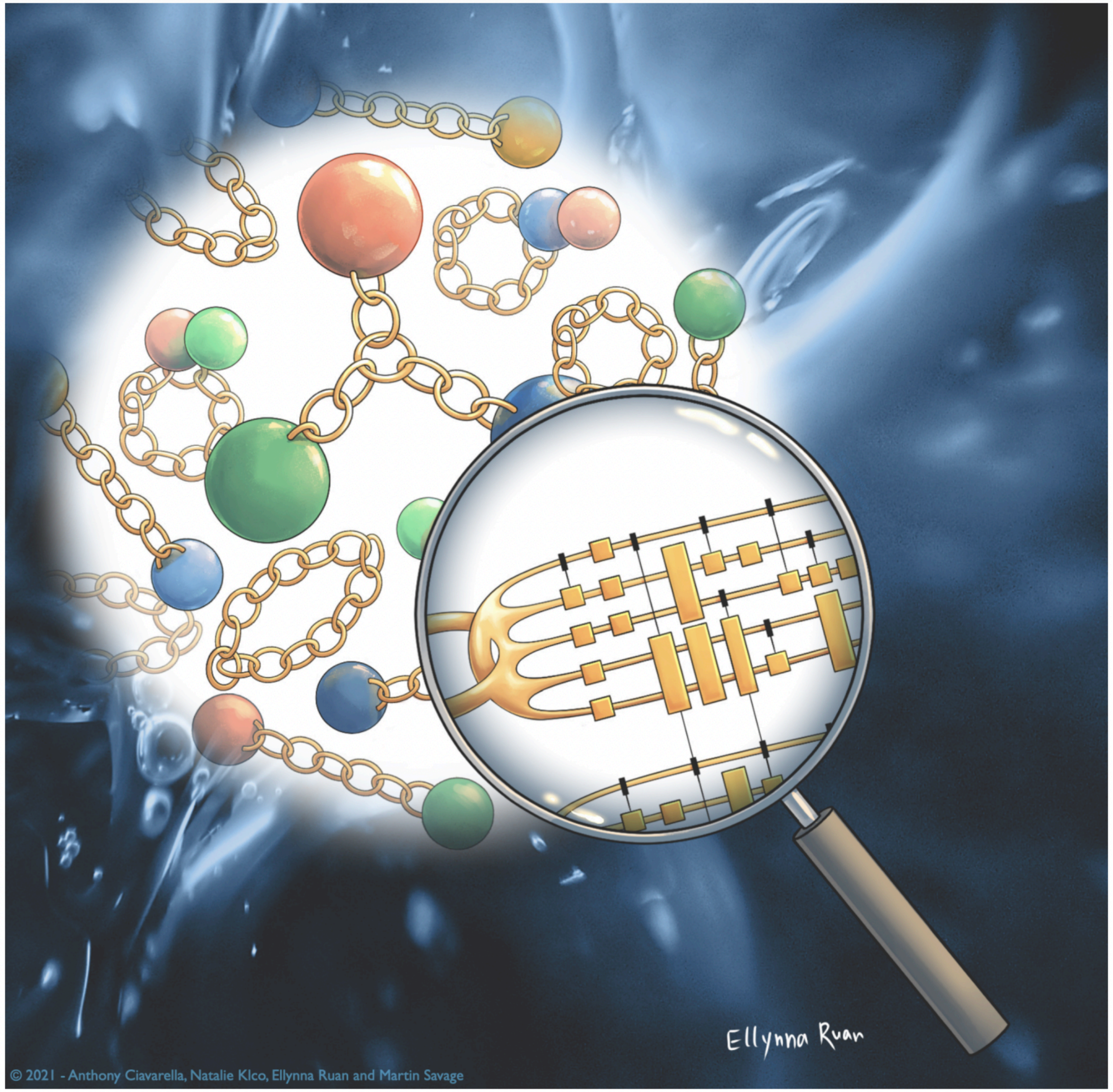


Exploring Partonic Strucutre by Quantum Circuits

第四届重味物理与量子色动力学研讨会

刘晓辉

北京師範大學
BEIJING NORMAL UNIVERSITY

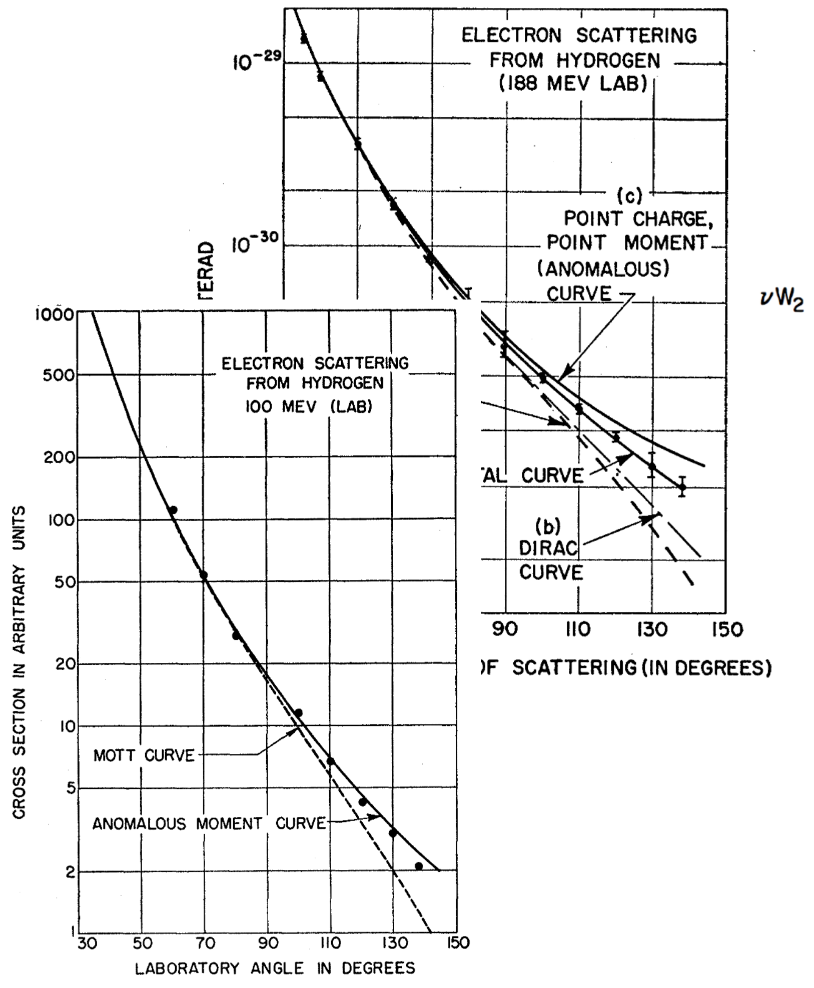
Outline

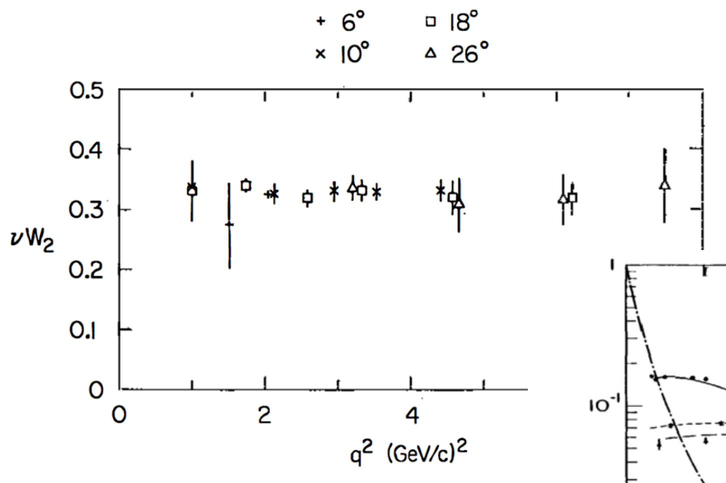
- PDFs
- Quantum computing for HEP
  - Fundamental skills
  - A toy model Li, et al, PRD letter 22
- Conclusion

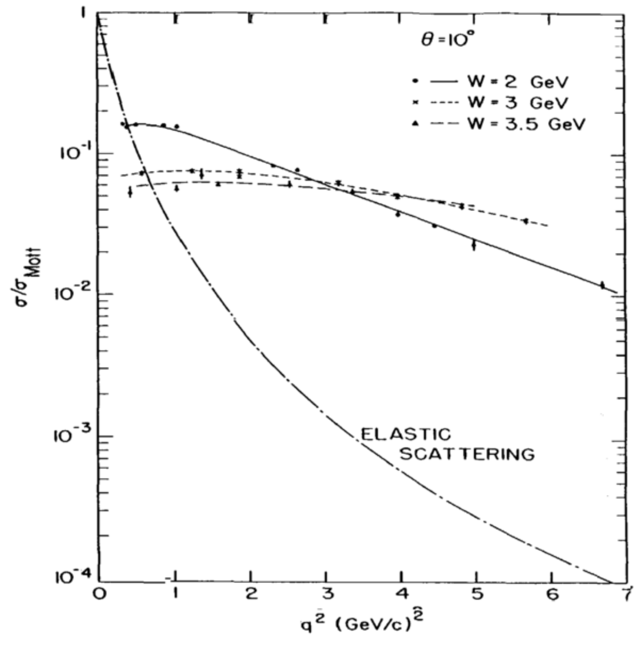
Proton Structures

Long last efforts

- …
- Parton model: Bjorken, Feynman 1969
- …
- QCD







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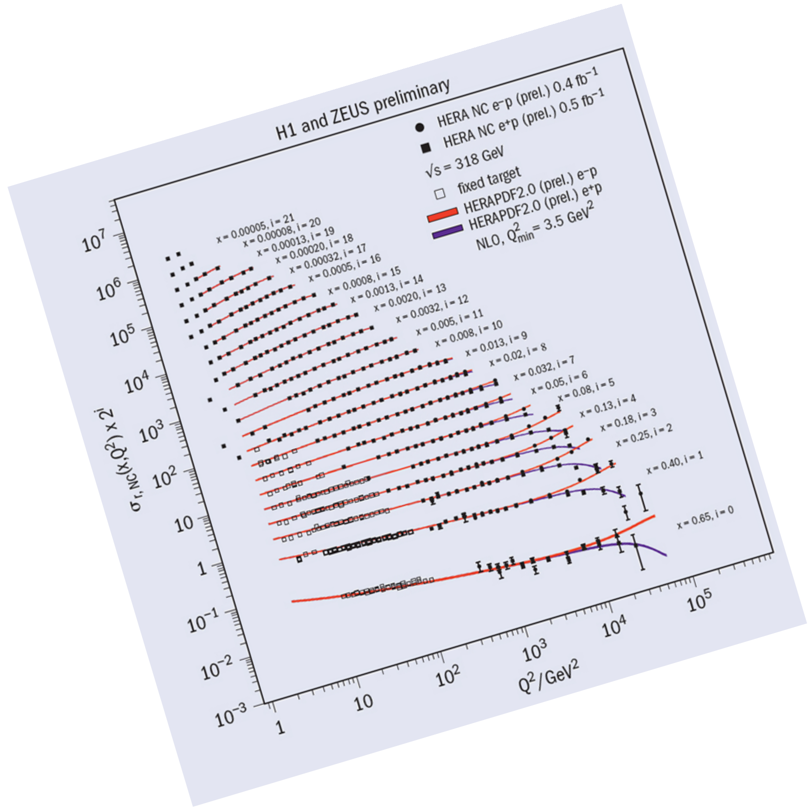

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1956

Today

LHC, Hera, EicC, EIC ..

