Lattice QCD using Large momentum effective theory

Lattice Parton Collaboration

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2022/08/10

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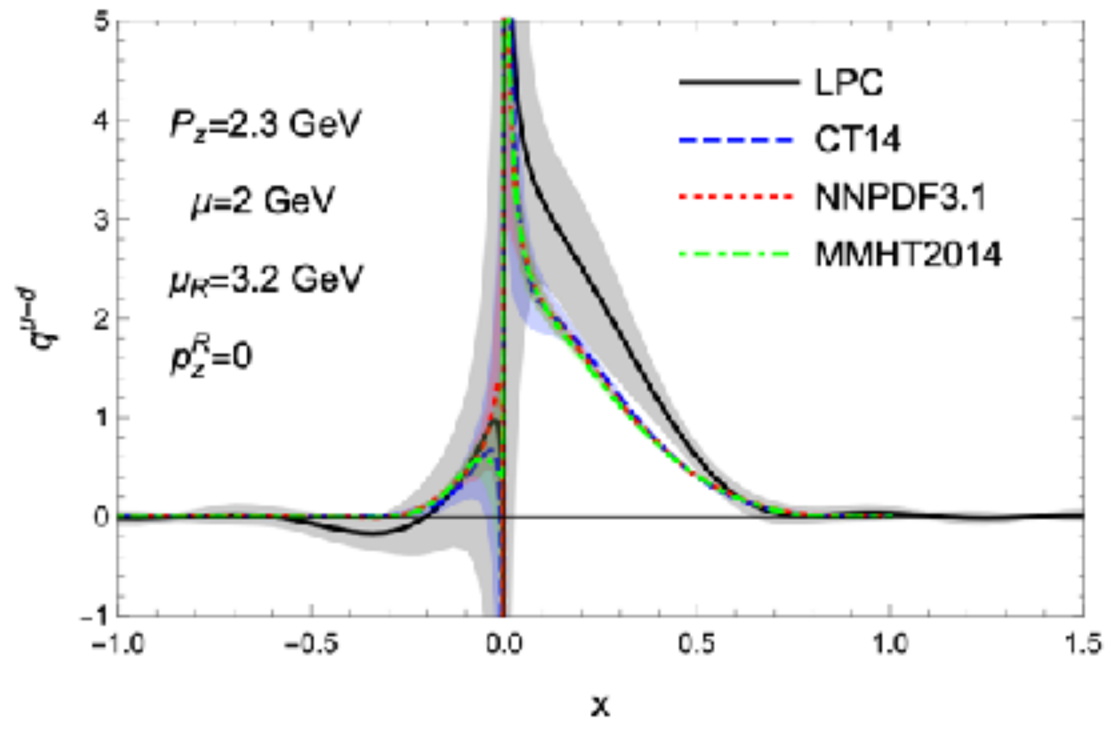
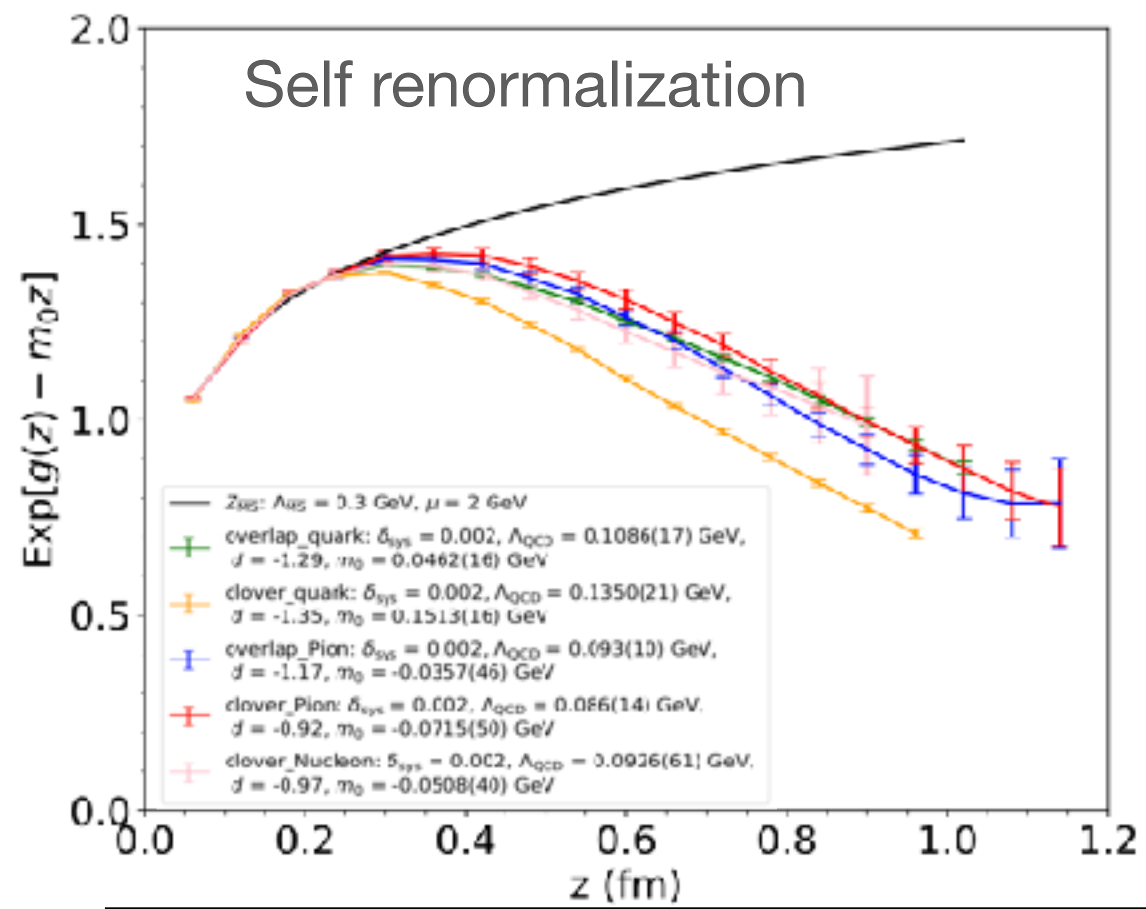
Lattice Parton Collaboration

- Understanding partonic structure of hadrons from lattice QCD

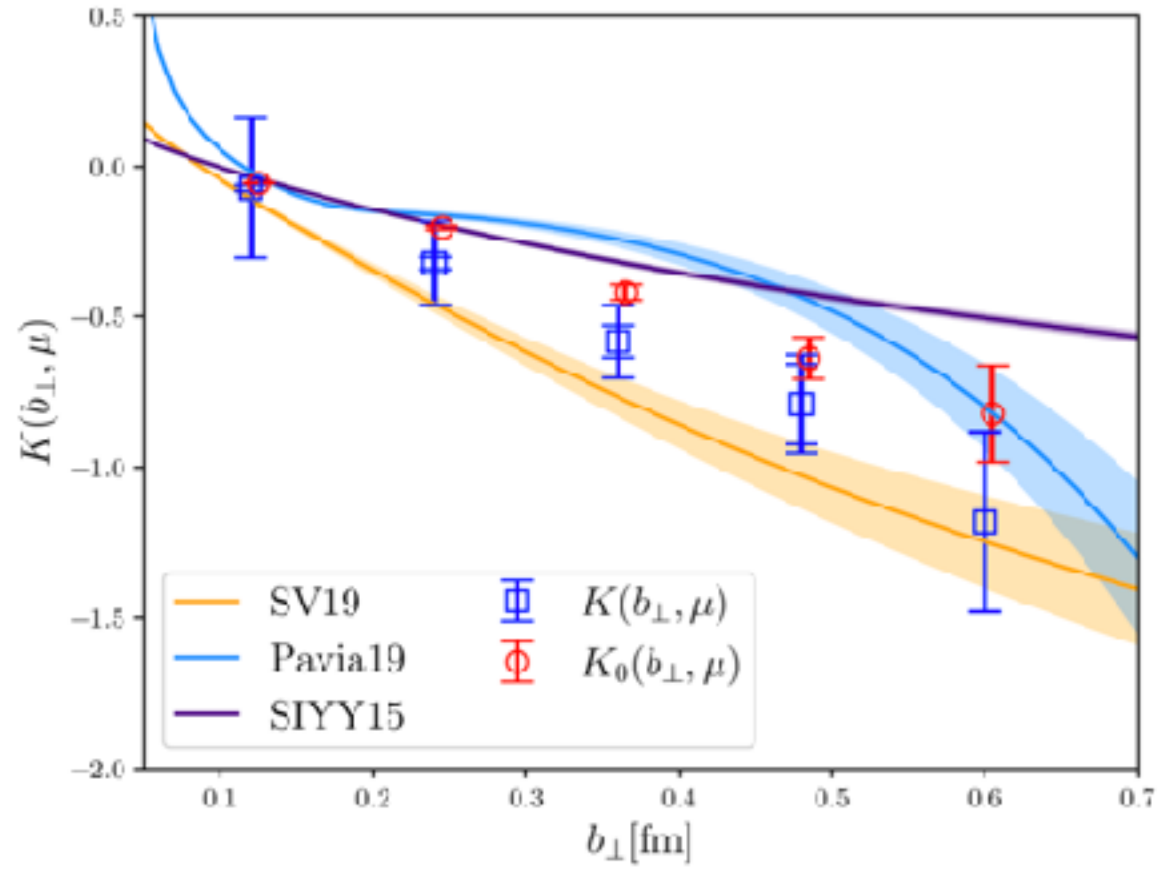


Unpolarized nucleon PDF

Y. Liu, et.al., LPC, PRD101(2020)034020



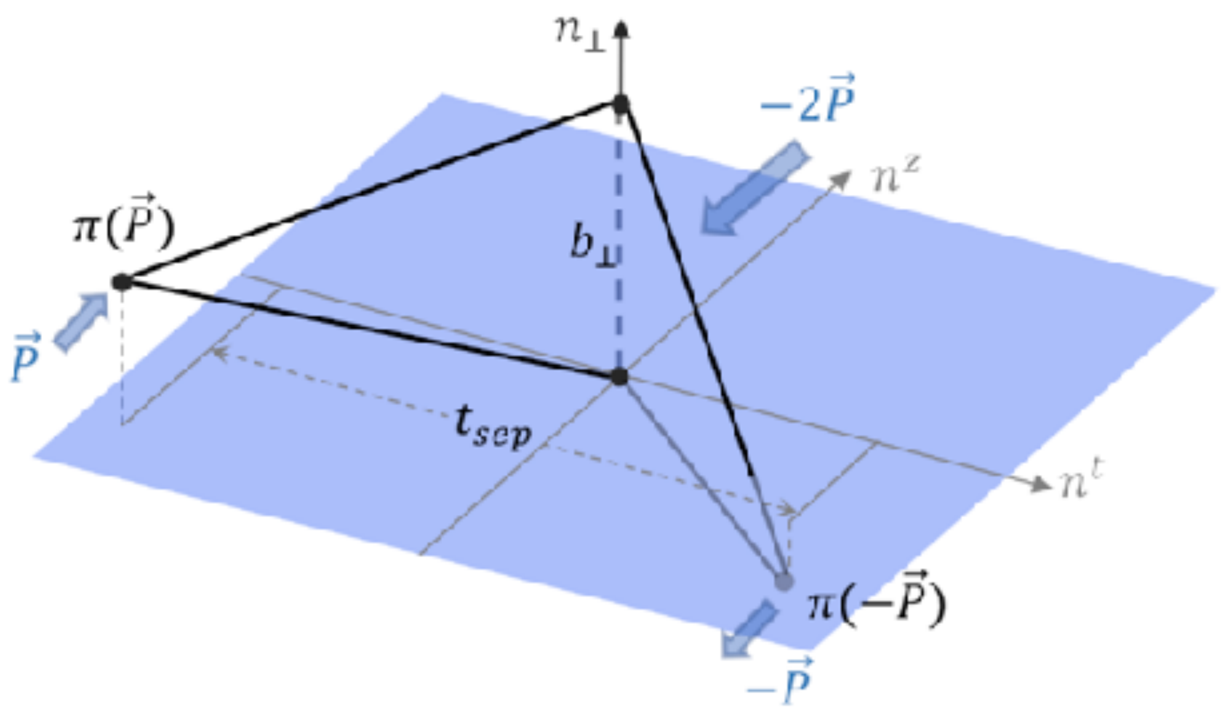
Y.-K. Huo. et.al., LPC, NPB969(2021)115443



