W mass at future e^+e^- colliders

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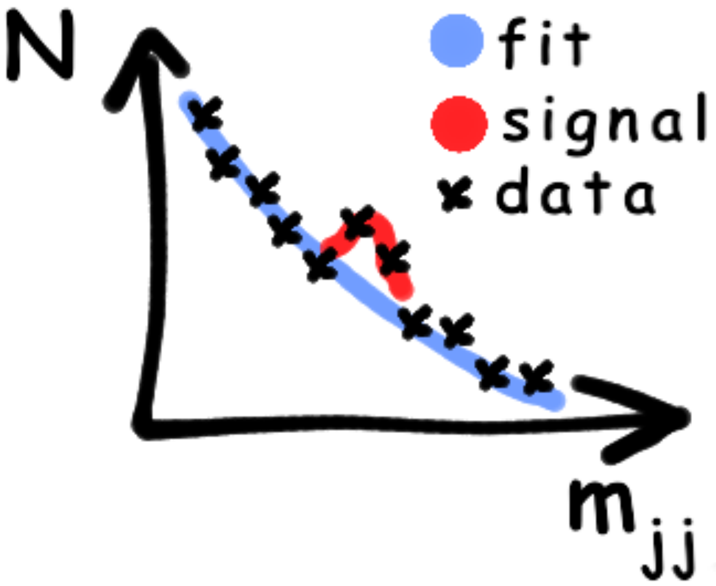
W Mass Workshop
IHEP, THU, NJNU & PKU

Outline

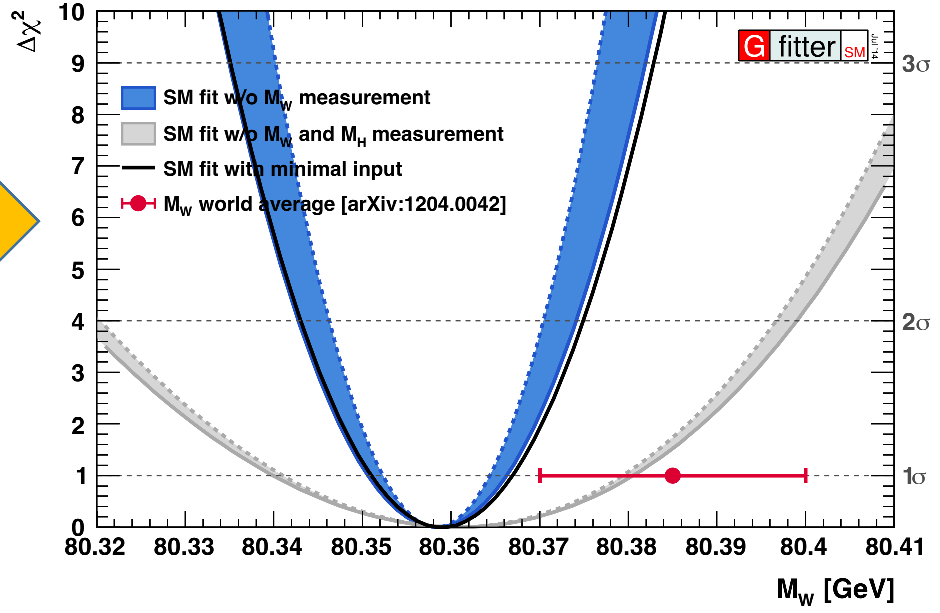
- Introduction
- Measurement methods
- Direct measurement
- Threshold scan
- Summary & discussion

Introduction

- After the discovery of Higgs boson, the new physics could be probed via
  - Direct searches:
    - Based on a theory hypothesis, looking for excesses in the mass spectrum.
    - e.g. Heavy Di-boson resonance searches



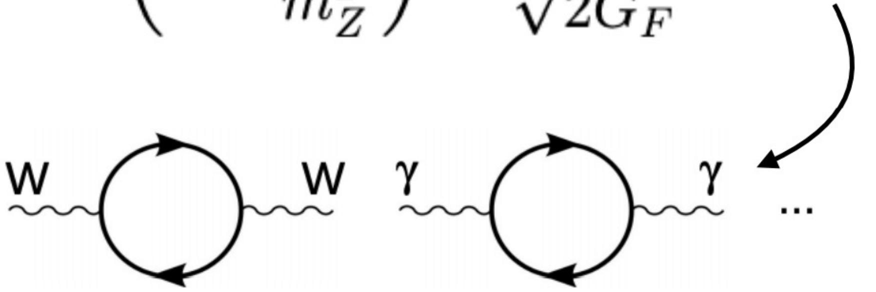
  - Indirect searches, indirect constraints:
    - Precisely measure SM properties, compare with SM predictions, check the consistency
    - The differences can come for contributions from new particles
    - E.g. W boson mass measurements



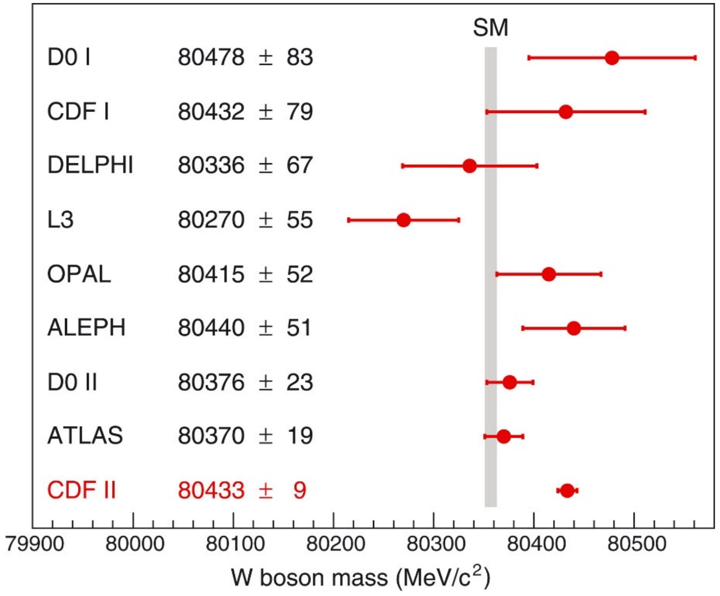
Introduction

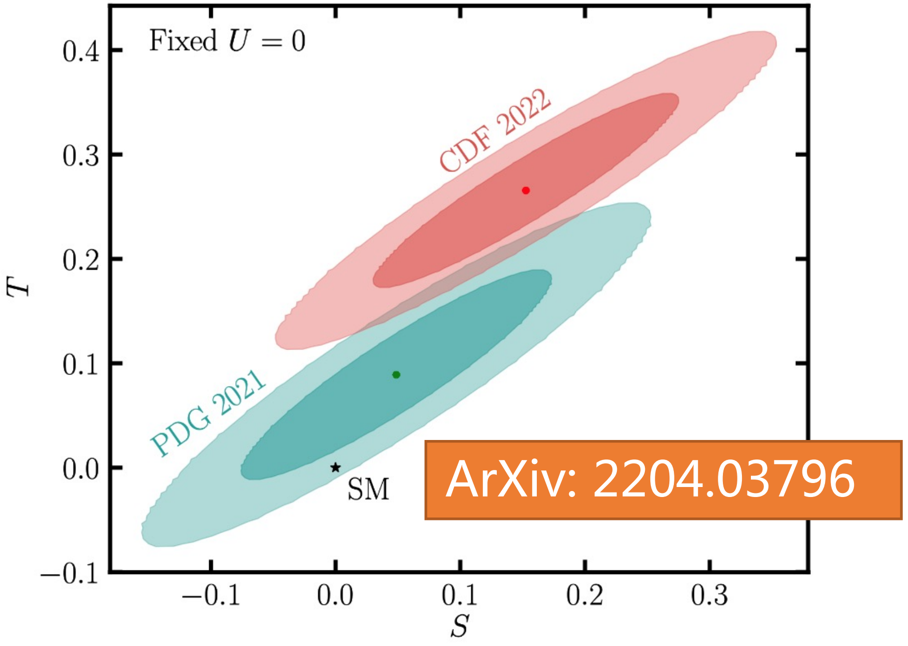
- EW precisions play important role to test the consistency of the SM via global fit.
  - e.g. Given M_Z, α, and G_F , the W mass is predictable

$$m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta)$$



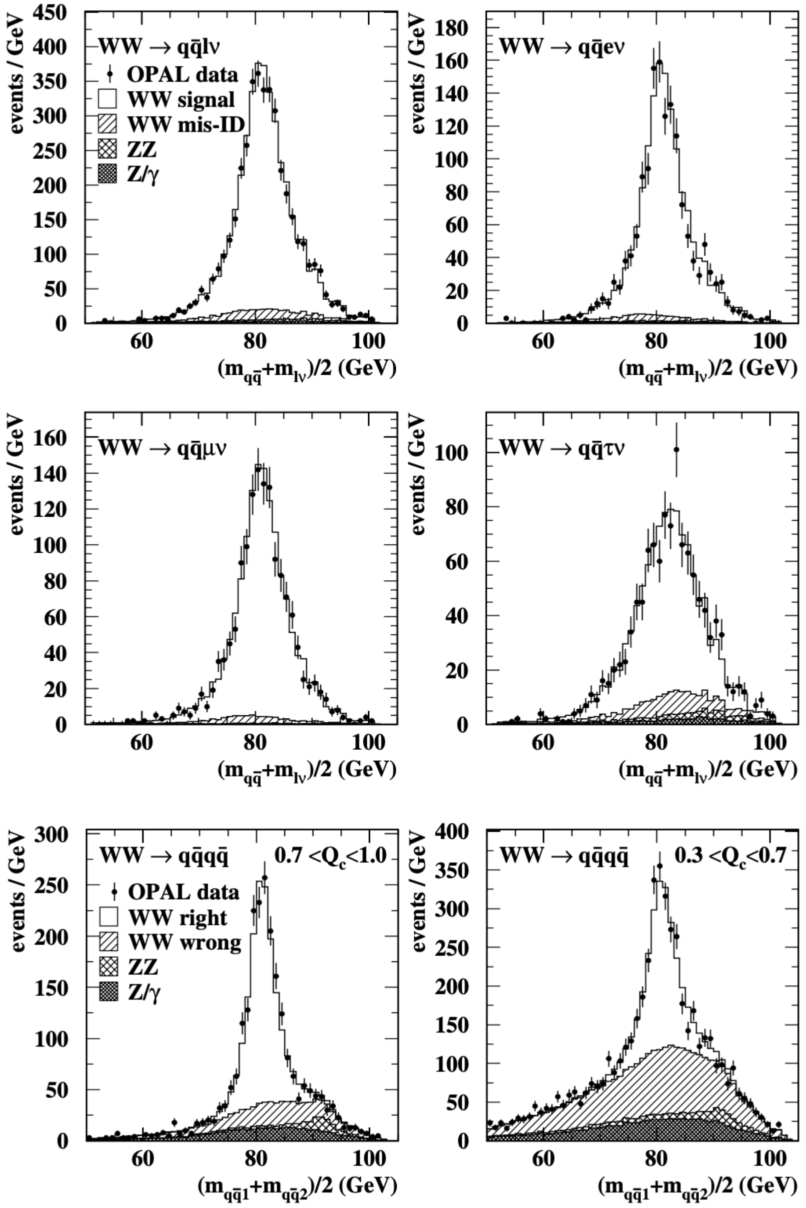
- Various ways to measure m_W ...





