proxy of χ_{11}^{BS}/χ_2^S

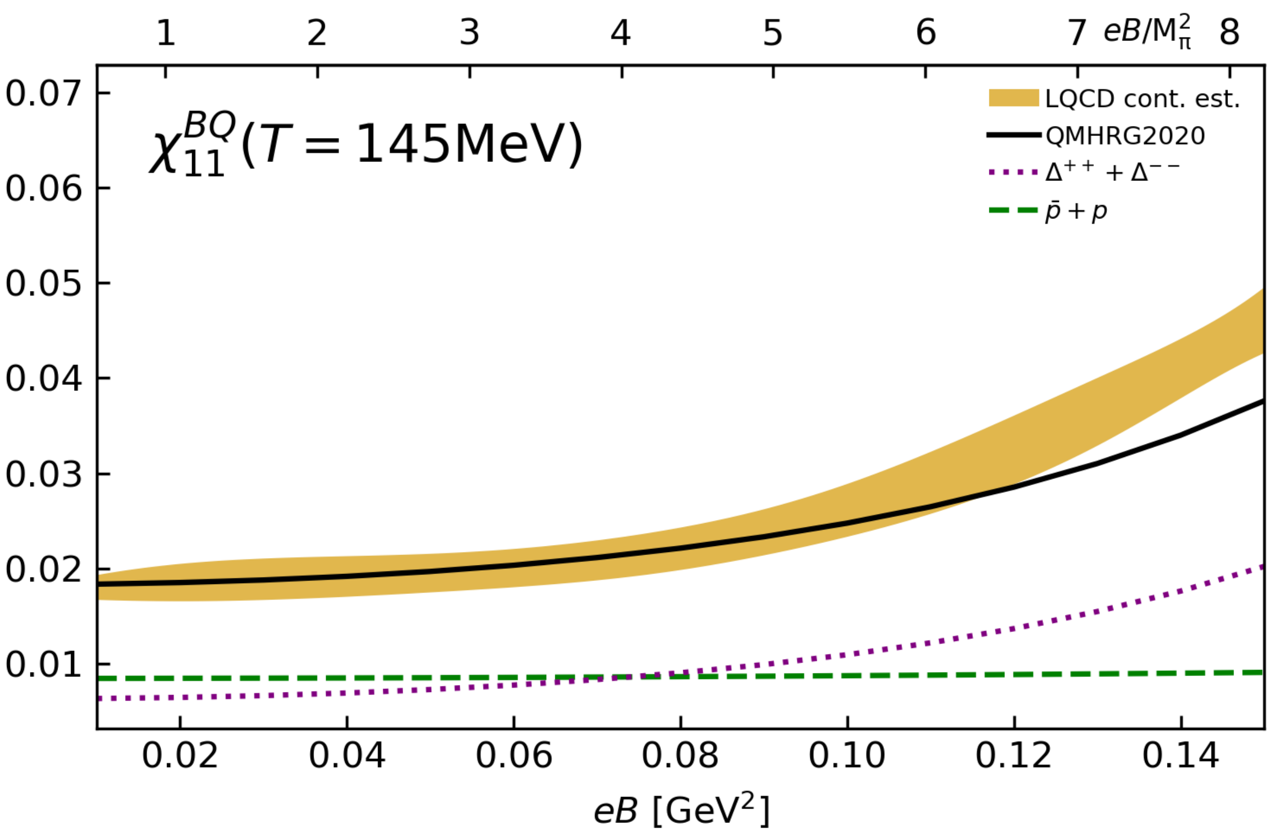


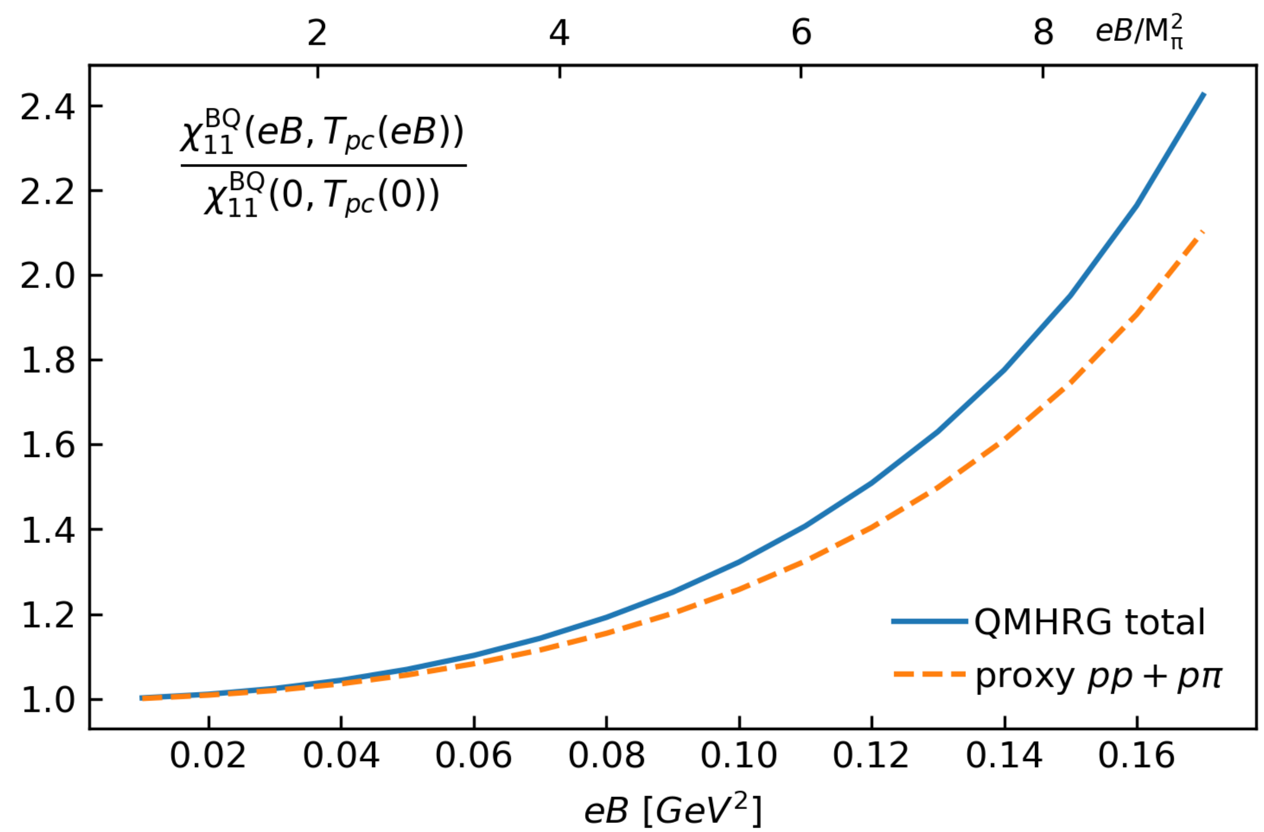
Smaller *eB* ⟷ Larger *eB*

Larger *eB* ⟷ Smaller *eB*

STAR, Phys.Rev.C 100 (2019) 1, 014902

Hadron resonance gas model





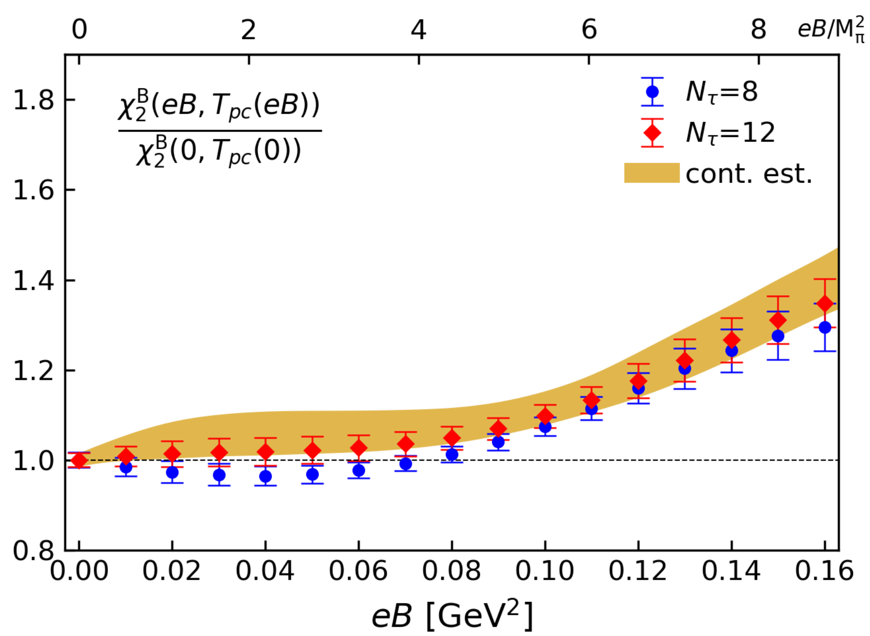
Preliminary

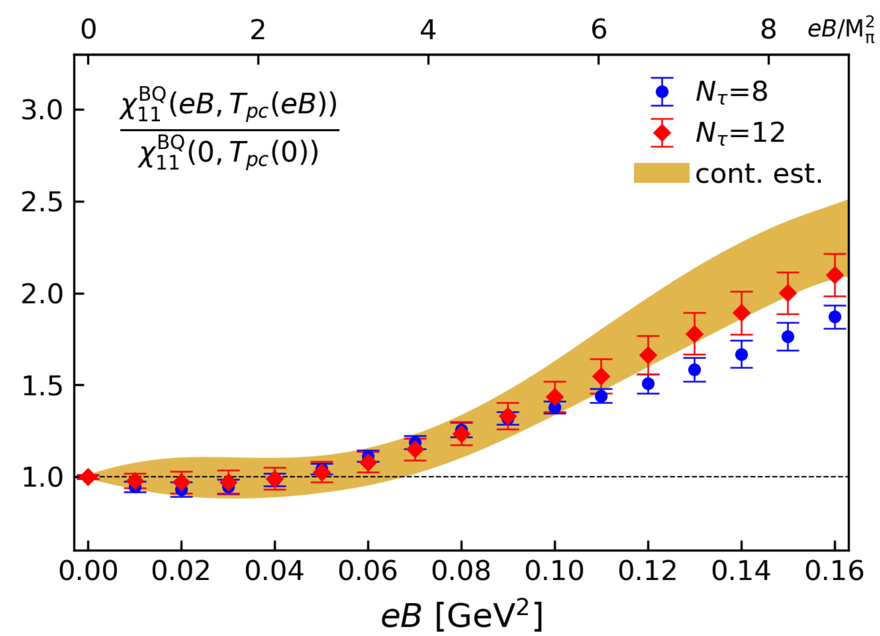
$$\frac{p_c^{M/B}}{T^4} = \frac{\left|q_i\right| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{e^{k\mu_i/T}}{k} \mathrm{K}_1\left(\frac{k\varepsilon_0}{T}\right)$$

$$\varepsilon_0 = \sqrt{m_i^2 + 2\left|q_i\right| B\left(l + 1/2 - s_z\right)}$$

Summary and outlook

- The 2nd order fluctuations and correlations of B,Q & S are strongly affected by *eB*
- R_{cp} like quantity could be useful to detect the existence of the magnetic field in HIC





work in progress

- Construct suitable proxies for the fluctuations via HRG

Thank you for your attention!

