上海交通大学
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SJTU

Some Progress on LQCD Analysis of Charmed Hadrons

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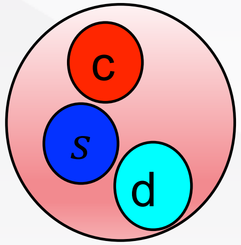
第四届重味物理与量子色动力学研讨会

饮 水 思 源 • 爱 国 荣 校

Outline

- ✓ Charm Quark Physics
- ✓ Theoretical tools for charm
- ✓ LQCD
  - ✓ Ξ_c Decays: form factors and $|V_{cs}|$
  - ✓ Hidden Charm Hexaquark
- ✓ Summary

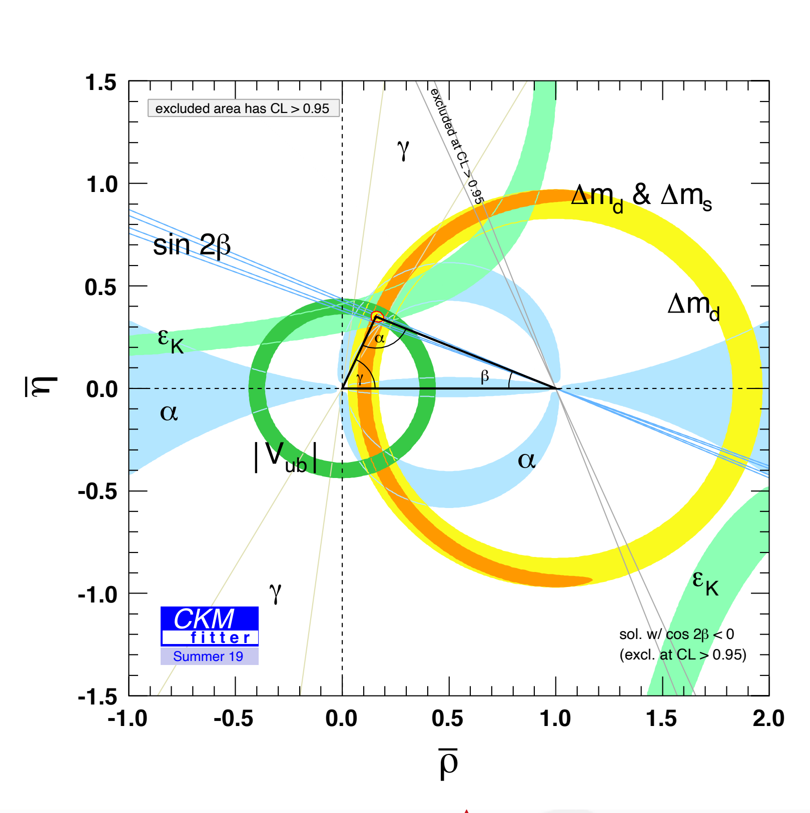


Charm Quark Physics: SM test

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$VV^{\dagger} = 1 \rightarrow \sum_k V_{ik}V_{jk}^{*} = \delta_{ij}$$

$$V_{ub}V_{ud}^{*} + V_{cb}V_{cd}^{*} + V_{tb}V_{td}^{*} = 0$$



Charm Quark Physics: SM test

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

PDG2021

First row:

$$|V_{ud}| = 0.97370 \pm 0.00014$$
$$|V_{us}| = 0.2245 \pm 0.0008$$
$$|V_{ub}| = 0.00382 \pm 0.00024$$

10^{-4} *accuracy* $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0006$

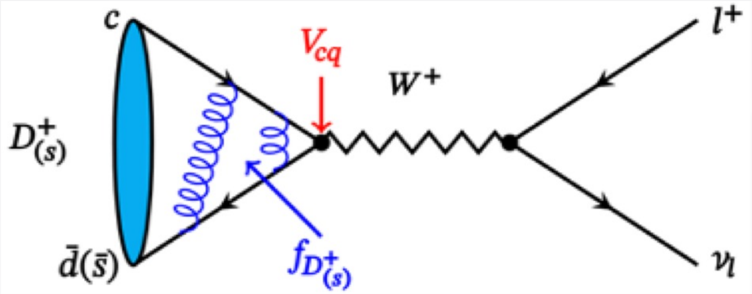
Second row:

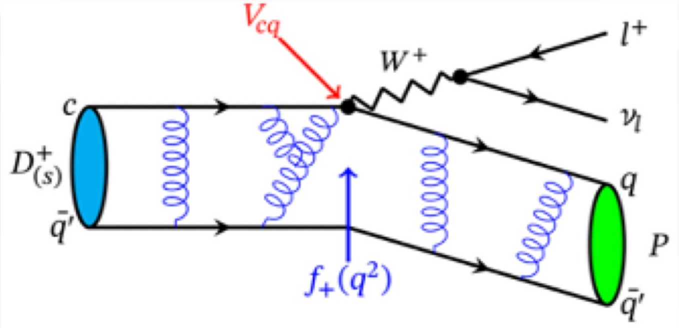
$$|V_{cd}| = 0.221 \pm 0.004$$
$$|V_{cs}| = 0.987 \pm 0.011$$
$$|V_{cb}| = 0.0410 \pm 0.0014$$

10^{-2} *accuracy* $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$

High precision determination of the second row provide strong tests of CKM unitarity.

$$|V_{cd}| = 0.221 \pm 0.004$$
$$|V_{cs}| = 0.987 \pm 0.011$$
$$|V_{cb}| = 0.0410 \pm 0.0014$$





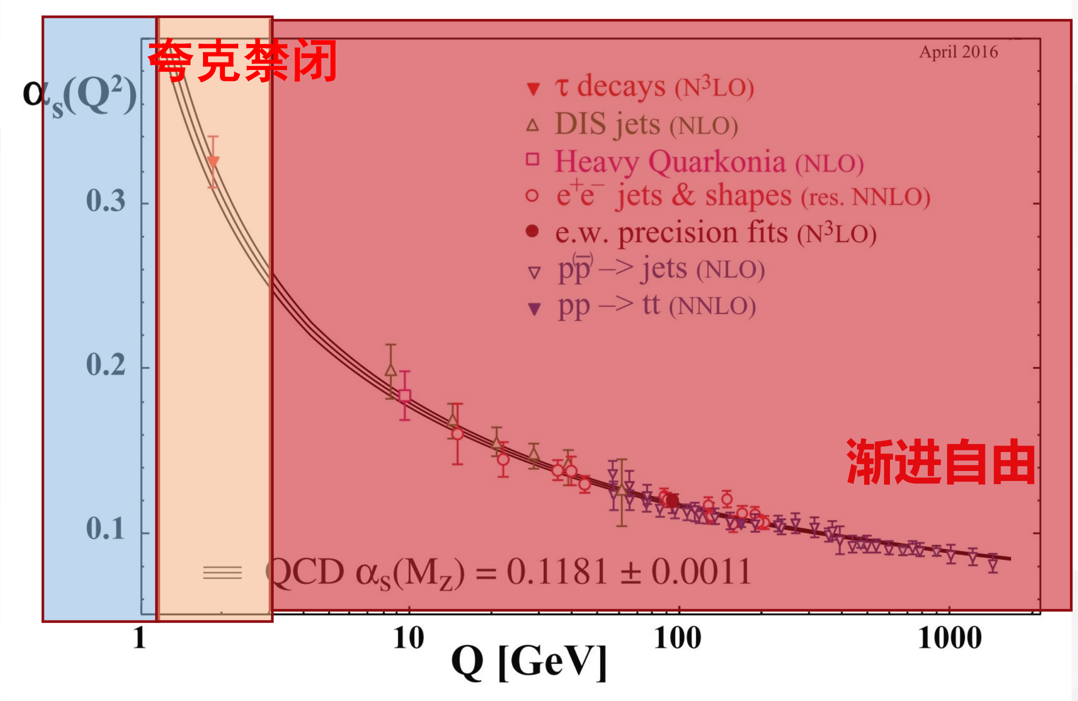
PDG2021

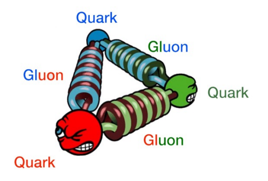
determinations of $|V_{cs}|$ can be obtained using the PDG values for the mass and lifetime of the D_s, the masses of the leptons, and $f_{D_s} = (249.9 \pm 0.5)$ MeV [14]. The average of these determinations gives $|V_{cs}| = 0.992 \pm 0.012$, where the error is dominated by the experimental uncertainty. In semileptonic D decays, lattice QCD calculations of the $D \to K\ell\nu$ form factor are available [14]. Using $f_+^{DK}(0) = 0.765 \pm 0.031$ and the average [24] of CLEO-c [28], Belle [29], BaBar [48], and recent BESIII [26, 49] measurements of $D \to K\ell\nu$ decays, one obtains $|V_{cs}| = 0.939 \pm 0.038$, where the dominant uncertainty is from the theoretical calculation of the form factor. Averaging the determinations from leptonic and semileptonic decays, we find

$$|V_{cs}| = 0.987 \pm 0.011\,. \qquad (12.10)$$

Theoretical difficulties in Charm

Charm Physics Scale





QCD perturbation theory: α_s

Chiral perturbation theory: $p/(4\pi f_\pi)$

CP violation in $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$

LHCb, PRL108, 111602 (2012)

$$\Delta A_{CP} = [-0.82 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})]\,\%$$