Measurement method – I

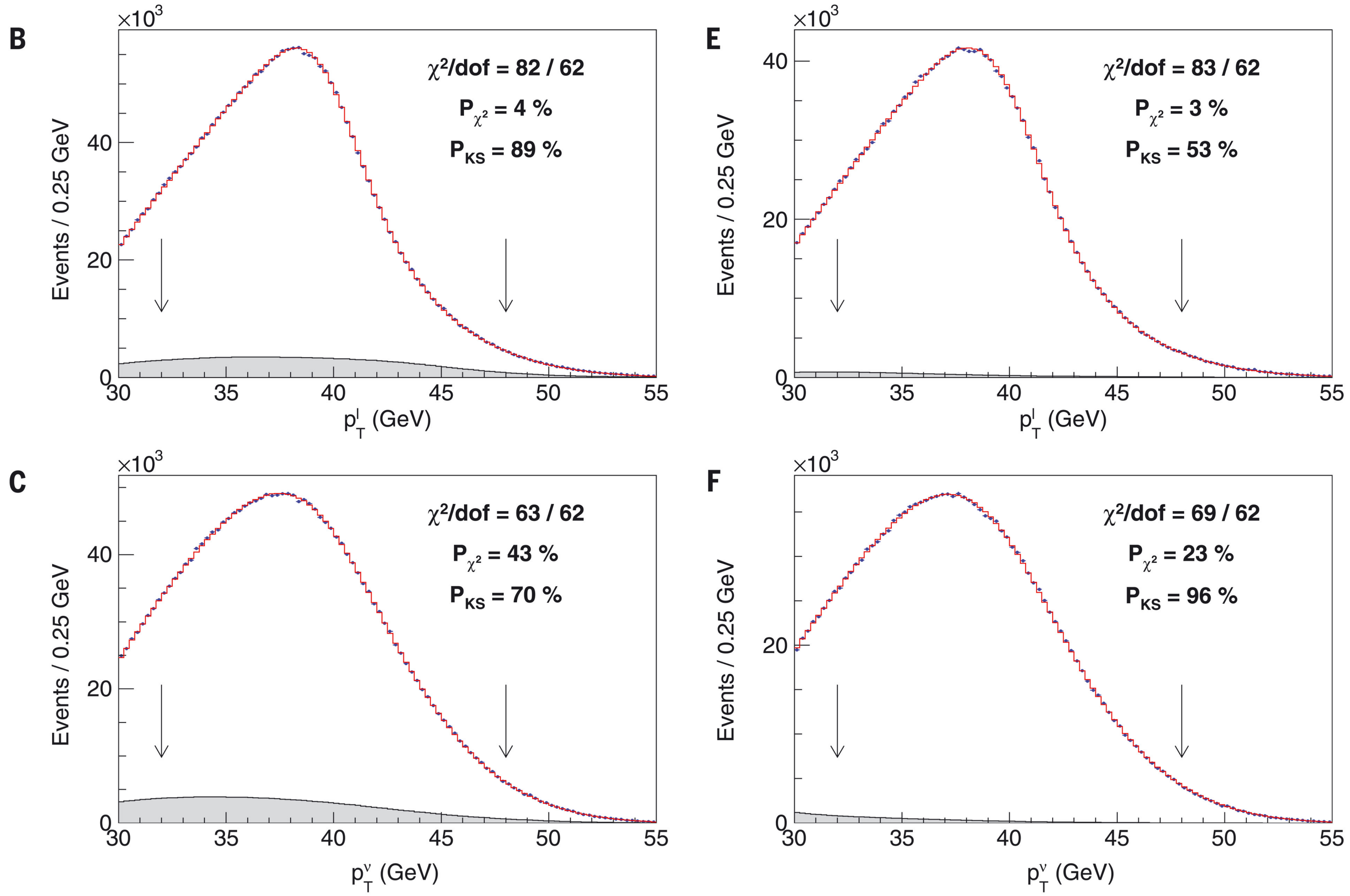
- Direct reconstruction
- Pros
  - Large statistics
  - Kinematics fits (ee collision)
- Cons
  - Jet energy resolution and calibration
  - Fragmentation models
  - PDFs in pp collision
  - Color reconnection
- Final combination of LEP
  - Stat. 25 MeV
  - Syst. 22 MeV



Measurement method – II

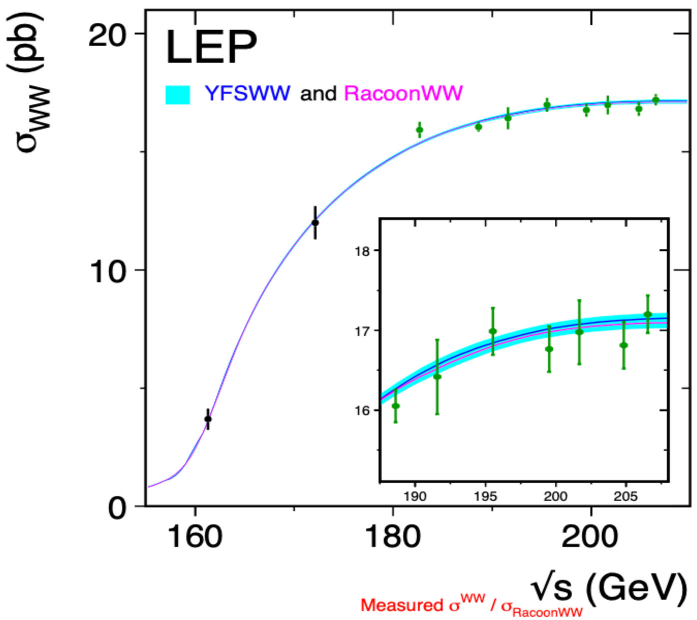
CDF II

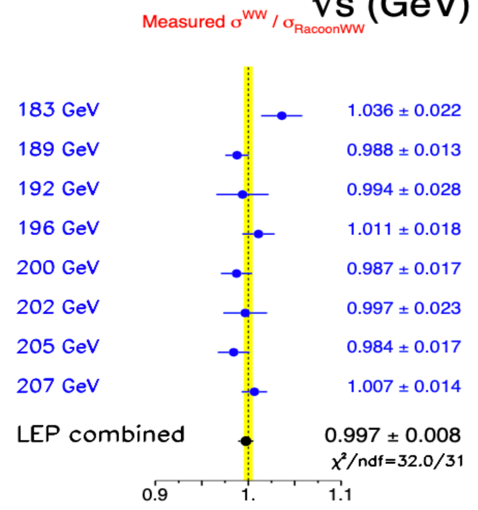
- Lepton spectrum
- Pros
  - Large statistics
  - Kinematics fits
- Cons
  - Momentum calibration
  - PDFs in pp collision



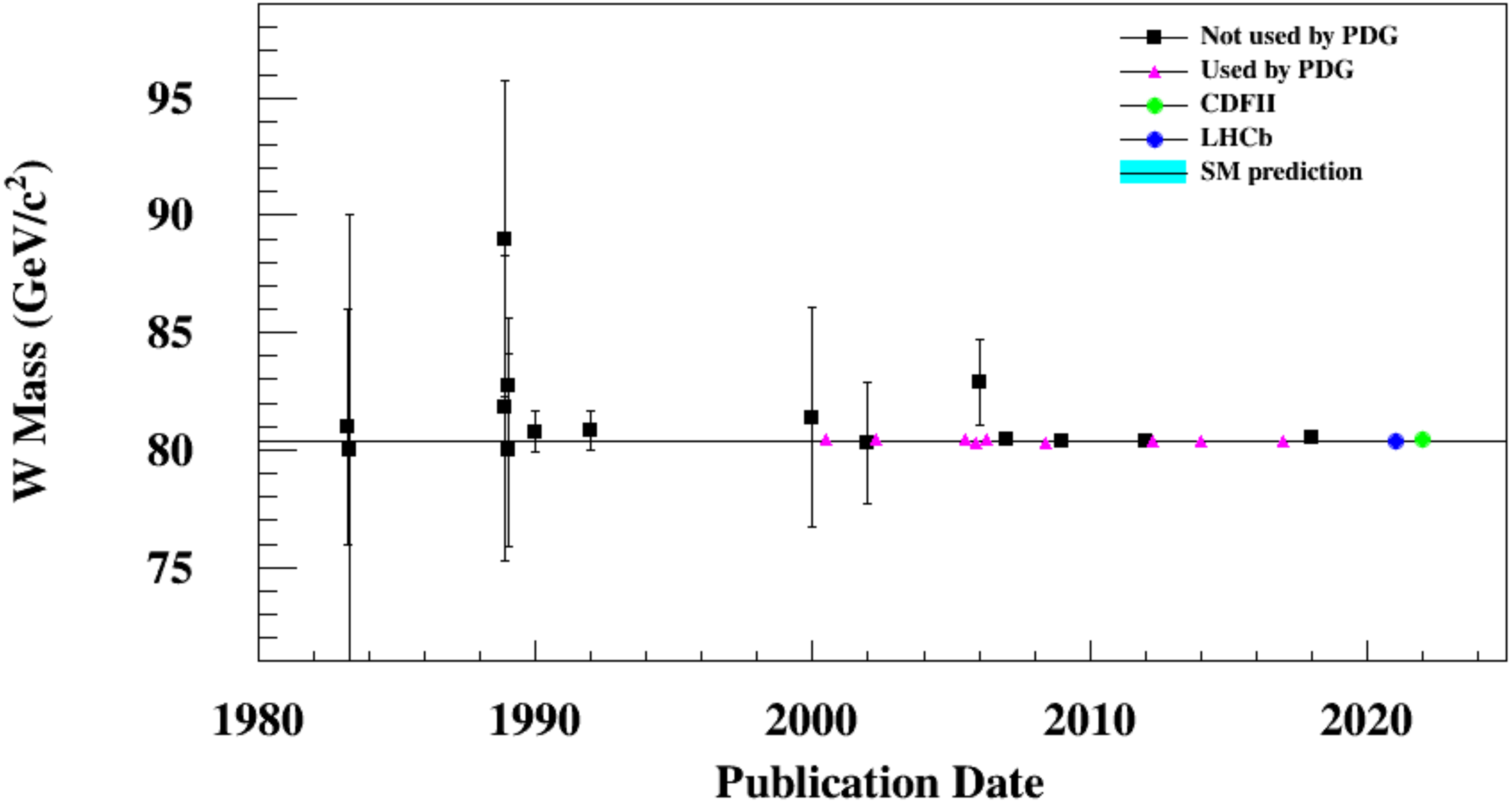
Measurement method – III

- Threshold scan around 160 GeV
  - Comparing the measured cross section with theoretical calculation and extracting the W mass and width
- Simple counting experiment
- Less systematics w.r.t direct method
- Challenge: beam energy calibration
- LEP
  - Stat. 200 MeV
  - Syst.   30 MeV