Backup

B pointing to the z direction

$$u_x\left(n_x, n_y, n_z, n_\tau\right) = \begin{cases} \exp\left[-iqa^2BN_xn_y\right] & (n_x = N_x - 1) \\ 1 & \text{(otherwise)} \end{cases}$$

$$u_y\left(n_x, n_y, n_z, n_\tau\right) = \exp\left[iqa^2Bn_x\right]$$

$$u_z\left(n_x, n_y, n_z, n_\tau\right) = u_t\left(n_x, n_y, n_z, n_\tau\right) = 1$$

No sign problem !

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

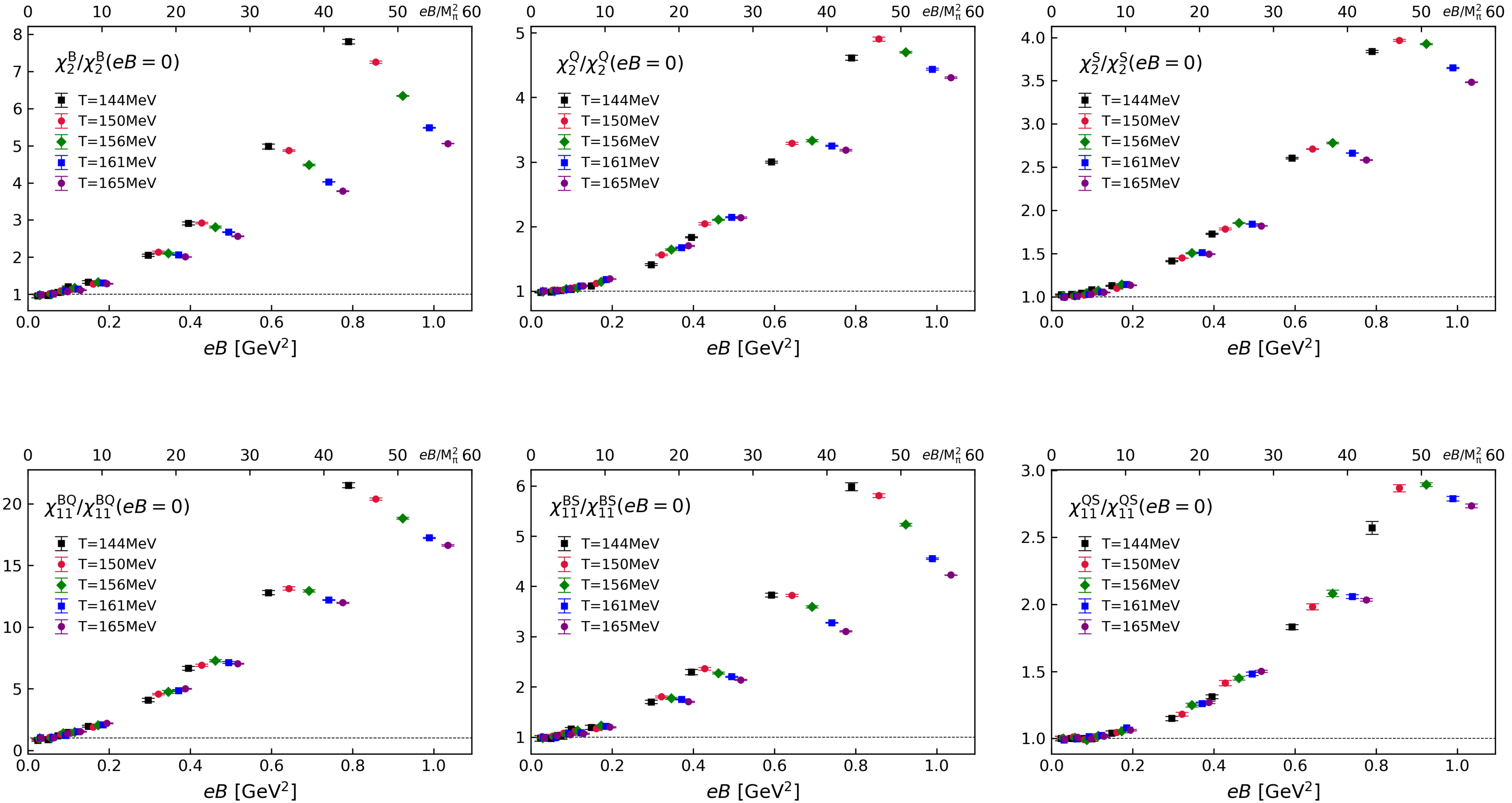
Quantization of the magnetic field

$$q_u = 2/3e$$
$$q_d = -1/3e$$
$$q_s = -1/3e$$

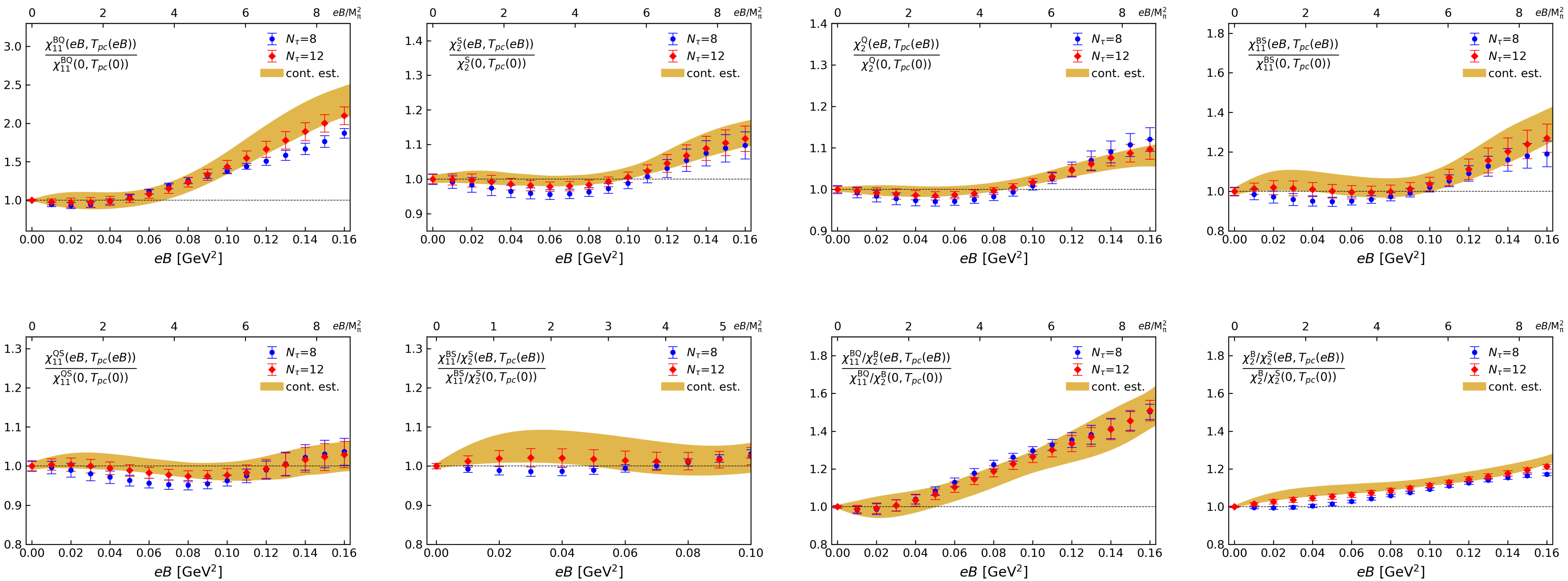
→

$$eB = \frac{6\pi N_b}{N_xN_y}a^{-2}$$

Second order fluctuation in $N_\tau = 8$ case



$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV,with HISQ action


