

Spin Polarization induced by magnetic field and rotation

Mei HUANG (黄梅)



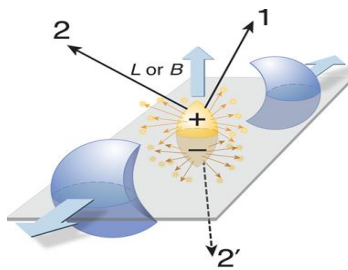
中国科学院大学

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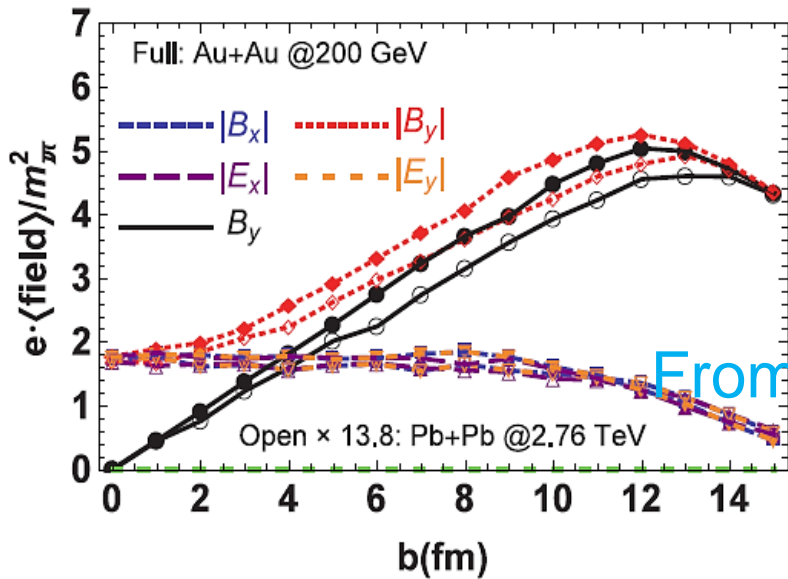
中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会，

2022年8月8-11日

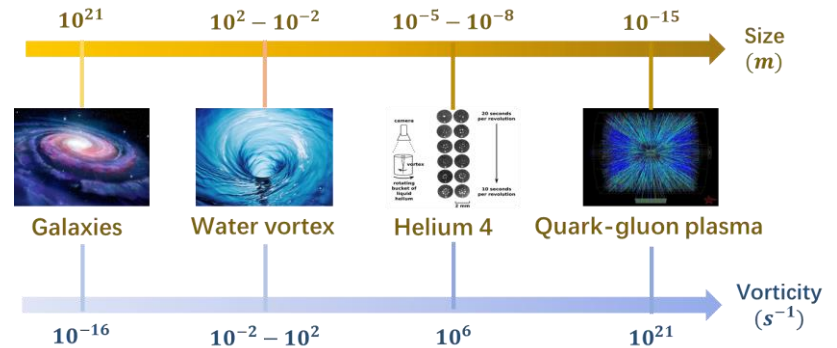
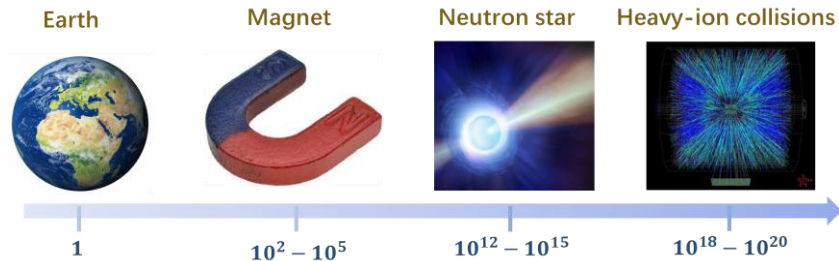
MAGNETIC FIELDS



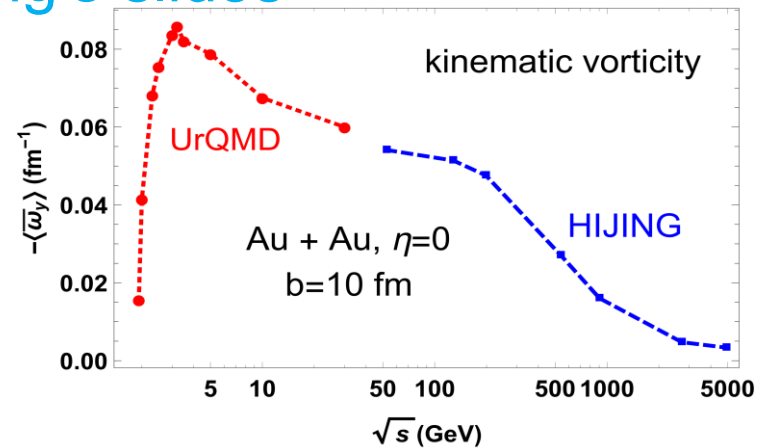
ROTATION



HIJING (Deng-XGHuang PRC2012)



From Xuguang's slides

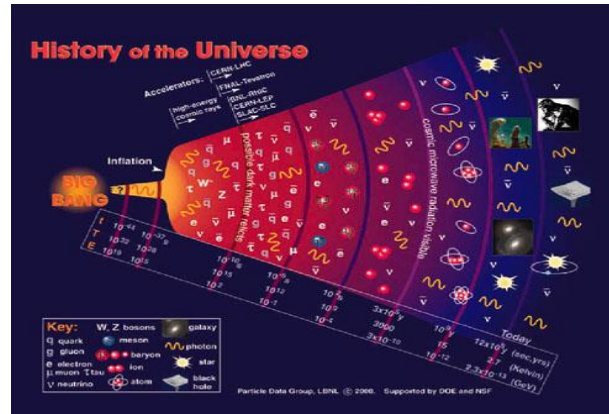
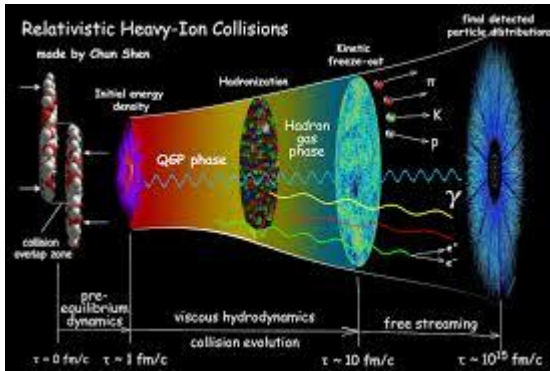


Deng-XGHuang PRC2016

Deng-XGHuang-GLMa-Zhang PRC2020

QCD matter under extreme conditions

$$T, \mu_B, \mathbf{B}, \mathbf{E} \cdot \mathbf{B}, \omega, \mu_I, L$$



LHC,RHIC,FAIR,NICA,HIAF

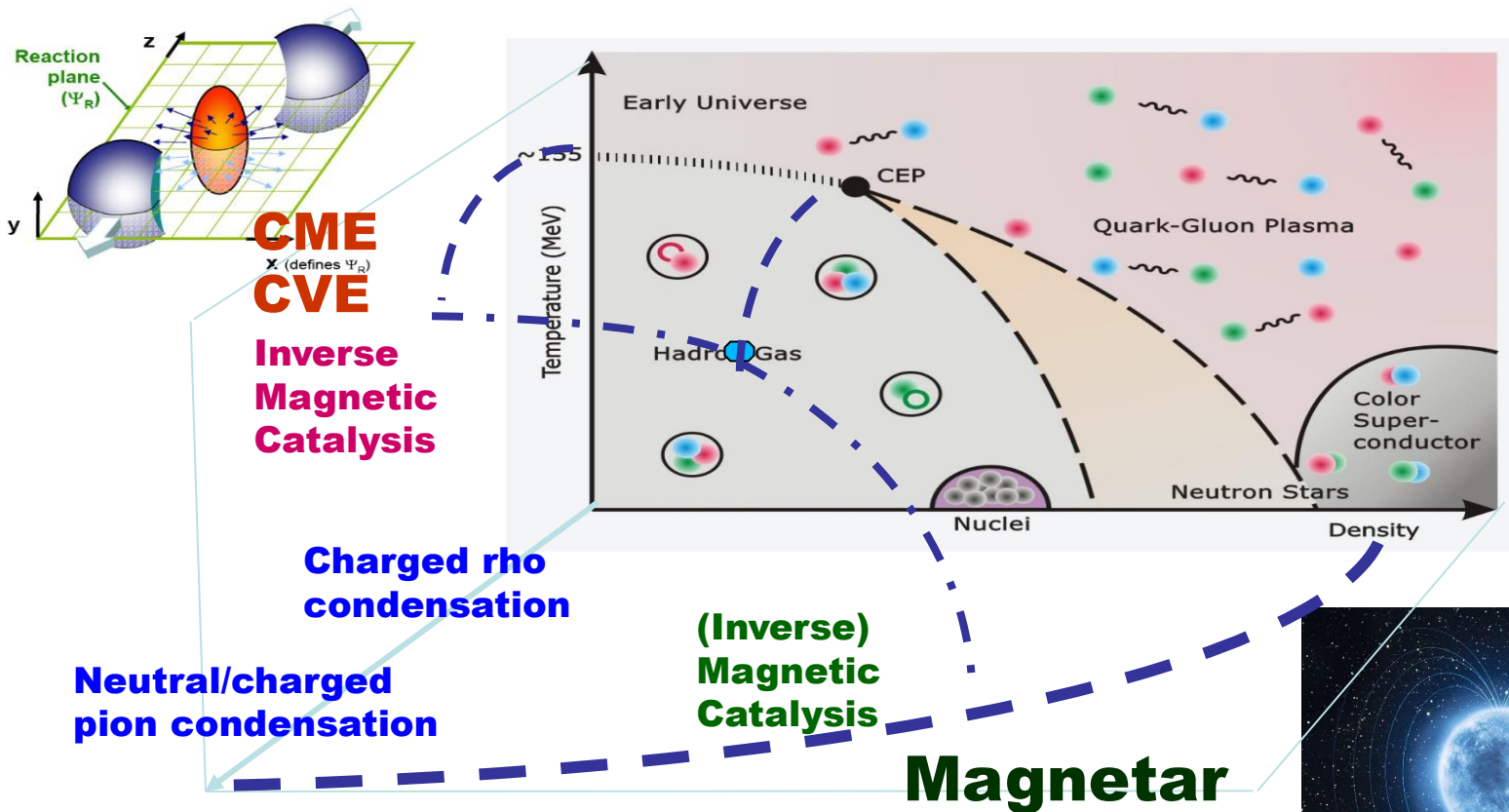
Early universe

Neutron star



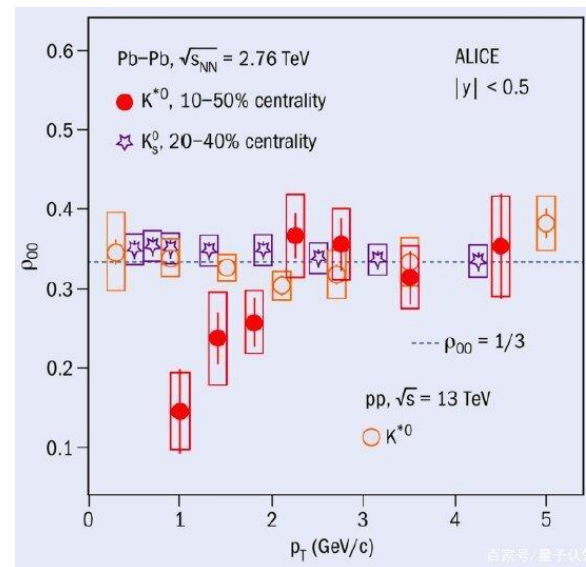
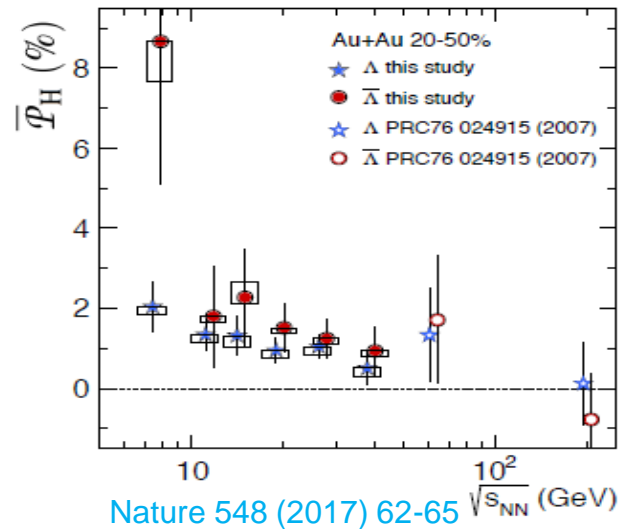
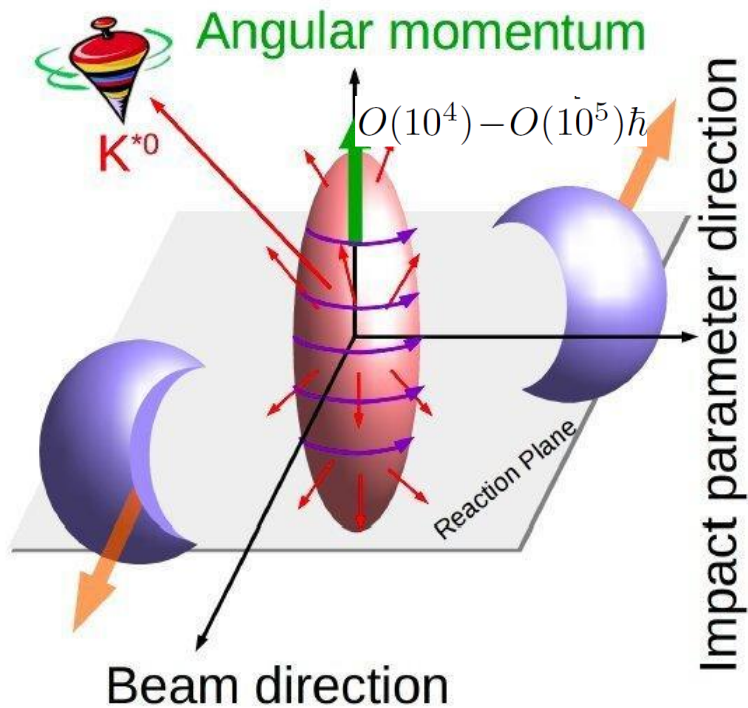
Neutron star merge \rightarrow BH

Explored QCD phase diagram *by theorists*



$$\mathbf{B}, \mathbf{E} \cdot \mathbf{B}, \omega, \mu_I, \mathbf{L}$$

What happens under rotation?



Physical Review Letters (2020).
DOI: 10.1103/PhysRevLett.125.012301

What happens under magnetic fields?

- 1) Magnetic catalysis at zero temperature;
- 2) Inverse magnetic catalysis around T_c ;
- 3) Diamagnetism at low T paramagnetism at high T ;
- 4) Meson mass under magnetic field does not show point particle behavior;
- 5) Charged Vector meson condensation?