Cheng, Chiang, 1201.0785, -0.25%
Li, Lu, Yu, 1203.3120, -0.1%

Brod, Grossman, Kagan, Zupan: Large penguins

LHCb, PRL122, 211803 (2019)

$$\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4},$$

Theoretical tools for Charm

- ✓ **Quark model:**
- ✓ **QCD sum rules:**
- ✓ **Factorization-Assisted Topological-Amplitude:**

  Li, Lu, Yu, PRD 86, 036012 (2012)

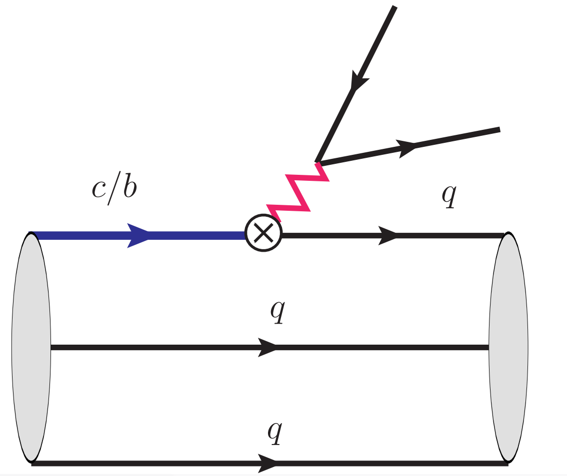
  Yu, Wang, Li, PRL 119, 181802(2017)
- ✓ **SU(3) symmetry:**
- ✓ **Lattice QCD**

Theoretical tools for Charm

✓ **SU(3) symmetry:**

$$\Gamma(\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell) = \Gamma(\Xi_c^0 \to \Xi^- \ell^+ \nu_\ell) = \frac{3}{2}\Gamma(\Lambda_c^+ \to \Lambda^0 \ell^+ \nu_\ell)$$

channel	branching ratio(%)	
	experimental data	SU(3) symmetry
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	3.6 ± 0.4 [33]	3.6 ± 0.4 (input)
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5 [33]	3.5 ± 0.5 (input)
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	2.3 ± 1.5 [33]	12.17 ± 1.35
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.54 ± 0.35 [4, 5]	4.10 ± 0.46
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44 [4]	3.98 ± 0.57



He, Huang, Wang, Xing, 2110.04179

BES-III: PRL 115, 221805(2015)
Belle: PRL 127, 121803(2021)
ALICE: 2105.05187

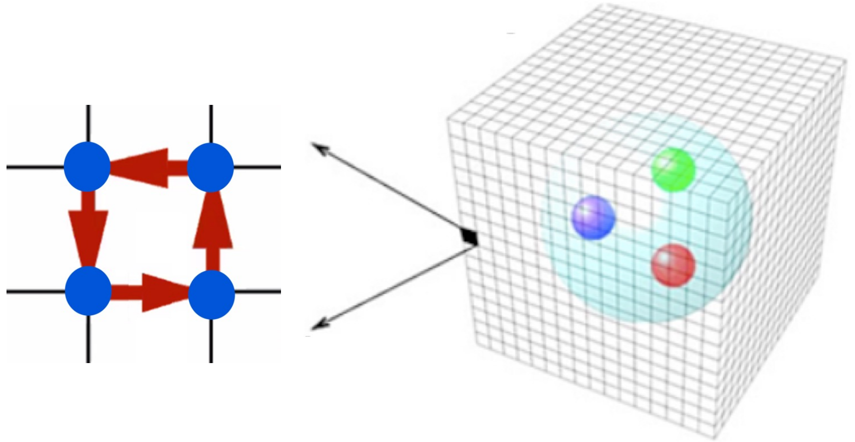
Lattice QCD

- **Numerical simulation in discretized 4D Euclidean space-time;**

- **Lattice QCD: action**

$$S_E^{\rm latt} = -\sum_{\Box} \frac{6}{g^2} \mathrm{Re}\, \mathrm{tr}_N(U_{\Box,\mu\nu}) - \sum_q \bar{q}(D_\mu^{\rm lat}\gamma_\mu + am_q)q$$

Wilson gauge action   Lattice fermion action



- **Correlation functions:**

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \frac{\int [\mathcal{D}\mathcal{U}] \prod_q [\mathcal{D}q_q]\,[\mathcal{D}\bar{q}_q]\, e^{-S_E^{\rm latt}}\, \mathcal{O}(U,q,\bar{q})}{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q]\,[\mathcal{D}\bar{q}_q]\, e^{-S_E^{\rm latt}}}$$

$$= \frac{\int [\mathcal{D}U] e^{-S_{\rm glue}^{\rm latt}} \prod_q \det\left(D_\mu^{\rm latt}\gamma_\mu + am_q\right) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}U] e^{-S_{\rm glue}^{\rm latt}} \prod_q \det\left(D_\mu^{\rm latt}\gamma_\mu + am_q\right)}$$

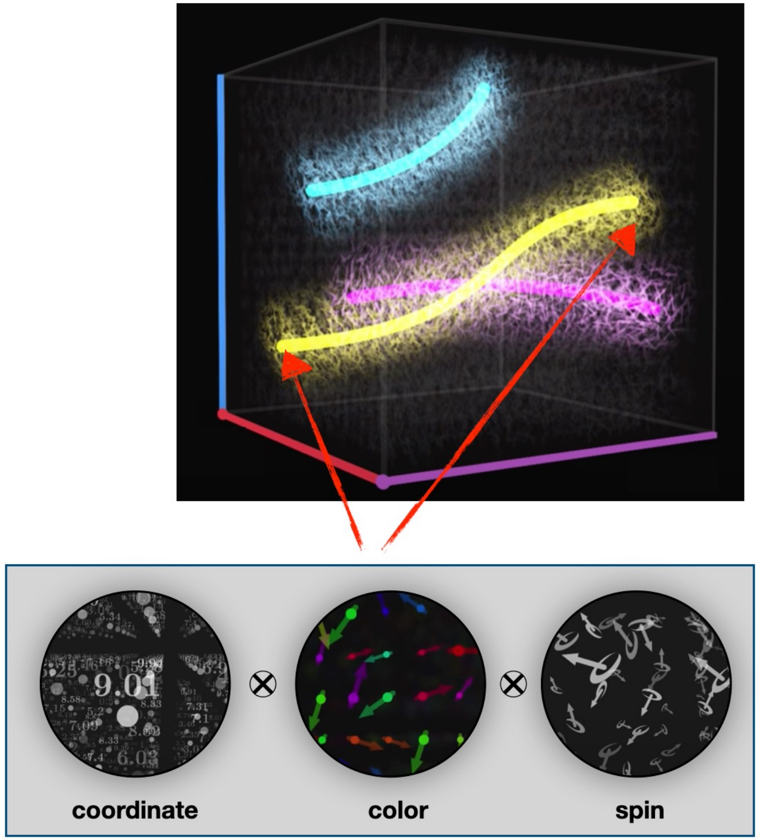
Lattice QCD

- **Monte Carlo:**
  - The integration is performed for all link variables: $n_s^3 \times n_t \times N_{\text{color}} \times N_{\text{spin}}$
  - Importance sampling:

$$e^{-S_{\text{glue}}^{\text{latt}}}(U)\prod_q \det\left(D_\mu^{\text{latt}}(U)\gamma_\mu + am_q\right)$$

  - Therefore

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \frac{1}{N_{\text{conf}}}\sum_{k=1}^{N_{\text{conf}}} \tilde{\mathcal{O}}\left(U^{(k)}\right)$$

