Higher Twist Transverse Momentum Dependent Parton Distribution Functions in the MIT Bag Model

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  - MIT bag model
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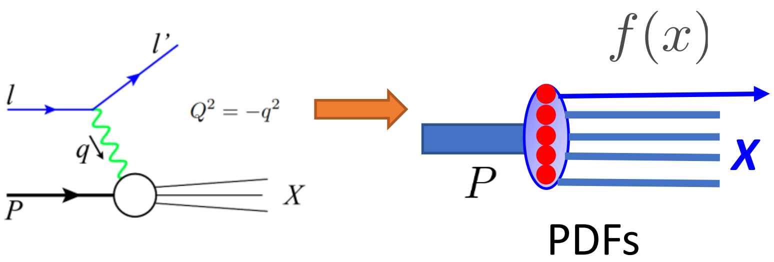
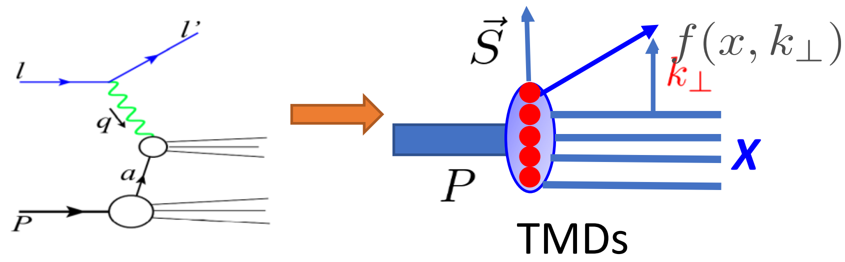
Introduction

Transverse momentum dependent parton distribution functions (TMDs)





Leading twist TMDs		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon polarization	U	f_1 = Number density		$h_1^\perp$ = - Boer-Mulders
	L		g_{1L} = - Helicity	$h_{1L}^\perp$ = - Longi-transversity
	T	$f_{1T}^\perp$ = - Sivers	$g_{1T}^\perp$ = - Trans-helicity	h_{1T} = - Transversity $h_{1T}^\perp$ = - Pretzelocity

Nucleon Spin   Quark Spin

Leading twist azimuthal asymmetries for semi-inclusive process: $e^- p \rightarrow e^- q X$

$$\langle \sin(\phi-\phi_S)\rangle_{UT} = \frac{|\vec{k}_\perp|}{2M}\frac{f_{1T}^\perp}{f_1}$$

$$\langle \cos(\phi-\phi_S)\rangle_{LT} = \frac{|\vec{k}_\perp|}{2M}\frac{C(y)}{A(y)}\frac{g_{1T}^\perp}{f_1}$$

Introduction

Higher twist effects (twist-2, twist-3, twist-4)

Leading twist azimuthal asymmetries

$$\langle \sin(\phi-\phi_S)\rangle_{UT} = \frac{|\vec{k}_\perp|}{2M}\frac{f_{1T}^\perp}{f_1}\left(1-\frac{M^2}{Q^2}\alpha_{UT}\right)$$

$$\langle \cos(\phi-\phi_S)\rangle_{LT} = \frac{|\vec{k}_\perp|}{2M}\frac{C(y)}{A(y)}\frac{g_{1T}^\perp}{f_1}\left(1-\frac{M^2}{Q^2}\alpha_{LT}\right)$$

$$\alpha_{UT} = \alpha_{UU} - 8x^2\frac{E(y)}{A(y)}\frac{f_{3T}^\perp}{f_{1T}^\perp} + 4x\frac{\mathrm{Re} f_{+3ddT}^\perp}{f_{1T}^\perp}$$

$$\alpha_{LT} = \alpha_{UU} - 4\kappa_M^2 x\frac{\mathrm{Re} f_{+3ddT}^{\perp 3}}{g_{1T}^\perp}$$

$$\alpha_{UU} = 8x^2\frac{E(y)}{A(y)}\frac{f_3}{f_1} - 4x\frac{\mathrm{Re} f_{+3dd}}{f_1}$$

Higher twist azimuthal asymmetries

$$\langle \cos\phi\rangle_{UU} = -\kappa_M\frac{|\vec{k}_\perp|}{M}\frac{B(y)}{A(y)}\frac{xf^\perp}{f_1}$$

$$\langle \sin\phi\rangle_{UL} = -\kappa_M\frac{|\vec{k}_\perp|}{M}\frac{B(y)}{A(y)}\frac{xf_L^\perp}{f_1}$$

$$\langle \cos 2\phi\rangle_{UU} = -\kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}\frac{E(y)}{A(y)}\frac{xf_{-3d}^\perp}{f_1}$$

$$\langle \sin 2\phi\rangle_{UL} = \kappa_M^2\frac{|\vec{k}_\perp|^2}{M^2}\frac{E(y)}{A(y)}\frac{xf_{+3dL}^\perp}{f_1}$$

⋮

There are three impacts of the higher twist TMDs

① Twist-2 azimuthal asymmetries corrected by twist-4 factors
② Higher twist azimuthal asymmetries arising
③ Quark-gluon-quark correlators contributions involved

S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

Estimation of the Higher twist effects

Relationships at g=0
i.e. Wandzura-Wilczek-type approximation

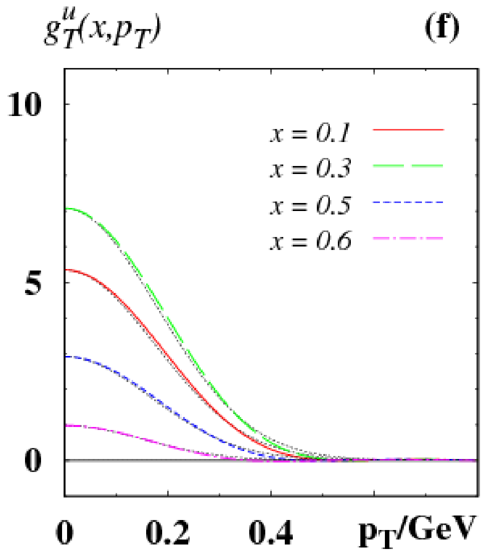
$$x^2 f_3 = -\frac{k_\perp^2}{2M^2} x f^\perp = -\frac{k_\perp^2}{2M^2} f_1$$

$$x^2 g_{3L} = -\frac{k_\perp^2}{2M^2} x g_L^\perp = -\frac{k_\perp^2}{2M^2} g_{1L}$$

$$x^2 g_{3T}^\perp = x g_T = -\frac{k_\perp^2}{2M^2} x g_T^\perp = -\frac{k_\perp^2}{2M^2} g_{1T}^\perp$$

$$x^2 f_{3T}^\perp = -x f_T = -\frac{k_\perp^2}{2M^2} x f_T^\perp = -\frac{k_\perp^2}{2M^2} f_{1T}^\perp$$

Model calculation



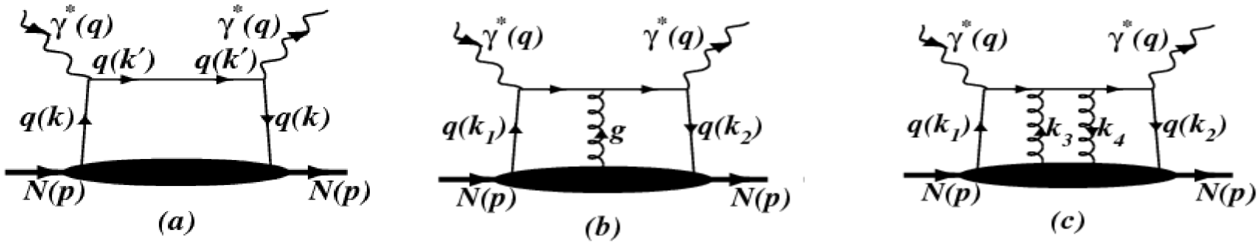
A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P.J. Mulders, M. Schlegel, JHEP 02 (2007) 093.
S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan, PRD 81, 074035 (2010).

TMDs in the MIT bag model

quark-quark and quark-gluon-quark correlators

R. K. Ellis, W. Furmanski, R. Petronzio, NPB 207, 1 (1982); NPB 212, 29 (1983).
Z.T. Liang, X.N. Wang, PRD 75, 094002 (2007).



$\langle P \| \bar{\psi}(0)\mathcal{L}(0,y)\psi(y) \| P\rangle$	$\langle P \| \bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y) \| P\rangle$	$\langle P \| \bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)D_{\perp\sigma}(y)\psi(y) \| P\rangle$
twist-2, twist-3, twist-4	twist-3, twist-4 …	twist-4 …

in MIT bag model ✓ ✓/? ? ? ?
T-even/T-odd

$$\langle P \,|\bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y)|\, P\rangle$$

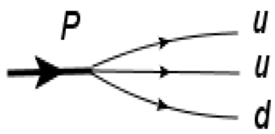
- $\psi(y)$ → **The quark field**
- $|P\rangle$ → **The proton state**
- $D_{\perp\rho}(0)\mathcal{L}(0,y)$ → **The gauge link and the covariant derivative**

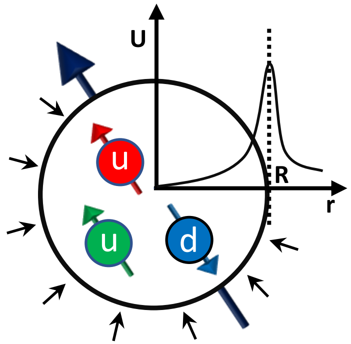
TMDs in the MIT bag model

MIT bag model

① The proton state

$$|P\Lambda\rangle = \frac{1}{\sqrt{3}}\left(a^{\dagger}_{u,\Lambda}a^{\dagger}_{u,\Lambda}a^{\dagger}_{d,-\Lambda} - a^{\dagger}_{u,\Lambda}a^{\dagger}_{u,-\Lambda}a^{\dagger}_{d,\Lambda}\right)|EB\rangle$$





② The quark field

$$\psi_m(\vec{r},t) = \phi_m(\vec{r})\, e^{-i\omega t/R}$$

$$\phi_m(\vec{r}) = \frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0\left(\frac{\omega r}{R}\right)\chi_m \\ j_1\left(\frac{\omega r}{R}\right) i\vec{\sigma}\cdot\vec{e}_r\chi_m \end{pmatrix}$$

j_i are spherical Bessel functions

A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V. Weisskopf, PRD 9, 3471 (1974).
R. Jaffe, PRD 11, 1953 (1975).

