Beam energy calibration with inverse-Compton scattering method

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Outline

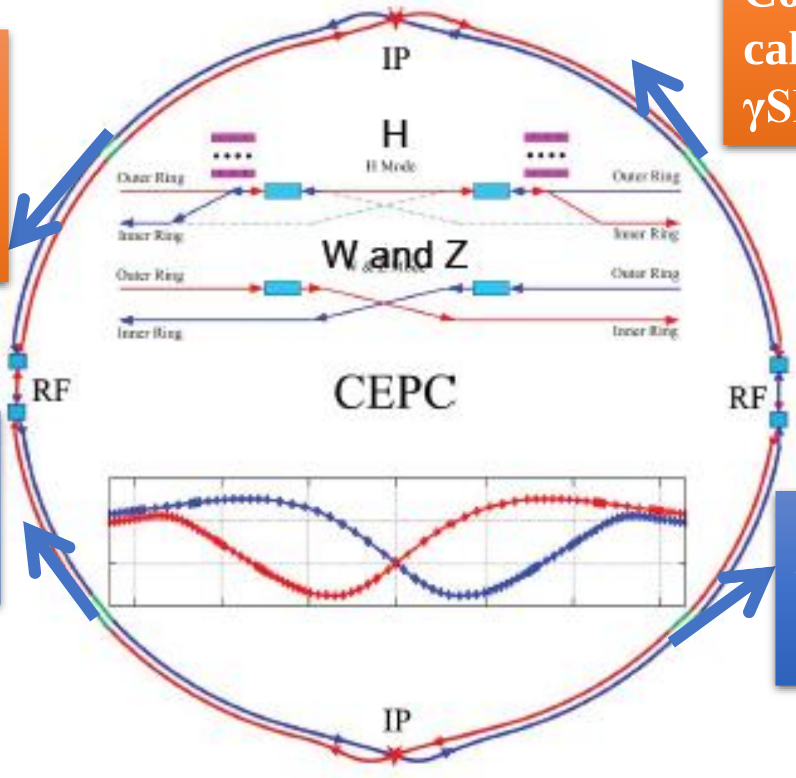
1. Configuration of CEPC/FCC
2. Laser-Compton scattering method
3. Microwave-Compton scattering method

Configuration of beam-energy calibration system @CEPC/FCC

- **Laser-Compton Scattering method**
- **Microwave-Compton scattering method**
- **γSR beamline and applications**

Microwave-Compton beam calibration system+ γSR beamline

Microwave-Compton beam calibration system+ γSR beamline



Laser-Compton beam calibration system

Laser-Compton beam calibration system

CEPC collider ring (100km)

Outline

1 Configuration of CEPC/FCC

2 Laser-Compton scattering method

3 Microwave-Compton scattering method

Configuration

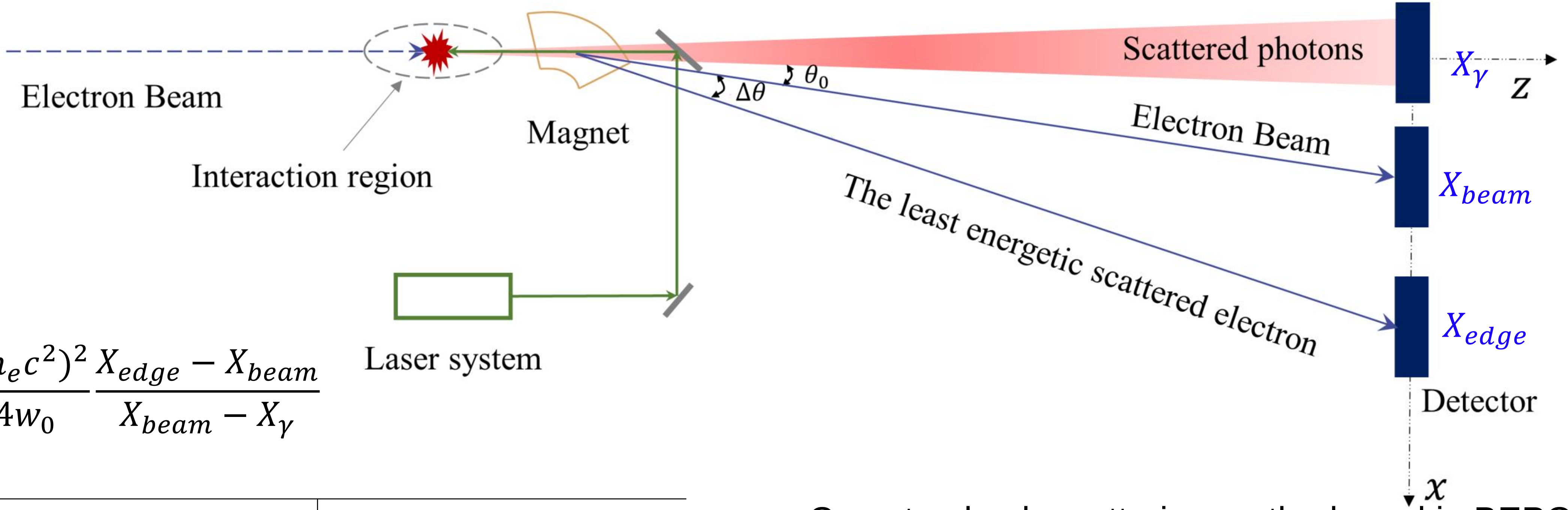
Principle

测到的与对撞点能量的关系

能量精度

Laser-Compton Method of calibration of beam energy

➢ Method: Compton back-scattering combining a bending magnet



$$E_{beam} = \frac{(m_e c^2)^2}{4w_0} \frac{X_{edge} - X_{beam}}{X_{beam} - X_\gamma}$$

Electron beam		**Nd:YAG Laser system**	
Energy (GeV)	120	λ(nm)	532
N_e	15×10^{10}	Energy(J)	0.1
Collision angle α		~ 2.35 mrad	
Compton scattering cross section		202 mb	

- Compton back-scattering method used in BEPC by measuring the energy of scattered photons with accuracy is 2×10^{-5}.

