强子的有效相互作用模型构造和分波分析

FDC-PWA 程序包的物理内容

高能所，王建雄

第四届重味物理与量子色动力学研讨会，湖南长沙

2022 年 7 月 28 日

Motivation (FDC)

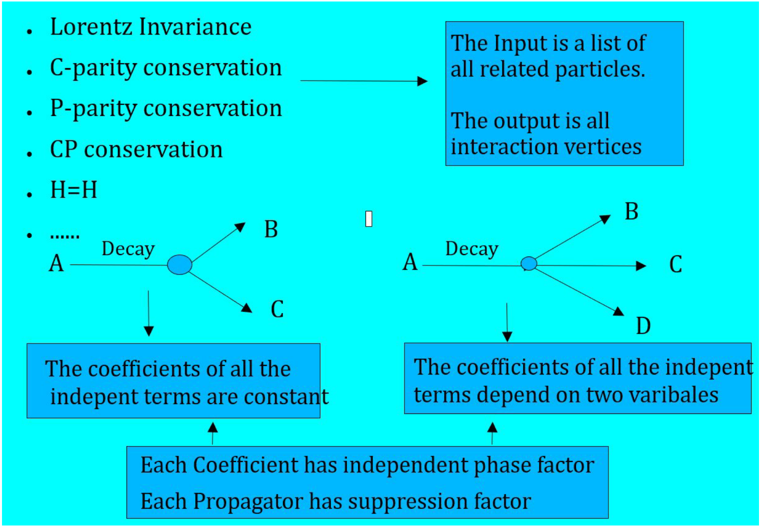
- Feynman Diagram Calculation(FDC), The project was started from 1993.
- PROGRESS IN FDC PROJECT, Nucl.Instrum.Meth.A534:241-245,2004
- FDC Homepage: http://www1.ihep.ac.cn/wjx
-
    - FDC-SM-and-Many-Extensions
    - FDC-NRQCD
    - FDC-MSSM
    - FDC-PWA
    - FDC-LOOP
- FDC-PWA was started from 1998 and constructed for BES partial wave analysis. PWA: to decide the mass, width and branch ratio of a resonant

Motivation （FDC-PWA）

- To work with high spin states (0, 1/2, 1, 3/2, 2, 5/2, 3 , 7/2, 4, 9/2) and construct effective Lagrangians.
- The expression of the effective interaction vertices and the propagators for the high spin states are quite lengthy.
- The related amplitudes and amplitude squares are complicated.
- There are many free parameters in the effective Lagrangian and these parameters will be fixed when the generated program is used to do Likelihood fitting of experimental data.
- To generate a complete set of the Fortran sources to do the partial wave analysis on experimental data.

Motivation (FDC-PWA)

The Rule to Construct Effective Lagrangian For PWA



Motivation（强子的有效相互作用模型构造）

- 分波分析的洛伦兹协变物理模型构造：20 多年前当时搞明白了，做了分波分析程序的自动化产生的程序包 FDC-PWA，但是 note 写的很简单，也没有写成文章，最近十几年，BES 物理分析使用的过程中，发现了不少问题，结果我也看不懂以前的 note 了，就对程序做了一些改动；
-
- 遇到最多的问题是：电荷共轭的两个顶点表达式（费曼规则）不一样？应该一样？这样电荷共轭的衰变道的衰变分宽度就不一样？
-
- 去年底及今年几个月，我把以前的构造方法，所有的表述，完全搞清楚啦，发现了一些有趣的事情，现在按照我的方式改正了 FDC-PWA 程序包中这些问题。BES 物理分析组分析了很多过程，表明现在这个版本程序完全解决了以前的问题。

Motivation（强子的有效相互作用模型构造）

现在的方案从场论的形式理论出发：

- 从场算符和产生淹没算符的 C，P 宇称变换性质，粒子的 C，P 宇称定义出发（在多年的研究生粒子物理课上讲过基本粒子，但只包括自旋 0，1/2，1），推广到包括高自旋粒子态（会发现找不到资料讨论高自旋粒子态 C，P 宇称内容），讨论怎么构造强相互作用的 C，P，T 不变，厄米共轭的，洛伦兹协变的强子物理模型。
-
- 是一个完全场论化形式的东西，需要到处讲一讲，并跟各位专家们进行讨论。

Motivation（强子的有效相互作用模型构造）

需要厘清的问题：

- 高自旋粒子场算符和其产生湮没算符的 C，P，T 宇称变换性质怎么推广？
- 粒子的 C,P 量子数是从场算符定义，还是从产生湮没算符（粒子态）定义？
- 电荷共轭不变的模型，对于各种相互作用项有什么要求？
- 一对相互电荷共轭的相互作用顶点表达式（费曼规则）应该一样吗？
- 相互作用哈密顿量的厄米共轭的要求对于构造模型带来什么要求？
- 辐射衰变顶点的电磁流守恒怎么构造？(counter term)