History of the W mass measurements



Future e^+e^- colliders

CEPC & FCC-ee

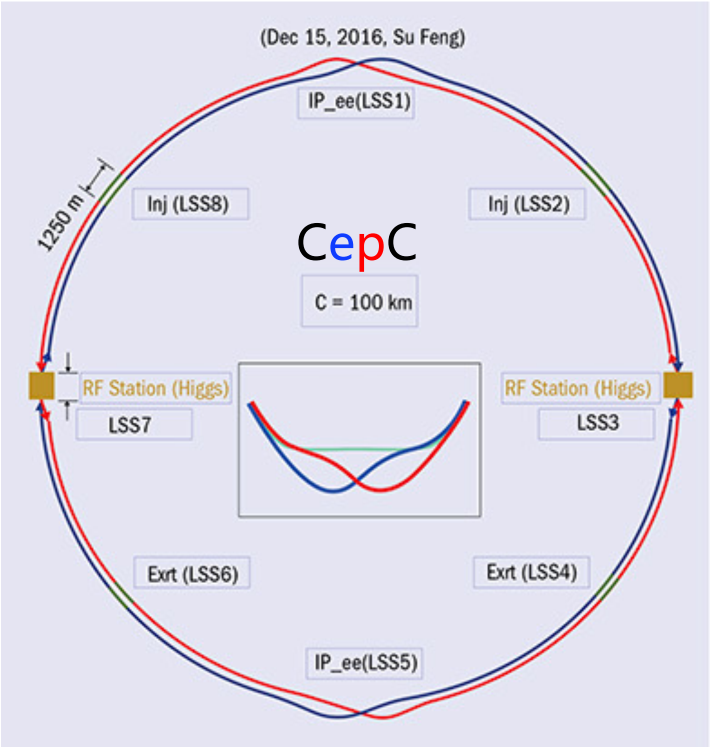
- Two circular e+e- collider were proposed after the discovery of Higgs boson
- Similar design philosophy and similar performances
- Similar physics programs

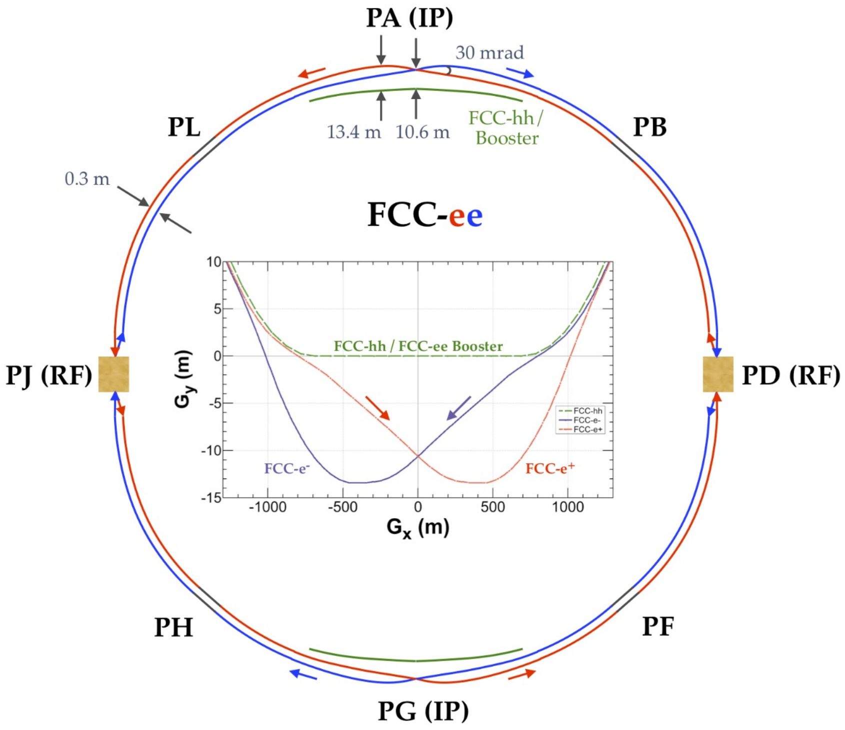
✓ Higgs, EW, QCD, and flavor physics and the new physis BSM

✓ Active R&D activities

✓ Released the CDRs in 2018 and 2019, respectively

✓ FCC-ee is the first prior of the European Strategy of Particle Physics and received strong support





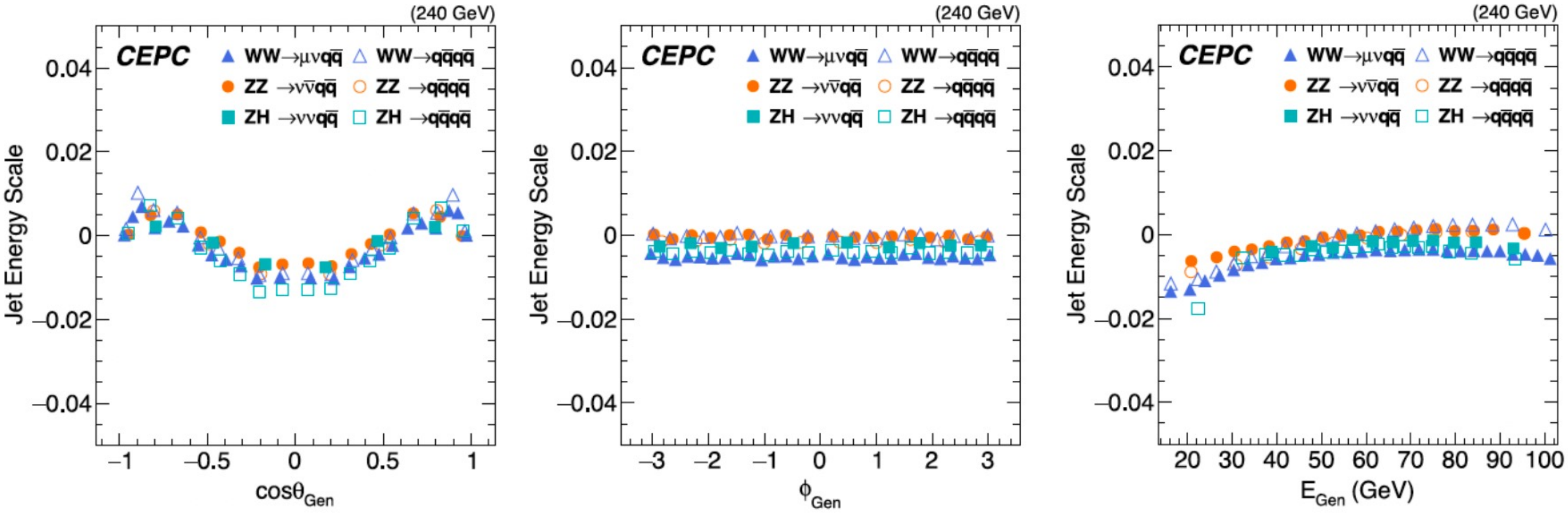
Physics programs of CEPC & FCC-ee

Operation scenarios	CEPC		FCC-ee	
	Luminosity (ab^{-1})	# of evts (10^6)	Luminosity (ab^{-1})	# of evts (10^6)
Z	100	$3x10^6$	192	$6x10^6$
WW	6	100	6-12	100-200
Higgs	20	4	5.1	1
Top	1	0.5	1.7	0.85

Slightly different, not fixed yet

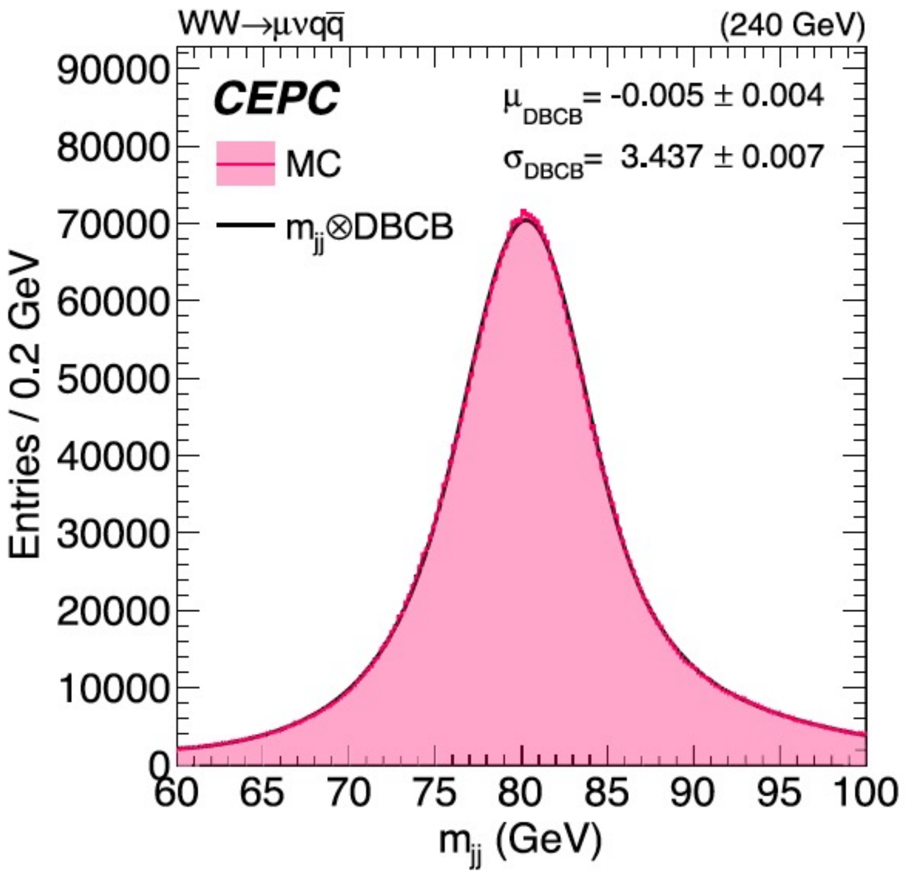
Direct reconstruction

- CEPC expected to accumulate 20 iab integrated luminosity at 240 GeV. On top of 4 Million Higgs, it will generate 60 M lvqq events
- evqq slightly more than muvqq with the contribution from single W process. The W boson mass can be reconstructed with
  - Relative Mass resolution of 4%
  - Jet energy scale: ~1% before angular/energy correction, 0.1% after correction.