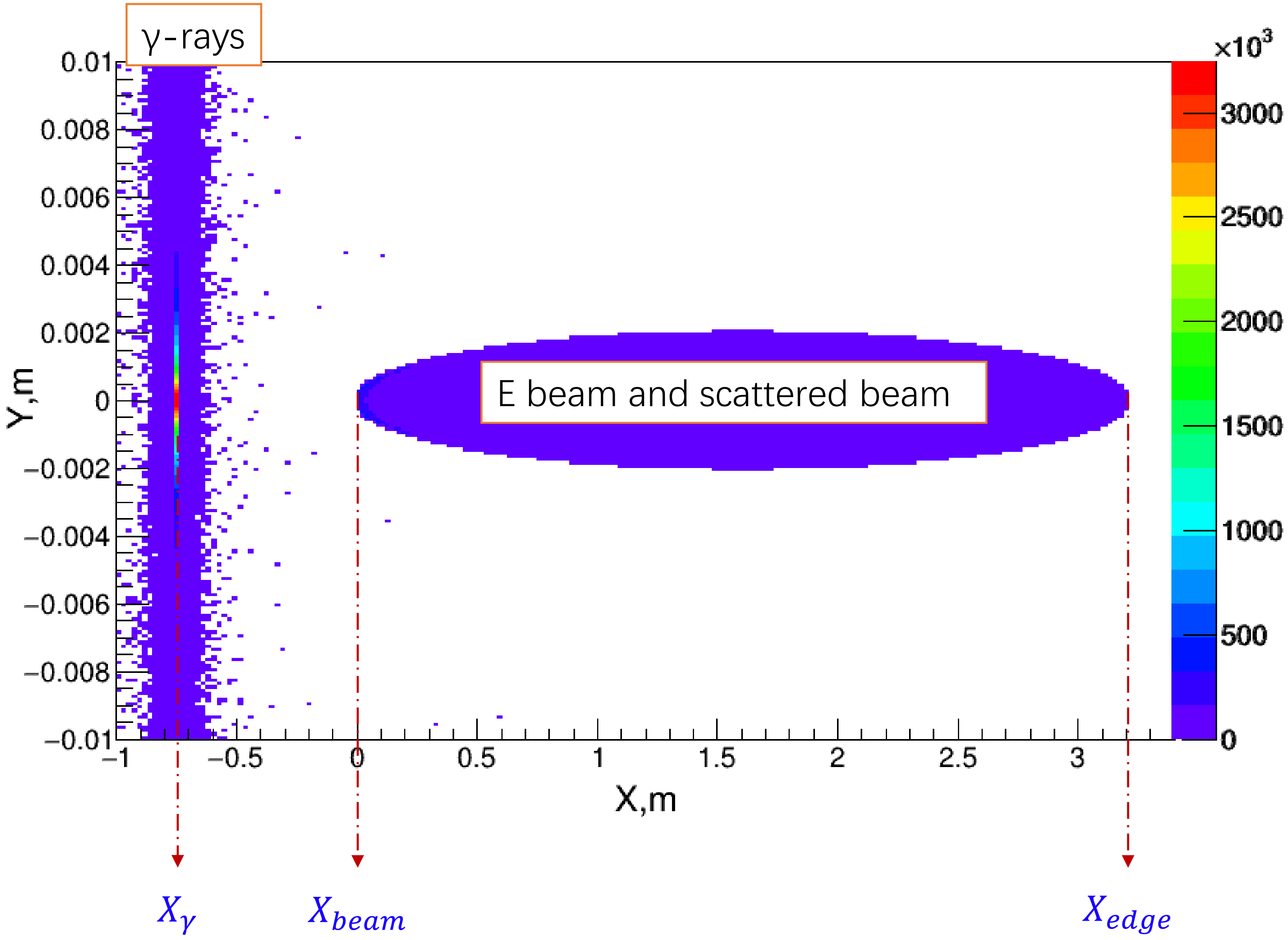
  https://doi.org/10.1016/j.nima.2011.08.050

- The technique is “non-destructive”: $\sim 10^6$ Compton scattered particles in one collision.

Spatial distribution of scattered particles

- Beam energy can be calibrated by:
  - Position of the main electron beam particles(X_{beam}).
  - Position of scattered photons(X_γ).
  - Position of the scattered electrons with the least energy(X_{edge}).

$$E_{beam} = \frac{(m_e c^2)^2}{4w_0} \frac{X_{edge} - X_{beam}}{X_{beam} - X_\gamma}$$

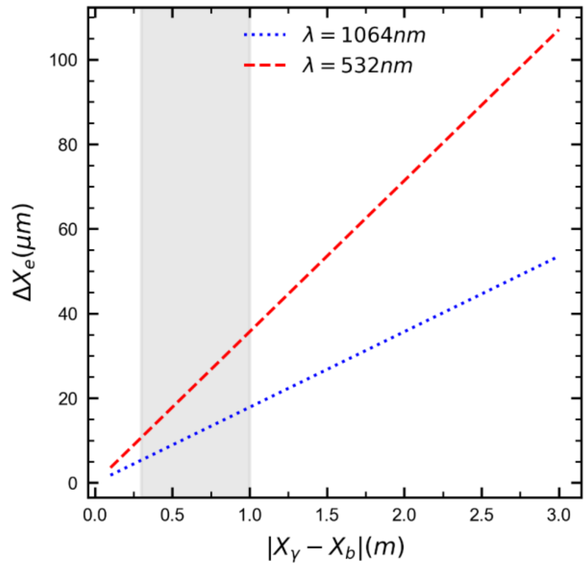


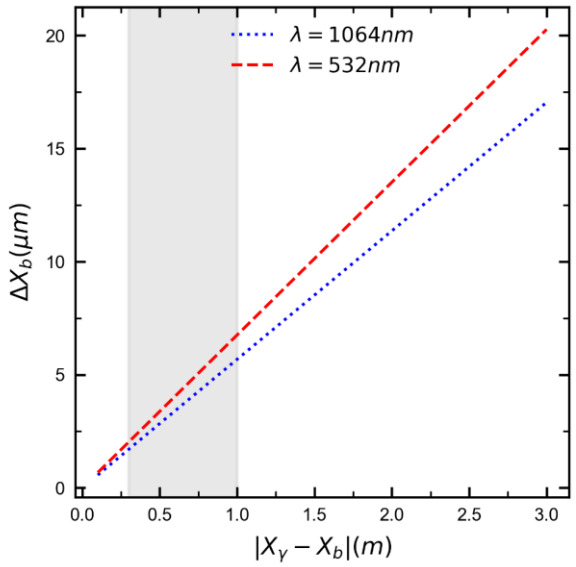
arXiv:1803.09595, 2018.