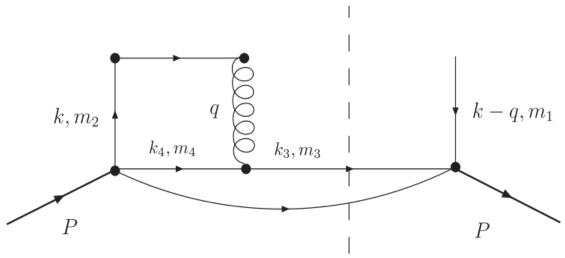
③ The gauge link and the covariant derivative

$$\mathcal{L}(z,y) = \mathcal{L}^{\dagger}(\infty;z)\,\mathcal{L}(\infty;y),$$

$$\mathcal{L}(\infty;y) = \mathcal{P}e^{-ig\int_{y^-}^{\infty} d\xi^- A^+(y^+,\xi^-,\vec{y}_\perp)}$$

$$D_\rho = -i\partial_\rho + gA_\rho$$

One-gluon-exchange approximation



F. Yuan, PLB 575, 45 (2003).
A. Courtoy, S. Scopetta, V. Vento, PRD 79, 074001 (2009).

TMDs in the MIT bag model

Higher twist TMDs

(twist-2, twist-3, twist-4)

Definition

Quark-quark correlator

$$\Phi_{\alpha}^{(0)} = \cdots + \cdots + \frac{M^2}{p^+} n_\alpha \left(f_3 - \frac{\varepsilon_\perp^{kS}}{M} f_{3T}^\perp \right)$$

Quark-gluon-quark correlator

$$\varphi_{\rho\alpha}^{(1)} = p^+ \bar{n}_\alpha k_{\perp\rho} f_d^\perp \cdots + M^2 g_{\perp\rho\alpha} f_{3d} \cdots$$

Quark-two-gluon-quark correlator

$$\varphi_{\rho\sigma\alpha}^{(2)} = p^+ \bar{n}_\alpha M^2 g_{\perp\rho\sigma} f_{3dd} \cdots$$

K. Goeke, A. Metz, M. Schlegel, PLB 618 90 (2005).
S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

Expression in the MIT bag model

$$\begin{aligned} f_3 &= \frac{\sqrt{2}\,(p^+)^2}{2M^2(2\pi)^3} E_P \sum_{m_1 m_2 \Lambda} C_{q,\Lambda'\Lambda}^{m_1 m_2} \delta_{\Lambda\Lambda'} \bar{\varphi}_{m_1}(\vec{\tilde{k}}) \gamma^- \varphi_{m_2}(\vec{\tilde{k}}) \\ &\quad - g^2 E_p \frac{(p^+)^2}{M^2} \sum_{m_1 m_2 m_3 m_4 \Lambda} C_{q,\Lambda'\Lambda}^{m_1 m_2 m_3 m_4} \delta_{\Lambda\Lambda'} \int \frac{d^3 k_1}{(2\pi)^6} \int \frac{d^3 k_2}{(2\pi)^2} \frac{2}{\vec{k}_1^2} \\ &\quad \times P\left(\frac{1}{k_{1z}}\right) \mathrm{Re}\left[\bar{\varphi}_{m_1}(\vec{\tilde{k}} - \vec{k}_1)\gamma^- \varphi_{m_2}(\vec{\tilde{k}}) \bar{\varphi}_{m_3}(\vec{k}_2 + \vec{k}_1)\gamma^+ \varphi_{m_4}(\vec{k}_2)\right] \end{aligned}$$

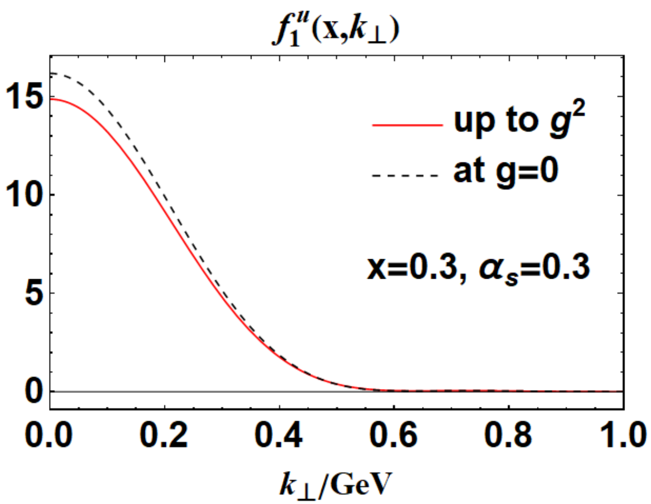
$$\begin{aligned} f_{3T}^\perp &= -g^2 \frac{(p^+)^2}{M k_y} E_p \sum_{m_1 m_2 m_3 m_4 \Lambda} C_{q,\Lambda'\Lambda}^{m_1 m_2 m_3 m_4} \delta_{\Lambda,-\Lambda'} \int \frac{d^2 k_{1\perp}}{(2\pi)^5} \int \frac{d^3 k_2}{(2\pi)^2} \\ &\quad \frac{1}{\vec{k}_{1\perp}^2} \mathrm{Im}\left[\bar{\varphi}_{m_1}(\vec{\tilde{k}} - \vec{k}_{1\perp})\gamma^- \varphi_{m_2}(\vec{\tilde{k}}) \bar{\varphi}_{m_3}(\vec{k}_2 + \vec{k}_1)\gamma^+ \varphi_{m_4}(\vec{k}_2)\right] \end{aligned}$$

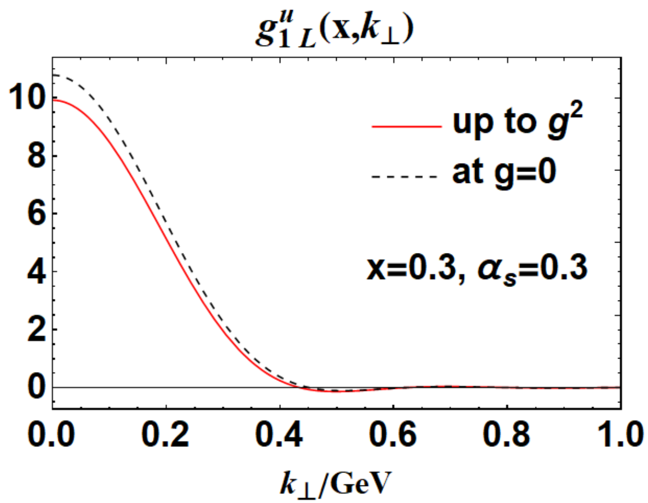
T-even: none gluon term
+ one-gluon-exchange term

T-odd: only one-gluon-exchange term

TMDs in the MIT bag model

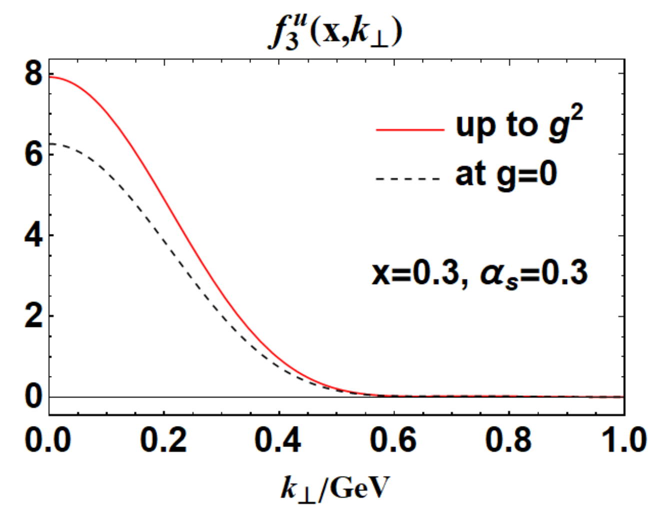
Higher twist TMDs from the quark-quark correlator (T-even)

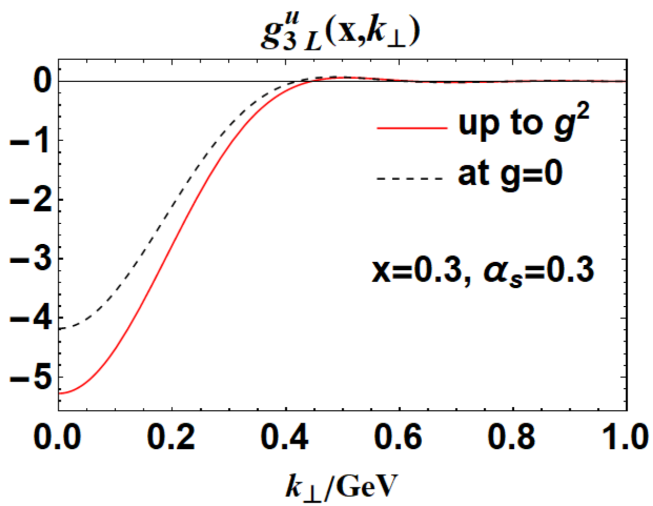




Twist-2

H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan, PRD 81, 074035 (2010).