Reconstruct W mass from lvqq events at 240 GeV

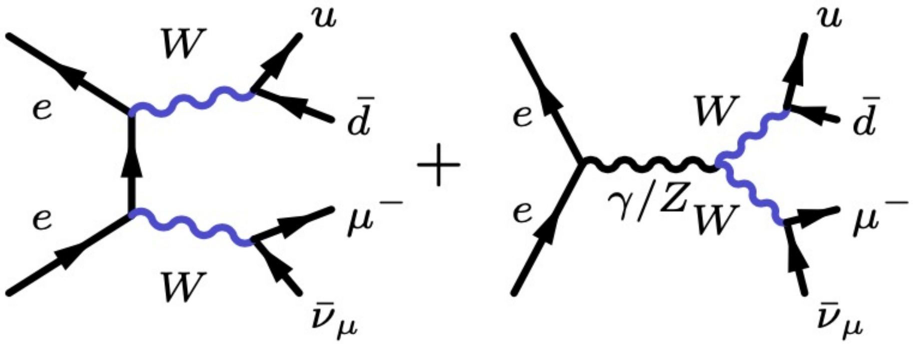
- Meanwhile, we are anticipating ~1 billion ee→qq events at 240 GeV to be used for jet energy calibration.
- Thus, we expect 2 - 3 MeV uncertainty on W mass using two lvqq processes at 240 GeV.
- Dominated by systematics
- Reference:
  - ➢ From JINST 16 P07037: < 10 MeV with only muvqq of 5 iab integrated luminosity



W mass from lvqq events at 240 GeV

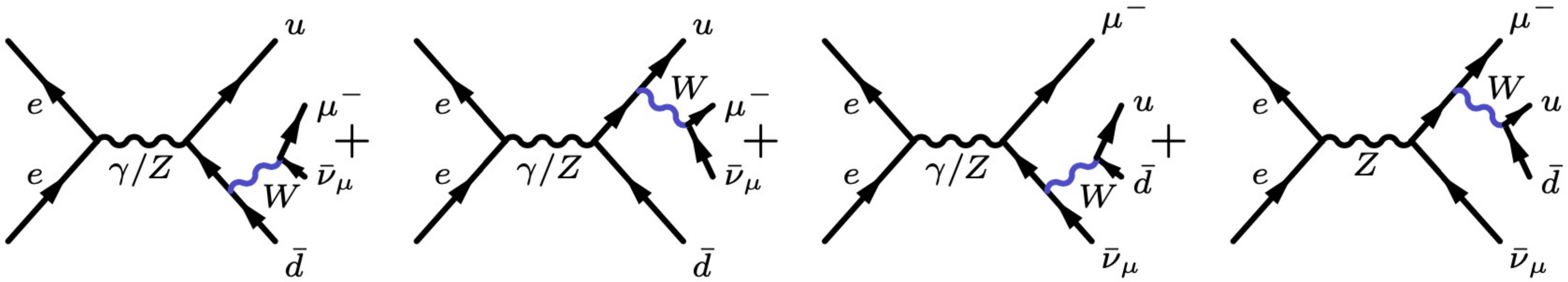
W threshold scan

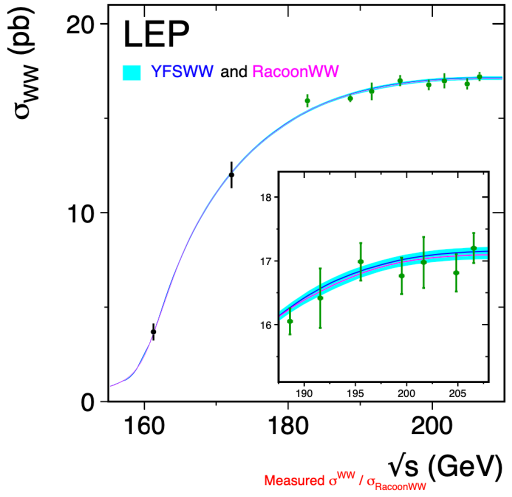
Signals (CC03)



10 diagrams in total

CC03 +





- Why scan the threshold? A simple counting experiment

  Line shape depends on the W mass/width

$$\sigma_{WW}(m_W, \Gamma_W, \sqrt{s}) = \frac{N_{obs}}{L\epsilon P} \qquad (P = \frac{N_{WW}}{N_{WW}+N_{bkg}})$$

m_W, Γ_W can be extracted by comparing the σ_{WW} with the prediction

- How?

$\Delta m_W, \Delta\Gamma_W$						
N_{obs}	L	ϵ	N_{bkg}	E	σ_E	……

Most of them depend on $\sqrt{s}$, so it is an optimization problem:

Which energy points should be chosen and allocation of luminosity

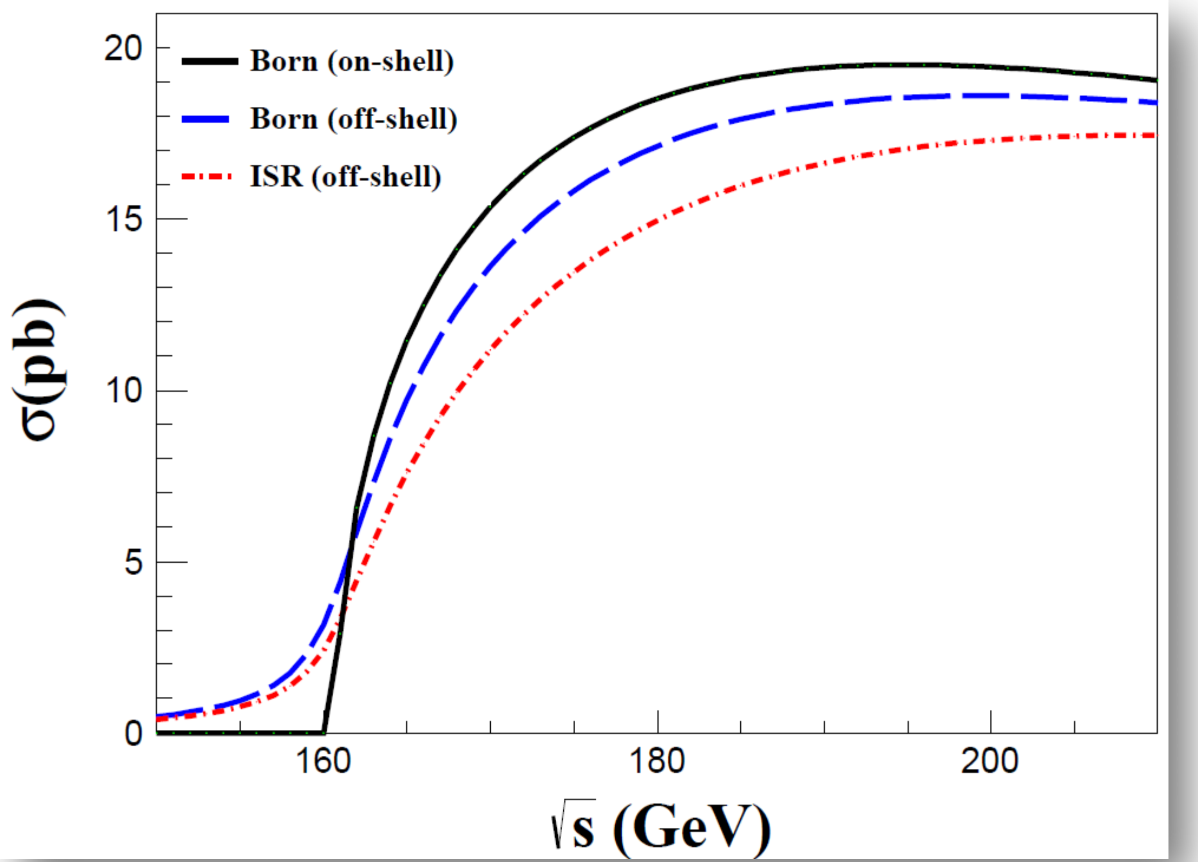
Theoretical Tool

- The σ_{WW} is a function of $\sqrt{s}$, m_W and Γ_W, calculated with the GENTLE (CC03)

- The ISR correction considered by convoluting the Born cross sections with QED structure function, with the precision up to NLO(α^2) and O(β^3)

  (implemented by ourselves)

- All expected to improved in future



Statistical and systematic uncertainties

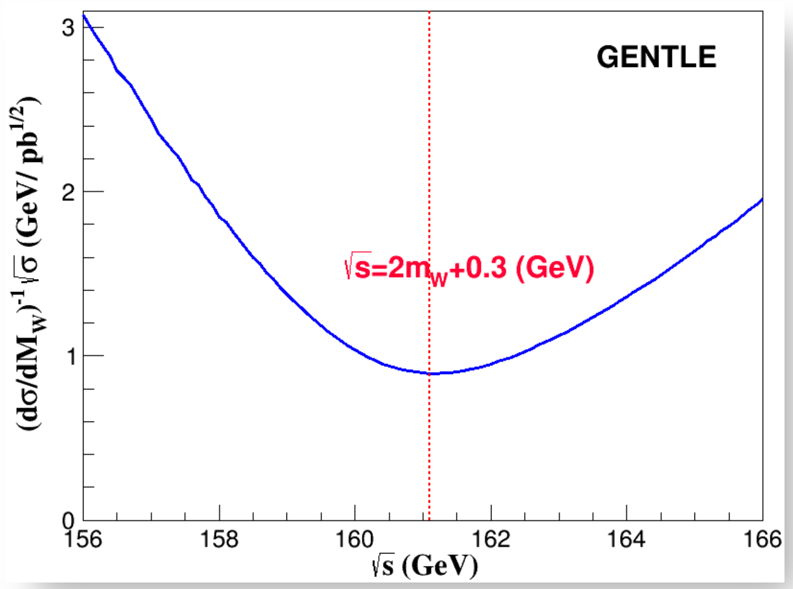
Statistical uncertainties with a single energy point

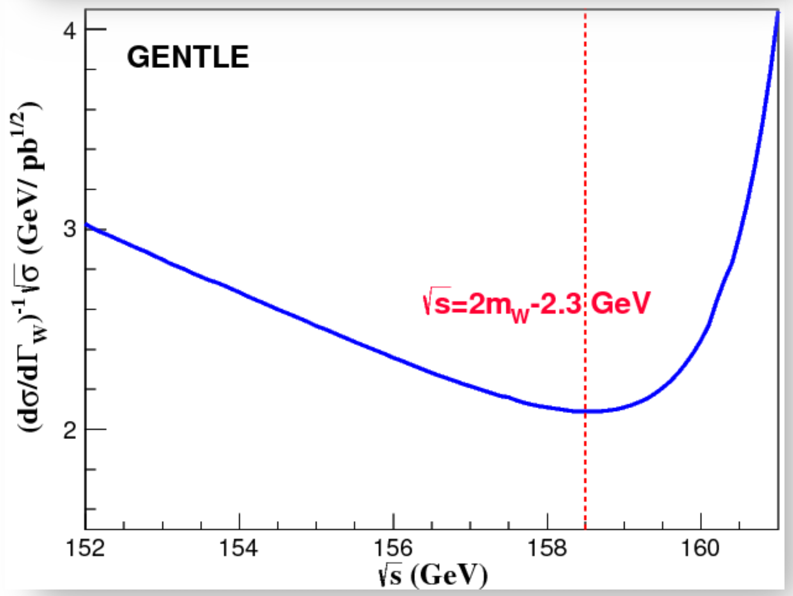
- $\Delta\sigma_{WW} = \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$ $\left(P = \frac{N_{WW}}{N_{WW}+N_{bkg}}\right)$
- $\Delta m_W = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$
- $\Delta\Gamma_W = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$

