Hadronic contributions to HVP and LBL: from amplitude analysis

Lingyun Dai

Hunan University

with S.Q.Kuang, S.J.Wang, Y. L. Ye, W. Qin, J. Portoles, *et.al.*

Based on: PRD88 (2013) 056001; PLB736(2014)11;
PRD90 (2014) 036004; PRD94 (2016) 116061;
PRD95 (2017) 056007; PRD97 (2018) 036012;
JHEP03(2021)092, RPP84(2021)076201, *et.al.*

第四届重味物理与量子色动力学研讨会

长沙，2022.07

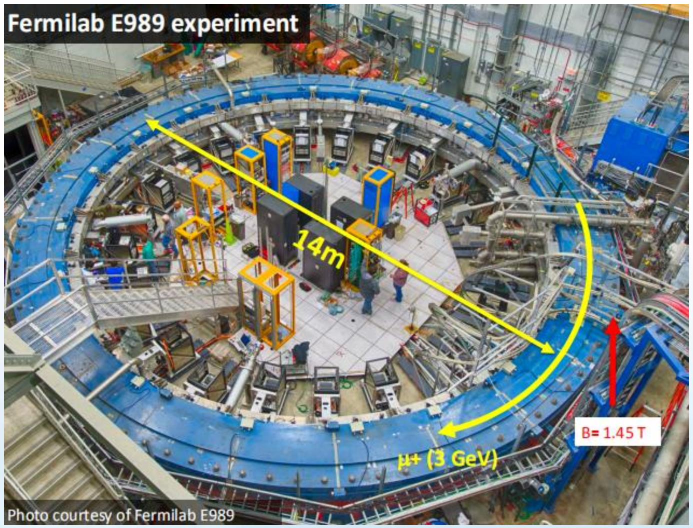
湖南大學
HUNAN UNIVERSITY

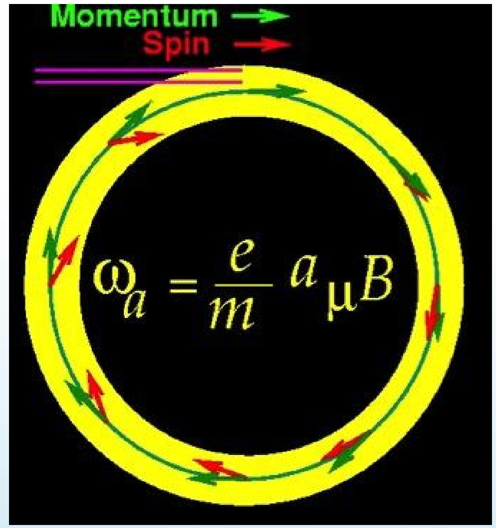
Outlines

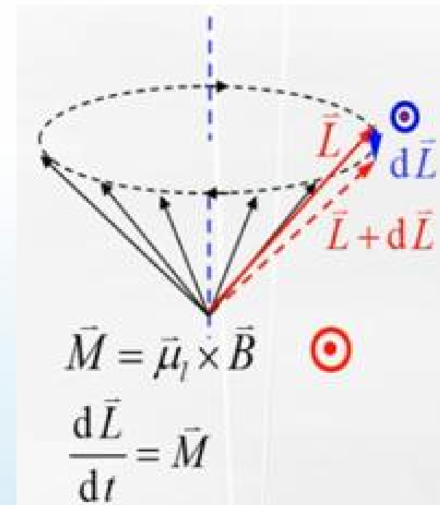
1. Introduction
2. FSI: HVP
3. FSI: HLBL
4. Summary

Introduction: muon g-2

- The most precise indicator of new physics





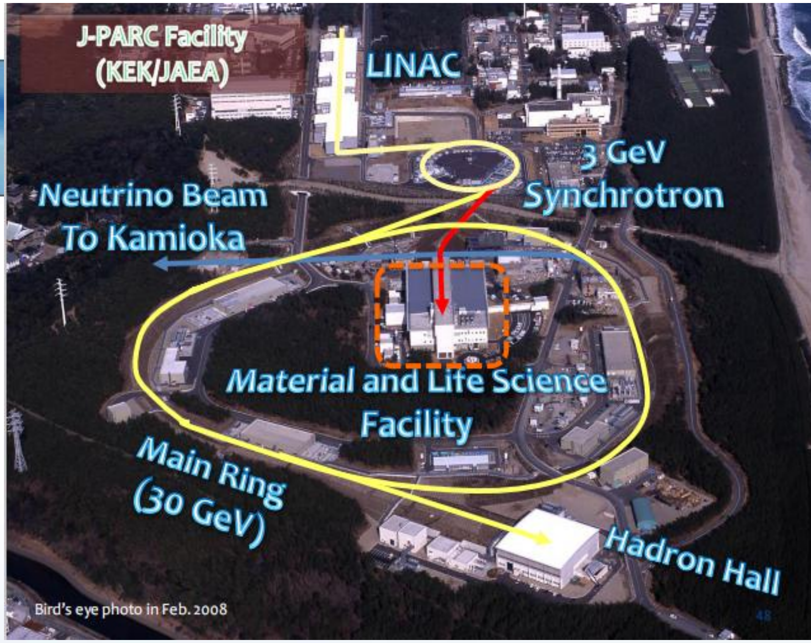


- muon spin precession $\omega_a = \frac{e}{m_\mu} a_\mu B$
- proton spin precession $\omega_p = \mu_p B$
- muon magnetic moment $\mu_\mu = g\frac{e}{2m_\mu} = (1 + a_\mu)\frac{e}{m_\mu}$

$$\vec{\mu}_S = g\frac{q}{2m}\vec{S} \quad a = \frac{g-2}{2}$$

$$a_\mu = \frac{\omega_a/\omega_p}{\omega_a/\omega_p - \mu_\mu/\mu_p}$$

Tsutomu Mibe, talk at cLFV school



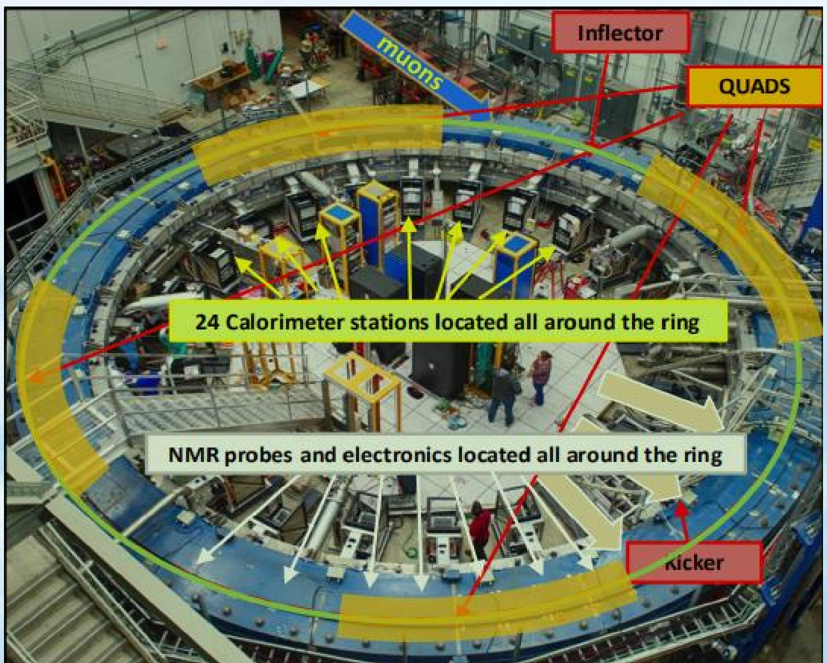
J-PARC

BNL E821 J-PARC E3

g-2: 0.46 ppm → 0.37 ppm (→0.1ppm)

50 times of number of events as large as BNL's to 0.46ppm

2001，2009，2025?



FNAL

Run1: only 6% of full statistics used now

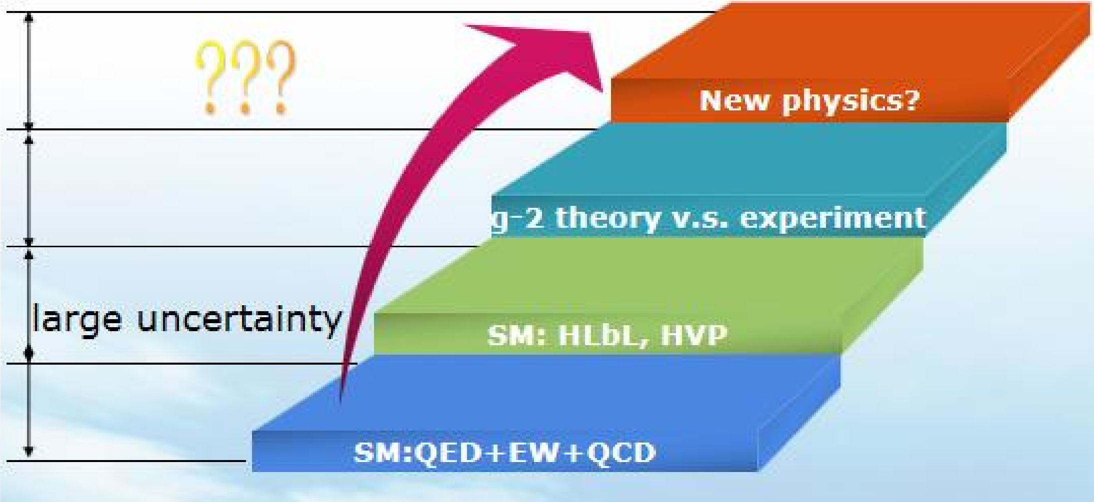
Run2-3: analyzing, factor 2 improvment

Run4: 13 times as large as BNL's

Run5: 20 times as large as BNL's

2017，2021，2025……

uncertainty from SM



$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm QCD}$$

- HVP, HLbL?

	values ($\times 10^{-11}$)
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
$a_\mu^{\rm SM}$-$a_\mu^{\rm exp}$	251(59)

Phys.Rev.Lett.126, 141801 (2021)
Phys.Rev.D 73, 072003 (2006).