1) Magnetic catalysis at zero temperature

S.P. Klevansky and R. H. Lemmer ('89); H. Suganuma and T. Tatsumi ('91);
V. P. Gusynin, V. A. Miransky and I. A. Shovkovy ('94, '95, '96,...)

$$\mathcal{L} = \bar{\Psi} i\gamma^\mu D_\mu \Psi + \frac{G}{2} \left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\Psi)^2 \right]$$

$$D_\mu = \partial_\mu - ieA_\mu^{\text{ext}}, \quad \mathbf{A}^{\text{ext}} = (0, Bx^1, 0)$$

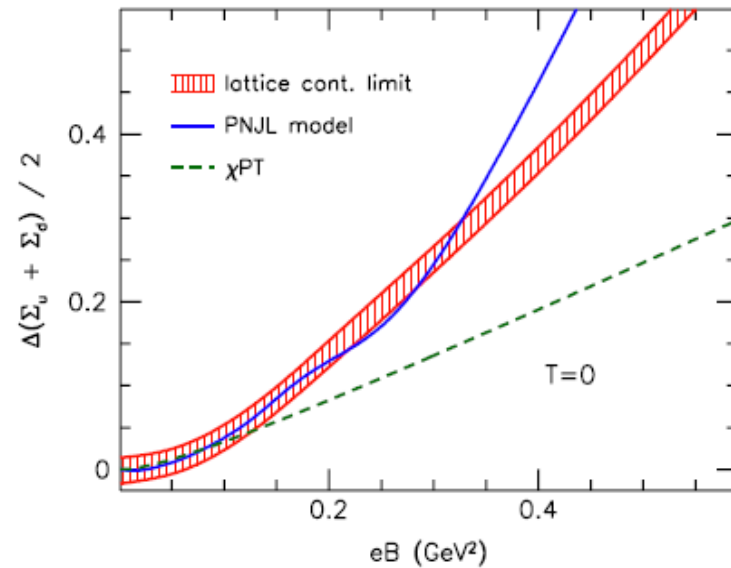
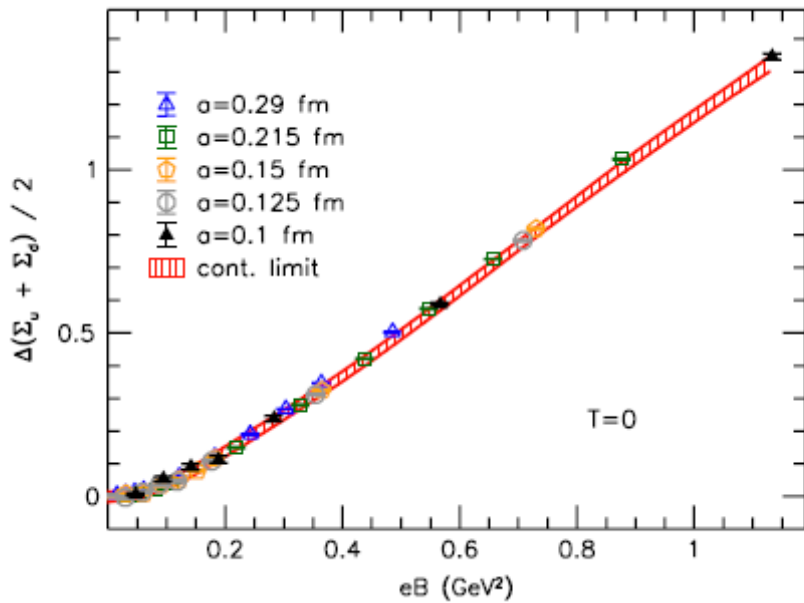
$$m = G \text{tr}[S(x,x)] \approx \frac{Gm}{(2\pi)^2} \left(\Lambda^2 + |eB| \ln \frac{|eB|}{\pi m^2} + O(m^2) \right)$$

$$m \propto \exp\left(-\frac{2\pi^2}{G|eB|}\right)$$

nonzero mass for arbitrary small G

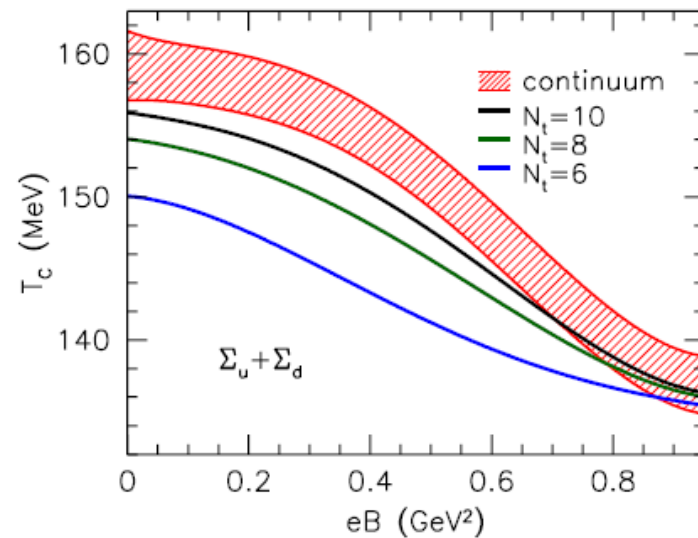
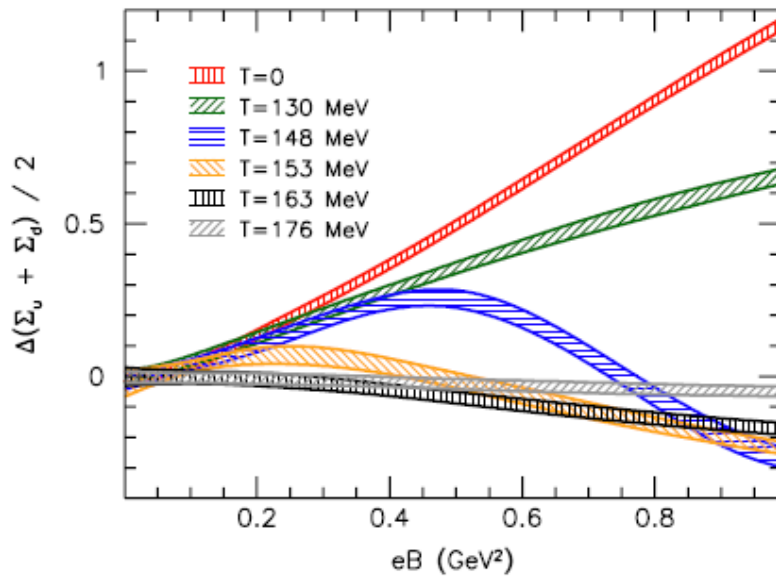
1) Magnetic catalysis at zero temperature

Bali et.al. arXiv:1206.4205 [hep-lat]



2) Inverse magnetic catalysis around the critical temperature

Bali et.al. arXiv:1206.4205 [hep-lat]



Surprise !!!

Some important information is missing in our understanding of chiral phase transition, which is enhanced by magnetic field!

How to understand inverse magnetic catalysis ?

1) Magnetic inhibition K. Fukushima, Y. Hidaka, PRL 110, 031601 (2013)

Contribution from neutral pions

2) Contribution from sea quarks

Bruckmann et.al. arXiv:1303.3972

3) Polyakov holomoly

Nowak et.al. arXiv:1304.6020

4) Chirality imbalance

Sphaleron transition Jingyi Chao, Pengcheng Chu, MH,
arXiv:1305.1100, PRD88(2013)

Instanton-anti-instanton pairing condensate

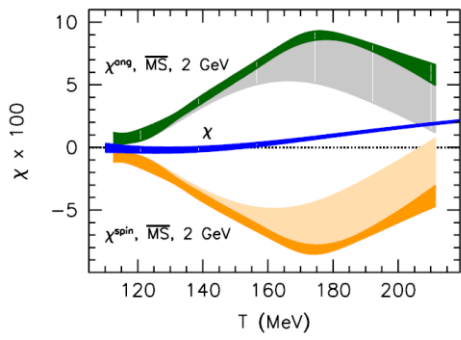
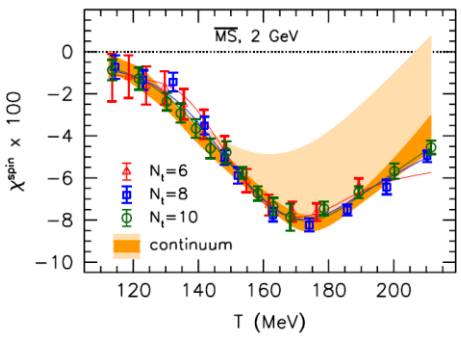
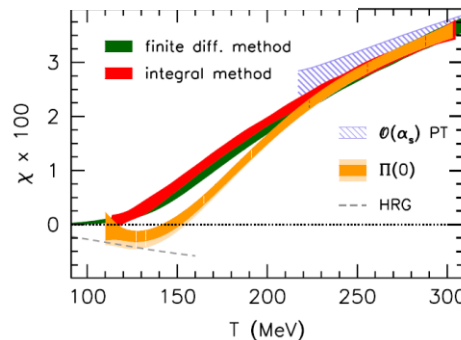
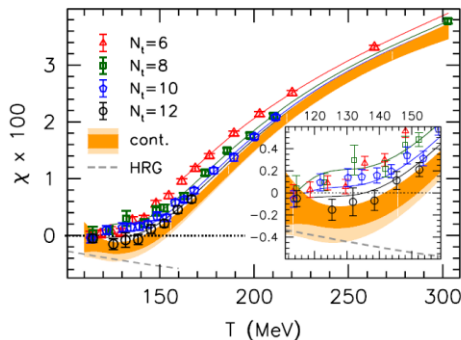
Lang Yu, Hao Liu, MH, arXiv:1404.6969,PRD90(2014)

5) Anomolous Magnetic catalysis

3) Diamagnetism!!!???

Diamagnetism at low temperature while strong paramagnetism at high temperature

$$-\frac{\partial^2 \Omega(T, eB)}{\partial (eB)^2} \Big|_{eB=0}$$



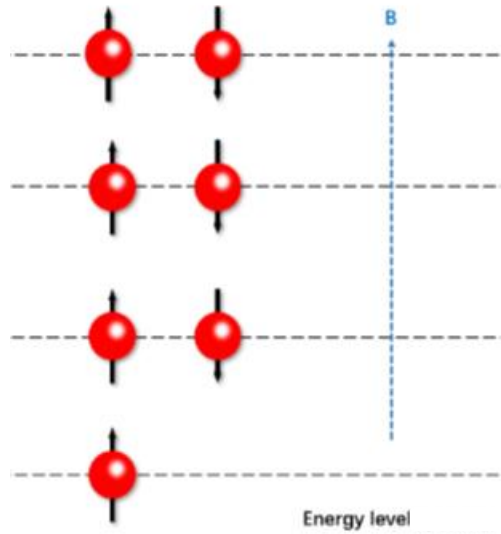
Regensburg-Lattice, PRD:1209.6015

Latest Regensburg-Lattice paper: 2004.08778

$$\xi_f = \frac{q_f/e}{2m_f} \left(\frac{\partial \langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle}{\partial (eB)} + \frac{\partial \langle \bar{\psi}_f L_{xy} \psi_f \rangle}{\partial (eB)} \right) \Big|_{eB=0},$$

$$\xi^S = \sum_f \frac{(q_f/e)^2}{2m_f} \tau_f, \quad \xi^L = \sum_f \frac{q_f/e}{2m_f} \frac{\partial \langle \bar{\psi}_f L_{xy} \psi_f \rangle}{\partial (eB)},$$

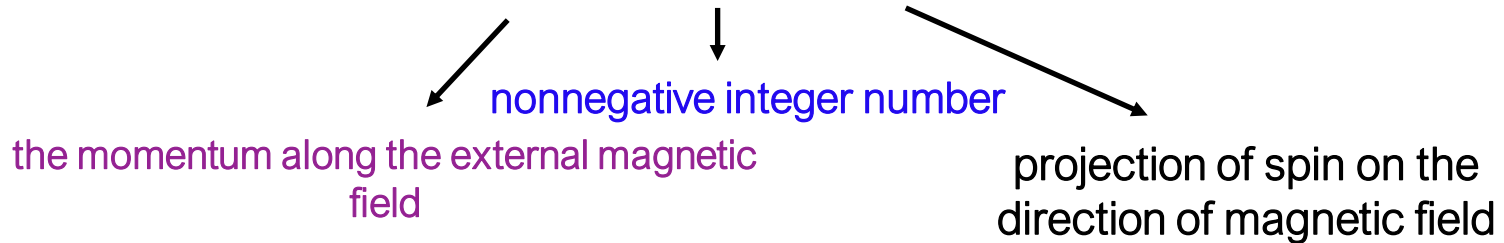
$$\langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle = q_f B \cdot \langle \bar{\psi}_f \psi_f \rangle \cdot \chi_f \equiv q_f B \cdot \tau_f,$$



Easy to understand paramagnetism!

-Energy of relativistic point-particle in the external magnetic field B:

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - 2\text{sgn}(q)s_z + 1)|qB| + m^2$$



-Masses of ρ mesons and π in magnetic field:

$$m_{\pi^\pm}^2(B) = m_{\pi^\pm}^2 + eB \quad \text{becomes larger}$$

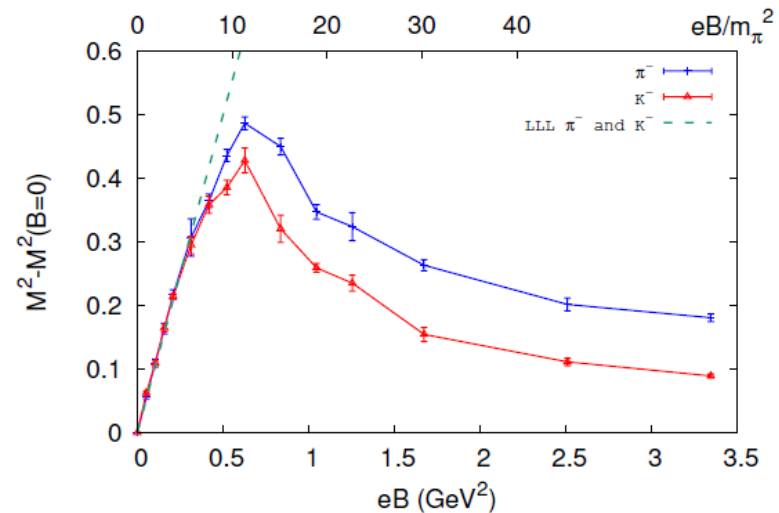
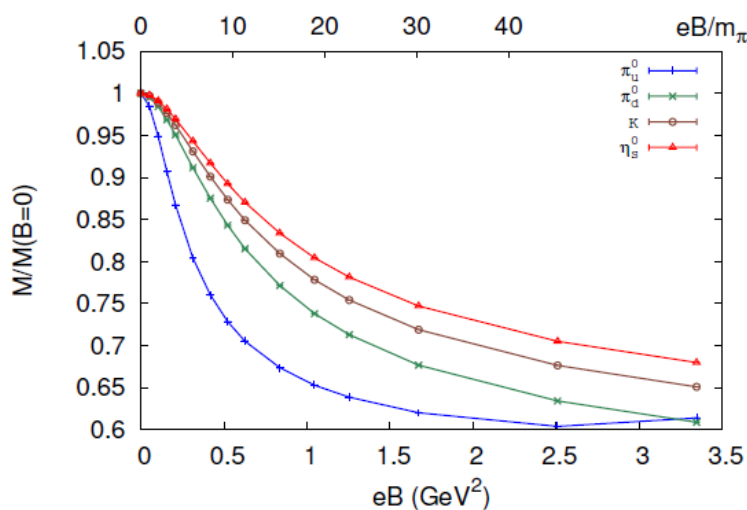
$$m_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - eB \quad \text{becomes lighter}$$

$$\text{where } m_{\rho^\pm} = 768\text{MeV}, \quad m_{\pi^\pm} = 140\text{ MeV}$$