Asumming L=3.2 ab^{-1}, and ϵ=0.8, P=0.9

- Δm_W=0.6 MeV, $\Delta\Gamma_W$=1.4 MeV

(at the most sensitive energy points)





Statistical uncertainty

- ➢If more points, we can measure both m_W and Γ_W.
- ➢ The χ^2 defined as:

$$\chi^2 = \sum_i \frac{(N_{\text{fit}^i} - N^i_{\text{obs}})^2}{N^i_{\text{obs}}} = \frac{(\mathcal{L}\epsilon P)^i(\sigma^i_{\text{fit}} - \sigma^i_{\text{obs}})^2}{\sigma^i_{\text{obs}}}$$

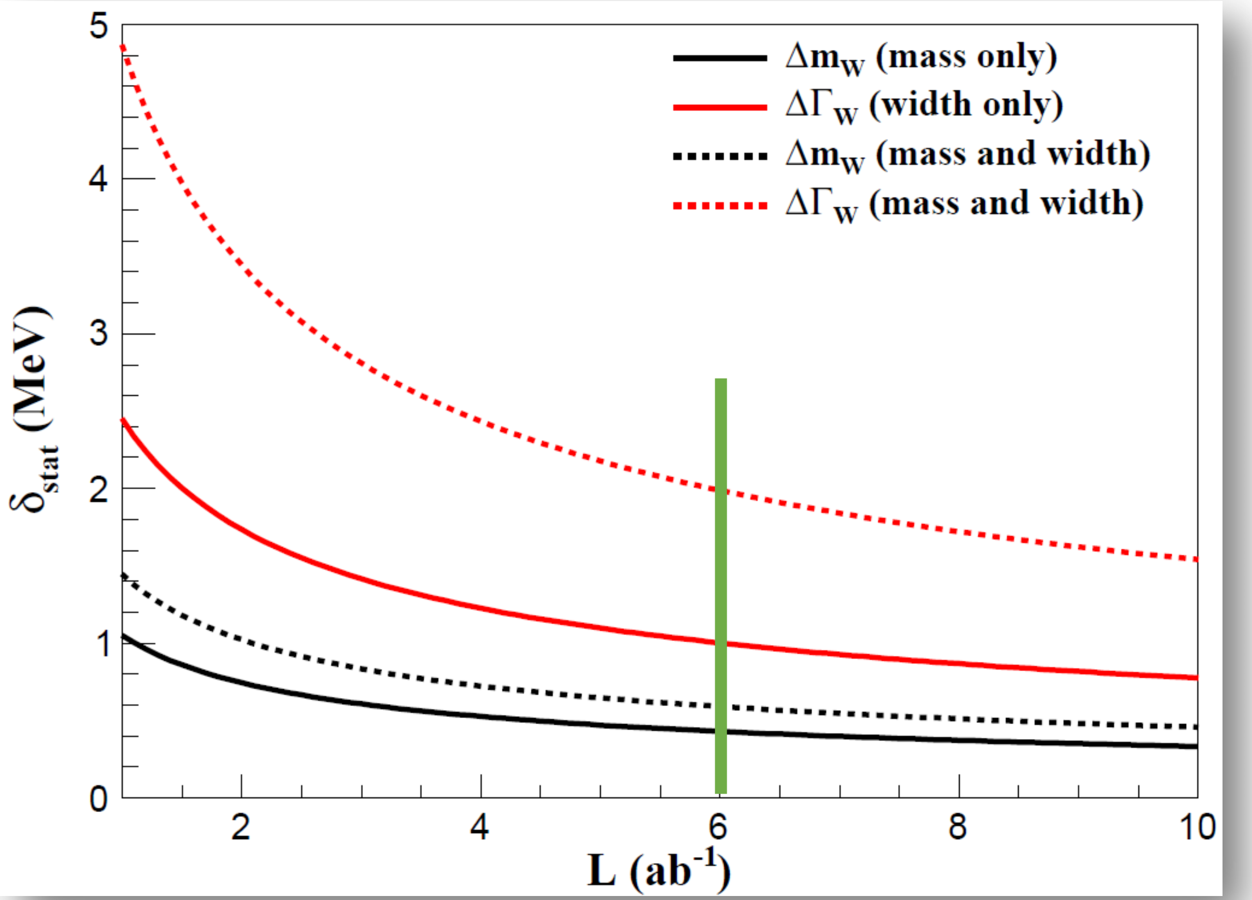
and the covariance matrix

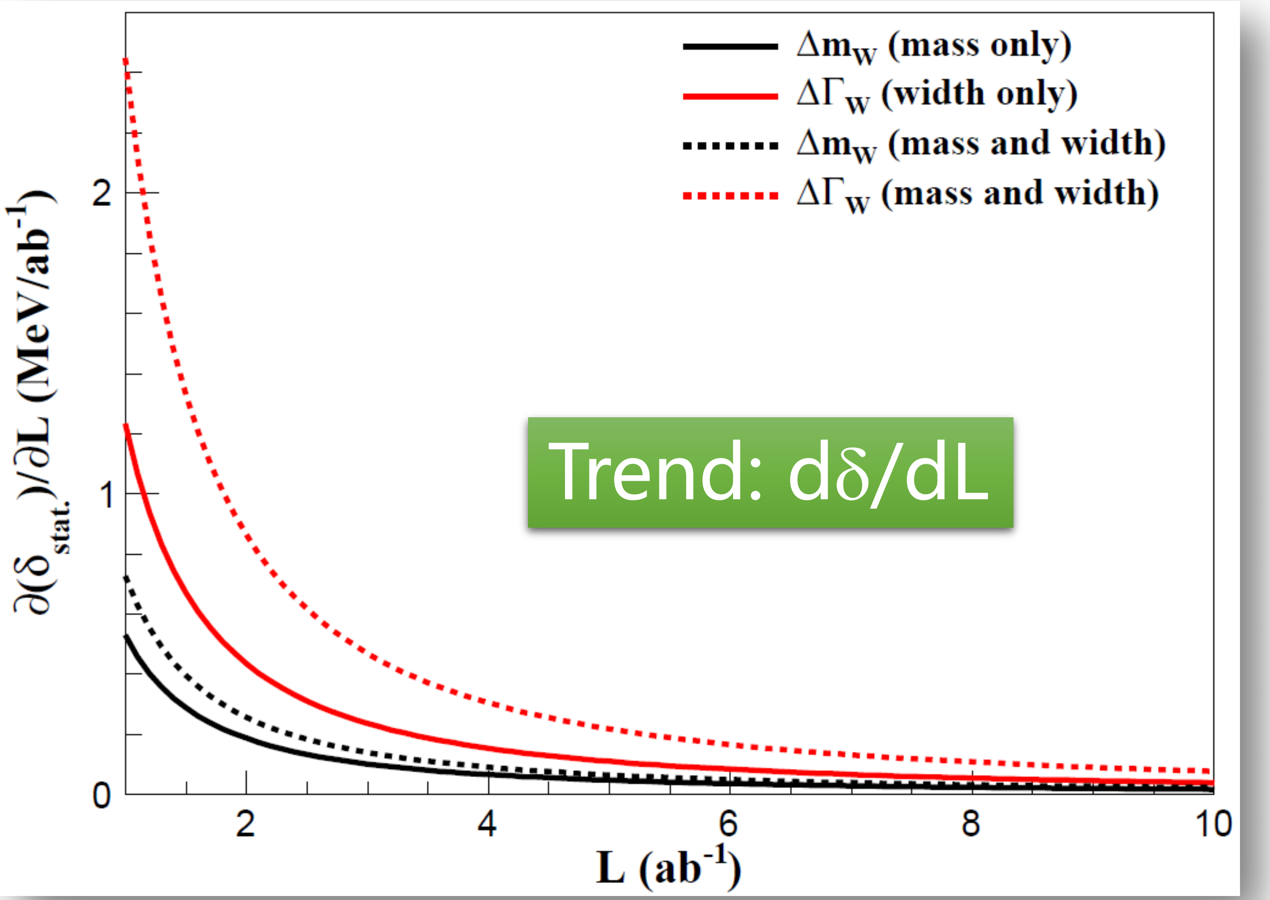
$$V = \frac{1}{2} \times \begin{pmatrix} \frac{\partial^2 \chi 2}{\partial m_W^2} & \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} \\ \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} & \frac{\partial^2 \chi 2}{\partial m^2_{\Gamma_W}} \end{pmatrix}^{-1} = \sum_i \begin{pmatrix} \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}}\left(\frac{\partial \sigma}{\partial m_W}\right)^2 & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} \\ \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}}\left(\frac{\partial \sigma}{\partial m_W}\right)^2 \end{pmatrix}^{-1}$$

- ➢When the number of parameters reduce to 1:

$$\Delta m_W = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

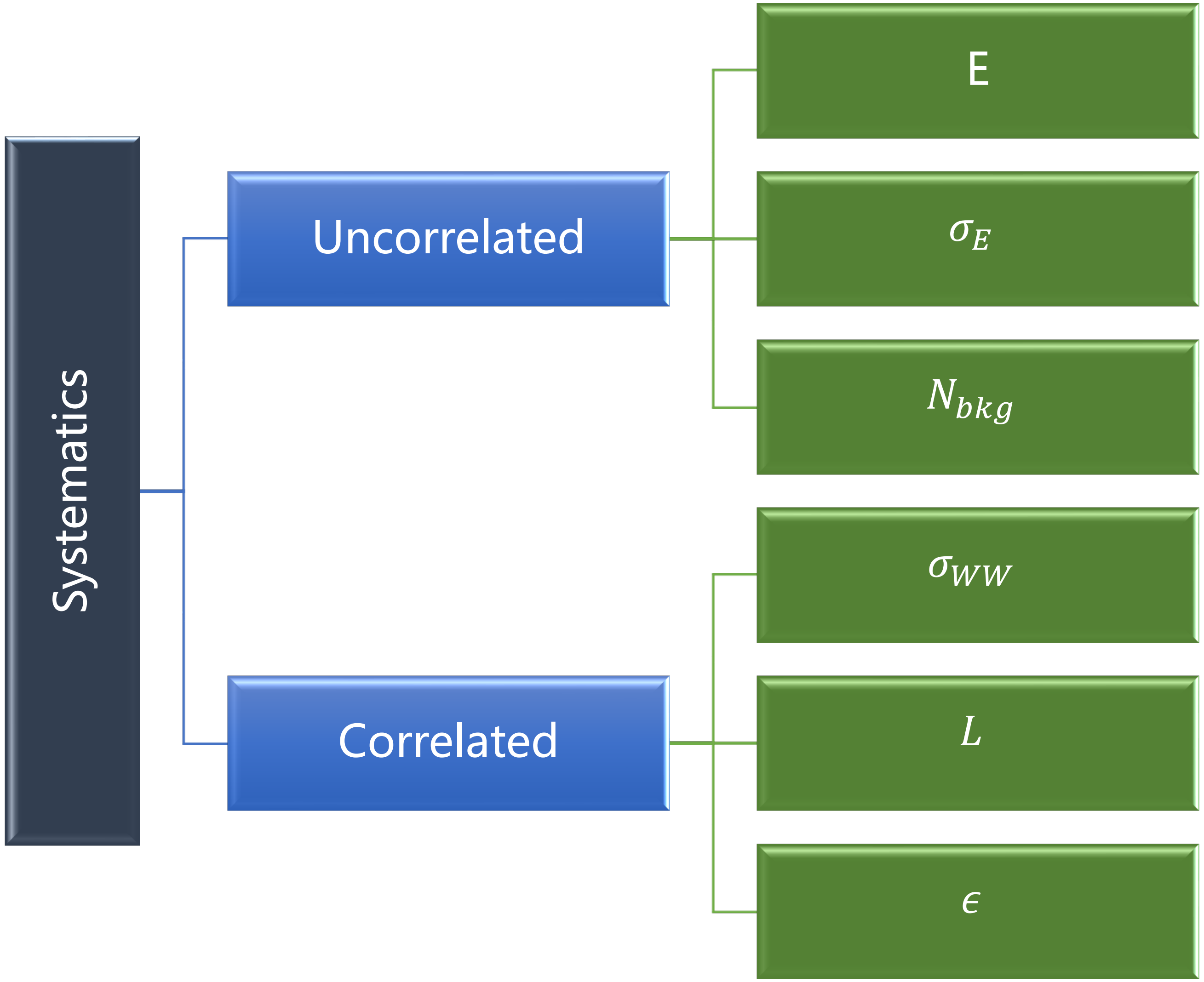
Statistical uncertainties vs $\int \mathcal{L}$