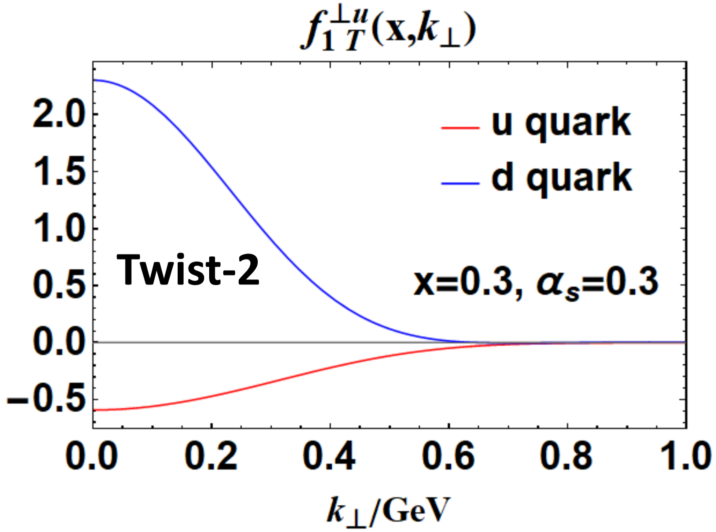


Twist-4

On the same order of magnitude as twist-2 TMDs

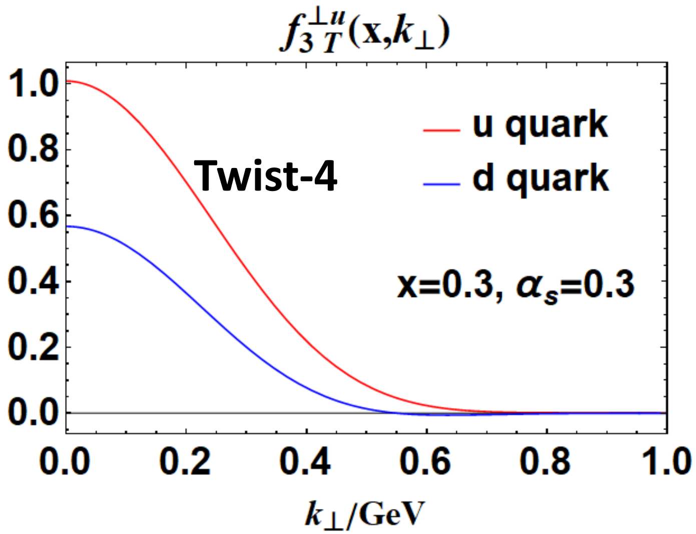
TMDs in the MIT bag model

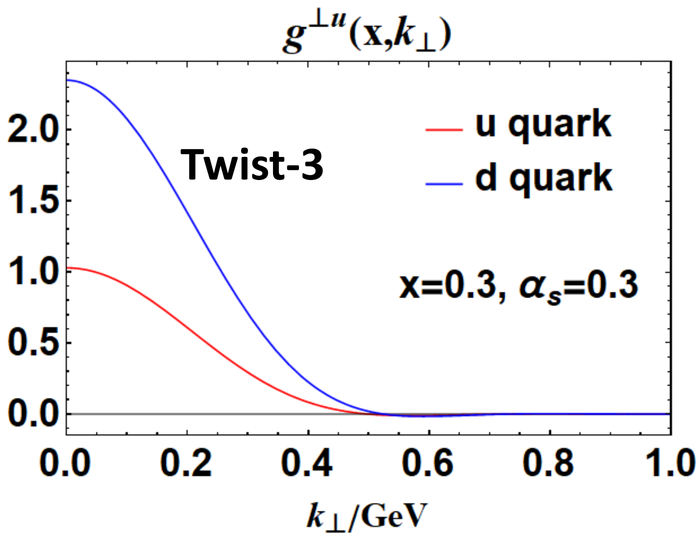
Higher twist TMDs from the quark-quark correlator (T-odd)



F. Yuan, PLB 575, 45 (2003).
A. Courtoy, S. Scopetta, V. Vento, PRD 79, 074001 (2009).

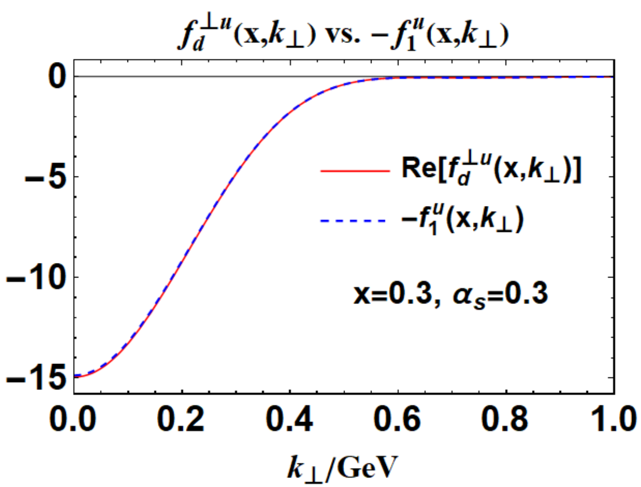
All of them on the same order of magnitude.

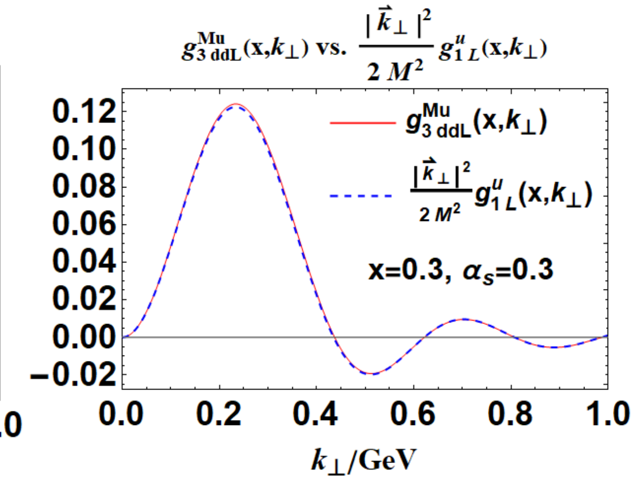




TMDs in the MIT bag model

Higher twist TMDs from the quark-gluon-quark correlator (T-even)





Relationships at g=0

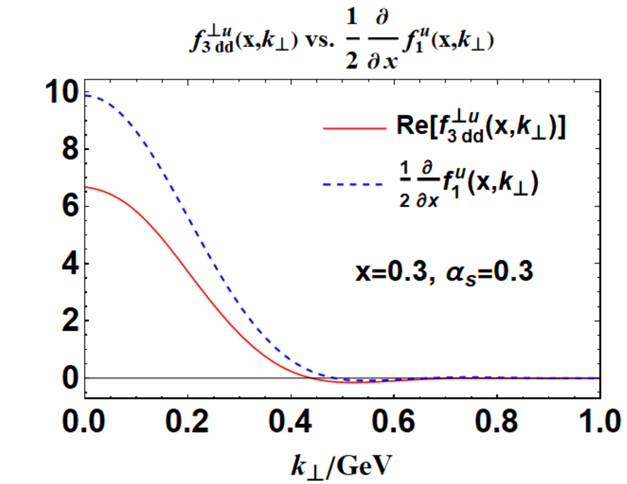
$$f_d^\perp = -f_1$$

$$g_{3ddL}^M = \frac{\left|\vec{k}_\perp\right|^2}{2M^2} g_{1L}$$

$$\mathrm{Re}\left[f_{3dd}^\perp\right] = \frac{1}{2}\frac{\partial}{\partial x} f_1$$

$$\vdots$$

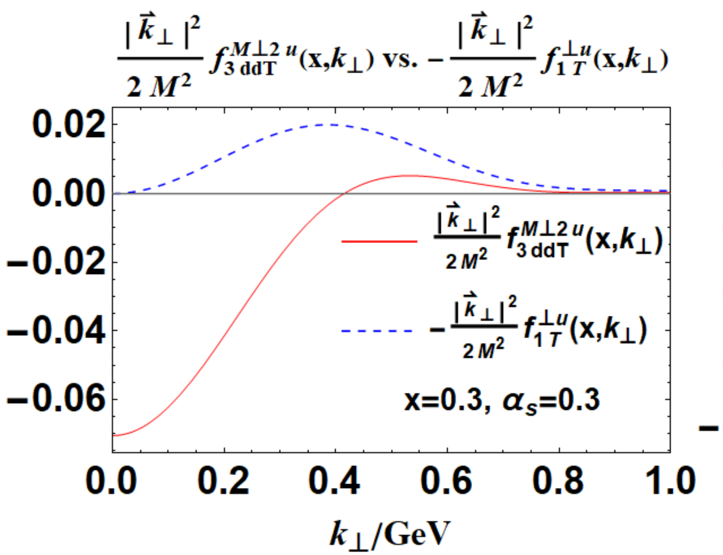
S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

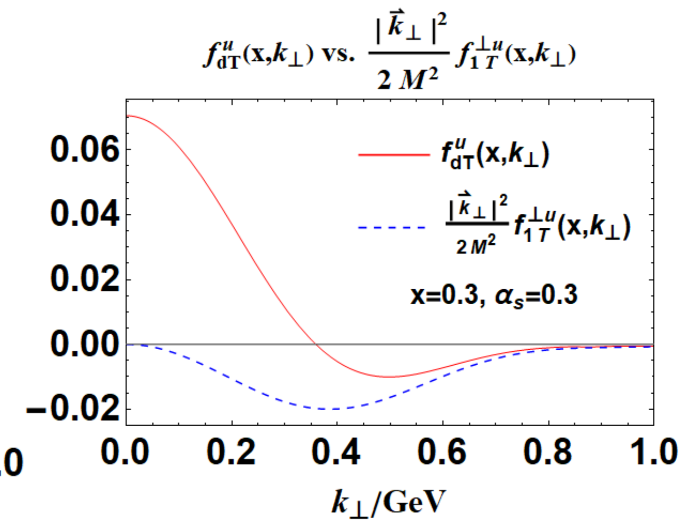


This approximation (g=0) is very efficient for the T-even TMDs, except for f_{dd}-type TMDs.