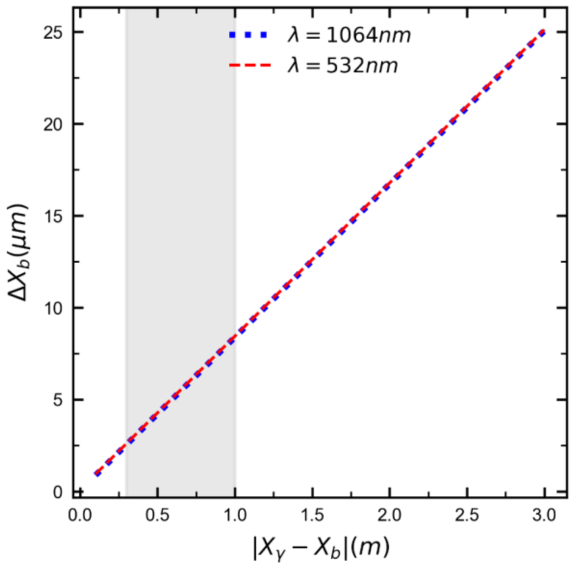
Requirement of measurement accuracy

$1MeV$ ← $$\frac{\Delta E_{beam}}{E_{beam}} = \sqrt{(\frac{\Delta X_{edge}}{|X_{edge} - X_{beam}|})^2 + (\frac{|X_\gamma - X_{edge}|\Delta X_{beam}}{|X_{beam} - X_\gamma||X_{edge} - X_{beam}|})^2 + (\frac{\Delta X_\gamma}{|X_{beam} - X_\gamma|})^2}$$

- The requirement for the measurement of positions: ΔX_{edge}, ΔX_{beam}, ΔX_γ

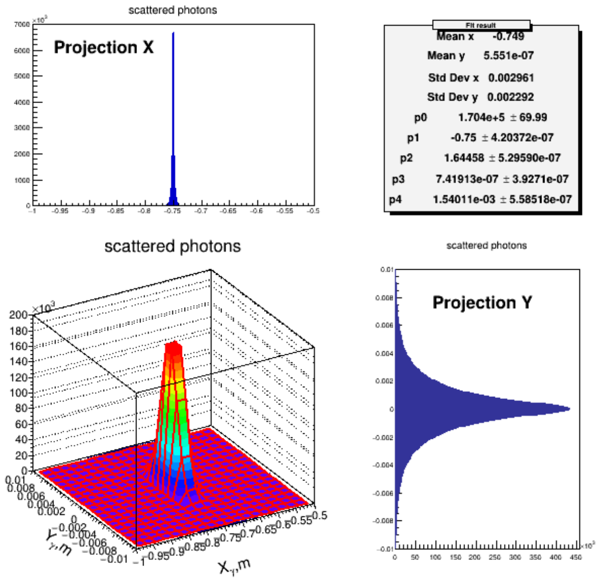


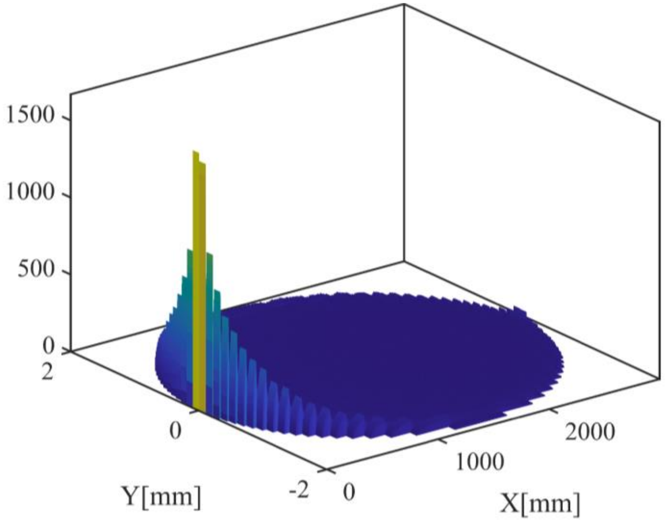


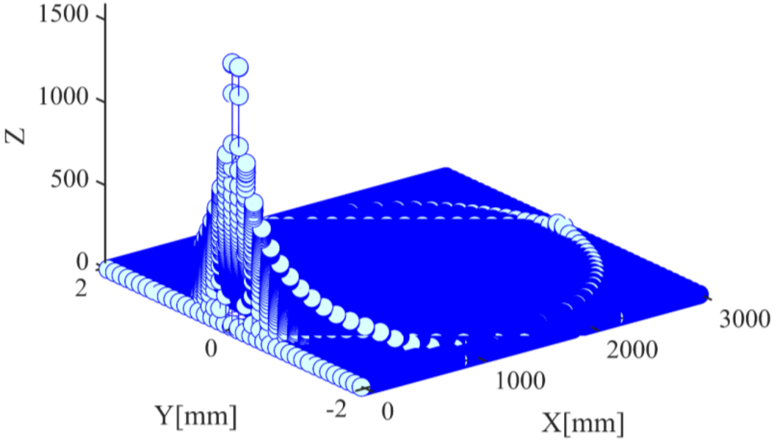


Statistical error

- The distance between electron-laser interaction point(IP) and detector is L_1
- The distance between magnet and detector is L_2
- Tens of seconds of data taking is necessary.





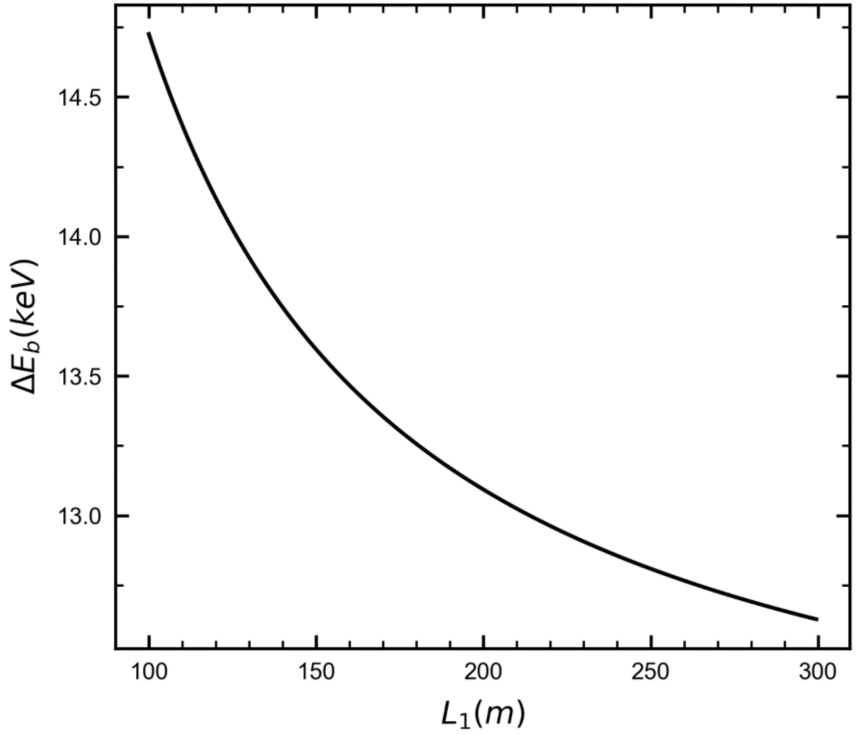


	$L_1 = 100m, L_2 = 80m$	$L_1 = 200m, L_2 = 180m$	$L_1 = 300m, L_2 = 280m$
Pixel size	$100\mu m \times 50\mu m$	$500\mu m \times 100\mu m$	$2mm \times 200\mu m$
$X_\gamma + \Delta X_\gamma[mm]$	-299.762±8.905$\times 10^{-5}$	-674.460±4.475 $\times 10^{-5}$	-1049.16±1.134$\times 10^{-4}$
$X_{beam} + \Delta X_{beam}[mm]$	-0.0011±1.8492 $\times 10^{-4}$	-0.0009±7.3215$\times 10^{-4}$	-0.0015±0.0018
$X_{edge} + \Delta X_{edge}[mm]$	1284.1928±0.0037	2889.4319±0.0132	4494.6437±0.0314
$E_b[GeV]$	119.9999	120.0003	119.9991
$\Delta E_b[MeV]$	0.356	0.573	0.875