Proton Structures

Long last efforts

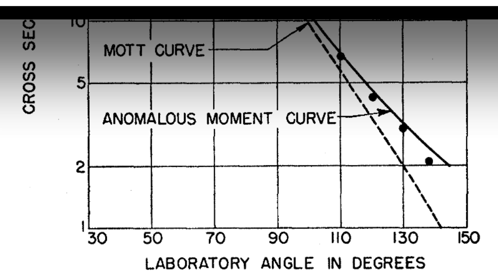
- …
- Parton model: E
- …
- QCD

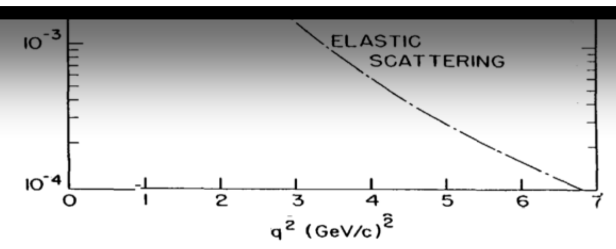
- Well described by 2-pt light-like correlator

$$f_{q/p}(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathscr{W}(0, y^-) \psi(y^-) | p \rangle$$

- Non-perturbative
  - Global fitting, $\sigma \sim \hat{\sigma}_{parton} f(x)$
  - LaMeT, $f_{quasi} = C \otimes f + \mathcal{O}\left(P_z^{-1}\right)$,
    f_{quasi} : space-like correlation, C: perturbative calculable

Ji, 2013 + many, especially our colleagues





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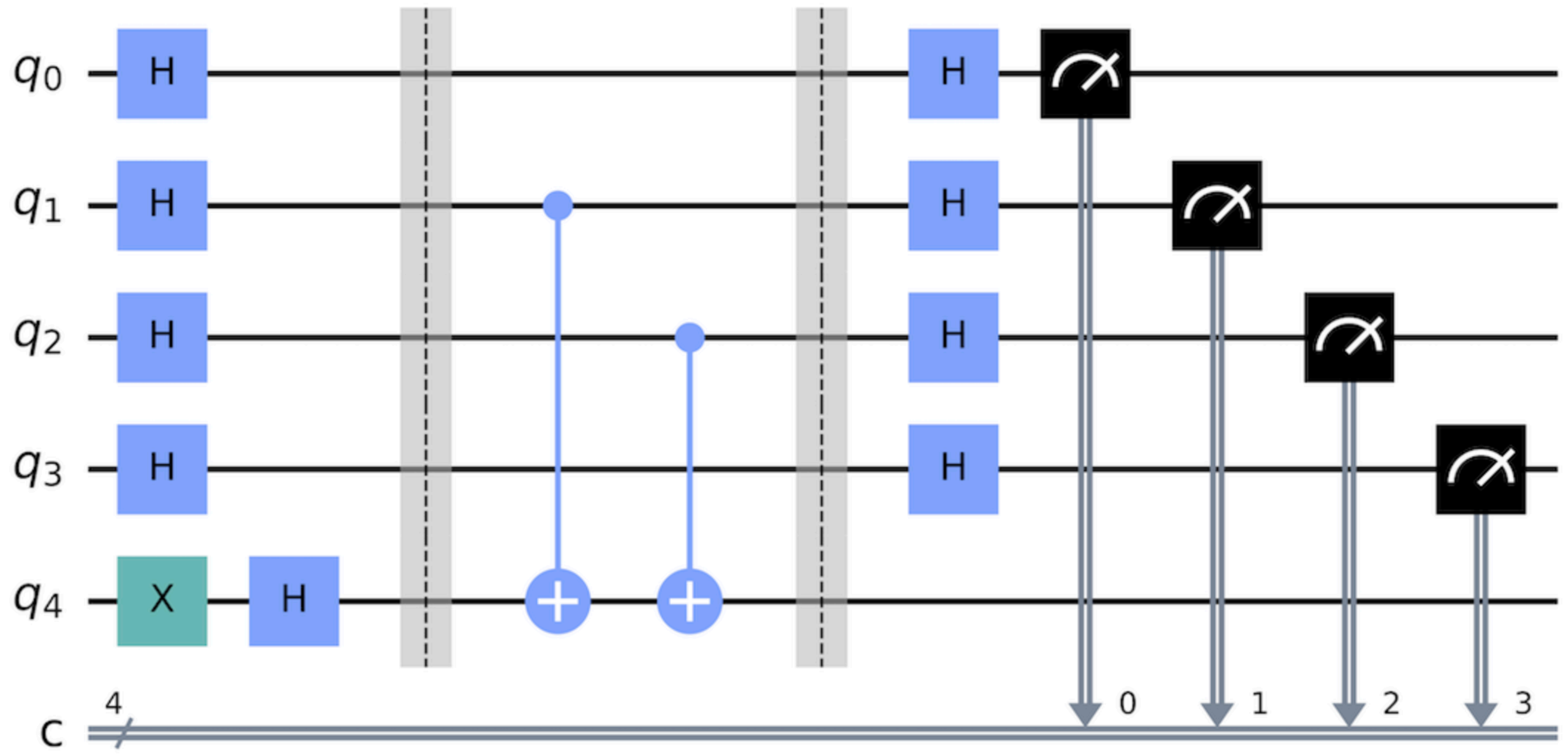
Quantum Computing