CS-Kernel with 1-loop matching

M. Chu. et.al., LPC, accepted by PRD

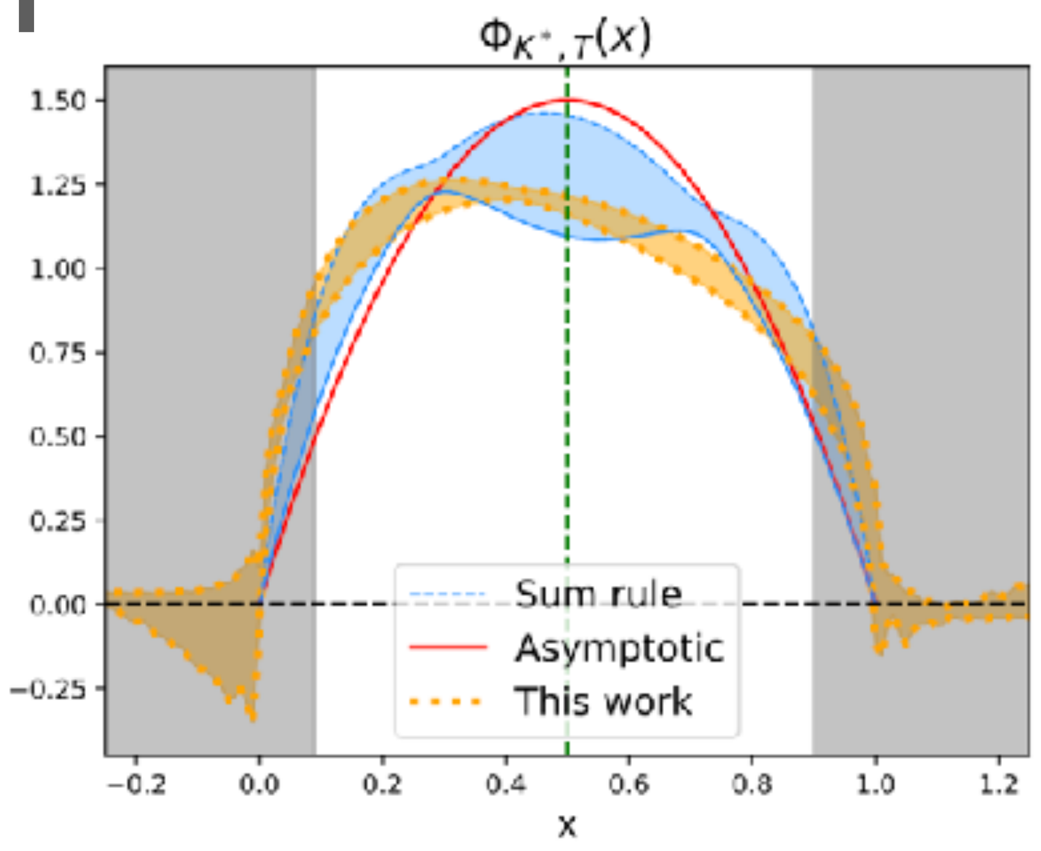
2020



TMD Soft function

Q.-A. Zhang. et.al., LPC, PRL125(2020)192001

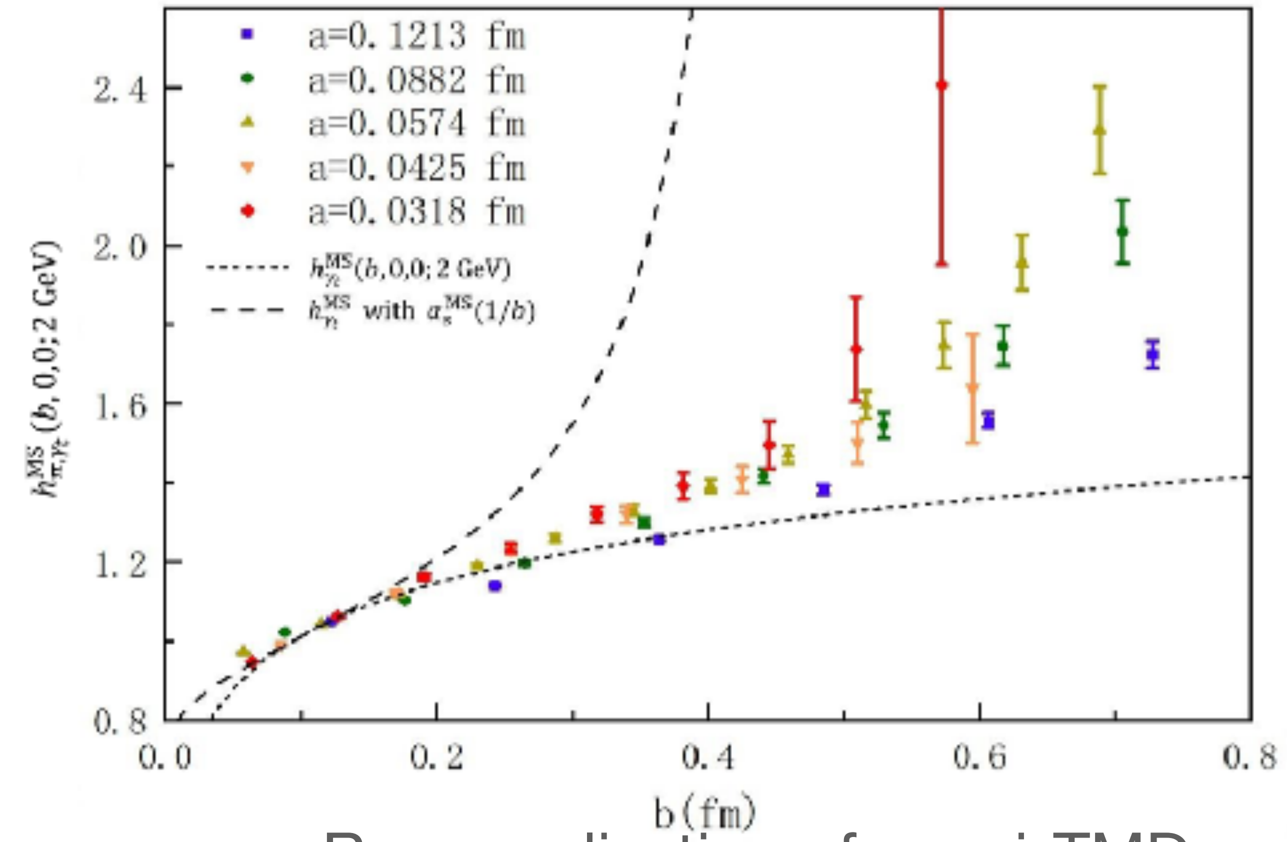
2021



Vector meson DA

J. Hua. et.al., LPC, PRL127(2021)062002

2022

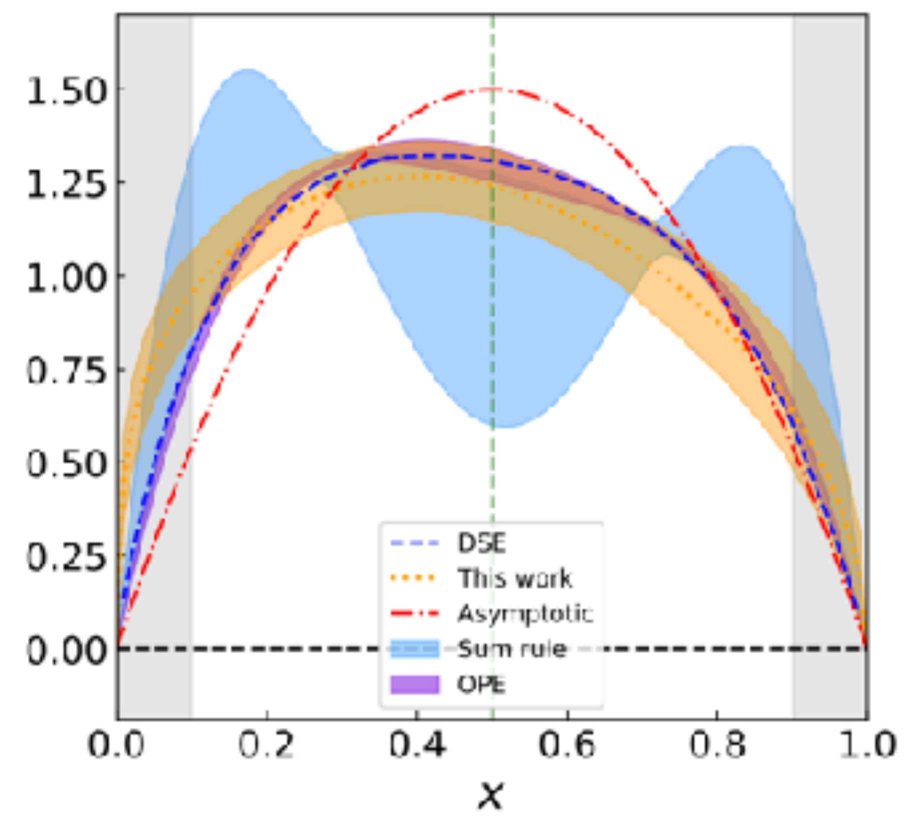


Renormalization of quasi-TMD

K. Zhang. et.al., LPC, accepted by PRL

Lattice Parton Collaboration

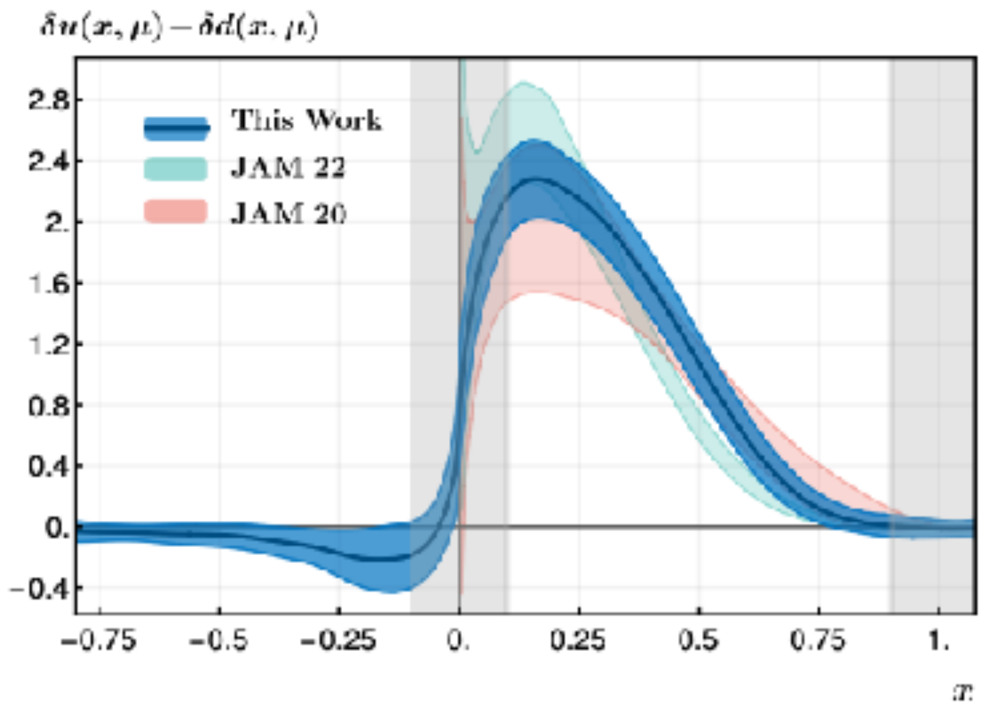
- Understanding partonic structure of hadrons from lattice QCD



Kaon DA

J. Hua, et.al., LPC, under review

2022



Transversity PDF

F. Yao. et.al., LPC, under review

- **Permanent members:**
  - Xiangdong Ji (U. Maryland)
  - Peng Sun (IMP/CAS)
  - Andreas Schaefer (U. Reg.)
  - Wei Wang (SJTU)
  - **Yi-Bo Yang (ITP/CAS, spokesperson)**
  - Jian-hui Zhang (BNU)
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  - Fei Yao (BNU)
  - Kuan Zhang (ITP/CAS)

……

TMD：

3-D structure of nucleon

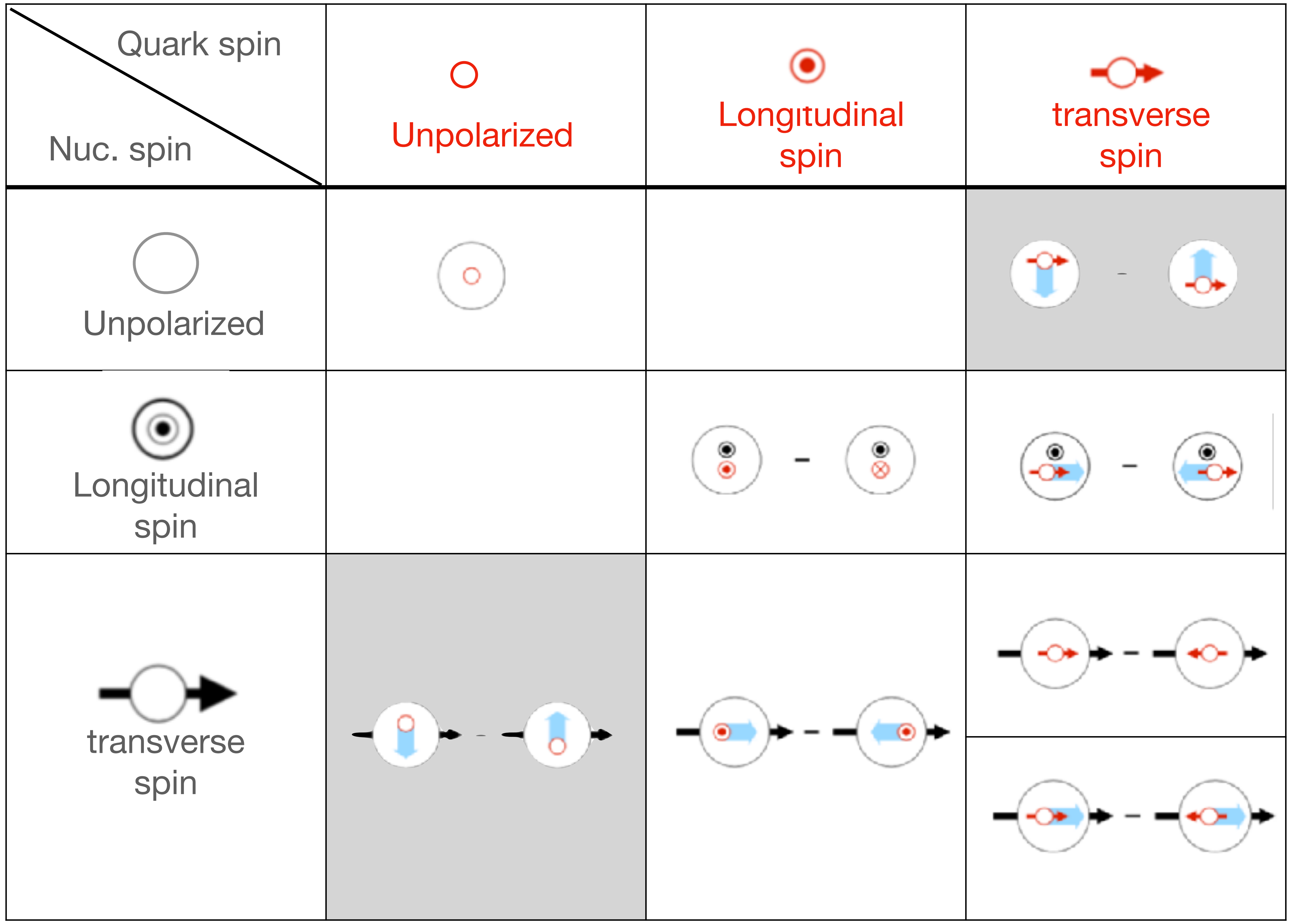
QCD Factorization for Semi-Inclusive Deep-Inelastic Scattering at Low Transverse Momentum

Xiangdong Ji,[1] Jian-Ping Ma,[2,1] and Feng Yuan[1]
[1]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Institute of Theoretical Physics, Academia Sinica, Beijing, 100080, P. R. China*
(Dated: October 25, 2018)

Abstract

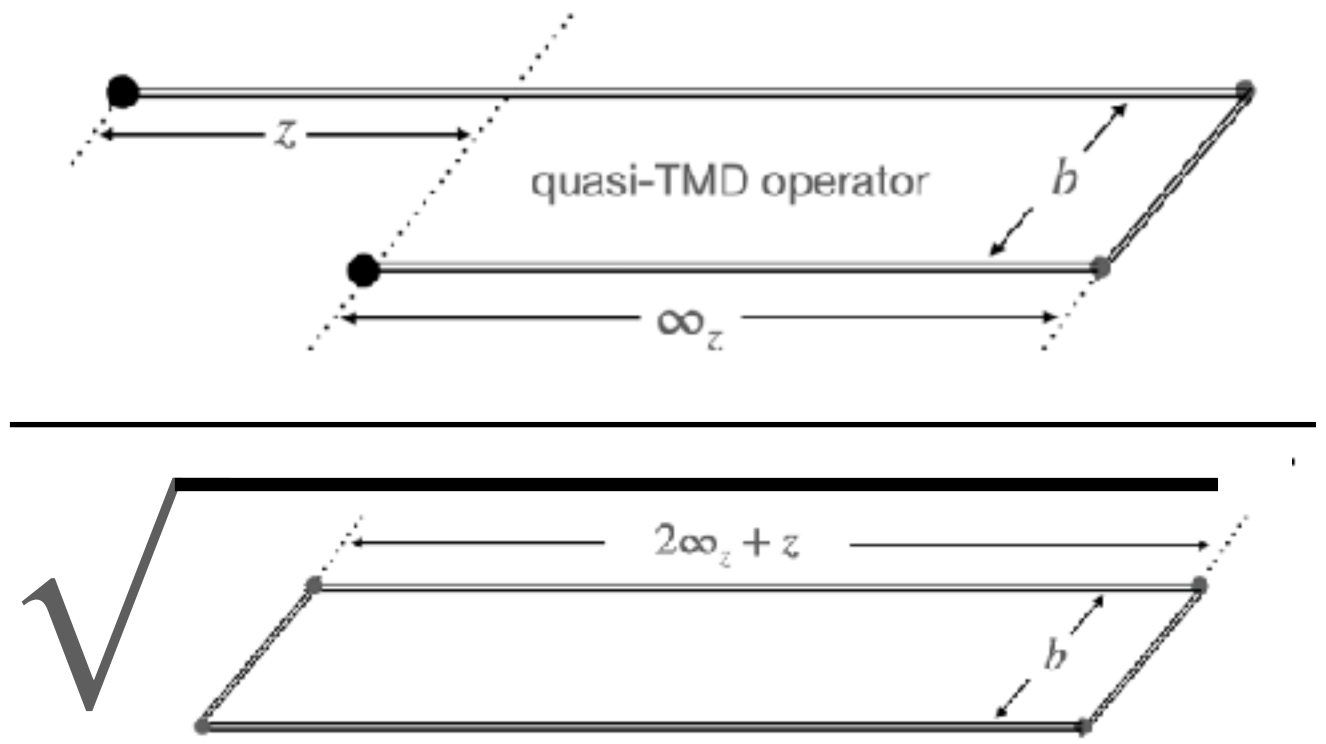
We demonstrate a factorization formula for semi-inclusive deep-inelastic scattering with hadrons in the current fragmentation region detected at low transverse momentum. To facilitate the factorization, we introduce the transverse-momentum dependent parton distributions and fragmentation functions with gauge links slightly off the light-cone, and with soft-gluon radiations subtracted. We verify the factorization to one-loop order in perturbative quantum chromodynamics and argue that it is valid to all orders in perturbation theory.

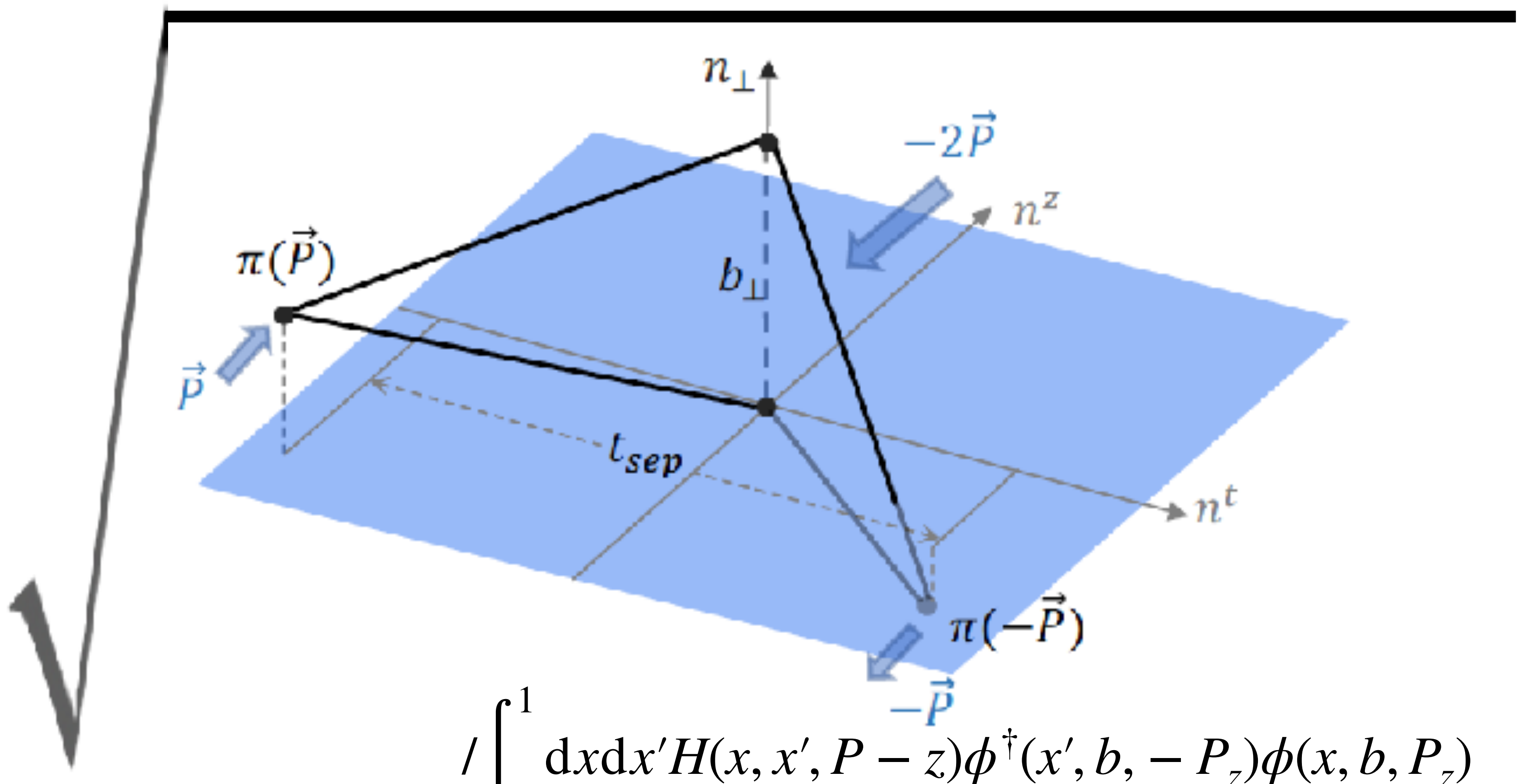
X. Ji, J. Ma, F. Yuan, PRD71 (2005) 034005



