Light-Nuclei Production and Critical Point

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QCD phase diagram

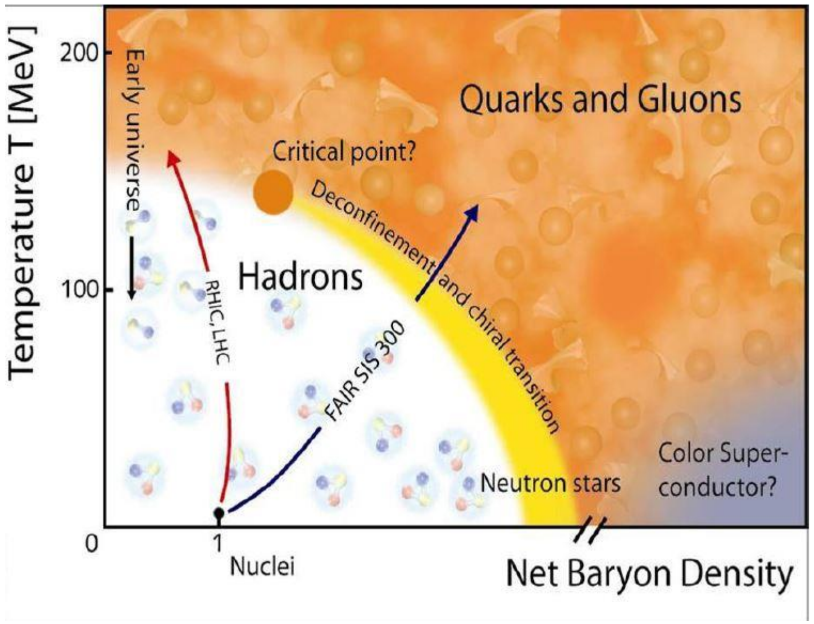
- Lattice QCD (small μ_B finite T):
  - Crossover
- Effective models(large μ_B)
  - 1st order phase trans.

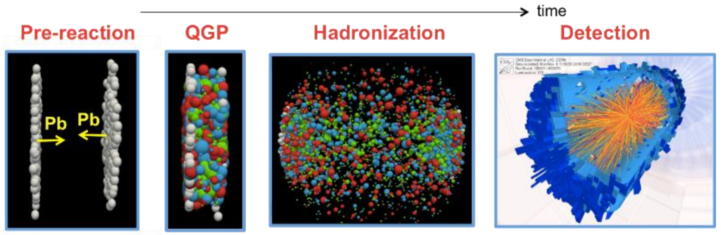
→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

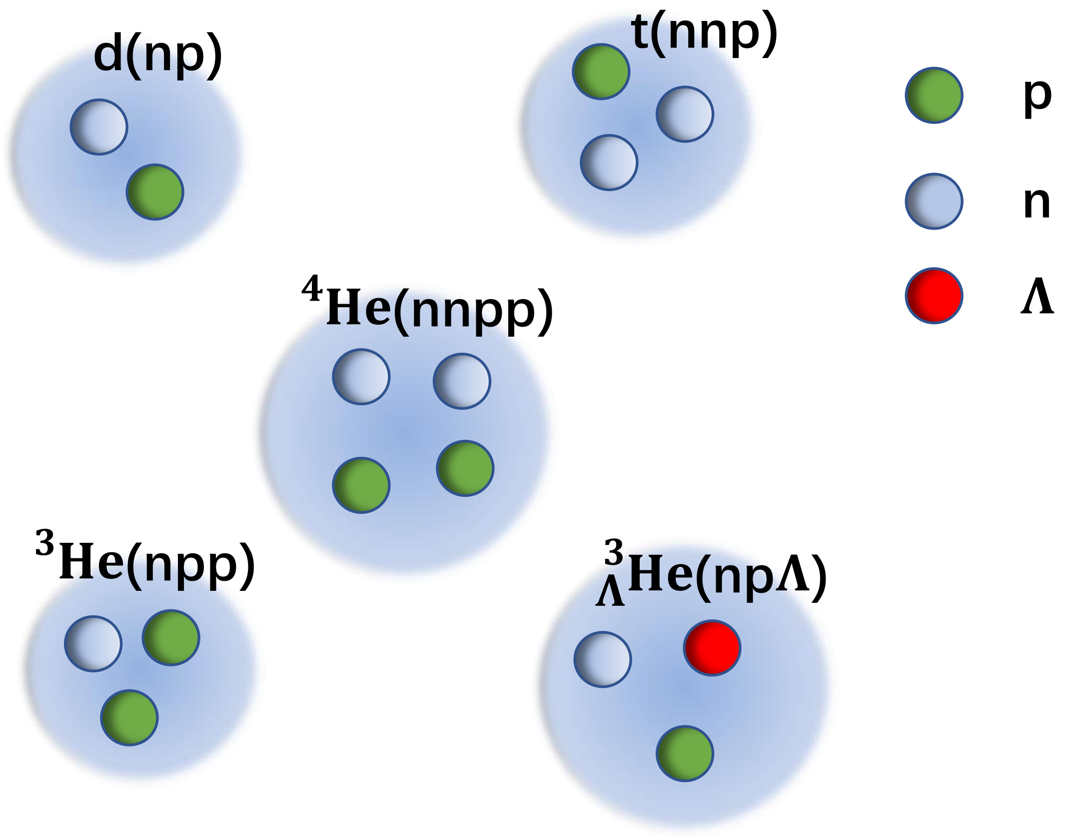
→ Heavy-ion collisions :