Phys.Rept.887(2020)1

Science and Technology Cooperation Program in High Energy Physics. This review benefited from discussions with O. Catà, N. Christ, L.Y. Dai, H. Davoudiasl, S. Fayer, S. Ganguly, A. Gasparian, S. Hashimoto, T. Iijima, K. Kampf, D. Kawall, I. Larin, Z. Pagel, M. Petschlies, A. Rebhan, K. Schilcher, K. Shimomura, E. Shintani, D. Steffen, S. Tracz, C. Tu, and T. Yamazaki.

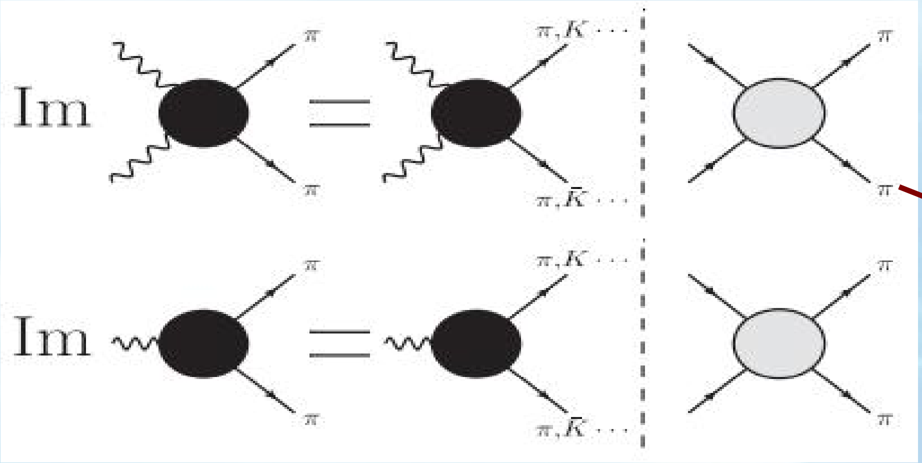
Methods from SM

- LQCD
- Data-driven solutions from experiment
- Amplitude analysis: model independent

  - Only one physical amplitude!
  - It should satisfy the fundamental QFT principles
  - It should be compatible with the data

why FSI ?

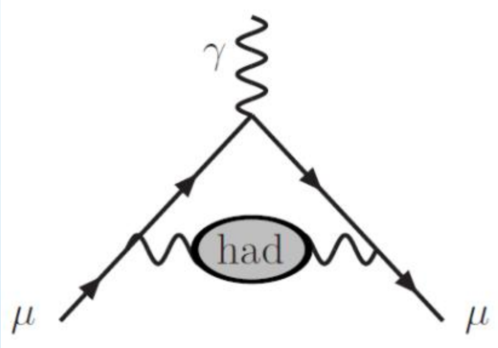
- Most resonances decay into light pseudoscalars
- FSI needs to be taken into account to perform an amplitude analysis
- Methods: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*



Yao, Dai, Zheng, Zhou, RPP84(2021)076201

2、HVP

- QCD: high energy region
- Dispersive approach: Roy, KT, PKU, etc., difficult to deal with multi-body rescattering
- ChPT: works in the low energy region
- RChT: extend to resonance region



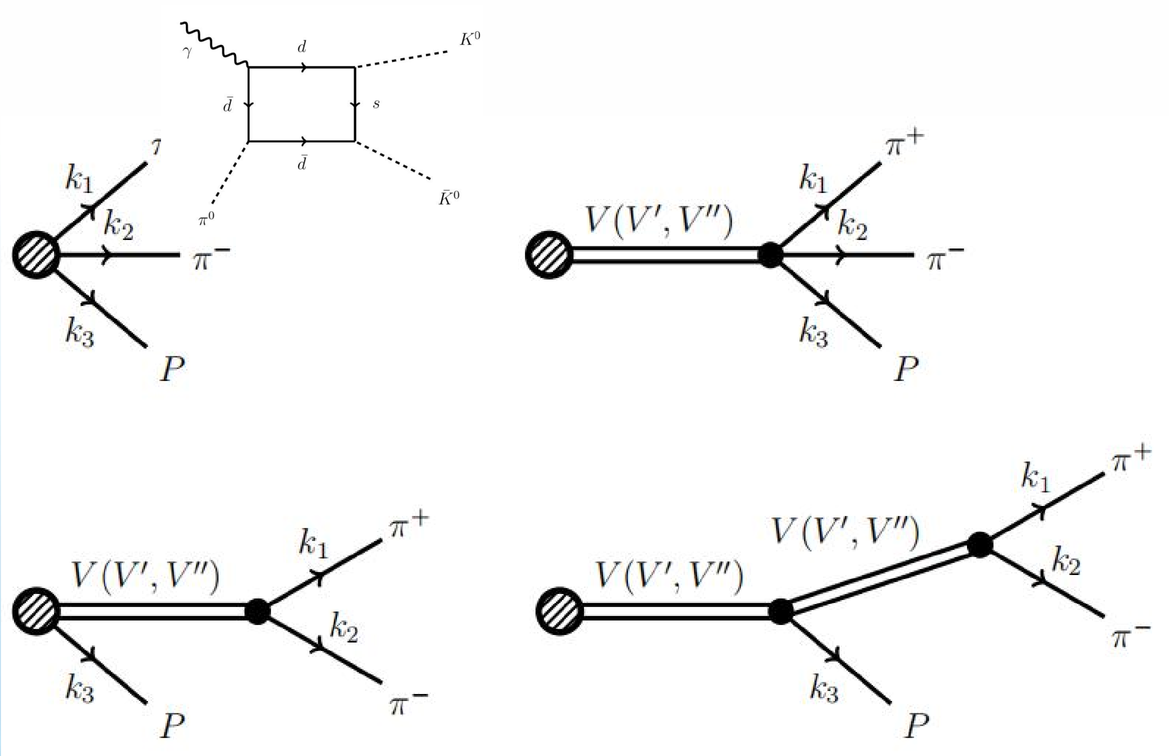
- resonances included as new degrees of freedom
- QCD high energy constraints to reduce LECs
- 1/Nc expansion

Dai *et.al.*, PRD99 (2019) 114015

Building amplitudes

- RChT in the resonance region, excited states?

- V', V'' has the same topologies as the ground states
- ππ-KK FSI part: ChPT matching with Omens functions



Guerrero, et.al., PLB 412 (1997) 382

Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- Combined analysis on lots of channels.
  - ππ-KK FSI part by matching with Omnes function
  - ρ-ω mixing, origined from Gasser&Leutwyler's
- Not much freedom for Fit

$$
\begin{aligned}
F_V^\pi = &\left(1 + \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\rho, \Gamma_{\rho,}, Q^2)\right.\right.\\
&\left. + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho',}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho'',}, Q^2)\right)\\
&\left(\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta\right) \cos\delta\\
&- \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\omega, \Gamma_{\omega,}, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega',}, Q^2)\right.\\
&\left.\left. + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega'',}, Q^2)\right) \left(\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega\right) \sin\delta^\omega \right)\\
&\exp\left[\frac{-s}{96\pi^2 F^2} \left(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\right)\right]
\end{aligned}
$$

=1, from QCD as well as disersion relation constraints

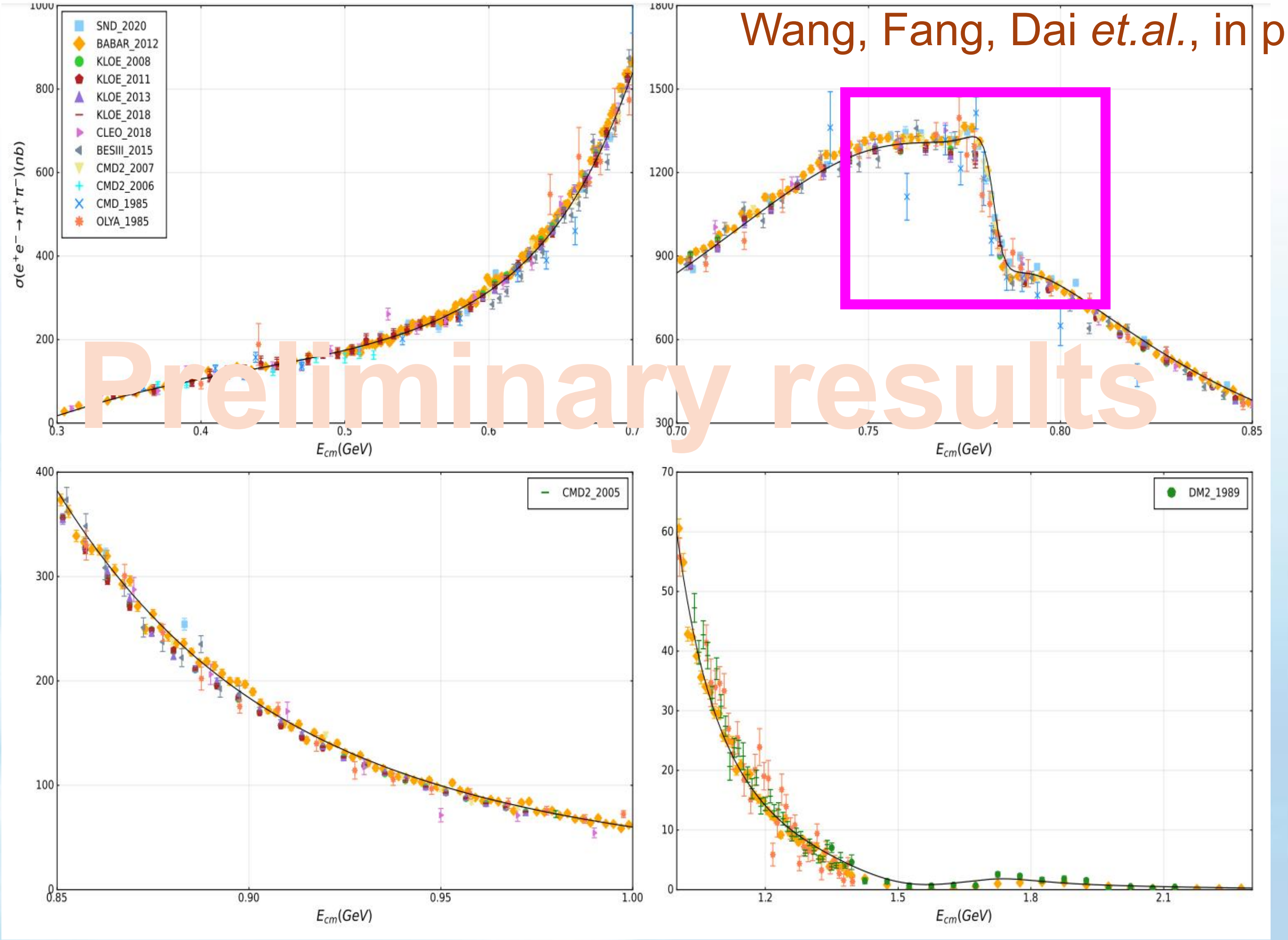
Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

