北京師範大學
BEIJING NORMAL UNIVERSITY

Single Inclusive Jet Production in pA Collisions at NLO in the small-x regime

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With Kexin Xie, Zhong-bo Kang and Xiaohui Liu

The 4th Workshop of Heavy flavors physics and QCD, Chang Sha

29th July, 2022

Based on

[HL, K.Xie, Z.Kang, X.Liu, arXiv:2204.03026]

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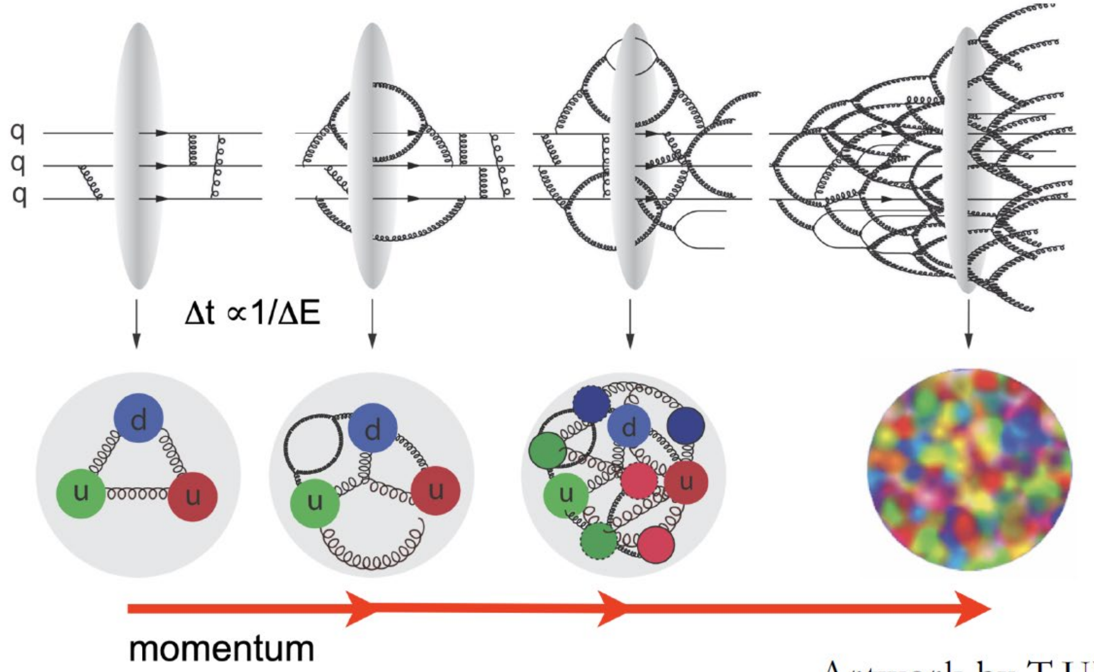
Outline

- **Review of CGC effective theory**
- **forward jet production in pA**
  - Motivation and Difficulties
  - Subtraction method
  - result
- **Outlook**