  - changing $\sqrt{s_{NN}}$,mapping $T-\mu$:
    RHIC(BES),NICA,FAIR,J_PARC….





Light Nuclei Cluster



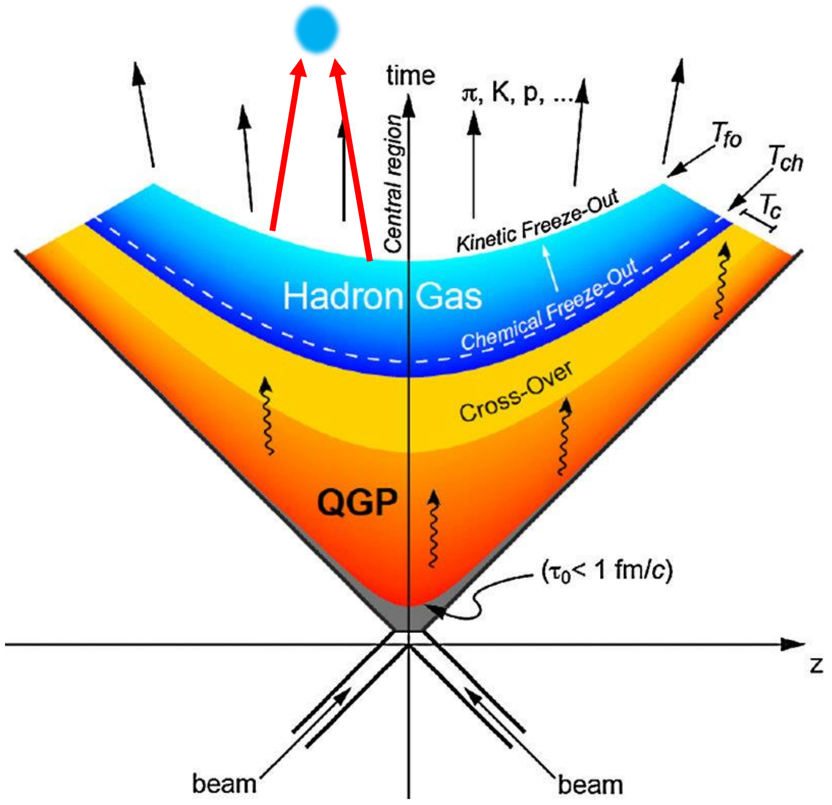
Loosely bounded objects (~MeV)

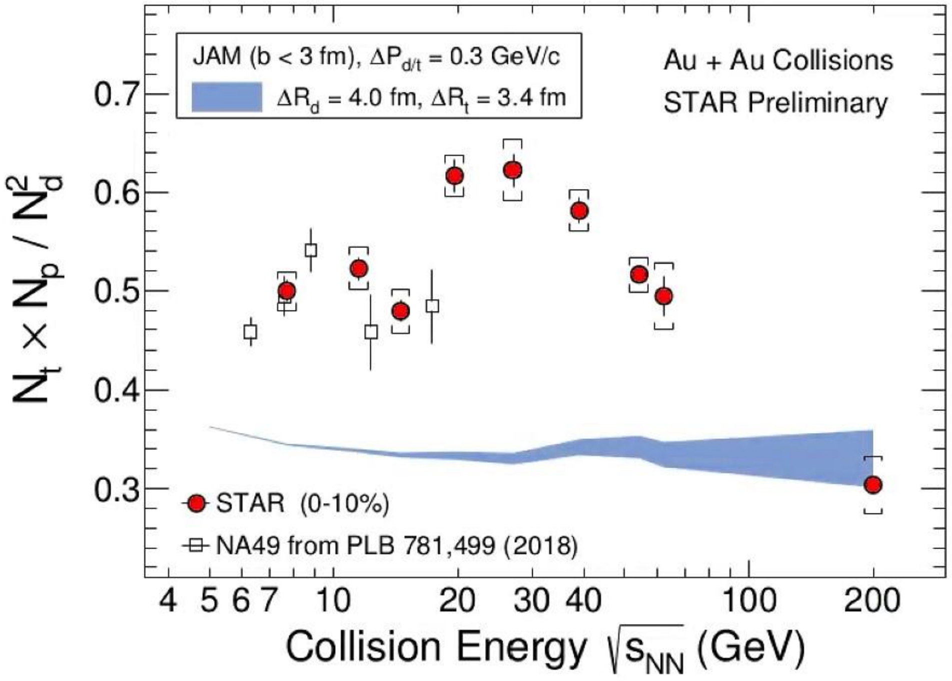
Nucleons close each other in phase-space (homogeneous):