Systematics matter in this case, let's check …

Systematic uncertainty



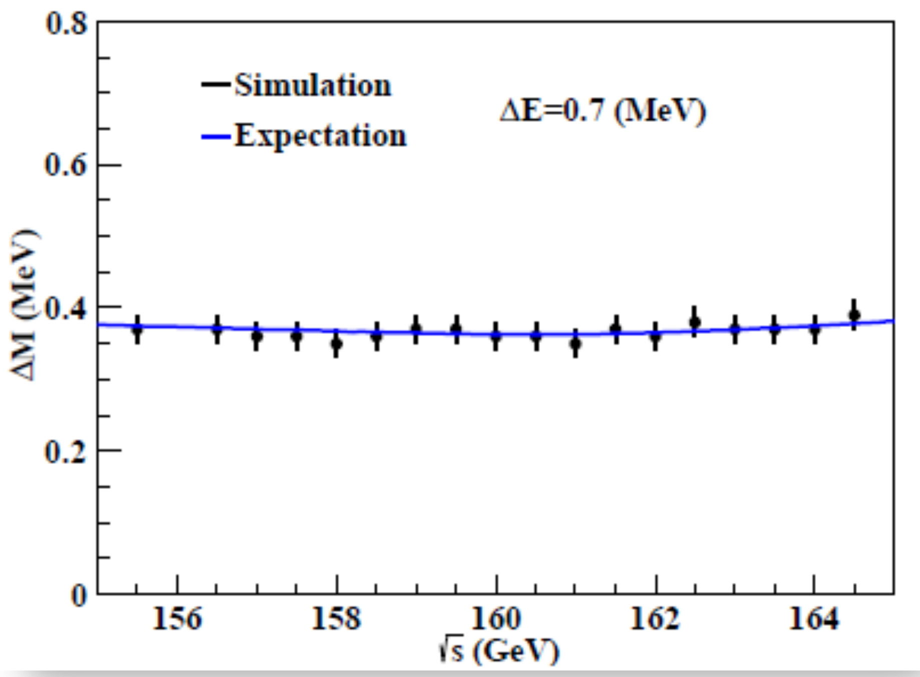
Energy calibration

- With ΔE, the total energy becomes:

$$E = G(E_p, \Delta E) + G(E_m, \Delta E)$$

(two beams)

- $\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \frac{\partial \sigma_{WW}}{\partial E} \Delta E$
- **The Δm_W proportional to ΔE,**
- **Almost independent on $\sqrt{s}$.**



Energy spread and the W width

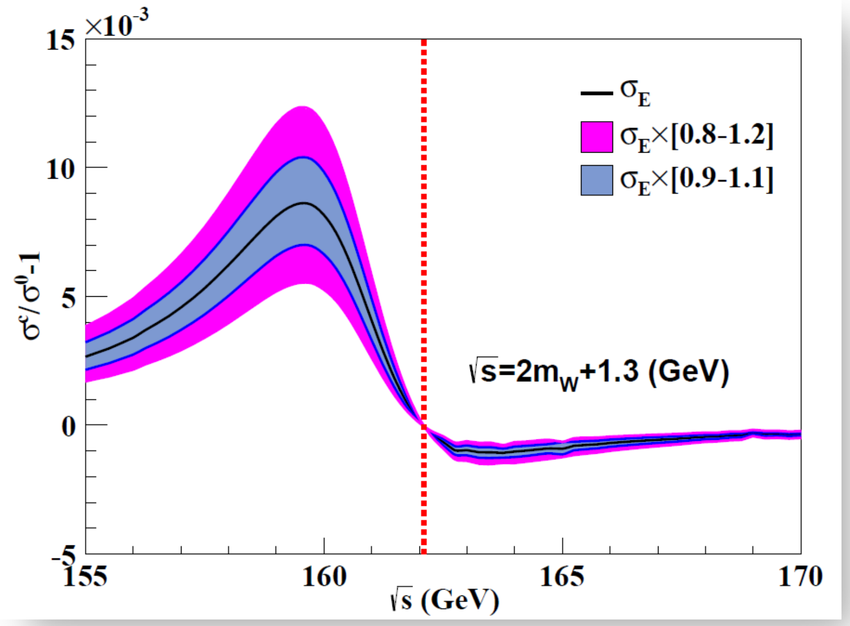
Two special Energy points

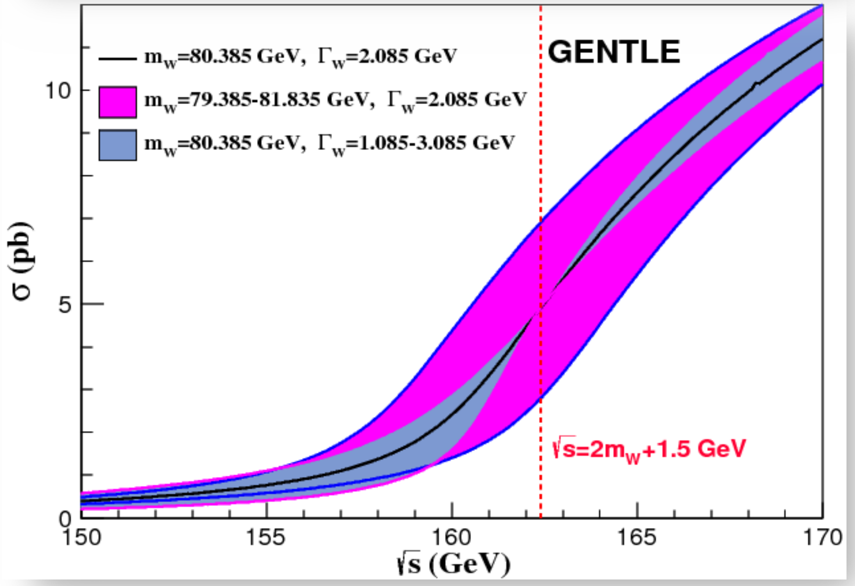
- With E_{BS}, the σ_{WW} becomes:

$$\sigma_{WW}(E) = \int_0^{\infty} \sigma_{WW}(E')\times G(E,E')dE'$$

$$= \int \sigma(E')\times \frac{1}{\sqrt{2\pi}\delta_E} e^{\frac{-(E-E')^2}{2\sigma_E^2}} dE'$$

- **The m_W insensitive to δ_E at around 162.3 GeV**
- **Similar for the W width at ~ 162.5 GeV**





Backgrounds

The effect of background are in two different ways

1. Stat. part: $\Delta m_W(N_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{\sqrt{L\epsilon_B\sigma_B}}{L\epsilon}$

2. Syst. part: $\Delta m_W(\sigma_B) = \frac{\partial m_W}{\partial \sigma_{WW}} \cdot \frac{L\epsilon_B\sigma_B}{L\epsilon} \cdot \Delta\sigma_B$

With L=3.2ab^{-1}, $\epsilon_B\sigma_B = 0.3$ pb, $\Delta\sigma_B = 10^{-3}$:

$\boldsymbol{\Delta m_W(N_B)} \sim$ **0.2 MeV**, $\epsilon \cdot P$ and σ_B are combined.

Improving MC and event selection could reduce it

The S/B ratios at different energies are taken into account in the study

Correlated sys. uncertainty

- The correlated sys. uncertainty includes: ΔL, $\Delta \epsilon$, $\Delta \sigma_{WW}$...
- Since $N_{obs} = L \cdot \sigma \cdot \epsilon$, these uncertainties affect σ_{WW} in the same way.
- We use the total correlated sys. uncertainty in the optimization:

$$\delta_c = \sqrt{\Delta L^2 + \Delta \epsilon^2}$$

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c \text{ , } \Delta \Gamma_W = \frac{\partial \Gamma_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Correlated sys. uncertainty

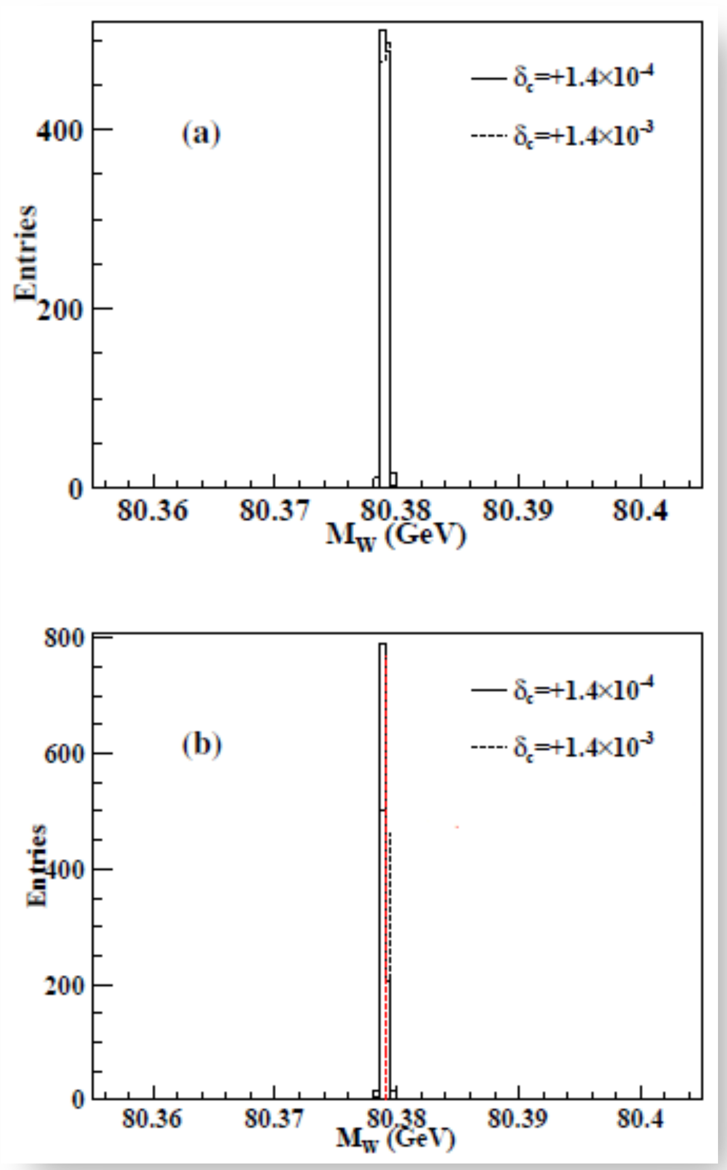
To consider the correlation, the scale factor method is used,

$$\chi^2 = \sum_i^n \frac{(y_i - h\cdot x_i)^2}{\delta_i^2} + \frac{(h-1)^2}{\delta_c^2},$$

where y_i, x_i are the true and fit results, h is a free parameter, δ_i and δ_c are the independent and correlated uncertainties.

