Small-x evolution of the gluon GPD E_g

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Outline:

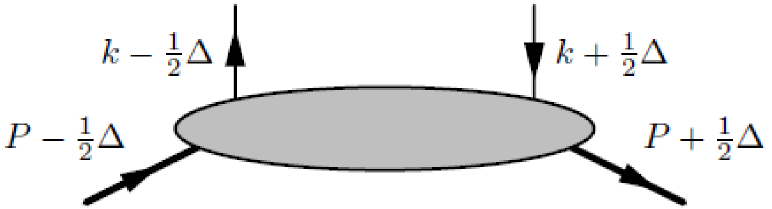
1: Background

2: The derivation of the evolution equation

3: Phenomengical consequences

4: Summary

Generalized Parton Distributions(GPDs)



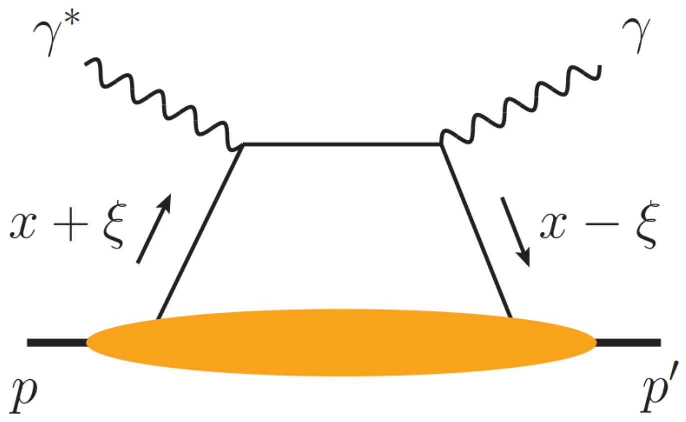
$$P = \frac{p+p'}{2} \qquad \Delta = p' - p$$

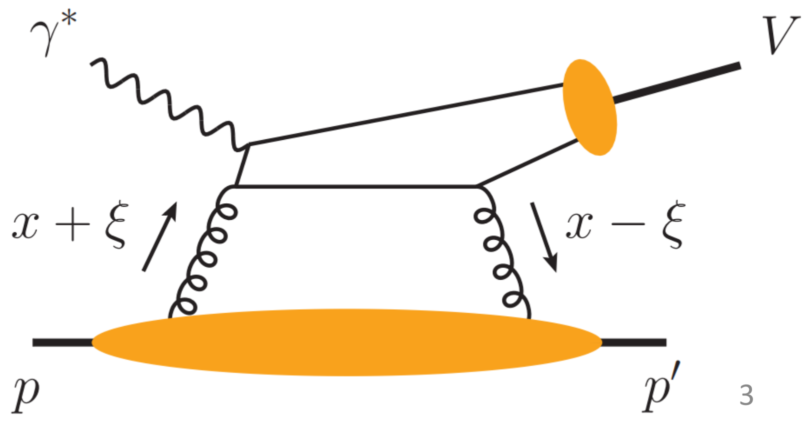
$$\int \frac{d\lambda}{2\pi} e^{ix(Pz)} n_{-\alpha} n_{-\beta} \left\langle p' \left| G^{\alpha\mu}\left(-\frac{z}{2}\right) G^{\beta}_{\mu}\left(\frac{z}{2}\right) \right| p \right\rangle \Big|_{z=\lambda n_-}$$

$$= \frac{1}{2}\left[H^g \, \bar{u}(p') \, \not{n}_- u(p) + E^g \, \bar{u}(p') \frac{i\sigma^{\alpha\beta} n_{-\alpha}\Delta_\beta}{2\, m_N} u(p) \right]$$

D. Muller, 94
X. D. Ji, 97
A. V. Radushkin, 97

x, ζ, t





Some properties of GPDs

- Form factors

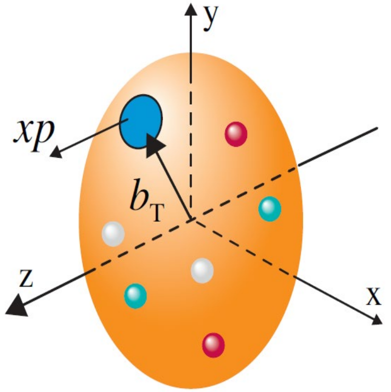
$$\sum_q e_q \int dx\, H^q(x,\xi,t) = F_1^p(t)\,, \qquad \sum_q e_q \int dx\, E^q(x,\xi,t) = F_2^p(t)$$