Systematic uncertainty

- Considering the measurement of magnet strength and drift distance.
- The relative error is assumed to be $\Delta B/B \approx 10^{-4}$ and $\Delta L/L \approx 10^{-4}$

$$\Delta E = \sqrt{\Delta E_B^2 + \Delta E_{L_1}^2 + \Delta E_{L_2}^2}$$



- More systematic error sources need to be considered.
- Extrapolating the center-of-mass energy needs to be discussed later.

Comparison of the key parameters for different models in CEPC

	Higgs mode	Z mode	WW scan	$t\bar{t}$ scan
E_{beam}/GeV	120	45	80	175
X_{edge}/m	6.16352	9.29686	7.10343	5.57276
X_{beam}/m	1.87935	5.00178	2.81903	1.28868
$\delta X_{edge}/m$	2.6×10^{-5}			
$\delta X_{beam}/m$	6×10^{-8}			
$\delta E_{beam}/MeV$	1.0	0.3	0.6	1.8

- The statistical uncertainties of beam energy are not included here

doi.10.1063/1.5132975

Measuring the center-of-mass energy

$$< \sqrt{s} >= 2\sqrt{E_+E_-}cos\frac{\alpha}{2}$$

- Potential corrections of c.m. energy
  - The correlated effects of dispersion
  - Collision offsets
  - Difference between the electron and positron beams
- Beam energy uncertainties from surroundings
  - Tidal effect → collider orbit circumference
  - Railway → magnetic field

Ref: [1] Assmann R, LEP Energy Working Group. Calibration of centre-of-mass energies at LEP 2 for a precise measurement of the W boson mass[J]. arXiv preprint hep-ex/0410026, 2004.
[2] Müller, Anke-Susanne. "Measurements of beam energy." (2009).
[3] Alain Blondel (Geneva U. and CERN and Paris U., VI-VII), Patrick Janot (CERN), Jörg Wenninger (CERN), Ralf Aßmann (DESY), Sandra Aumon (CERN) et al

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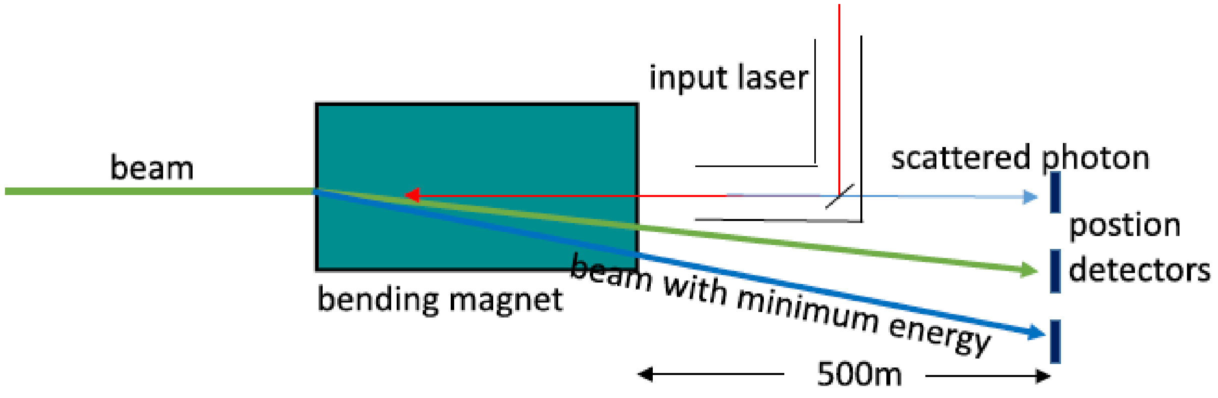
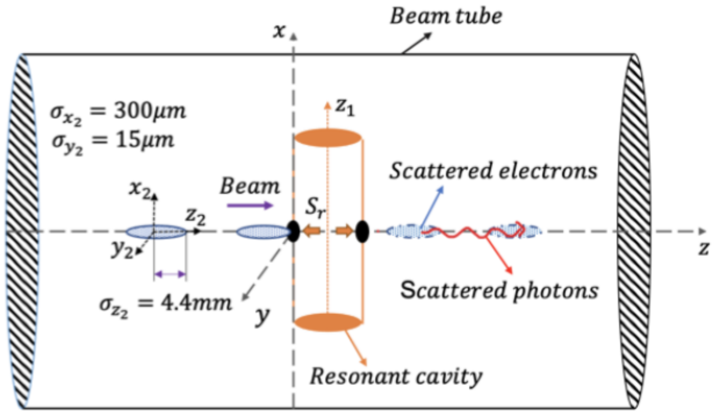
Laser Compton backscattering



With some proper corrections, the beam energy uncertainty of the Higgs mode is around 2 MeV.

- **Independent extraction device.**
- **Separately detect the positions of scattered electrons, scattered photons and unscattered beams.**

Microwave-beam Compton backscattering



Simple model of cavity and beam

- **Use synchrotron radiation lead wire.**
- **Detection of the maximum energy of scattered photons by a HPGe detector.**

If the beam energy is calibrated within 10MeV, it will be interesting and worth doing.