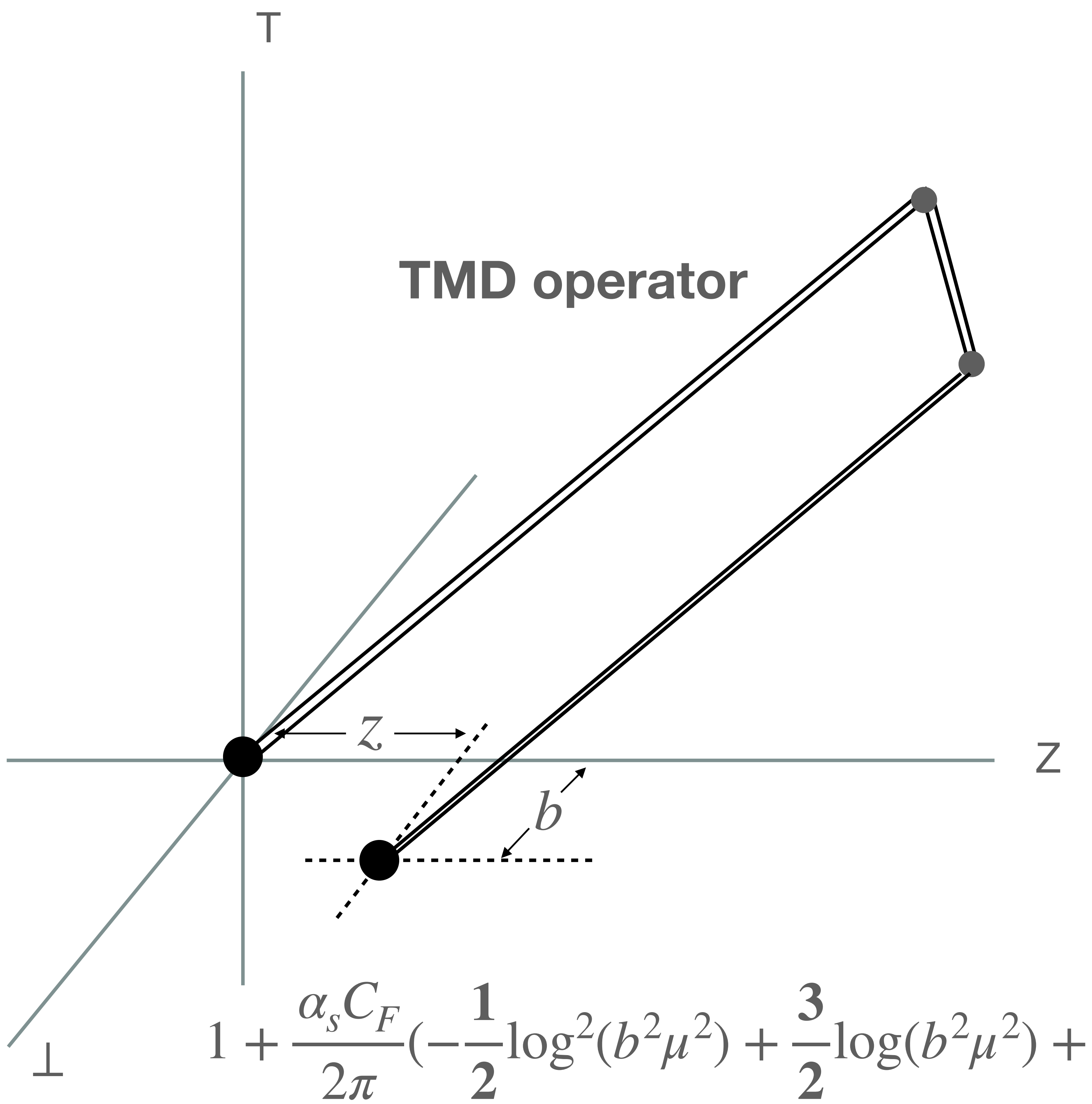
From quasi-TMD to TMD

- Renormalize the quasi-TMD operator properly;
- Remove the additional soft gluon contribution (TMD soft function).





$$/\int_0^1 \mathrm{d}x\mathrm{d}x' H(x, x', P-z)\phi^\dagger(x', b, -P_z)\phi(x, b, P_z)$$



$$1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\log^2(b^2\mu^2) + \frac{3}{2}\log(b^2\mu^2) + \ldots\right)$$

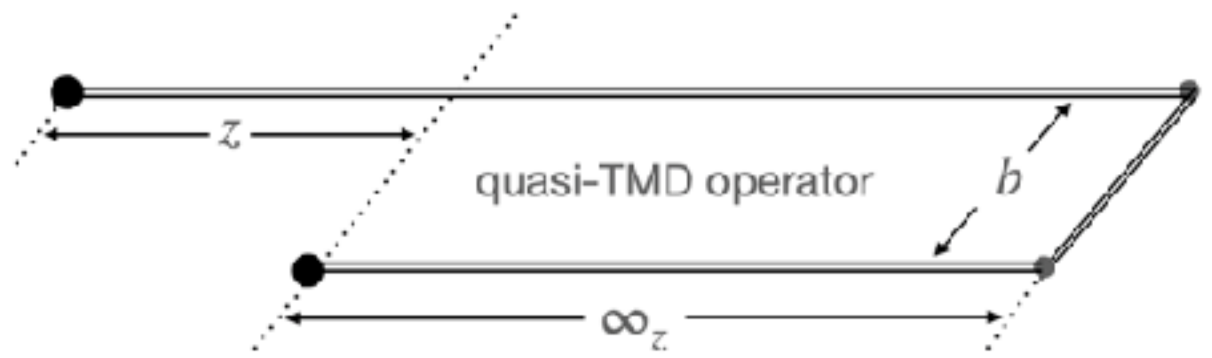
$$= \frac{1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\log^2(b^2\mu^2) + \frac{9}{2}\log(b^2\mu^2) + \ldots\right)}{1 + \frac{\alpha_s C_F}{2\pi}\left(2\log(b^2\mu^2) + \ldots\right)}\left(1 + \frac{\alpha_s C_F}{2\pi}\left(-\log(b^2\mu^2) + \ldots\right)\right)$$

$$= 1 + \frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\log^2(b^2\mu^2) + \frac{3}{2}\log(b^2\mu^2) + \ldots\right)$$

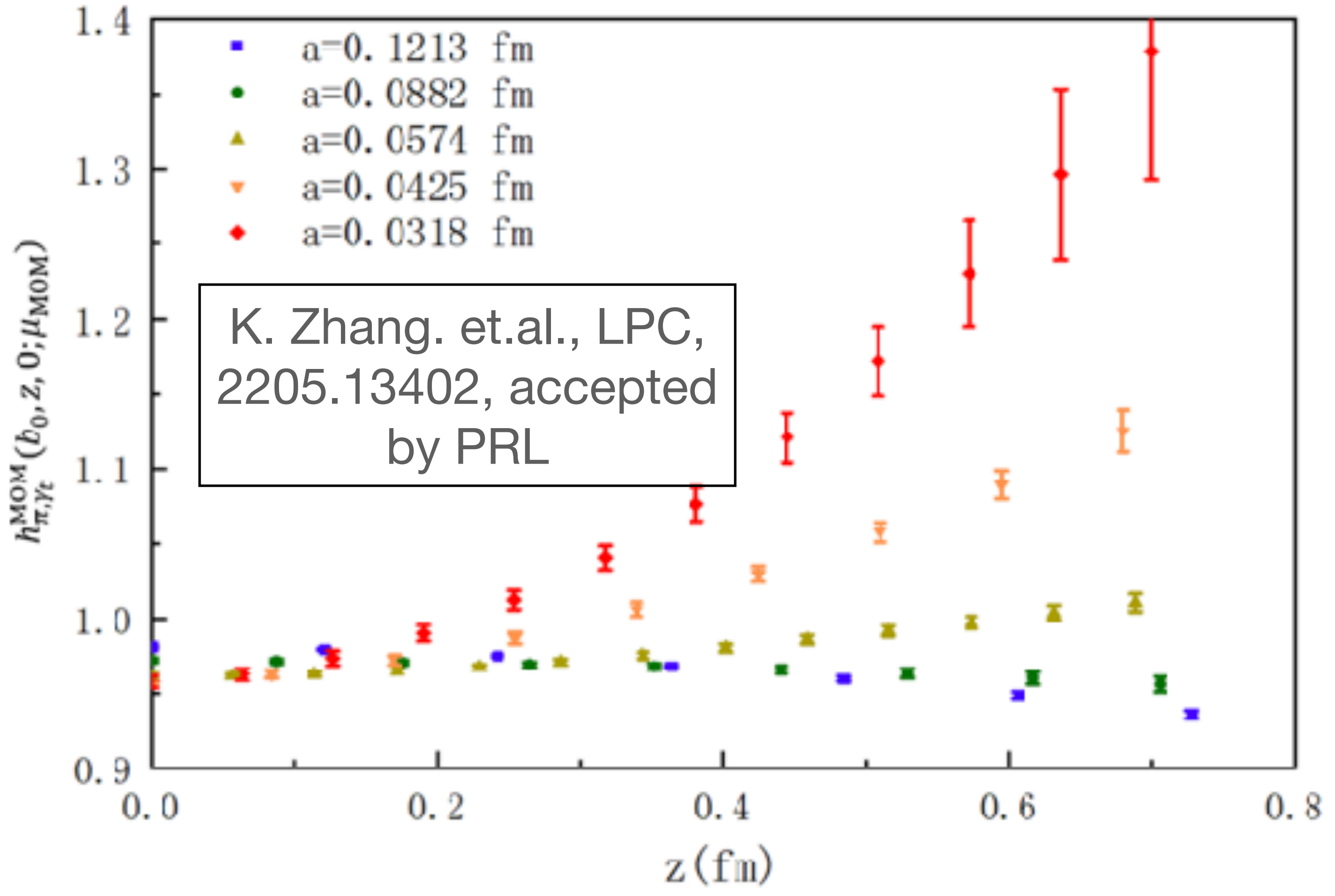
X.Ji. et.al., PLB(2020)135946

From quasi-TMD to TMD

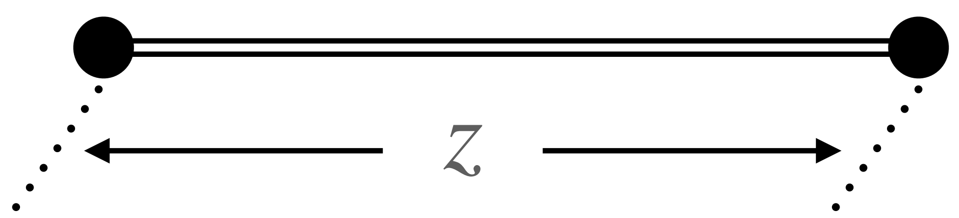
- Renormalize the quasi-TMD operator properly;



RI/MOM
renormalization

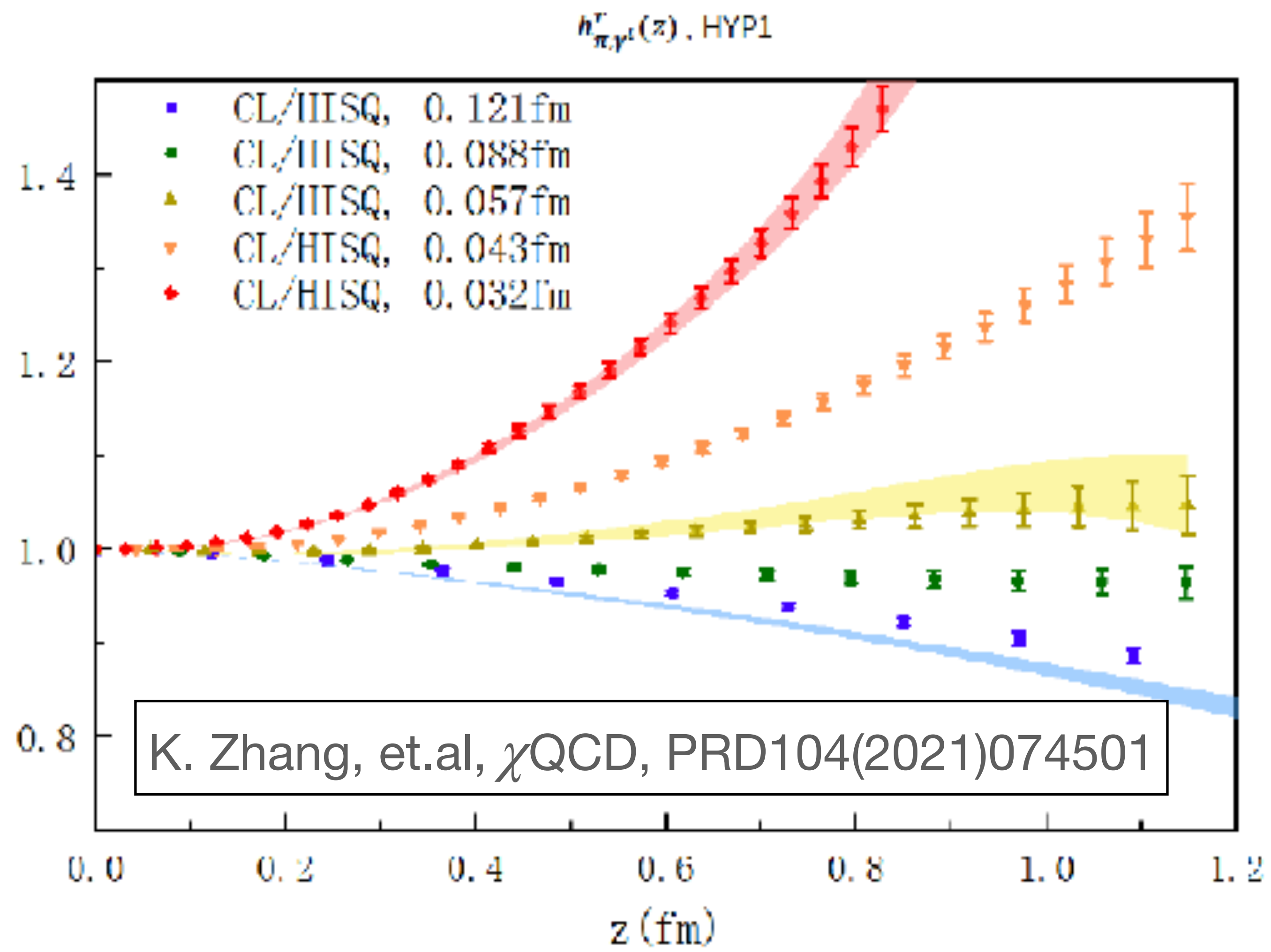


$O_{\gamma_t}(z) = \bar{q}(0)\gamma_t U_z(0,z)q(z)$



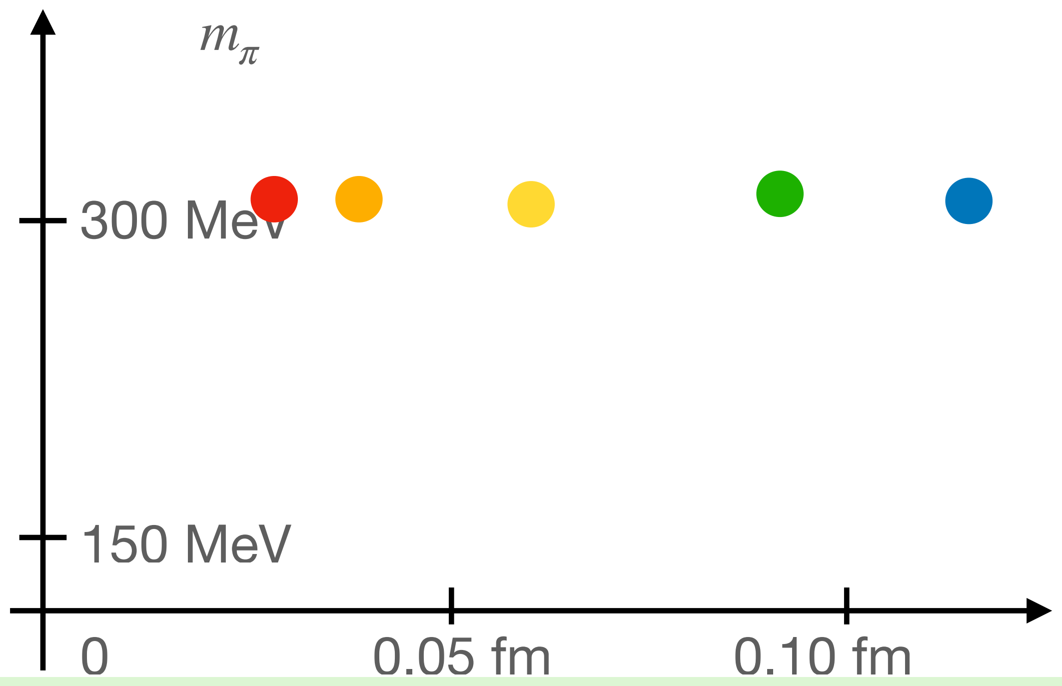
quasi-PDF operator

RI/MOM
renormalization



- The standard RI/MOM scheme doesn't work for either quasi-PDF or quasi-TMD, due to residual linear divergence $e^{\mathcal{O}(\alpha_s^2 \frac{z}{a})}$;
- Continuum limit doesn't exist with the RI/MOM renormalization!