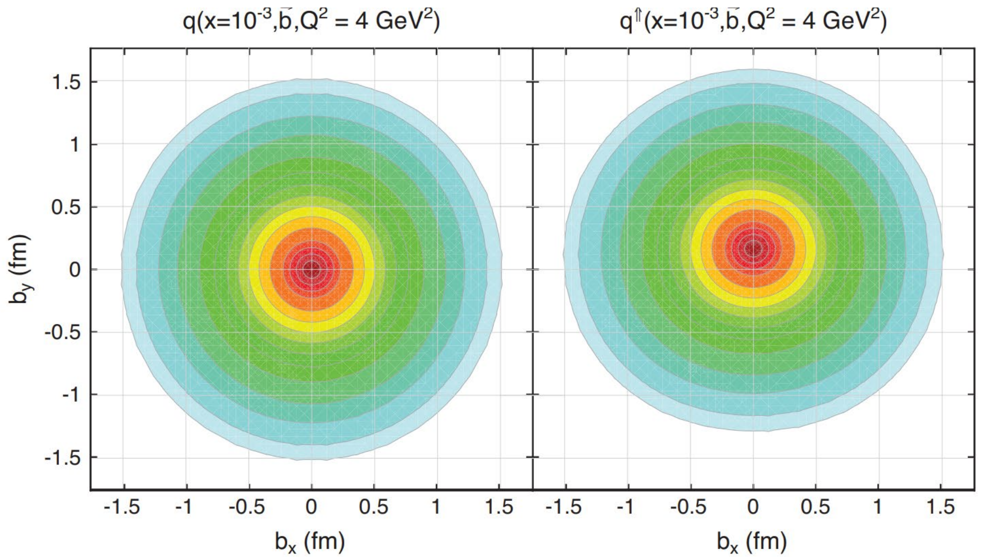
- Transverse spatial distribution

Soper 77 & Burkardt 2000

$$\mathcal{H}^q(x,\vec{b}_T^2) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2}\, e^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, H^q(x,0,-\vec{\Delta}_T^2)$$

$$f_q(z,b_{\perp,q}) = \mathcal{H}_q(z,b_{\perp,q}^2) + \frac{1}{M}\epsilon_\perp^{ij} b_{\perp,q}^i S_\perp^j \frac{\partial \mathcal{E}_q(z,b_{\perp,q}^2)}{\partial b_{\perp,q}^2}$$