]G. Tang, S. Chen, Y. Chen, Z. Duan, and C. Zhang, Review of Scientific Instruments 91, 033109 (2020).; Meiyu Si, Yongsheng Huang,⁎, Shanhong Chen, et al., Nuclear Inst. and Methods in Physics Research, A 1026 (2022) 166216,

Microwave-beam Compton backscattering

Head-to-head collision $\alpha = \pi$:

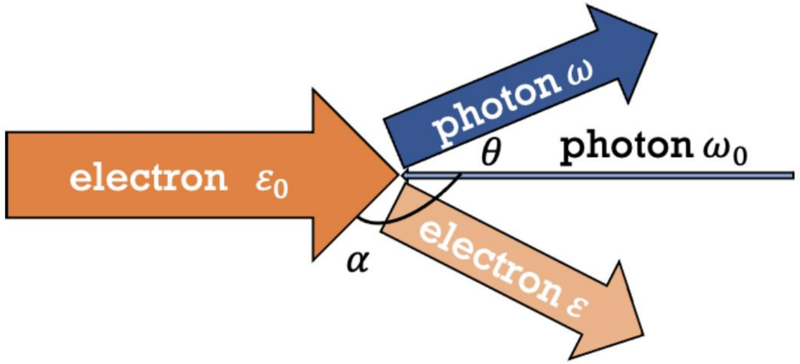


Figure 1. Compton backscattering process

Considering $\varepsilon_0 \gg m \gg \omega_0$

$$\omega_{max} = \frac{\varepsilon_0^2 sin^2(\frac{\alpha}{2}) + \frac{m^2}{4} cos\alpha}{\varepsilon_0 sin^2(\frac{\alpha}{2}) + \frac{m^2}{4\omega_0}}$$

$$\varepsilon_0 = \frac{\omega_{max}}{2}(1 + \sqrt{1 + \frac{m^2}{\omega_0 \omega_{max} sin^2(\frac{\alpha}{2})}})$$

Scattered photons:

$$\frac{dN_\gamma}{dt} = L\sigma \longrightarrow \frac{dN_\gamma}{d\omega dt} = L\frac{d\sigma}{d\omega}$$

Table I. CEPC parameters in Higgs mode.

	Higgs
Beam energy ε_0 (GeV)	120
Bunch number B	242
Particles/bunch $N_2(10^{10})$	15
Bunch spacing (ns)	680
Beam current I (mA)	17.4
Bending radius ρ (km)	10.7
Beam size $\sigma_{x_2}/\sigma_{y_2}(\mu m)$	200-450/5-20
Bunch lengthσ_{z_2}(mm)	4.4

The HPGe detector has a good calibration of gamma energy within 1 to 10MeV.

The energy of the scattered photons is chosen to be in the range of (8–20 MeV) compared with the synchrotron radiation background.

Choosing $\omega_{max} = 9MeV$

The energy of initial photons: $\omega_0 = 4.08 \times 10^{-5} eV$

The wavelength of initial photons : $\lambda = 3.04cm$

The frequency of initial photons : $f = 10GHz$ → **Microwave band!**

$E = \frac{hc}{\lambda}, f = \frac{c}{\lambda}$

System error:

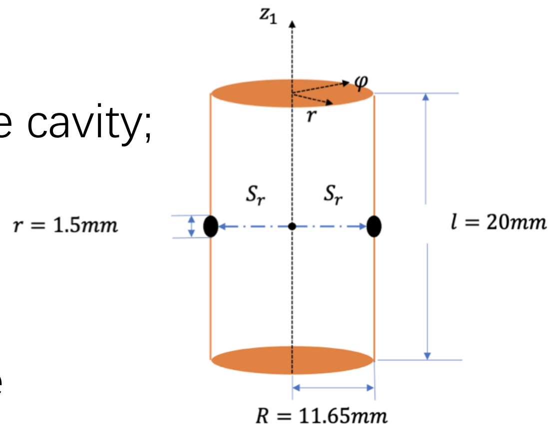
$$\delta\varepsilon_0 = \sqrt{(\frac{\partial\varepsilon_0}{\partial\omega_{max}})^2(\delta\omega_{max})^2 + (\frac{\partial\varepsilon_0}{\partial\omega_0})^2(\delta\omega_0)^2 + (\frac{\partial\varepsilon_0}{\partial\alpha})^2(\delta\alpha)^2}$$

- **the laser positioning accuracy is up to 5×10^{-7};**
- **the stability of the high-frequency microwave source itself can reach $10^{-5} \sim 10^{-6}$;**
- **assuming the detector can reach the order of 10^{-4} under good calibration;**
- **The measurement accuracy of the beam energy can reach the 6MeV@120GeV (5×10^{-5})**

Resonant cavity

Choosing the TM_{010} mode of the standing wave cavity;

$$\lambda = \frac{2\pi}{K} = 2.613R$$

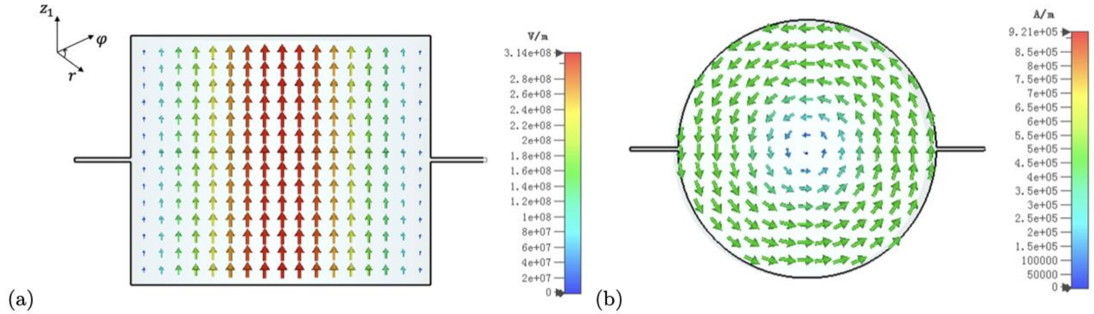


The electric field only has the longitudinal field in z1 direction, the magnetic field only has the transverse field in the ϕ direction.

$$E_{z_1} = E_m J_0(K_c r)e^{j\omega t}$$

$$H_\varphi = jE_m \frac{1}{\eta} J_1(K_c r)e^{j\omega t}$$

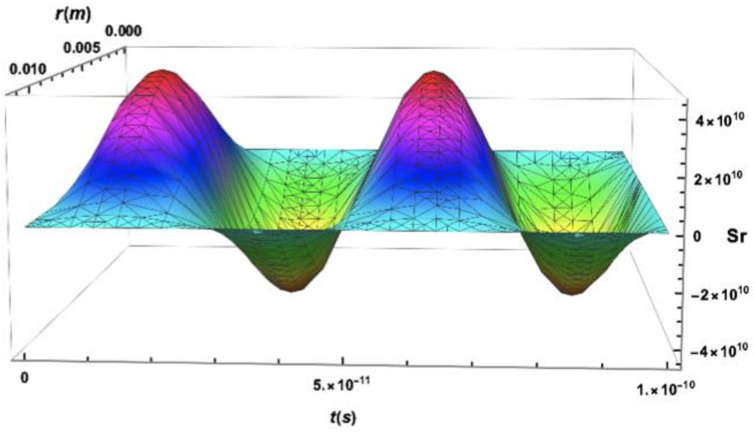
$$E_r = E_\varphi = H_r = H_z = 0$$



The Poynting vector:

$$S_r = -E_z \times H_\varphi = \frac{E_m^2 J_0(K_c r)J_1(K_c r)sin(\omega t)cos(\omega t)}{\eta}$$