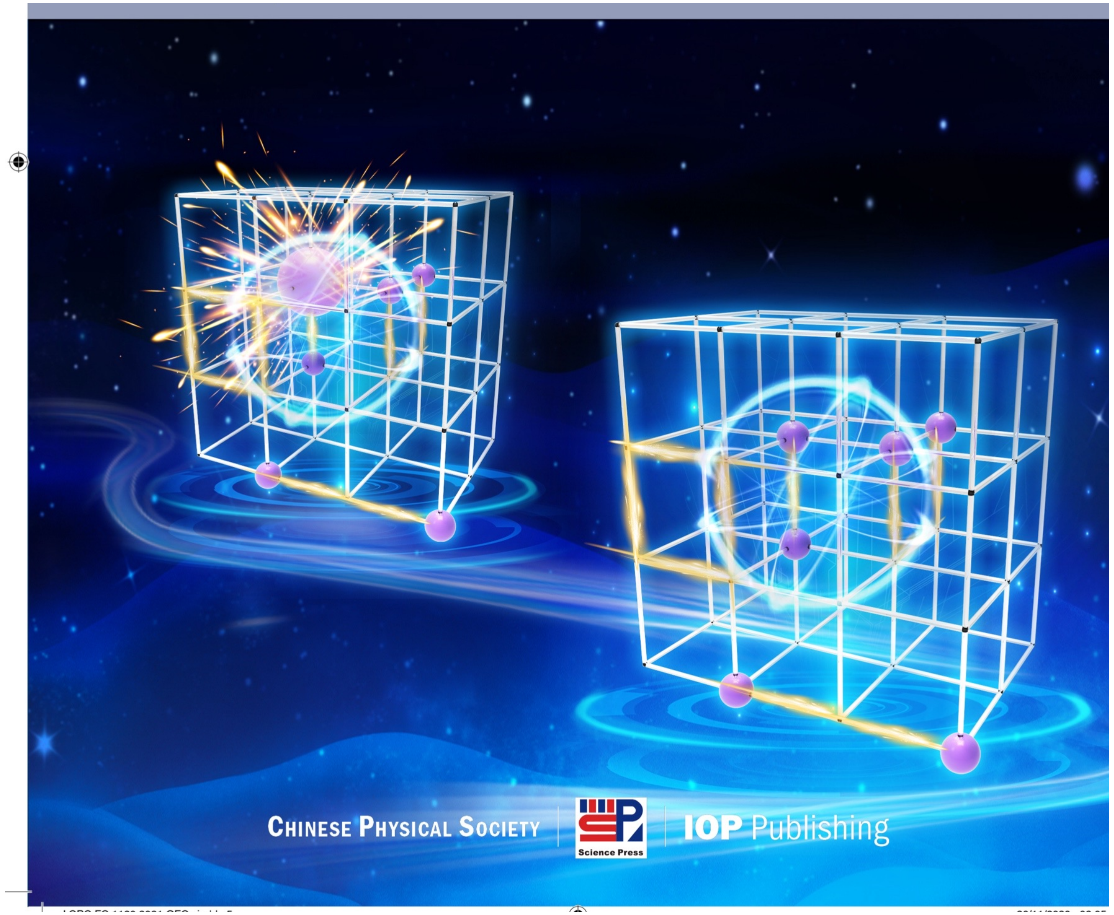


- **Have achieved great successes in calculating hadron masses, decay constants, α_s, form factors and so on.**

Lattice QCD: new configurations

Lattice size	Lattice spacing	Pion mass	N cfg
24x72	0.108fm	280-290MeV	2000+
32x96	0.08fm	280-300MeV	1000+
48x144	0.055fm	280MeV	producing
48x96	0.108fm	200MeV	producing
48x96	0.108fm	140MeV	prepare

Liuming Liu, Peng Sun, Wei Sun, Yibo Yang

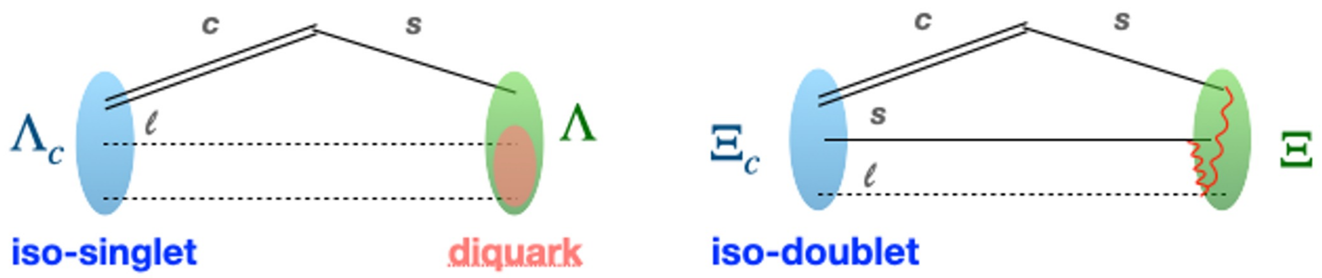


Q.A.Zhang,et.al., Chin.Phys.C 46 (2022) 7, 011002

Ξ_c Decays

- Ξ_c contains more **versatile decay modes**

  - $\Xi_c \to \Xi$ contain different QCD dynamics with $\Lambda_c \to \Lambda$;



  - A **different pattern** between inclusive and exclusive decays of Λ_c and D:

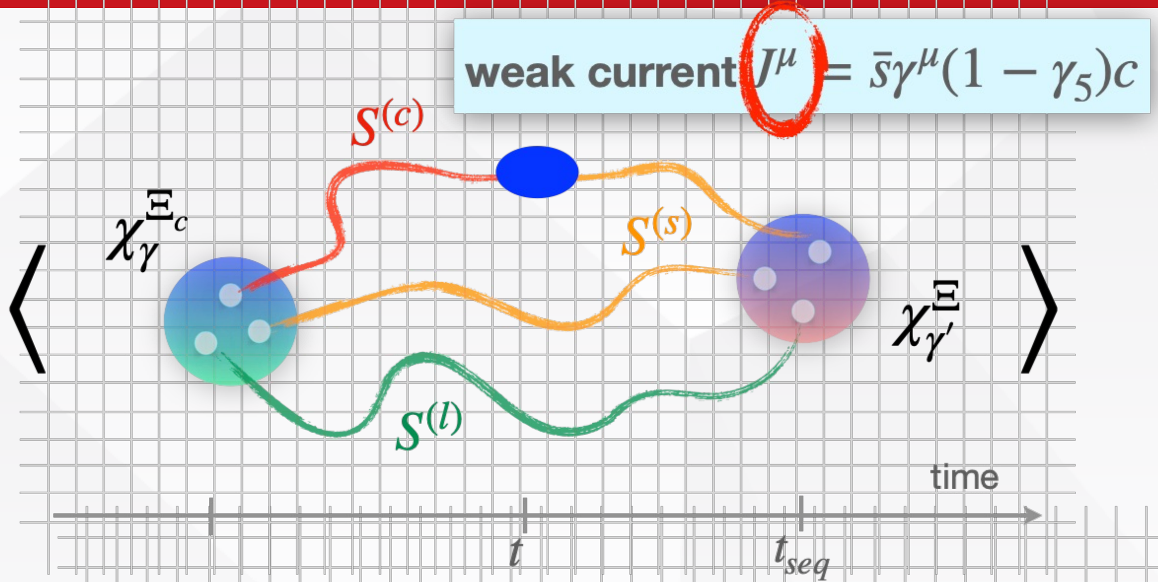
$$\frac{\mathscr{B}(\Lambda_c^+ \to X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%}{\mathscr{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20)\,\%} \quad \sim 1$$

$$\frac{\mathscr{B}(D^0 \to X e^+ \nu_e) = (6.49 \pm 0.11)\,\%}{\mathscr{B}(D^0 \to K^- e^+ \nu_e) = (3.542 \pm 0.035)\,\%} \quad \sim 2$$