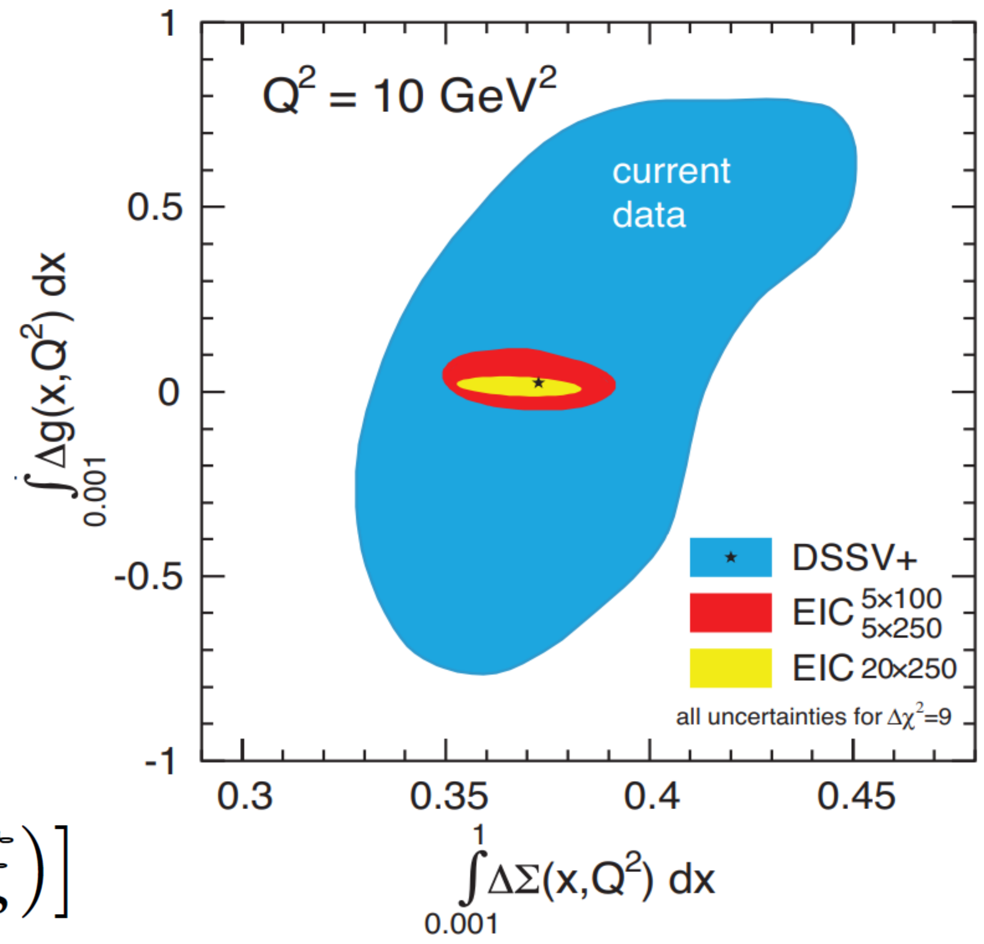
Proton spin decomposition

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

Canonical V.S. Kinematical

Mv+eA Mv

Jaffe, Manohar 1990
Ji, 1997
X. S. Chen et.al. 2008

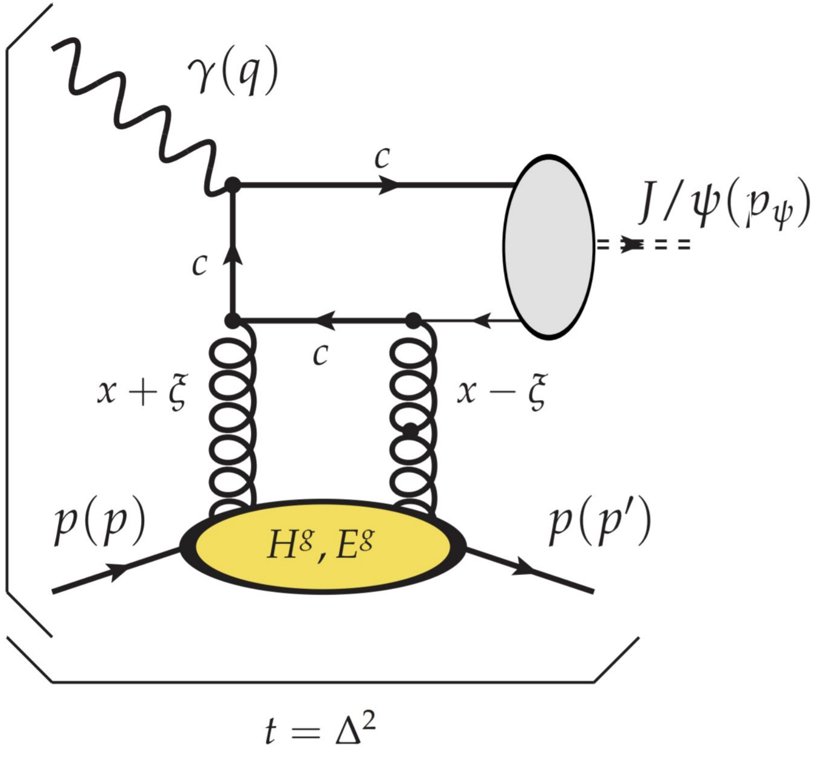


- Ji's sum rule

$$J_g = \frac{1}{2}\int_0^1 dx\ x\left[H_g(x,\xi) + E_g(x,\xi)\right]$$