The oscillation period T = 5 × 10^{-11}s = 50ps



System design

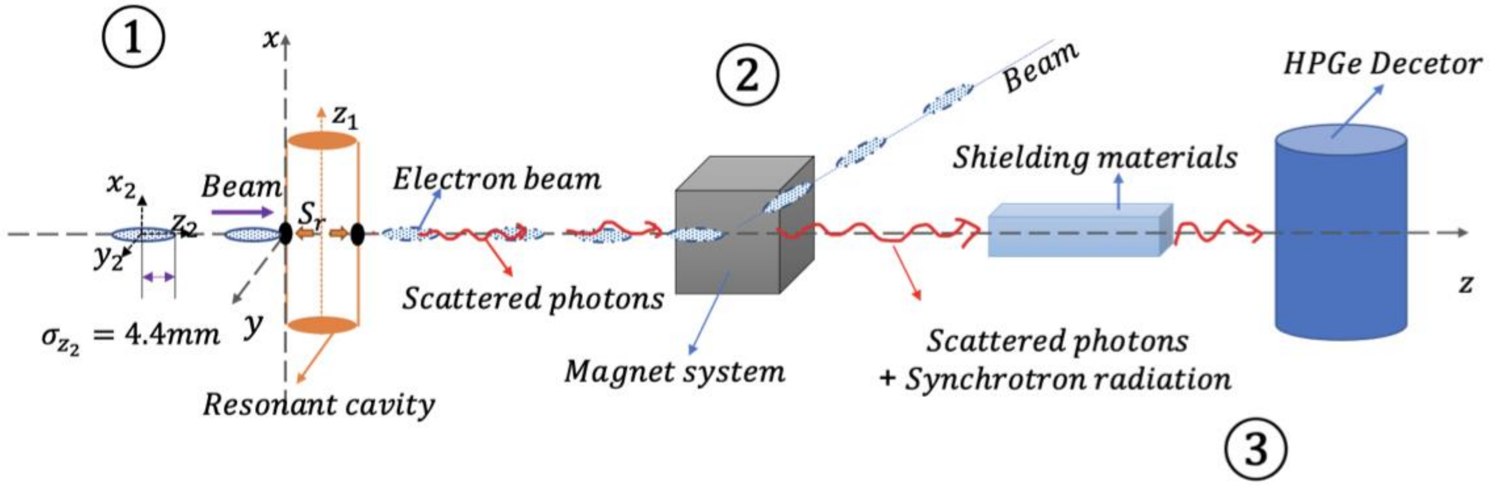
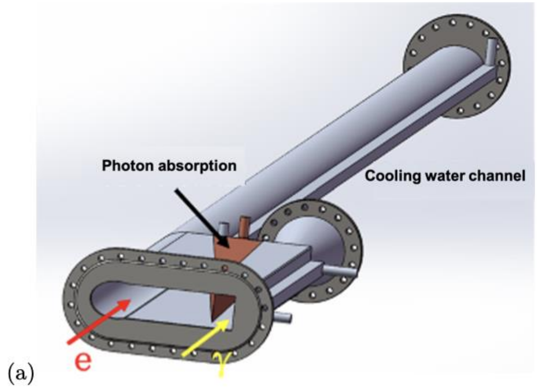


Figure 7. The design of the microwave measurement method.



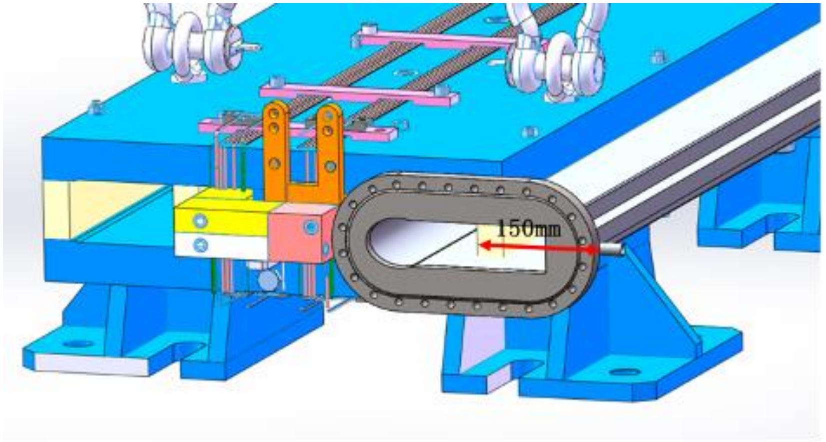


Figure 6. The separation system on CEPC between photons and electrons.

Calculation process

Interaction cross section

The Sunyaev-Zel'dovich effect (SZE) is a small spectral distortion of the cosmic microwave background (CMB) spectrum caused by the scattering of the CMB photons off a distribution of high energy electrons.

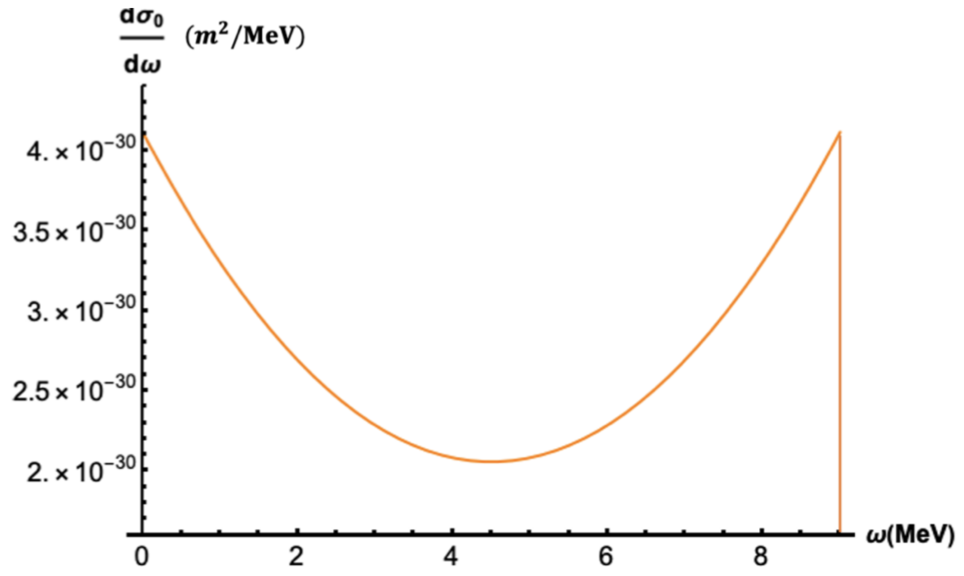
In free space

The local space in the resonant cavity.

$F^{(1)}_{(TM_{010})} = \mathbf{0.608484}$

The differential cross section:

$$\frac{d\sigma_0}{d\omega} = |F^{(1)}_{(TM010)}|^2 \cdot 2\pi \frac{r_e^2}{\kappa^2(1+u)^3} \frac{\varepsilon_0}{(\varepsilon_0 - \omega)^2} \{\kappa[1+(1+u)^2] - 4\frac{u}{\kappa}(1+u)(\kappa - u)\}$$



For the 9MeV(ω_{max}):

The cross section is about **0.04barn/MeV**.