Solution for quasi-PDF



- At 2018, we thought the RI/MOM works fine based on the result at a=0.121 fm;

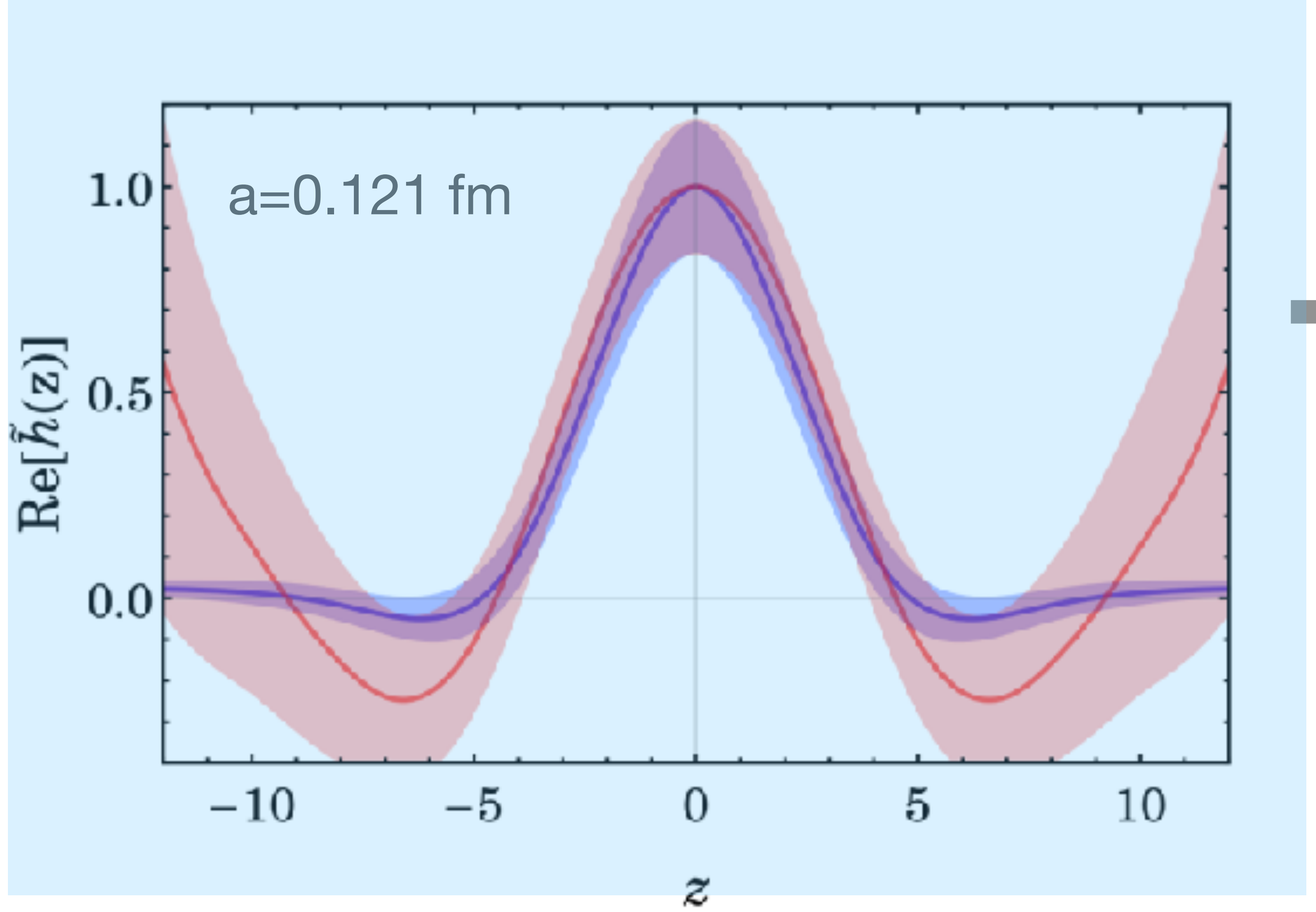
J. Chen, et.tal, LP3, PRD97 (2018) 014505

- At 2020, we confirmed that RI/MOM doesn't really work when the lattice spacing is small enough.

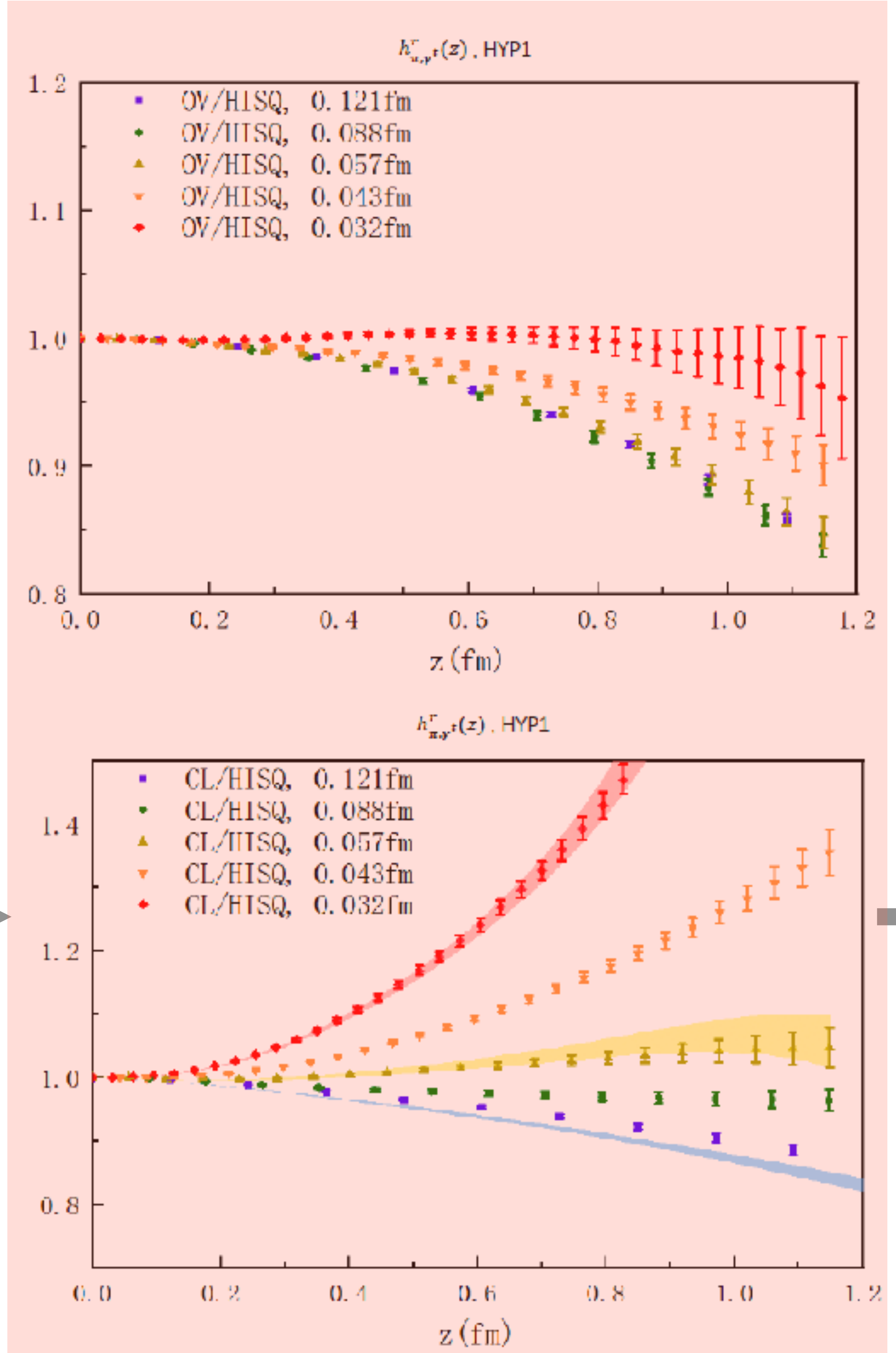
K. Zhang, et.tal, χQCD, PRD104 (2021) 074501

- **At 2021, we proposed the "self renormalization" to extract and remove the divergence by fit the matrix elements at different lattice spacing.**

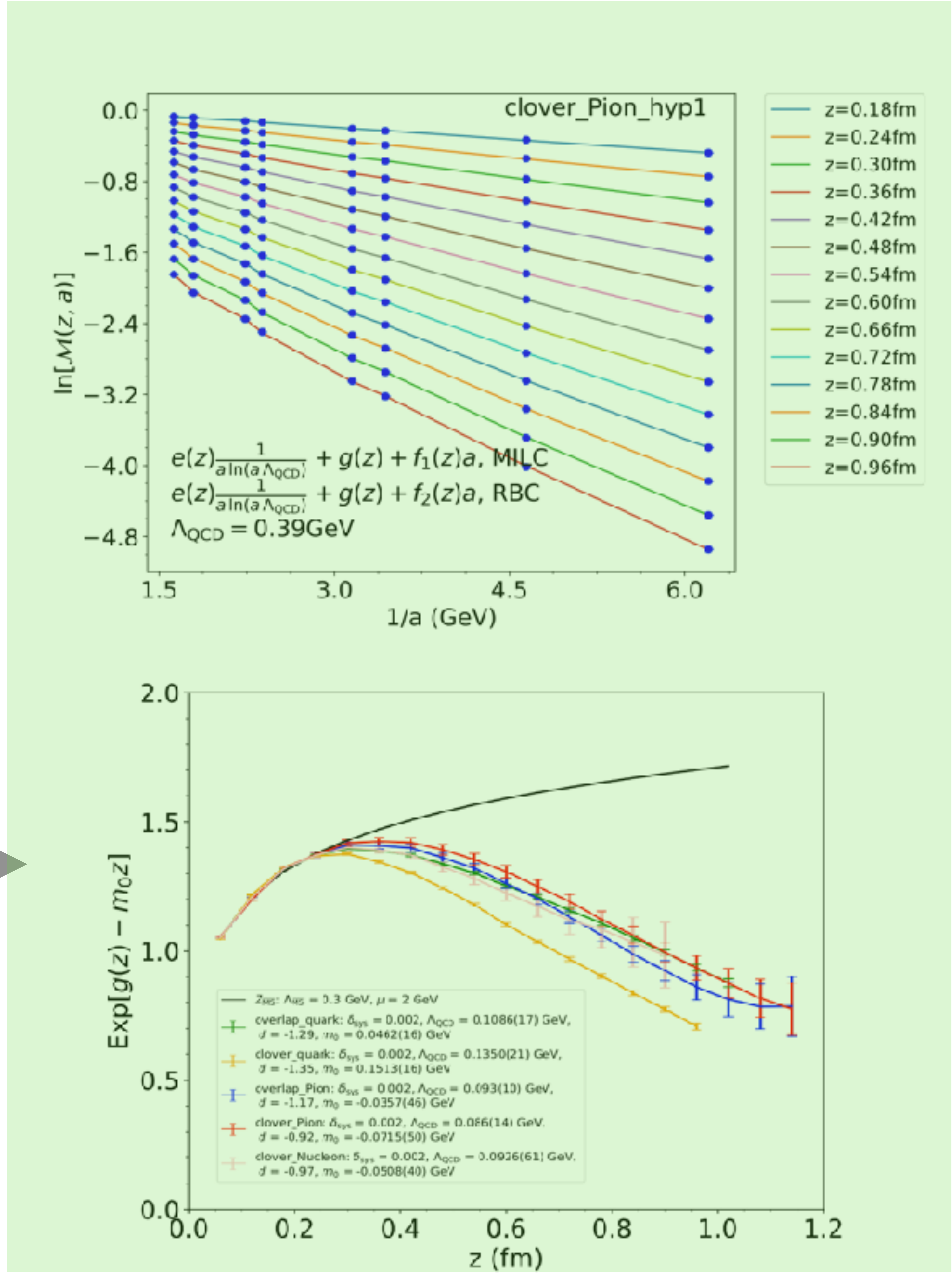
Y.-K. Huo. et.al., LPC, NPB969(2021)115443