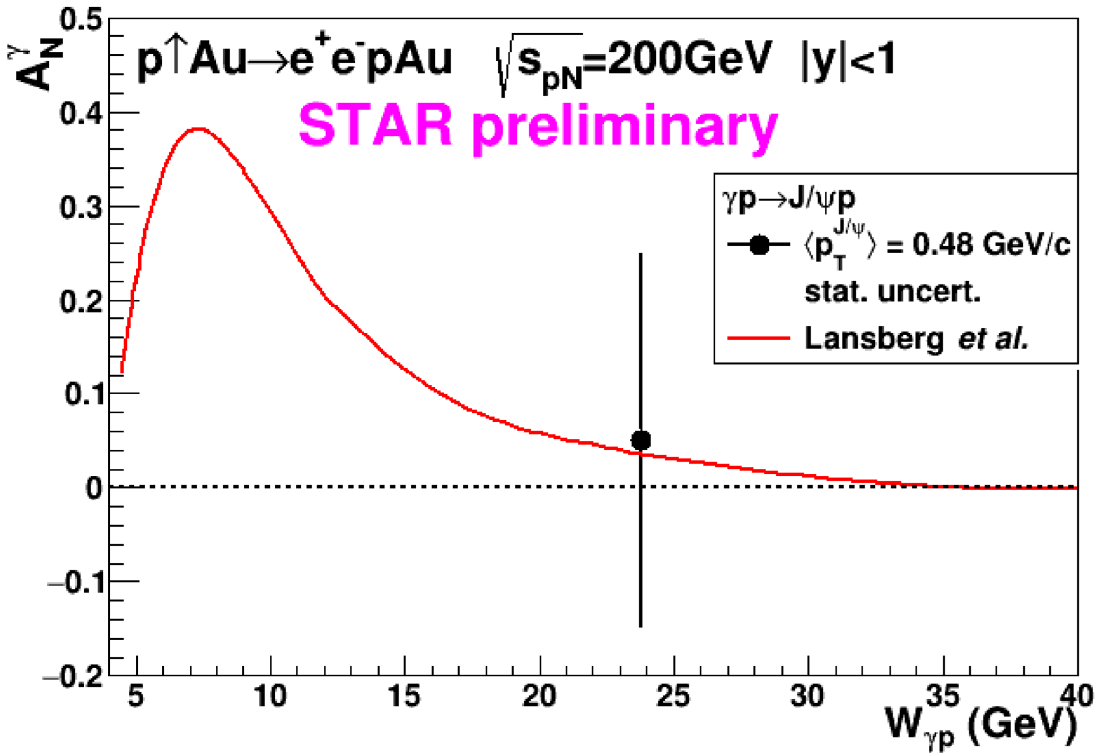
Ji, 1997

TSSA in exclusive J/psi production



$$A_N^\gamma = \frac{\frac{1}{2m_N}(1+\xi)|\Delta_T|\sin(\phi_{\vec{\Delta}})\,\Im(\mathcal{H}^g\mathcal{E}^{g\,\star})}{(1-\xi^2)|\mathcal{H}^g|^2 + \frac{\xi^4}{1-\xi^2}|\mathcal{E}^g|^2 - 2\xi^2\Re(\mathcal{H}^g\mathcal{E}^{g\,\star})}$$