TMDs in the MIT bag model

Higher twist TMDs from the quark-gluon-quark correlator (T-odd)





Relationships at g=0

$$f_{dT} = \frac{\left|\vec{k}_\perp\right|^2}{2M^2} f_{1T}^{\perp}$$

$$f_{3ddT}^{M\perp2} = -f_{1T}^{\perp}$$

$$\vdots$$

S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

The deviation of this approximation (g=0) is very significant for the T-odd TMDs.

TMDs in the MIT bag model

Twist-2 azimuthal asymmetries in $e^- p \to e^- q X$ (T-even)

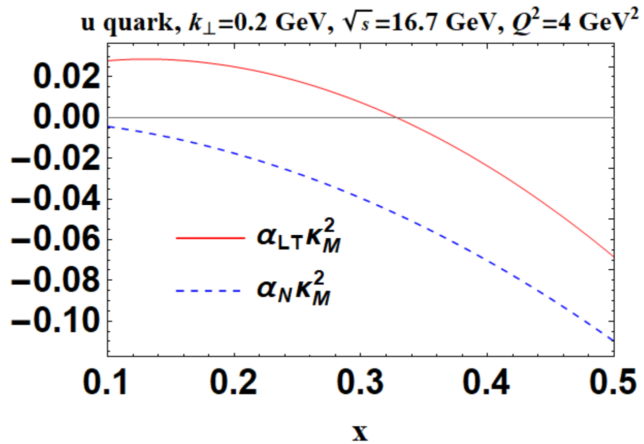
$$\langle \cos(\phi - \phi_S) \rangle_{LT} = \frac{|\vec{k}_\perp|}{2M} \frac{C(y)}{A(y)} \frac{g_{1T}^\perp}{f_1} \left(1 - \alpha_{LT} \kappa_M^2\right)$$

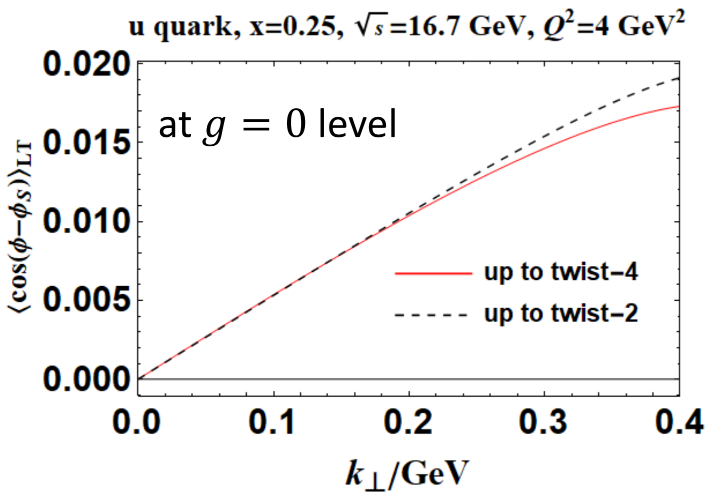
twist-2 and twist-4

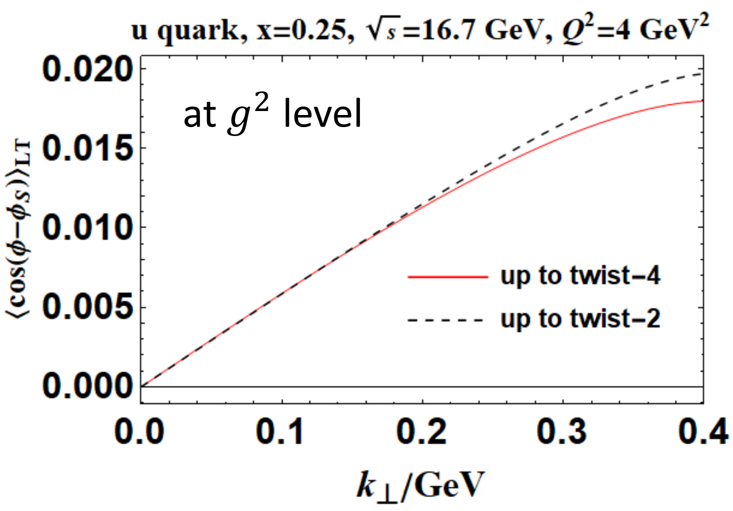
$$\alpha_{LT} = \alpha_N + \alpha_{UU} - 4\kappa_M^2 x \frac{\mathrm{Re} f_{+3ddT}^{\perp 3}}{g_{1T}^\perp}$$

$$\alpha_N = -2x^2 \frac{E(y)}{A(y)} \qquad \kappa_M = \frac{M}{Q}$$

$$\alpha_{UU} = \frac{E(y)}{A(y)} \frac{2x^2 f_3 + 4x \mathrm{Re} f_{-3d} - 2 f_{-3dd}^M}{f_1} - 4x \frac{\mathrm{Re} f_{+3dd}}{f_1}$$







Twist-4 contributions vary between 2% and 5%.

TMDs in the MIT bag model

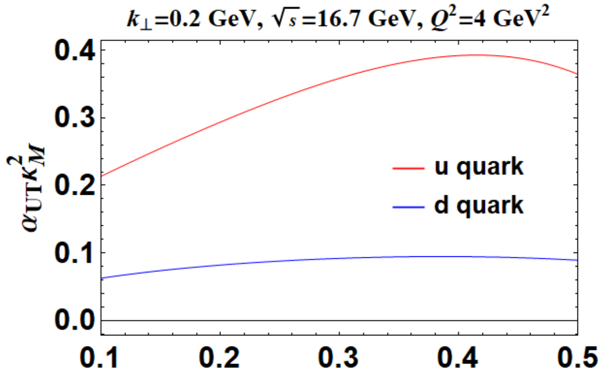
Twist-2 azimuthal asymmetries in $e^- p \to e^- q X$ (T-odd)

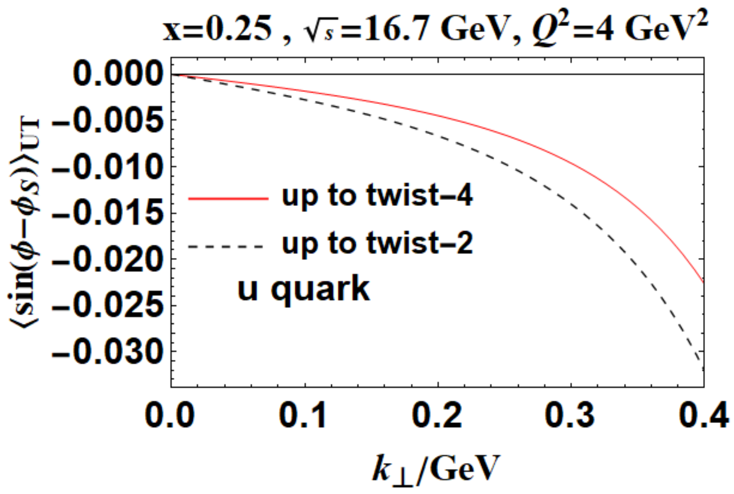
$$\langle \sin(\phi - \phi_S) \rangle_{UT} = \frac{|\vec{k}_\perp|}{2M} \frac{f_{1T}^\perp}{f_1} \left(1 - \alpha_{UT} \kappa_M^2\right)$$

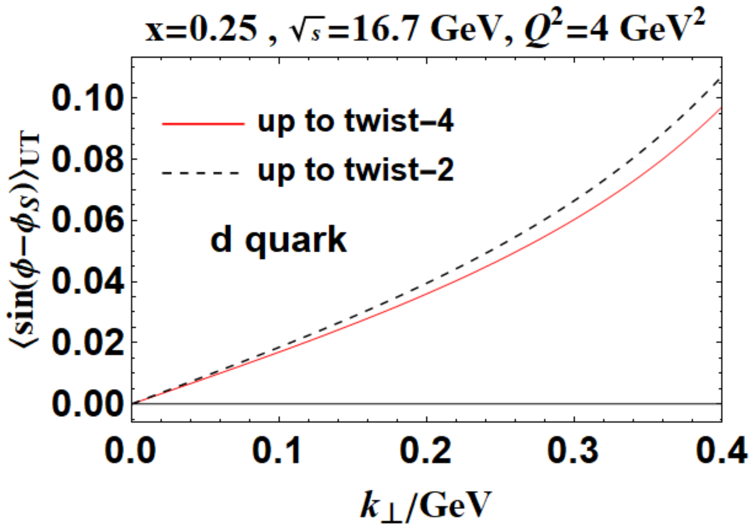
twist-2 and twist-4

$$\alpha_{UT} = \alpha_{UU} - \frac{E(y)}{A(y)} \frac{\left(2x^2 f_{3T}^\perp + 4x \mathrm{Re} f_{-3dT}^\perp - 2 f_{-3ddT}^{M\perp}\right)}{f_{1T}^\perp} + 4x \frac{\mathrm{Re} f_{+3ddT}^\perp}{f_{1T}^\perp}$$

$$\alpha_{UU} = \frac{E(y)}{A(y)} \frac{2x^2 f_3 + 4x \mathrm{Re} f_{-3d} - 2 f_{-3dd}^M}{f_1} - 4x \frac{\mathrm{Re} f_{+3dd}}{f_1}$$