$\frac{\chi_{11}^{BQ}(eB, T_{pc}(eB))}{\chi_{11}^{BQ}(0, T_{pc}(0))}$
$\frac{\chi_{2}^{S}(eB, T_{pc}(eB))}{\chi_{2}^{S}(0, T_{pc}(0))}$
$\frac{\chi_{2}^{Q}(eB, T_{pc}(eB))}{\chi_{2}^{Q}(0, T_{pc}(0))}$
$\frac{\chi_{11}^{BS}(eB, T_{pc}(eB))}{\chi_{11}^{BS}(0, T_{pc}(0))}$
$\frac{\chi_{11}^{QS}(eB, T_{pc}(eB))}{\chi_{11}^{QS}(0, T_{pc}(0))}$
$\frac{\chi_{11}^{BS}/\chi_{2}^{S}(eB, T_{pc}(eB))}{\chi_{11}^{BS}/\chi_{2}^{S}(0, T_{pc}(0))}$
$\frac{\chi_{11}^{BQ}/\chi_{2}^{B}(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\chi_{2}^{B}(0, T_{pc}(0))}$
$\frac{\chi_{2}^{B}/\chi_{2}^{S}(eB, T_{pc}(eB))}{\chi_{2}^{B}/\chi_{2}^{S}(0, T_{pc}(0))}$
N_τ=8
N_τ=12
cont. est.
eB [GeV^2]
eB/M_π^2