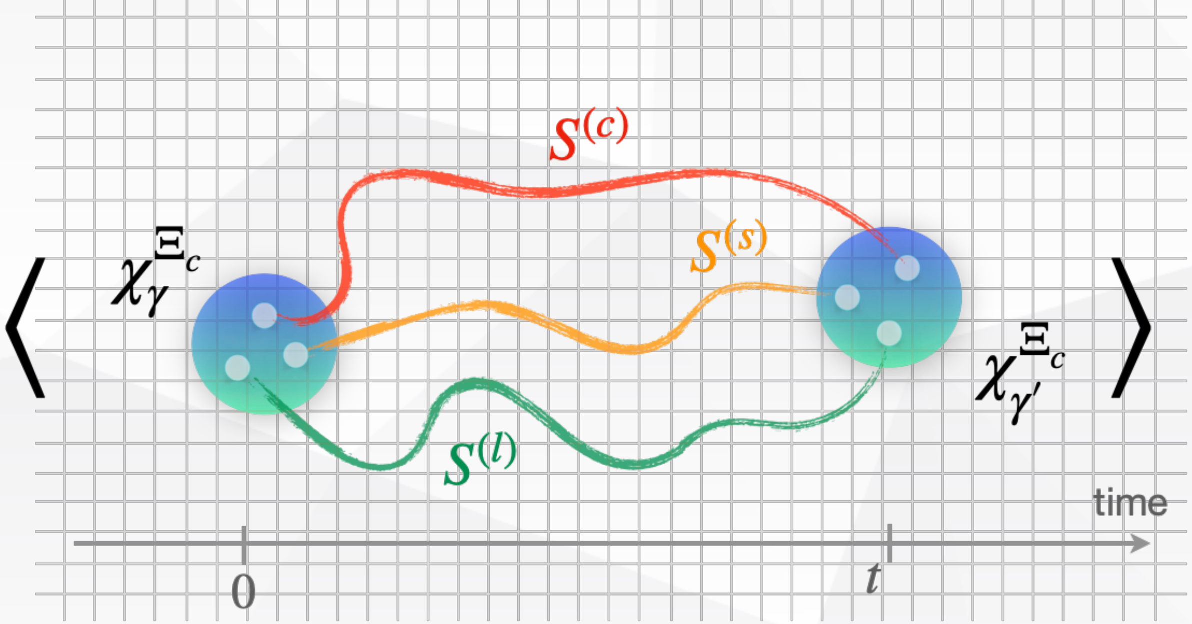
M. Ablikim et al. [BESIII], PRL121, 251801 (2018)

Form factor on the LQCD

$C_3(q^2, t, t_{seq}) \sim T^{\gamma\gamma'}$



$C_2^{\Xi_c}(t) \sim I^{\gamma\gamma'}$

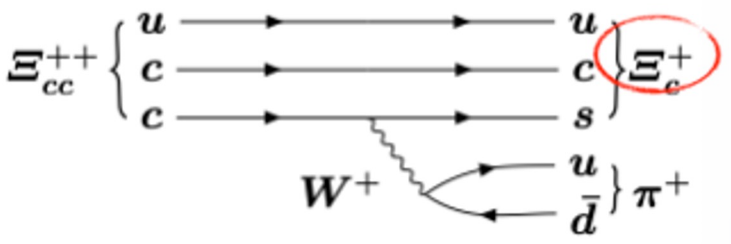


Ξ_c Decays

- **Importance for the experimental researches of heavy baryons:**

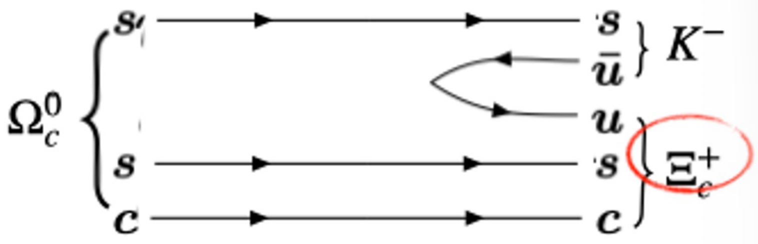
-Studies of doubly-charmed baryon Ξ_{cc}^{++} decay

R. Aaij et al. [LHCb], PRL121, 162002 (2018)



-Precision measurement of the lifetime of Ξ_b^0

R. Aaij et al. [LHCb], PRL113, 032001 (2014)



-Discovery of new exotic hadron candidates Ω_c

R. Aaij et al. [LHCb], PRL118, 182001 (2017)

-......

Ξ_c Decays

✓ Experimental

Belle

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$$

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\,\%$$

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

ALICE

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$$

J. Zhu on behalf of the ALICE collaboration, PoS ICHEP2020 (2021) 524.

✓ Theoretical

QCD SR

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\,\%$$

Z. X. Zhao, arXiv:2103.09436 [hep-ph].

LF QM

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\,\%$$

C. Q. Geng et al, arXiv:2012.04147 [hep-ph].

LCSR

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\,\%$$

Y. L. Liu et al, J. Phys. G 37, 115010 (2010).

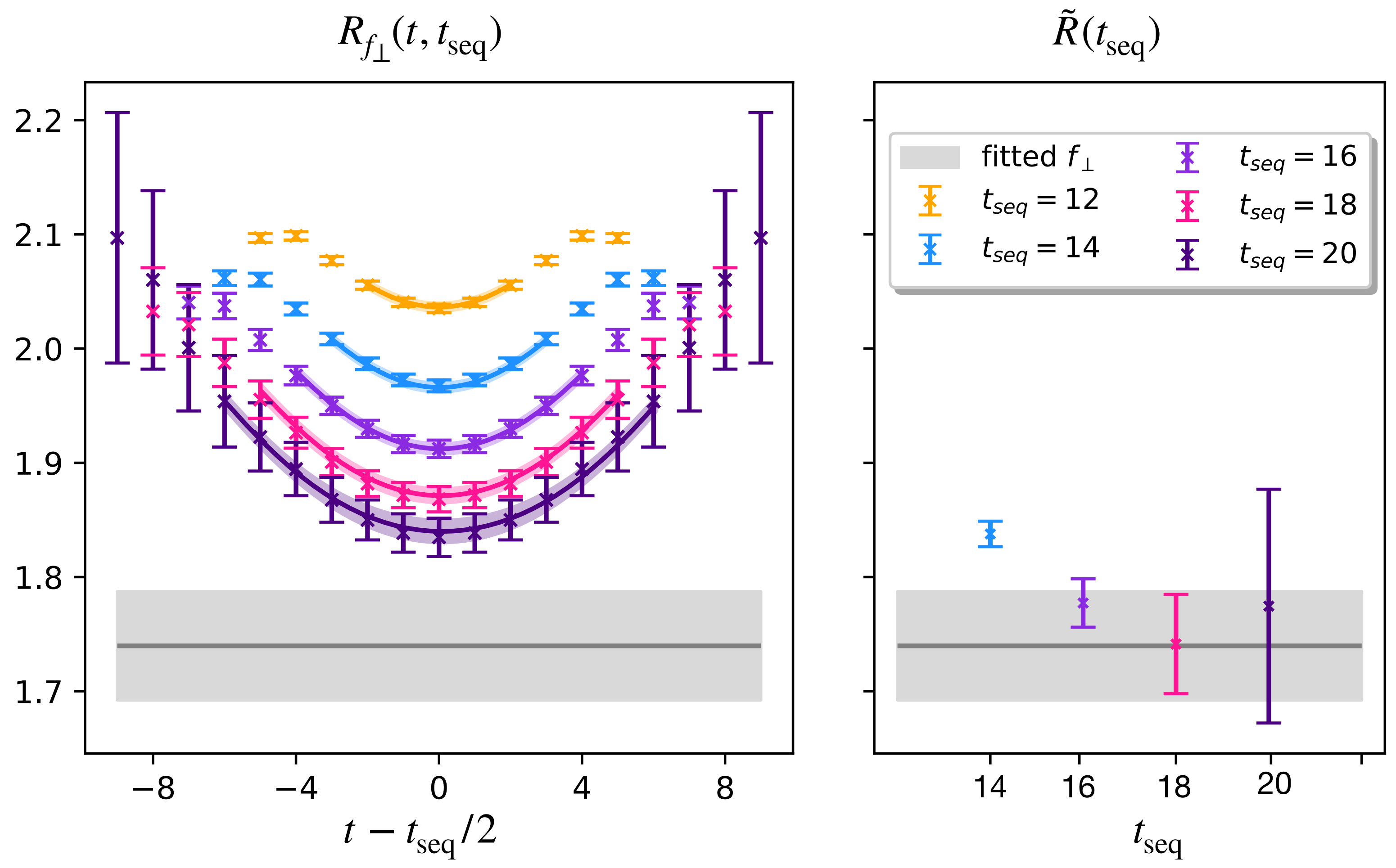
Lattice ?