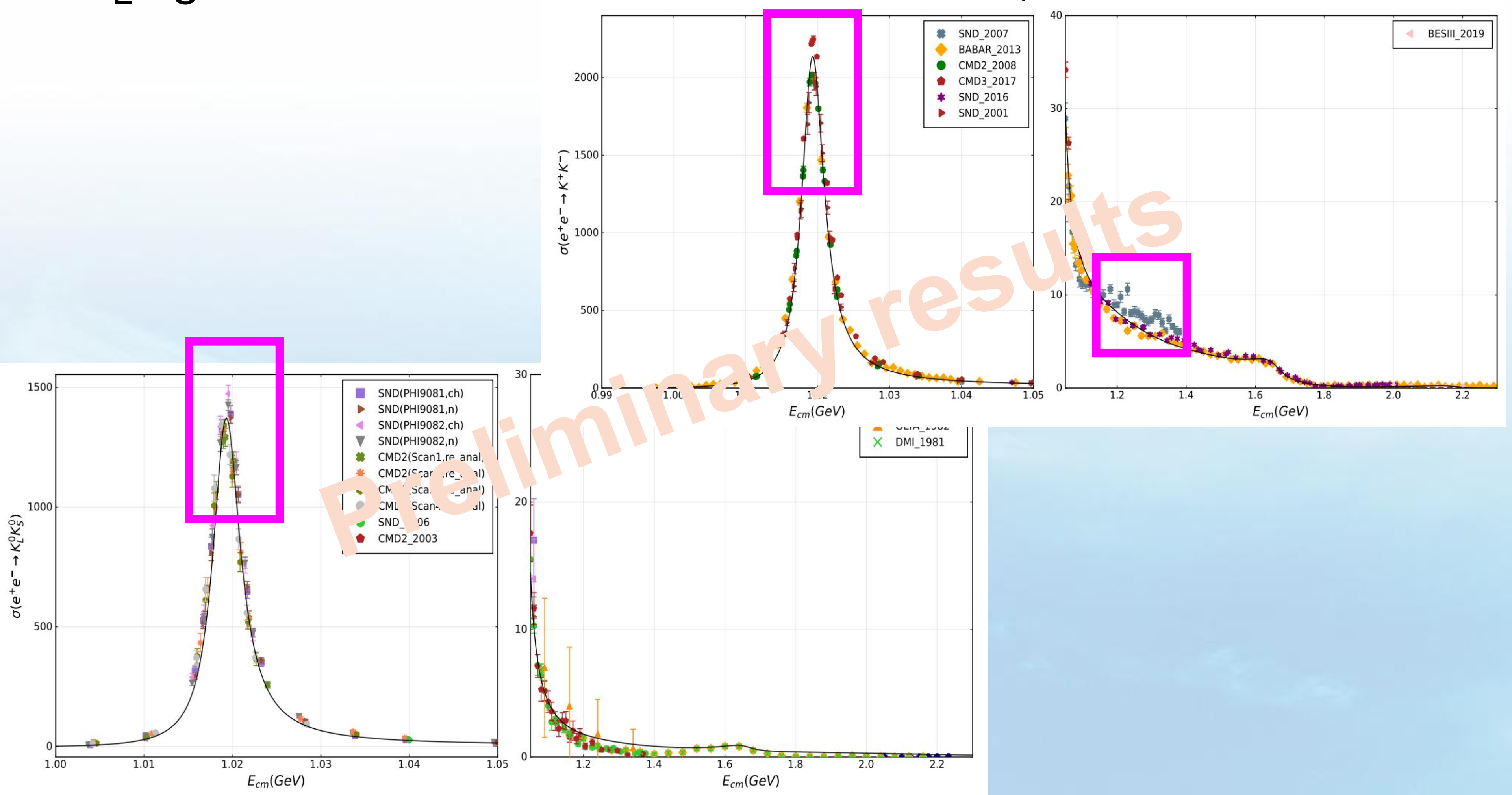
Fit

- $\pi\pi$: now closer to KLOE and BESIII's

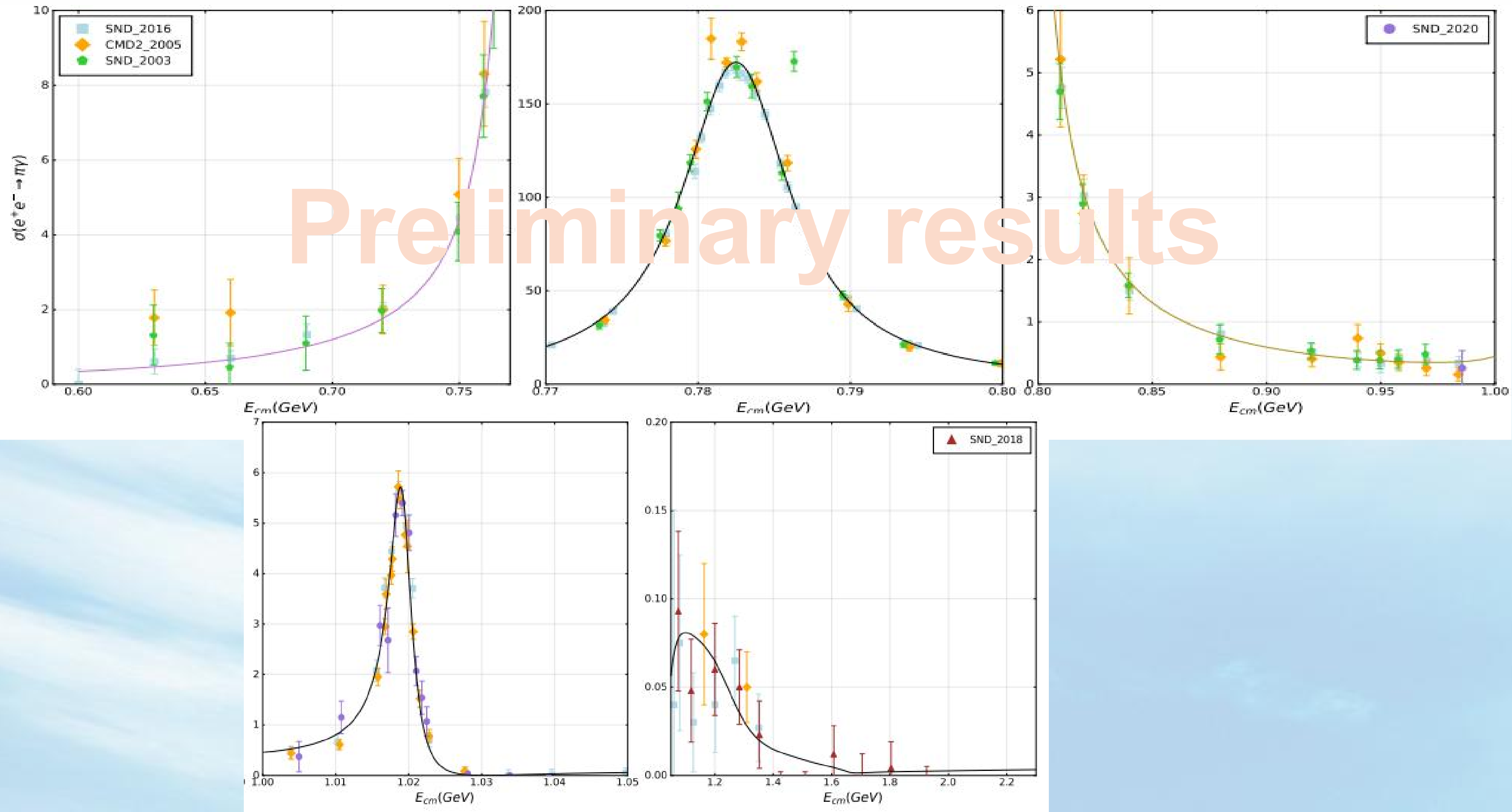
Wang, Fang, Dai *et.al.*, in preparation