Neutral particles should keep constant mass

4) Meson mass under magnetic field does not show point particle behavior;

$$m_{\sigma}(eB) \approx 2m(eB)$$



Heng-Tong Ding, Sheng-Tai Li ; Swagato Mukherjee,
Akio Tomiya, Xiao-Dan Wang, arXiv:2001.05322

Point-particle: Neutral particles should keep constant mass

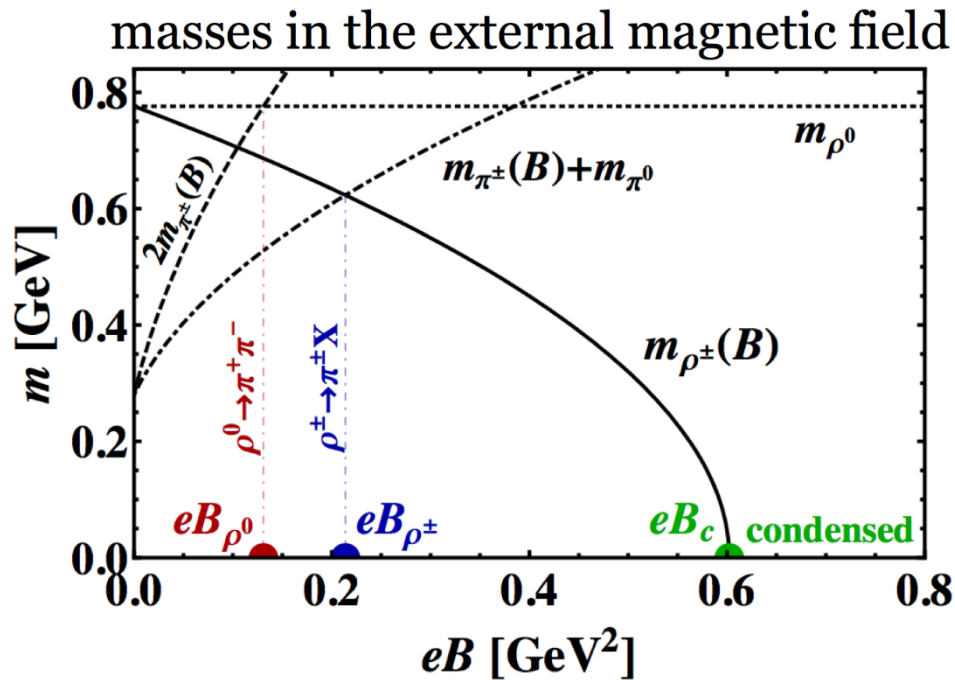
Quark model and lattice: Neutral hadrons don't keep constant mass

5) Vacuum Superconductor or not?

The charged rho becomes massless and condensate at a critical magnetic fields :

$$eB_c = m_{\rho^\pm}^2$$

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003 [arXiv:1101.0117 [hep-ph]]



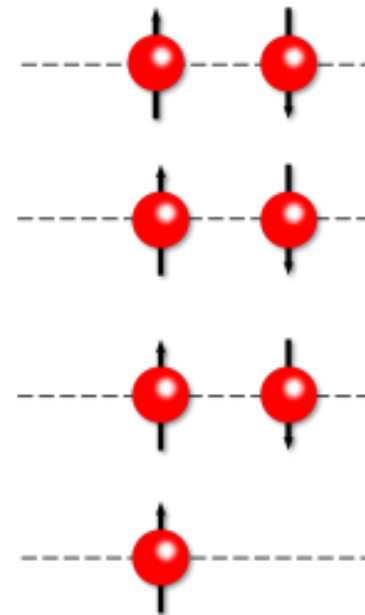
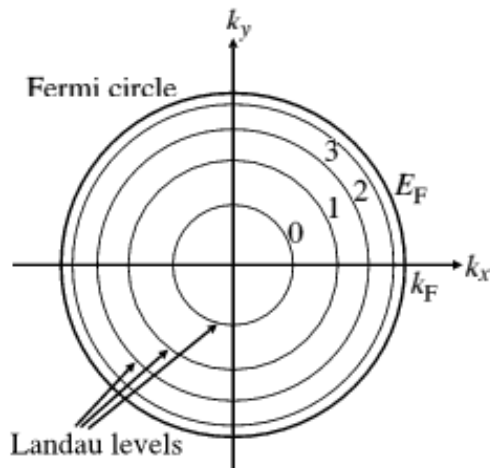
The pions become heavier while the charged vector mesons become lighter in the external magnetic field

The $\rho^\pm \rightarrow \pi^\pm \pi^0$ decay stops at a critical eB

Spin polarization induced by magnetic fields

Landau Level, dimension reduction

$$\int \frac{d\vec{p}^3}{(2\pi)^3} \longrightarrow \frac{|q_f B|}{2\pi} \sum_n \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi}$$



For positive charged quark, LLL spin up

1) Quark polarization in vacuum

$$\mathcal{L} = \bar{\psi}(i \not{D} - \hat{m})\psi + G_S [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2] - G_V [(\bar{\psi}\gamma^\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi)^2] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

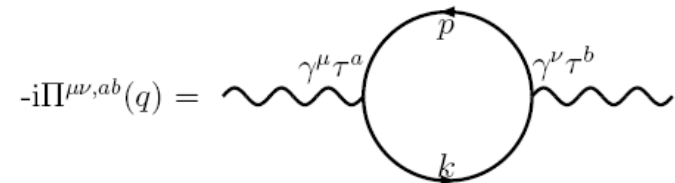
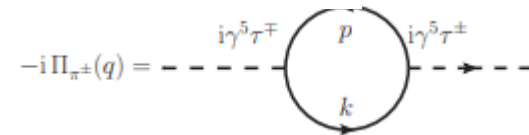
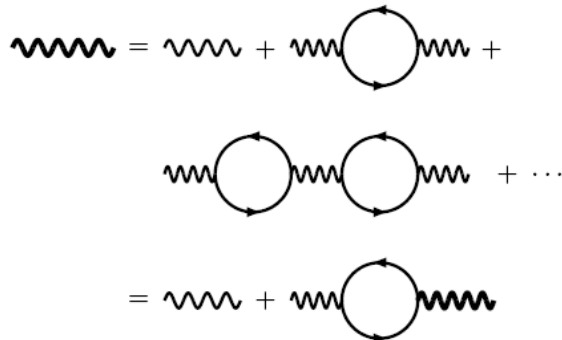
Hao Liu, Lang Yu, M.H. Phys.Rev.D 91 (2015) 1, 014017, e-Print: 1408.1318 1318

Hao Liu, Lang Yu, Xinyang Wang, Mei Huang, Phys.Rev.D 97 (2018) 7, 076008 • e-Print: 1801.02174

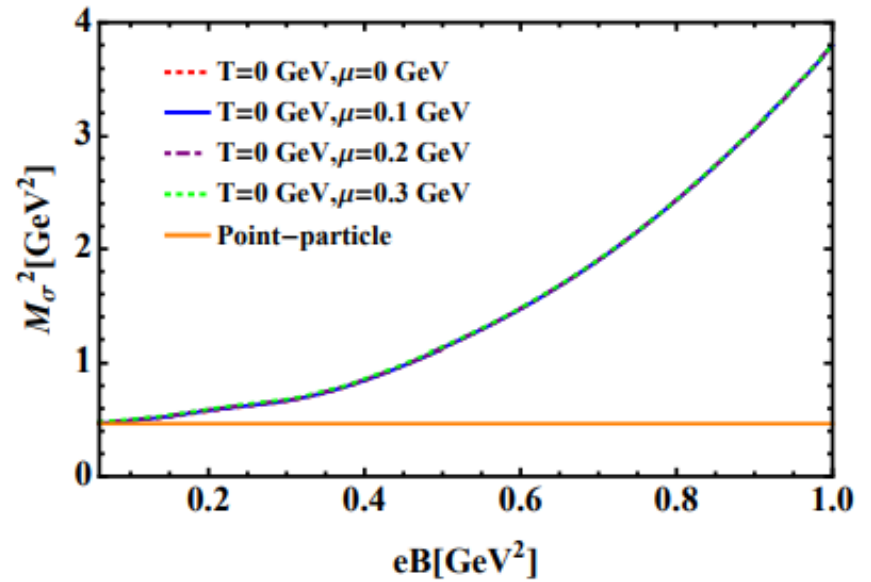
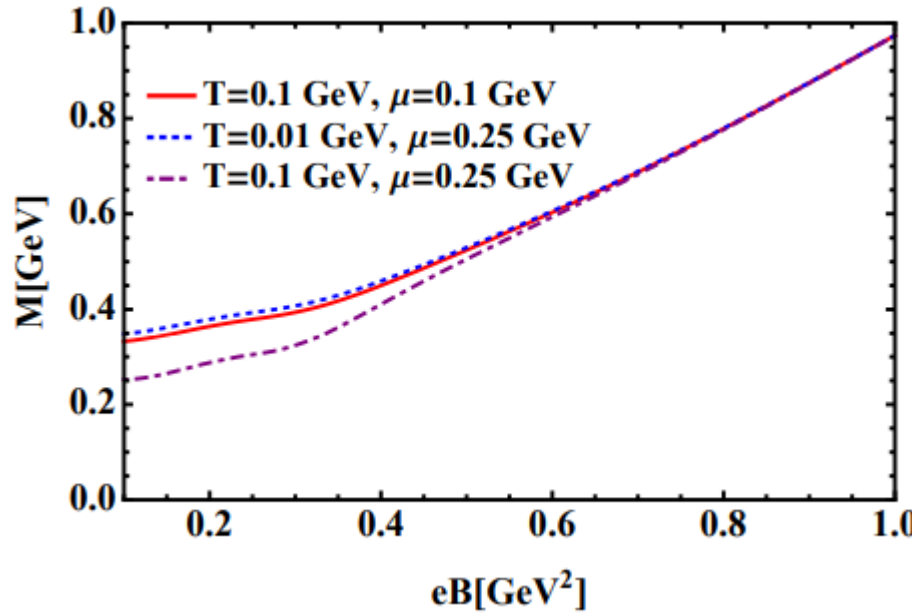
$$\tilde{S}_Q(k) = i \exp\left(-\frac{\mathbf{k}_\perp^2}{|QeB|}\right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(QeB, k)}{k_0^2 - k_3^2 - M^2 - 2|QeB|n},$$

with

$$D_n(QeB, k) = (k^0\gamma^0 - k^3\gamma^3 + M) \left[(1 - i\gamma^1\gamma^2\text{sign}(QeB))L_n\left(2\frac{\mathbf{k}_\perp^2}{|QeB|}\right) - (1 + i\gamma^1\gamma^2\text{sign}(QeB))L_{n-1}\left(2\frac{\mathbf{k}_\perp^2}{|QeB|}\right) \right] + 4(k^1\gamma^1 + k^2\gamma^2)L_{n-1}^1\left(2\frac{\mathbf{k}_\perp^2}{|QeB|}\right).$$

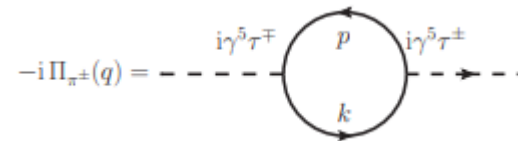
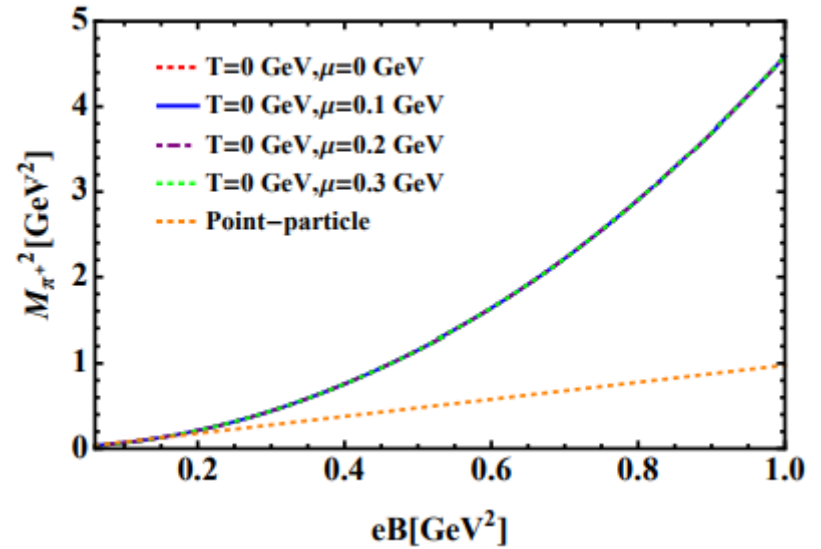
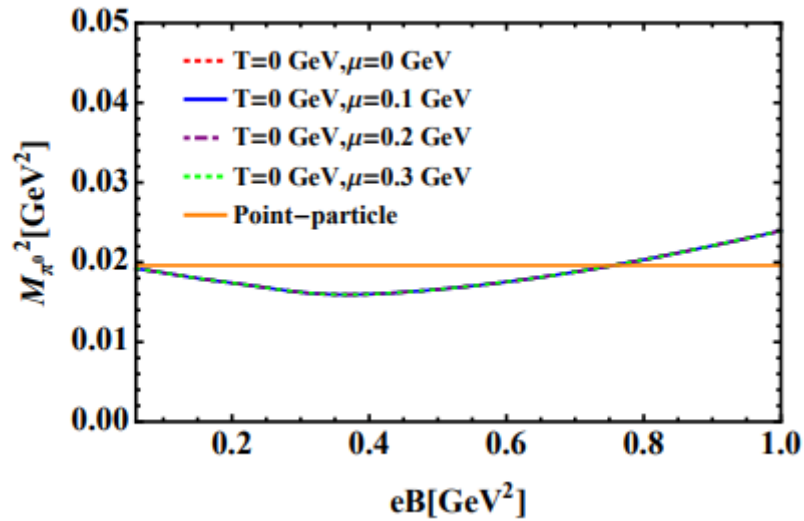


1) Magnetic catalysis



Hao Liu, Lang Yu, Xinyang Wang, Mei Huang,
Phys.Rev.D 97 (2018) 7, 076008 • e-Print: 1801.02174

1) quark loop corrections



Hao Liu, Lang Yu, Xinyang Wang, Mei Huang,
Phys.Rev.D 97 (2018) 7, 076008 • e-Print: 1801.02174

Charged vector meson condensation?