Quantum Computing

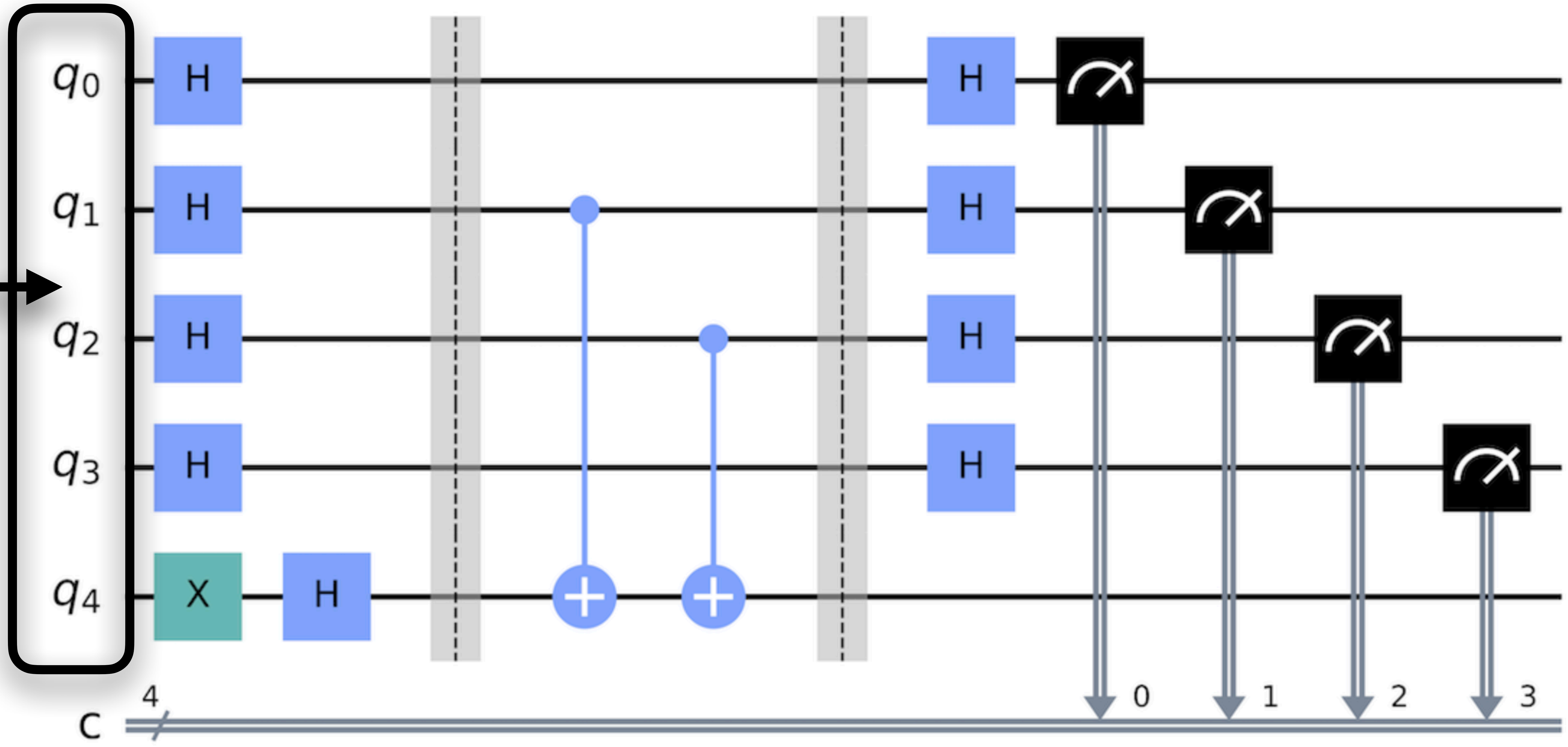


IBM QISKIT

Quantum Computing

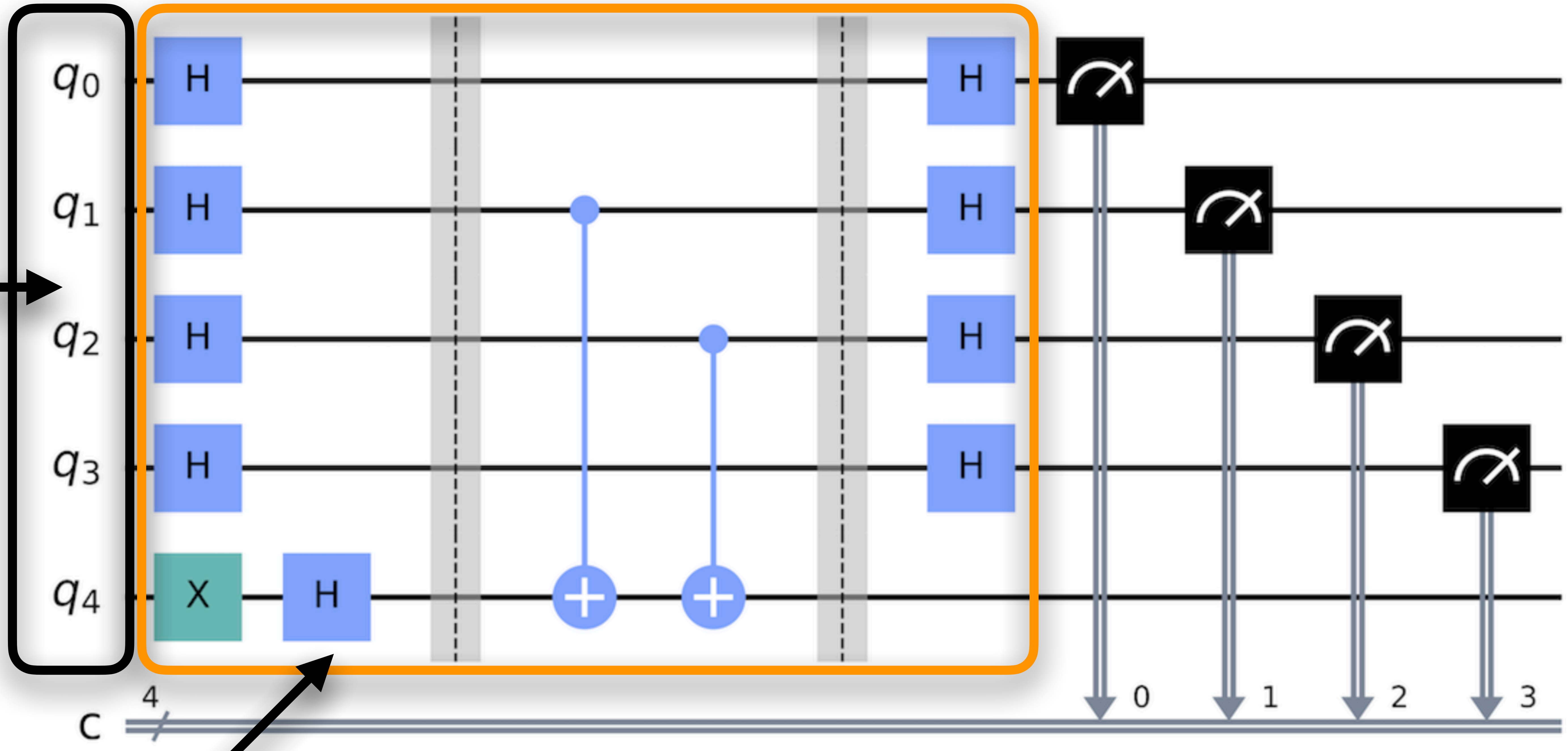
Input qubits

$q_i = \alpha|0\rangle + \beta|1\rangle$



IBM QISKIT

Quantum Computing



Input qubits

$q_i = \alpha|0\rangle + \beta|1\rangle$

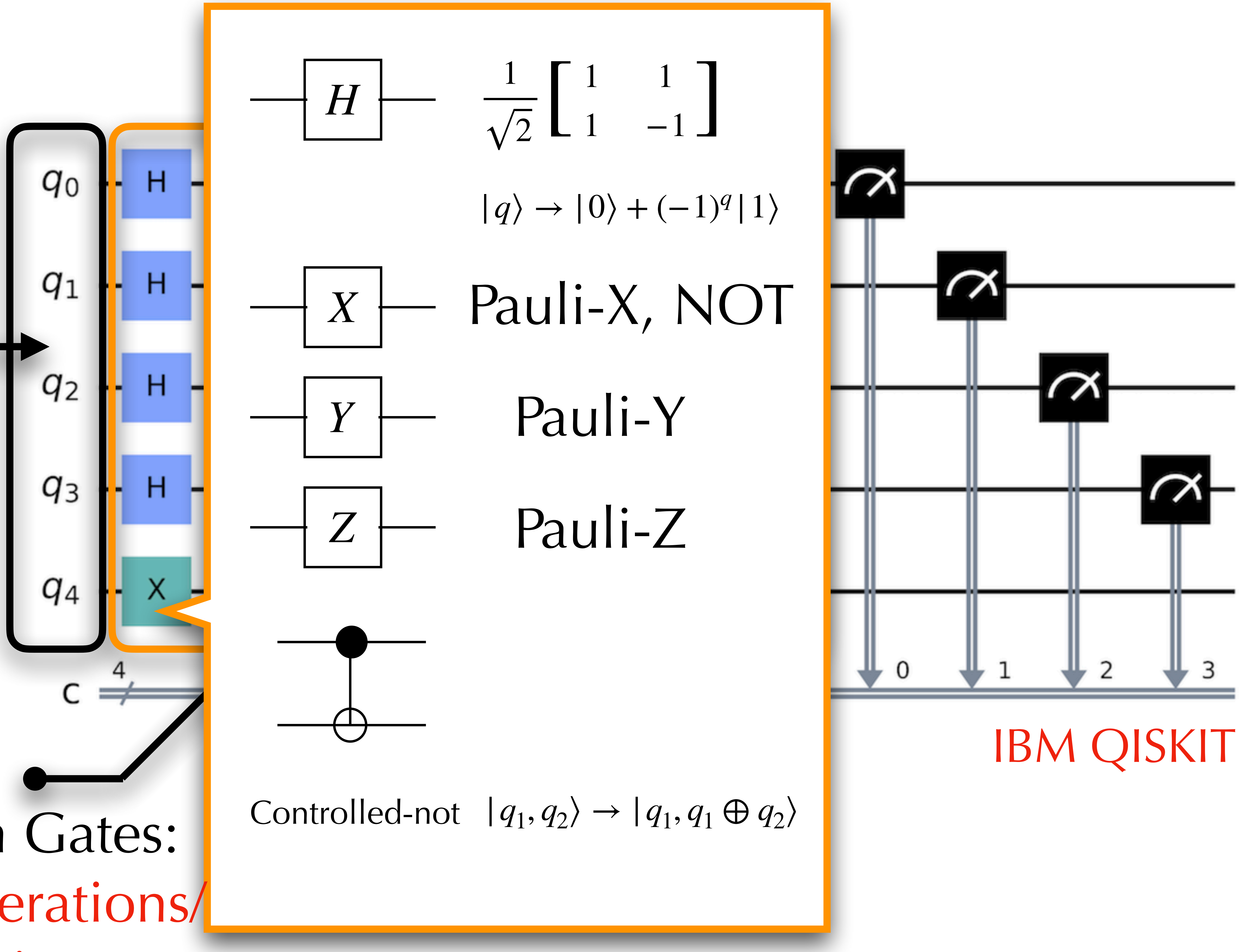
Quantum Gates:
unitary operations/
evolution

IBM QISKIT

Quantum Computing

Input qubits

$q_i = \alpha|0\rangle + \beta|1\rangle$



Quantum Gates:
unitary operations/
evolution

IBM QISKIT

Quantum Computing

Input qubits

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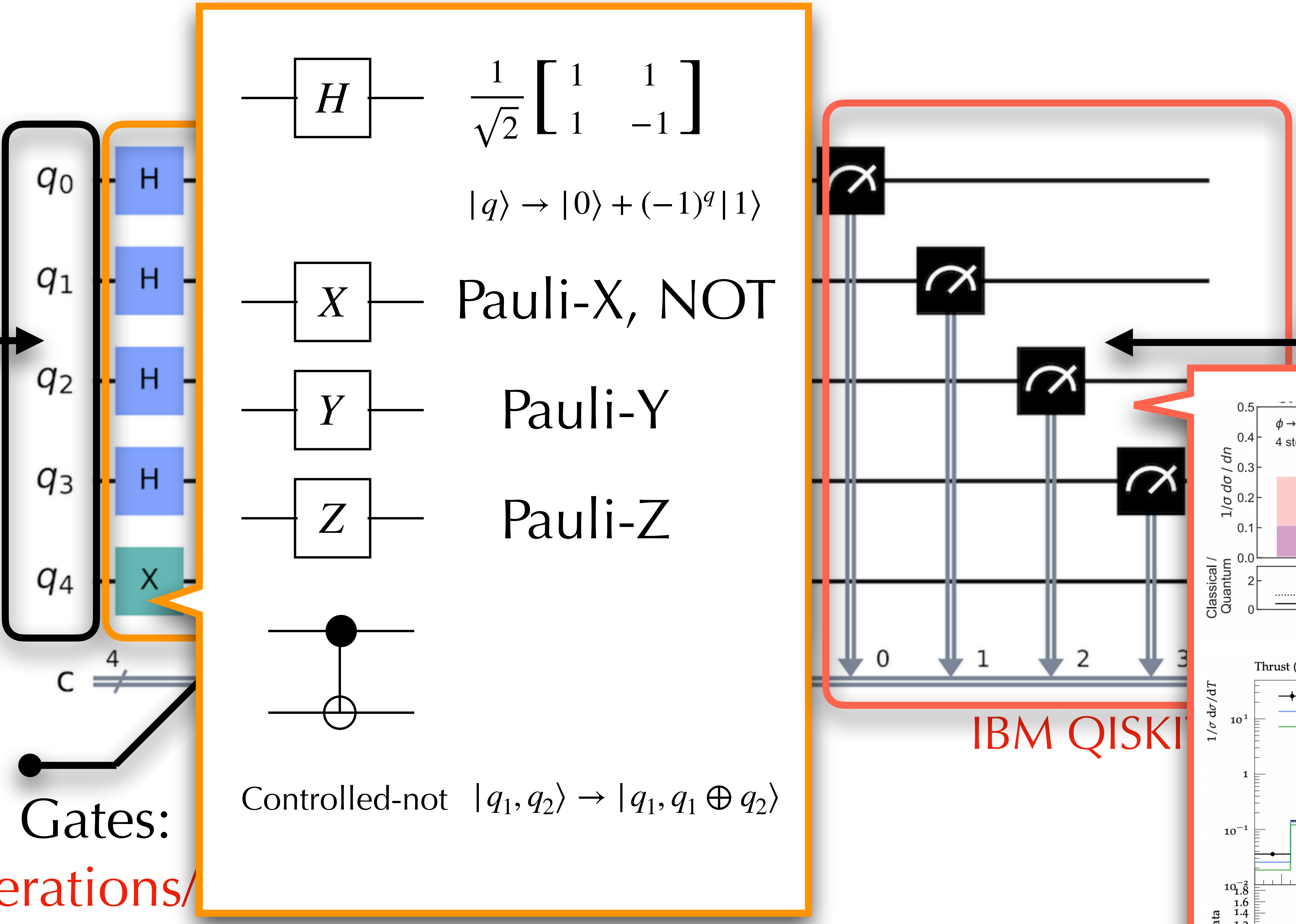
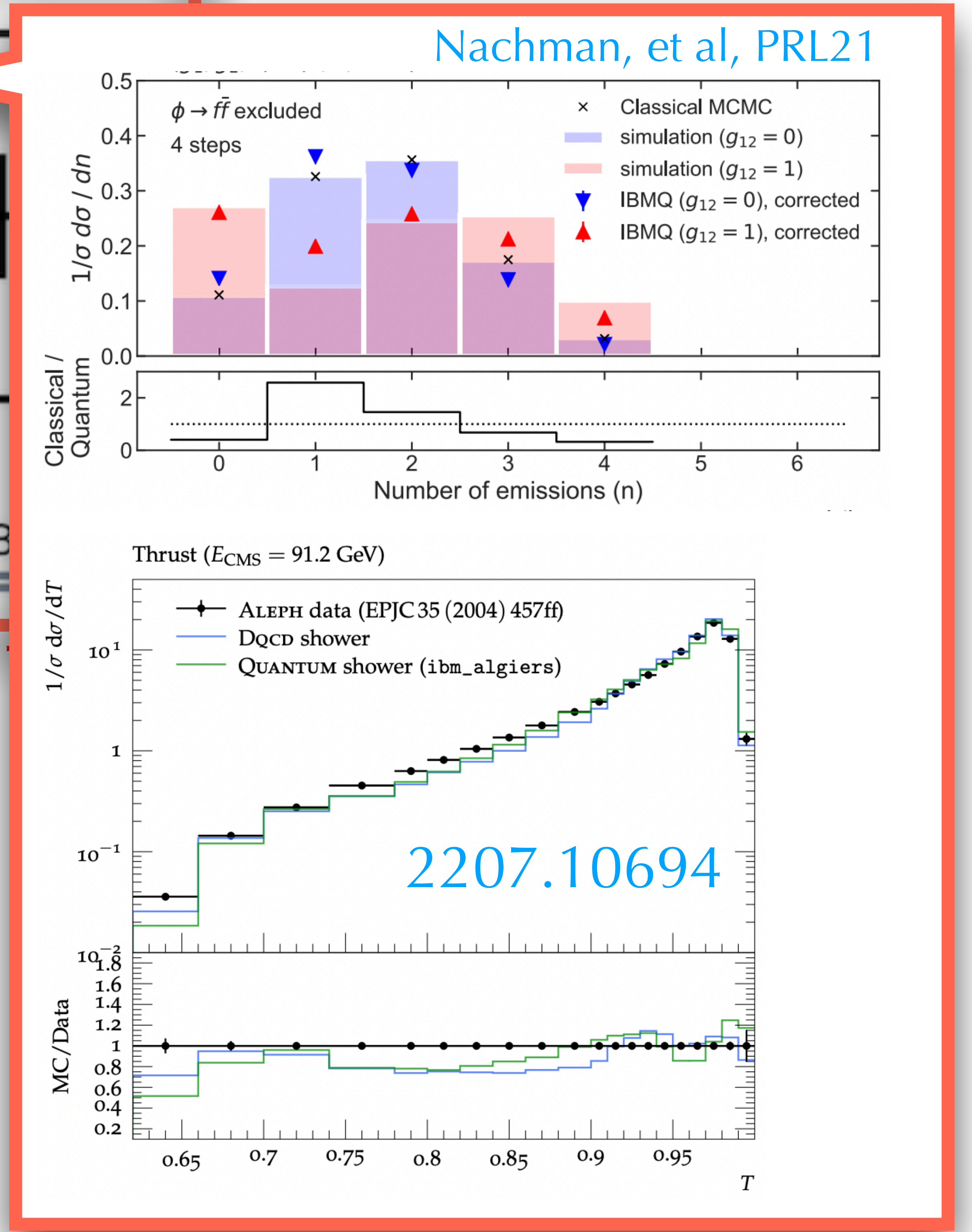


Measurements

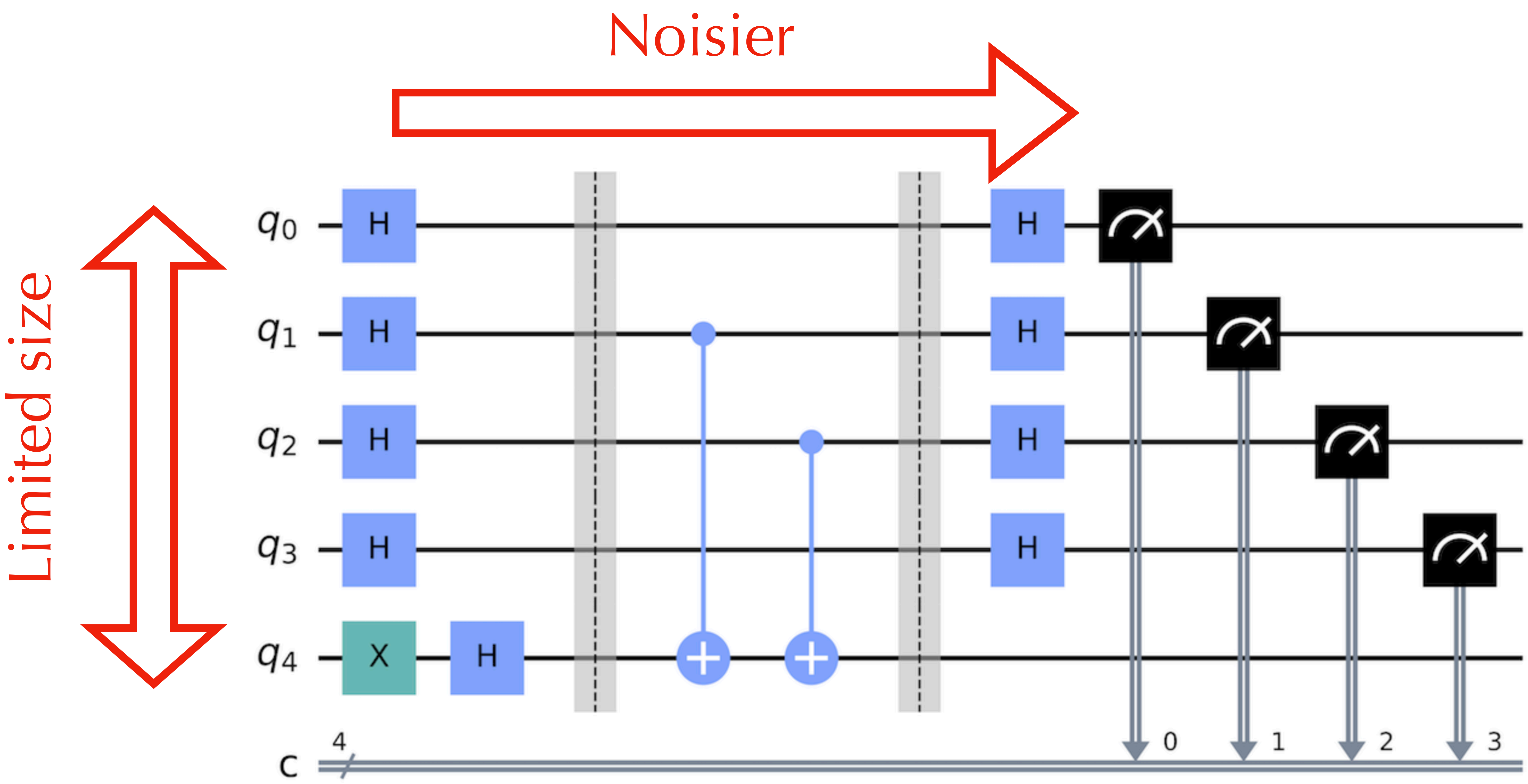


Quantum Gates:

unitary operations/

evolution

Quantum Computing



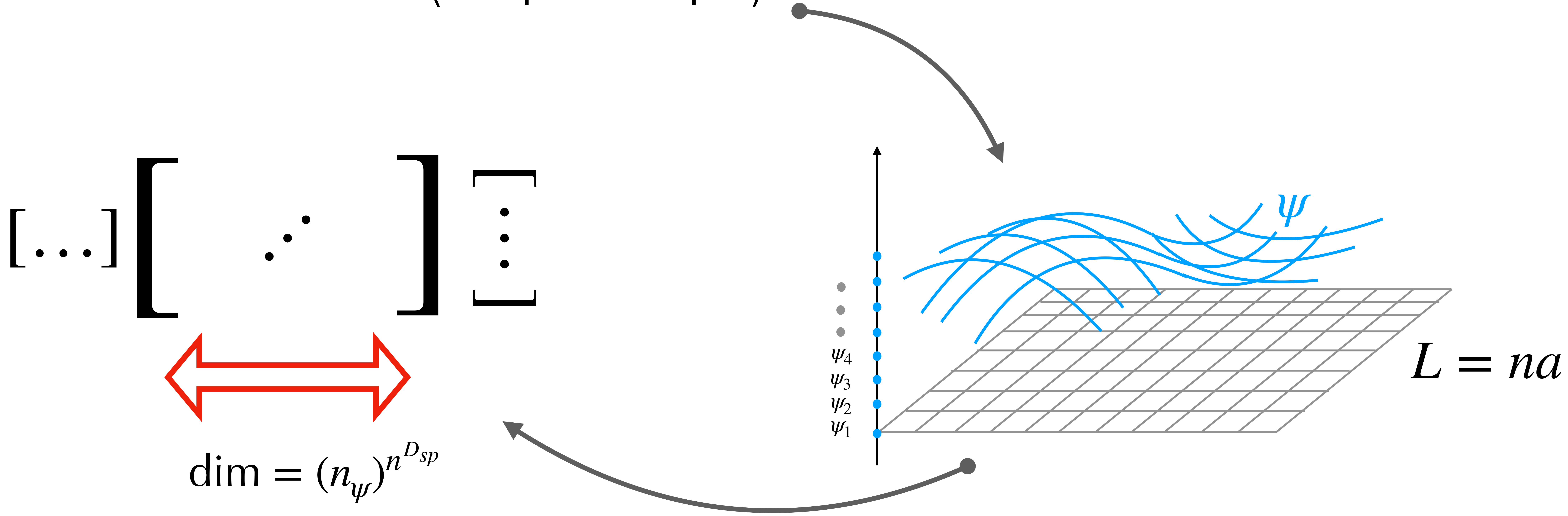
Noisy Intermediate-Scale Quantum (NISQ):

- low qubits limit the problem size ~ $\mathcal{O}(10^2)$
- decoherence limits the fidelity and the circuit depth ~ $\mathcal{O}(10^2)$