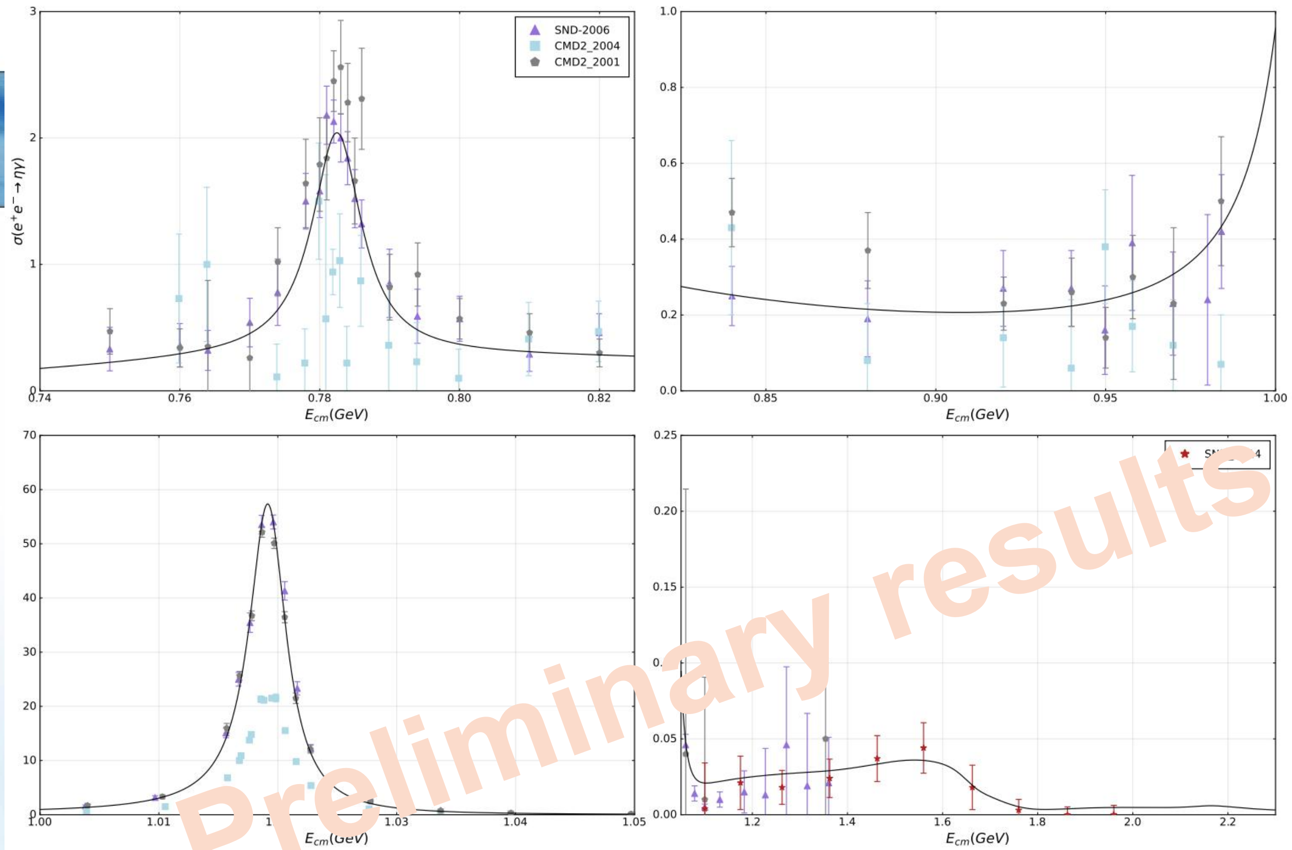


- KK: data in the ϕ 'peak' have large discrepancy
- K_LK_S: further direct constraints on $\pi\pi$, KK channels



- πγ: helps to constrain ππ, KK channels
- ηγ: helps to constrain KK





$a_\mu^{K_L^0K_S^0}\| \leq 1.1$ GeV	-	-	-	-	11.45
$a_\mu^{K_L^0K_S^0}\| \leq 1.8$ GeV		12.82±0.06±0.18±0.15	-	-	12.32
$a_\mu^{K_L^0K_S^0}\| \leq 2.3$ GeV	-	-	-	-	12.42
$a_\mu^{\pi^0\gamma}\| \leq 0.63$ GeV	-	-	-	-	0.1457
$a_\mu^{\pi^0\gamma}\| \leq 1$ GeV	-	-	-	-	4.54
$a_\mu^{\pi^0\gamma}\| \leq 1.8$ GeV		4.41±0.06±0.04±0.07	-	-	4.619
$a_\mu^{\pi^0\gamma}\| \leq 2.3$ GeV	-	-	-	-	4.620
$a_\mu^{\eta\gamma}\| \leq 1$ GeV	-	-	-	-	0.613
$a_\mu^{\eta\gamma}\| \leq 1.8$ GeV		0.65±0.02±0.01±0.01	-	-	0.631
$a_\mu^{\eta\gamma}\| \leq 2.3$ GeV	-	-	-	-	0.633

- Ours differs significantly from FNAL's.
- Data driven +ChEFT+FSI v.s. LQCD's?

$708.7(5.3) \times 10^{-10}$

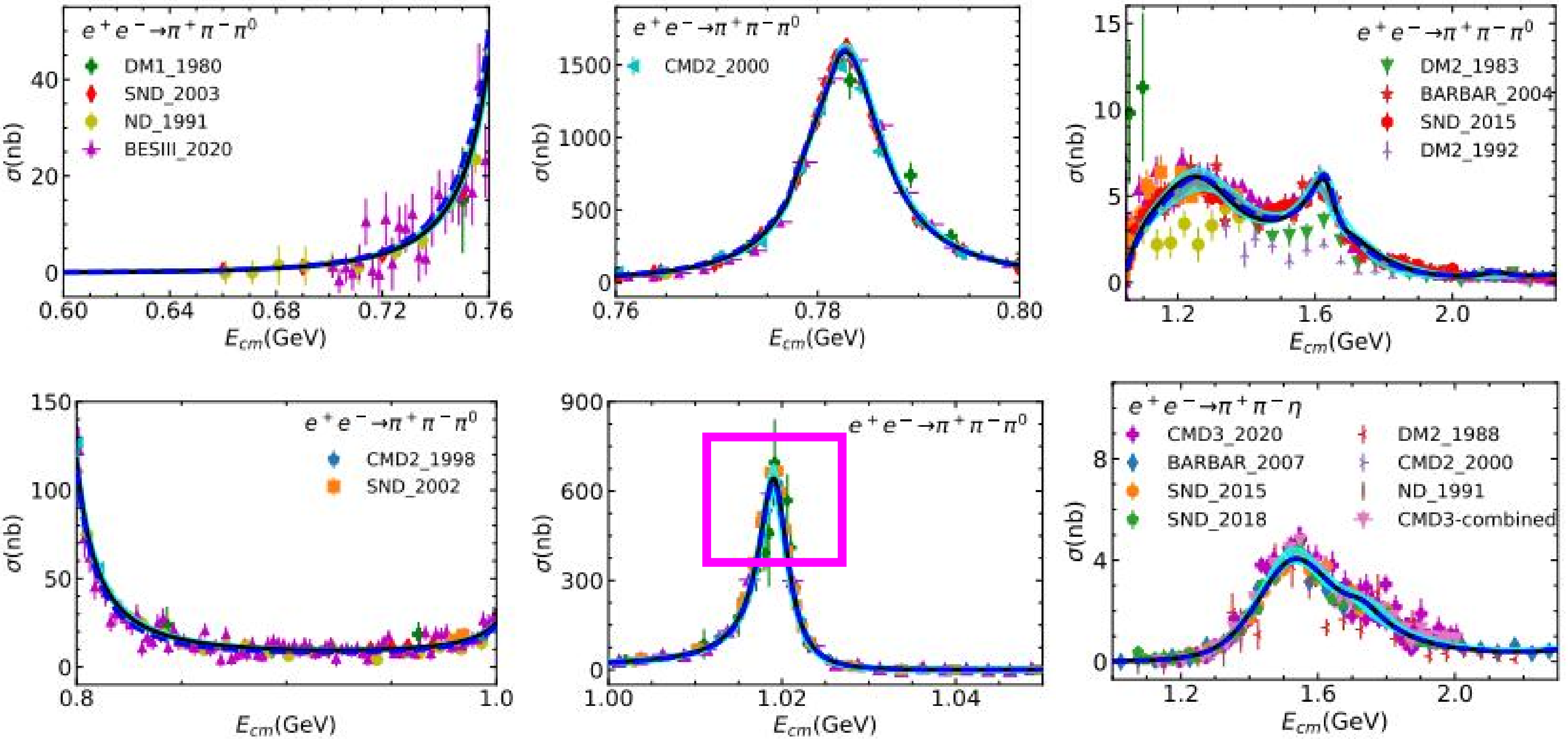
Nature 593 (2021) 7857, 51-55; arxiv:2206.06582

- Future experiments?

Three body final states?

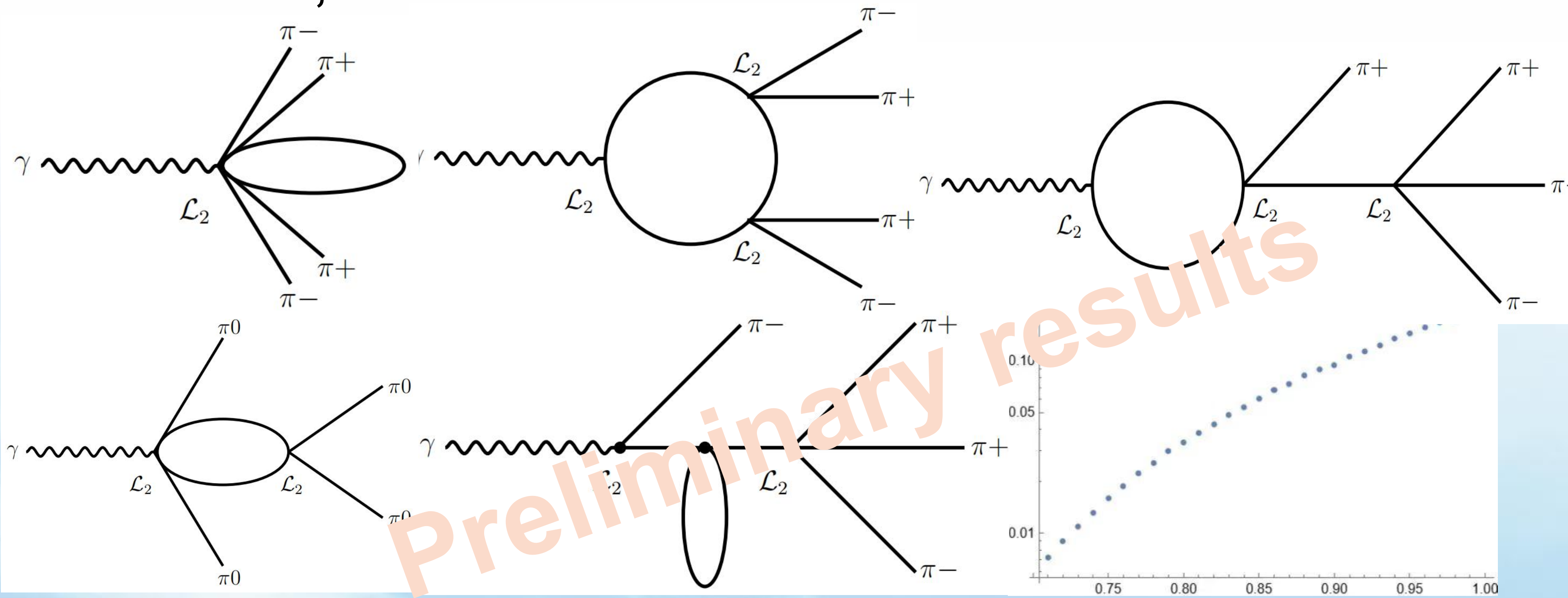
- $\pi\pi\pi$: needs more precise data in the ω ϕ region
- $\pi\pi\eta$: check our model

Qin, Dai, Portoles, JHEP03(2021)092



Four body final states?