Gluon Saturation and CGC

The gluon density increases with Bjorken x decreases



Artwork by T.Ullrich

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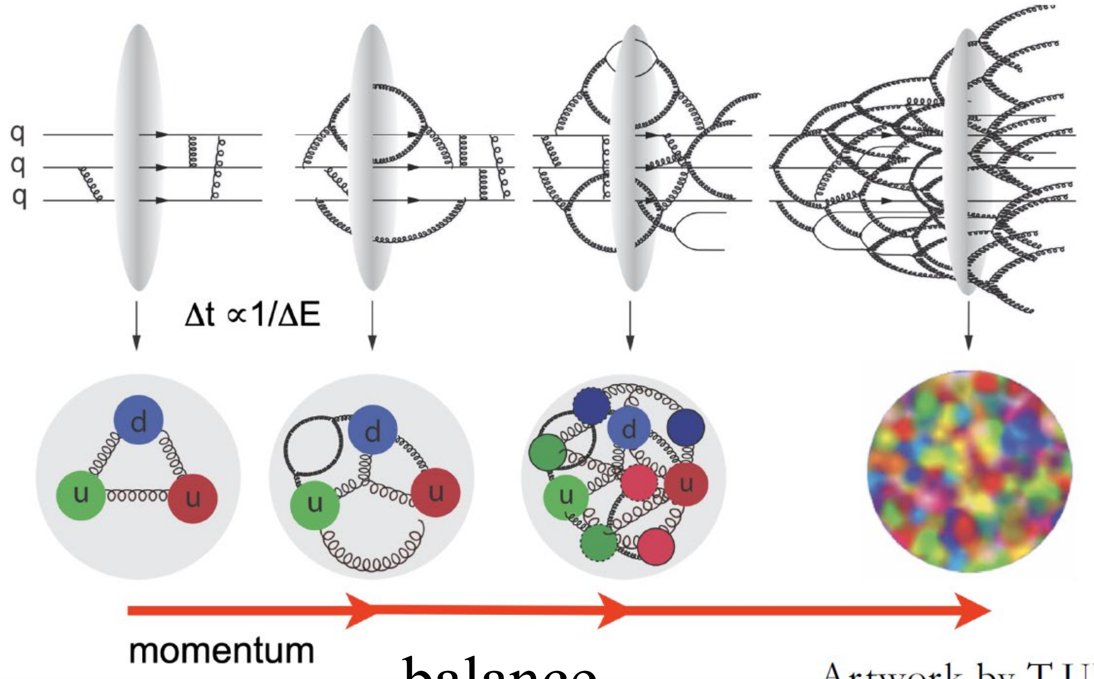


Artwork by T.Ullrich

balance

splitting recombination

Gluon saturation

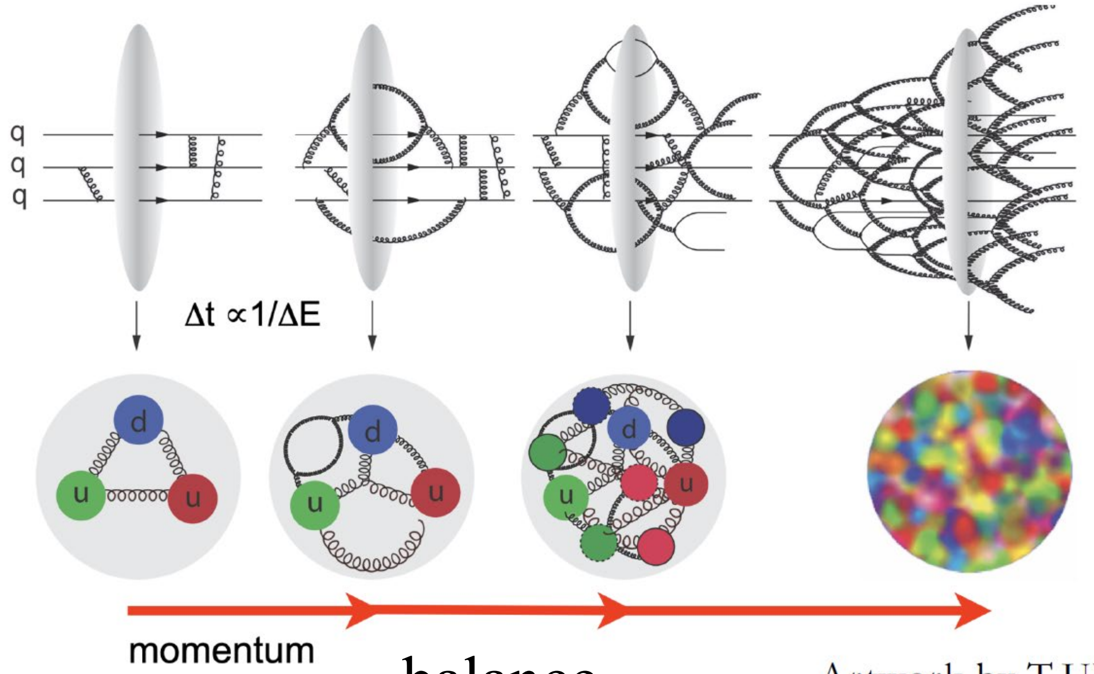
Non-linear evolution

CGC (Color Glass Condensate) is one of the most appropriate theories for saturation