Ξ_c Decays on the Lattice

	$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	c_{sw}	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3 \times 72$	0.108	1.161	-0.2770	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	-0.2295	300	0.1318	650

Zhang, Hua, et.al., 2103.07064, To appear in Chinese Physics C

Form factor on the LQCD



$$R_{f_\perp} \equiv \frac{R_V(\gamma_5\gamma^x, \gamma^y)}{4m_{\Xi_c} N_z \hat{p}} = f_\perp \left(\frac{\left(1 + c_1 e^{-\Delta E_1 t} + c_2 e^{-\Delta E_2 \left(t_{seq} - t\right)}\right)\left(1 + c_1 e^{-\Delta E_1 \left(t_{seq} - t\right)} + c_2 e^{-\Delta E_2 t}\right)}{\left(1 + d_1 e^{-\Delta E_1 t_{seq}}\right)\left(1 + d_2 e^{-\Delta E_2 t_{seq}}\right)} \right)^{1/2}$$

Ξ_c Decays on the Lattice

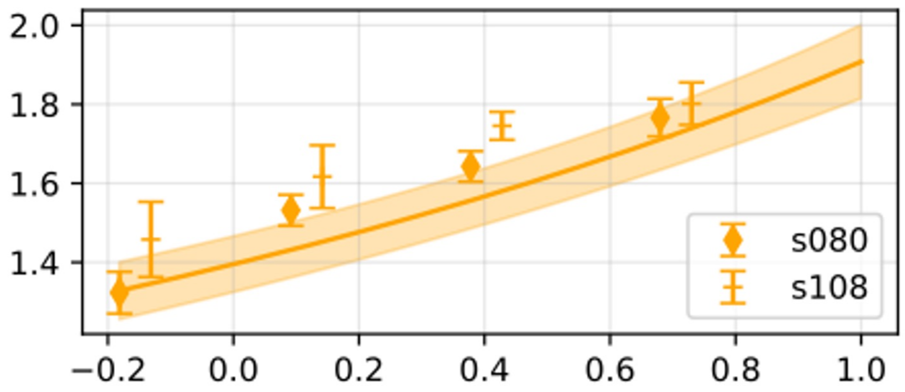
- Extrapolate to the **continuum limit** (shaded regions);

- z-expansion parametrization to obtain the q^2-distribution:

$$f(q^2) = \frac{1}{1 - q^2 / (m^f_{\text{pole}})^2} \sum_{n=0}^{n_{\max}} \left(c_n^f + d_n^f a^2\right) \left[z(q^2)\right]^n$$

- Use **D_s meson pole mass** for $m^{f_\perp}_{\text{pole}}$, …

- Consider the **discretization effects** by estimating the d_n^f terms.

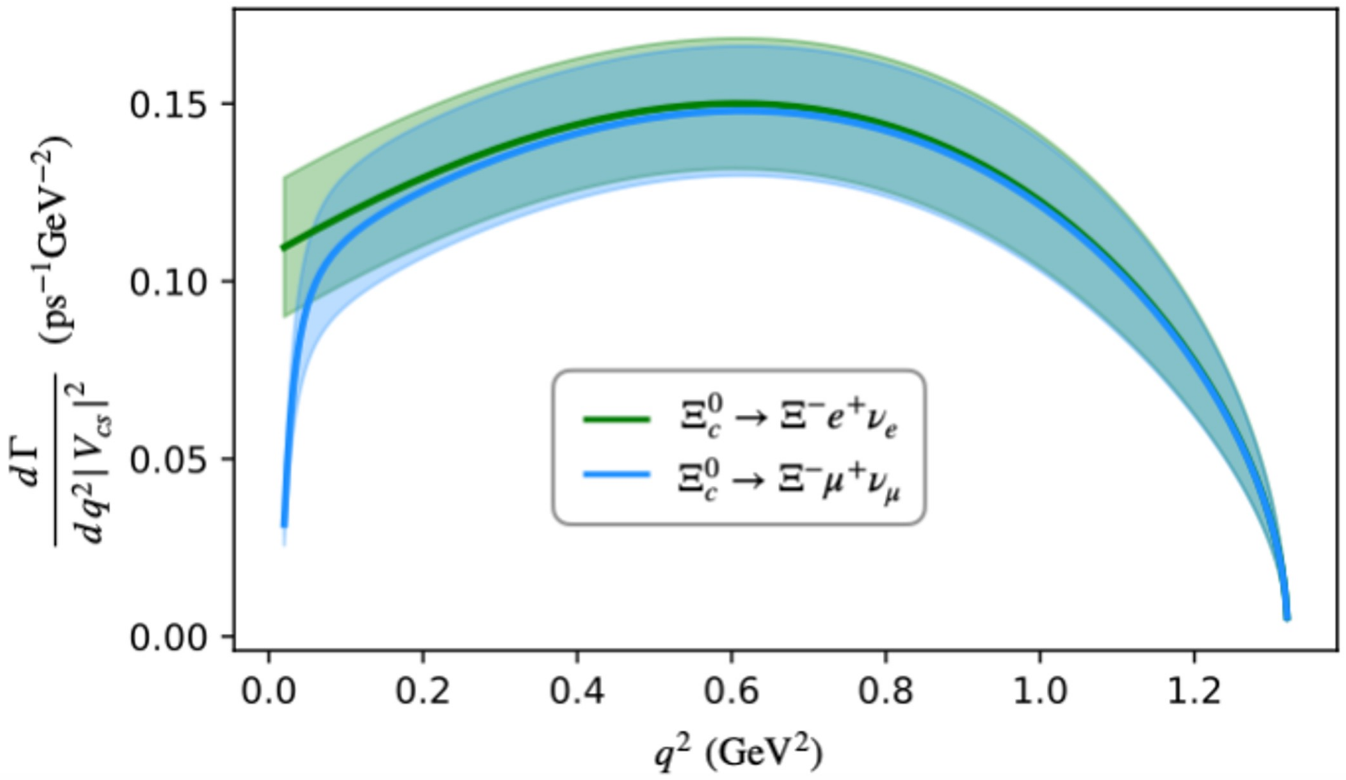


Fit results for the z-expansion parameters

	c_0	c_1	c_2
$f_\perp$	1.51±0.09	-1.88±1.21	1.71±0.49
f_0	0.64±0.09	-1.83±1.22	0.56±0.51
f_+	0.77±0.07	-4.09±1.18	0.35±0.49
$g_\perp$	0.56±0.07	-0.35±1.26	0.15±0.29
g_0	0.63±0.07	-1.37±1.36	0.15±0.29
g_+	0.56±0.08	0.00±1.38	0.14±0.29

Ξ_c Decays on the Lattice

-The differential decay widths of $\Xi_c^0 \to \Xi^- l^+ \nu_l$:



Ξ_c Decays on the Lattice

-Predicted decay branching fractions:

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = 2.38(0.30)(0.32)(0.07)\ \%$$

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = 2.29(0.29)(0.30)(0.06)\ \%$$

$$\mathcal{B}\left(\Xi_c^+ \to \Xi^0 e^+ \nu_e\right) = 7.18(0.90)(0.96)(0.20)\ \%$$

$$\mathcal{B}\left(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu\right) = 6.91(0.87)(0.91)(0.19)\ \%$$

- Statistical errors
- Systematic errors from continuum extrapolation
- Systematic errors from renormalization

$(2.38 \pm 0.44)\ \%$

-Compare with PDG, experiment and theory:

PDG	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.8 \pm 1.2)\ \%$	✓
Belle	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\ \%$	✓
ALICE	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\ \%$	✓
QCD SR	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\ \%$	✓
LF QM	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\ \%$	✓
LCSR	$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\ \%$	✓

Fitted well with all data (within 1-σ) !

-From Belle measurements: *Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].*

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$$

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\,\%$$

From the uncertainty of $\Xi_c^0 \to \Xi^- \pi^+$

$$|V_{cs}| = 0.834 \pm (0.051)_{\text{stat.}} \pm (0.056)_{\text{syst.}} \pm (0.127)_{\text{exp.}}$$

Theo. error ~ 8.9% Exp. error ~ 15.2%

-From ALICE measurements: *J. Zhu, PoS ICHEP2020 (2021) 524.*

$$\mathcal{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$$

$$|V_{cs}| = 0.983 \pm (0.060)_{\text{stat.}} \pm (0.065)_{\text{syst.}} \pm (0.167)_{\text{exp.}}$$

Exp. error ~ 17.0%

-Compare with PDG result:

$$|V_{cs}| = 0.987 \pm 0.011$$

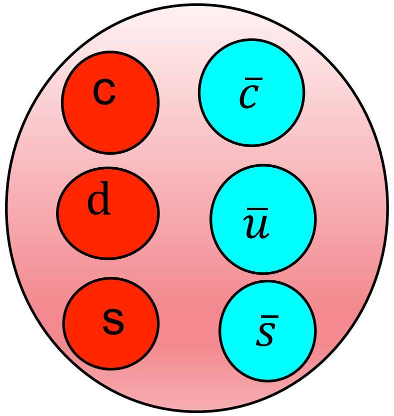
$$|V_{cs}| = 0.939 \pm 0.038 \quad D \to K\ell\nu$$