- **Phase-space**
- nucleons interaction

Light nuclei in heavy ion collisions

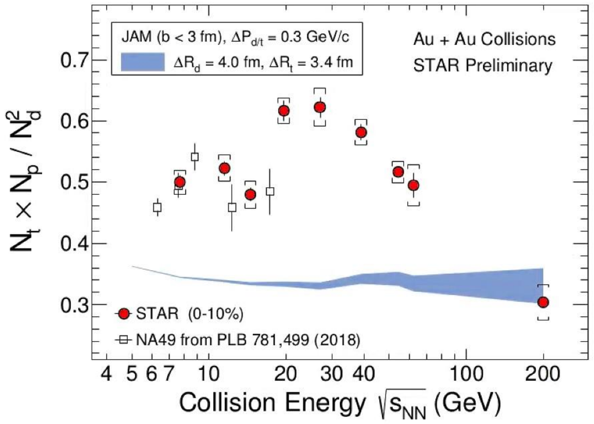
H. Liu et al., Phys. Lett. B805, 135452 (2020)



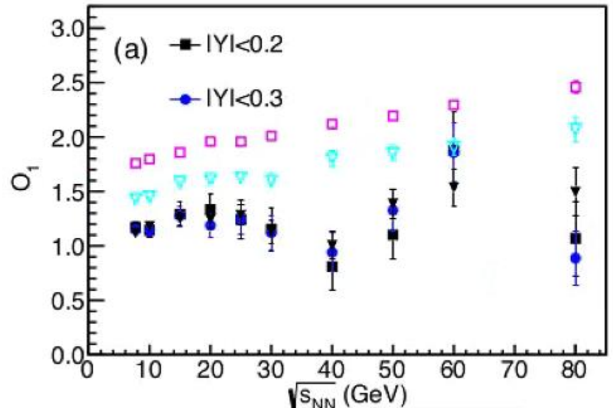


Non-monotonic v.s. $\sqrt{s_{NN}}$

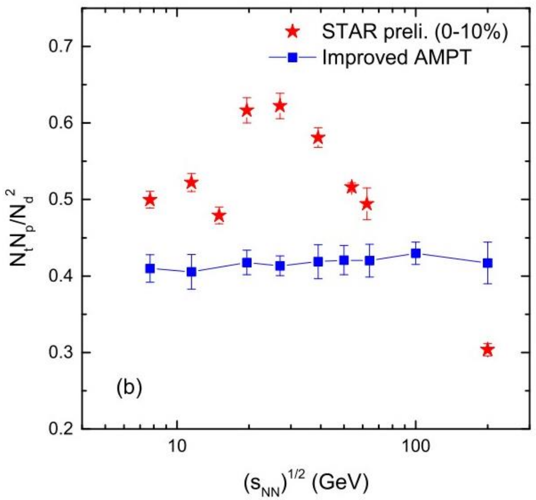
Dynamical models with light nuclei ratio



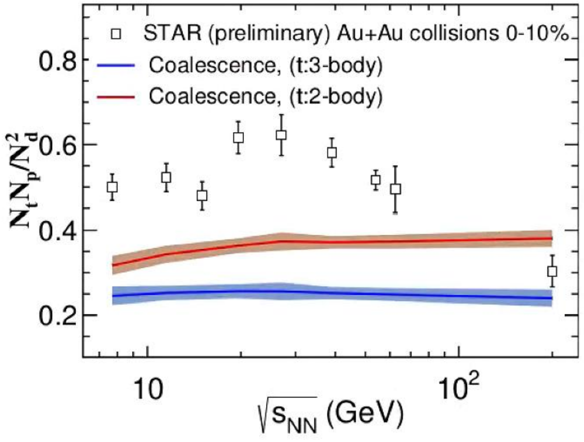
Hui Liu et al., PLB (2020)