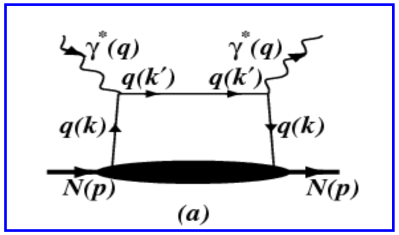


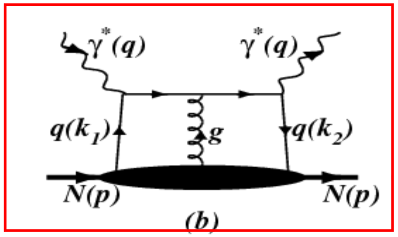
Twist-4 contributions are significant: u quark 30%, d quark 8%

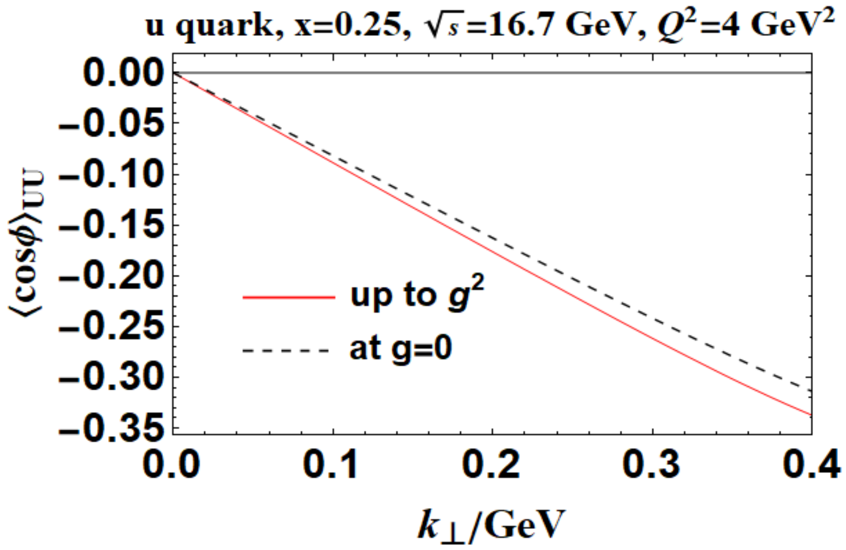
TMDs in the MIT bag model

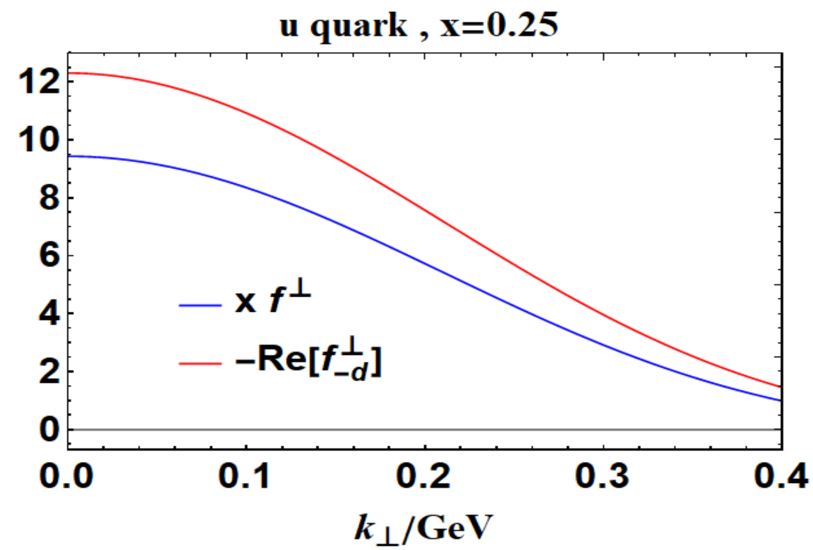
Twist-3 azimuthal asymmetries in $e^- p \to e^- q X$

$$\langle \cos\phi \rangle_{UU} = -\kappa_M \frac{\left|\vec{k}_\perp\right|}{M} \frac{B(y)}{A(y)} \frac{x f^\perp - \mathrm{Re}\left[f^\perp_{-d}\right]}{2 f_1}$$





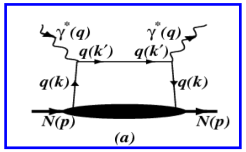


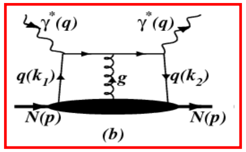


Contributions from quark-gluon-quark correlator can be comparable with those from quark-quark correlator.

TMDs in the MIT bag model

Twist-3 azimuthal asymmetries in $e^- p \to e^- q X$

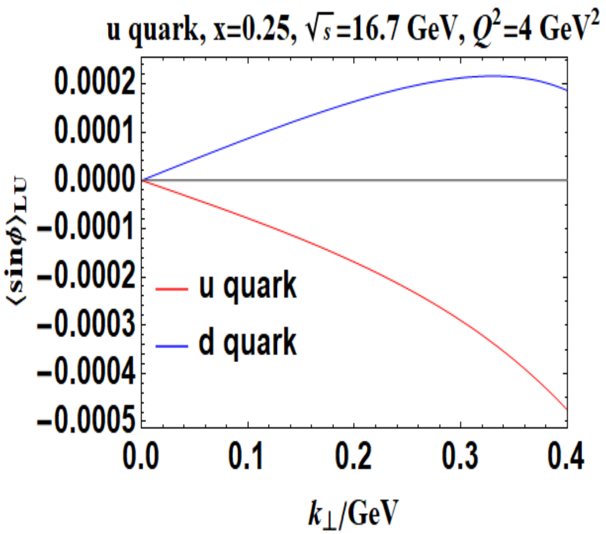




$$\langle \sin\phi\rangle_{LU} = \kappa_M \frac{|\vec{k}_\perp|}{M}\frac{B(y)}{A(y)}\frac{x g^\perp + \mathrm{Im}\left[f_{-d}^\perp\right]}{2 f_1}$$

$$\langle \cos\phi_S\rangle_{LT} = -\kappa_M \frac{D(y)}{A(y)}\frac{x g_T + \mathrm{Im}\left[f_{-dT}\right]}{2 f_1}$$

$$\langle \sin(2\phi-\phi_S)\rangle_{UT} = -\kappa_M \frac{|\vec{k}_\perp|^2}{2M^2}\frac{B(y)}{A(y)}\frac{x f_T^\perp - \mathrm{Re}\left[f_{-dT}^\perp\right]}{2 f_1}$$



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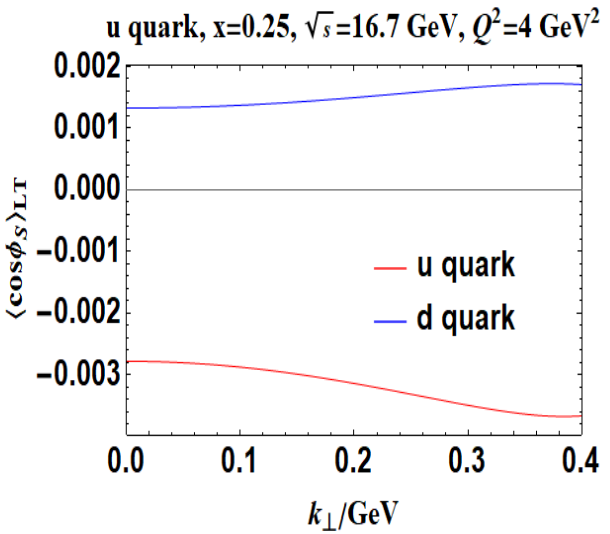

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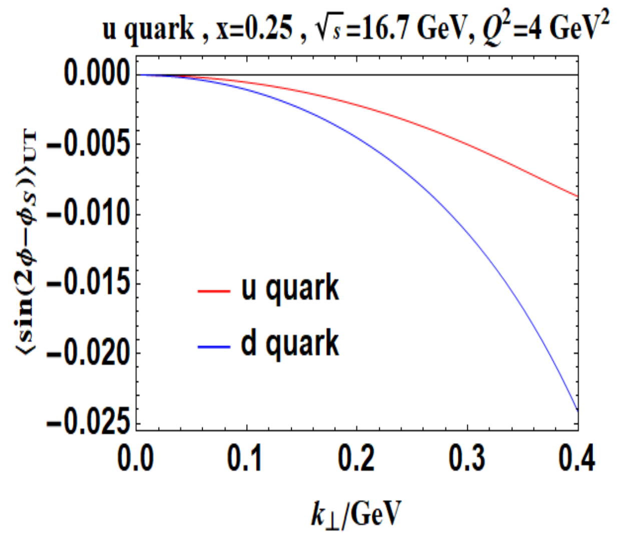

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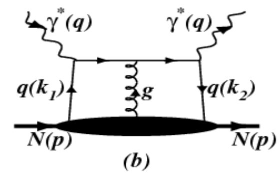
Twist-3 azimuthal asymmetries vary between 0.01 % and 20%.