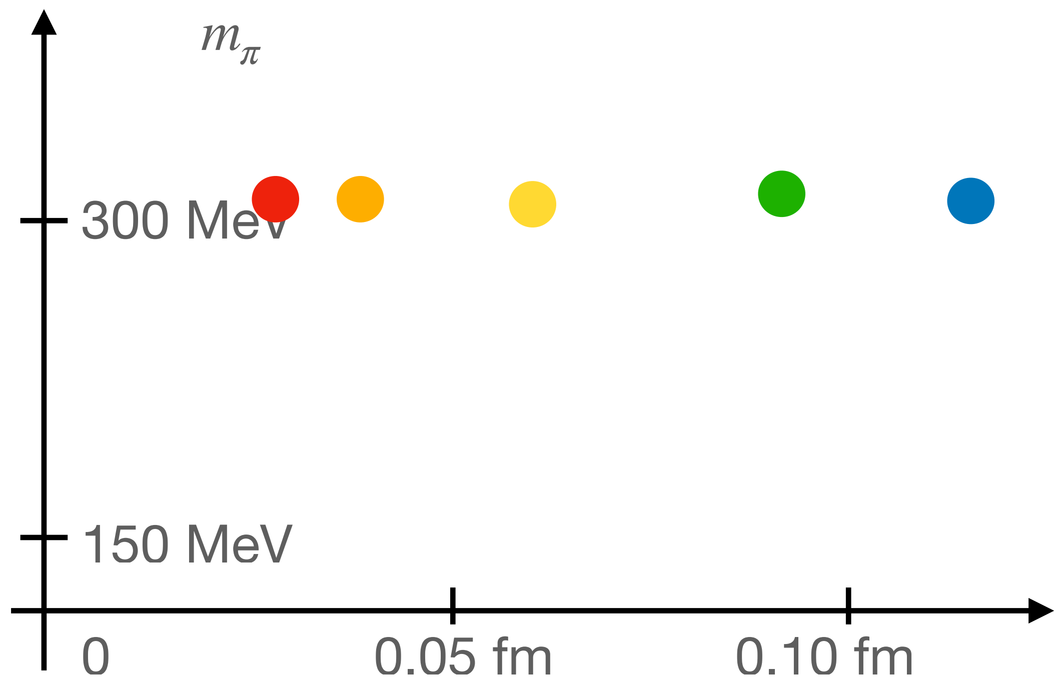
2018-2020

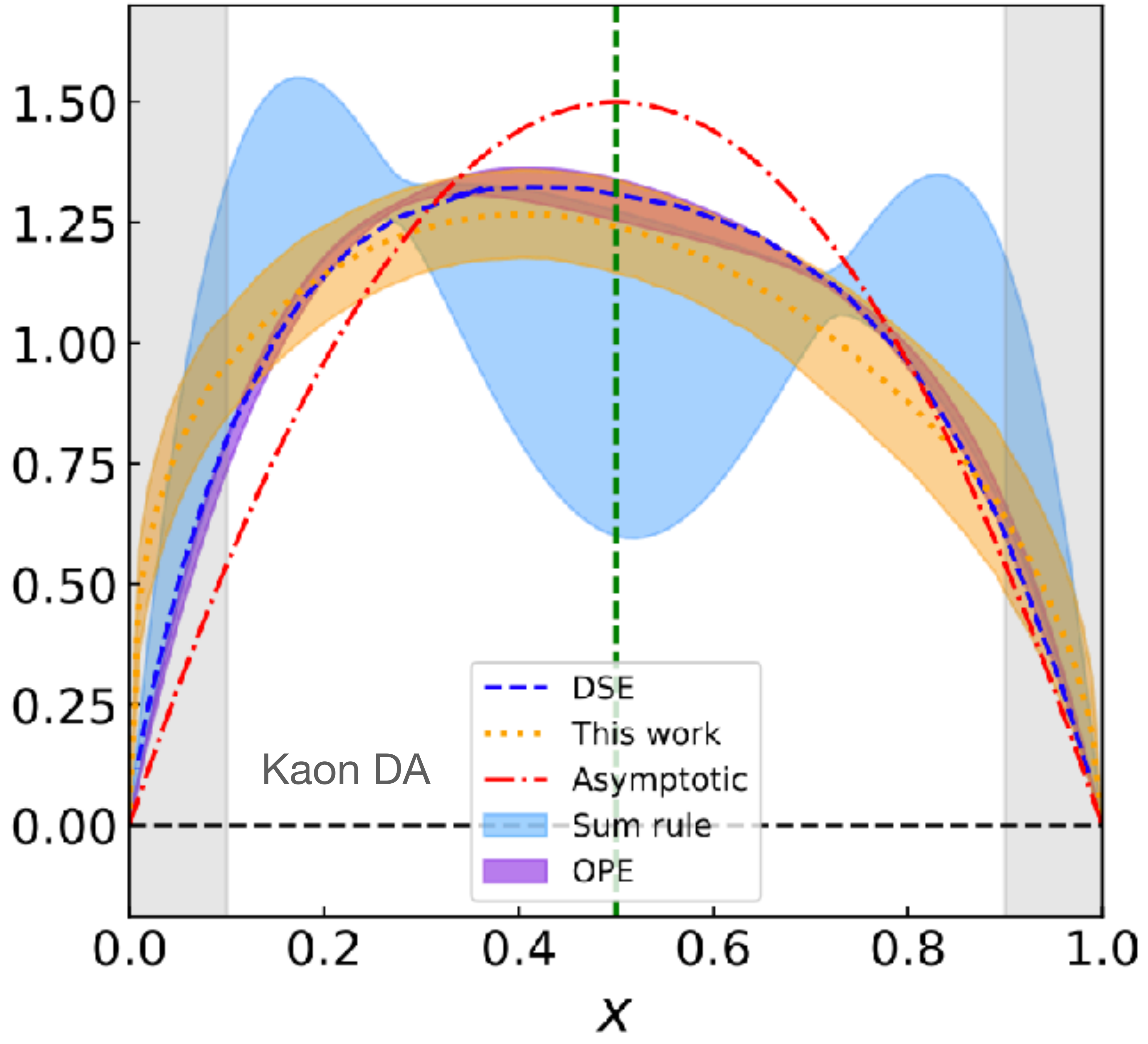


2020-2021

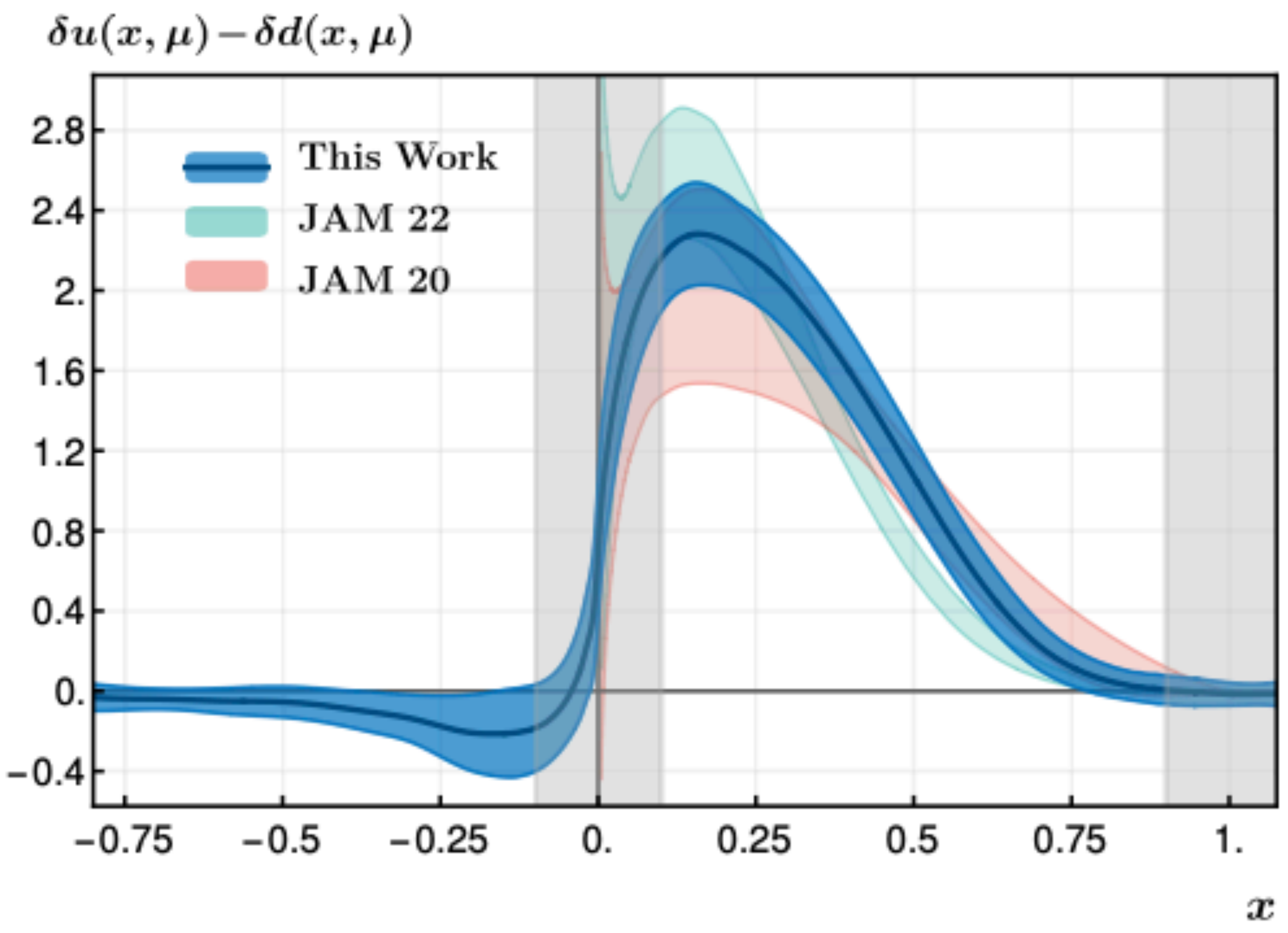


Solution for quasi-PDF





J. Hua, et.al., LPC, 2201.09173, under review

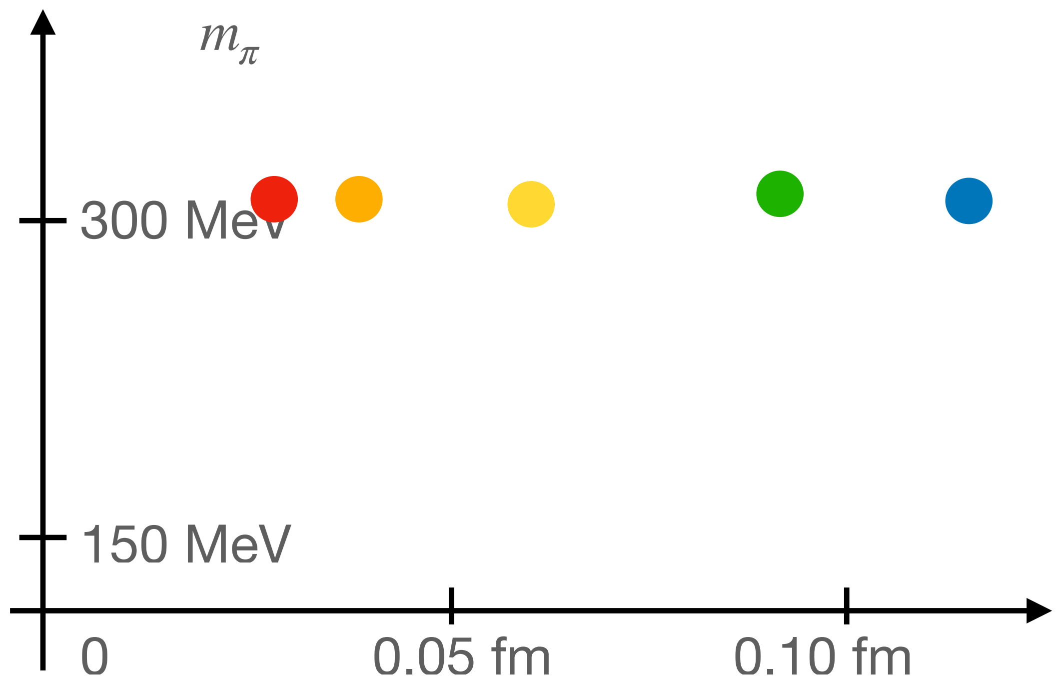


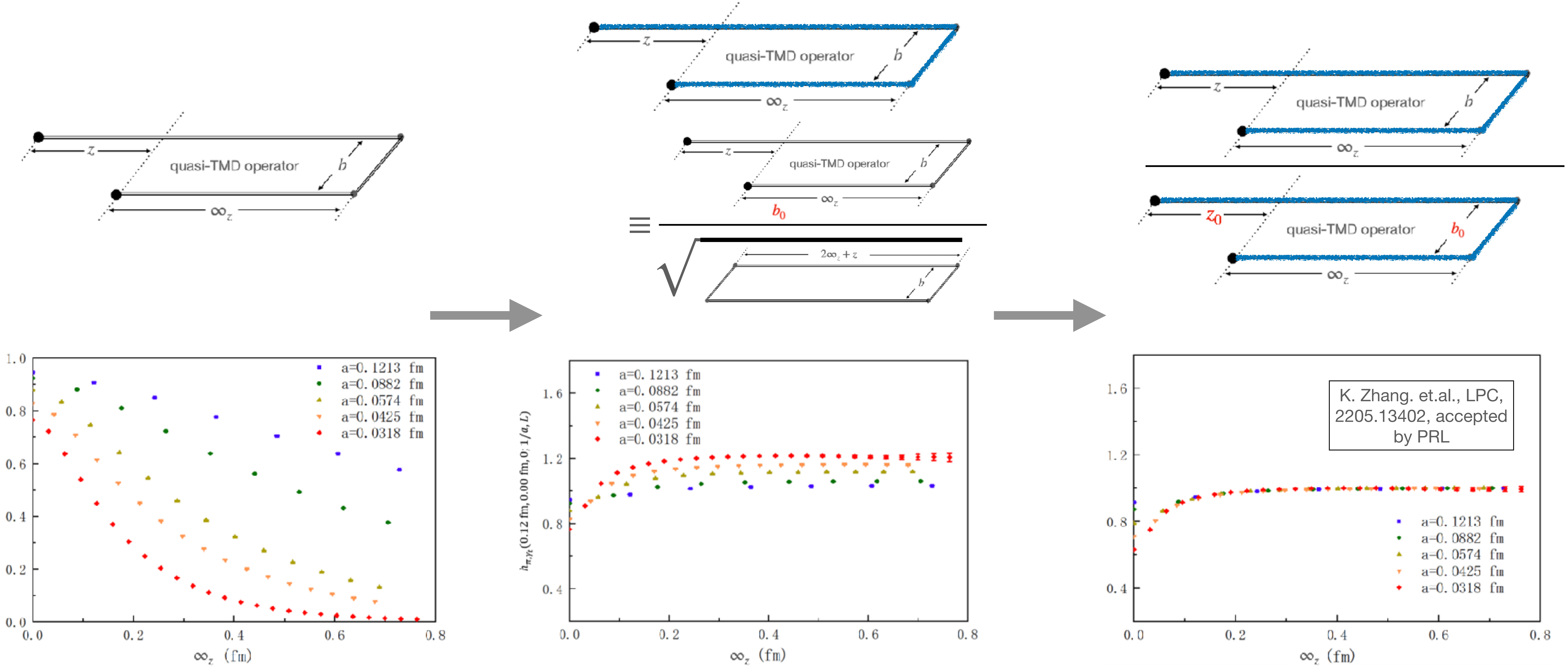
Transversity PDF

F. Yao. et.al., LPC, under review

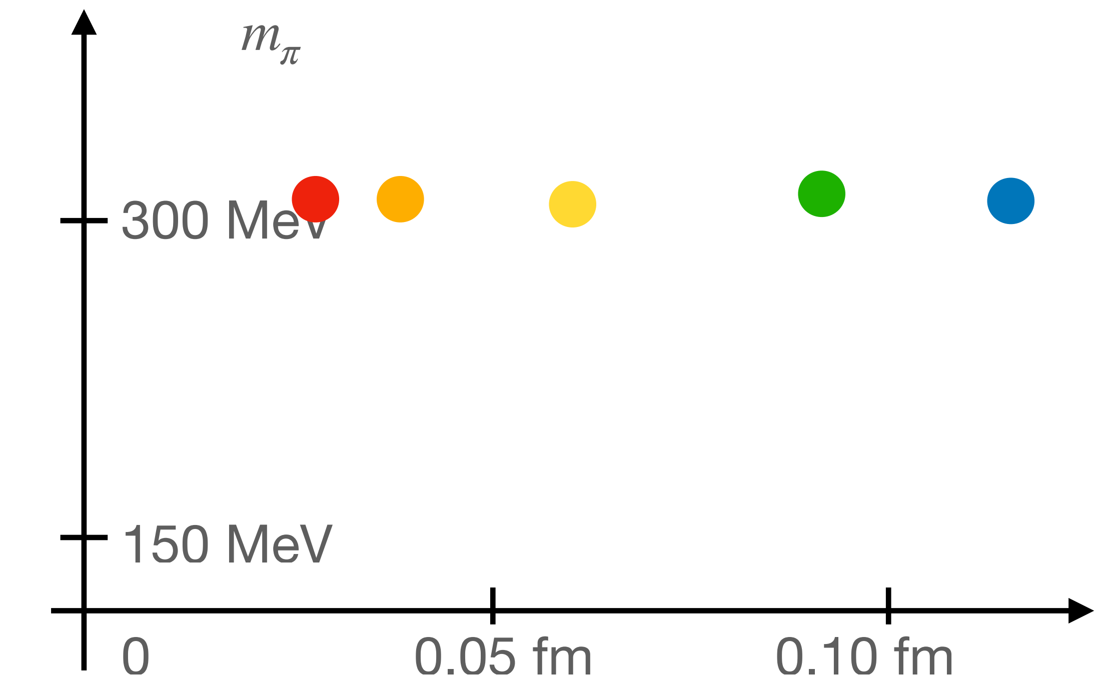
- We applied the “self renormaliaztion” to the meson DA and nucleon transversity PDF.
- It works well in both the cases.

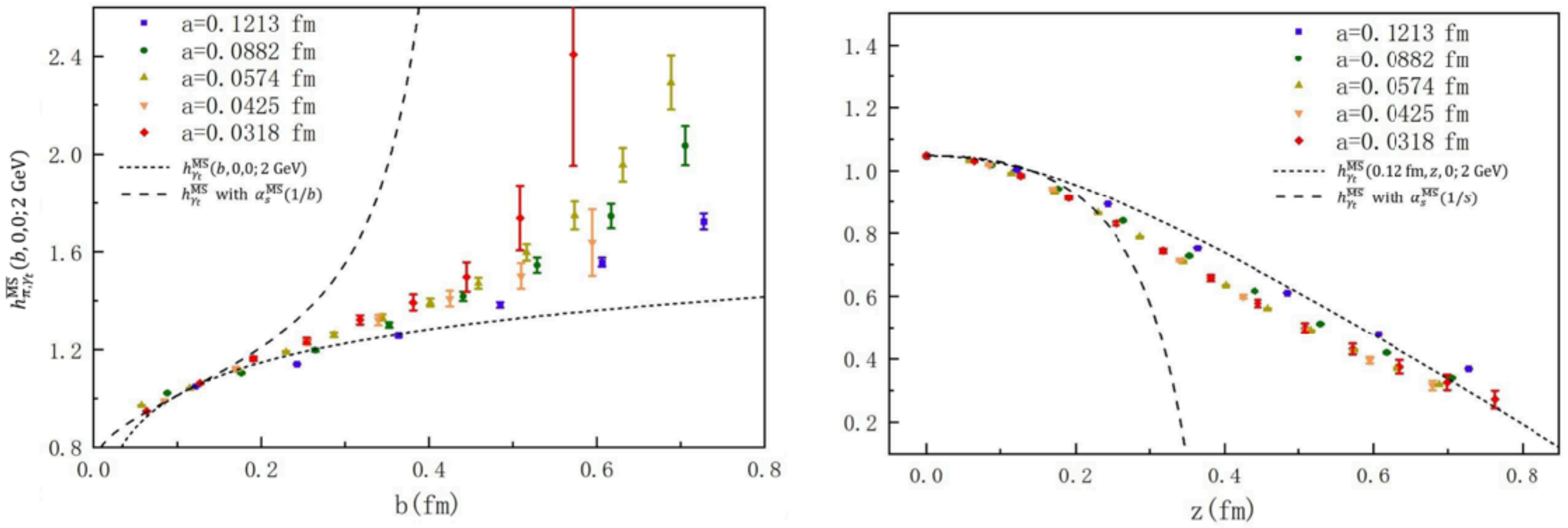
Solution for quasi-TMD





Solution for quasi-TMD

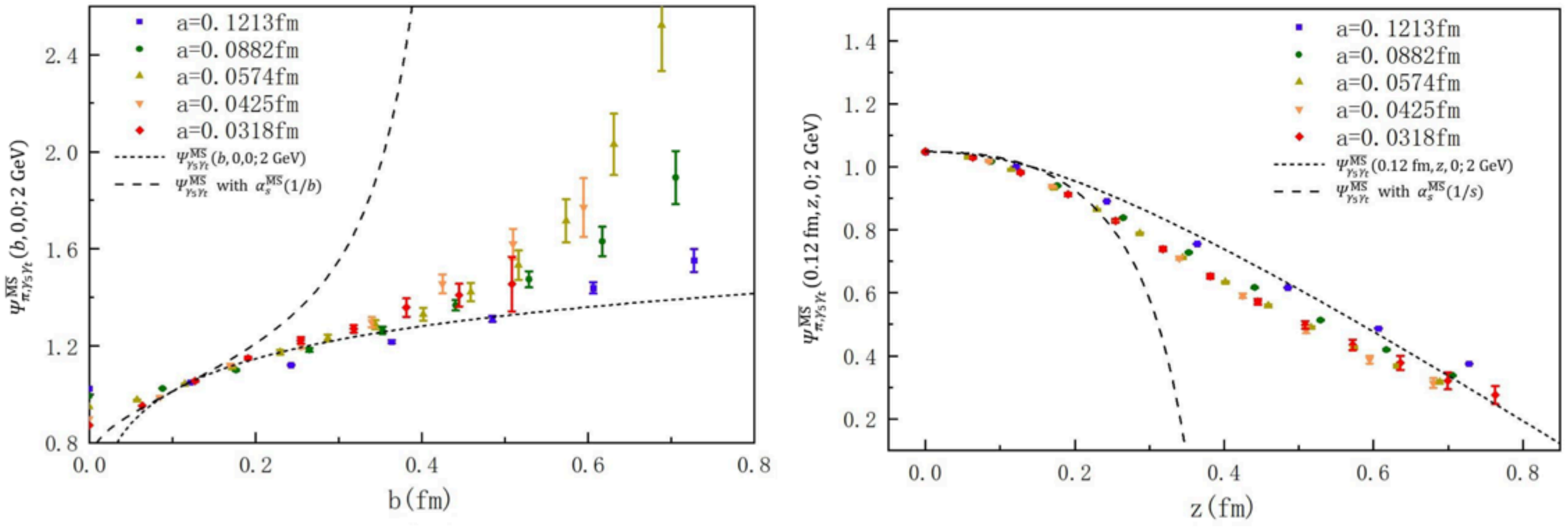




Renormalized quasi-TMD **PDF** matrix element

$$h^{\overline{MS}}_{\chi,\gamma_t}(b,z,P_z;\mu) = h^{\overline{MS}}_{\gamma_t}(b_0,0,0;\mu)\,\frac{h_{\chi,\gamma_t}(b,z,P_z;1/a)}{h_{\pi,\gamma_t}(b_0,z_0=0,0,1/a)}$$

This solution works well for both quasi-TMD PDF and WF, with different gamma matrices.