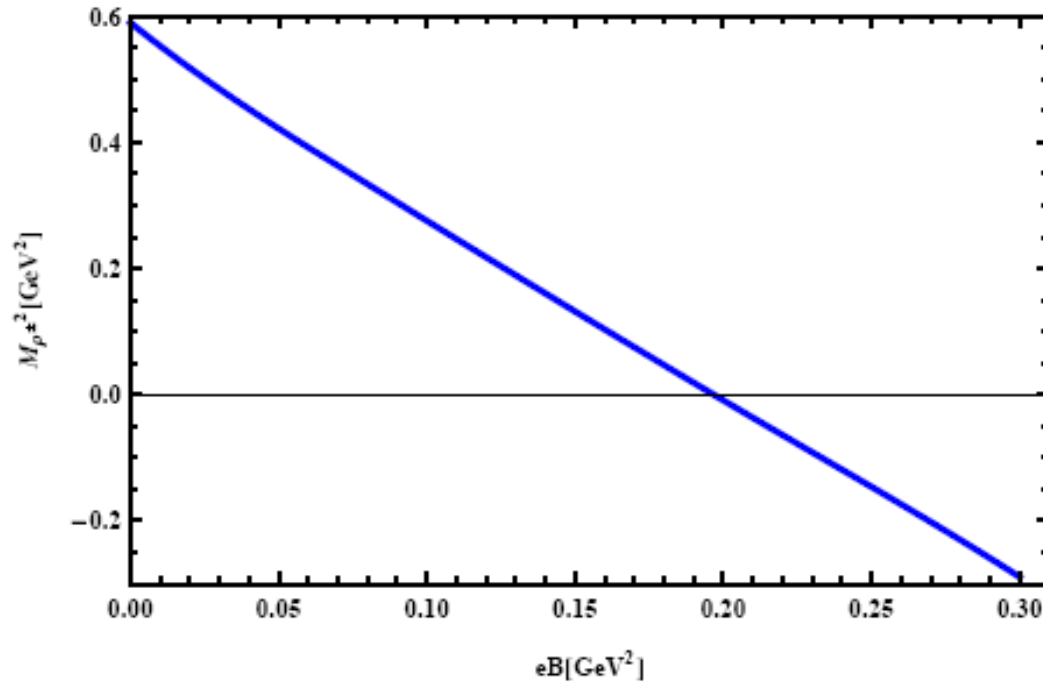
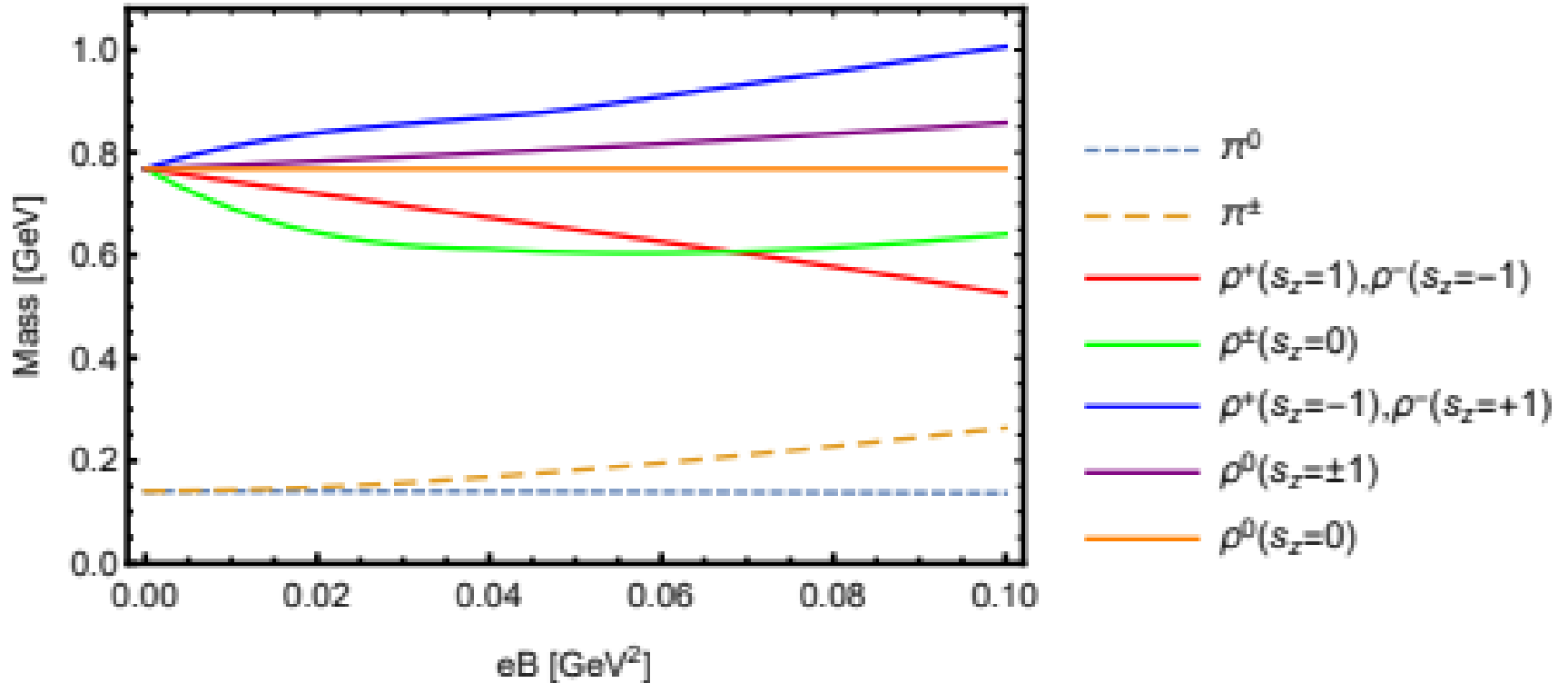


FIG. 4: The mass square of charged ρ^\pm with spin component $s_z = \pm 1$ as a function of eB .

$$eB_c \simeq 0.2 \text{GeV}^2$$

*Hao Liu, Lang Yu, Mei Huang,
Phys.Rev.D 91 (2015) 1, 014017*

Zeeman Splitting



Kun Xu, Shuzhe Shi, Hui Zhang, Defu Hou, Jinfeng Liao, MH,
 Phys.Lett.B 809 (2020) 135706 • e-Print: 2004.05362

Considering the quark loop polarization, one can get

- 1) Magnetic catalysis;
- 2) Zeeman Splitting;
- 3) vacuum superconductor

What else can happen?

Anomalous magnetic moment of quarks (AMM)

Anomalous magnetic moment (AMM) driven by
chiral symmetry breaking!

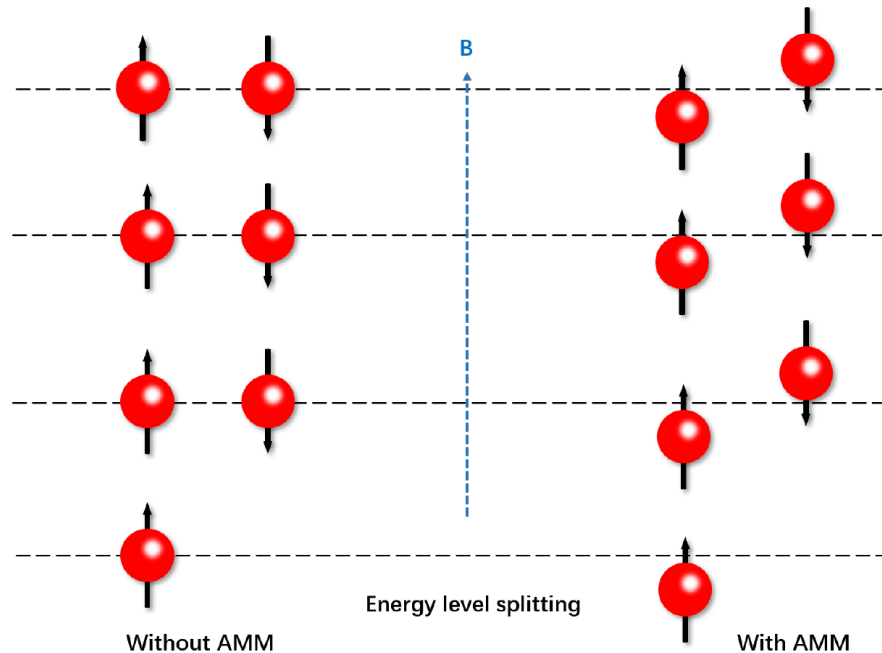
E. J. Ferrer and V. de la Incera, Phys. Rev. Lett. 102, 050402 (2009);

L. Chang, Y. Liu and C. D. Roberts, Phys. Rev. Lett. 106, 072001 (2011)

Anomalous magnetic moment of quarks (AMM)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_0 + \kappa_f q_f F_{\mu\nu} \sigma^{\mu\nu})\psi + G_S \left\{ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 \vec{\tau}\psi)^2 \right\}$$

$$E_k^2 = p_z^2 + \left\{ \sqrt{M^2 + (2k + 1 - s\xi)|qB|} - s\kappa qB \right\}^2$$



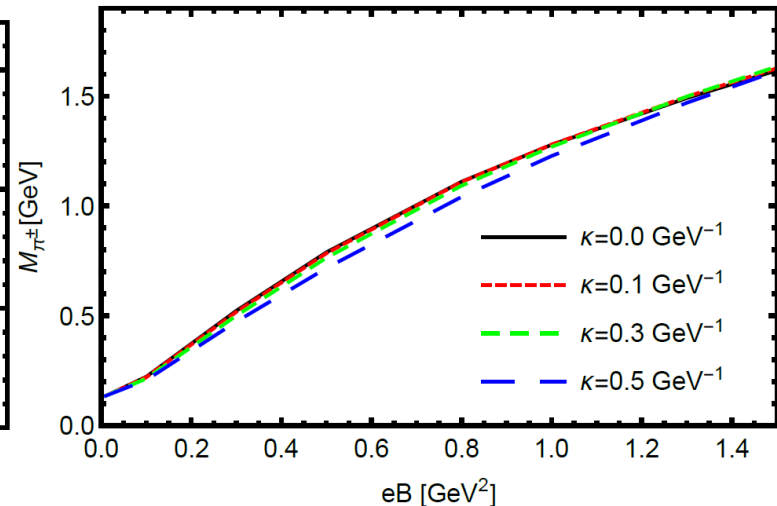
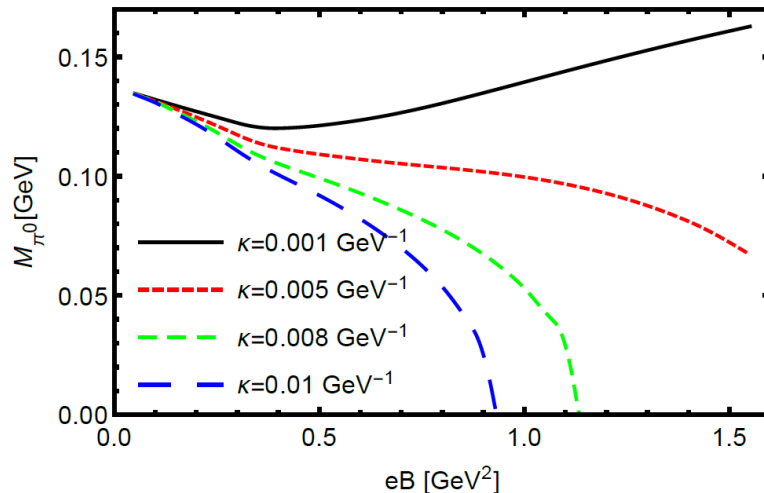
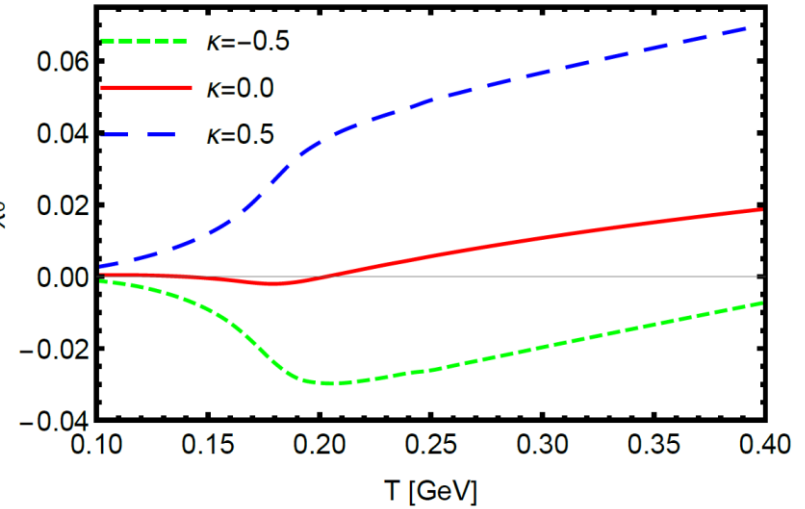
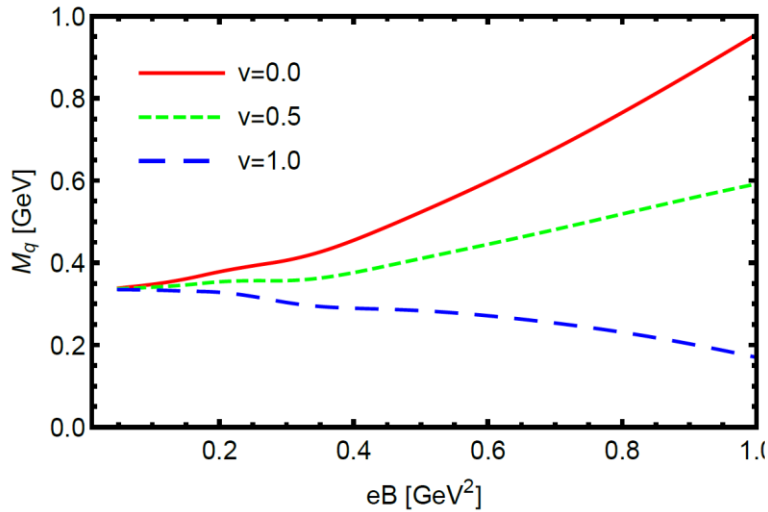
Kun Xu, Jingyi Chao, Mei Huang, arXiv:2007.13122, PRD 103 (2021) 7, 076015

Shijun Mao, Dirk Rischke, arXiv:1812.06684, Phys. Lett. B 792, 149 (2019)

Jingyi Chao, Yu-Xin Liu, Lei Chang, 2007.14258 [hep-ph]

Jingyi Chao, Yu-Xin Liu, Lei Chang, 2106.08168 [hep-ph]

Constant AMM- \rightarrow IMC, paramagnetism, neutral pion decreases, charged pion increases



Magnetic field dependent AMM & Tensor type spin polarization

$$\mathcal{L}_{\text{TSP}} = \bar{\psi} (i\gamma^\mu D_\mu - m_0) \psi + G_S \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\boldsymbol{\tau}\psi)^2 \right] + G_T \left[(\bar{\psi}\Sigma^3\psi)^2 + (\bar{\psi}i\gamma^5\Sigma^3\psi)^2 \right]$$