TMDs in the MIT bag model

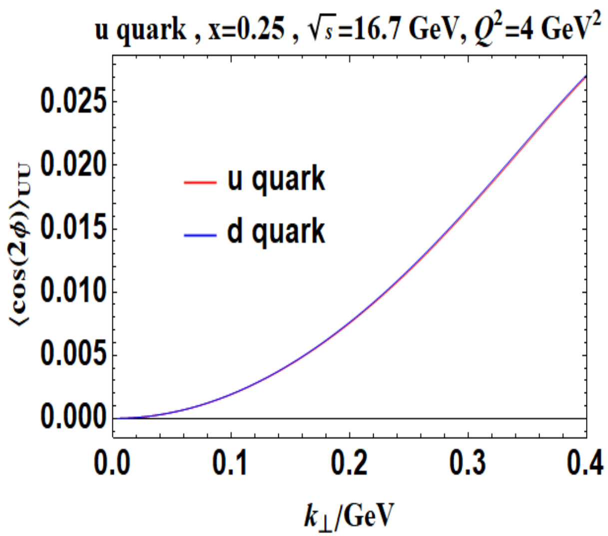
Twist-4 azimuthal asymmetries in $e^- p \to e^- q X$



$$\langle \cos 2\phi \rangle_{UU} = -\kappa_M^2 \frac{|\vec{k}_\perp|^2}{M^2} \frac{E(y)}{A(y)} \frac{x f^{\perp}_{-3d}}{f_1}$$

$$\langle \sin 2\phi \rangle_{UL} = \kappa_M^2 \frac{|\vec{k}_\perp|^2}{M^2} \frac{E(y)}{A(y)} \frac{x f^{\perp}_{+3dL}}{f_1}$$

$$\langle \sin(3\phi - \phi_S) \rangle_{UT} = -x\kappa_M^2 \frac{|\vec{k}_\perp|^3}{2M^3} \frac{E(y)}{A(y)} \frac{f^{\perp 4}_{+3dT} - f^{\perp 2}_{-3dT}}{f_1}$$



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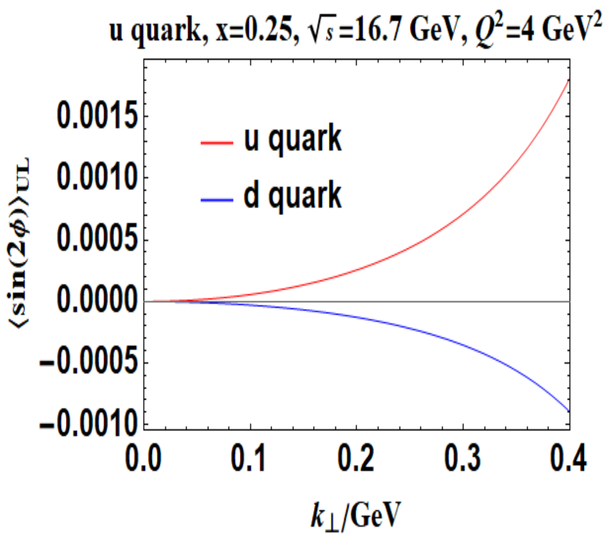
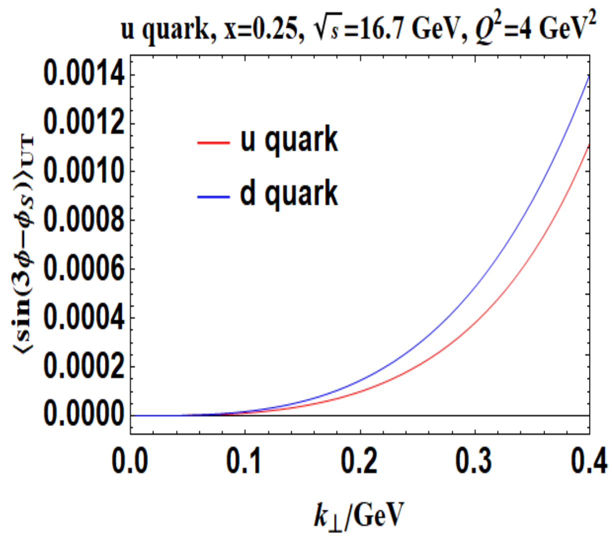


~ 0.01%



~ 0.01%

Twist-4 azimuthal asymmetries vary between 0.01 % and 1 %.

Summary and outlook

- We calculate the chiral-even TMDs up to twist-4 in the MIT bag model.

	twist-2	twist-3	twist-4
quark-quark correlator	4	8 (4+4)	4
quark-gluon-quark correlator	/	8	48

- Twist-4 TMDs defined via quark-quark correlator are about the same size as the twist-2 TMDs in order of magnitude.

- The twist-4 TMDs provide considerable contributions to the leading twist azimuthal asymmetries at $\sqrt{s} = 16.7\ \mathrm{GeV}, Q^2 = 4\ \mathrm{GeV}^2$, especially for the T-odd type azimuthal asymmetries.

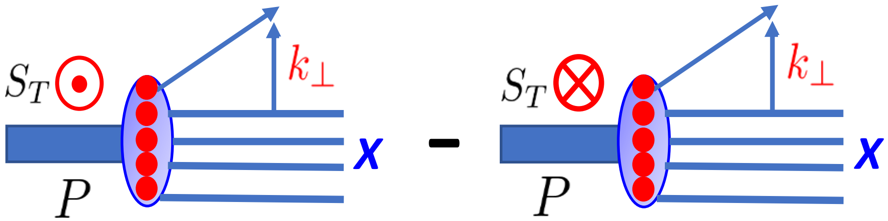
- Higher twist azimuthal asymmetries vary between 0.01 % and 20 % at $\sqrt{s} = 16.7\ \mathrm{GeV}, Q^2 = 4\ \mathrm{GeV}^2$.

- Higher twist effects from the Quark-gluon-quark correlators are comparable with those from quark-quark correlator.

Thanks for your attention!

backup

T-odd effect: simulating the effect of the gauge link



$$f_q(x, k_\perp; S_T) = f_q(x, k_\perp) + \left(\hat{k}_\perp \times \hat{p}\right) \cdot \vec{S}_T f_{1T}^{\perp}(x, k_\perp)$$

D.W. Sivers, PRD 41, 83 (1990).
D.W. Sivers, PRD 43, 261 (1991).

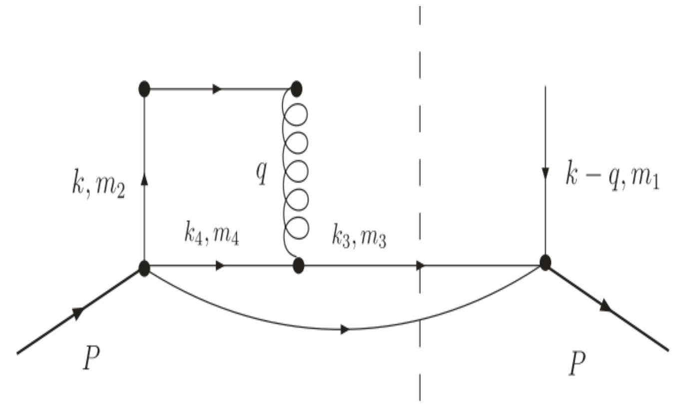
$$\hat{\Phi}^{(0)}(x, k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ik\cdot y} \times \langle P | \bar{\psi}(0)\psi(y) | P \rangle$$

$$\Downarrow$$

$$\hat{\Phi}^{(0)}(x, k_\perp) = \int \frac{p^+ dy^- d^2 y_\perp}{(2\pi)^3} e^{ik\cdot y} \times \langle P | \bar{\psi}(0)\mathcal{L}(0, y)\psi(y) | P \rangle$$

S.J. Brodsky, D.S. Hwang, I. Schmidt, PLB 530, 99 (2002).
J.C. Collins, PLB 536, 43 (2002).

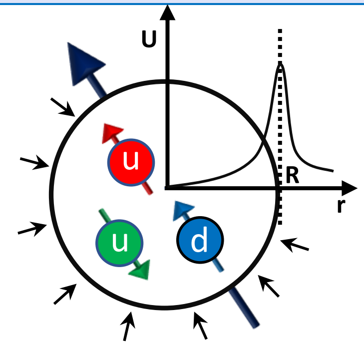
One-gluon-exchange approximation

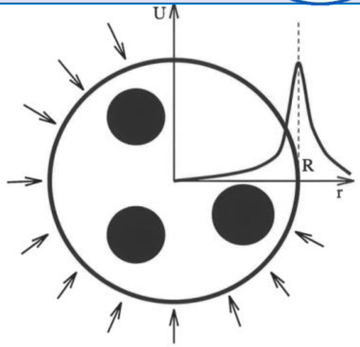


helicity flip: $\delta_{m_1 - m_2}$ vs. $\delta_{m_1 - m_2}$ or $\delta_{m_3 - m_4}$

F. Yuan, PLB 575, 45 (2003).
A. Courtoy, S. Scopetta, V. Vento, PRD 79, 074001 (2009).

MIT bag model





Lagrangian density for the MIT bag model

$$\mathcal{L} = \left(i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - B\right)\theta\left(R-r\right) - \frac{1}{2}\bar{\psi}\psi\delta\left(R-r\right)$$

E. Roionov, *The MIT bag model in nuclear and particle physics*, 1997.

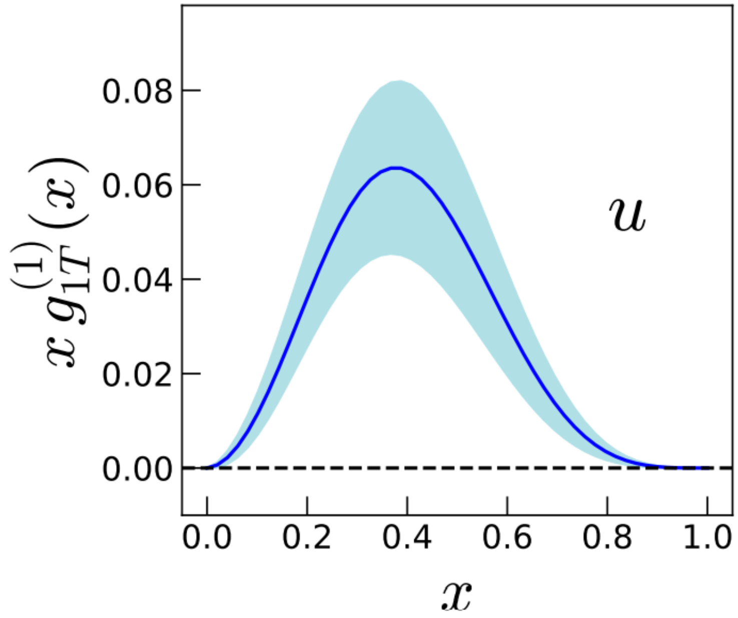
$$\begin{aligned} i\gamma^{\mu}\partial_{\mu}\psi &= 0 \qquad \text{for } r < R \\ i\gamma^{\mu}\eta_{\mu}\psi &= \psi \qquad \text{for } \vec{r} \in S \\ -\frac{1}{2}\eta_{\mu}\partial^{\mu}\left(\bar{\psi}\psi\right) &= B \qquad \text{for } \vec{r} \in S \end{aligned}$$