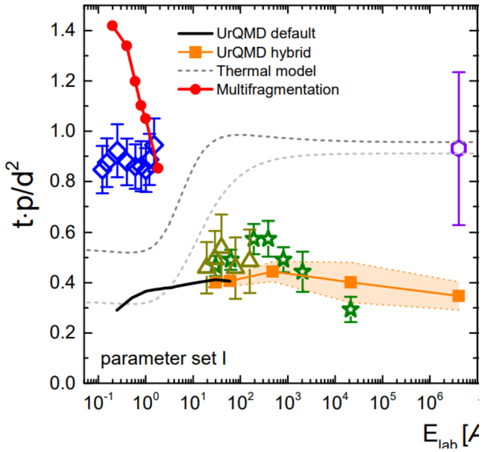
X.Deng et al., PLB (2020)



K.Sun et al., PRC (2021)



W. Zhao et al., PRC (2018)



P.Hillmann et al., 2109.05972

And others….

Phase-space produced in HIC

No clear non-monotonic on the model so far

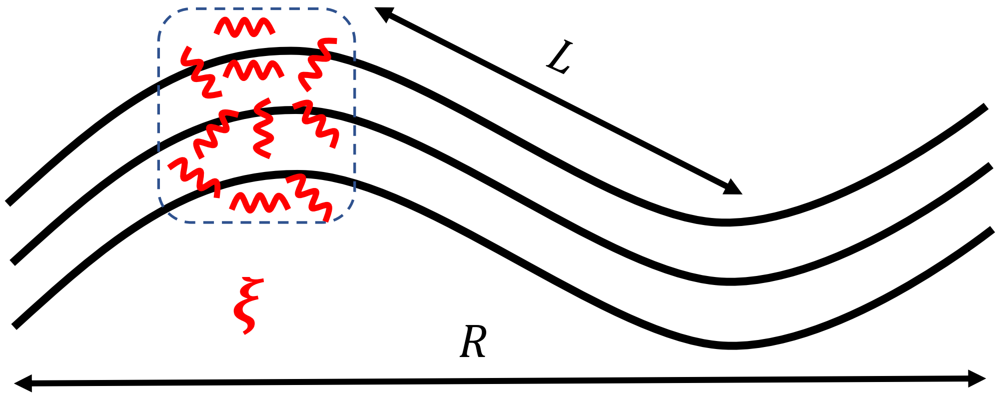
Scales for light-nuclei production near critical point

Nucleons close to each other in r space have similar momentum p

=>Homogeneity length $L \sim 1/\partial_\mu u^\mu$

$R, L \gg \xi$, when not so close to critical regime.

Background is large for N_A



R : Fireball size

L :homogeneity length

ξ: correlation length

Light Nuclei Yield Ratio (Background~~+Critical~~):

Canceling the background

SW, K.Murase, S.Tang, H.Song, 2205.14302

Light-nuclei yield ratio (Background)

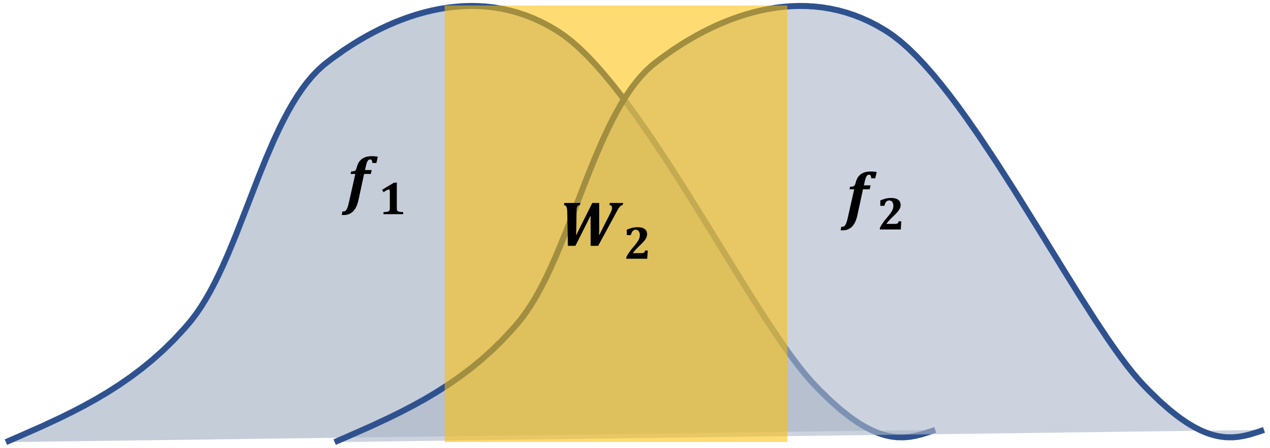
SW, K.Murase, S.Tang, H.Song, 2205.14302