For the Gaussian consideration, the scale factor can reduce the effect.

For the non-Gaussian case, the bias of the m_W is negligible



Optimization of the experiment

Data taking scheme

One point

- Smallest Δm_W, $\Delta\Gamma_W$ (stat.)
- Large sys. uncertainties
- Only for m_W or Γ_W, without correlation

Two points

- Measure m_W and Γ_W simultanously
- Without the correlation

Three points or more

- Measure m_W and Γ_W simultaneously, with the correlation

One single point (only $\boldsymbol{m_W}$ measured)

There are two special energy points :

- The one most statistical sensitivity to m_W:

  Δm_W(stat.) ~0.59 MeV at E=161.2 GeV

  ($\Delta\Gamma_W$ and ΔE_{BS} significant)

- The one Δm_W(stat)~0.65 MeV at $E \approx$ 162.3 GeV

  ($\Delta\Gamma_W, \Delta E_{BS}$ negligible)

$\sqrt{s}$(GeV)	161.2	162.3
E	0.36	0.37
σ_E	0.20	-
σ_B	0.17	0.17
δ_c	0.24	0.34
Γ_W	7.49	-
Stat.	0.59	0.65
Δm_W(MeV)	7.53	0.84

Assuming $\Delta L\ (\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$, ΔE=0.7MeV, $\Delta\sigma_E$=0.1, $\Delta\Gamma_W$=42MeV)

Two energy points

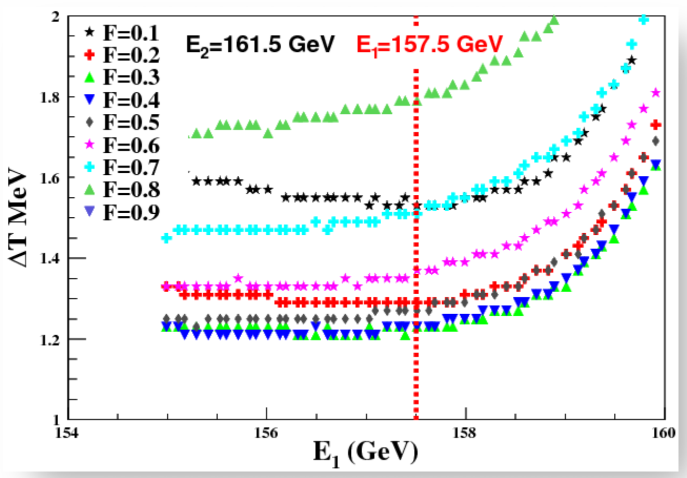
- To measure Δm_W and $\Delta\Gamma_W$, optimize the energies and the luminosity fractions:

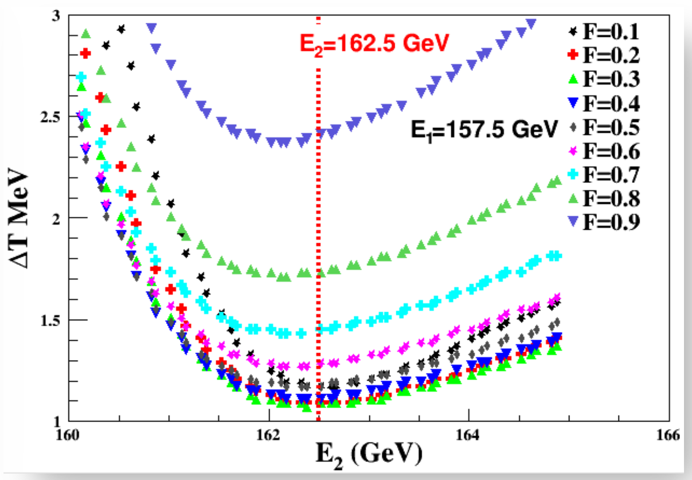
  1. $E_1, E_2 \in [155,\ 165]$ GeV, $\Delta E = 0.1$ GeV
  2. $F \equiv \left(\frac{L_1}{L_2}\right) \in (0, 1)$, $\Delta F = 0.05$

- Define the objective function: $T = \mathrm{m_W} + 0.1\Gamma_W$ to optimize the parameters (assuming m_W is more important than Γ_W).

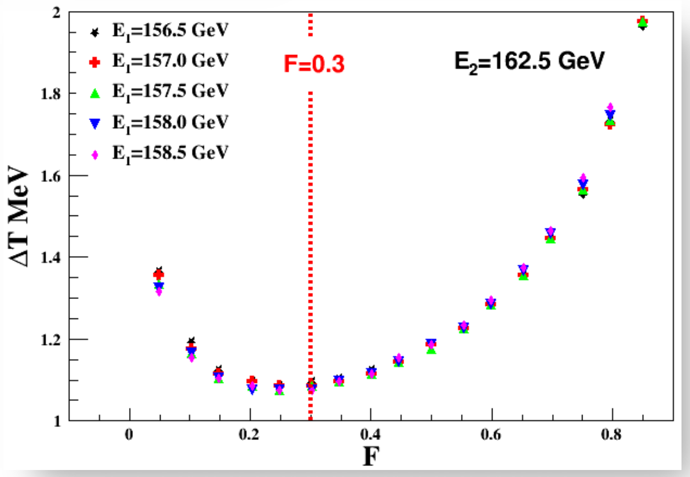
Two energy points

- Simple scan used here
  - E_1=157.5 GeV (~ best for width)
  - E_2=162.5 GeV (~ $\frac{\partial \sigma_{WW}}{\partial \Gamma_W}=0$, $\frac{\partial \sigma_{WW}}{\partial E_{BS}}=0$)
  - F=0.3





Assuming:
$\Delta L(\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$ $\sigma_E = 1\times 10^{-3}, \Delta E = 0.7$MeV , $\Delta\sigma_E = 0.1\%$



(MeV)	E	σ_E	σ_B	δ_c	Stat.	Total
Δm_W	0.38	-	0.21	0.33	0.80	0.97
$\Delta \Gamma_W$	0.54	0.56	1.38	0.20	2.92	3.32

Taking data at three or more energy points

Similar optimization procedure

E_1	157.5 GeV
E_2	162.5 GeV
E_3	161.5 GeV
F_1	0.3
F_2	0.9

$\Delta m_W \sim 0.98$ MeV
$\Delta\Gamma_W \sim 3.37$ MeV

$\Delta L(\Delta\epsilon) < 10^{-4}, \Delta\sigma_B < 10^{-3}$
$\sigma_E = 1\times10^{-3}, \Delta E = 0.7$MeV
$\Delta\sigma_E = 0.1$

- Study shows the above energy points are optimal
- Certainly, more points, more robust
- From point of view of experiment, extra point(s) below threshold is necessary for background study, etc

Summary of uncertainties

Data-taking scheme	mass or width	δ_{stat} (MeV)	δ_{sys} (MeV)				Total (MeV)
			ΔE	$\Delta\sigma_E$	δ_B	δ_c	
One point	Δm_W	0.65	0.37	–	0.17	0.34	0.84
Two points	Δm_W	0.80	0.38	–	0.21	0.33	0.97
	$\Delta\Gamma_W$	2.92	0.54	0.56	1.38	0.20	3.32
Three points	Δm_W	0.81	0.30	–	0.23	0.29	0.98
	$\Delta\Gamma_W$	2.93	0.52	0.55	1.38	0.20	3.37

Theoretical uncertainties

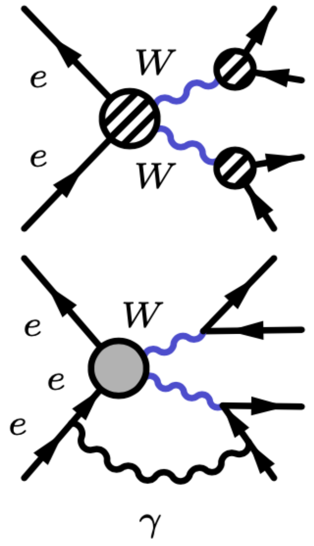
- W-pair production cross section
- + High order corrections
- Initial Radiative Correction (ISR)
- …

Borrowed from Christian Schwinn's slides

W-pair production

NLO calculations in Double Pole Approximation

- Implemented in Monte-Carlo programs for LEPII:
  - Berends et al. 98;
  - Denner et al. 99
  - RacoonWW (Denner et al. 99),
  - YFSWW (Jadach et al. 99)