1. COSMOLOGY WITH THE SUNYAEV-ZEL'DOVICH EFFECT.
2. Quantization of standing wave field and calculation of microwave Compton scattering cross section; Meiyu Si, Shanhong Chen, Yongsheng Huang*, et al., Eur. Phys. J. D (2022) 76:63 https://doi.org/10.1140/epjd/s10053-022-00389-4

Luminosity and the number of scattered photons

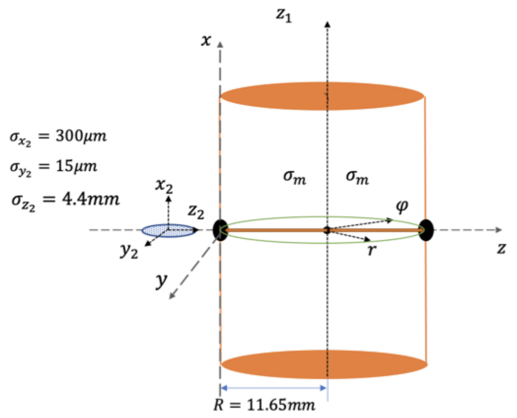
The areal density of microwave photons: (the unit is $N/(\mathrm{m}^2\cdot\mathrm{s})$)

$$\sigma_m(r) = \frac{S_r}{\omega_1} = \frac{E_m^2 J_0(K_c r) J_1(K_c r) sin(\omega t) cos(\omega t)}{\eta\omega_1} \qquad r = \sqrt{y^2 + (R-(ct))^2}.$$

$$f_2(x_2, y_2, z_2, t) = \frac{1}{2\pi\sigma_{x2}\sigma_{y2}\cdot\sqrt{2\pi}\sigma_{z2}} exp[-\frac{1}{2}(\frac{x_2^2}{\sigma_{x2}^2} + \frac{y_2^2}{\sigma_{y2}^2} + \frac{z_2^2}{\sigma_{z2}^2})]$$

The luminosity in the Compton scattering process:

$$L = N_2 \cdot 2Bf' \int \sigma_m(r) f_2(x_2, y_2, z_2, t) dxdydzdt \qquad B = 1, f' = 1$$



1. The left part of z1 experiences two complete wave packets;
2. The interaction time of the right half of z1 is 18.3625ps.

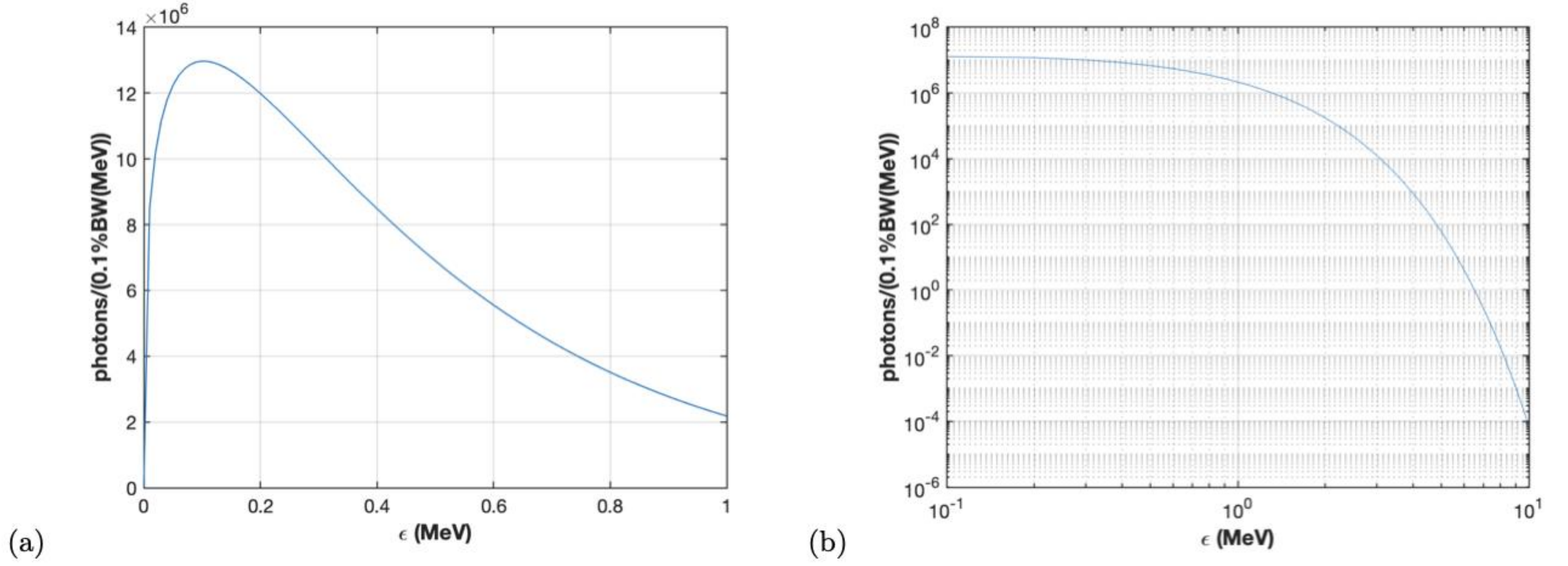
That is an electron bunch pass through the resonant cavity can generate at least **50459** scattered photons with energy of 9MeV.

Synchrotron radiation

Bending magnet :

$$\frac{dF_{bm}(y)}{d\theta} = 2.457 \times 10^{13} E(GeV) I(A) G(y)$$

Figure 4 shows the synchrotron radiation flux in 0.1%BW spread per bunch for the horizontal observation angle within 0.2mrad.



The energy range of photons is from 0 to 1MeV.

The energy range of photons is from 0 to 10MeV.

Shielding

To minimize the background noise from the synchrotron radiation.

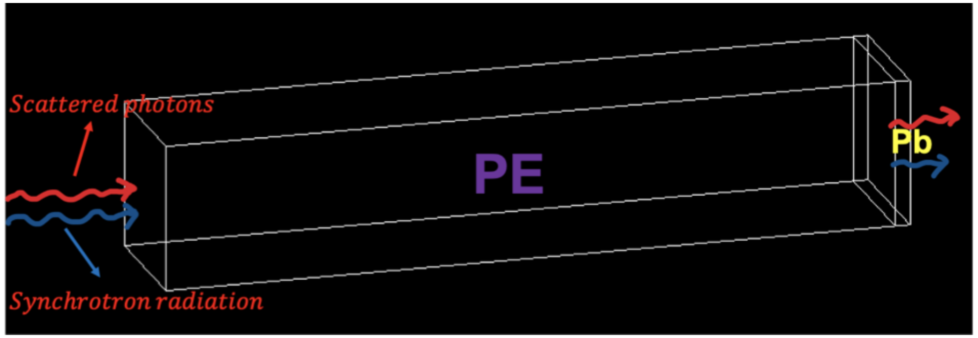


Figure 15. **The polyethylene (0.962g/cm^3) and the lead target (11.34g/cm^3).** A combination of 400cm polyethylene and 0.2cm lead are used to shield synchrotron radiation photons.