Lattice QCD setup

$m_s = m_s^{\text{phy}}, m_l = m_l^{\text{phy}}, m_\pi \sim 135\text{MeV}$

The N_σ is fixed to $32,48; N_\sigma = N_x = N_y = N_z$

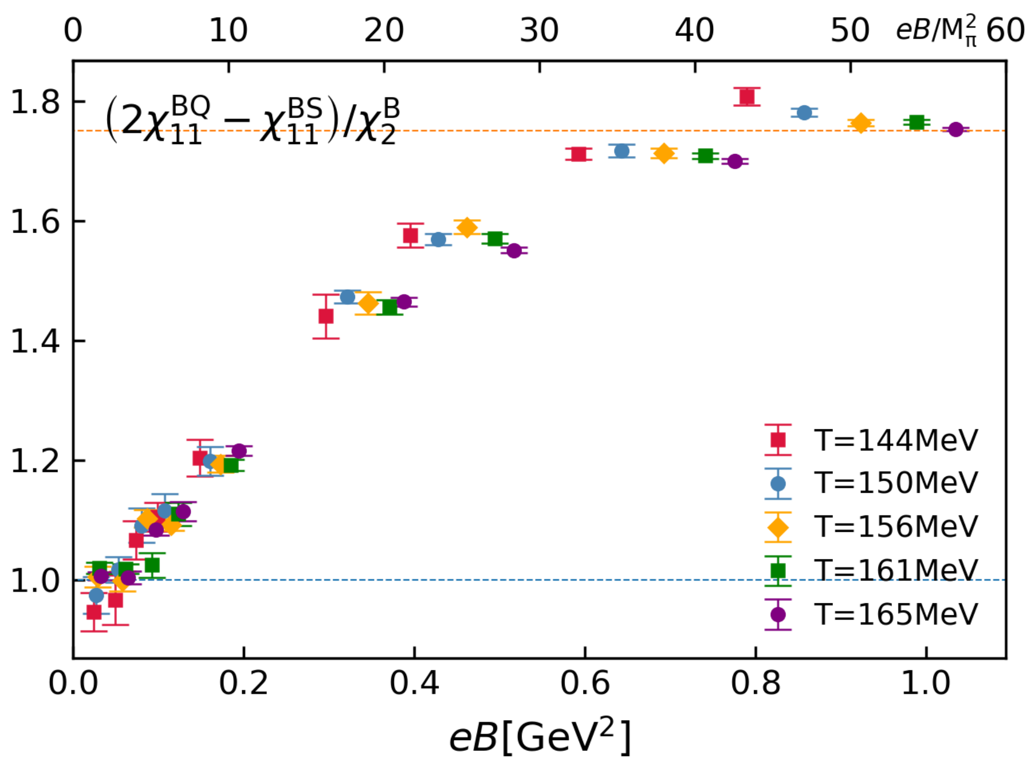
The N_τ is fixed to $8,12$

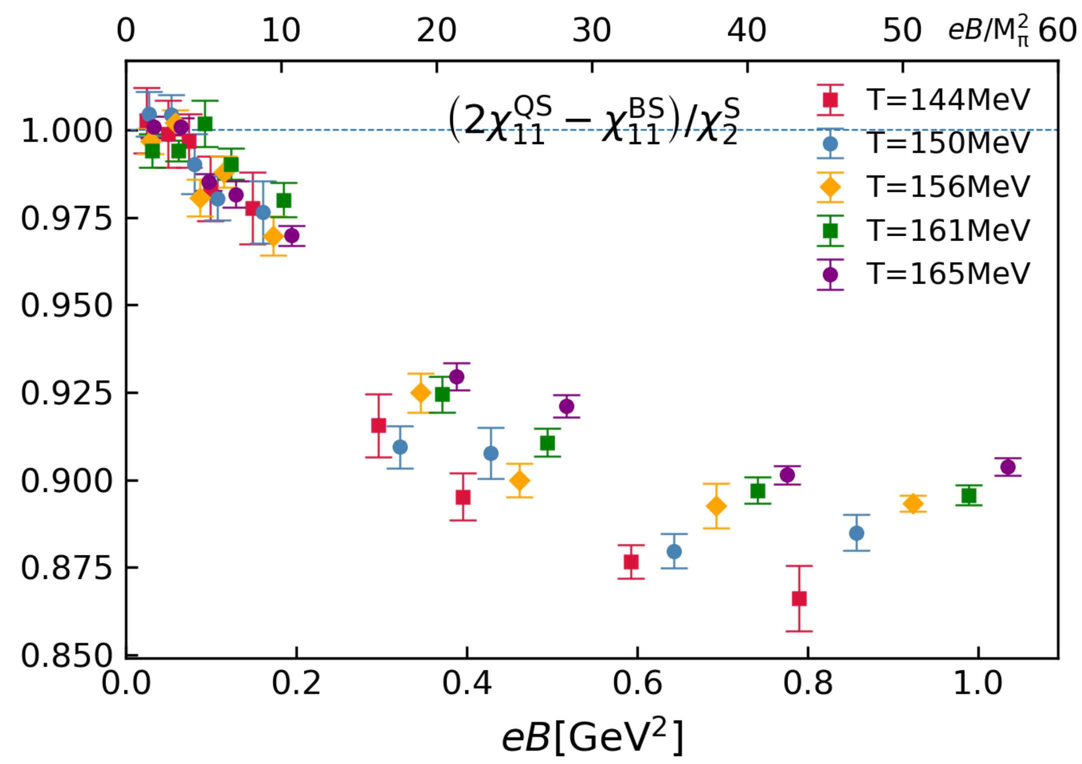
T window: $(144\text{MeV},165\text{MeV})$ around $(0.9T_{pc},1.1T_{pc})$

a is changed to get the targeted T , $T = \frac{1}{aN_\tau}$

eB window: $\left(0,1\text{GeV}^2\right)$

Isospin symmetry breaking in $N_\tau = 8$ lattice





Due to $\chi_{11}^{us} = \chi_{11}^{ds}$ at $eB = 0$ case, we get:

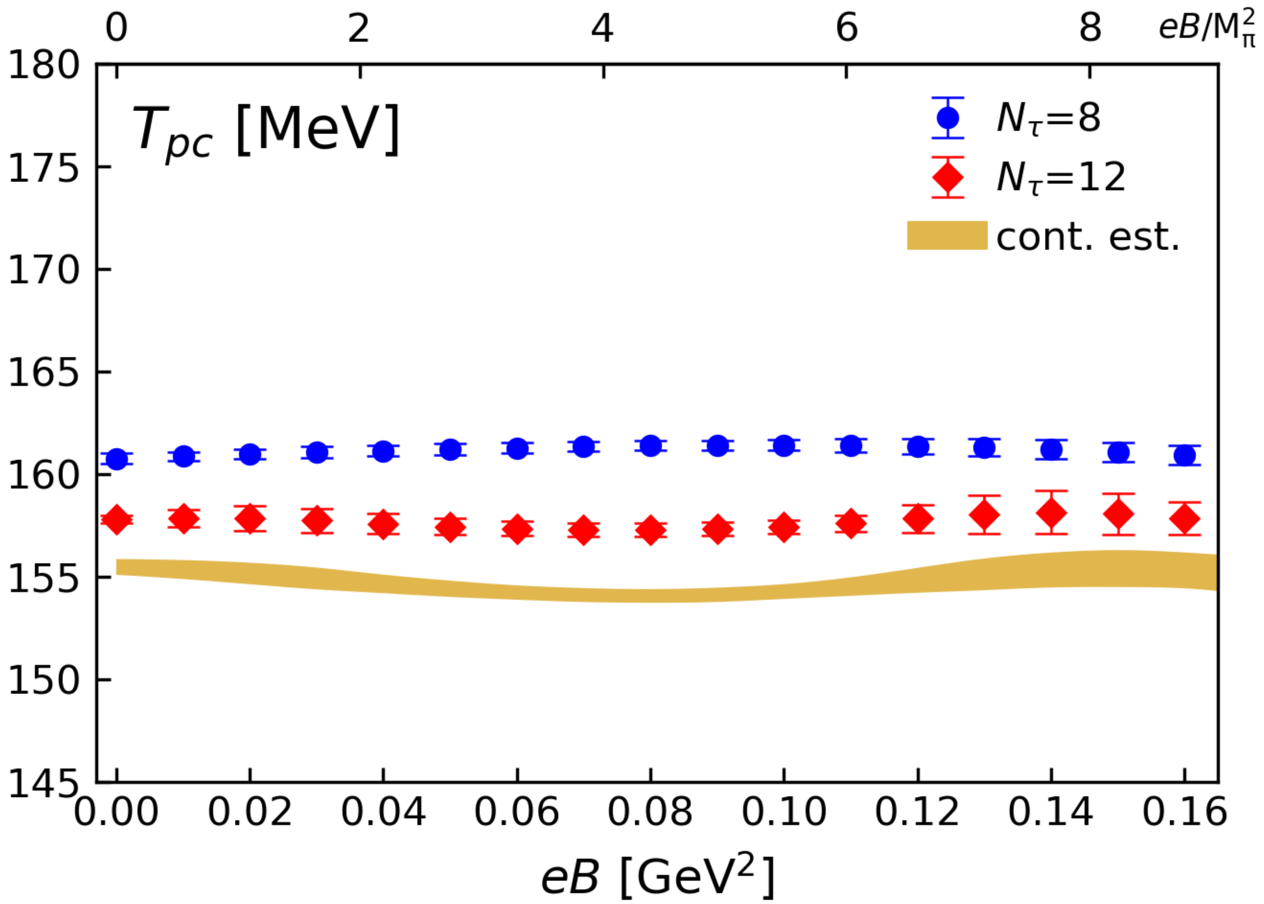
$$2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_2^S,$$
$$2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_2^B$$

Transition line on $T - eB$ plane

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right]$$

$$\chi^{\Sigma} = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma$$

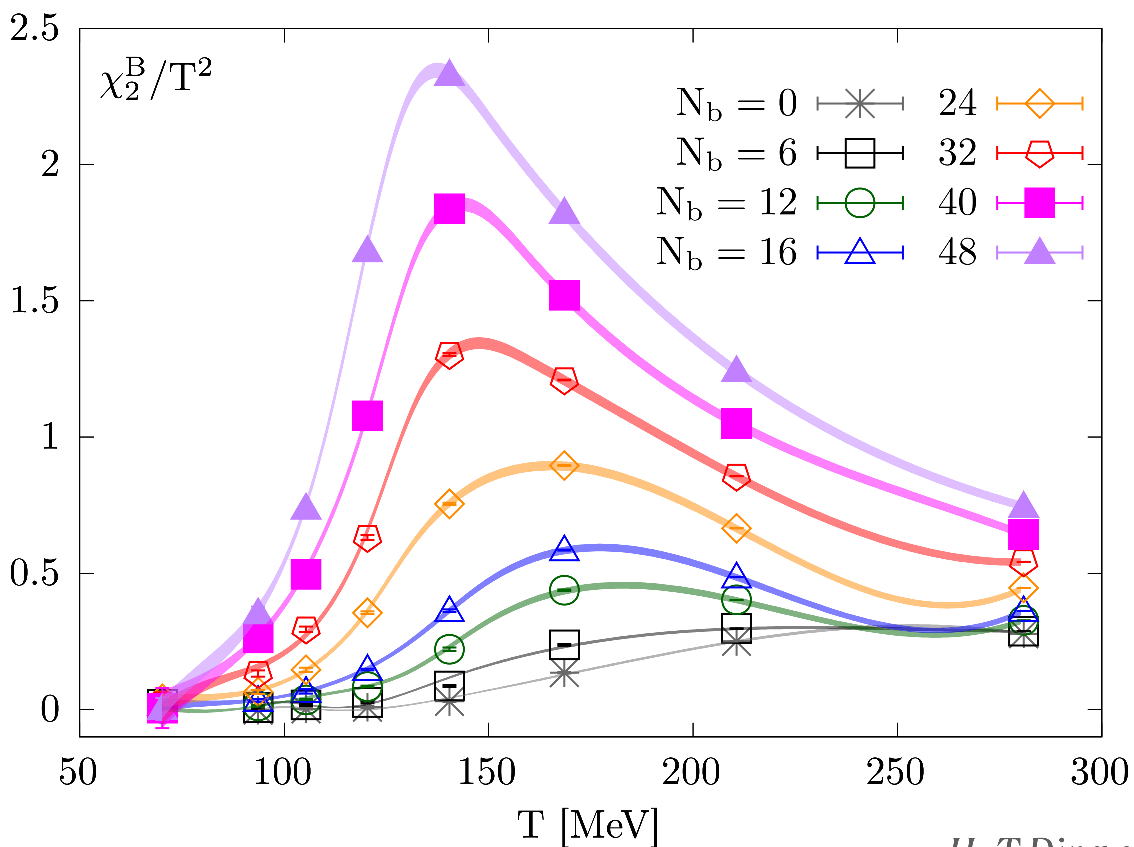
Finding the peak location of χ^{Σ} at each eB value