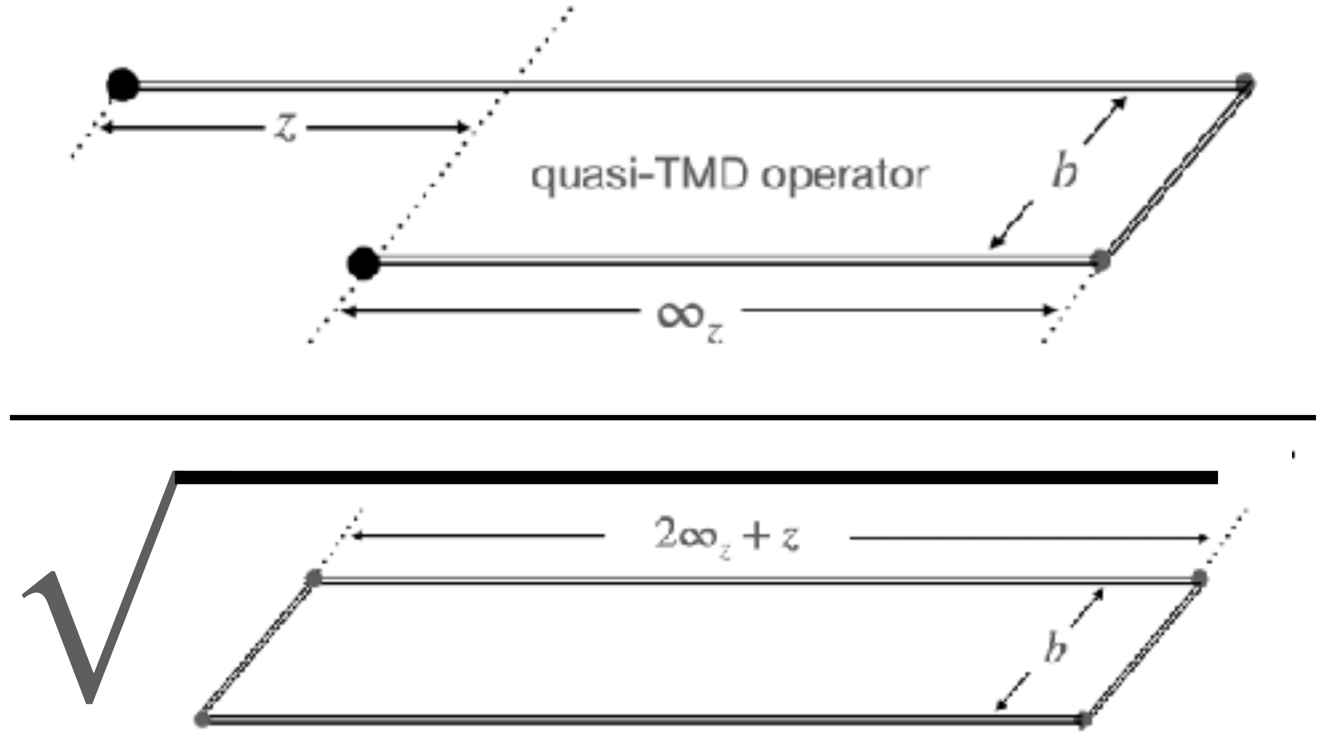
Renormalized quasi-TMD **WF** matrix element

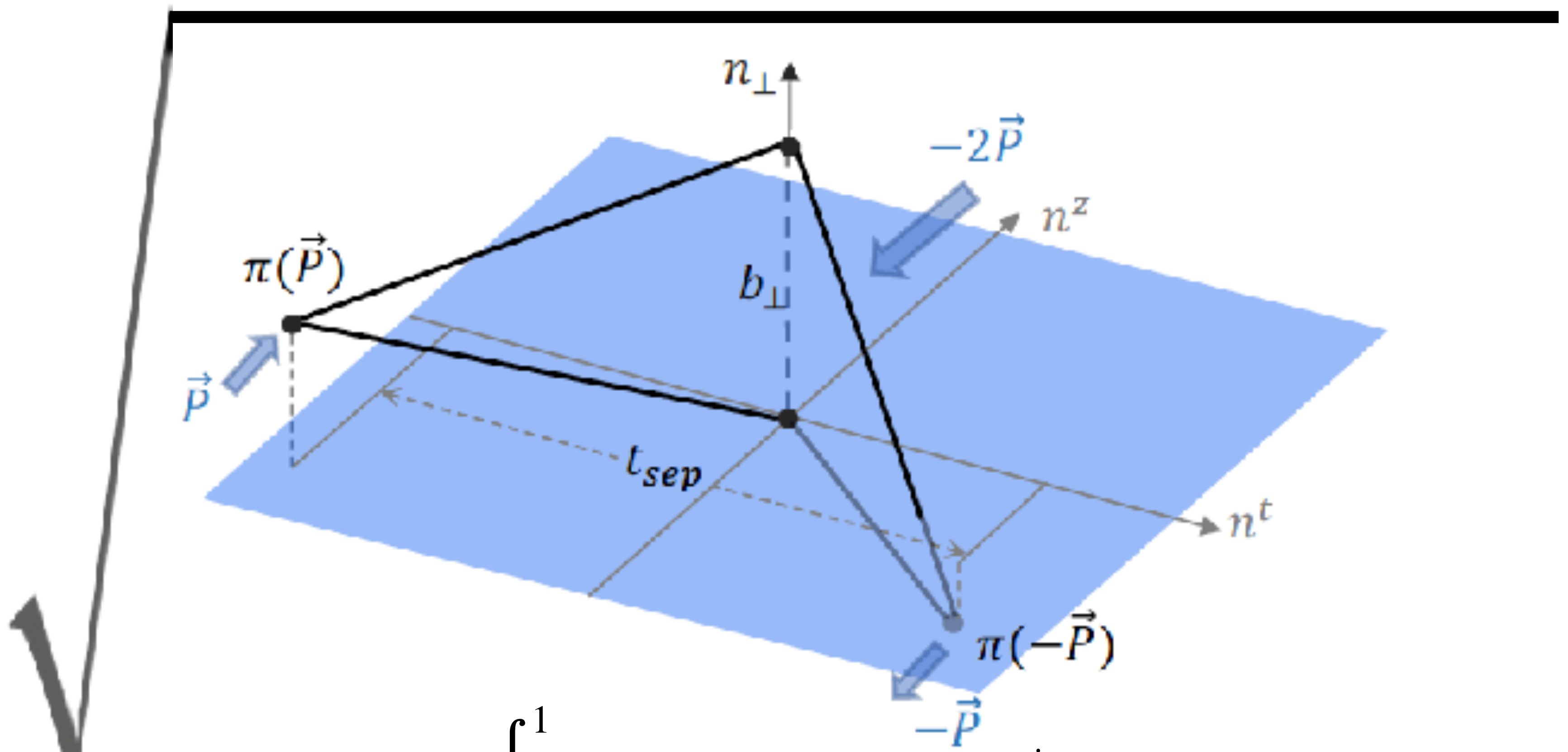
$$\Psi^{\overline{MS}}_{\pi,\gamma_5\gamma_t}(b,z,P_z;\mu) = \psi^{\overline{MS}}_{\gamma_5\gamma_t}(b_0,0,0;\mu)\,\frac{\Psi_{\pi,\gamma_5\gamma_t}(b,z,P_z;1/a)}{\Psi_{\pi,\gamma_5\gamma_t}(b_0,z_0=0,0,1/a)}$$

K. Zhang. et.al., LPC, 2205.13402, accepted by PRL

From quasi-TMD to TMD

- Renormalize the quasi-TMD operator properly;
- Remove the additional soft gluon contribution (TMD soft function).

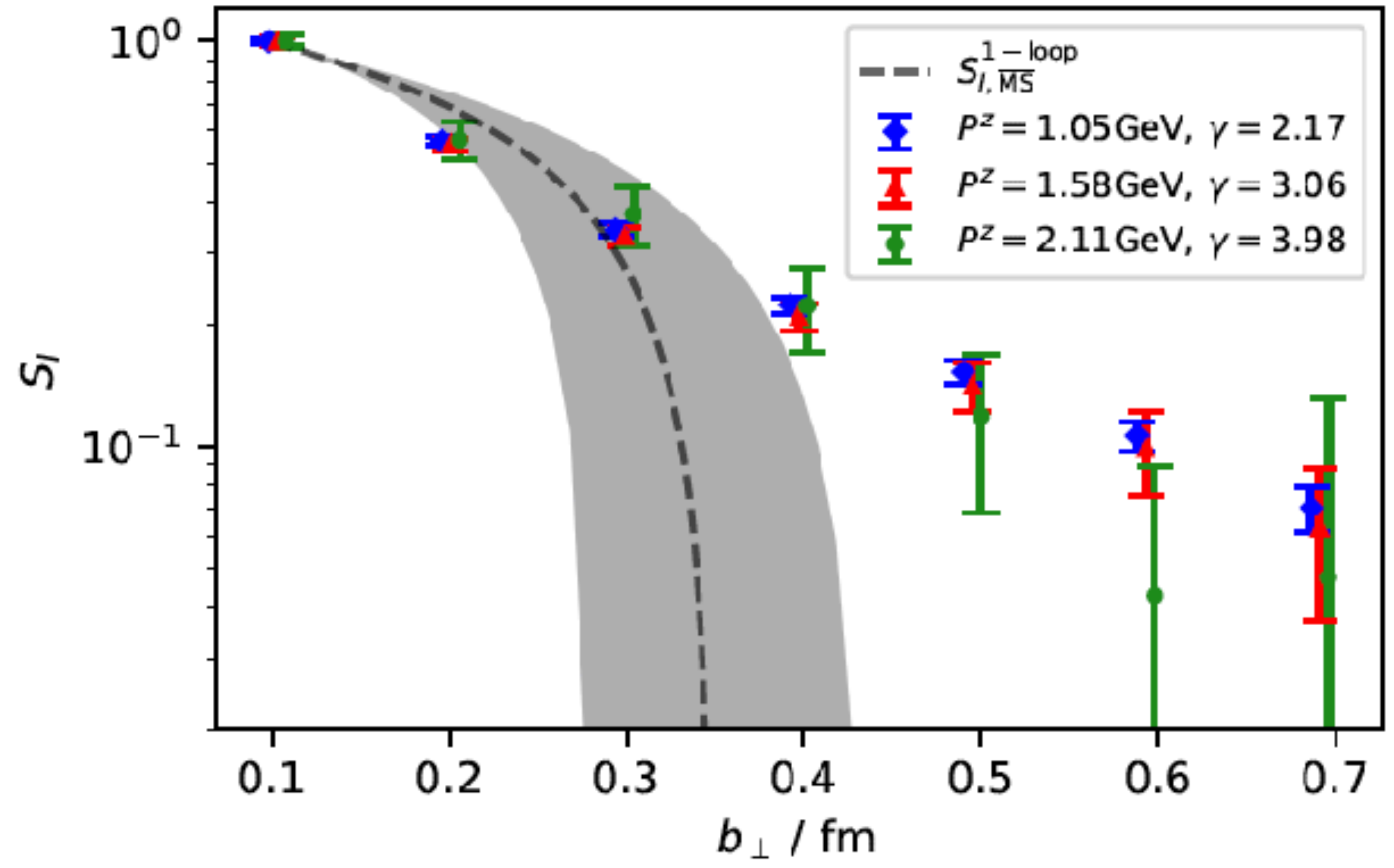




$/\int_0^1 \mathrm{d}x\mathrm{d}x' H(x,x',P-z)\phi^\dagger(x',b,-P_z)\phi(x,b,P_z)$

- The form factor looks like a back elastic scattering of pion, with two currents separated by a spacial distance $b_\perp$.
- Our lattice calculation shows that it is doable and converges with reasonable momentum.

Q.-A. Zhang. et.al., LPC, PRL125(2020)192001



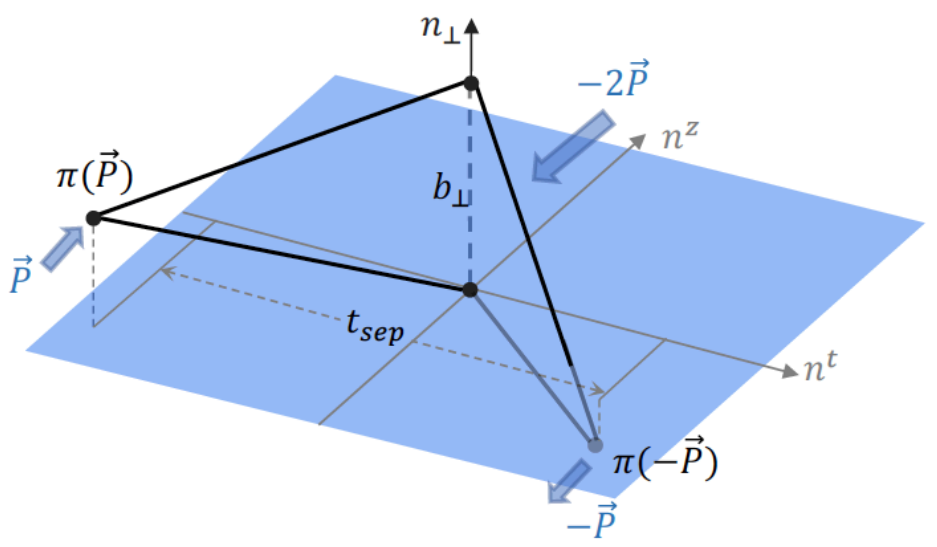
- More precisely, what we need is the rapidity-independent part of pion form factor $F(b,P_z)$:

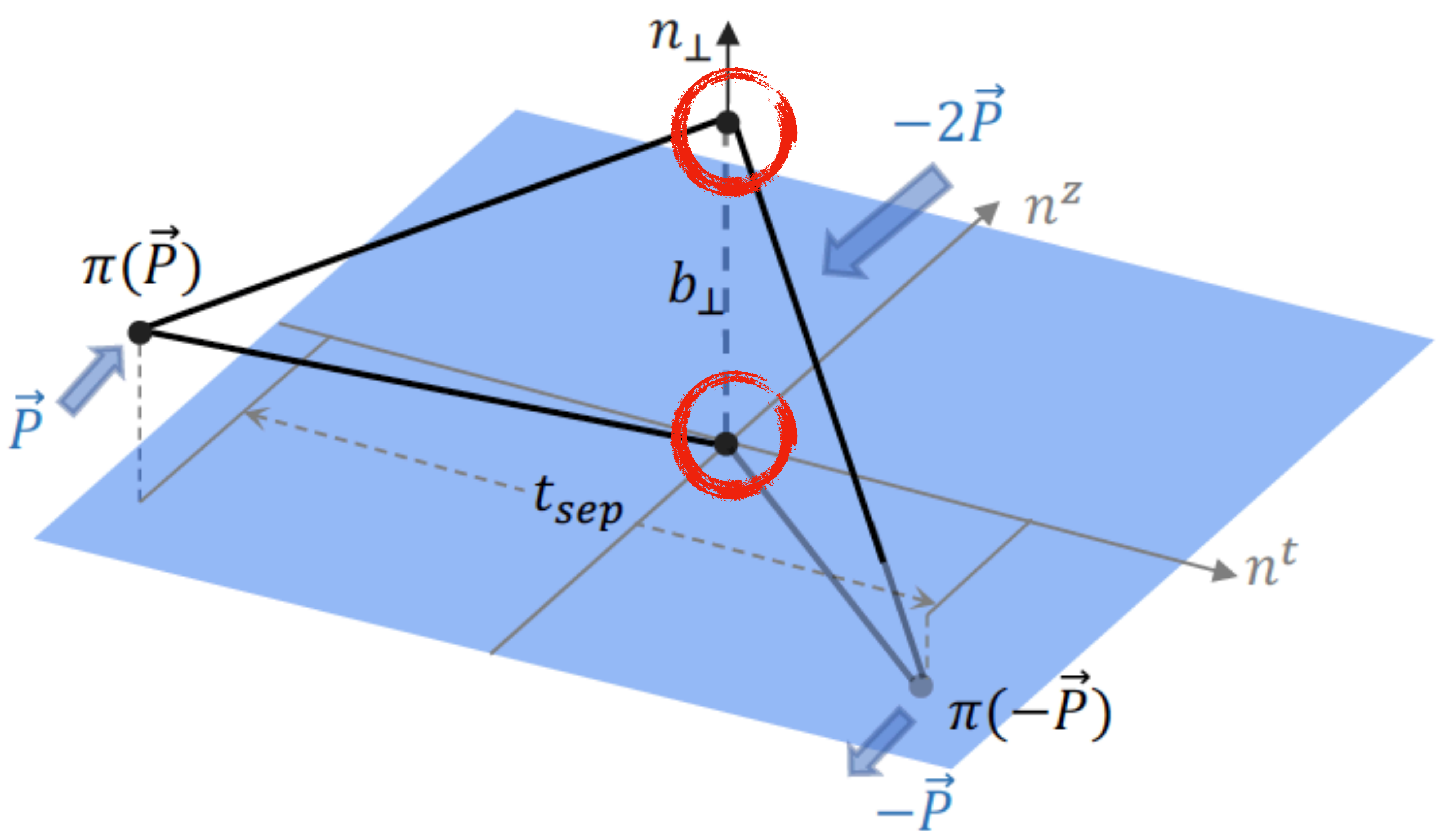
$$S_I(b) = \frac{F(b,P_z)}{\int_0^1 \mathrm{d}x\mathrm{d}x' H(x,x',P-z)\phi^\dagger(x',b,-P_z)\phi(x,b,P_z)} = \frac{F(b,P_z)}{|\phi(0,b,-P_z)|^2} + \mathcal{O}(\alpha_s) + \mathcal{O}(\frac{1}{P_z^2}),$$