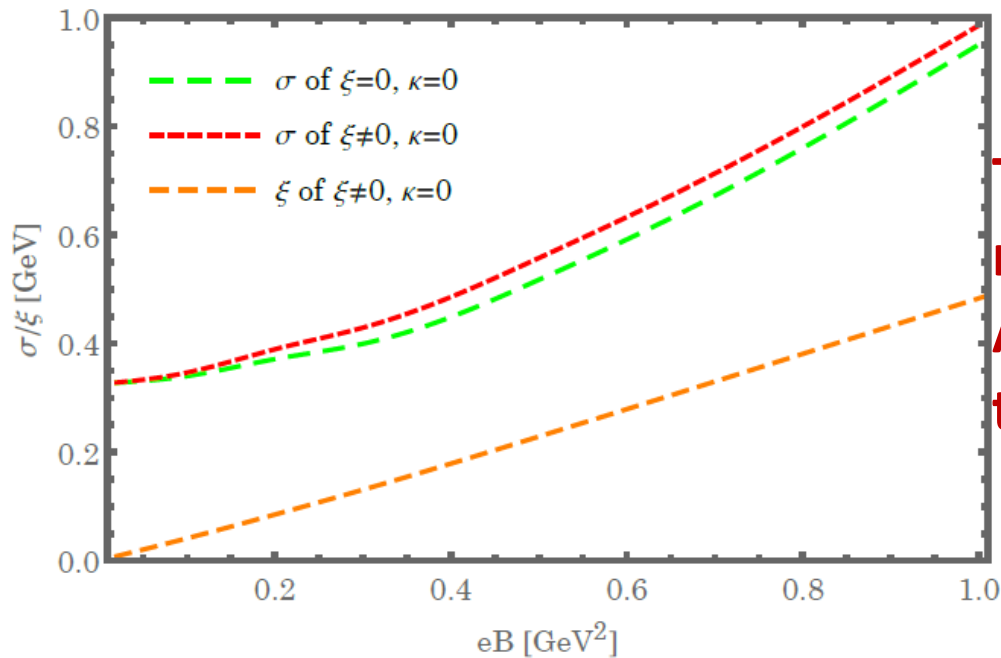
$$E_{f,n,s}^2 = \begin{cases} p_z^2 + \left(\sqrt{M_f^2 + 2n|q_f|B} - s\xi \right)^2, & n \geq 1. \\ p_z^2 + (M_f + \xi)^2, & n = 0. \end{cases} \quad \text{tensor type spin polarization}$$

$$\mathcal{L}_{\text{AMM}} = \bar{\psi} \left(i\gamma^\mu D_\mu - m_0 + \frac{1}{2}q\kappa F_{\mu\nu}\sigma^{\mu\nu} \right) \psi + G_S \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\boldsymbol{\tau}\psi)^2 \right]$$

$$\omega_{f,l,s}^2 = \begin{cases} p_z^2 + \left[\sqrt{M_f^2 + (2l+1-s\zeta)|q_f B|} - s\kappa q_f B \right]^2, & k \geq 1. \\ p_z^2 + (M_f - \kappa|q_f|B)^2, & k = 0. \end{cases} \quad \text{AMM}$$

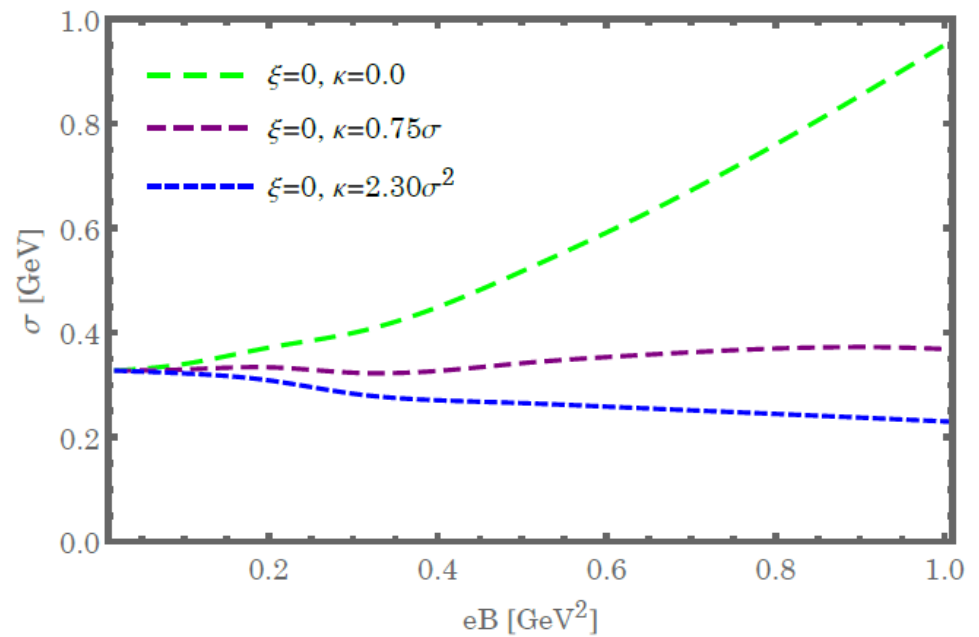
Fan Lin, Mei Huang, 2112.01051;

Fan Lin, Kun Xu, Mei Huang, 2202.03226, PRD2022



(a) Condensate σ and ξ (SP).

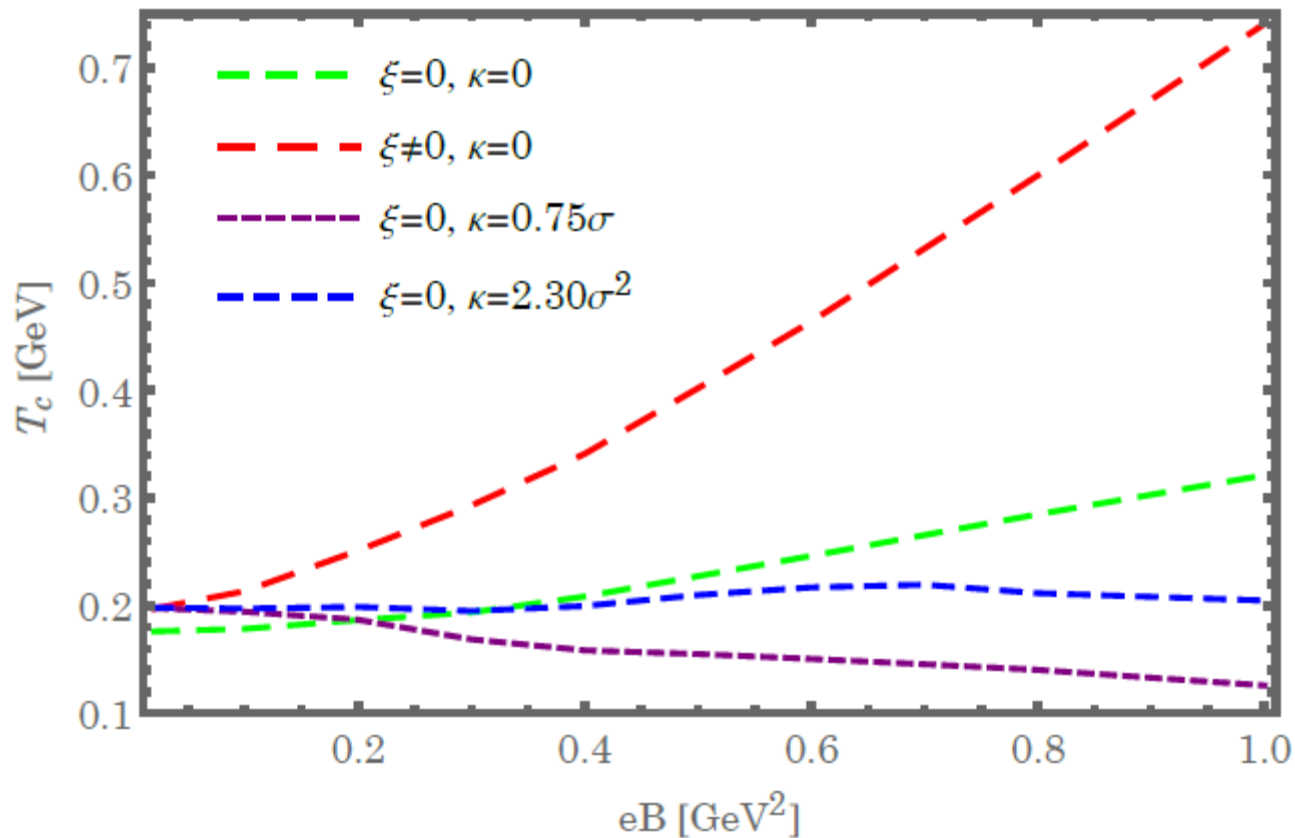
**Tensor spin polarization enhances magnetic catalysis;
AMM induces magnetic inhibition,
then induces IMC**



(b) Condensate σ with κ (AMM) .

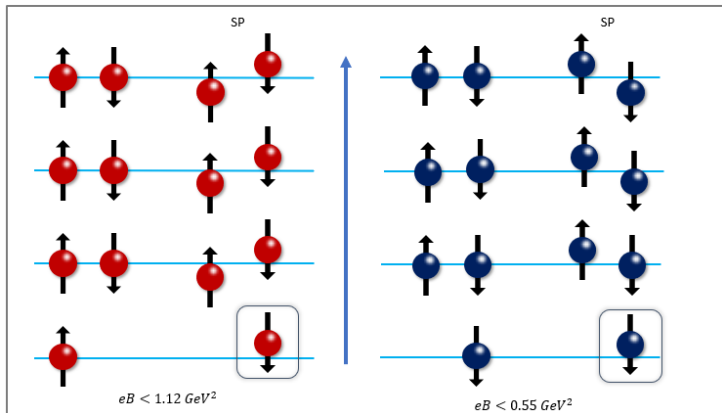
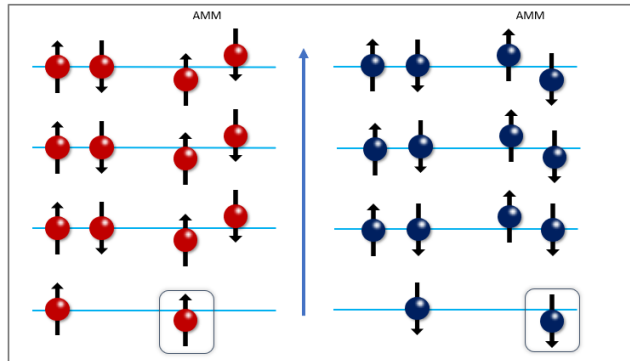
Tensor spin polarization enhances magnetic catalysis; AMM induces magnetic inhibition, then induces IMC

Fan Lin, Kun Xu, Mei Huang, 2202.03226, PRD2022

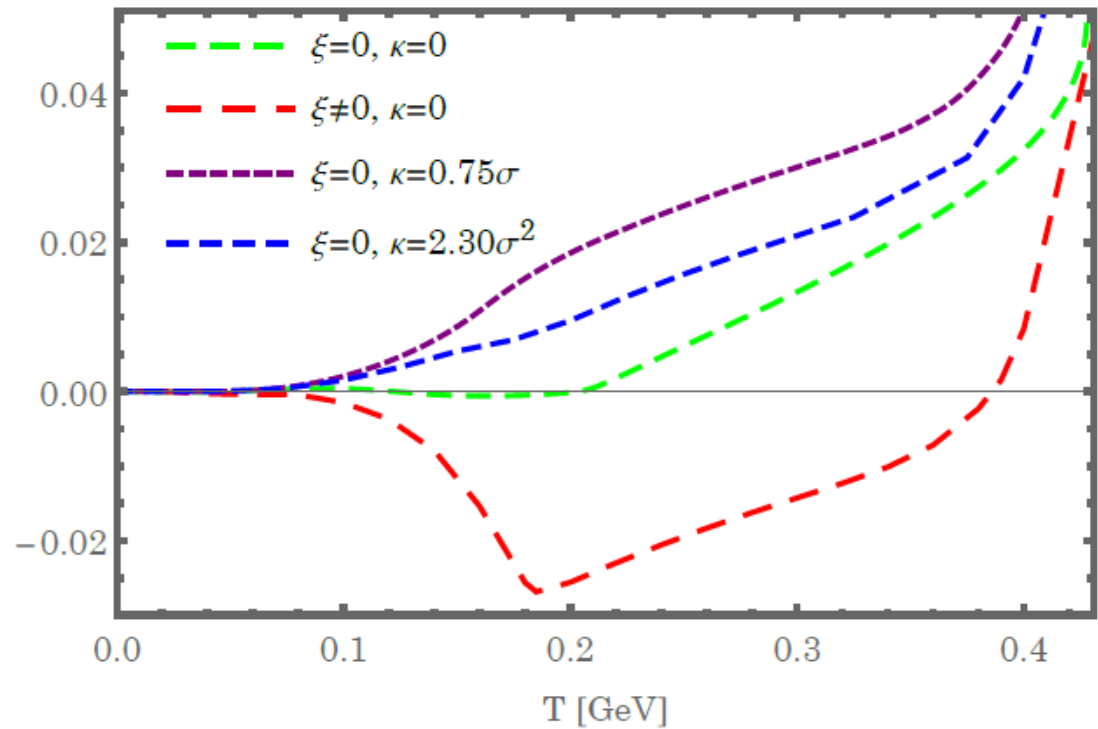


(a) Critical temperature T_c .

Tensor spin polarization induces diamagnetism; AMM induces paramagnetism



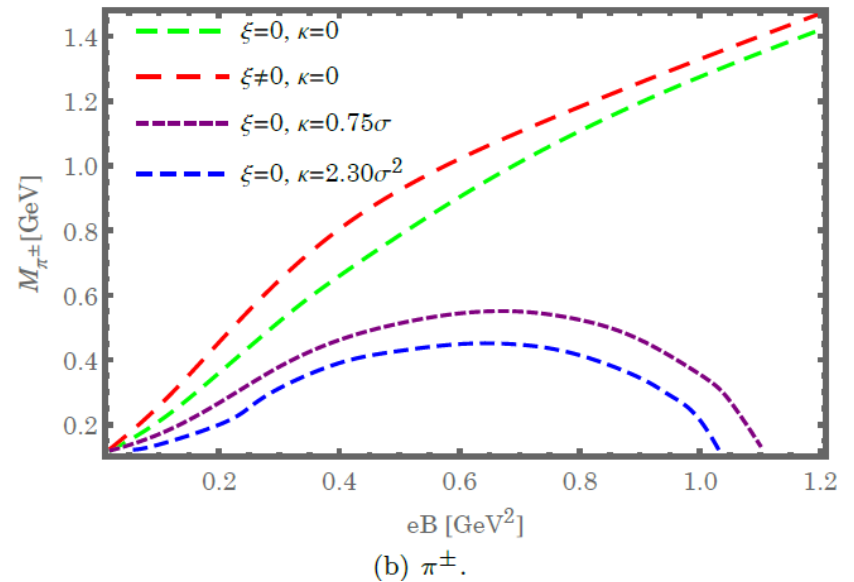
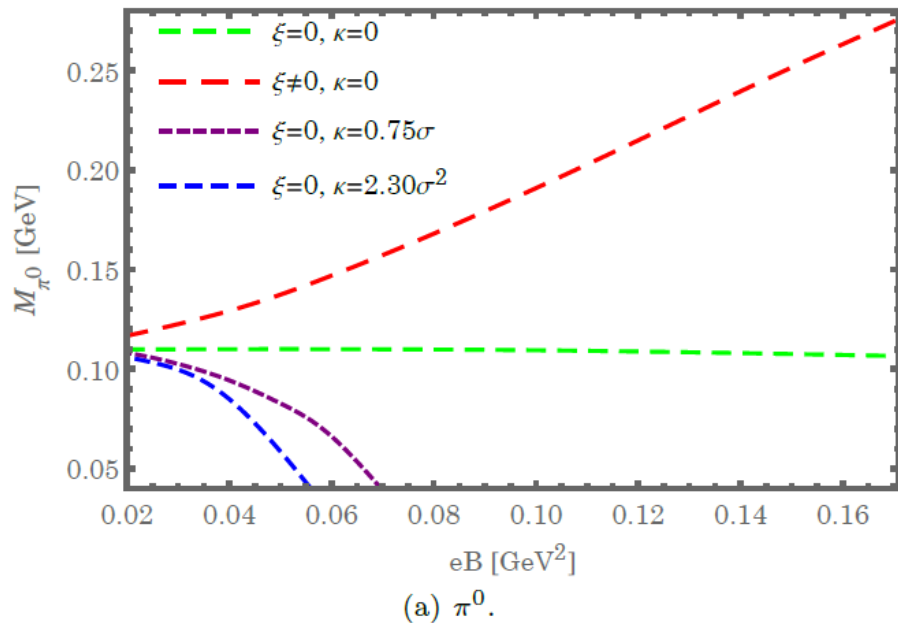
χ



(b) Magnetic susceptibility χ .