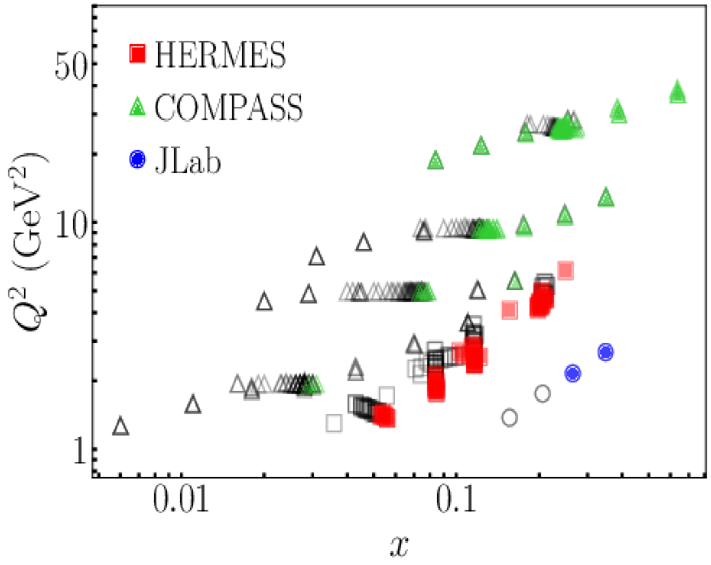
$$\begin{aligned} \psi_m\left(\vec{r},t\right) &= \phi_m\left(\vec{r}\right)e^{-i\omega t/R} \\ \phi_m\left(\vec{r}\right) &= \frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0\left(\frac{\omega r}{R}\right)\chi_m \\ j_1\left(\frac{\omega r}{R}\right)i\vec{\sigma}\cdot\vec{e}_r\chi_m \end{pmatrix} \\ N &= \left(\frac{\omega^4}{R^3\left(\omega^2-\sin^2(\omega)\right)}\right)^{1/2} \end{aligned}$$

A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V.F. Weisskopf, PRD 9, 3471 (1974).
R. Jaffe, PRD 11, 1953 (1975).

j_i are spherical Bessel functions

Energy scale





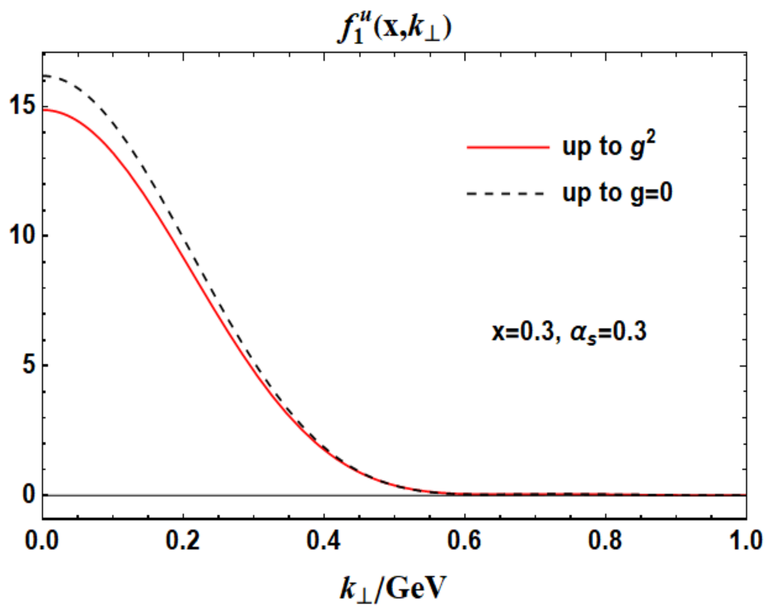
S. Bhattacharya, Z.B. Kang, A. Metz, G. Penn, D. Pitonyak, PRD 105, 034007 (2022).

Leading twsit TMDs

T-even TMDs vs. T-odd TMDs

T-even TMDs

T-odd TMDs



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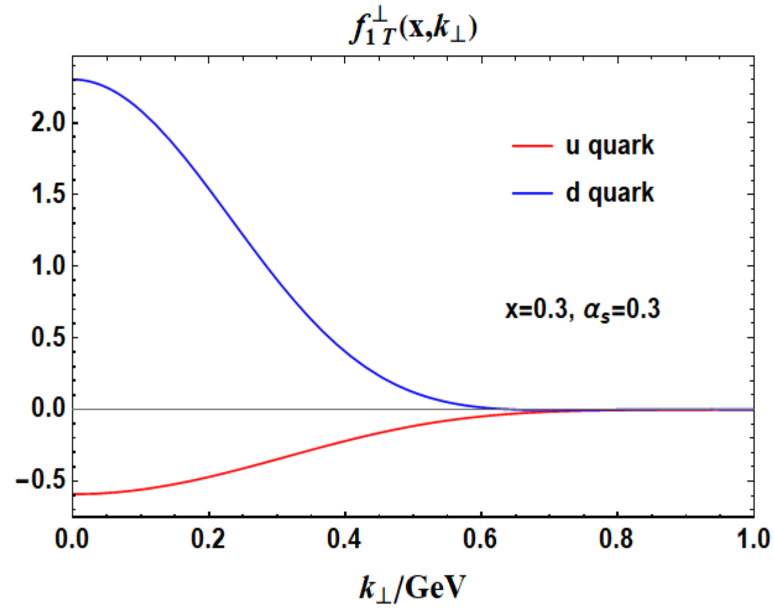
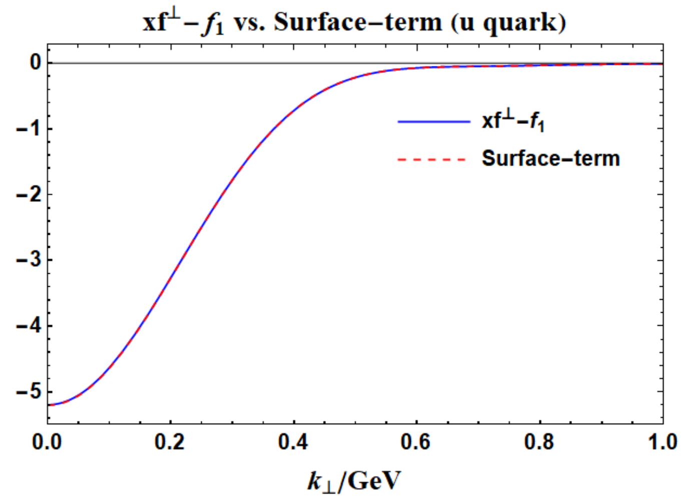
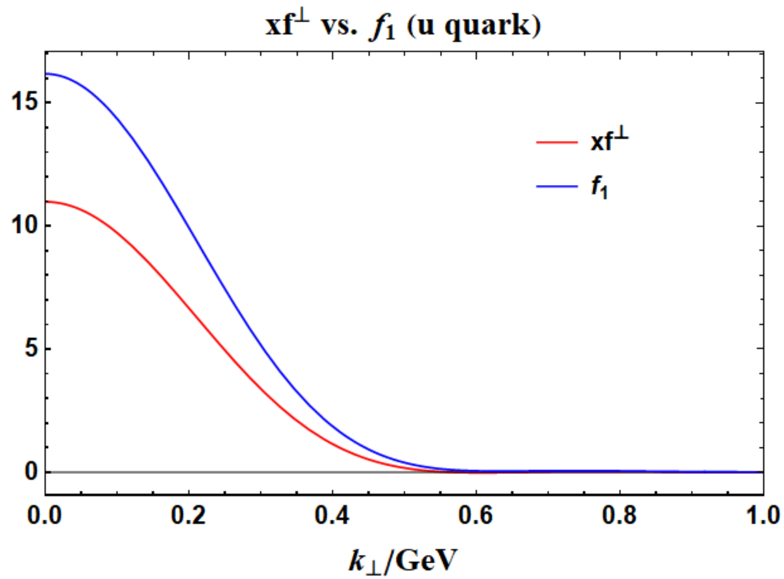