Effective theory of QCD

Gluon Saturation and CGC

The gluon density increases with Bjorken x decreases



Artwork by T.Ullrich

Balitsky-Kovchegov(BK) evolution equation

[I.Balitsky ,1997] [Y.Kovchegov,2000]

$$\frac{dS^{(2)}_{X_f}(b_\perp, b'_\perp)}{d\ln X_f} \propto S^{(2)}_{X_f}(b_\perp, b'_\perp) - \boxed{S^{(2)}_{X_f}(r_\perp, b'_\perp) S^{(2)}_{X_f}(b_\perp, r_\perp)}$$

balance

splitting ⟷ recombination

Gluon saturation ⇓ Non-linear evolution

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Gluon Saturation and CGC

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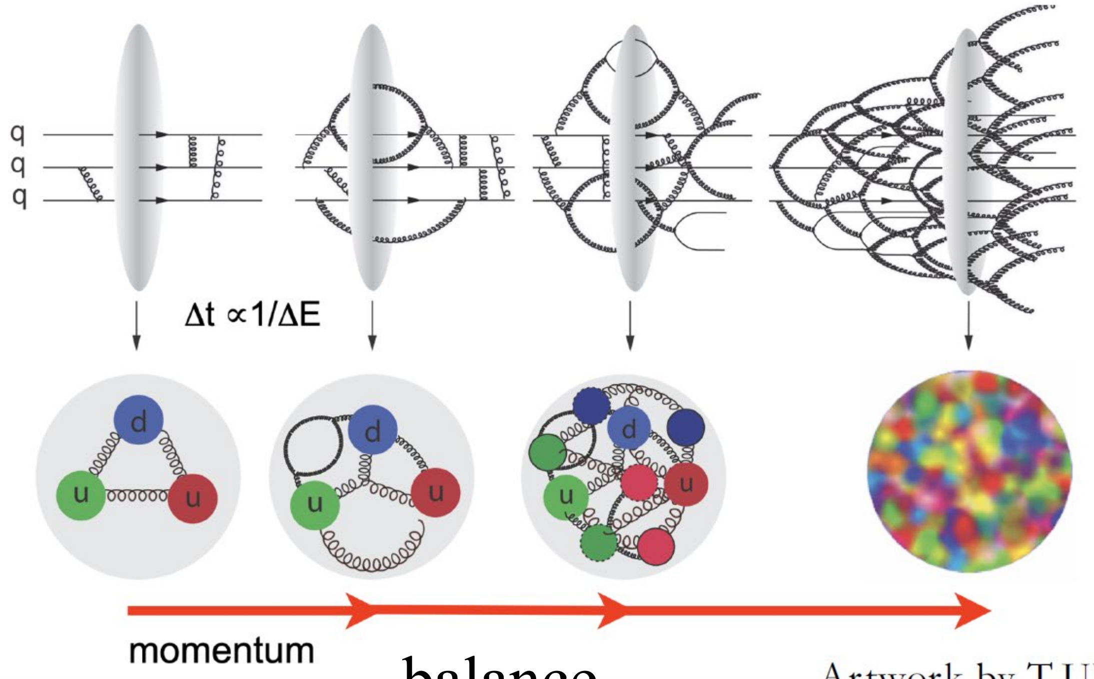

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Dipole amplitude(distribution in CGC)

$$S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{b}'_\perp) = \frac{1}{N_c} \langle Tr[W(\mathbf{b}_\perp) W^\dagger(\mathbf{b}'_\perp)] \rangle_{X_f}$$