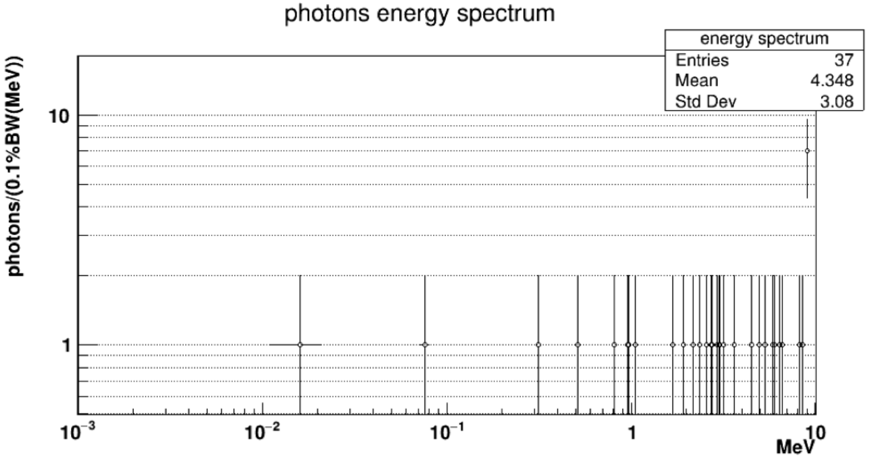


Figure 16. The photons energy spectrum of two photons sources after passing through the shielding material.

It is easy to distinguish between scattered photons and synchrotron radiation photons.

The effect of the hole radius on the field and frequency

Table III. The relation between the resonance frequency and the hole radius.

hole radius/mm	frequency/GeV
1.0mm	9.84790
1.5mm	9.84533
2.0mm	9.84026

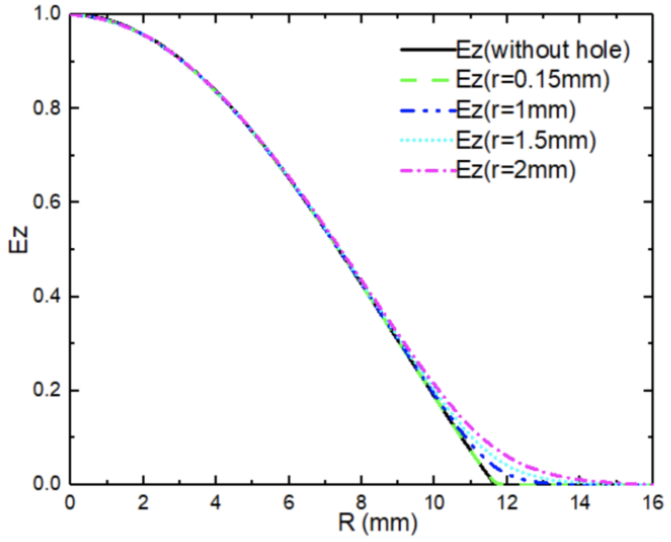


Figure 11. The normalized distribution of the field in the direction of radius in the cavity.

Almost no effect on the field, the effect on the frequency can be compensated.

The energy storage in the cavity is 0.001J.

$$W = \frac{\varepsilon_0}{2} \cdot 2\pi l E_m^2 \int_0^R J_0^2(\frac{2.405}{R} r) r dr = \frac{1}{2}\pi\varepsilon_0 R^2 l E_m^2 J_1^2(2.4)$$

The quality factor Q:

$$Q = \frac{R}{\delta(1+\frac{R}{l})}$$

Table II. The corresponding resonance frequency and Q value of the resonator cavity in theoretical calculation, simulation.

parameter	frequency(GeV)	Q value
Theoretical calculation	9.848975	11055.4
Simulation (without hole)	9.848976	11048.2
Simulation (hole radius 0.15mm)	9.848973	11043.8

Possible background

The effect of radiation in the field on the electron beam.

In the TM010 mode:

$$\begin{cases} E_z = E_m J_0(K_c r) \\ \bar{E}_z = \dfrac{\int_{-R}^{R} E_z dr}{2R} \end{cases} \Rightarrow \bar{E}_z = 6.11351 \times 10^6 \mathrm{V/m}$$

electric field:

$$\begin{cases} E = \gamma m_0 c^2 = 120GeV \\ F = q\bar{E}_z = \dfrac{\gamma m_0 c^2}{r} \end{cases}$$

Bending radius and critical energy:

$$\begin{cases} r = 19.629\text{km} \\ \epsilon_c = 2.218\dfrac{E^3}{r} = 195.257KeV \end{cases}$$

magnetic field:

$$H_\varphi = -E_m \frac{1}{\eta} J_1(K_c r) sin\omega t$$

Bending radius and critical energy:

$$\begin{cases} r = 28.8374\text{km} \\ \epsilon_c = 2.218\dfrac{E^3}{r} = 132.828KeV \end{cases}$$

Synchrotron radiation:

Bending radius: 10700m;
Critical energy: 352.8KeV

The same order compared with synchrotron radiation, it can be well shielded in front of the detector.

Summary

- Assuming the detector can reach the order of 10^{-4} under good calibration;
- The measurement accuracy of the beam energy can reach the 6MeV@120GeV ($\mathbf{5\times10^{-5}}$). Theoretically verified the feasibility of this program.
- To minimize the background noise from the synchrotron radiation, a combination of 400cm polyethylene and 0.2cm lead are used to shield synchrotron radiation photons.

Next step

- The design of the resonant cavity still needs more detailed considerations.
- If the detection accuracy of the HPGe detector is 10^{-3}, the uncertainty of energy measurement of beam energy $\delta\varepsilon_0 = 60\text{MeV}$. It is important to study the calibration method of the HPGe detector.
- The effect of scattering on the pipe and the loss of electron beam.

Conclusion

Laser-Compton

- **1D fitting: 1MeV@120GeV, 0.6@80GeV， 0.3@40GeV**
- **2D fitting： 0.4MeV@120GeV， <0.3MeV@80GeV**

Microwave-Compton

- **6MeV@120GeV; <6MeV@80GeV;**
- **A simple method+ γSR beamline**

Center of Mass

- **Potential corrections of c.m. energy**
- **Beam energy uncertainties from surroundings**

Thanks for your attentions!