- all three bosons are eigenstates of charge conjugation,

$$\hat{\mathcal{H}}_I^\dagger = \Gamma^\dagger \phi^A \phi^B \phi^C = \hat{\mathcal{H}}_I^C = \chi_A \chi_B \chi_C \Gamma \phi^A \phi^B \phi^C \tag{1}$$

where χ_A, χ_B and χ_C are the charge conjugation eigenvalues. Thus $\chi = \chi_A \chi_B \chi_C = 1$ and $\Gamma^\dagger = \Gamma$ should be satisfied.

- only particle C is the real boson, then

$$\hat{\mathcal{H}}_I^\dagger = \Gamma^\dagger \phi^{A\dagger} \phi^{B\dagger} \phi^C \, . \tag{2}$$

Obviously there are a pair of charge conjugation vertices, and they together yield a hermitian Hamiltonian density

$$\hat{\mathcal{H}}'_I = \hat{\mathcal{H}}_I + \hat{\mathcal{H}}_I^\dagger \tag{3}$$

This time the charge conjugation of (**??**) is

$$\hat{\mathcal{H}}_I^C = \chi_C \Gamma \phi^{A\dagger} \phi^{B\dagger} \phi^C \tag{4}$$

$\hat{\mathcal{H}}_I^C$ should be equal to $\mathcal{H}_I^\dagger$ since they both represent the charge conjugation of the origin vertex, Thus we get

$$\Gamma^\dagger = \chi_C \Gamma \tag{5}$$

模型构造 (*Hermiticity and Charge Conjugation Invariance*)

- Fermion-Fermion-Boson: A fermion spinor field transforms under charge conjugation as

$$\hat{C}\psi\hat{C}^{-1} = C\overline{\psi}^T, \qquad \hat{C}\overline{\psi}\hat{C}^{-1} = \psi^T C \tag{6}$$

The hermitian conjugate Hamiltonian density is

$$\hat{\mathcal{H}}_I^\dagger = \overline{\psi}^B \gamma^0 \Gamma^\dagger \gamma^0 \psi^A \phi^C \tag{7}$$

and the charge conjugation is

$$\hat{\mathcal{H}}_I^C = \chi_C \overline{\psi}^B (C\Gamma C^{-1})^T \psi^A \phi^C. \tag{8}$$

Since (7) and (8) both describe the charge conjugation of the original vertex, they should be equal. This means

$$\chi_C (C\Gamma C^{-1})^T = \gamma^0 \Gamma^\dagger \gamma^0 \tag{9}$$

or equivalently

$$C(\gamma^0 \Gamma^\dagger \gamma^0)^T C^{-1} = \chi_C \Gamma \tag{10}$$

The table below lists $C(\gamma^0\Gamma^\dagger\gamma^0)^TC^{-1}$

Γ	i	$\hat{p}^\mu$	γ^μ	γ_5	$g^{\mu\nu}$	$\sigma^{\mu\nu}$
$C(\gamma^0\Gamma^\dagger\gamma^0)^TC^{-1}$	$-$i	$\hat{p}^\mu$	γ^μ	$-\gamma_5$	$g^{\mu\nu}$	$\sigma^{\mu\nu}$

If there are two tensors Γ_1 and Γ_2 which satisfy

$$C(\gamma^0\Gamma_1^\dagger\gamma^0)^TC^{-1} = \chi_1\Gamma_1 \quad (11)$$

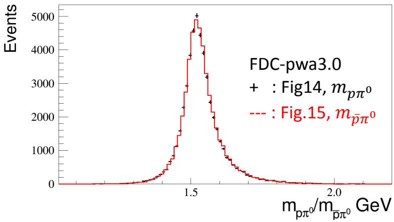
$$C(\gamma^0\Gamma_2^\dagger\gamma^0)^TC^{-1} = \chi_2\Gamma_2 \quad (12)$$

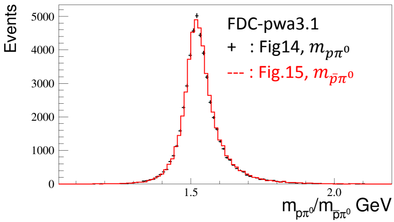
where $\chi_1, \chi_2 = \pm 1$, then their product $\Gamma_1\Gamma_2$ would have the property

$$\begin{aligned} C(\gamma^0(\Gamma_1\Gamma_2)^\dagger\gamma^0)^TC^{-1} &= C(\gamma^0\Gamma_2^\dagger\gamma^0\gamma^0\Gamma_1^\dagger\gamma^0)^TC^{-1} \\ &= C(\gamma^0\Gamma_1^\dagger\gamma^0)^TC^{-1}C(\gamma^0\Gamma_2^\dagger\gamma^0)^TC^{-1} \\ &= \chi_1\chi_2\Gamma_1\Gamma_2 \ . \end{aligned} \quad (13)$$