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

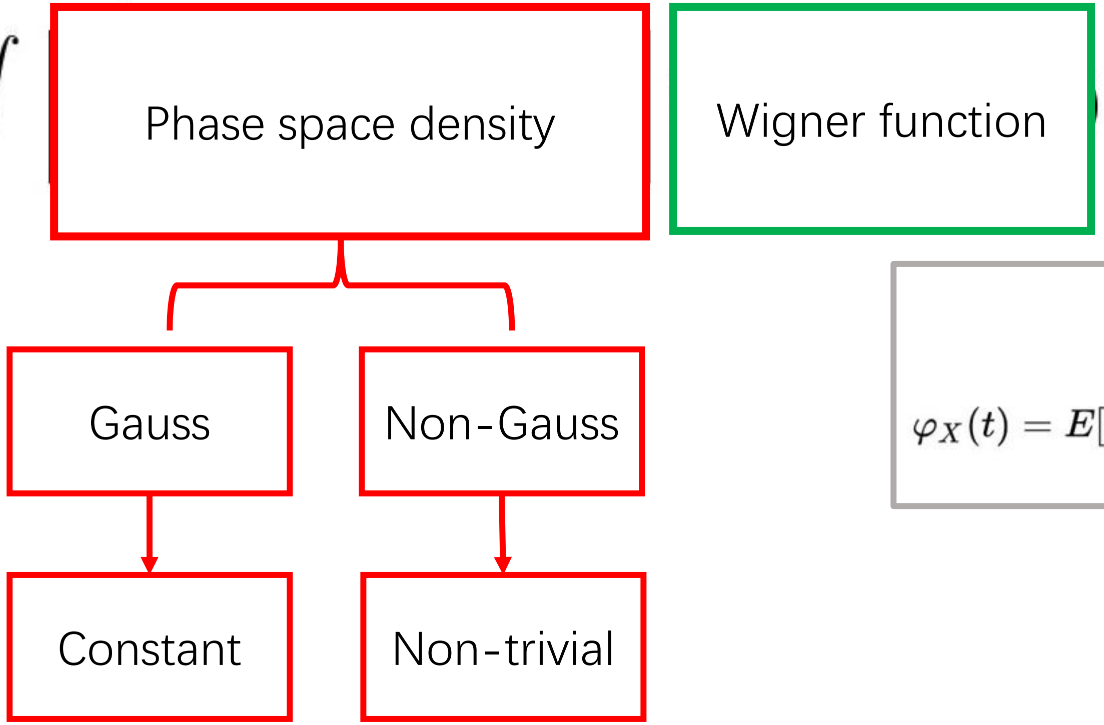
$N_A = g_A \int$ [Phase space density] [Wigner function]



Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

$$N_A = g_A \int$$



$$\varphi_X(t) = E[e^{itX}] = 1 + \frac{it\,\overbrace{E[X]}^{\text{一阶矩}}}{1} - \frac{t^2\,\overbrace{E[X^2]}^{\text{二阶矩}}}{2!} + \cdots + \frac{(it)^n\,\overbrace{E[X^n]}^{\text{n阶矩}}}{n!})$$

Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

$N_A = g_A \int$ [Phase space density] [Wigner function]

Phase space density → Gauss / Non-Gauss

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}}\right]^{A-1} \cdot \left[1 + \mathcal{O}(\{\mathcal{C}_\alpha\}_{|\alpha|\geq 3})\right]$$

Fireball size, Homog. Length

$$C_2 = 2\begin{pmatrix} \frac{\langle \boldsymbol{r}\boldsymbol{r}^{\mathrm{T}}\rangle}{\sigma_A^2} & \langle \boldsymbol{r}\boldsymbol{p}^{\mathrm{T}}\rangle \\ \langle \boldsymbol{p}\boldsymbol{r}^{\mathrm{T}}\rangle & \sigma_A^2\langle \boldsymbol{p}\boldsymbol{p}^{\mathrm{T}}\rangle \end{pmatrix}$$

$$\langle r\, r^T\rangle \sim \int drdp\, f(r,p) r r^T \sim R^2$$

$$\langle r\, p^T\rangle \; relates\ to\ \frac{1}{\partial_\mu u^\mu} \sim L$$

$$\langle p\, p^T\rangle \sim T_f^2$$

Light-nuclei yield ratio (Background)