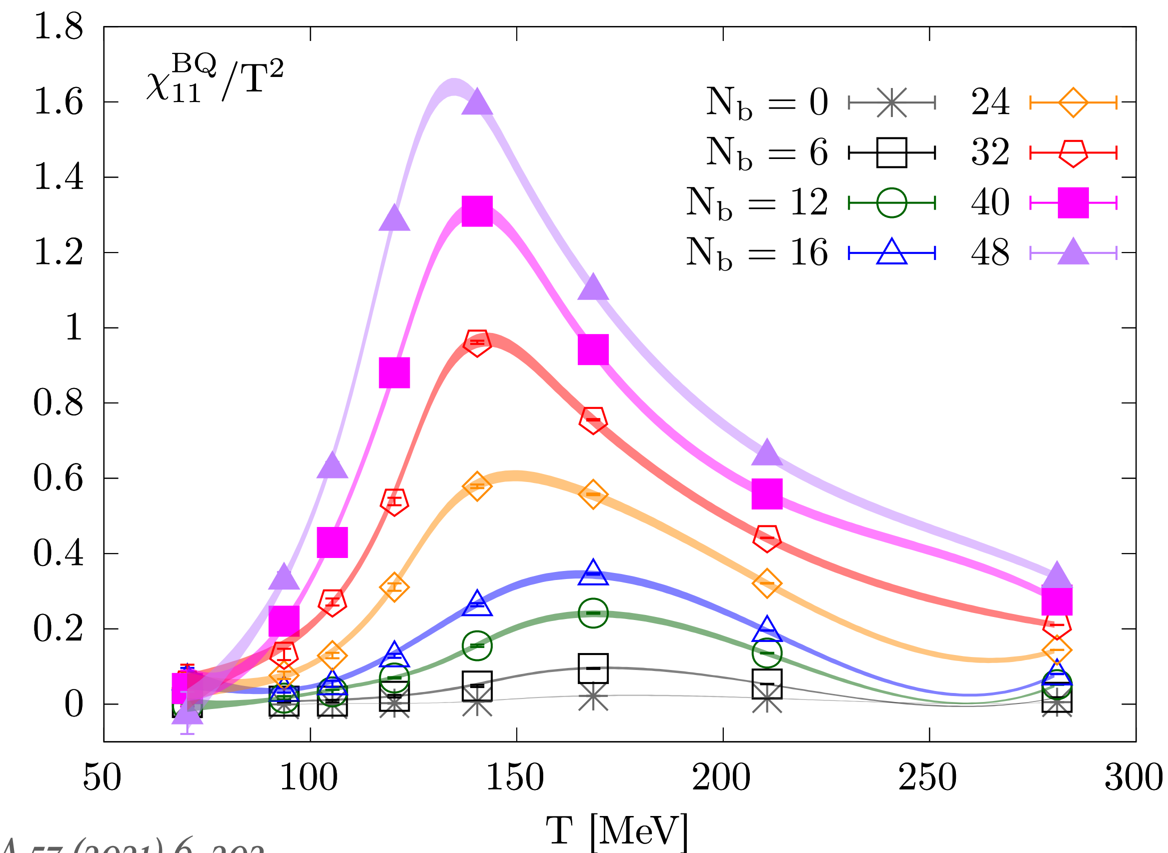


Second order fluctuation from Lattice QCD

No sign problem !

Nf=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, with $a^{-1} \approx 1.7$ GeV and HISQ action, fixed a approach ($T = a^{-1}/N_\tau$)





H.-T.Ding et al. , Eur.Phys.J.A 57 (2021) 6, 202

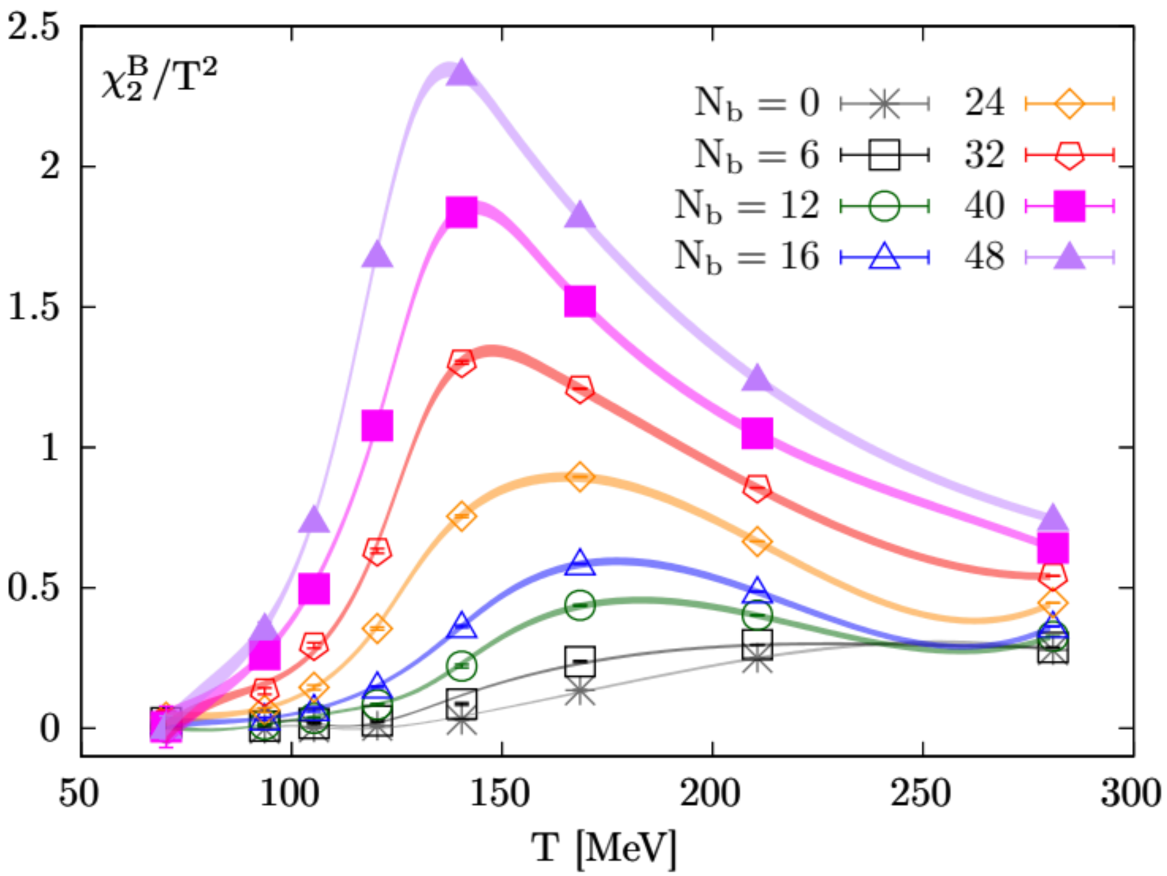
Peak locations shift to lower T in a stronger magnetic field.