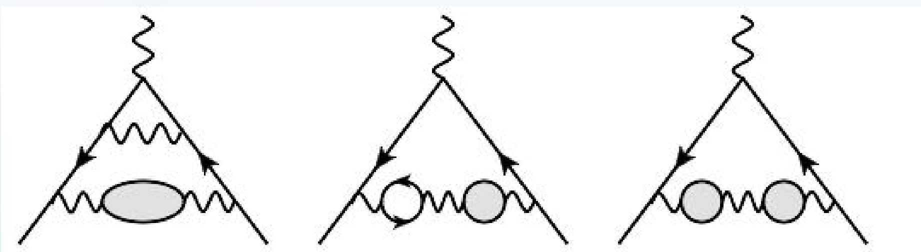
Four body final states are important: ππππ, ππKK channels,etc.

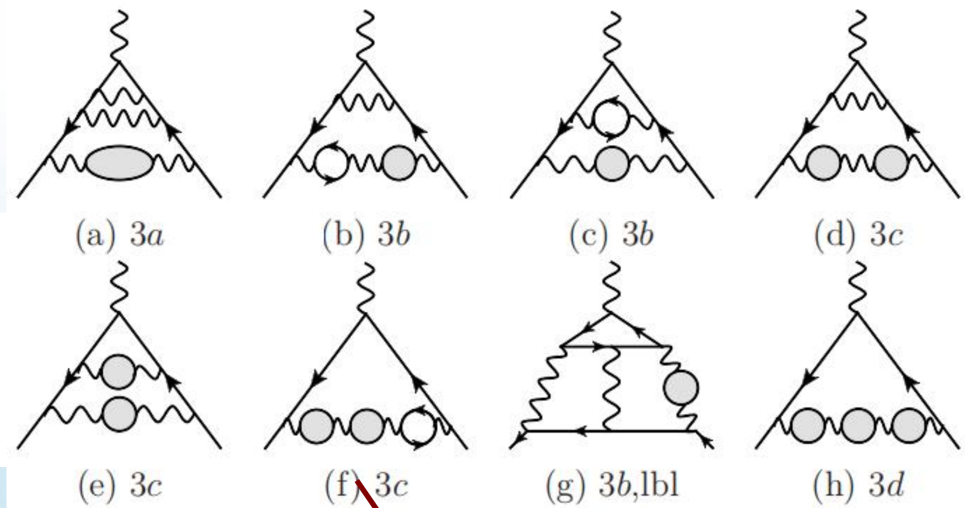


- ChPT’s << data, in resonance energy region
- FSI?
- Resonances?

HVP: NLO, NNLO?

- More channels (also high energy ones) to give a complete estimation?



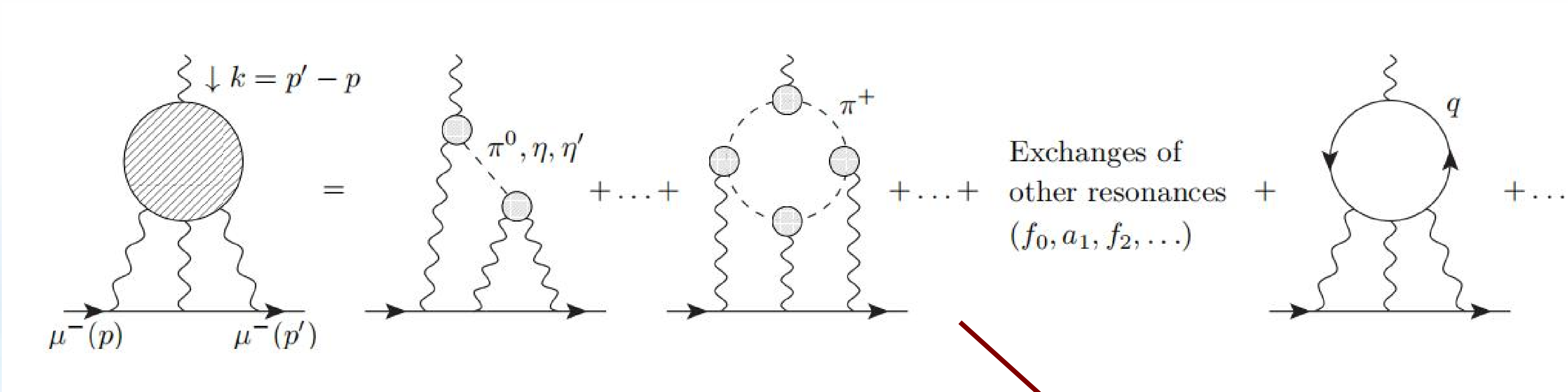


Three, four body final states.
Also refine results of NLO and NNLO.

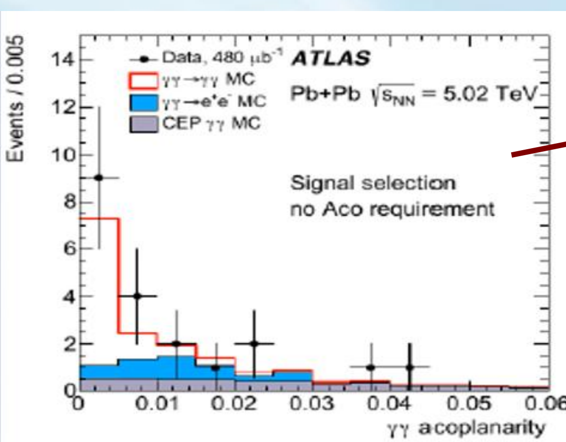
Kurz, et.al.
PLB 734 (2014) 144

3、HLBL

- γγ*→γ*γ* has the clean background, a typical example for amplitude analysis



Phys.Rept.887(2020)1



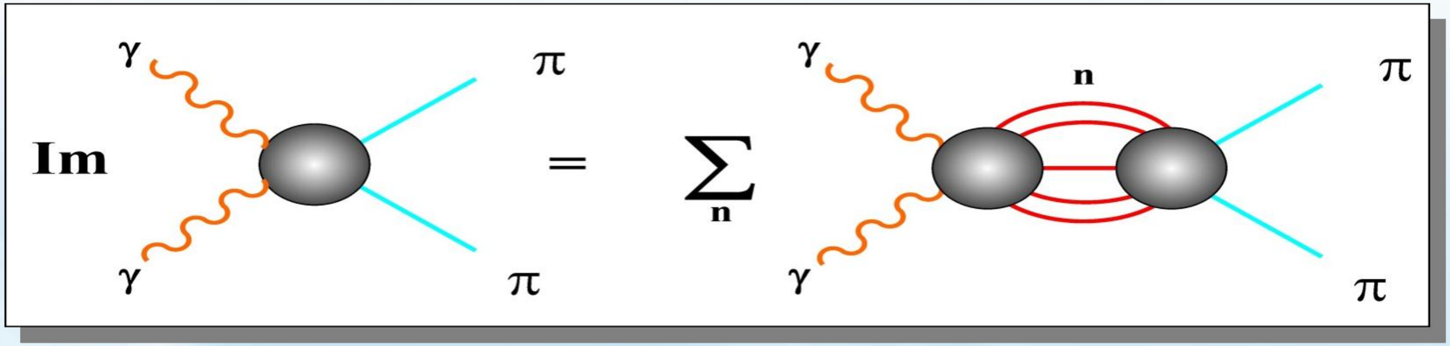
Natrure Phys. 13 (2017) 852,
γγ→γγ, only 13 events

Small yeild, but the result has already been used to set new limits on the Born infeld extension of the SM

Phys. Rev. Lett. 118 (2017) 261802

Building amplitudes

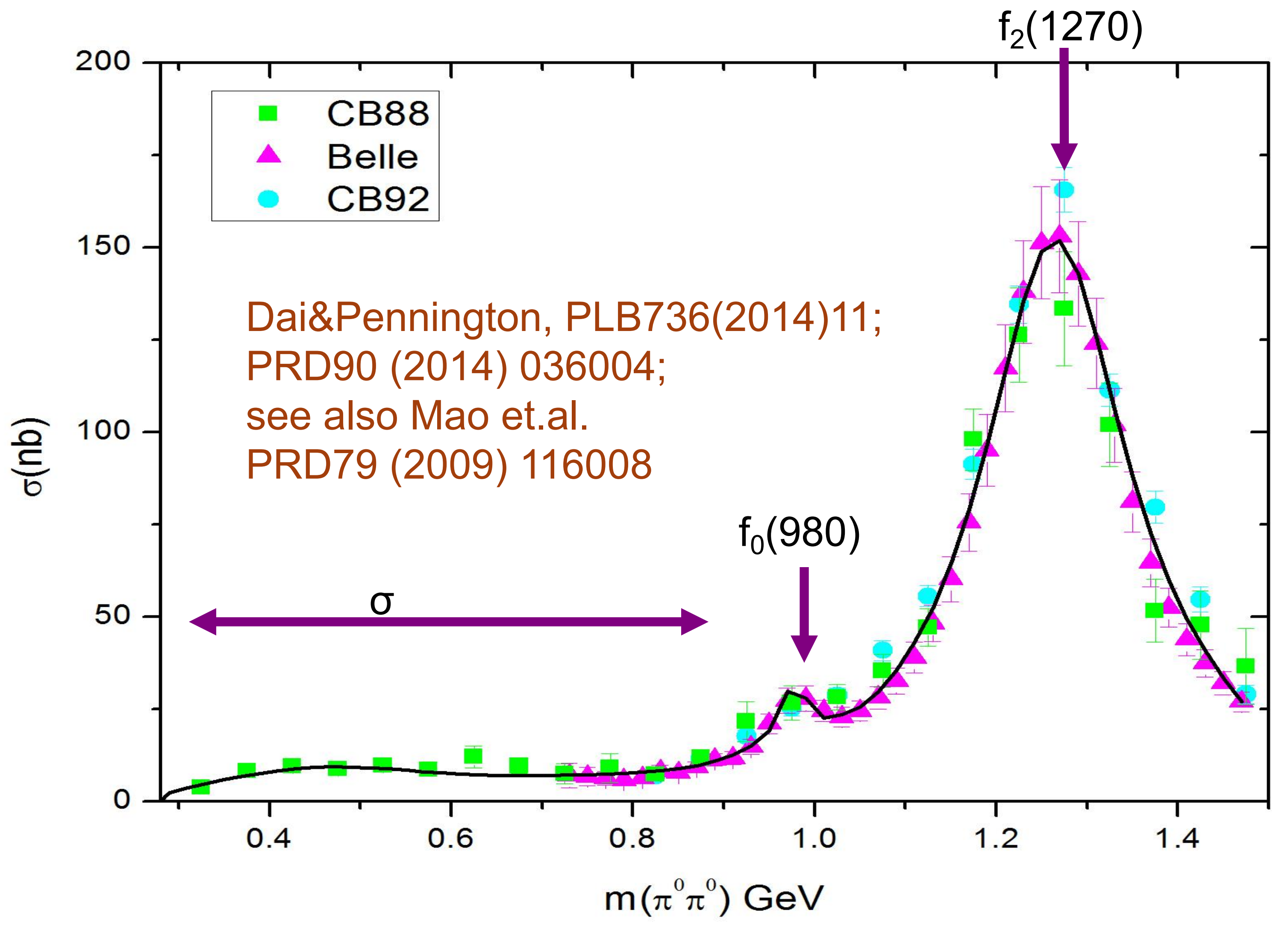
- Final State Interaction Theorem
- Dispersion relations
- ChPT constraints