Theoretical uncertainties:

- total ~ 8.9%
- statistical ~ 6.1%
- systematic from extrapolation ~ 6.5%
- systematic from renormalization ~ 1.5%

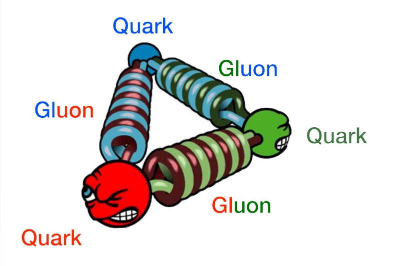
Experimental uncertainties:

- Belle ~ 15.2%
- ALICE ~ 17.0%

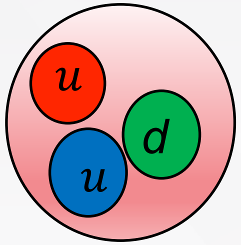


H. Liu,et.al., 2207.00183

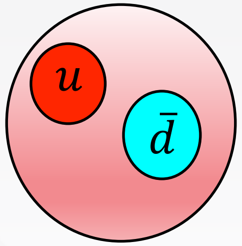
Hadron Exotics



- Ordinary Hadrons

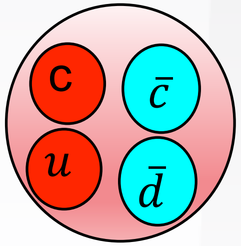


baryon

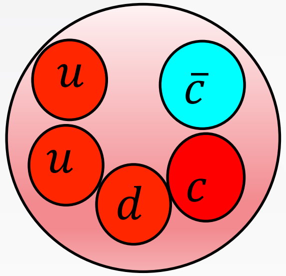


meson

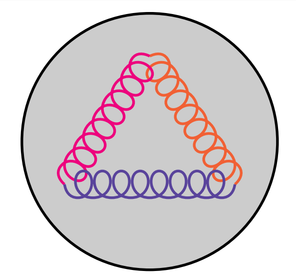
- Exotic Hadrons



tetra-quark



penta-quark

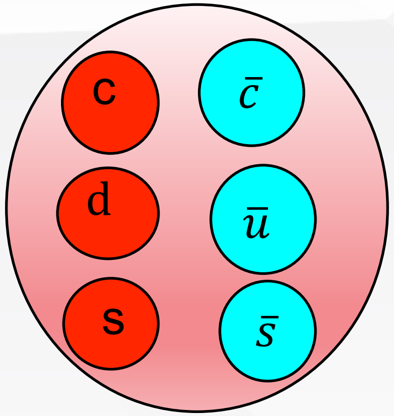


Glueball

$$|p >= |uud > + |uud\bar{q}q > + \cdots$$

$$< 0|O_{uud}\ |p > \neq 0$$

Hexaquark



$$0^{++} : \mathcal{O}_1 = \epsilon^{abc}\epsilon^{def}[u_a^T C\gamma_5 s_b][\bar{d}_d C\gamma_5 \bar{s}_e^T] \times [\bar{c}_f c_c]$$

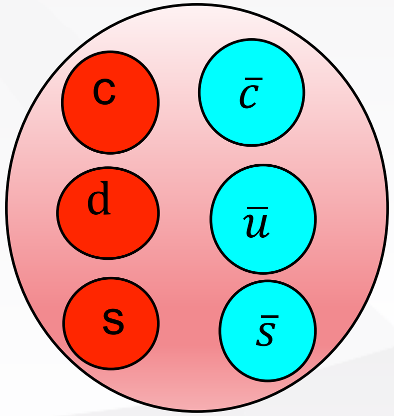
$$0^{-+} : \mathcal{O}_2 = \epsilon^{abc}\epsilon^{def}[u_a^T C\gamma_5 s_b][\bar{c}_d C\gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_5 c_c]$$

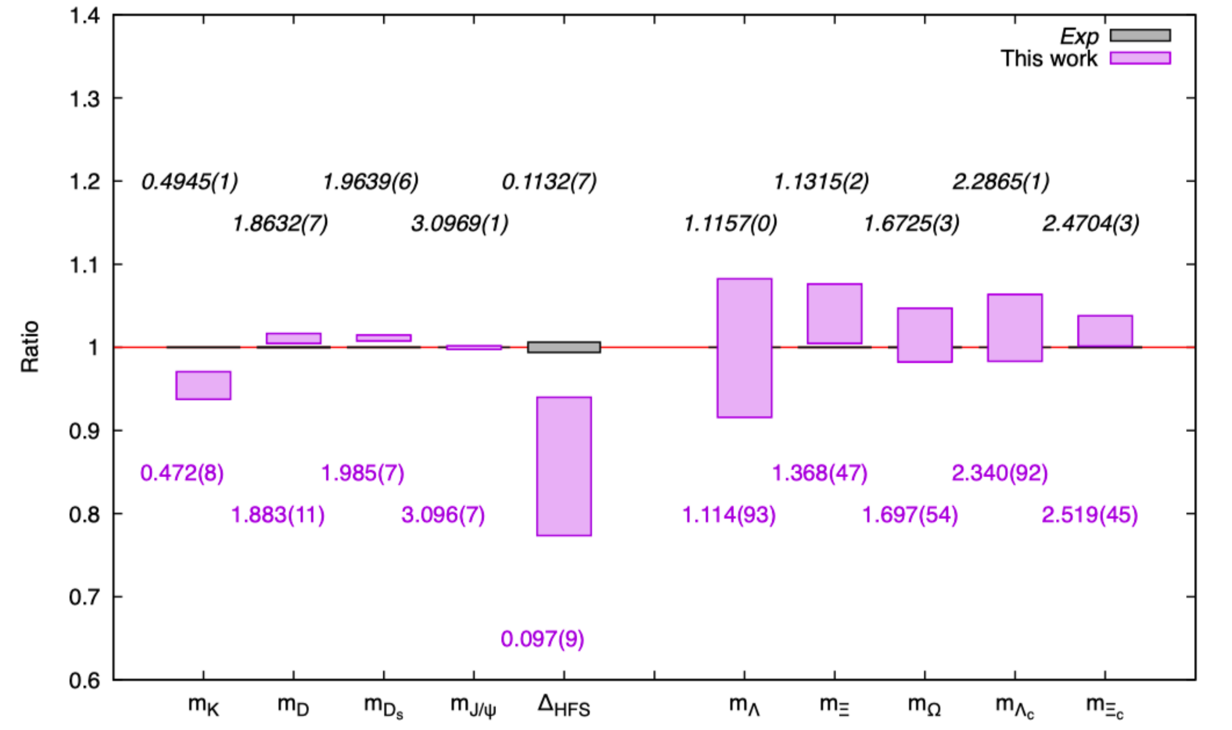
$$1^{++} : \mathcal{O}_3 = \epsilon^{abc}\epsilon^{def}[u_a^T C\gamma_5 s_b][\bar{d}_d C\gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_i \gamma_5 q_c]$$

$$1^{--} : \mathcal{O}_4 = \epsilon^{abc}\epsilon^{def}[u_a^T C\gamma_5 s_b][\bar{d}_d C\gamma_5 \bar{s}_e^T] \times [\bar{c}_f \gamma_i c_c]. \ ($$

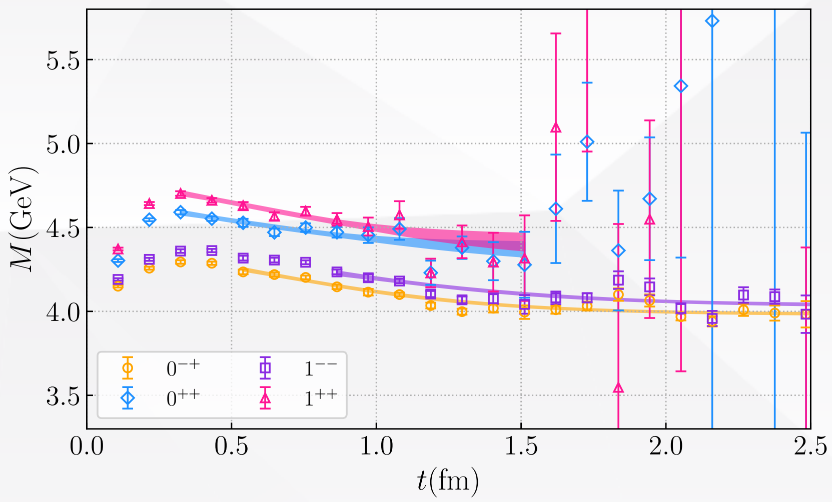
Hexaquark

	β	$L^3 \times T$	a	κ_l	m_π	κ_s	κ_c
C11P29S	6.2	$24^3 \times 72$	0.108	0.1343	285(2)	0.1327	0.1117
C11P22M	6.2	$32^3 \times 64$	0.108	0.1344	220(2)	0.1326	0.1116
C08P30S	6.41	$32^3 \times 96$	0.080	0.1326	301(3)	0.1316	0.1181
C06P30S	6.72	$48^3 \times 144$	0.054	0.1311	311(6)	0.1305	0.1227