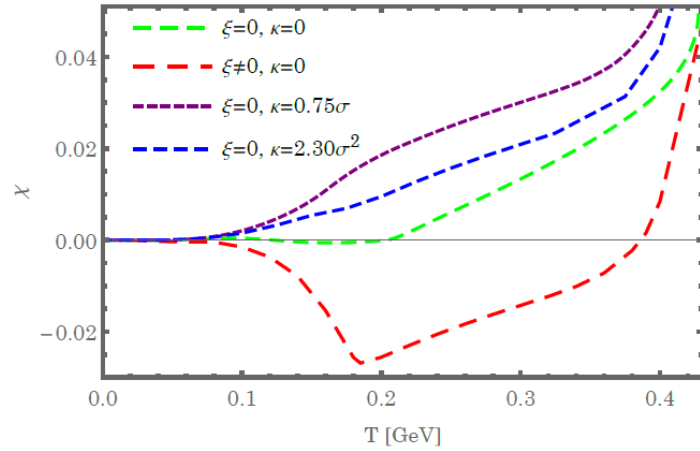
Tensor spin polarization: both neutral and charged pion masses increase with magnetic field;
Magnetic field dependent AMM: neutral pion mass decreases, charged pion masses nonmonotonic behavior.

Fan Lin, Kun Xu, Mei Huang, 2202.03226, PRD2022

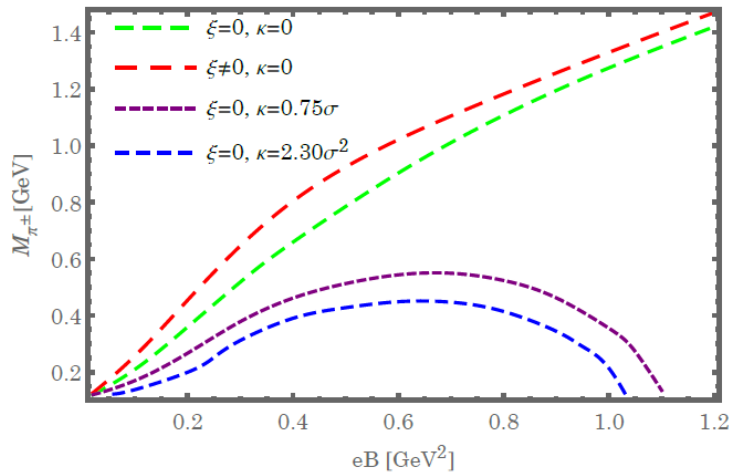


Contradiction between diamagnetism and IMC (magnetic inhibition)?

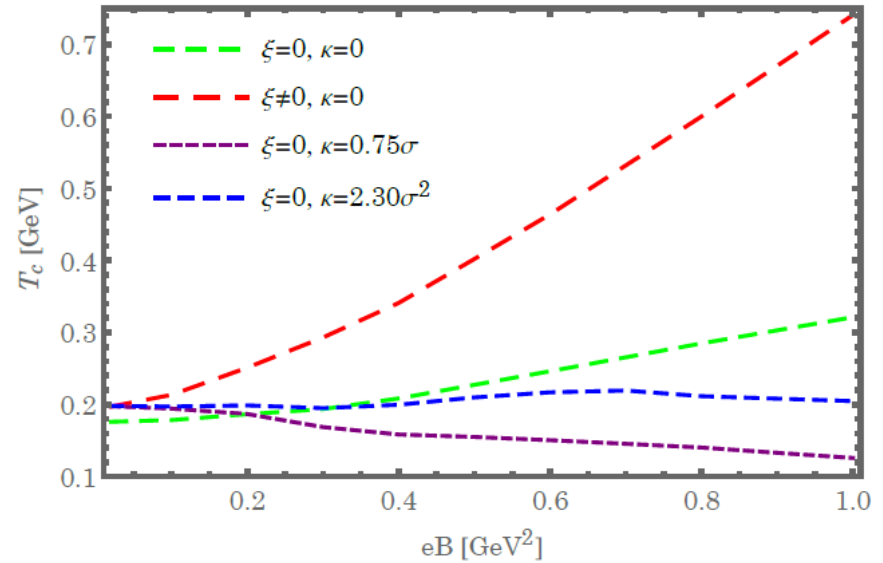
Fan Lin, Kun Xu, Mei Huang, 2202.03226, PRD2022



(b) Magnetic susceptibility χ .



(b) π^\pm .



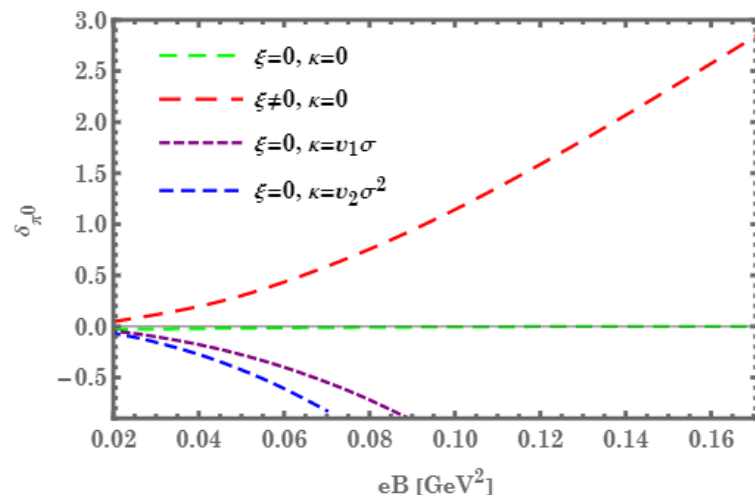
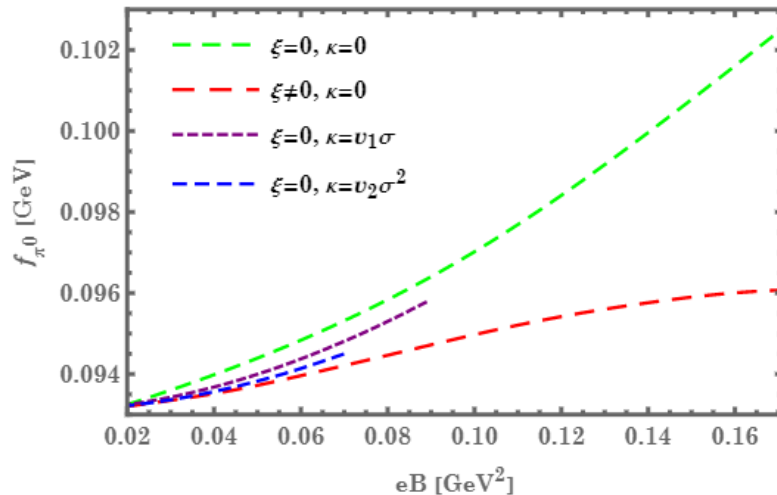
(a) Critical temperature T_c .

spin polarization: both neutral and charged pion masses increase with magnetic field;
Magnetic field dependent AMM: neutral pion mass decreases, charged pion masses nonmonotonic behavior.

Fan Lin, Kun Xu, Mei Huang, 2202.03226, PRD2022

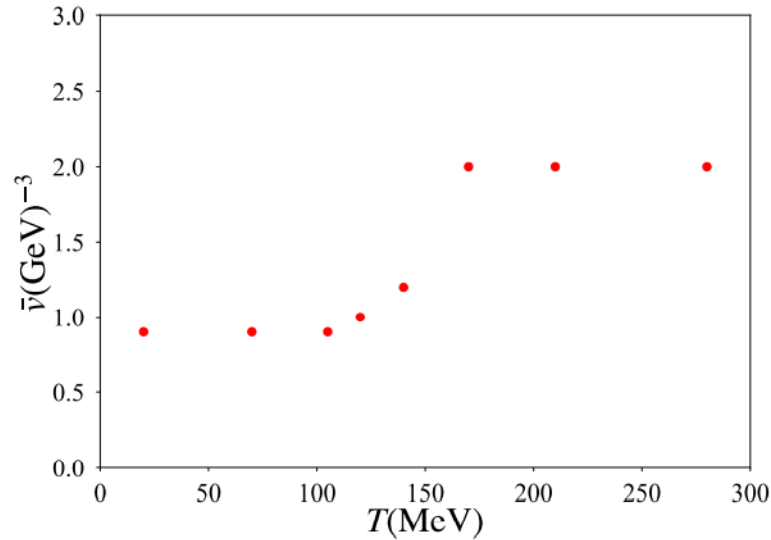
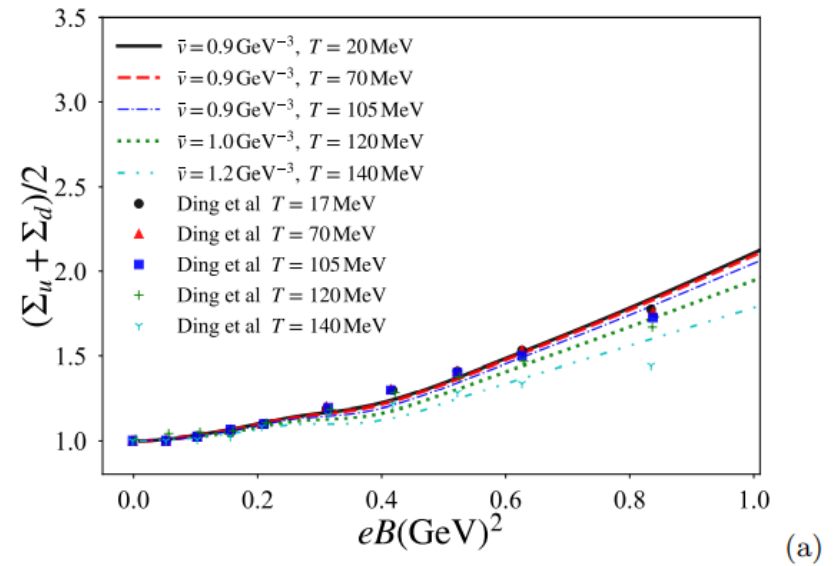
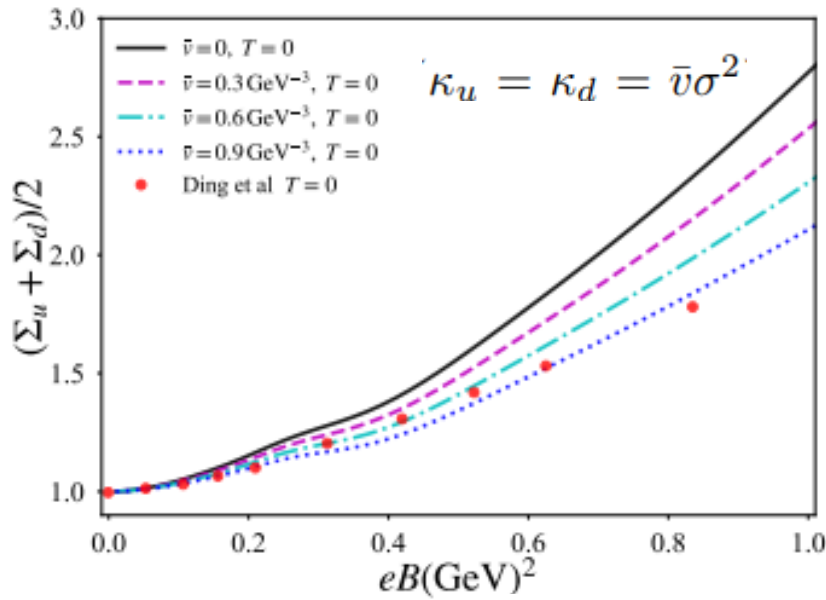
$$2f_\pi^2 m_\pi^2 \stackrel{\equiv}{=} -(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle$$

$$2m_{\pi^0}^2 f_{\pi^0}^2 = -(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle (1 + \delta_{\pi^0}).$$



IMC by Magnetic field dependent AMM

Mamiya Kawaguchi, Mei Huang, 2205.08169



Spin polarization induced by rotation

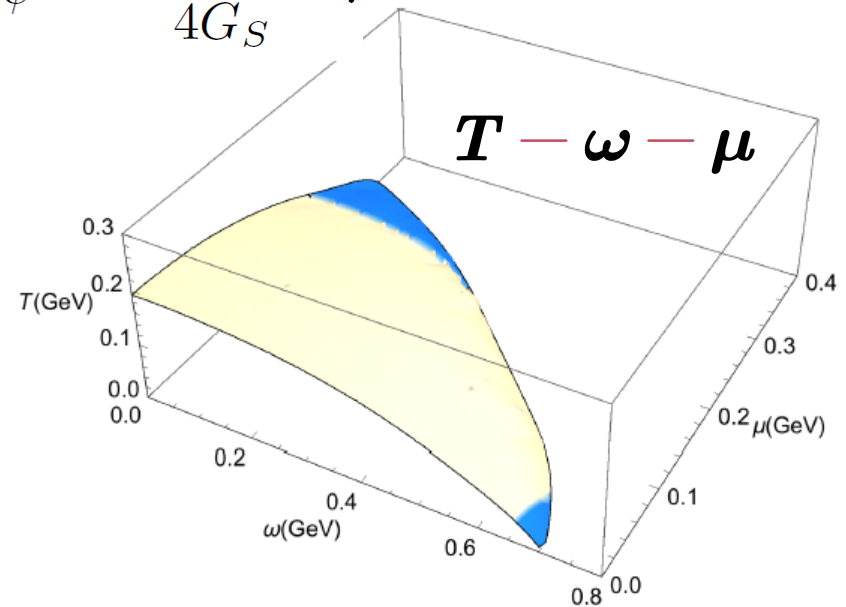
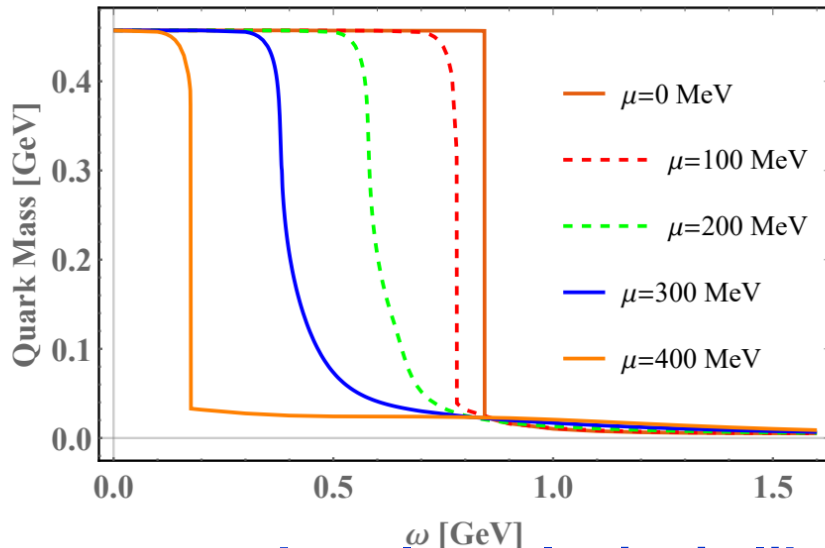
Chiral dynamics under rotation from NJL model