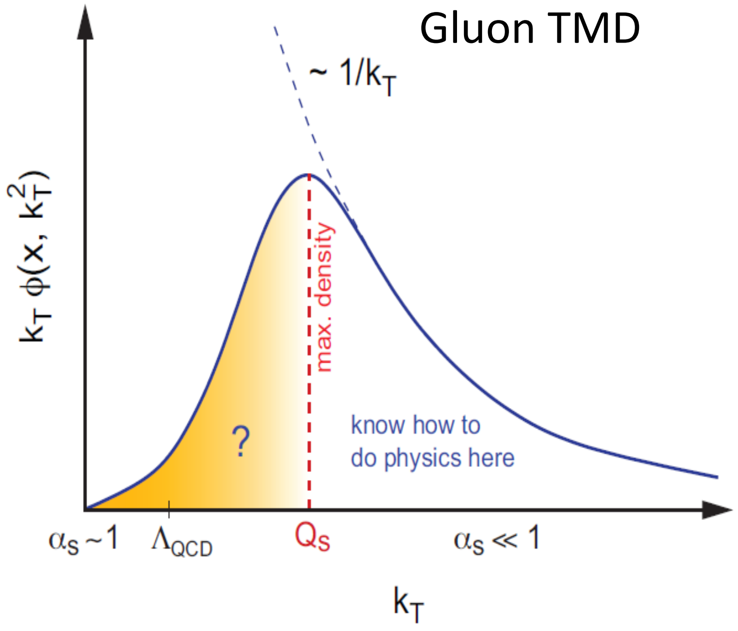
Koempel, Kroll, Metz, ZJ, 2012



Lansberg, Massacrier, Szymanowski, Wagner, 2018

Small x evolution equations

- DGLAP $\ln\frac{Q^2}{\mu^2}$
- BFKL(CCFM) $\ln\frac{1}{x}$
- BK(JIMWLK, GLR-MQ) $\ln\frac{1}{x}$



take into account the saturation effect $\frac{1}{N_c}\mathrm{Tr}U(b_\perp + r_\perp/2)U^\dagger(b_\perp - r_\perp/2)$

$$\partial_Y \mathcal{N}(\mathbf{x},\mathbf{y}) = \frac{\bar{\alpha}}{2\pi}\int d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\mathcal{N}(\mathbf{x},\mathbf{z}) + \mathcal{N}(\mathbf{z},\mathbf{y}) - \mathcal{N}(\mathbf{x},\mathbf{y}) - \mathcal{N}(\mathbf{x},\mathbf{z})\mathcal{N}(\mathbf{z},\mathbf{y})\right]$$

Balitsky, 1996
Kovchegov, 1997

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

- BK equation in momentum space:

$$\partial_Y \mathcal{N}(k_\perp,\Delta_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left\{\mathcal{N}(k'_\perp,\Delta_\perp) - \frac{1}{4}\left[\frac{(\frac{\Delta_\perp}{2}+k_\perp)^2}{(\frac{\Delta_\perp}{2}+k'_\perp)^2} + \frac{(\frac{\Delta_\perp}{2}-k_\perp)^2}{(\frac{\Delta_\perp}{2}-k'_\perp)^2}\right]\mathcal{N}(k_\perp,\Delta_\perp)\right\}$$

$$-\frac{\bar{\alpha}_s}{2\pi}\int d^2\Delta'_\perp \mathcal{N}(k_\perp + \frac{\Delta'_\perp}{2}, \Delta_\perp - \Delta'_\perp)\mathcal{N}(k_\perp + \frac{\Delta'_\perp - \Delta_\perp}{2}, \Delta'_\perp)$$

Forward limit

◆ Typical nucleon recoiled transverse momentum is reversely proportional to the radius of nucleon,

$$\mathcal{F}_{1,1}(k_\perp, \Delta_\perp) = \overline{\mathcal{F}}_{1,1}(k)(2\pi)^2 \delta^{(2)}(\Delta_\perp)$$

◆ The forward BK(for the unpolarized gluon TMD) reads,

$$\partial_Y \overline{\mathcal{F}}_{1,1}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \left\{ \overline{\mathcal{F}}_{1,1}(k'_\perp) - \frac{1}{2} \frac{k_\perp^2}{k'^2_\perp} \overline{\mathcal{F}}_{1,1}(k_\perp) \right\} - 4\pi^2 \alpha_s^2 \left[\overline{\mathcal{F}}_{1,1}(k_\perp) \right]^2$$