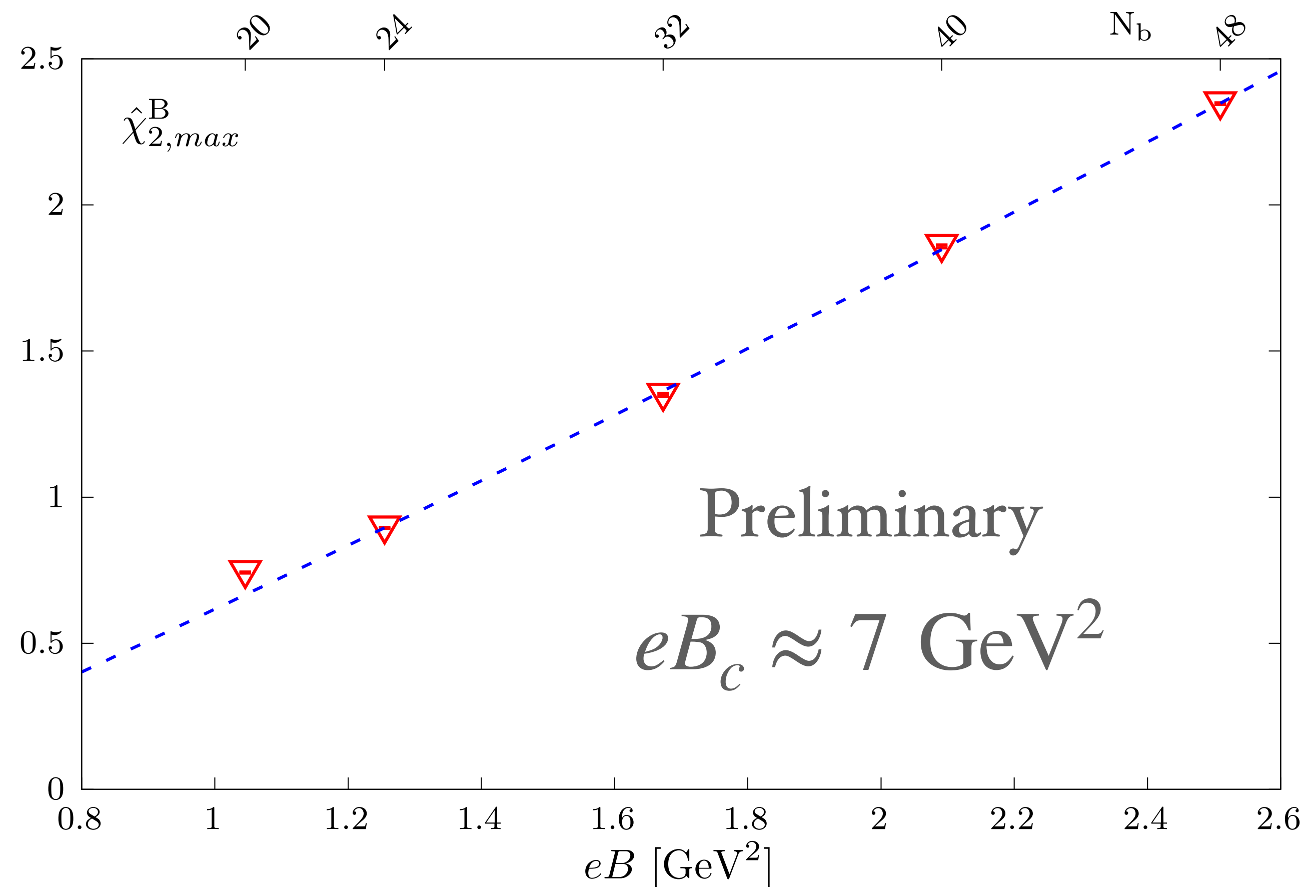
Peak height becomes higher in a stronger magnetic field.

Consistent with the reduction of Tpc in a stronger magnetic field

Close to the critical end point in T-eB plane?

An estimate of the location of CEP in T-eB plane





At $eB = 0$:

$$\chi_n^B \propto (-2\kappa_q)^{n/2} h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z)$$

Friman et al., Eur. Phys. J. C 71(2011)1694

$$\chi^B_{2,max} = b\,(eB_c - eB)^{(1-\alpha)/\beta\delta} + d$$

	$\beta\delta$	α	$(1-\alpha)/\beta\delta$
Z(2)	1.5654	0.1088	0.5693

1st order phase transition observed ~ 9 GeV^2 M. D'Elia et al. Phys.Rev.D 105 (2022) 3, 034511