Factorization framework

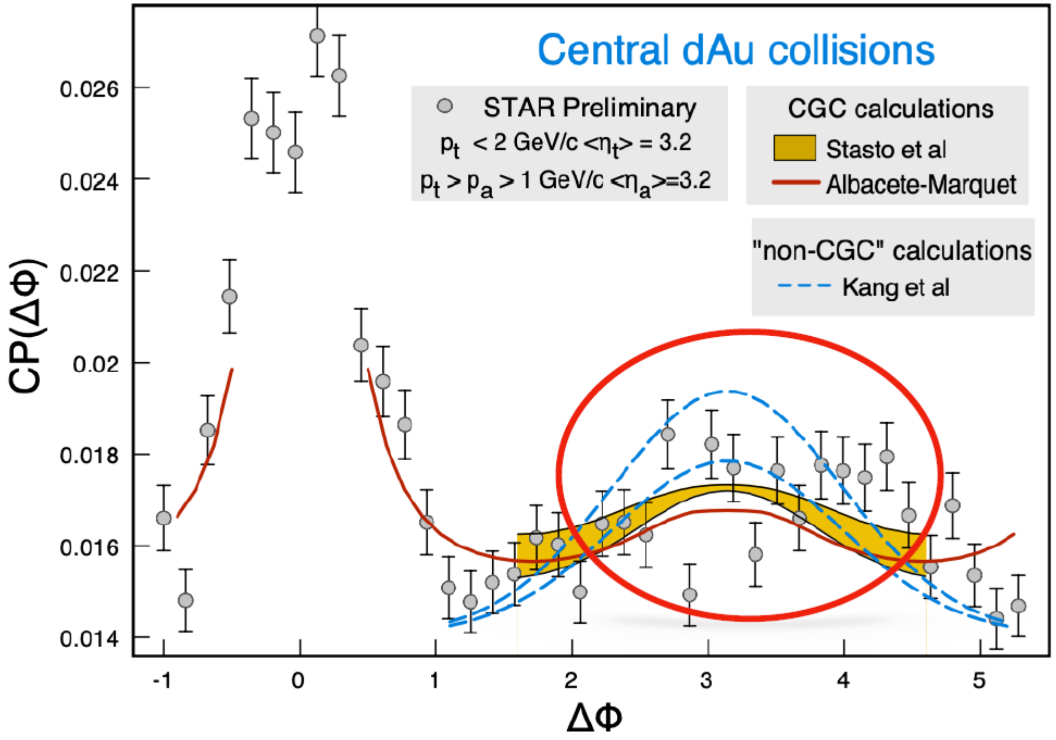
Distribution with non-linear evolution

CGC (Color Glass Condensate) is one of the most appropriate theories for saturation

Effective theory of QCD

Searching for deterministic evidence of saturation

One of the strong hints for saturation



[E. Braidot [STAR Collaboration], 2011.]

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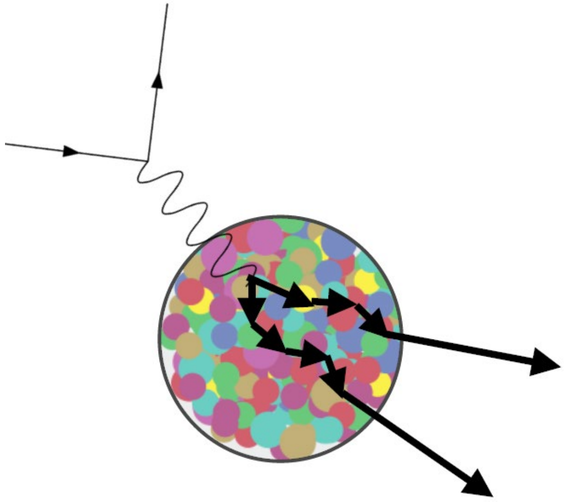
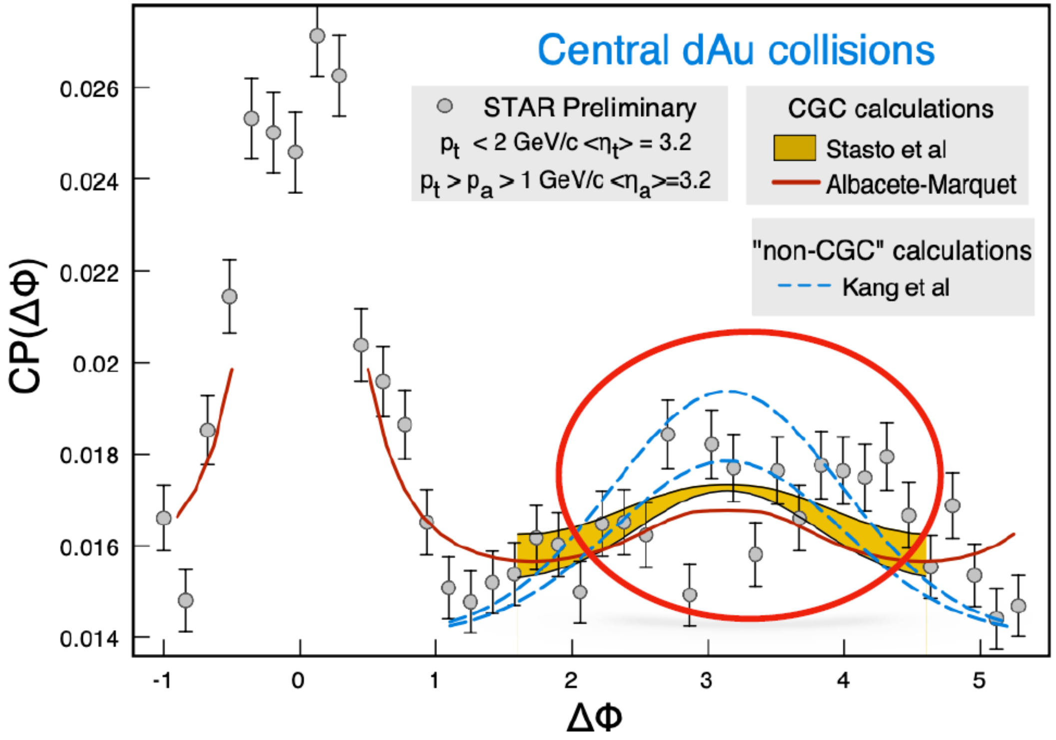