This implies that if a composite tensor $\Gamma_i = \Gamma_1 \cdots \Gamma_N$ does not satisfy (10), we can simply multiply an i and get a valid tensor $\mathrm{i}\Gamma_i$.

$$J/\psi \to p\,\bar{N}(1520), \bar{p}\,N(1520) \to p\bar{p}\pi^0$$





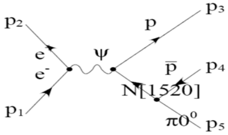


Fig. 14

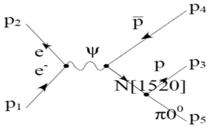


Fig. 15

Individual diagram can not identify the sign difference in the charged conjugate vertex!

9

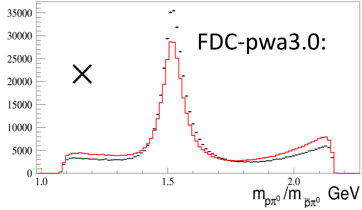
模型构造问题的例子（*Hermiticity and Charge Conjugation Invariance*）

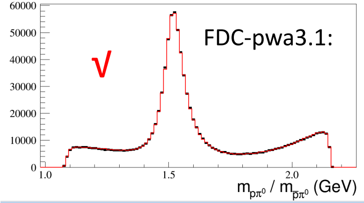
The Rule to Construct Effective Lagrangian For PWA

$$J/\psi \rightarrow p\,\bar{N}(1520), \bar{p}\,N(1520) \rightarrow p\bar{p}\pi^0$$

FDC-pwa3.0: $V_{52} = g\,(p2_\mu p2_\nu f82 + g_{\mu,\nu} f83 + \gamma_\nu p2_\mu f84)$

FDC-pwa3.0: $V_{53} = g\,(p2_\mu p2_\nu f82 + g_{\mu,\nu} f83 + \gamma_\nu p2_\mu f84)$





FDC-pwa3.1 $V_{52} = gi\,(-p2_\mu p2_\nu f80 - g_{\mu,\nu} f81 + \gamma_\nu p2_\mu f82)$

FDC-pwa3.1: $V_{53} = gi\,(p2_\mu p2_\nu f80 + g_{\mu,\nu} f81 + \gamma_\nu p2_\mu f82)$

- Vertices:
  If one of the outgoing bosons is a photon, the interaction Hamiltonian should also be gauge invariant. The gauge invariance condition is

$$k_\mu \Gamma^{\mu\alpha_1\ldots\alpha_m\beta_1\ldots\beta_n} = 0 \tag{14}$$

  or

$$\sum_i f_i \; k_\mu \Gamma_i^{\mu\alpha_1\ldots\alpha_m\beta_1\ldots\beta_n} = 0 \ . \tag{15}$$

- There are two way for the condition:
  - The condition is satisfied on mass shell.
  - The condition is satisfied on mass shell and off mass shell.
- Radiative decay processes: processs dependent counter term

新的费曼图产生程序（牛睿昌，昌翔宇）

FDC-PWA-TG

Big Model
initial particle
final particles
all vertices

FDC-PWA-TG
⟶

Sub Model
no redundant particles

⟶ Then do your works ...

- A C++ program: use pointers to efficiently manipulate a tree.
- Efficient: submodels are automatically created.
- Fast : generates all submodels for all 2-4 final processes in 83 ms.
- Usage :
  Writing a integrated program, using a command to generate all 2-4 final tree diagrams of a model and display them by HTML in 5 min.

Accelerate FDC with GPU-Tensorflow

- Compile amplitudes of fortran codes into a python modules
- Calculate the event amplitude in Tensorflow framework
- MLLH minimized with Minuit in pyROOT (python version of Minuit)
- Access fit results (signal yields and statistical uncertainty calculated based on resultant covariance matrix)
- Allow user to add mass and width as hyper-parameters in the fit
- Allow for simultaneous fit to multiple samples

4

FDC-PWA :

- $>$ gmodel
- $>$ doall
- $>$ cd fort
- $>$ make
- $>$ fit

总结

- 强子的有效相互作用模型构造
- 分波分析程序的自动化产生的程序包 FDC-PWA
- FDC-PWA 可以很方便的应用到各种对撞机上（BEPC，B-factory, LHCB) 的分波分析
- 今后计划：初态母粒子极化，新的模型构造方案（包括电磁流守恒的 counter term 引入）

End

The End
Thank you very much