Yin Jiang, Jinfeng Liao PRL2015

$$\mathcal{L} = \bar{\psi}[i\bar{\gamma}^\mu(\partial_\mu + \Gamma_\mu) - m]\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - G_V[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2].$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2} [\gamma^a, \gamma^b] \Gamma_{ab\mu} \quad \Gamma_{ab\mu} = \eta_{ac} (e_\sigma^c G_{\mu\nu}^\sigma e_b^\nu - e_b^\nu \partial_\mu e_\nu^c)$$

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + \gamma^0\omega\hat{J}_z) - M]\psi - \mu\psi^\dagger\psi - \frac{(M - m)^2}{4G_S}.$$

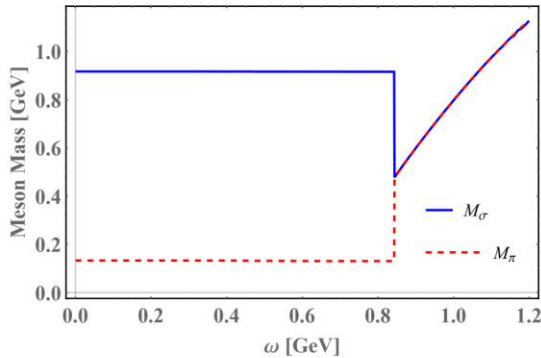


**Angular velocity is like the chemical potential,
1st order phase transition in two corners!**

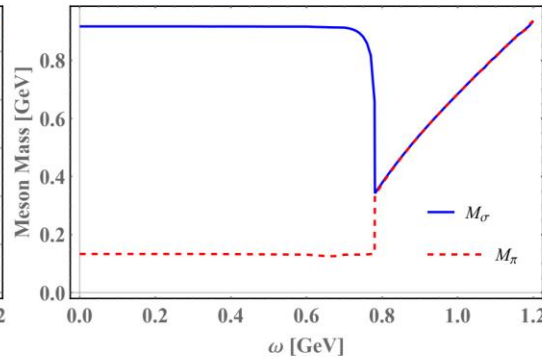
Minghua Wei, Ying Jiang,
M.H. 2011.10987

Xinyang Wang, Minghua Wei, Zhibin
Li, Mei Huang PRD2019 33

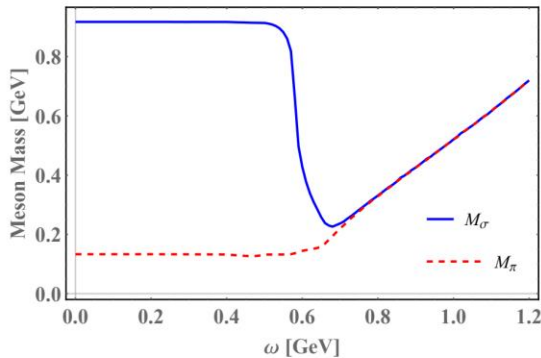
Scalar meson masses as functions of angular velocity



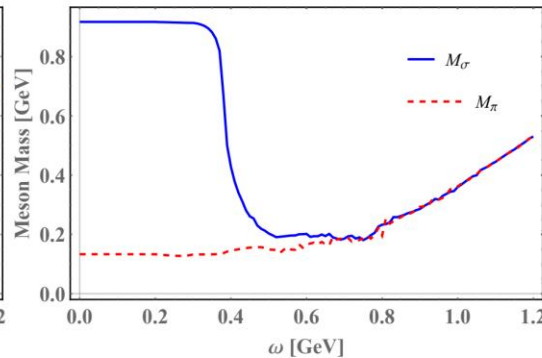
(a) scalar meson mass as a function of angular velocity at $\mu = 0 MeV$



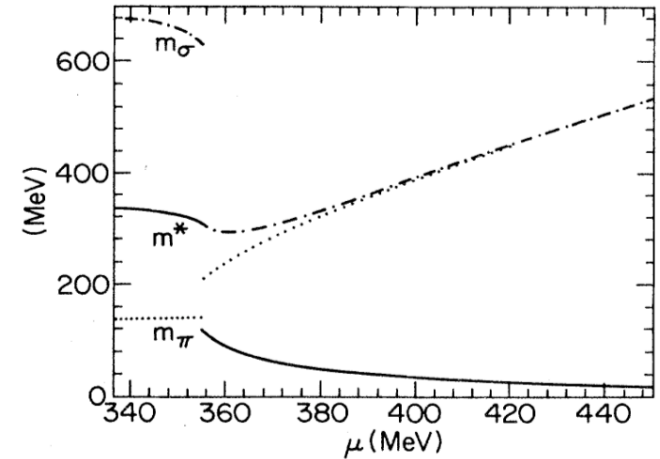
(b) scalar meson mass as a function of angular velocity at $\mu = 100 MeV$



(c) scalar meson mass as a function of angular velocity at $\mu = 200 MeV$



(d) scalar meson mass as a function of angular velocity at $\mu = 300 MeV$

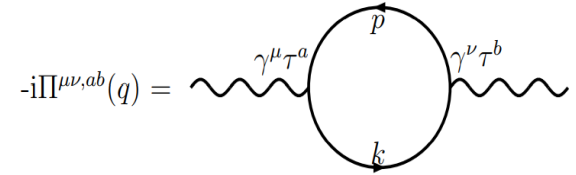


Minghua Wei, Ying Jiang,
M.H. 2011.10987

The effect of rotation on the scalar meson mass is similar to that of chemical potential !

Vector meson masses as functions of angular velocity

$$\Pi^{\mu\nu,ab}(q) = -i \int d^4\tilde{r} Tr_{sf c} [i\gamma^\mu \tau^a S(0; \tilde{r}) i\gamma^\nu \tau^b S(\tilde{r}; 0)] e^{q \cdot \tilde{r}}$$



$$D_\rho^{\mu\nu}(q^2) = D_1(q^2)P_1^{\mu\nu} + D_2(q^2)P_2^{\mu\nu} + D_3(q^2)L^{\mu\nu} + D_4(q^2)u^\mu u^\nu$$

$$P_1^{\mu\nu} = -\epsilon_1^\mu \epsilon_1^\nu, (S_z = -1 \text{ for } \rho \text{ meson})$$

$$P_2^{\mu\nu} = -\epsilon_2^\mu \epsilon_2^\nu, (S_z = +1 \text{ for } \rho \text{ meson})$$

$$L^{\mu\nu} = -b^\mu b^\nu, (S_z = 0 \text{ for } \rho \text{ meson})$$

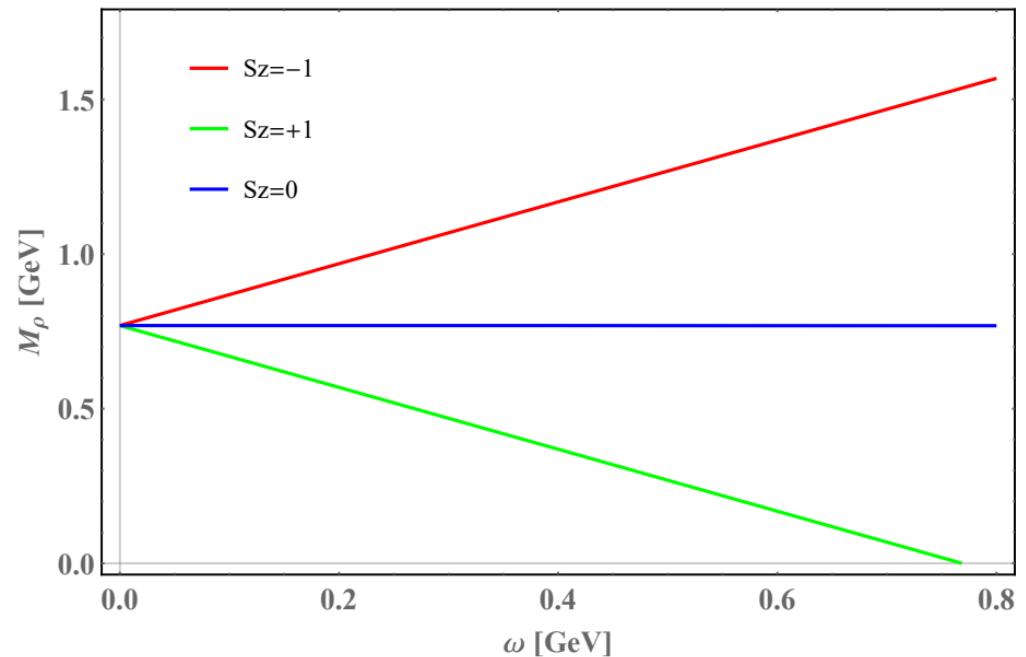
$$1 + 2G_V A_i^2 = 0$$

$$A_1^2 = -(\Pi_{11} - i\Pi_{12}), (S_z = -1 \text{ for } \rho \text{ meson})$$

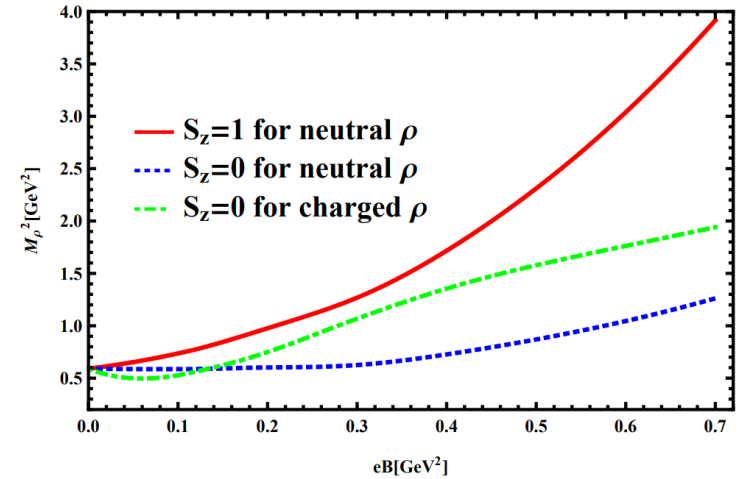
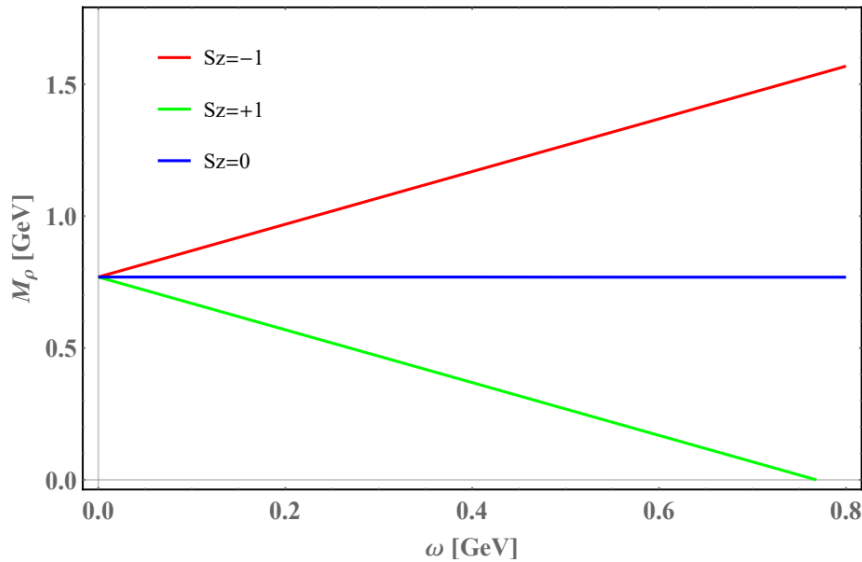
$$A_2^2 = -\Pi_{11} - i\Pi_{12}, (S_z = +1 \text{ for } \rho \text{ meson})$$

$$A_3^2 = \Pi_{33}, (S_z = 0 \text{ for } \rho \text{ meson})$$

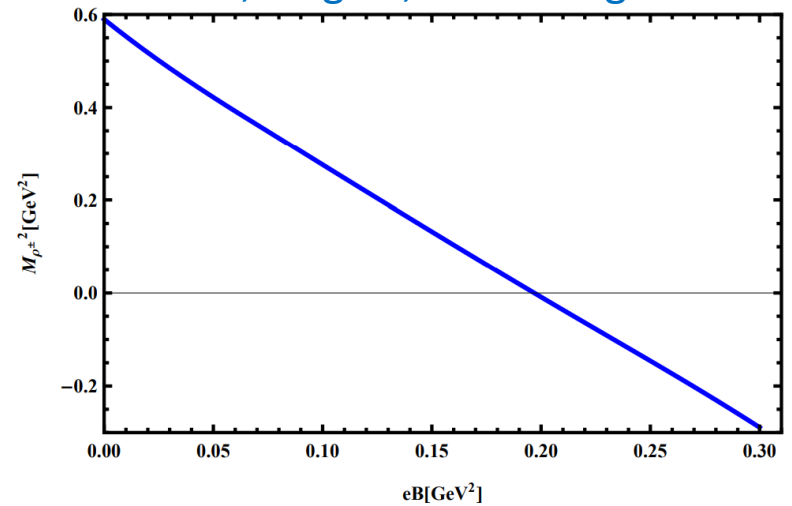
Zeeman splitting effect for different spin component!