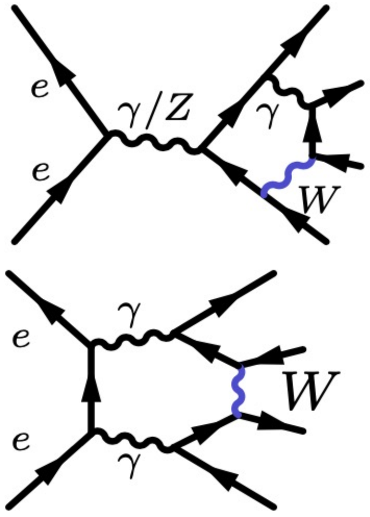


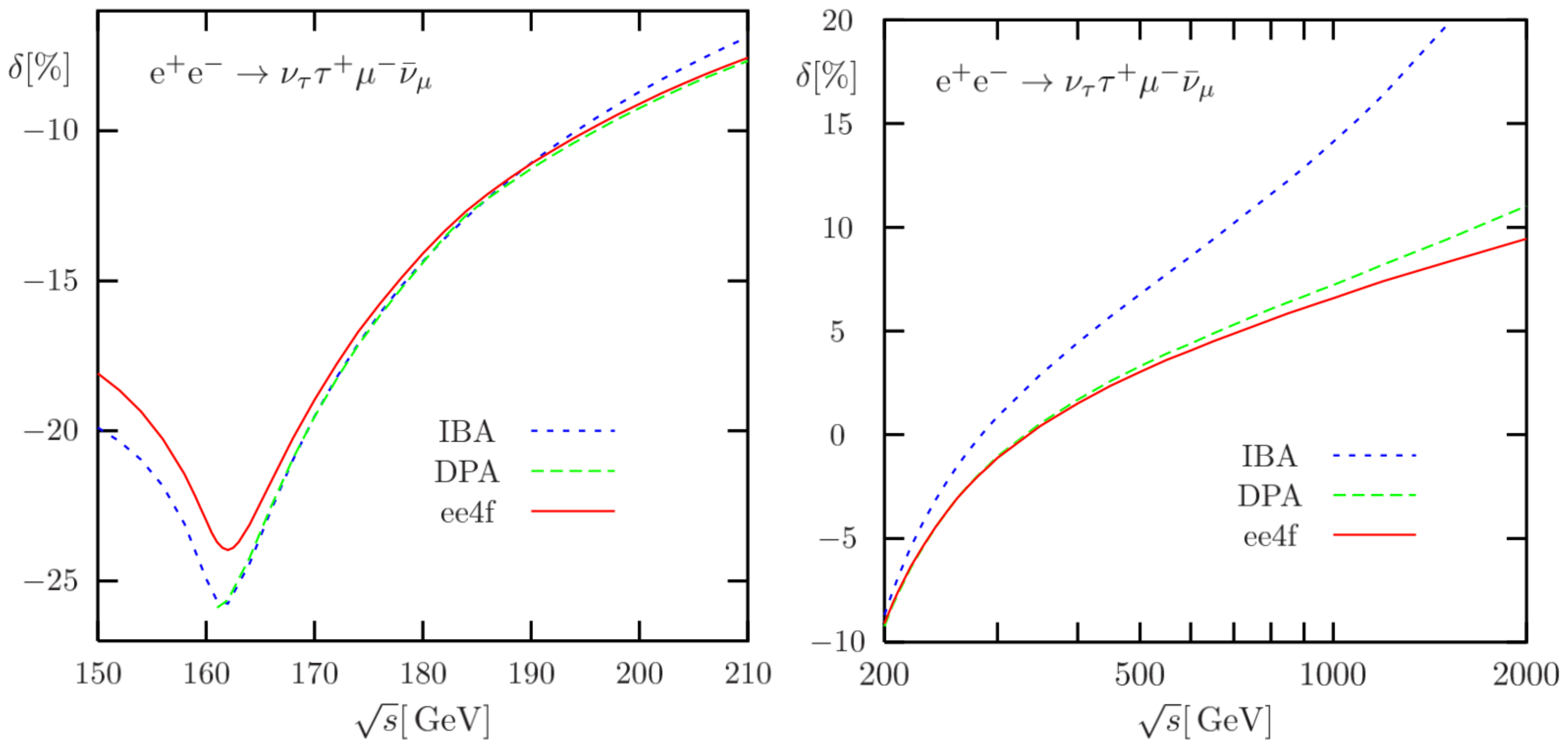
Theory developments after LEP2:

- Complete NLO calculation for charged current e+e− → 4f (Denner et al. 05)
- Log-enhanced NNLO corrections for hat(s) ≫ M_W (Kuhn et al. 07)
- NLO and leading NNLO correction in threshold expansion (Beneke et al. 07, Actis et al. 08)

W-pair production

Full NLO calculation for e+e− → 4f (Denner, Dittmaier, Roth, Wieders 05)





Sizable correction

Beyond NLO at threshold

Enhanced corrections in threshold limit $\beta = \sqrt{1 - \frac{4M_W^2}{s}} \to 0$:

soft threshold logarithms $\sim (\alpha \log^2 \beta)^n$, Coulomb correction $\sim (\alpha/\beta)^n$

EFT approach: (Beneke/Falgari/CS/Signer/Zanderighi 07)

expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \frac{k_W^2 - M_W^2}{M_W^2}$

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)			
$\sqrt{s}$ [GeV]	NLO_{EFT}	NLO_{ee4f} [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.38(4)	118.77	0.44	0.15
170	399.9(2)	404.5(2)	0.2	1.6

- non-resonant NLO corrections included in NLO_{ee4f} but not in NLO_{EFT}: $\Delta\sigma_{WW} \sim 1\%$
- leading NNLO corrections: $\Delta\sigma_{WW} \sim \mathcal{O}(‰) \Rightarrow [\delta M_W]_{\text{C2}} < 4\,\text{MeV}$

ISR

Important issue at electron positron colliders

ISR: resum leading logs

$$\beta_e = \frac{2\alpha}{\pi}\left(2\log\left(\frac{2M_W}{m_e}\right) - 1\right)$$

in electron structure functions: (Skrzypek 92)

$$\sigma_{\text{NLO}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{\text{LL}}(x_1)\Gamma_{ee}^{\text{LL}}(x_2)\left(\sigma_{\text{tree}} + \Delta\hat{\sigma}_{\text{NLO}}\right)$$

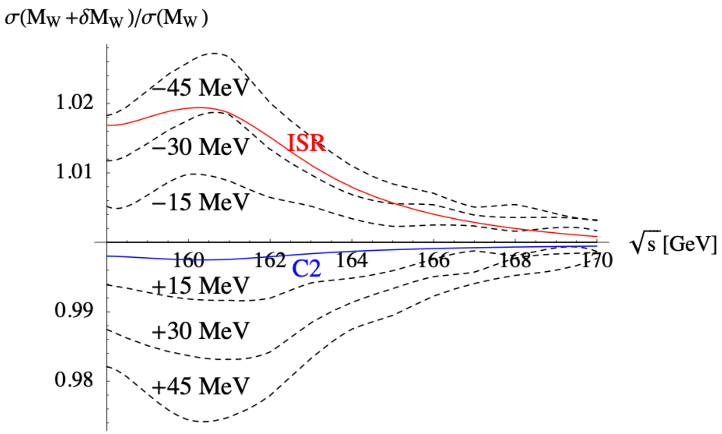
Estimate missing NLL $\mathcal{O}(\alpha\beta_e)$:

ISR for tree only $\Leftrightarrow$ also for NLO

Uncertainty $\sim 2\%$ at threshold

$\Rightarrow \quad [\delta M_W]_{\text{ISR}} \approx 30\,\text{MeV}$

$\Rightarrow$ NLL resummation important

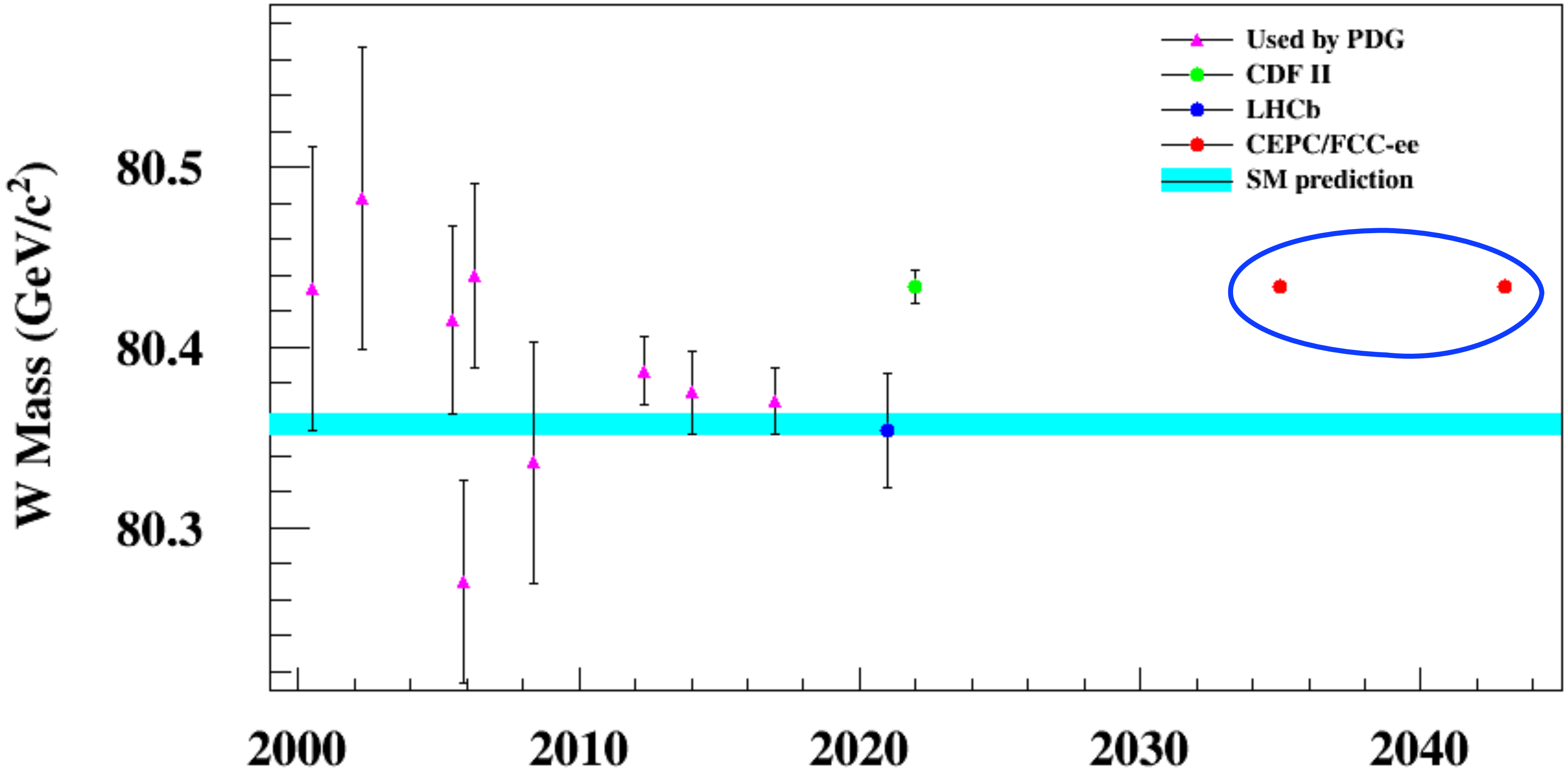


Summary

- m_W (Γ_W) could be measured with great precisions by threshold scan and direct reconstruction
- With most systematics taken into account except the theoretical ones, 1 MeV and 3 MeV uncertainties for W mass and width could be achieved, respectively.
- **Challenges for theorists : σ_{WW} of ~O(0.01)%**
  - **NNLO EW corrections to e+e− → 4f: needs new approaches**
  - **ISR uncertainty: needs NLL treatment**
- **Challenges for experimentalists**
  - **Beam energy calibration, Duan Zhe and Yongsheng will continue the story …**
  - **…**

Very, very optimistic perspective

Defend or kill the SM?   It’s a problem.



Thanks !

Correlated sys. uncertainty

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma_{WW} \cdot \delta_c$$

Two ways to consider to effect:

(a) Gaussian distribution

$$\sigma_{WW} = G(\sigma_{WW}^0, \delta_c \cdot \sigma_{WW}^0)$$

(b) Non-Gaussian (will cause shift)

$$\sigma_{WW} = \sigma_{WW}^0 \times (1 + \delta_c)$$

With $\delta_c = +1.4 \cdot 10^{-4} (10^{-3})$ at 161.2GeV

$\Delta m_W \sim 0.24$MeV (3MeV)