SW, K.Murase, S.Tang, H.Song, 2205.14302

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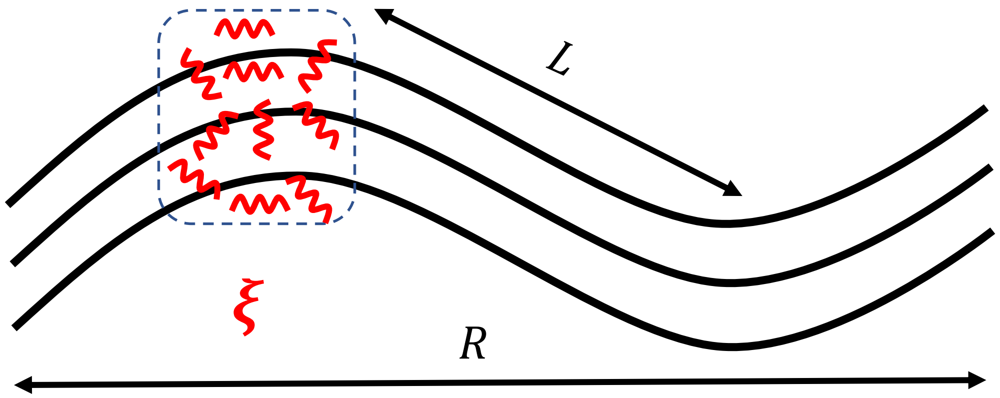


$\boldsymbol{R}$: Fireball size
$\boldsymbol{L}$:homogeneity length
ξ: correlation length

Light Nuclei Ratio Near QCD Critical Point:

(Background+Critical)

SW, K.Murase, S.Zhao, H.Song, to appear

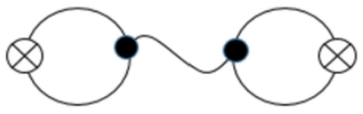
Critical contribution δf in phase-space

SW, K.Murase, S.Zhao, H.Song, to appear

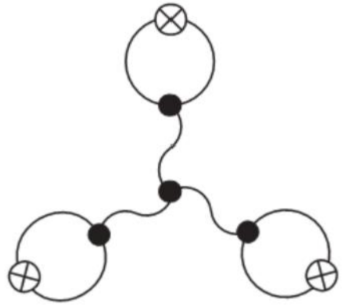
$$N_A \sim \left\langle (f_0 + \delta f)^A \right\rangle_\sigma \sim f_0^A + \left\langle (\delta f)^2 \right\rangle_\sigma^{\beta_2} + \left\langle (\delta f)^3 \right\rangle_\sigma^{\beta_3} + \cdots + \left\langle (\delta f)^A \right\rangle_\sigma^{\beta_4}$$

Critical δf: A constituent nucleons relates to $2,3,\ldots A$-point critical correlator

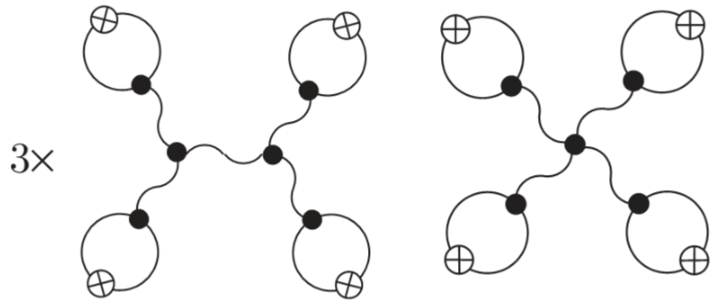
$\langle \delta f_1 \delta f_2 \rangle_\sigma \sim \Xi(A,2)$



$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma \sim \Xi(A,3)$



$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma \sim \Xi(A,4)$



Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, to appear

$$N_A = g_A \int$$ [Phase space density: Background+Critical] [Wigner func]

Characteristic func.

Fourier transf.