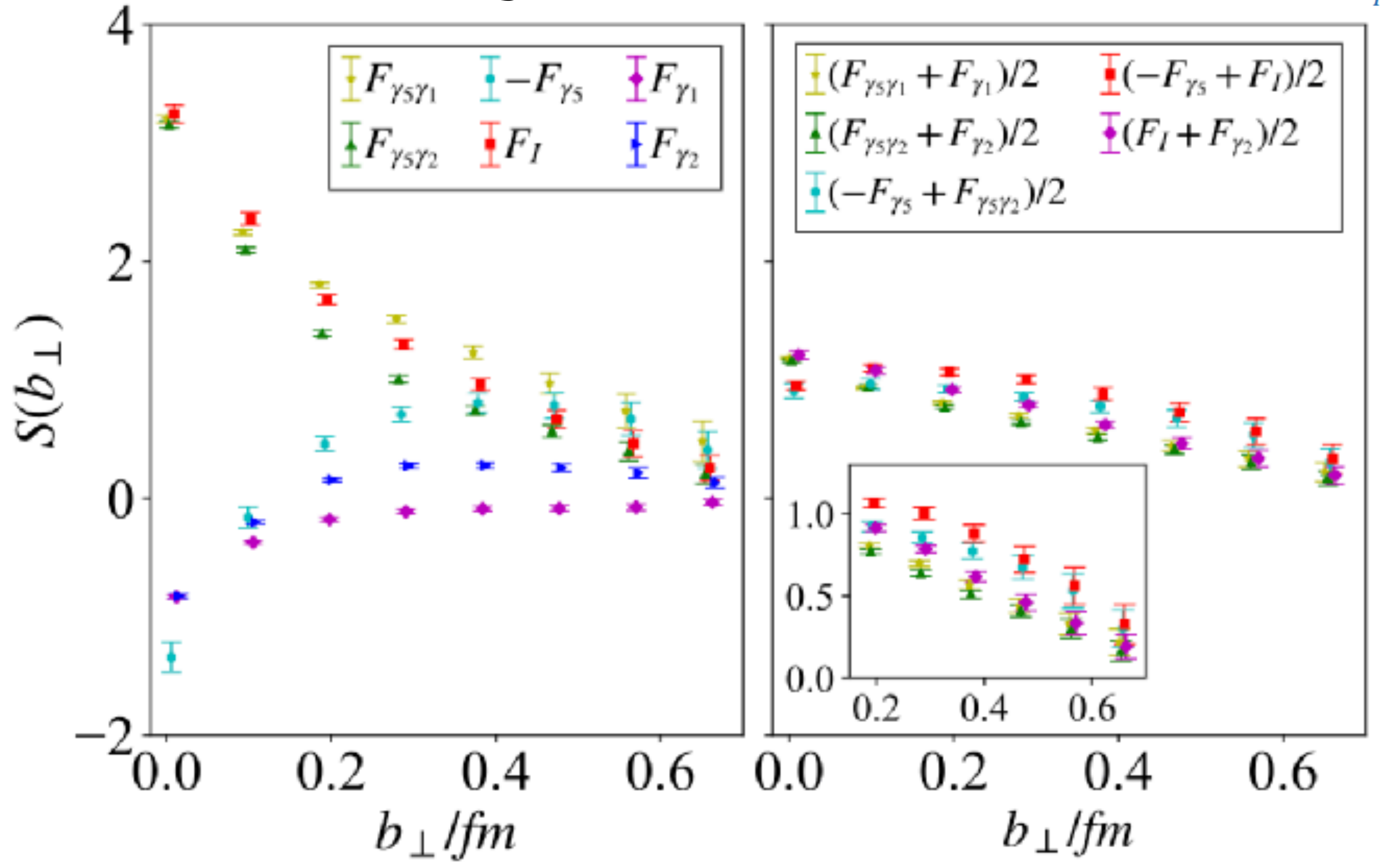
- where $\phi(x,b,P_z)$ is the TMD wave function which cancels the rapidity (momentum P_z) dependence.

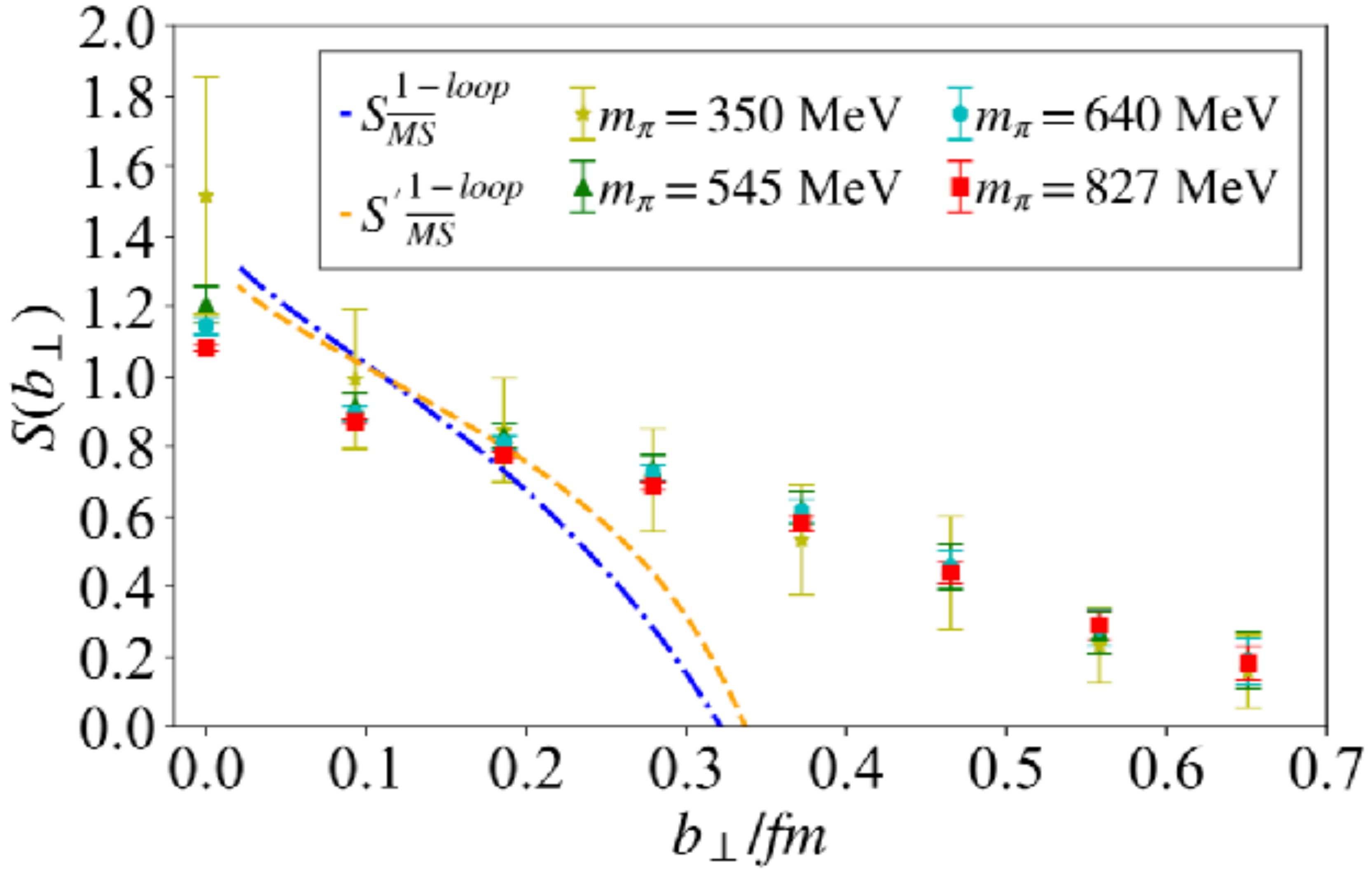
Intrinsic soft function

more systematic study









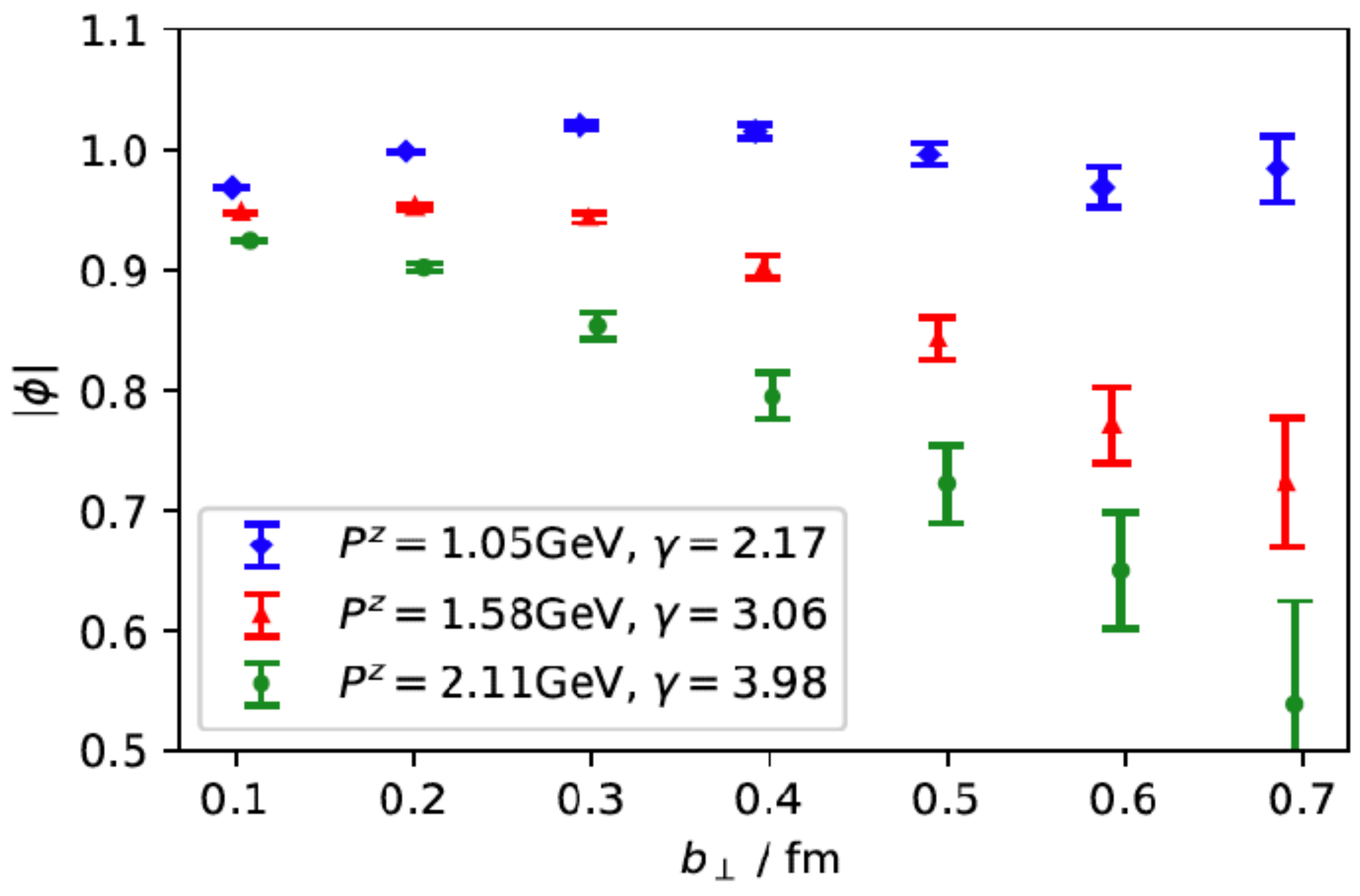
- The higher-twist effects can be very sensitive to the gamma matrices used in the currents, and proper combinations are helpful to suppress them.

- The combined results is independent to the pion mass within the statistical uncertainty.

Y. Li. et.al., ETMC/PKU, PRL128(2022)062002

Soft function

rapidity-dependent part



Q.-A. Zhang. et.al., LPC, PRL125(2020)192001

- Both the pion form factor $F(b, P_z)$ and quasi-TMD wave function $\Phi(z, b, P_z) \equiv \int \mathrm{d}x e^{ixzP_z}\phi(x, b, P_z) = \langle 0 | O_{\gamma_t\gamma_5}(z, b) | \pi(P_z)\rangle^R$ are rapidity (momentum P_z) dependent.

- Such a dependence should be universal and described by **the Collins-Soper kernel** $K(b)$:

$$\phi(x, b, P_z) = e^{\log\frac{P_z^2}{(P_z)'^2}K(b)}\phi(x, b, P_z').$$

- $K(b)$ can also extracted from the (quasi-)TMD-PDF.