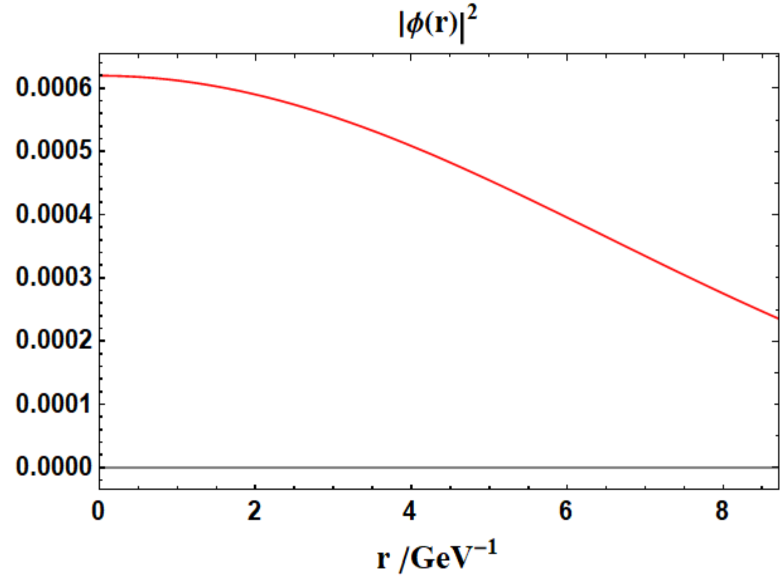
Surface effects

WW-type approximation

$$xf^{\perp} = f_1$$

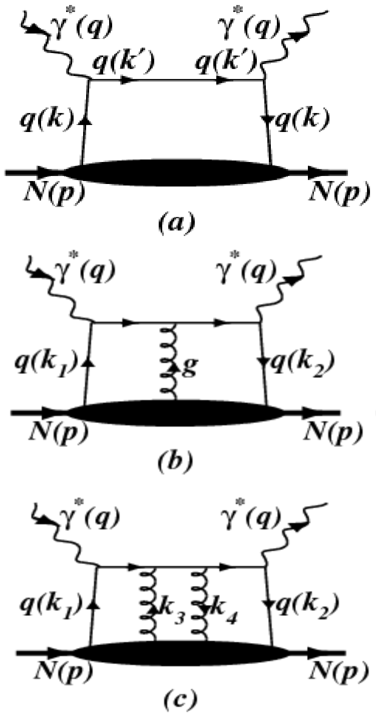






TMDs in the MIT bag model

Collinear expansion and quark-quark/quark-j-gluon-quark correlators



$$\hat{\Phi}^{(0)}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} e^{ik\cdot y} \times \langle P \,|\bar{\psi}(0)\mathcal{L}(0,y)\psi(y)|\, P\rangle$$

twist-2, twist-3, twist-4 …

$$\hat{\varphi}^{(1)}_{\rho}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} e^{ik\cdot y} \times \langle P \,|\bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y)|\, P\rangle$$

twist-3, twist-4 …

$$\hat{\varphi}^{(2)}_{\rho\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} \int_0^\infty ip^+dz^- e^{ik\cdot y} \times \langle P \,|\bar{\psi}(0)\mathcal{L}(0,z)D_{\perp\rho}(z)D_{\perp\sigma}(z)\mathcal{L}(z,y)\psi(y)|\, P\rangle$$

$$\hat{\varphi}^{(2,M)}_{\rho\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} e^{ik\cdot y} \times \langle P \,|\bar{\psi}(0)D_{\perp\rho}(0)\mathcal{L}(0,y)D_{\perp\sigma}(y)\psi(y)|\, P\rangle$$

twist-4 …

Z.T. Liang, X.N. Wang, PRD 75, 094002 (2007).
Y.K. Song, J.H. Gao, Z.T. Liang, X.N. Wang, PRD 83, 054010 (2011); 89, 014005 (2014).

$$\langle \boldsymbol{P} \,|\bar{\psi}(0)\boldsymbol{D}_{\perp\rho}(0)\mathcal{L}(0,y)\psi(y)|\, \boldsymbol{P}\rangle$$

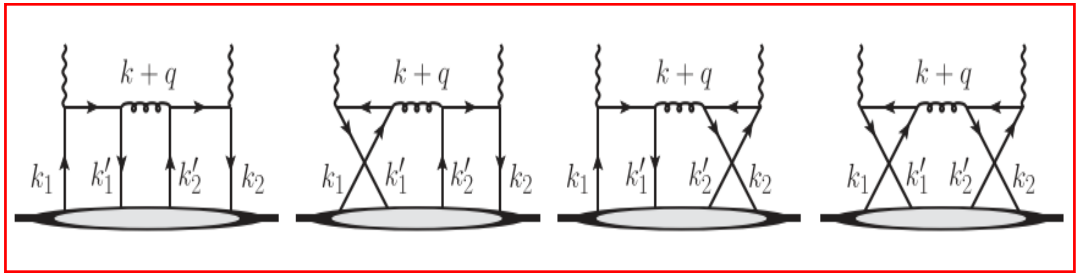
The quark field

The proton state

The gauge link and the covariant derivative

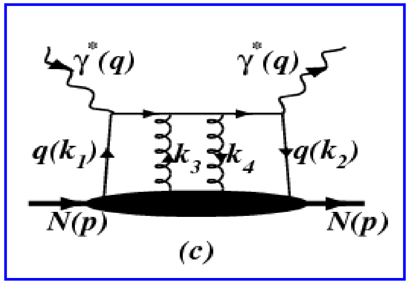
Contributions from the four-quark correlator

TMDs defined via the four-quark correlator



R. Ellis, W. Furmanski, R. Petronzio, NPB 207, 1 (1982).
J.W. Qiu, PRD 42, 30 (1990).
S.Y. Wei, Y.K. Song, K.B. Chen, Z.T. Liang, PRD 95, 074017 (2017).

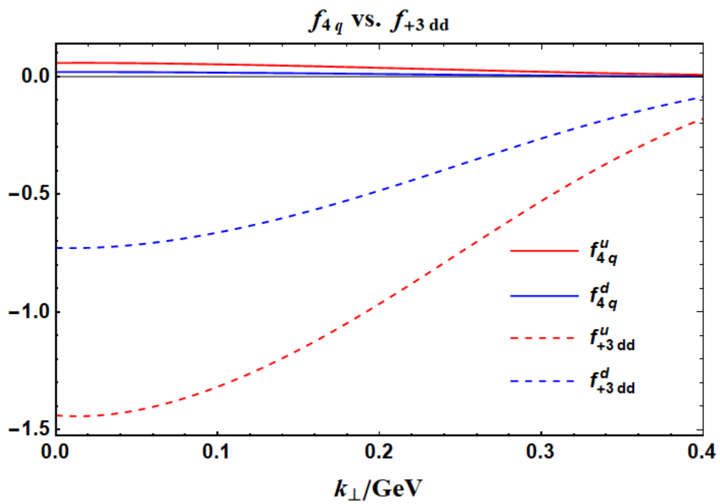
$$W_{UU,T} = xf_1 + 4x^2\kappa_M^2 \left(f_{+3dd} + f_{4q}\right)$$
$$W_{LL} = xg_{1L} + 4x^2\kappa_M^2 \left(f_{+3ddL} - f_{4qL}\right)$$
$$W_{UT,T}^{\sin(\phi-\phi_S)} = \frac{|\vec{k}_\perp|}{M}\left(xf_{1T}^\perp + 4x^2\kappa_M^2\left(f_{+3ddT}^\perp + f_{4qT}^\perp\right)\right)$$
$$W_{LT}^{\cos(\phi-\phi_S)} = \frac{|\vec{k}_\perp|}{M}\left(xg_{1T}^\perp + 4x^2\kappa_M^2\left(f_{+3ddT}^{\perp 3} - f_{4qT}^{\perp 2}\right)\right)$$

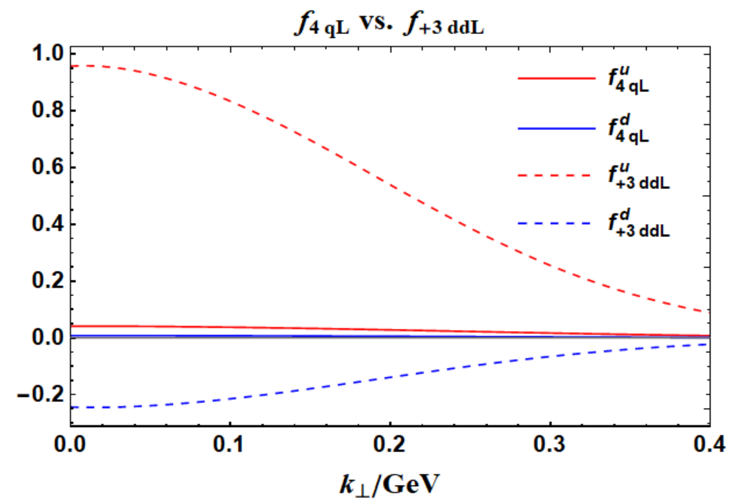


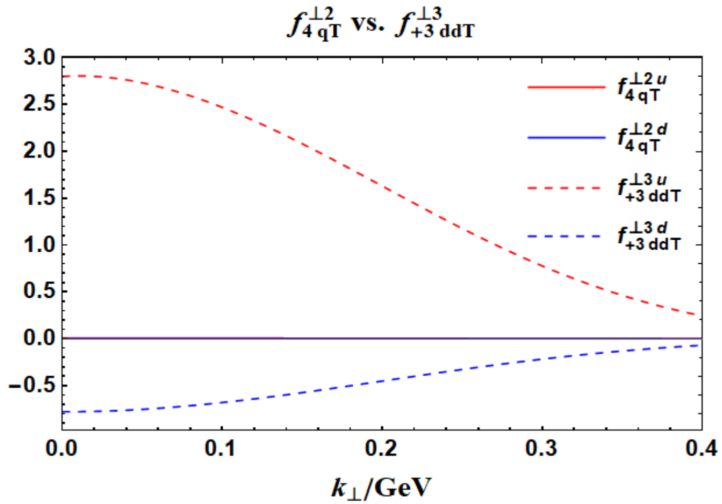
TMDs defined via four-quark correlator behave as addenda to those defined via the quark-two-gluon-quark correlator.

Contributions from the four-quark correlator

Four-quark correlator vs. quark-two-gluon-quark correlator







One might well neglect the contributions from the four-quark correlator.