Scale of ξ

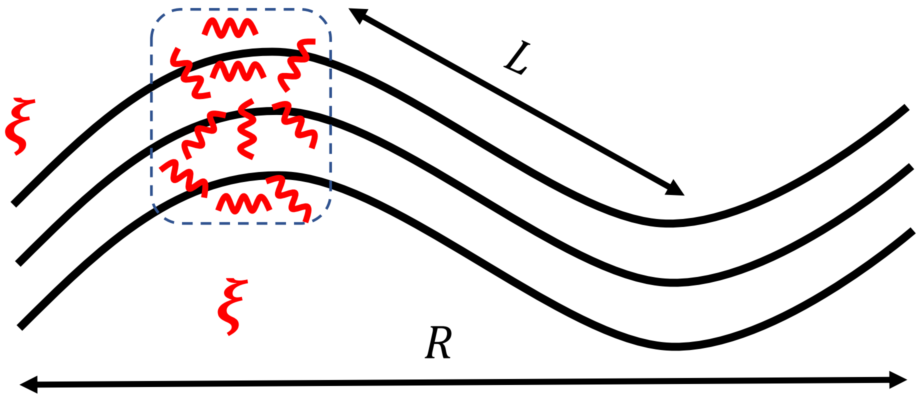
$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}}\right]^{A-1} \cdot [1 + \sum_{i=2}^{A} \Xi(A,i)]$$

Scales of R, L

Bkg

Cri

N_A share a common structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$
=> The ratios of N_A cancel Bkg and highlight Cri



R : Fireball size
L :homogeneity length
ξ: correlation length

Light nuclei yield: Background+Critical

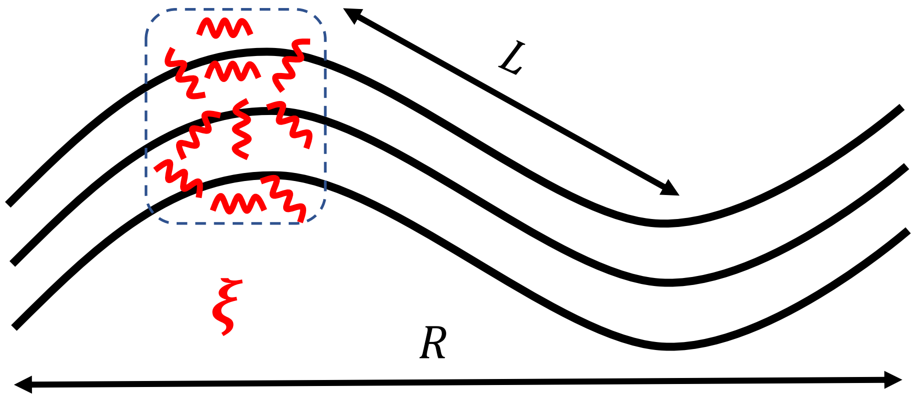
SW, K.Murase, S.Zhao, H.Song, to appear

$$R^{1-B,A-1}_{A,B} = \frac{N_p^{B-A} N_B^{A-1}}{N_A^{B-1}}$$

$$\tilde{R}(A,B) \equiv R^{1-B,A-1}_{A,B} - g_B^{A-1} g_A^{-(B-1)} \sim \mathcal{O}(\xi)$$

$$\tilde{R}(A,B,D) \equiv R^{1-B,A-1}_{A,B} - g_B^{A-1} g_D^{-(A-1)(B-1)/(D-1)} [R^{1-D,A-1}_{A,D}]^{(B-1)/(D-1)} \sim \mathcal{O}(\xi)$$

N_A share a common structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$
=> The ratios of N_A cancel Bkg and highlight Cri

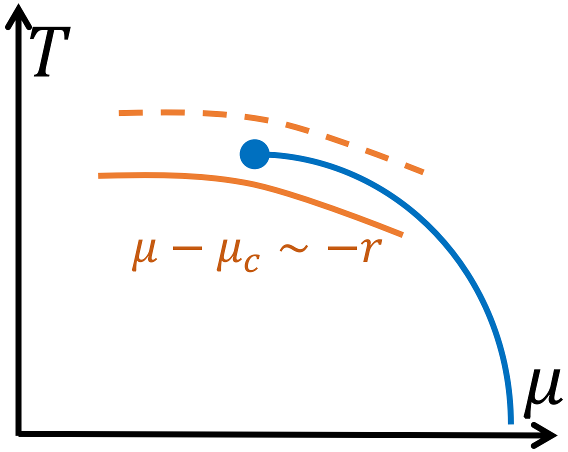


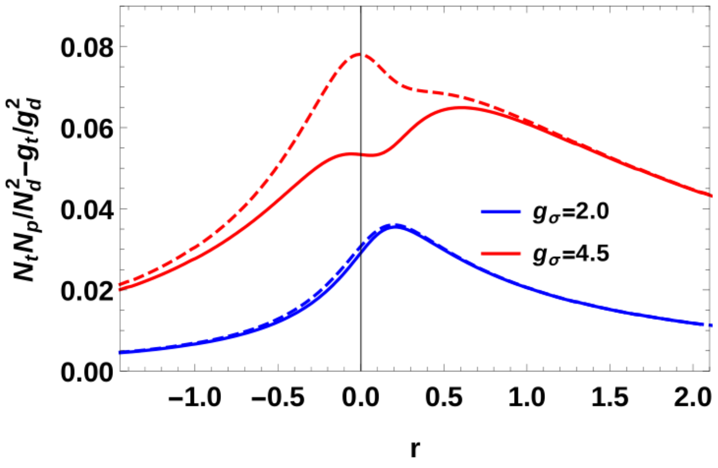
R : Fireball size
L :homogeneity length
ξ: correlation length