Generalized TMDs

- Parametrization

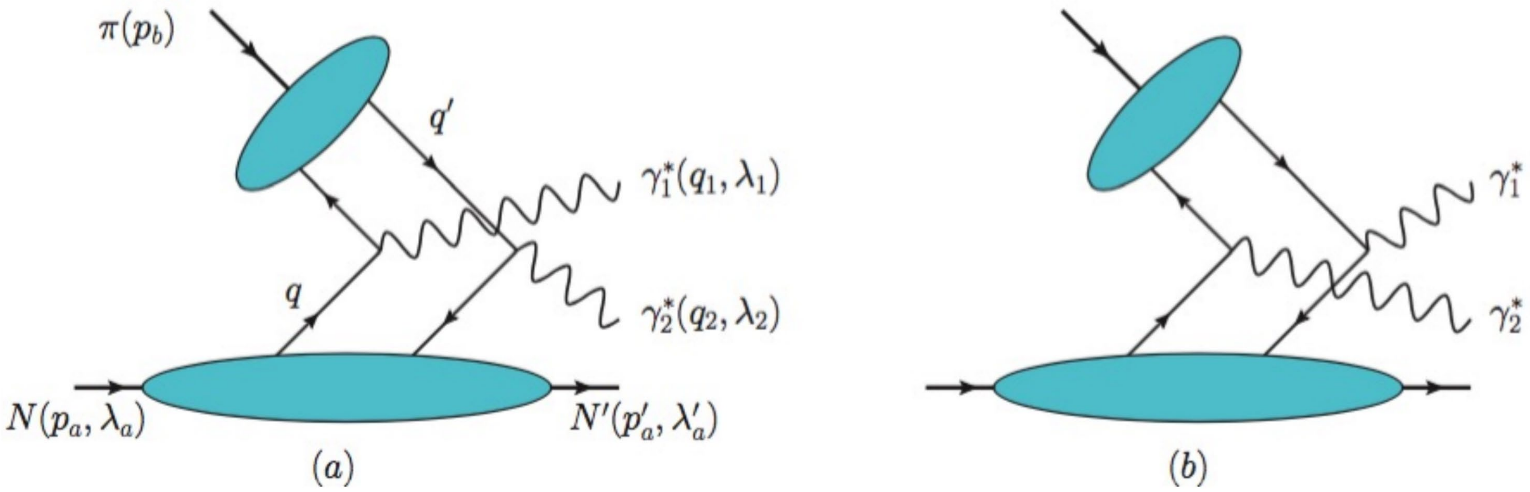
$$\mathcal{N}(k_\perp, \Delta_\perp, S_\perp) \approx \frac{\pi g^2}{2N_c}\left\{\mathcal{F}_{1,1}(k_\perp,\Delta_\perp) - ik_\perp \times S_\perp \frac{k_\perp\cdot\Delta_\perp}{M^4}\mathcal{F}_{1,2}(k_\perp,\Delta_\perp) - i\frac{\Delta_\perp\times S_\perp}{2M^2}\left[2\mathcal{F}_{1,3}(k_\perp,\Delta_\perp) - \mathcal{F}_{1,1}(k_\perp,\Delta_\perp)\right]\right\}$$

$$\mathcal{N}(k_\perp,\Delta_\perp) = \frac{1}{(2\pi)^2}\int d^2r_\perp e^{ik_\perp\cdot r_\perp}\frac{N(r_\perp,\Delta_\perp)}{r_\perp^2}$$

Assuming target transversely polarized.

- Exclusive double Drell-Yan process:

S. Bhattacharya, A. Metz, ZJ, 2017



The relation between GTMDs and GPDs

- Related to:

$$xH_g(x) = \int d^2k_\perp \; k_\perp^2 \frac{\partial^2}{\partial k_\perp^\alpha \partial k_\perp^\alpha} \mathcal{F}_{1,1}(k_\perp)$$

- Kt dependent angular momentum “density”

$$x\left[H_g(x,\xi) + E_g(x,\xi)\right] = \int d^2k_\perp \; k_\perp^2 \frac{\partial^2}{\partial k_\perp^\alpha \partial k_\perp^\alpha} \mathcal{J}(k_\perp)$$

with

Hatta, ZJ, 2022

$$\mathcal{J}(k_\perp) \equiv 2\mathcal{F}_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2}\mathcal{F}_{1,2}(k_\perp)$$