Quantum Computing for HEP

In HEP, we are dealing with n-pt correlates, i.e. S-matrix

$$\langle \text{out} | e^{-iH[\psi]t} | \text{in} \rangle$$



Quantum Computing for HEP

In HEP, we are dealing with n-pt correlates, i.e. S-matrix

$$\langle \text{out} | e^{-iH[\psi]t} | \text{in} \rangle$$

$(na)^{-1} \lesssim E \lesssim a^{-1}$

For the LHC $10^2\text{MeV} \lesssim E \lesssim 1\text{TeV}$

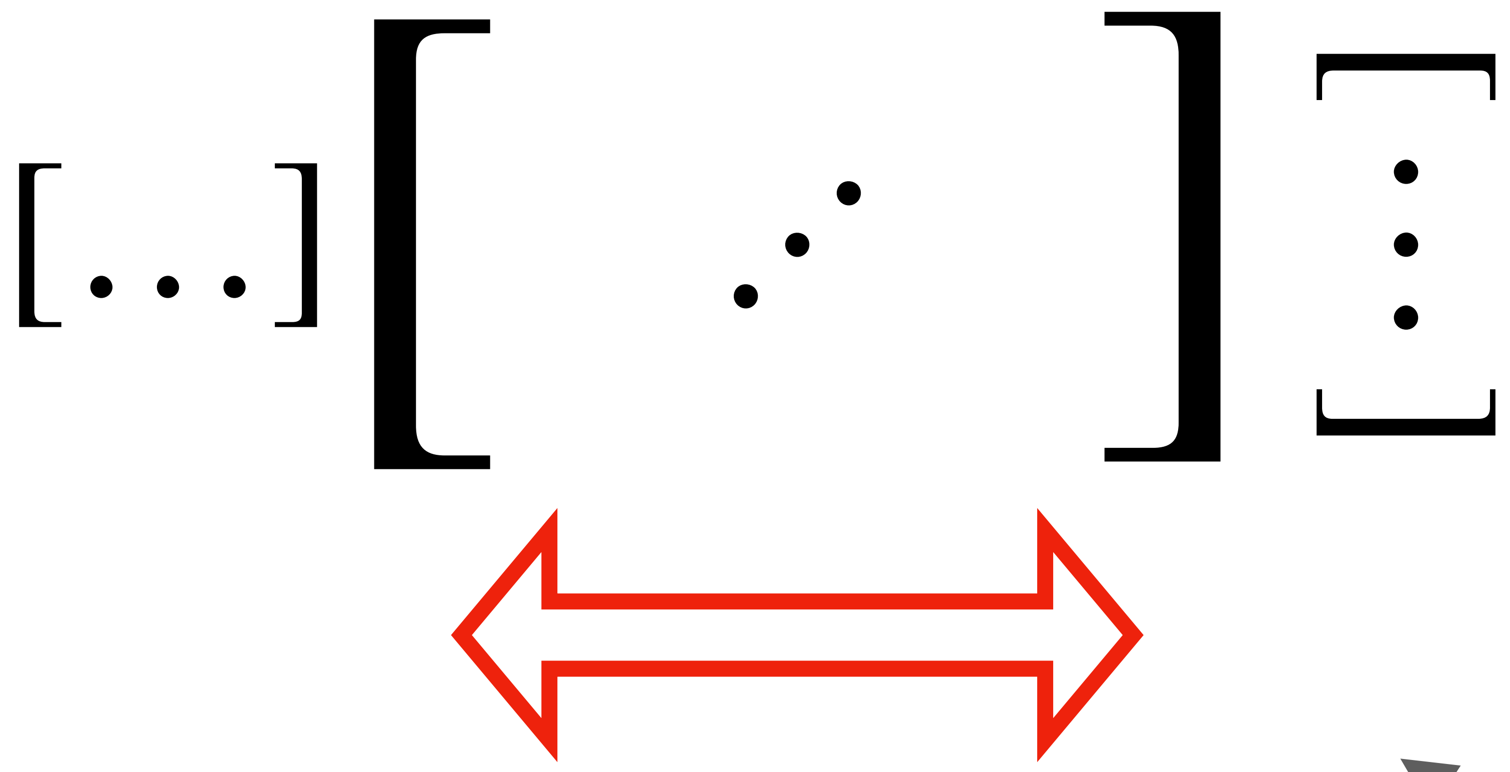
$$n^{D_{sp}} \sim 10^{12}$$

For the hadron: $10^2\text{MeV} \lesssim E \lesssim 1\text{GeV}$

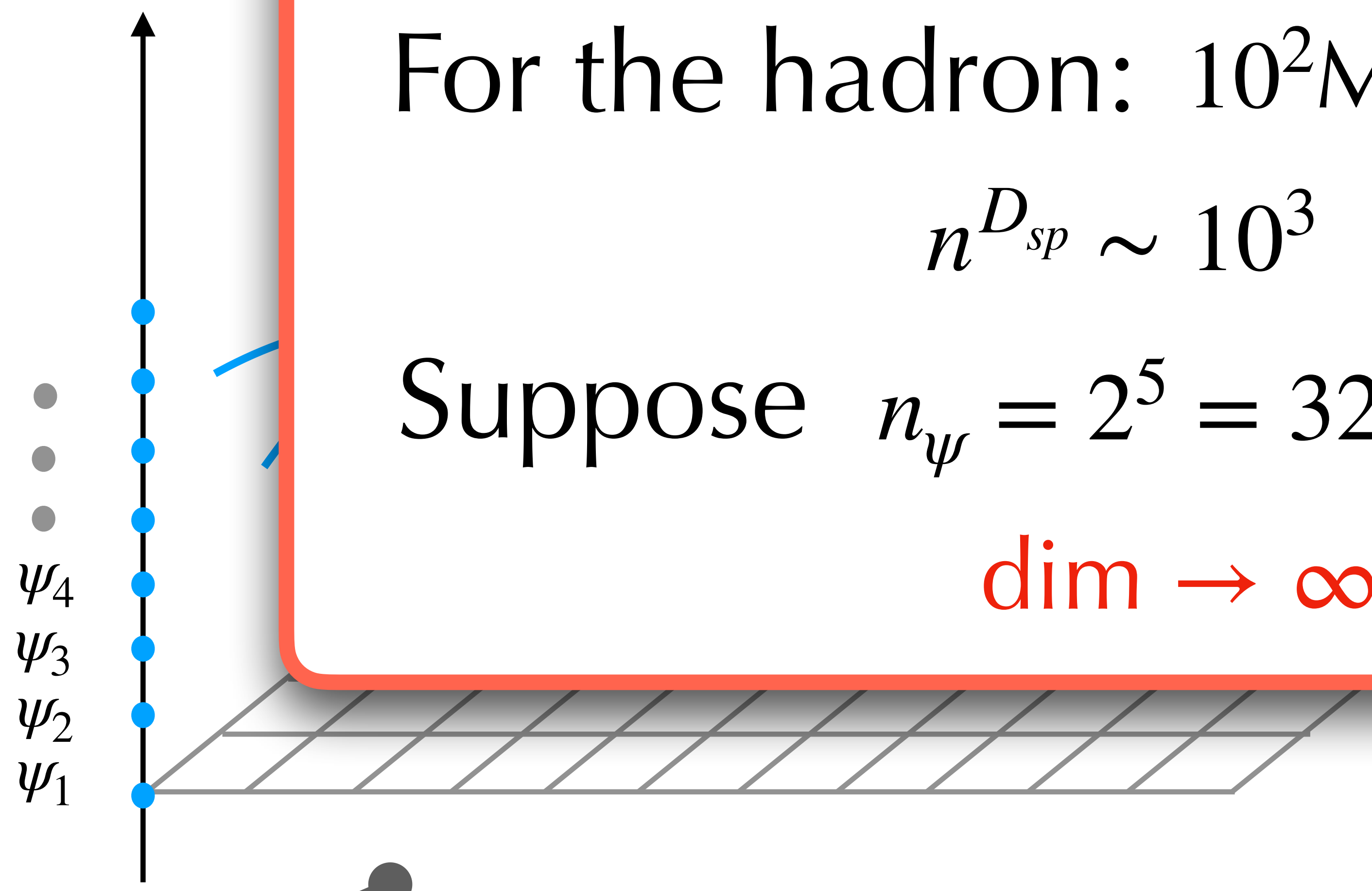
$$n^{D_{sp}} \sim 10^{3}$$

Suppose $n_\psi = 2^5 = 32$

dim $\to \infty$



dim $= (n_\psi)^{n^{D_{sp}}}$

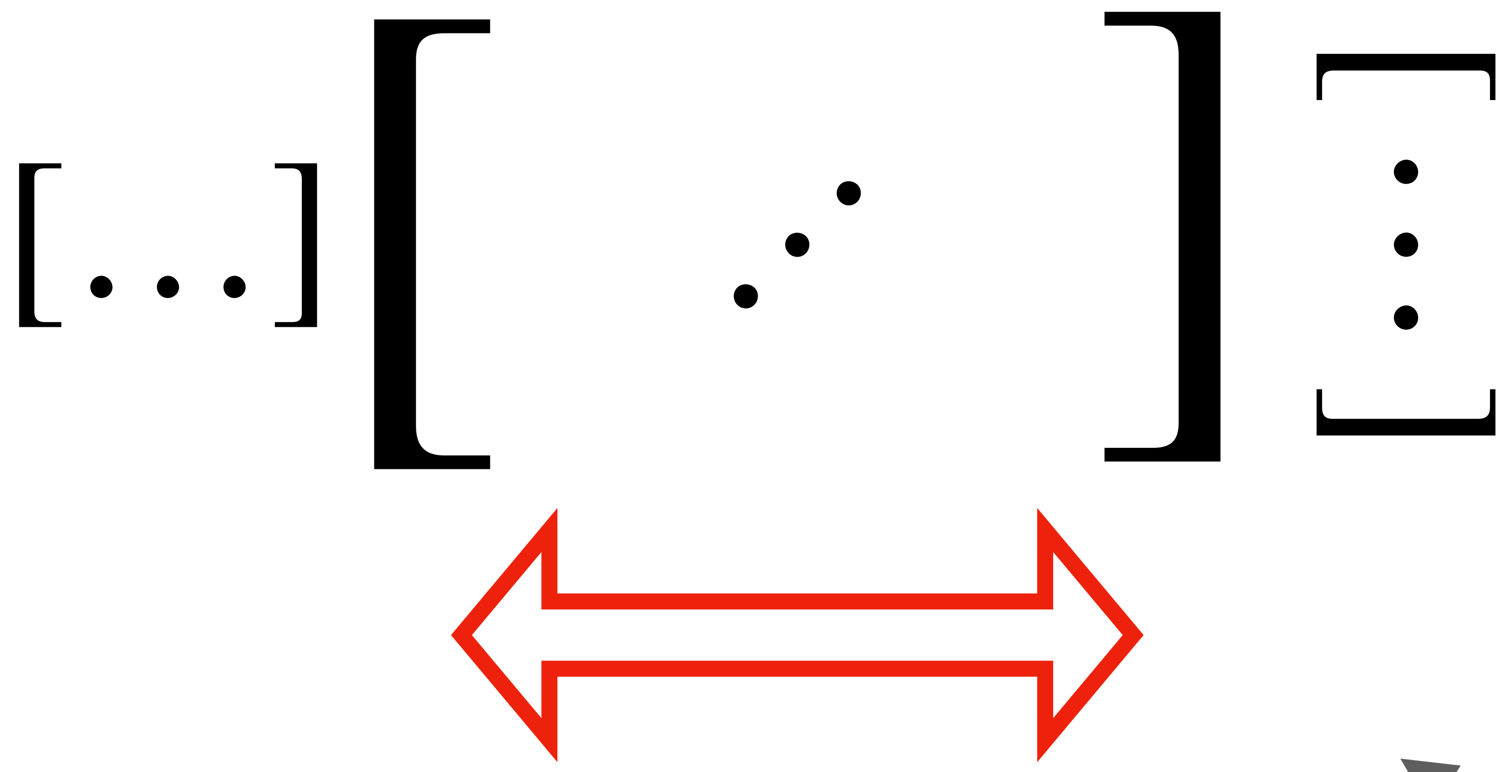


Classically: gigantic size, diagonalize H with infinite dim , impossible/hard !!

Quantum Computing for HEP

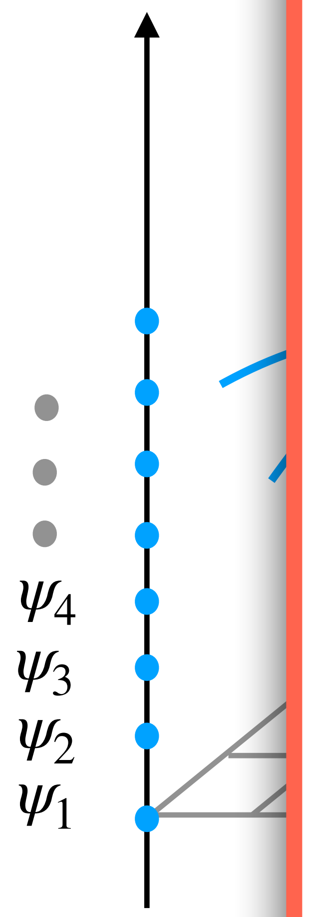
In HEP, we are dealing with n-pt correlates, i.e. S-matrix

$$\langle \text{out} | e^{-iH[\psi]t} | \text{in} \rangle$$



$$\text{dim} = (n_\psi)^{n^{D_{sp}}}$$

Quantum computing: reasonable size



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For the hadron: $10^2\text{MeV} \lesssim E \lesssim 1\text{GeV}$

$$n^{D_{sp}} \sim 10^{3}$$

Suppose $n_\psi = 2^5 = 32$

dim $\to \infty$

Require qubits

$n_{q,\text{LHC}} \sim 5 \times 10^{12}$ $n_{q,\text{hadron}} \sim 5000$

Quantum Computing for HEP

In HEP, we are dealing with n-pt correlates, i.e. S-matrix

$$\langle \text{out} | e^{-iH[\psi]t} | \text{in} \rangle$$

$$[\ldots]\begin{bmatrix} & & \cdot \\ & \cdot & \\ \cdot & & \end{bmatrix}$$

$$\text{dim} = (n_\psi)^{n^{D_{sp}}}$$

decompose to a set of gates, evolution much cheaper

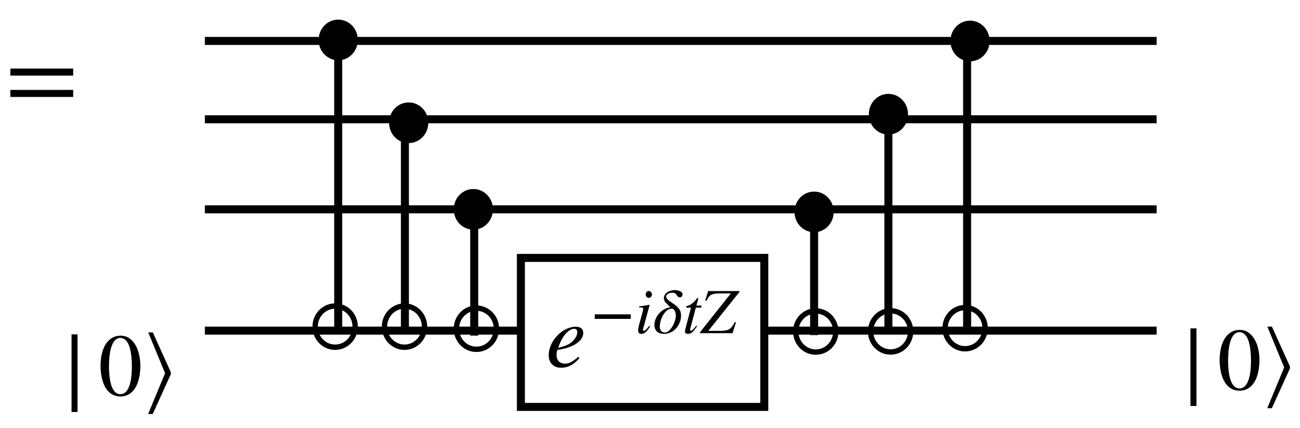
$$e^{-iHt} \approx \lim_{\delta t \to 0, N \to \infty} \left[e^{-iH\delta t}\right]_N$$ Trotter, 1959

e.g.

$$H = Z_1 \otimes Z_2 \otimes Z_3$$

Use the fact that $|\phi_1\rangle|\phi_2\rangle \ldots |0\rangle \to |\phi_1\rangle|\phi_2\rangle \ldots |\phi_1 \oplus \phi_2 \ldots\rangle$

$$e^{-iH\delta t} =$$



Others can be realized similarly

e.g. by using $e^{-i\delta t X_1 \otimes Z_2 \otimes \ldots} = H_1 e^{-i\delta t Z_1 \otimes Z_2 \otimes \ldots} H_1$

Quantum computing: reasonable size and operations (scales logarithmically)

A toy model

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2$$ (no gauge, 1+1)   Gross, Neveu, 1974

$$f(x) = \int dz^- e^{-ixM_h z^-} \langle h | \bar{\psi}(z^-)\gamma^+\psi(0) | h \rangle = \int dz^- e^{-ixM_h z^-} \langle h | e^{iHz}\bar{\psi}(0,-z)e^{-iHz}\gamma^+\psi(0) | h \rangle$$

- Map the QFT on to a qubits+gates system
- Prepare the external state $|h\rangle$
- Evolution

QuNu Collaboration

Li, et al, PRD letter 22

A toy model

- Map the QFT on to a qubits+gates system

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2 \quad \text{(no gauge, 1+1)}$$

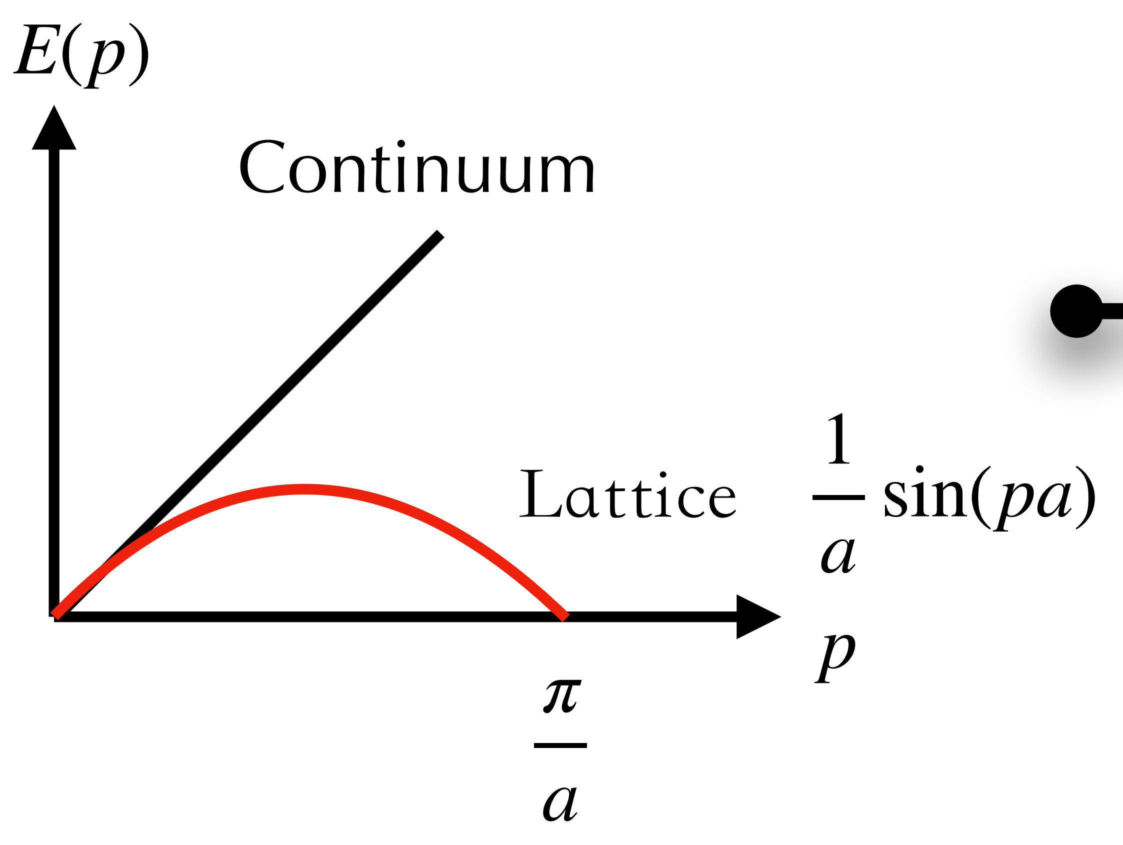
Gross, Neveu, 1974

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \to \begin{pmatrix} \phi_{2n} \\ \phi_{2n+1} \end{pmatrix}$$

Staggered fermion,
Put different fermion components, flavors on different sites

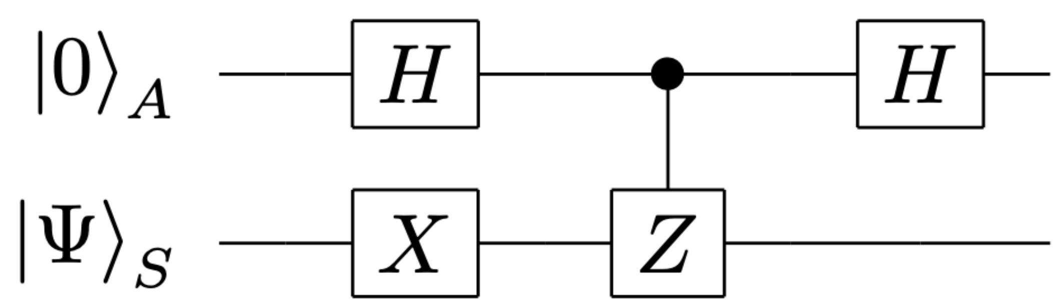
$$\phi_n = \prod_{i<n} Z_i (X + iY)_n$$

Jordan-Wigner



Fermion doubling!!

fields will be represented by a set of gates.