The ratios of N_A proportional to Cri
=> In the ratios of N_A, large scales R, L are unimportant but ξ matters

Example: in the Ising critical regime

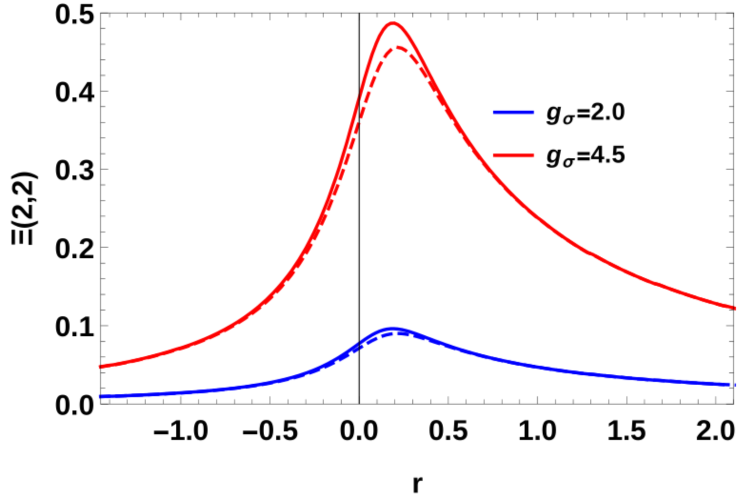
SW, K.Murase, S.Zhao, H.Song, to appear

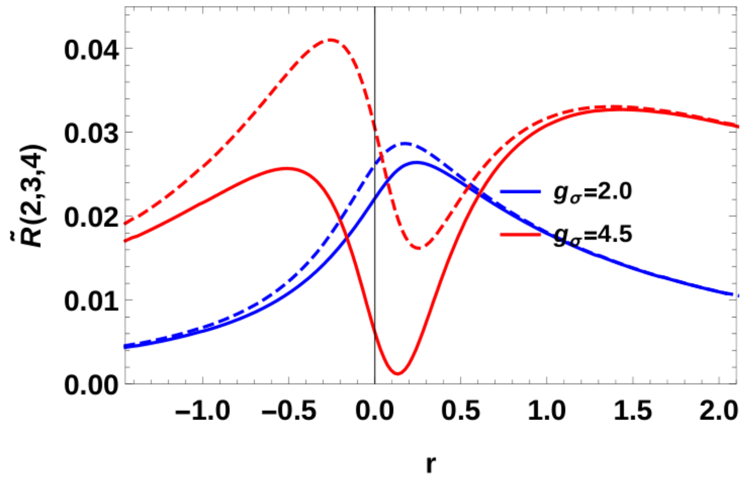




$$\frac{g_t}{g_d^2}\frac{3\Xi(3,2)-\Xi(3,3)-2\Xi(2,2)-\Xi(2,2)^2}{[1+\Xi(2,2)]^2}.$$

~2pt - 3pt - (2pt)^2





$$\tilde{R}(2,3,4) = \frac{\langle N_t\rangle_\sigma N_p}{\langle N_d\rangle_\sigma^2} - \frac{g_t}{g_{4He}^{2/3}}\left[\frac{\langle N_{4He}\rangle_\sigma N_p^2}{\langle N_d\rangle_\sigma^2}\right]^{2/3}$$

~2pt -4 (2pt)^2

Conclusion

- Using Characteristic Function to study the phase space distribution in light-nuclei production:
  - Lower order phase-space cumulants ($C_\alpha, |\alpha| < 3$) play similar role for different light-nuclei production N_A
    - => Fireball size R, Homogeneity length L play similar role.
    - => Higher order phase-space cumulants ($C_\alpha, |\alpha| \geq 3$) are important to light-nuclei yield ratios.
- Proper ratios of light nuclei largely cancel the effects from the scales of fireball size, homogeneity length, etc. But critical correlation length can not be canceled.
- $2{\sim}A$ point correlators contribute to N_A, square and higher order terms of 2-point correlator result in dip inside the peak near the critical point.