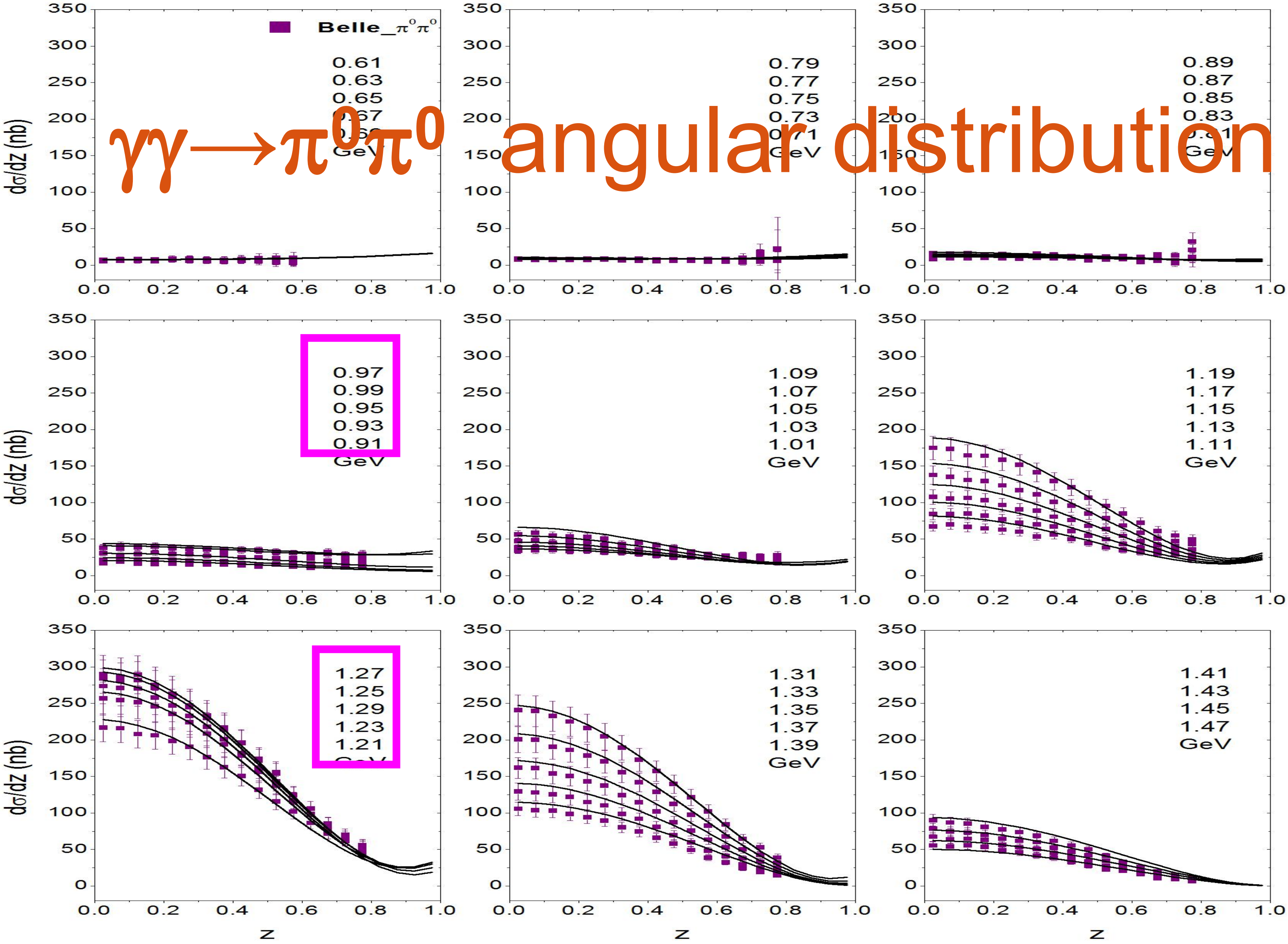
$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2 (s'-s)} - \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\, \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2 (s'-s)}$$

Solved by ChPT

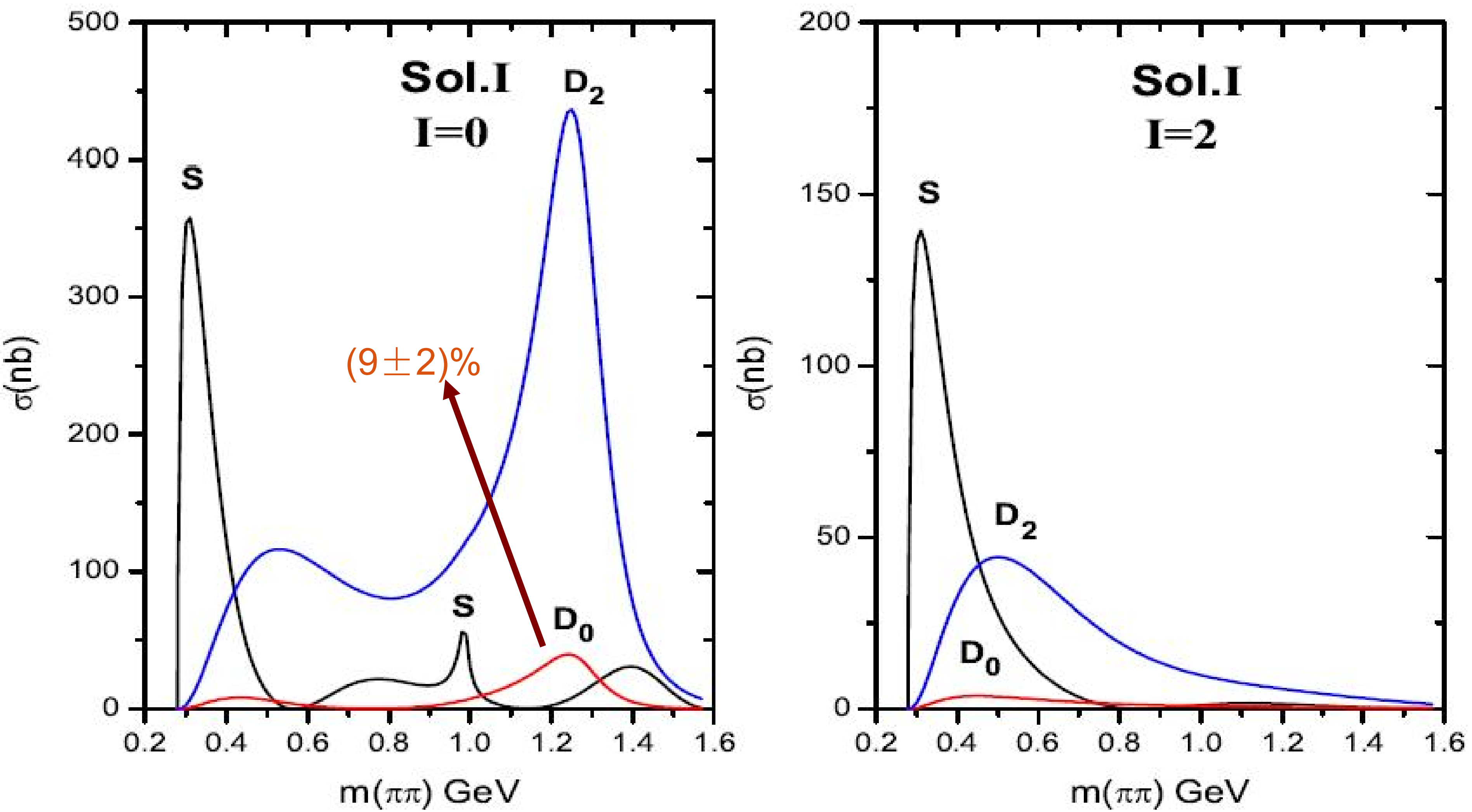
$\gamma\gamma \rightarrow \pi^0\pi^0$ integrated cross section



The angular distribution is helpful to seperate each partial wave.



γγ→ππ individual partial waves



Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.
- 4π channel's contribution is significant for HLBL
- I=0:150–200 nb, I=2: 50nb

evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25 †*	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

BESIII? BelleII?