P. Shanahan. et.al., PRD102(2020)014511

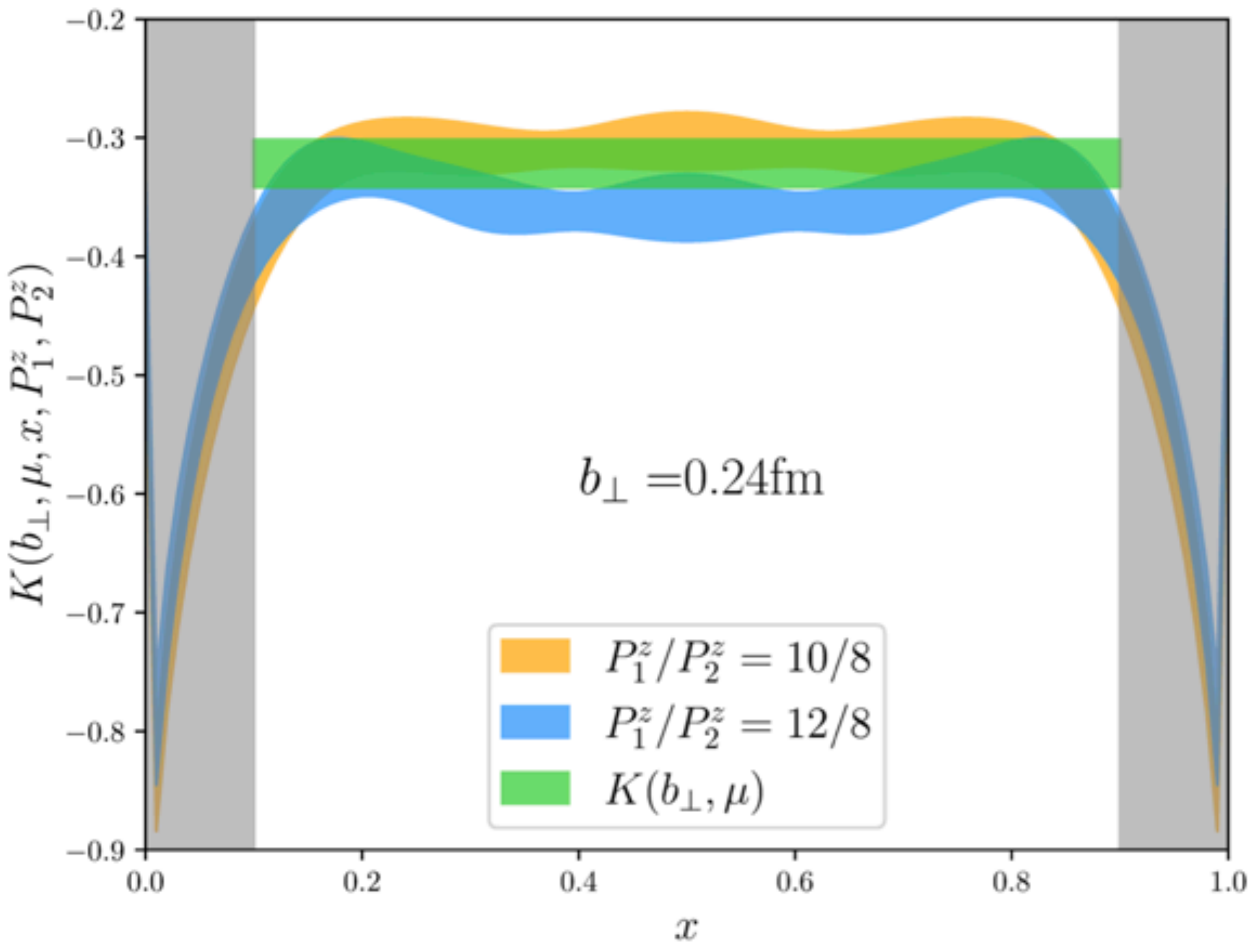
M. Schlemmer. et.al., JHEP(2021)004

P. Shanahan. et.al., arXiv:2107.11930

Collins-Soper kernel

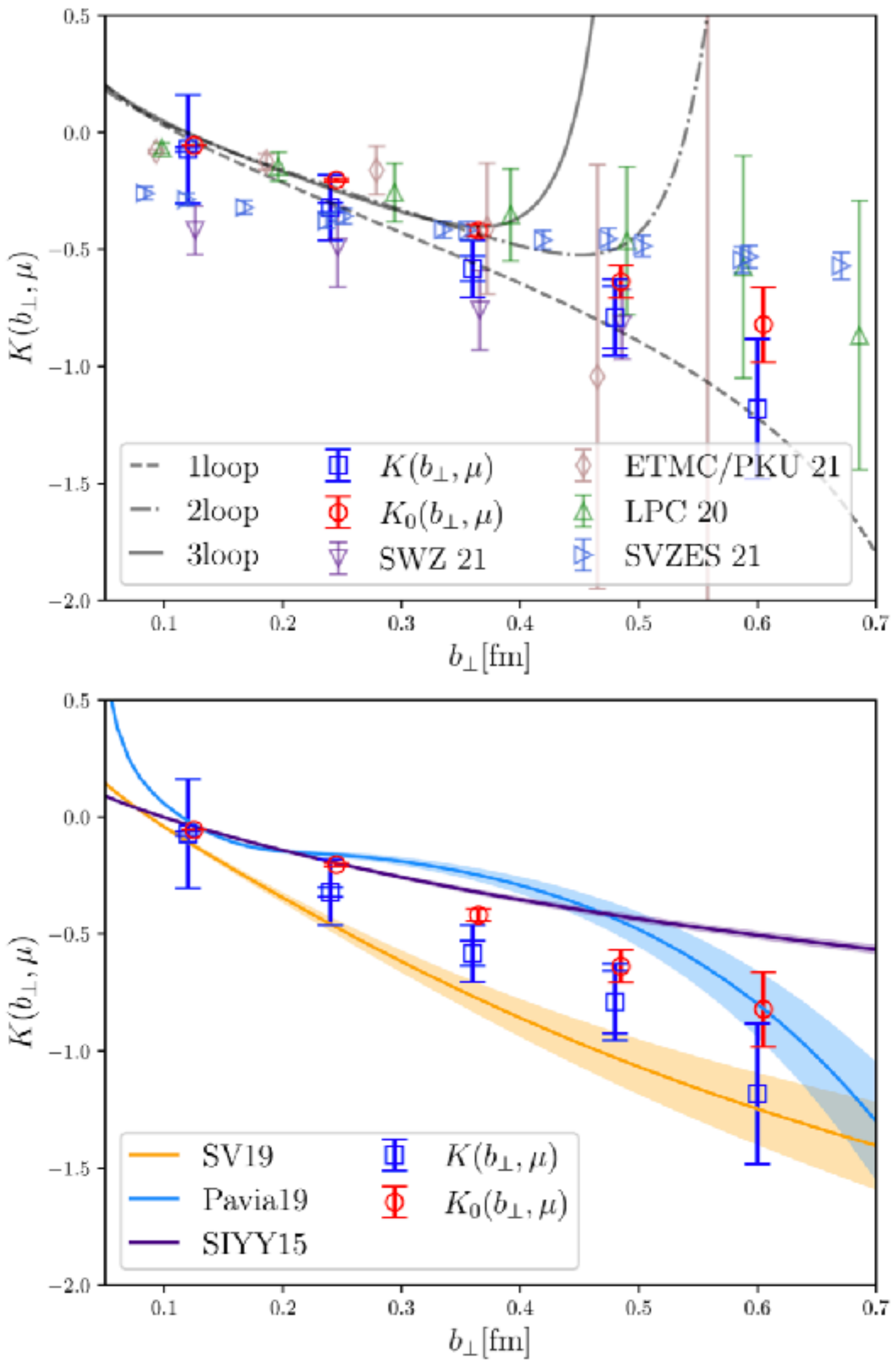
from TMD WF with 1-loop correction

$$K(b) = \frac{\log\frac{\phi(x,b,P_z)}{\phi(x,b,P_z')}}{\log\frac{P_z^2}{(P_z)'^2}}$$



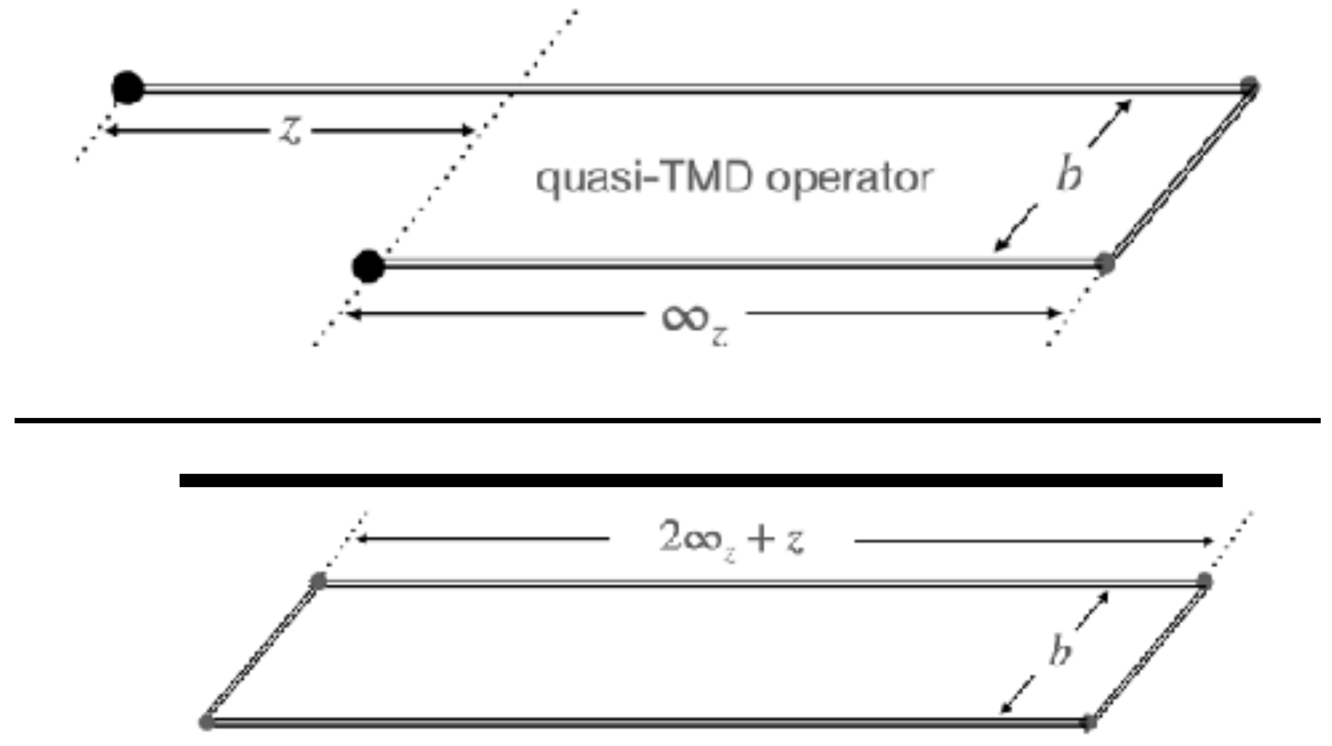
M. Chu. et.al., LPC, 2204.00200, accepted by PRD

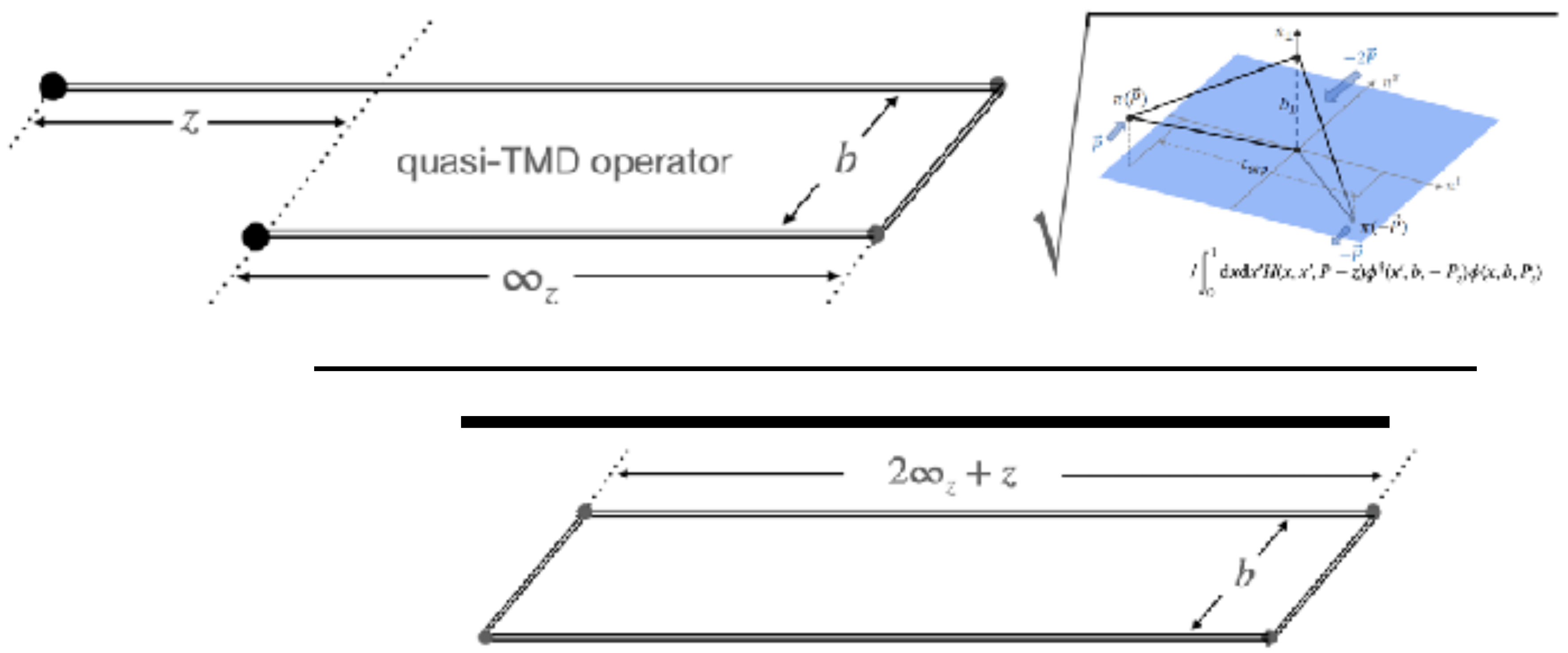
- The lattice ensemble (MILC2+1+1, $48^3 \times 64$, $a = 0.12$ fm, $m_\pi = 130$ MeV) used here is the same as that in SWZ 21, which obtained the CS kernel from TMD PDF with systematic analysis;
- Our result shows a good cancellation on the x-dependence
- More investigations at smaller lattice spacing and larger momentum are essential and in progress.

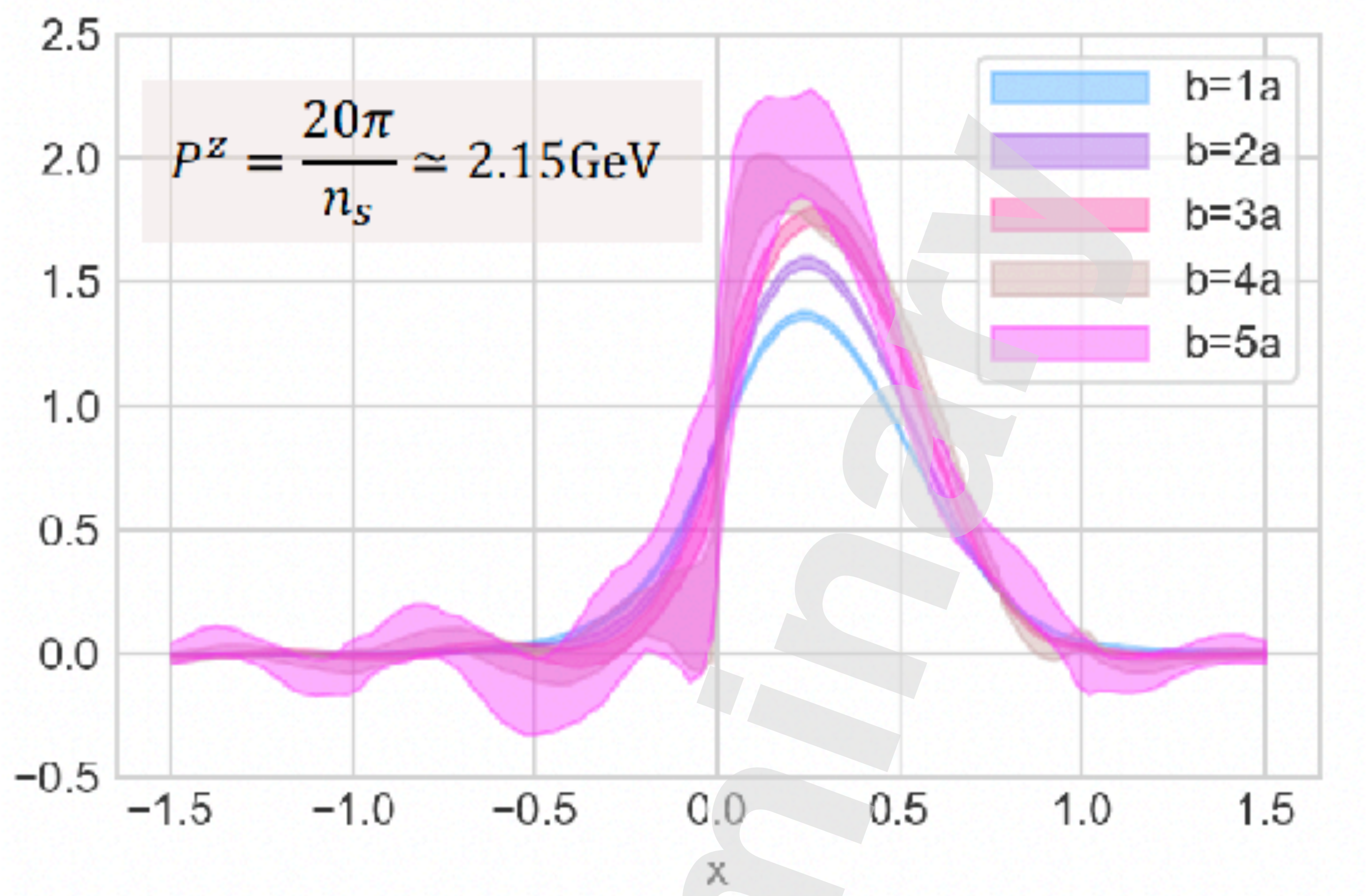


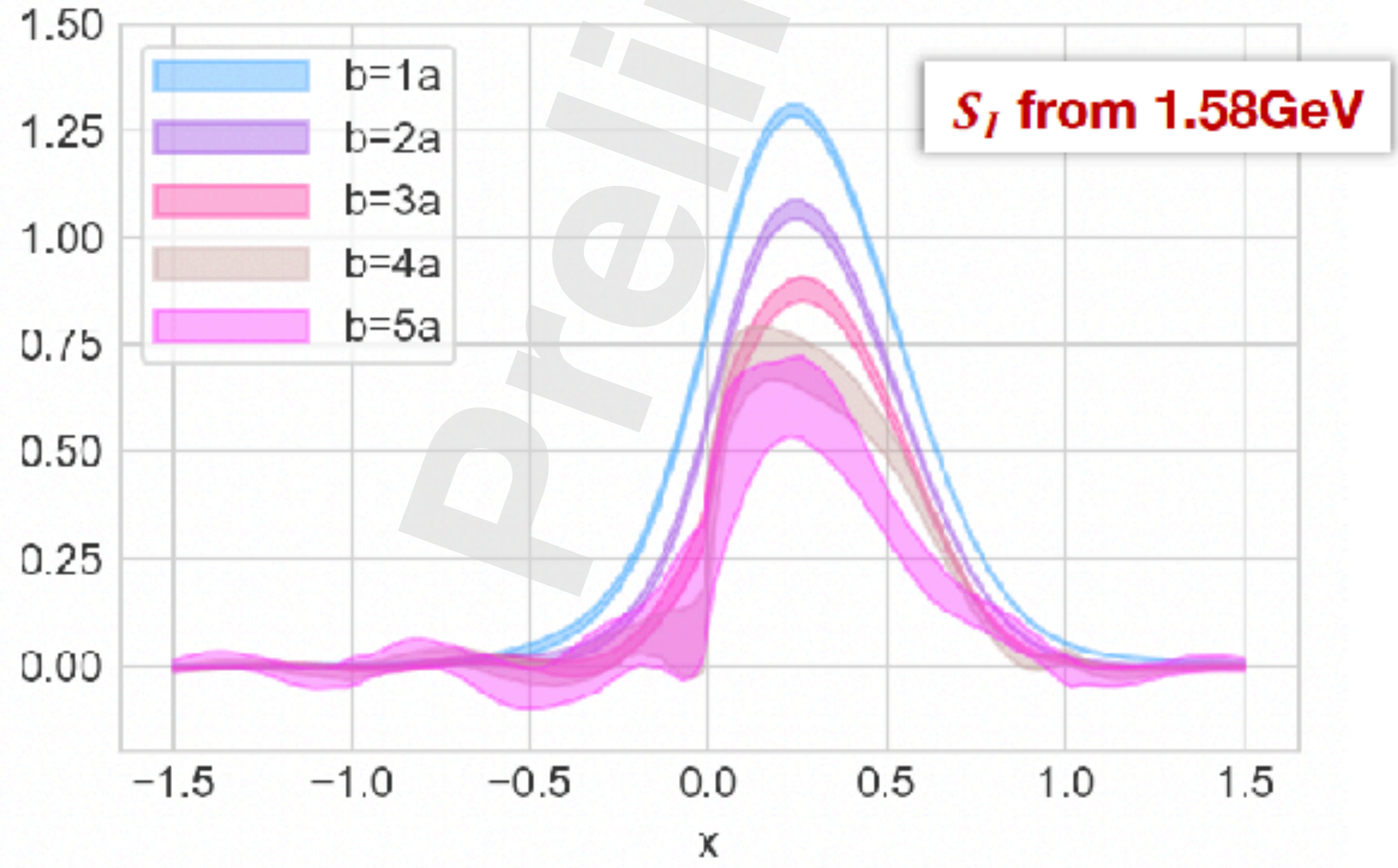
Preliminary TMD PDF result

- The soft function is essential to make the TMD PDF at large $b_\perp$ to be smaller than that at $b_\perp = 0$, and then ensure a proper definition in the momentum $k_\perp$ space.
- Further data analysis and systematic uncertainty treatment are in progress.









Q.A. Zhang. et.al., LPC, in preparation

Summary

- TMD PDF describe the 3D quark spin distribution in the nucleon with different polarizations.
- Short distance renormalization with Wilson loop works well for the renormalization of the quasi-TMD operators;
- The soft function and CS-kernel extracted from both TMD-WF and TMD-PDF are keeping improved.
- The final prediction of the TMD PDF is in progress.