Quantum circuit for $\psi_1(0)$

Li, et al, PRD letter 22

A toy model

- Map the QFT on to a qubits+gates system

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2 \quad \text{(no gauge, 1+1)}$$

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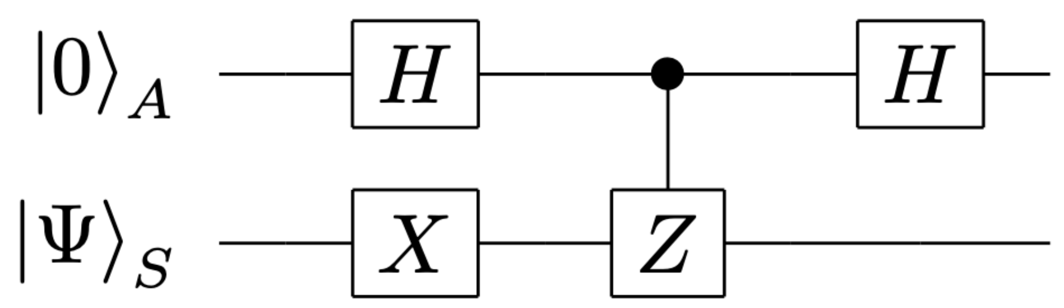
Jordan-Wigner

$$f(x) \to \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi^\dagger_{-2z+i} e^{-iHz} \phi_j | h \rangle$$

fields will be represented by a set of gates.

$$H = \underbrace{\sum_{n=\text{even}} \frac{1}{4} \left[X_n Y_{n+1} - Y_n X_{n+1} \right]}_{H_1} + H_2 + H_3 + H_4$$



Quantum circuit for $\psi_1(0)$

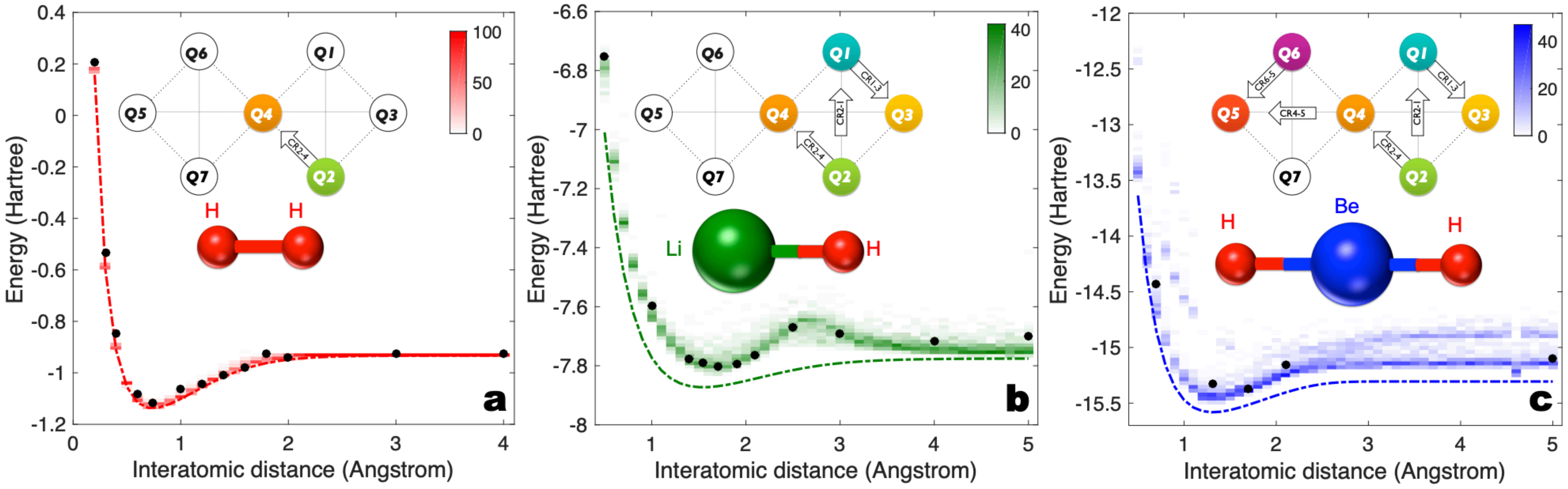
Li, et al, PRD letter 22

A toy model

- Prepare the external state $|h\rangle$

  Prepare the state by Variational-Quantum-Eigensolver (VQE) 2103.08505 + …

Potential energy surfaces



show its power in quantum chemistry 1704.05018v2

Li, et al, PRD letter 22

A toy model

- Prepare the external state $|h\rangle$

  Prepare the state by Variational-Quantum-Eigensolver (VQE) 2103.08505 + …

$|\psi_\Omega\rangle = |0101\ldots01\rangle$

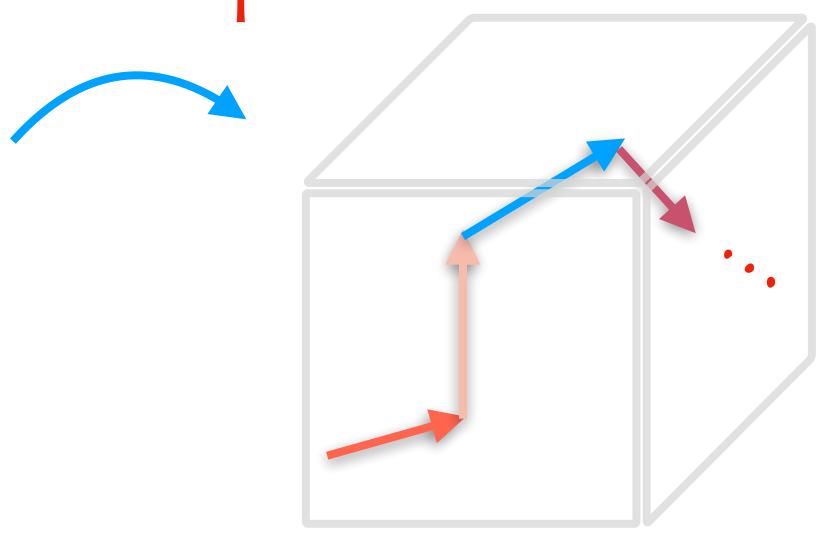
$|\psi_h\rangle = |1001\ldots01\rangle + |0110\ldots01\rangle + \ldots$

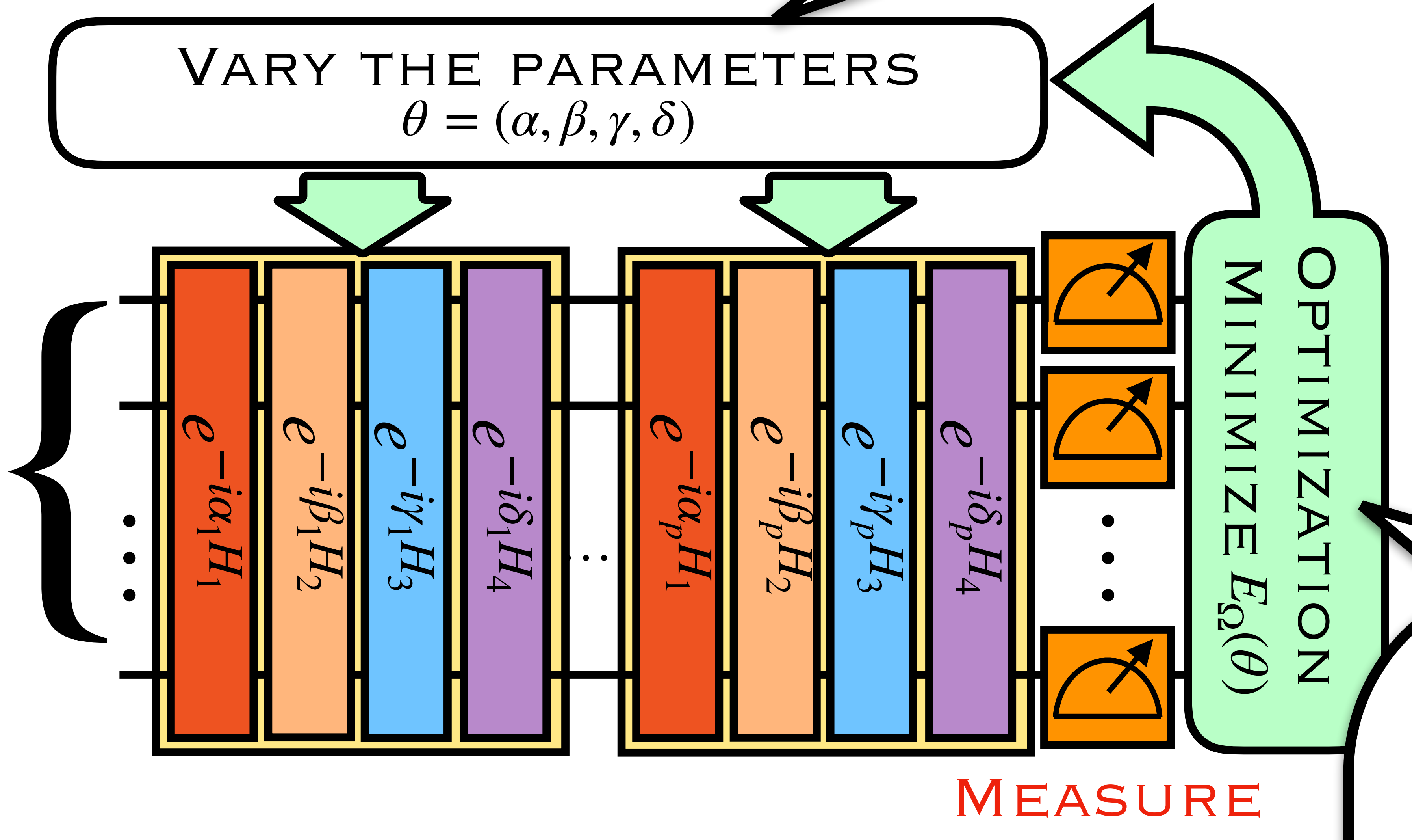
1904.07358

$\psi_i(\theta)\rangle = \prod e^{iH_i\theta_i}|\psi_i\rangle \quad [H_i, H_{i+1}] \neq 0$