Spin-dependent small x evolution equation

- Project to the different spin correlation structures,

$$\partial_Y \left(k_\perp \times S_\perp \frac{k_\perp^i}{M^2} \mathcal{F}_{12}(k_\perp) + \epsilon^{ij} S_\perp^j \left(\mathcal{F}_{13}(k_\perp) - \frac{1}{2}\mathcal{F}_{11}(k_\perp) \right) \right) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \Bigg[k'_\perp \times S_\perp \frac{k'^i_\perp}{M^2} \mathcal{F}_{12}(k'_\perp)$$
$$+ \frac{\epsilon^{ij} S_\perp^j}{2} (2\mathcal{F}_{13}(k'_\perp) - \mathcal{F}_{11}(k'_\perp)) - \frac{k_\perp^2}{2k'^2_\perp} \left(k_\perp \times S_\perp \frac{k_\perp^i}{M^2} \mathcal{F}_{12}(k_\perp) + \frac{\epsilon^{ij} S_\perp^j}{2} (2\mathcal{F}_{13}(k_\perp) - \mathcal{F}_{11}(k_\perp)) \right) \Bigg]$$
$$-4\pi^2 \alpha_s^2 \left(k_\perp \times S_\perp \frac{k_\perp^i}{M^2} \mathcal{F}_{1,2}(k_\perp) + \frac{\epsilon^{ij} S_\perp^j}{2} (2\mathcal{F}_{1,3}(k_\perp) - \mathcal{F}_{1,1}(k_\perp)) \right) \overline{\mathcal{F}}_{1,1}(k_\perp),$$

- Read off the coefficients of $k_\perp \times S_\perp$ $\frac{\epsilon^{ij} S_\perp^j}{2}$

$$\partial_Y \mathcal{F}_{1,2}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \left[-\frac{k_\perp^2}{2k'^2_\perp} \mathcal{F}_{1,2}(k_\perp) + \frac{2(k_\perp \cdot k'_\perp)^2 - k_\perp^2 k'^2_\perp}{(k_\perp^2)^2} \mathcal{F}_{1,2}(k'_\perp) \right] - 4\pi^2 \alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp) \mathcal{F}_{1,2}(k_\perp)$$

and,

$$\partial_Y \mathcal{F}_{1,3}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \left[-\frac{k_\perp^2}{2k'^2_\perp} \mathcal{F}_{1,3}(k_\perp) + \frac{k_\perp^2 k'^2_\perp - (k_\perp \cdot k'_\perp)^2}{k_\perp^2} \frac{\mathcal{F}_{1,2}(k'_\perp)}{M^2} + \mathcal{F}_{1,3}(k'_\perp) \right] - 4\pi^2 \alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp) \mathcal{F}_{1,3}(k_\perp)$$

Small x evolution of Eg

- Combine the evolution equations for $F_{1,2}$ and $F_{1,3}$

$$\partial_Y \mathcal{J}(k_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left[\mathcal{J}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp}\mathcal{J}(k_\perp)\right] - 4\pi^2\alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{J}(k_\perp)$$

Note: $\mathcal{E} \equiv -\mathcal{F}_{1,1} + \mathcal{J}$

- Small x evolution equation for kt dependent Eg,

$$\partial_Y \mathcal{E}(k_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp}\mathcal{E}(k_\perp)\right] - 4\pi^2\alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{E}(k_\perp)$$

Hatta, ZJ, 2022

- In the dilute limit:

$$xE_g(x) \sim xG(x) \propto \left(\frac{1}{x}\right)^{\bar{\alpha}_s 4 \ln 2}$$

Numerical results

- The MV model (x_0=0.01) $Y = \ln \frac{x_0}{x}$

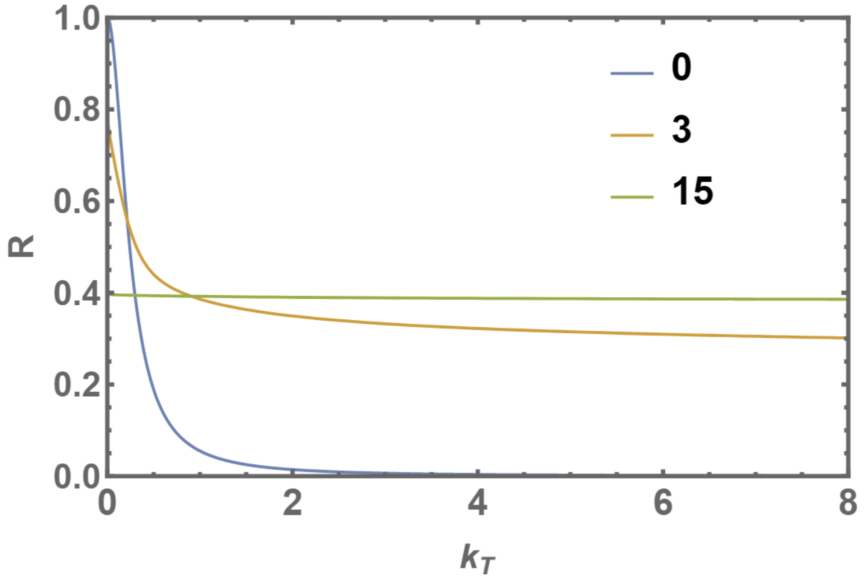
$$\mathcal{F}_{1,1}(Y=0,k_\perp) = \frac{N_c A_\perp}{2\pi^2 \alpha_s} \int \frac{d^2 r_\perp}{(2\pi)^2 r_\perp^2} e^{-ik_\perp \cdot r_\perp} \left\{ 1 - \exp\left[-\frac{r_\perp^2 Q_{s0}^2}{4} \ln\left(\frac{1}{r_\perp \Lambda_{\rm mv}} + e \right) \right] \right\}$$

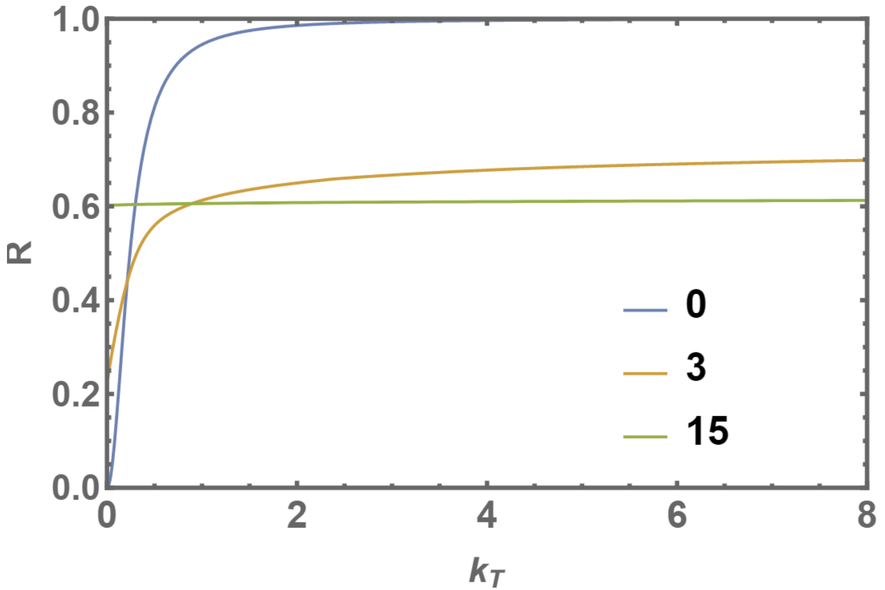
- Two toy models:

$$\mathcal{E}(Y=0,k_\perp) = \frac{\Lambda_{\rm mv}^2}{k_\perp^2 + \Lambda_{\rm mv}^2} \mathcal{F}_{1,1}(Y=0,k_\perp)$$

$$\mathcal{E}(Y=0,k_\perp) = \frac{k_\perp^2}{k_\perp^2 + \Lambda_{\rm mv}^2} \mathcal{F}_{1,1}(Y=0,k_\perp)$$

$$R \equiv \frac{\mathcal{E}(x,k_\perp)}{\mathcal{F}_{1,1}(x,k_\perp)}$$





Summary

- Small x evolution equation for gluon GPD Eg
- OAM from small x is not negligible
- Sizable TSSA in exclusive processes

- Quark GPD Eq at small x?
- Next to leading log behavior?

Thank you !

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