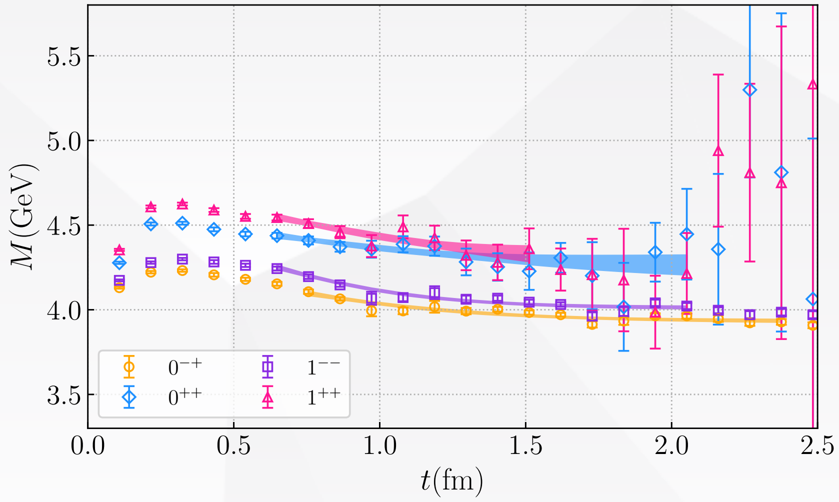




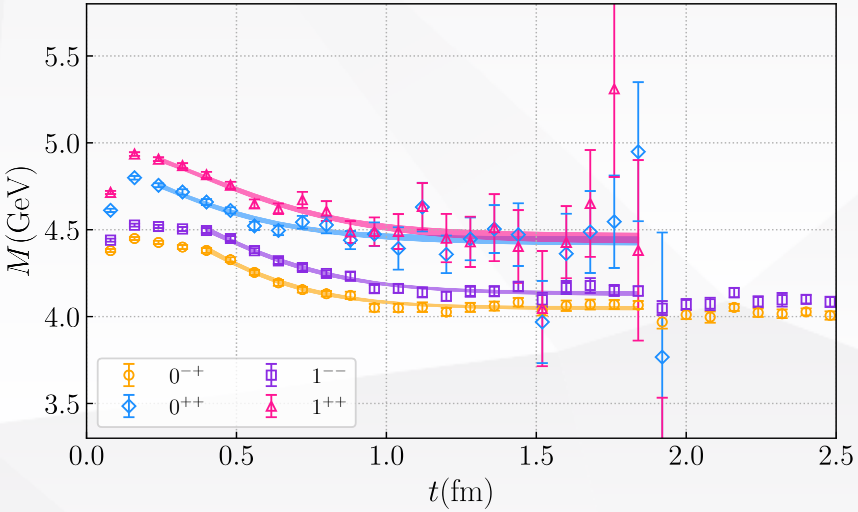
Hexaquark



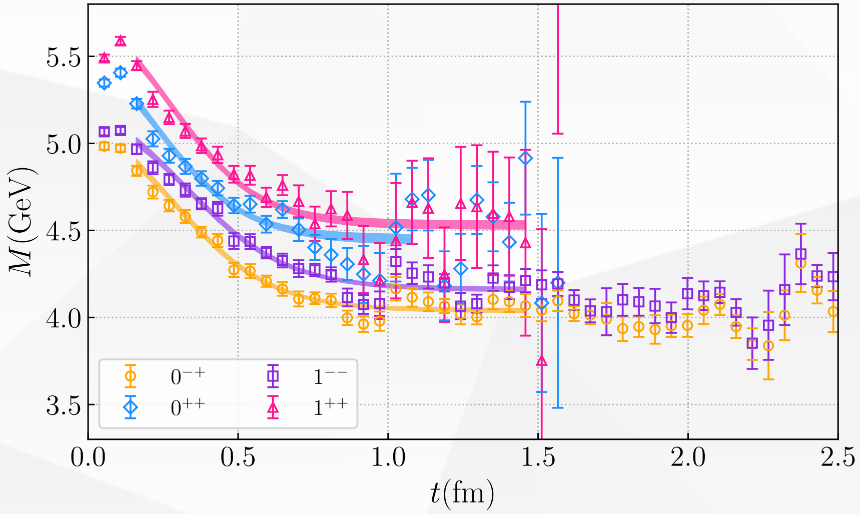
C11P29S



C11P22M

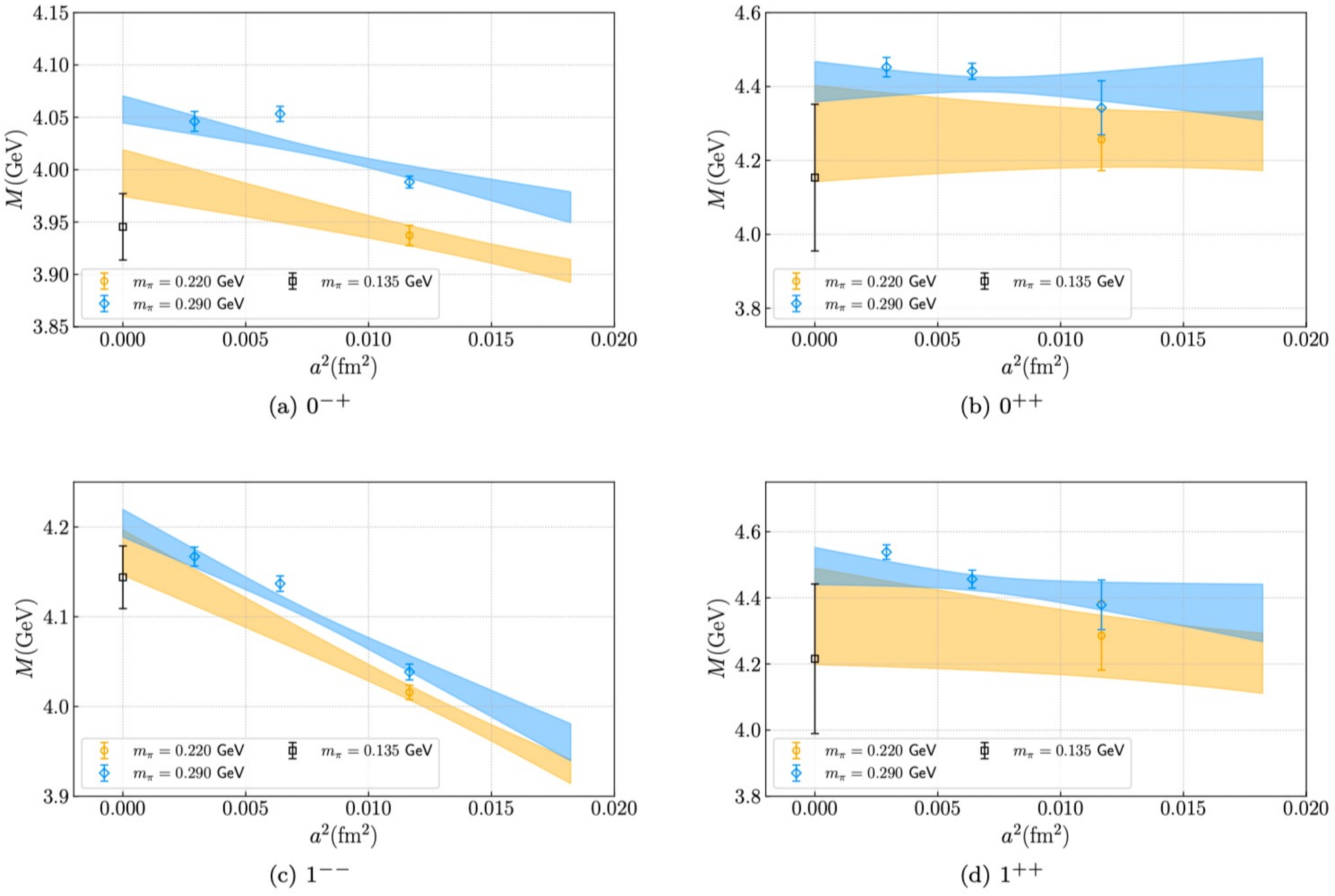


C08P30S



C06P30S

Hexaquark



(a) 0^{-+} (b) 0^{++} (c) 1^{--} (d) 1^{++}

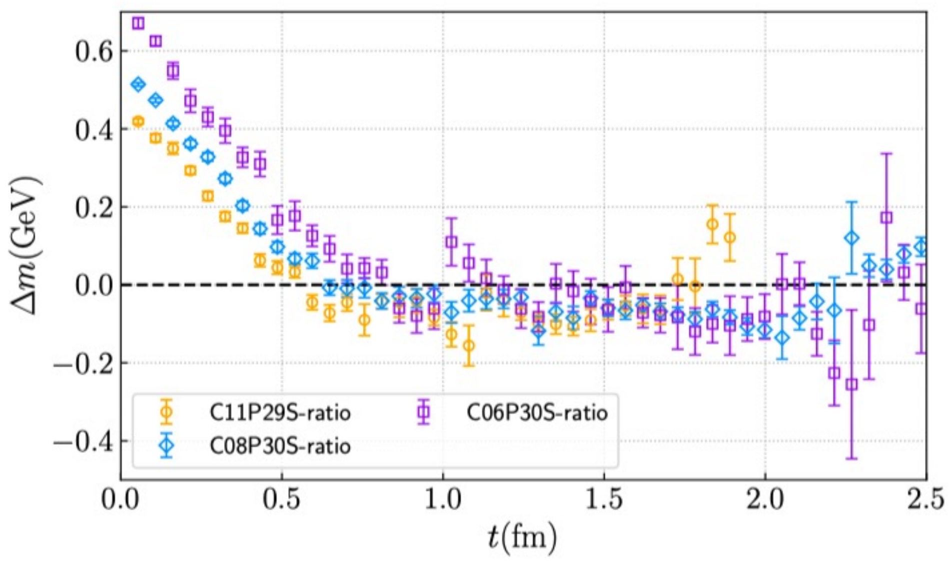
$I(J^{PC})$	$1(0^{-+})$	$1(1^{--})$	$1(0^{++})$	$1(1^{++})$
mass(GeV)	3.945(32)	4.144(35)	4.15(20)	4.22(23)

$$m_H(m_\pi, a) = m_{H,\text{phys}} + g_1^H(m_\pi^2 - m_{\pi,\text{phys}}^2) + g_2^H a^2,$$

物理延拓

Hexaquark

$I(J^{PC})$	$1(0^{-+})$	$1(1^{--})$	$1(0^{++})$	$1(1^{++})$
mass(GeV)	3.945(32)	4.144(35)	4.15(20)	4.22(23)



the 0^{-+} hexaquark might mix with $K^+K^-\eta_c$: 3.971 GeV

$$\Delta m \equiv \log \frac{C_{2,O_2}}{C_{2,K}^2 C_{2,\eta_c}}$$

Summary

- ✓ Charmed Hadrons: testing SM, probing NP, understanding QCD

- ✓ A set of new configurations have been generated and can be used for multi-purpose phenomenological analysis.

- ✓ The first lattice QCD calculation of $\Xi_c \to \Xi$ form factors and CKM matrix element:

  $|V_{cs}| = 0.834 \pm 0.051 \pm 0.056 \pm 0.127.$ Belle data

  $|V_{cs}| = 0.983 \pm 0.060 \pm 0.065 \pm 0.167.$ ALICE data

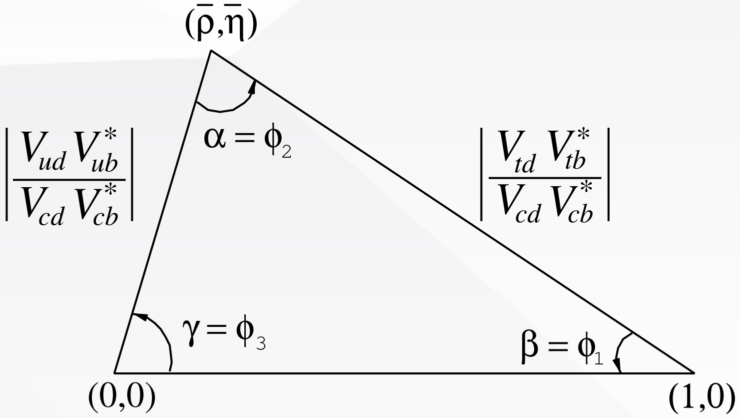