Span the Hilbert space

Conserve quantum numbers





$$E(\theta) = \sum_i^k w_i \langle \psi_i(\theta)|H|\psi_i(\theta)\rangle$$

Minimize the cost function

Li, et al, PRD letter 22

A toy model

- Evolution

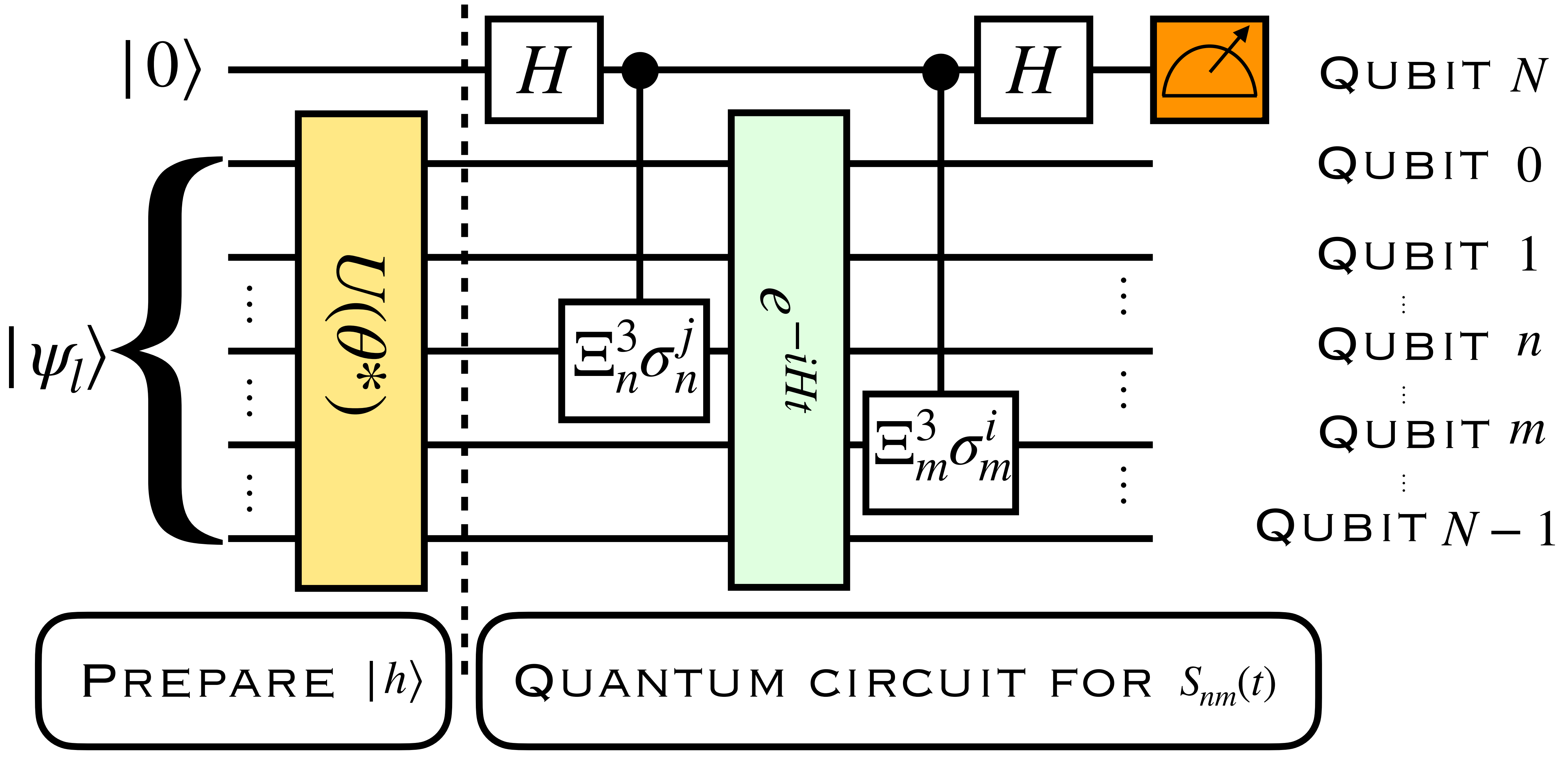
Evaluate evolution

$$S_{mn}(t) = \langle h | e^{iHt} \Xi^3_m \sigma^i_m e^{-iHt} \Xi^3_n \sigma^j_n | h \rangle$$

hermitian but not unitary

$$S_{mn}(t) = p_{mn}(t) - \frac{1}{2}$$

Pedernales et al., PRL, 2014



Encoding the S_{mn} to the auxiliary field

Trace out the $|h\rangle$ field

$$f(x) \to \sum_{i,j} \sum_{z} \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi^\dagger_{-2z+i} e^{-iHz} \phi_j | h \rangle$$

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A toy model

- Evolution

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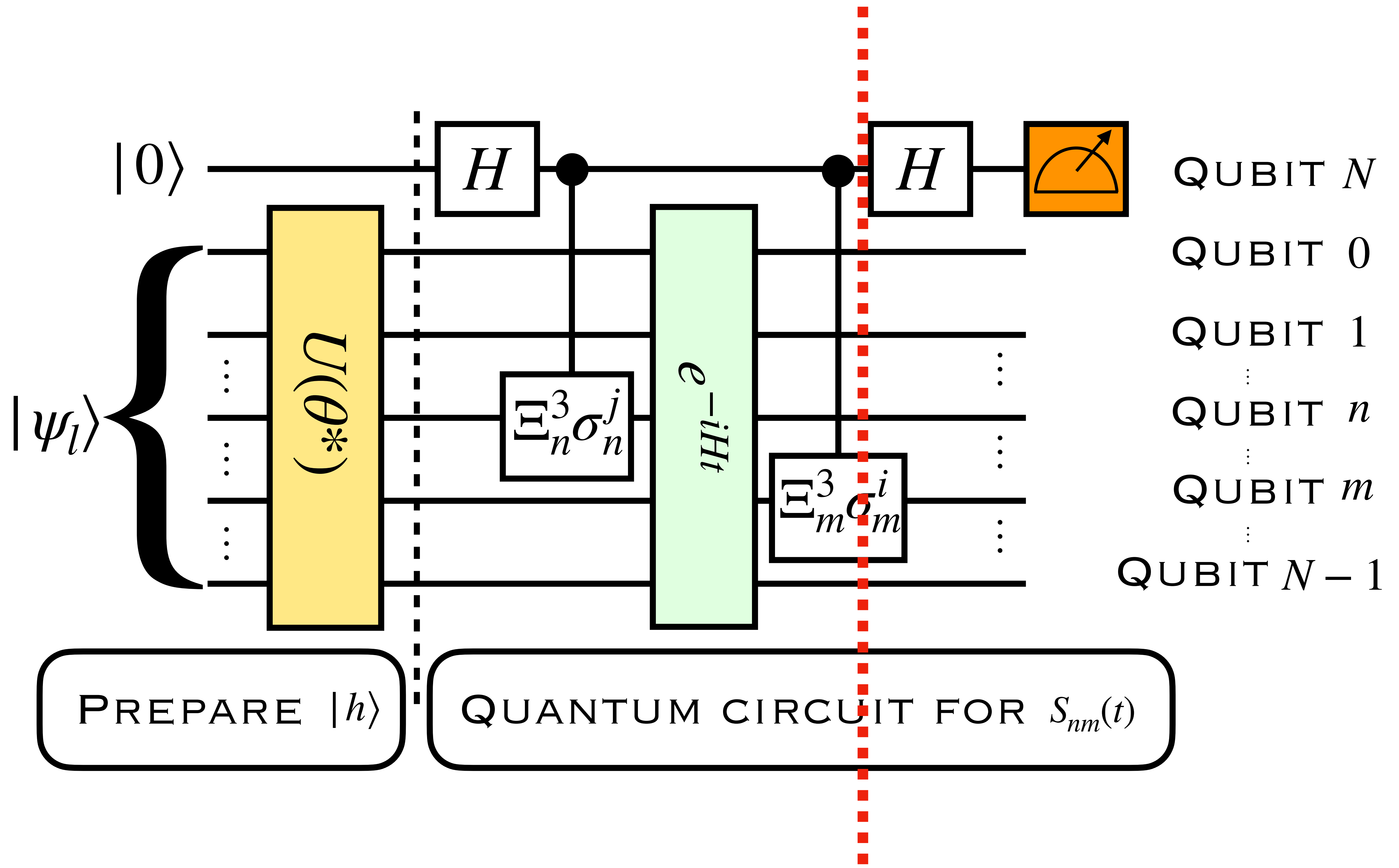

$$|0\rangle|h\rangle \rightarrow e^{-iMt}\left[|0\rangle|h\rangle + |1\rangle e^{iMt}\psi^{\dagger}e^{-iHt}\psi|h\rangle\right]$$

$$\rho_a = \mathrm{Tr_h}[\ldots] \rightarrow \mathrm{a}|0\rangle\langle 0| + |1\rangle\langle 0|\mathrm{S} + |0\rangle\langle 1|\mathrm{S}^{\dagger} + |1\rangle\langle 1|(1-\mathrm{a})$$

Li, et al, PRD letter 22

A toy model

- Evolution

Evaluate evolution

$$S_{mn}(t) = \langle h | e^{iHt} \Xi^3_m \sigma^i_m e^{-iHt} \Xi^3_n \sigma^j_n | h \rangle$$

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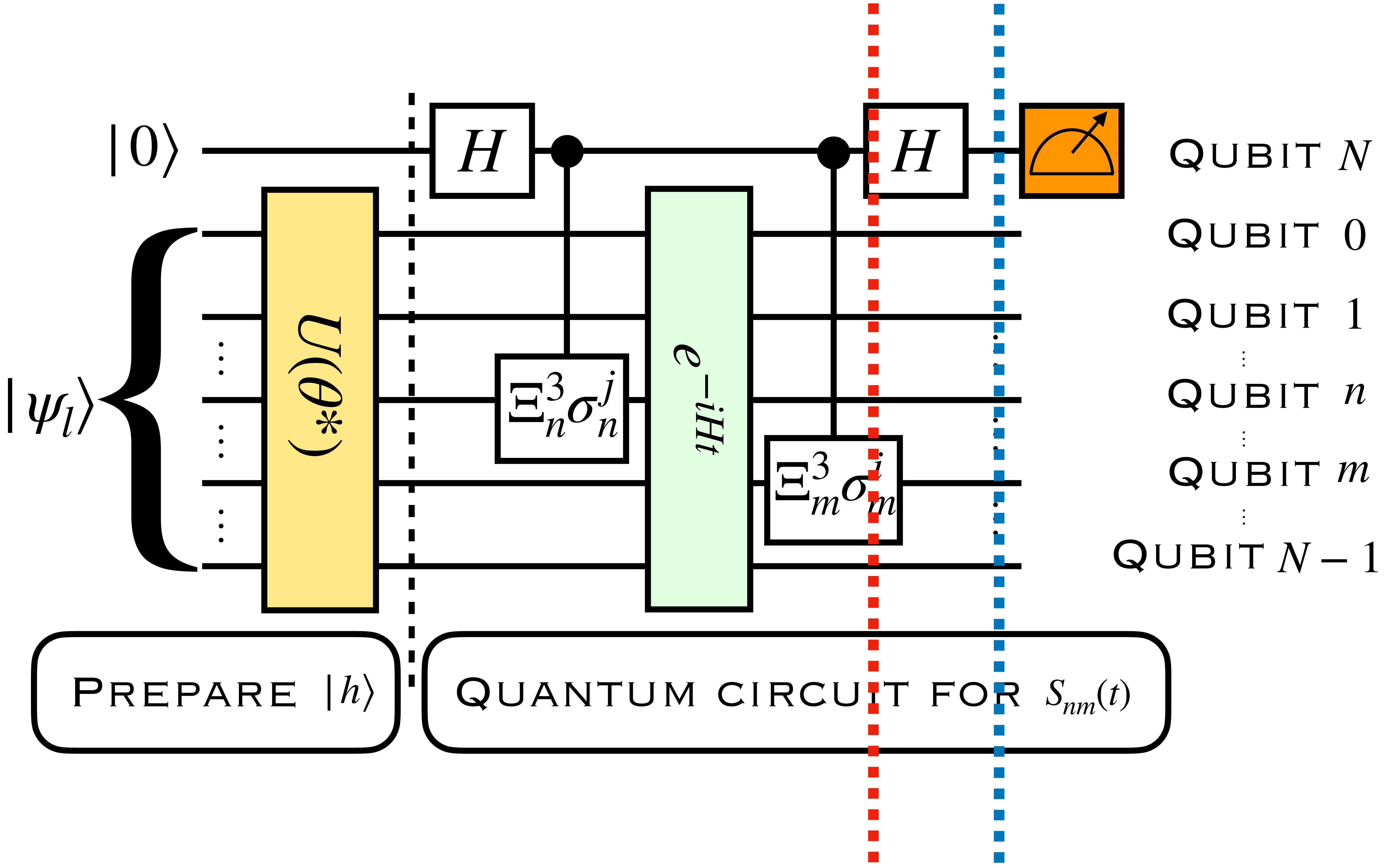


$$|0\rangle |h\rangle \to e^{-iMt} \left[|0\rangle |h\rangle + |1\rangle e^{iMt} \psi^\dagger e^{-iHt} \psi |h\rangle \right]$$