Dai&Pennington, PRD95 (2017) 056007;

Polarizabilities

Polarizabilities may also play important role on LbL sumrule

K.T.Engel et.al. PRD86 (2012) 037502

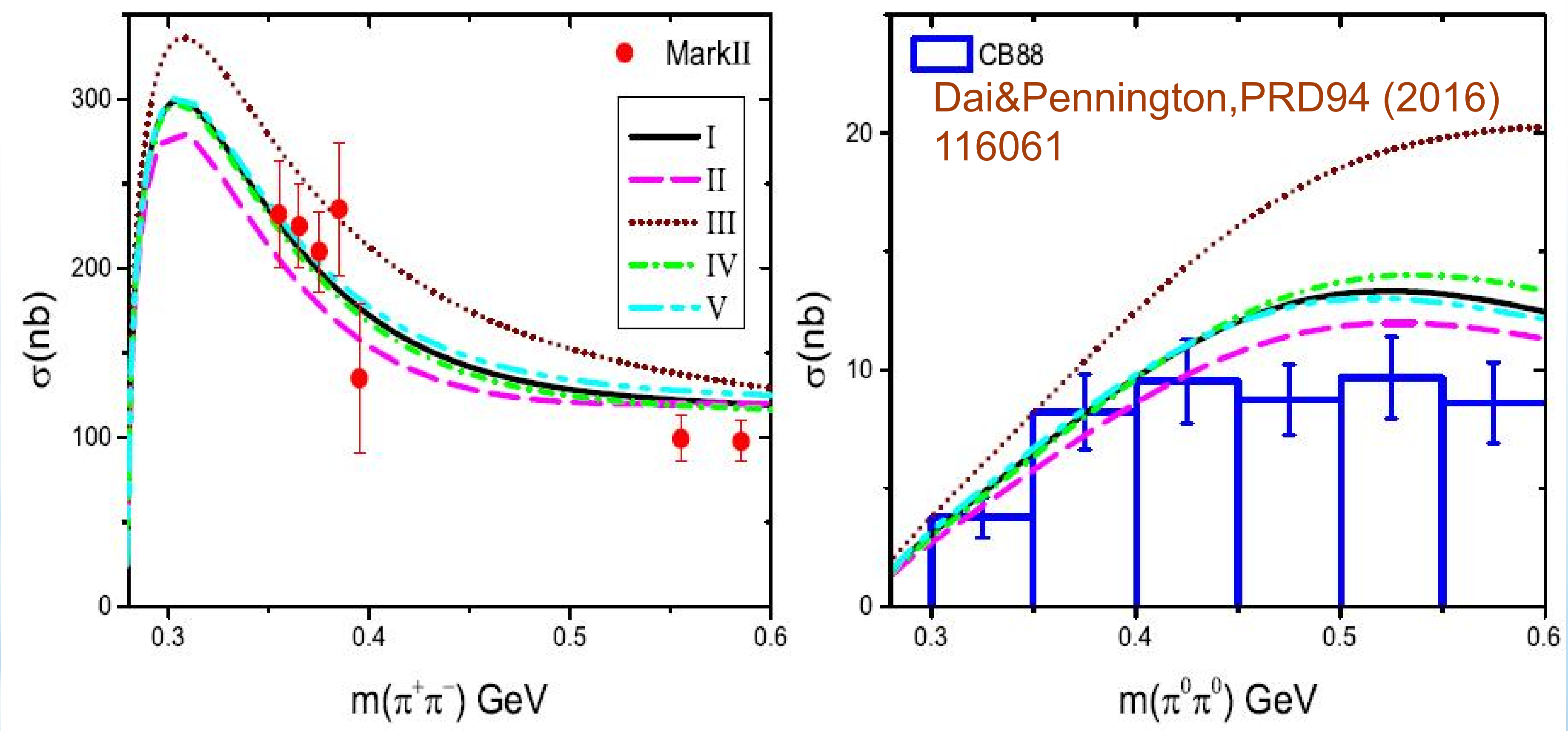
Polarizabilities $\lambda = 0$	Model I	Model II	Model III	Model IV	Model V	ChPT + Resonance Model
$(\alpha_1 - \beta_1)_{\pi^+}$	**4.0 ± 1.2 ± 1.4**	**0.0**	**11.6**	**4.0**	**4.0**	5.7±1.0
$(\alpha_2 - \beta_2)_{\pi^+}$	15.7±1.1	13.0±1.1	20.9±1.1	13.2±3.4	18.1±2.5	16.2[21.6]
$(\alpha_1 - \beta_1)_{\pi^0}$	-0.9±0.2	-0.8±0.1	-1.1±0.2	-0.8±0.2	-1.0±0.2	-1.9±0.2
$(\alpha_2 - \beta_2)_{\pi^0}$	20.6±0.8	17.8±0.8	26.0±0.8	18.6±2.4	22.4±1.8	37.6±3.3
$\lambda = 2$						
$(\alpha_1 + \beta_1)_{\pi^+}$	0.26±0.07	0.26±0.07	0.26±0.07	0.17±0.51	0.42±0.22	0.16[0.16]
$(\alpha_2 + \beta_2)_{\pi^+}$	-1.4±0.5	-1.4±0.5	-1.4±0.5	-0.9±3.5	-2.4±1.5	-0.001
$(\alpha_1 + \beta_1)_{\pi^0}$	0.60±0.06	0.60±0.06	0.60±0.06	-0.04±0.52	0.90±0.17	1.1±3.3
$(\alpha_2 + \beta_2)_{\pi^0}$	-3.7±0.4	-3.7±0.4	-3.7±0.4	0.4±3.4	-5.5±1.1	0.04

fixed by Adler zero and $(\alpha_1-\beta_1)_{\pi^+} = 4.0$

easiest one to be measured by experiment

Polarizabilities

Polarizabilities plays important role on HLbL DRs



$(\alpha_1\text{-}\beta_1)_{\pi+}$ = 11.6, has been exclude by CB's data, JLAB's new measurement?

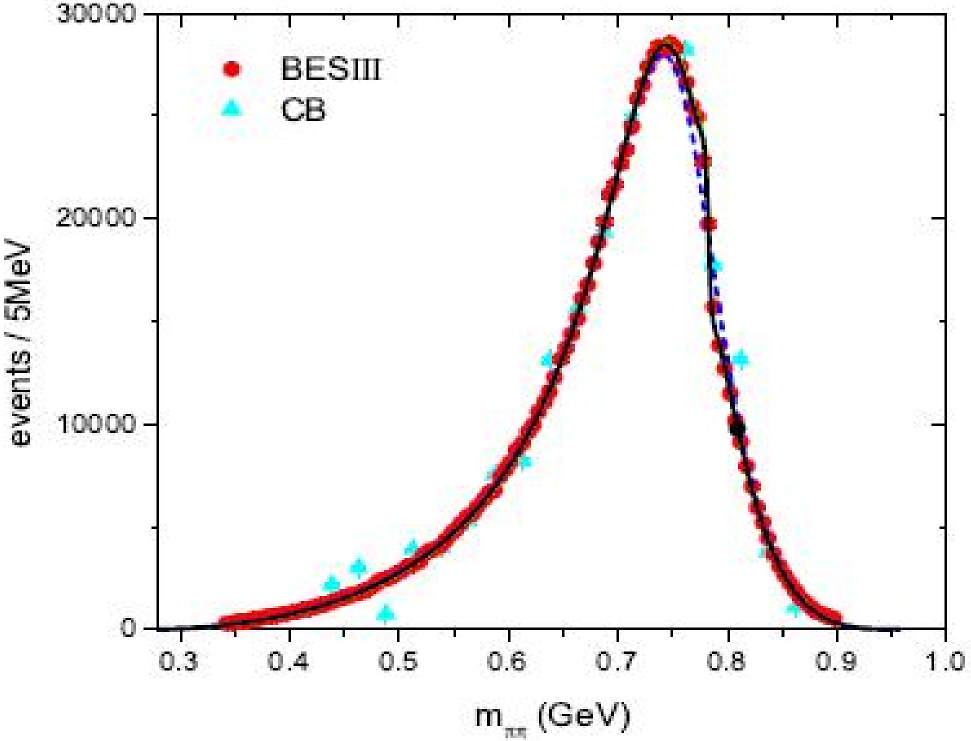
HLbL

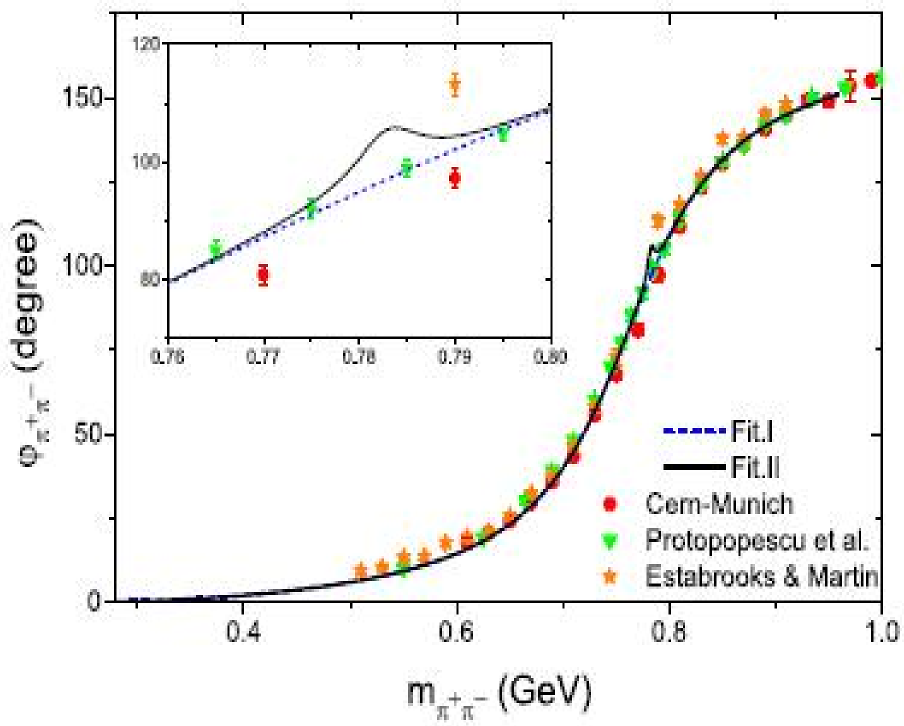
- $\pi^+\pi^-$ P-wave phase-shift should take into consideration of isospin violation