- ✓ Hidden-charm hexaquark candidates on LQCD: 4 ensembles. Tightly-bounded

- ✓ More exciting analyses of charmed hadrons:
  - ✓ Computational resources
  - ✓ New LQCD ensembles
  - ✓ More human resources

backup

Charm Quark Physics: Inputs for B decays

$$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$$



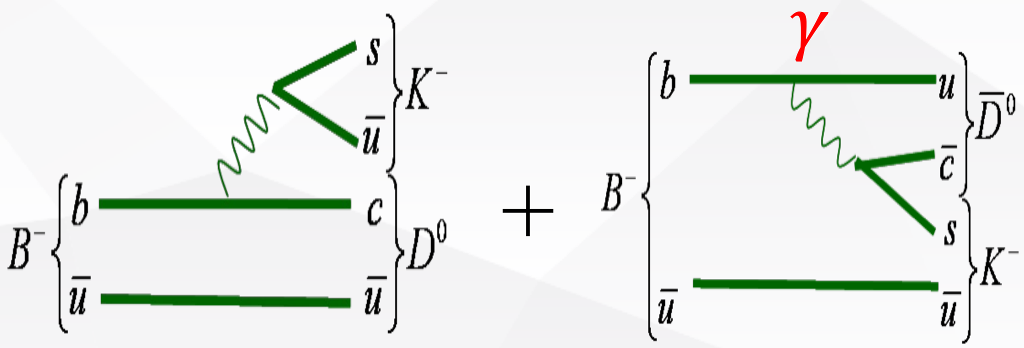
PDG2021

$$\alpha = \left(84.9^{+5.1}_{-4.5}\right)^\circ$$

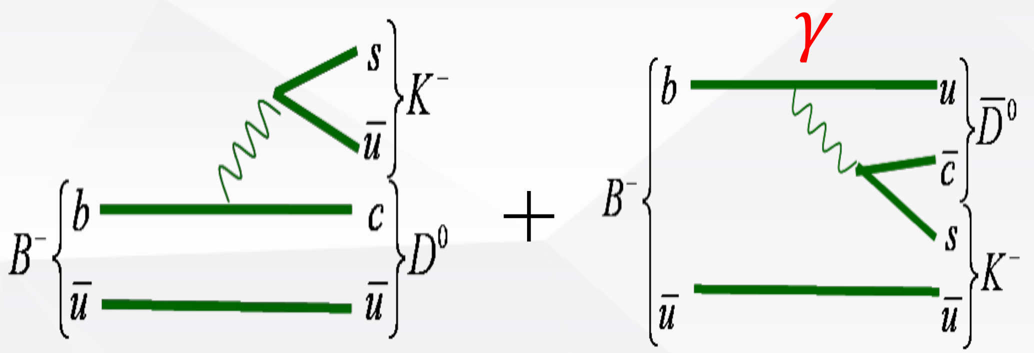
$$\beta = (22.2 \pm 0.7)^\circ \qquad \sin(2\beta) = 0.699 \pm 0.017$$

$$\gamma = \left(72.1^{+4.1}_{-4.5}\right)^\circ$$

$$\alpha + \beta + \gamma = \left(179^{+7}_{-6}\right)^\circ$$



Charm Quark Physics: Inputs for B decays



Direct CPA in D decays ($D^0 \to K^+K^-/\pi^+\pi^-$) can give an important corrections to γ: ($0.5^\circ - 5^\circ$)
[WW, PRL110, 061802(2013)
Martone and Zupan，PRD87， 034005(2013)
Bhattacharya, et.al, PRD87, 074002 (2013)]

- ✓ GLW: CP eigenstate [PLB 265, 172 (1991); PLB 253, 483 (1991)]
- ✓ ADS: Doubly-Cabibbo-suppressed [PRL78, 3257 (1997)]
- ✓ Dalitz: multi-body D decays [PRD68, 054018 (2003)]

The knowledge of charmed hadron decays at BESIII and other experimental facilities are prerequisites.