Vector meson masses as functions of angular velocity



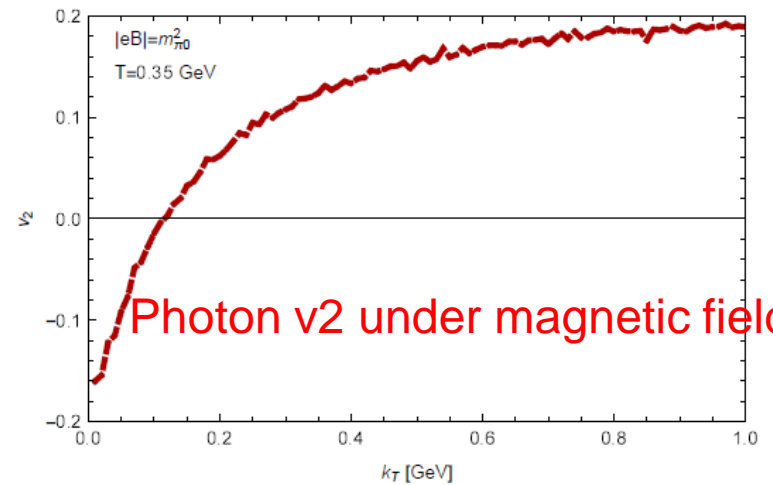
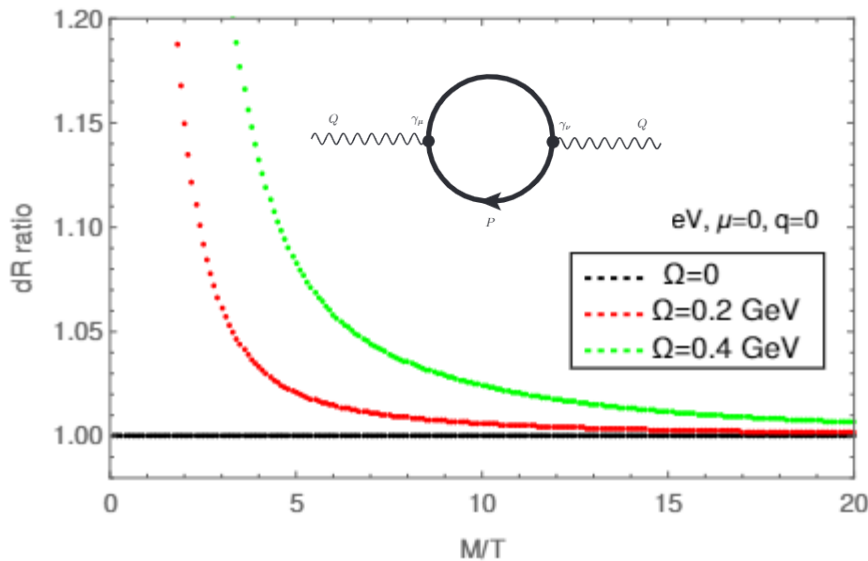
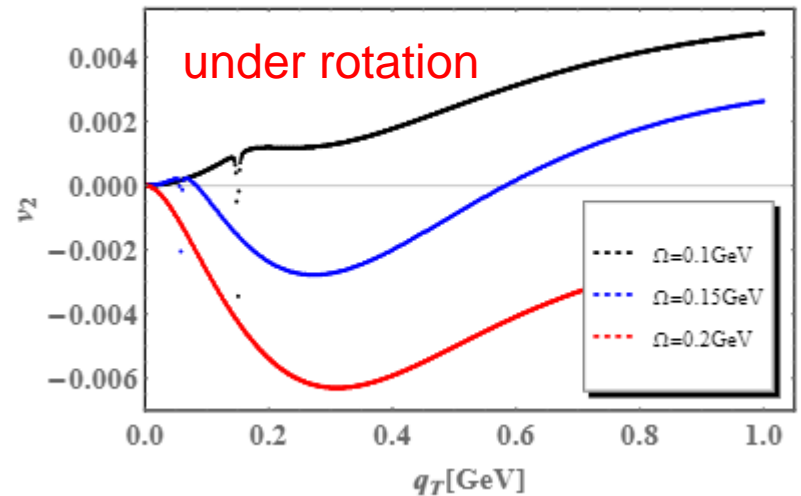
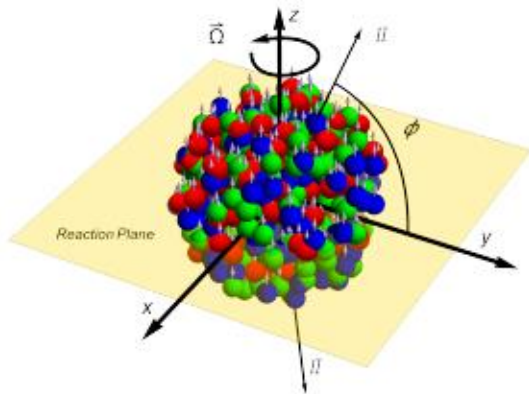
Hao Liu, Lang Yu, Mei Huang PRD2014



The effect of rotation on spin component of vector meson is similar to that of the magnetic field on charged vector mesons !

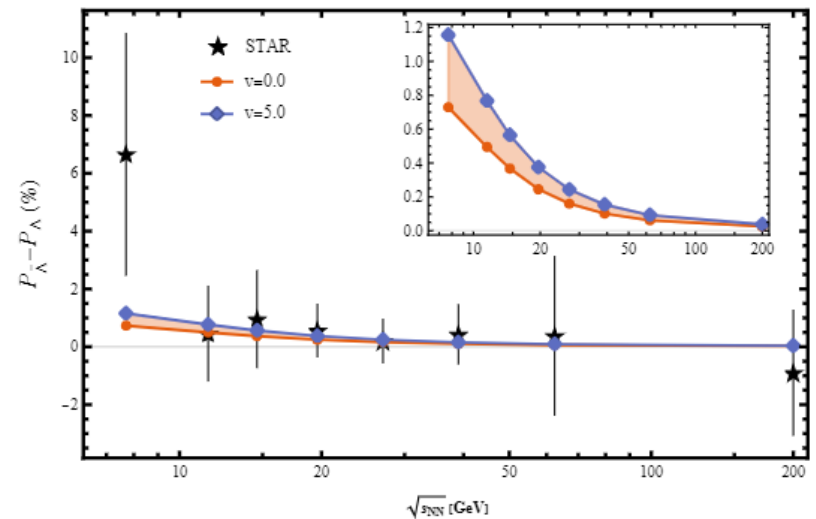
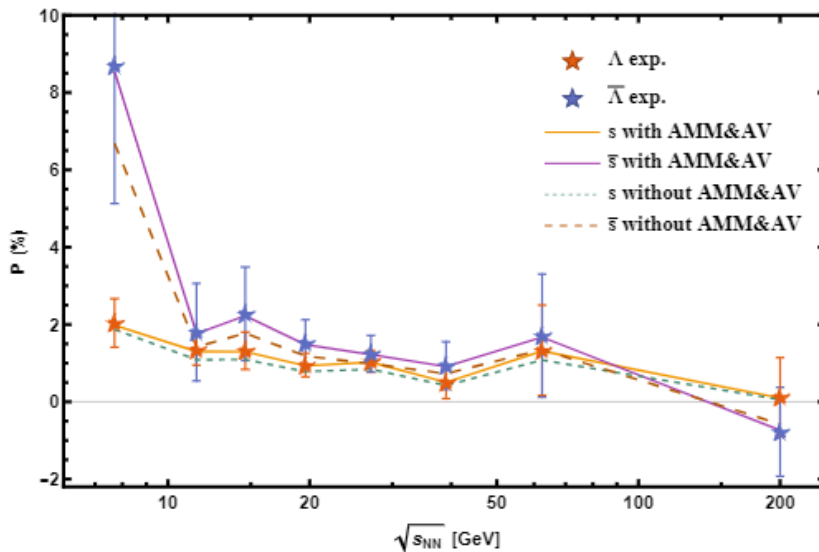
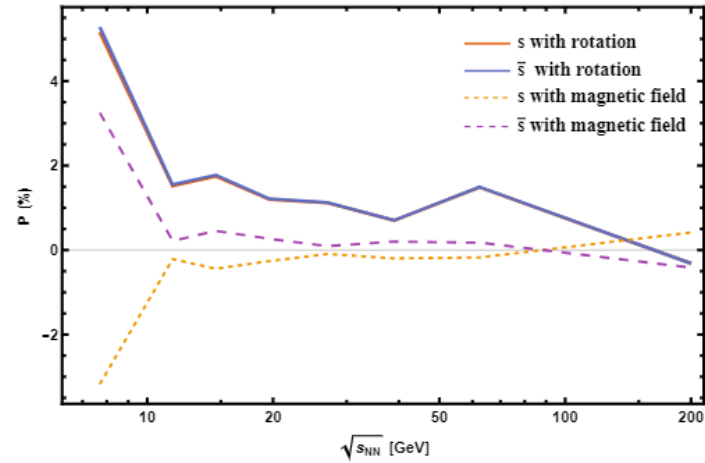
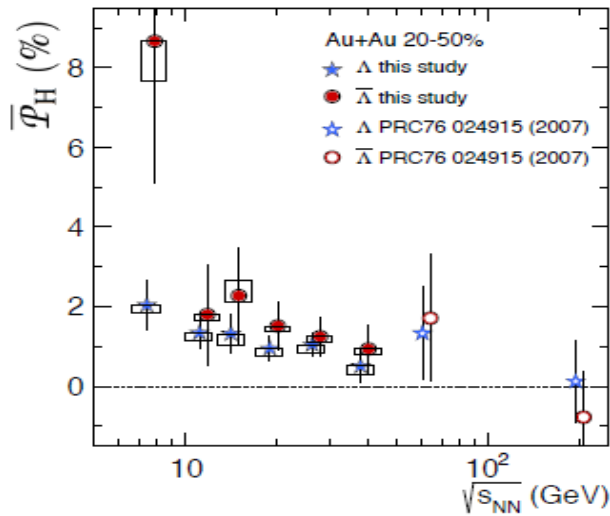
Dilepton rate and ellipticity under roation

Minghua Wei, Aminul Chowdhury, Mei Huang, arXiv: 2111.05192

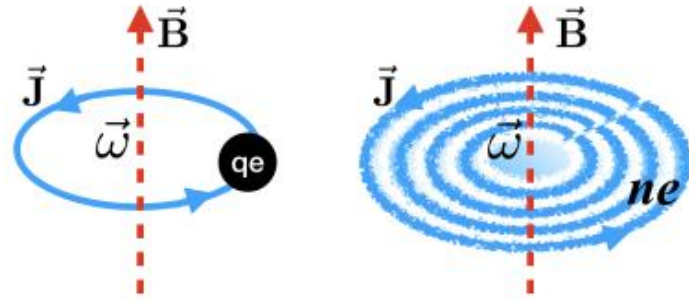
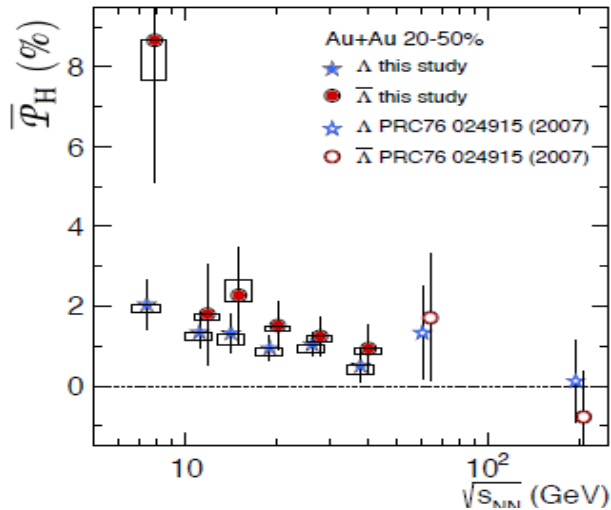


Global polarization induced by rotation, polarization difference of charged particles induced by magnetic field. SP and AMM plays important role.

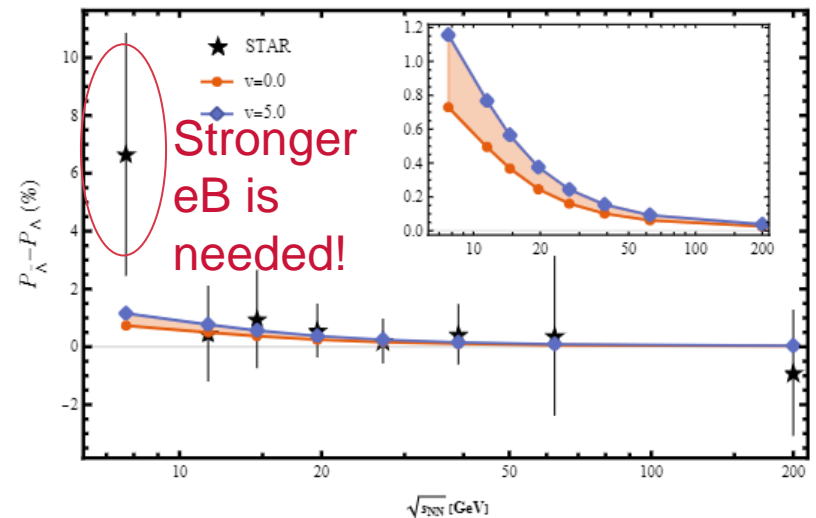
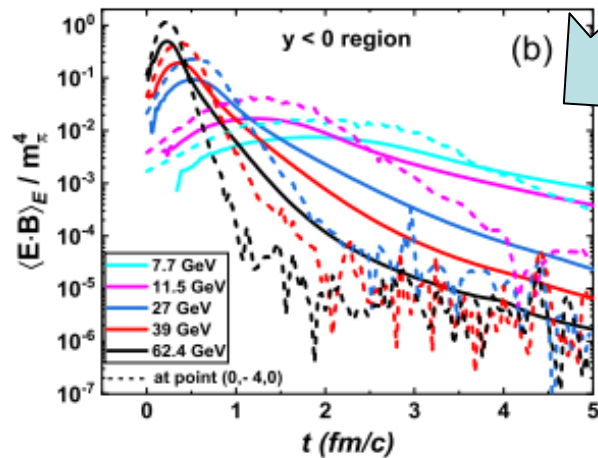
Kun Xu, Fan Lin, Anping Huang, M.H., 2205.02420



Global polarization induced by rotation, polarization difference of charged particles induced by magnetic field. SP and AMM plays important role.



X. Guo, J. Liao and E. Wang, Sci. Rep. 10, no.1, 2196 (2020) doi:10.1038/s41598-020-59129-6 [arXiv:1904.04704 [hep-ph]]



Irfan Siddique, Xin-Li Sheng, Qun Wang, Phys. Rev. C 104 (2021) 3, 034907

Kun Xu, Fan Lin, Anping Huang, M.H., 2205.02420

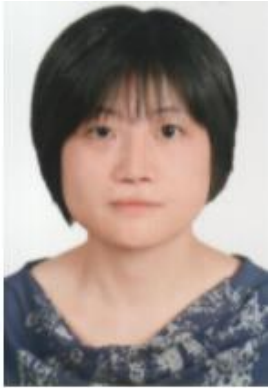
Summary

1, Both rotation and magnetic field induces polarization, global polarization induced by rotation, polarization difference of charged particles is induced by magnetic field.

2, TSP and AMM plays important role.

2, Self-consistent understanding on diamagnetism, IMC and meson spectra under magnetic field still need further studies.

Thanks for my collaborators on this topic!



巢静宜, 江西师大



余浪, 吉林大学



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国科大



Mamiya Kawaguchi,
国科大



刘好, 上海嘉定中学



许坤, 国科大



魏明华, 国科大



林凡, 国科大