Dai et.al.,PRD97 (2018) 036012

$\gamma^* \rightarrow \pi^+\pi^-$

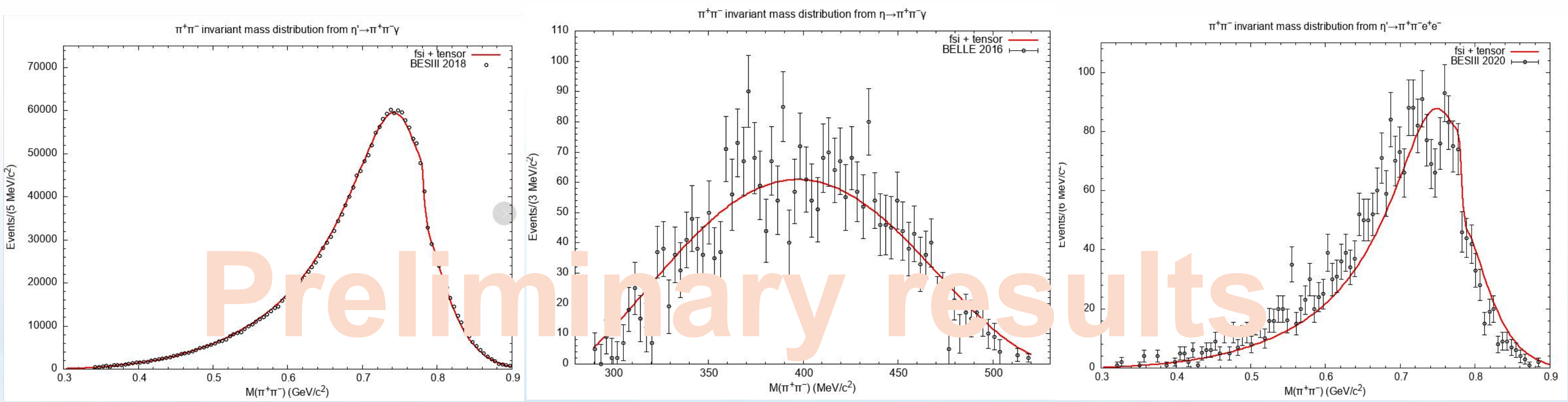
$\gamma^*\gamma \rightarrow \pi^+\pi^-$





TFFs

- TFFs



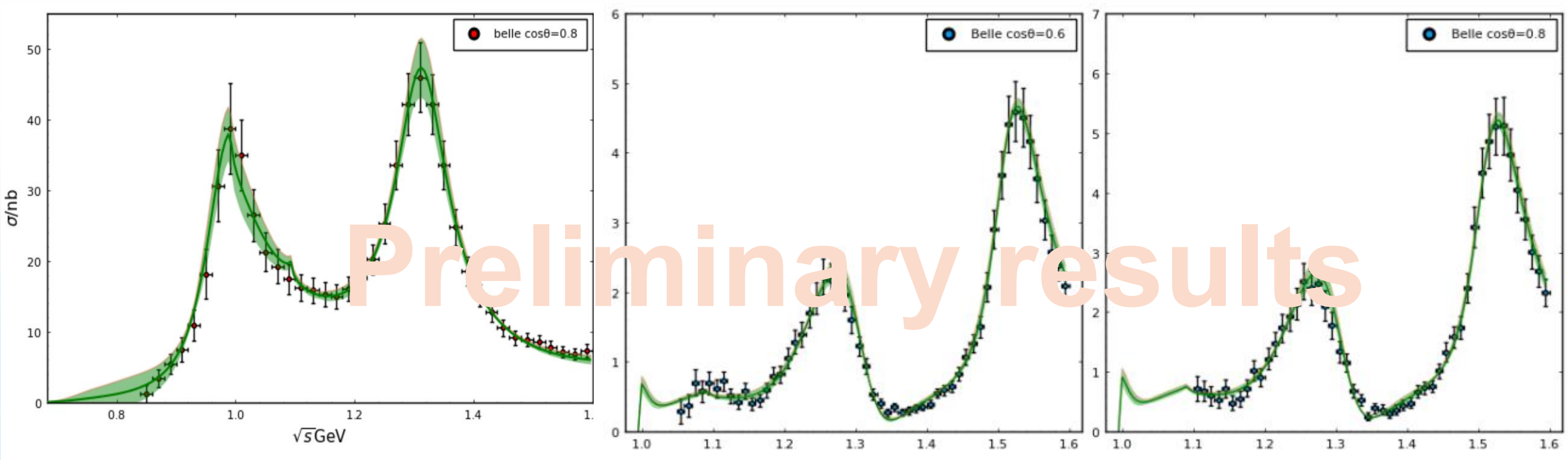
Ye, *et.al.*, in preparation

- HLbL contribution from pseudoscalar poles

$$a_\mu^{\mathrm{LbL};\pi^0} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty \mathrm{d}Q_1\mathrm{d}Q_2 \int_{-1}^{+1} \mathrm{d}t\,\sqrt{1-t^2}\,Q_1^3 Q_2^3\ \left[F_1\,P_6\,I_1(Q_1,Q_2,t) + F_2\,P_7\,I_2(Q_1,Q_2,t)\right]$$

Other γγ collisions

- πη-KK-πη' coupled channel scatterings



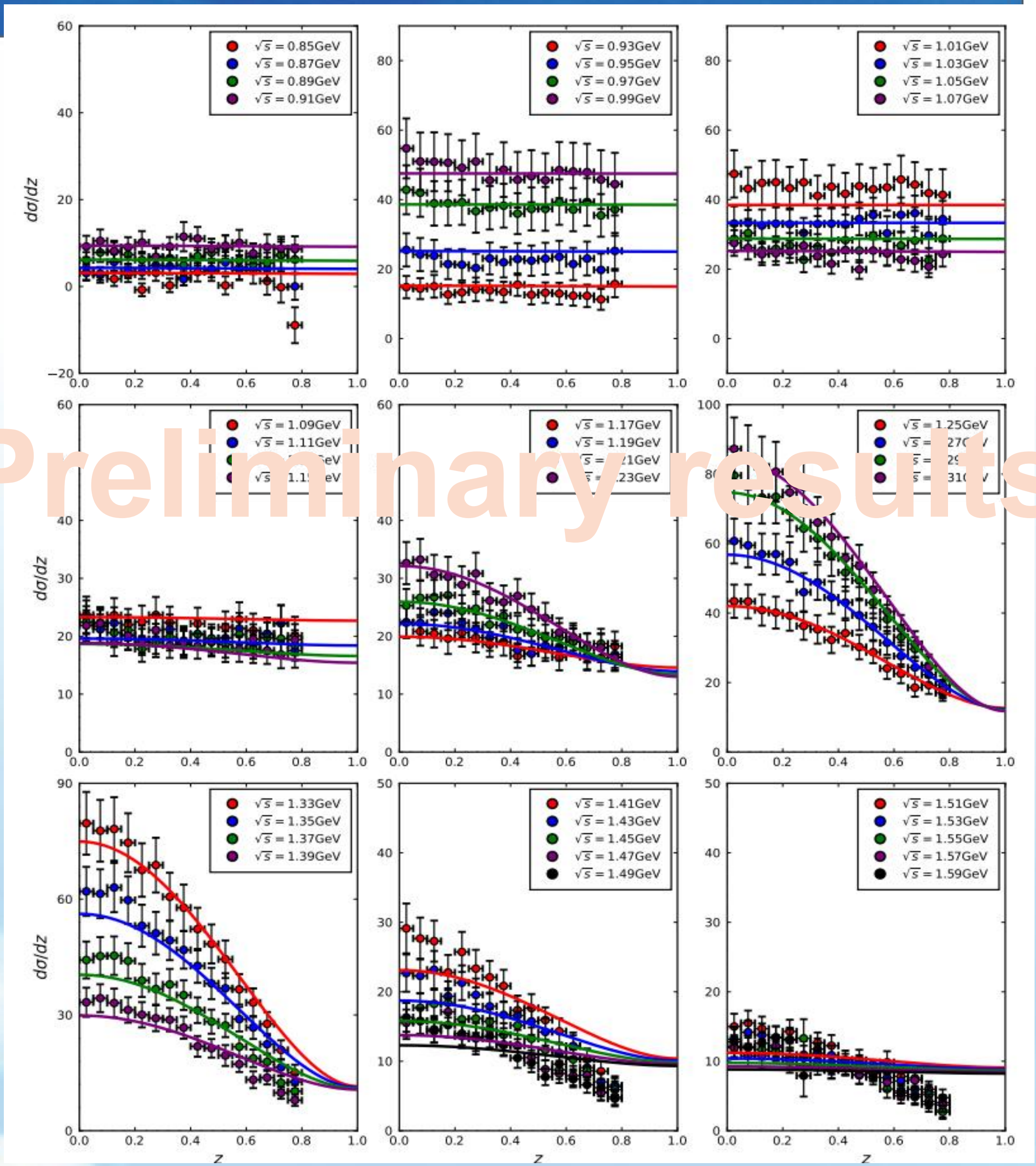
Kuang, Dai *et.al.*, in preparation

- DR+ChEFT constraints
- AMP: FSI

Experiment	Process	Data-points	χ^2_{average}
Belle/Crystal ball	$\gamma\gamma \to \pi^0\eta$	680	
CB(AGS)/A2 MAMI-B	$\eta \to \pi^0\gamma\gamma$	21	
TPC/Argus/Belle	$\gamma\gamma \to K^+K^-$	18	
TASSO/CELLO	$\gamma\gamma \to \bar{K}^0K^0$	5	
Belle	$\gamma\gamma \to \bar{K}^0_SK^0_S$	315	
BESIII	$\eta' \to \pi^0\gamma\gamma$	13	

angular distribution

- $a_0(980)$?
- HLBL constraints for I=1



4、Summary

FSI

Amplitude analsysis connects QFT principles and Exp. FSI needs to be considered when performing amplitude analysis.

HVP

Ours has a significant discrepancy with the latest FNAL's. Processes of multi-body channels needs to be studied.

HLBL

We have strong constraints to HLBL amplitudes. 4π's can not be ignored. $\pi\pi\pi\pi$, $\pi\pi KK$?

Next?

Further study of light hadrons is neccessary to give a more reliable answer to muon g-2; Discrepancy between LQCD v.s. data driven+ChEFT+FSI?

Thank You For your patience !