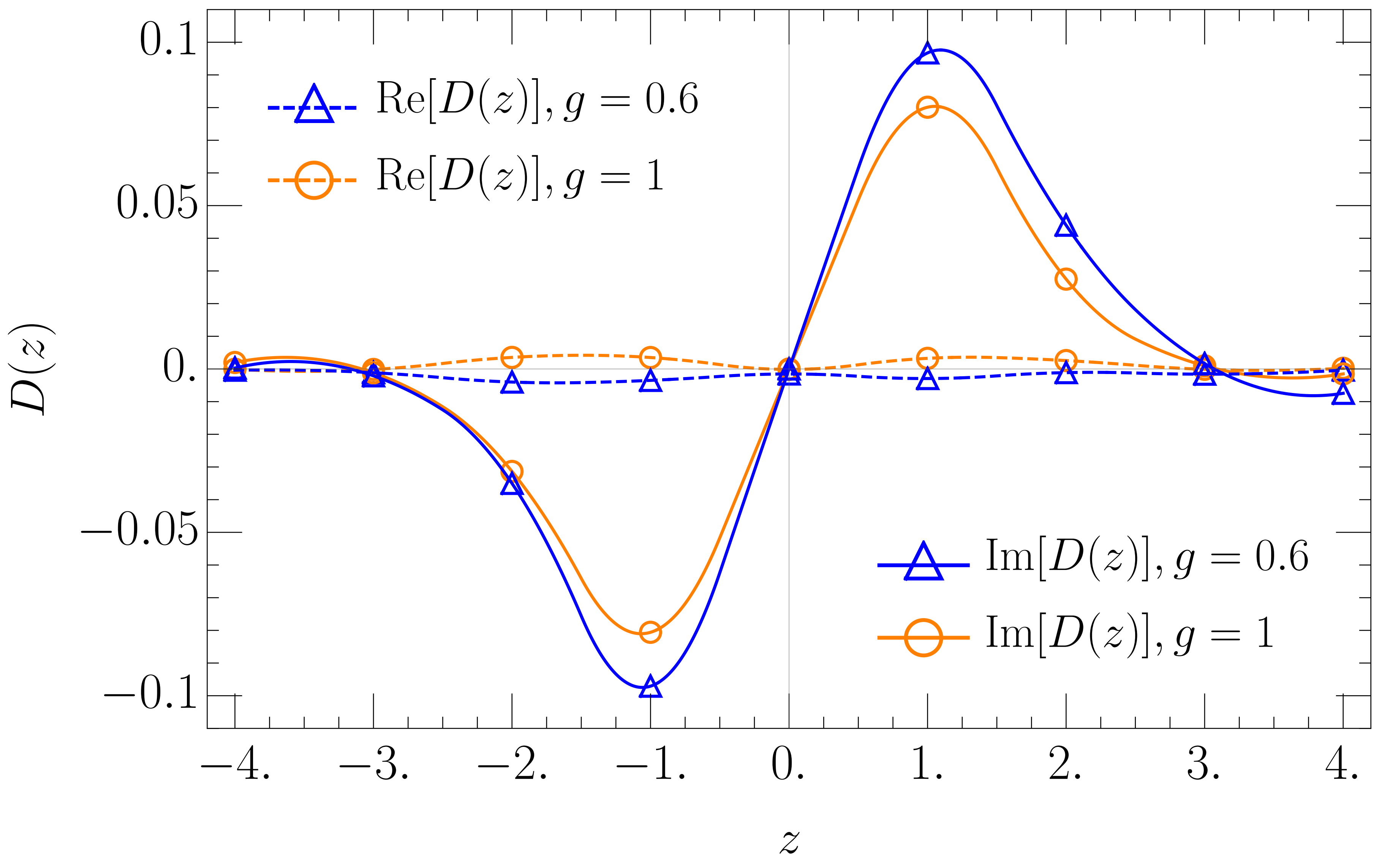
$$\rho_a = \mathrm{Tr_h}[\ldots] \to \mathrm{a}|0\rangle\langle 0| + |1\rangle\langle 0|\mathrm{S} + |0\rangle\langle 1|\mathrm{S}^\dagger + |1\rangle\langle 1|(1-\mathrm{a})$$

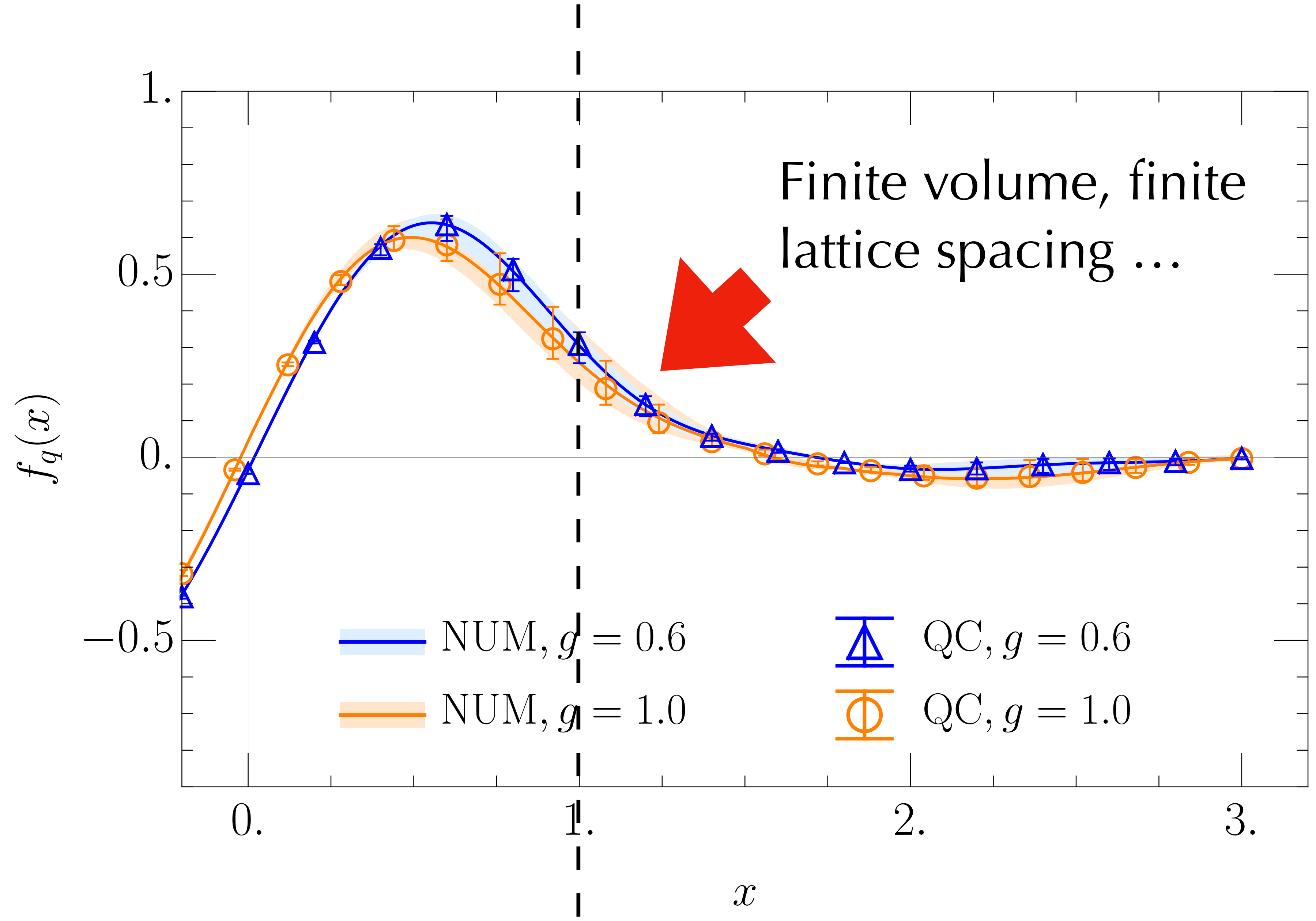
$$\mathrm{Tr}[\mathrm{H}\rho_\mathrm{a}\mathrm{H}] \to \rho_{00} = 1 + 2\mathrm{Re}[\mathrm{S}]$$

Li, et al, PRD letter 22

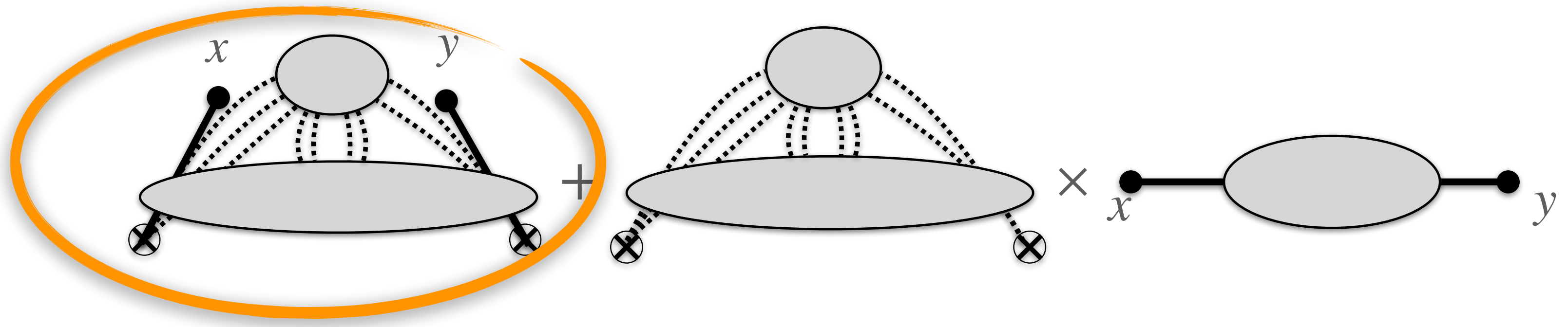
A toy model

- Results





$${}_H\langle h \,|\, T[\psi_H(x)\,\psi_H(y)] \,|\, h\rangle_H =$$



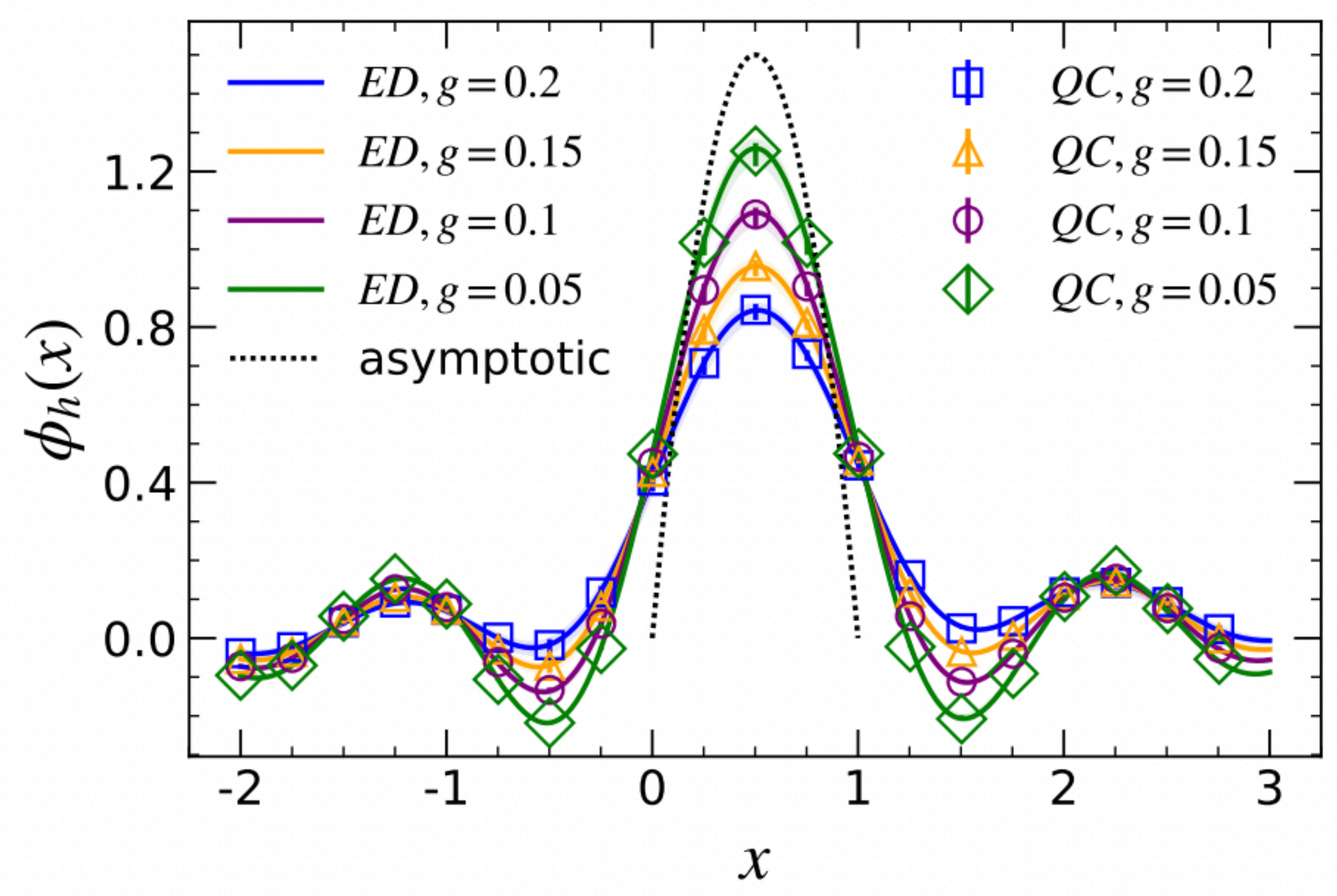
$$f(x) \to \sum_{i,j} \sum_{z} \frac{1}{4\pi} e^{-ixM_h z} \langle h \,|\, e^{iHz} \phi^{\dagger}_{-2z+i} e^{-iHz} \phi_j \,|\, h\rangle$$

Li, et al, PRD letter 22

A toy model: another application

- Light cone distribution amplitude

$$\text{F.T.}\langle 0|\psi(x)\psi(y)|\pi\rangle$$



Li, et al., 2207.13258

Conclusion

- A possible new tool for HEP
- Still at the proof-of-principle (toy) stage
- Very young field, young scientists can have major impacts

> "Start training at the undergraduate level. This is a really new field and so we have ample relatively simple yet nevertheless interesting projects to which undergraduate students can contribute. This is even true in theory, where it is often believed that one cannot do anything useful until after taking a course in QFT. We should make sure to take advantage of this exciting opportunity to engage very young people and start the training early."

Thanks