Away-side peak of the di-hadron correlation

Prediction based on CGC describe the data well

Searching for deterministic evidence of saturation

One of the strong hints for saturation



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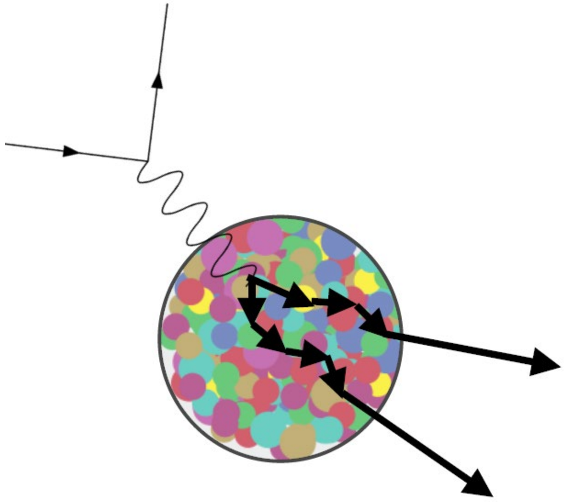


Away-side peak of the di-hadron correlation

Prediction based on CGC describe the data well

The same data sets can also be explained by collinear twist calculation

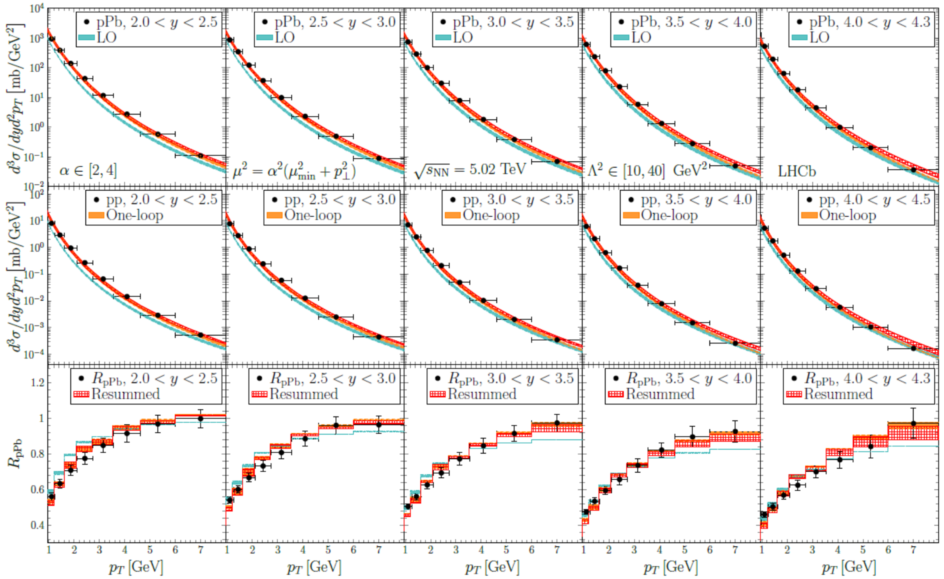
How to distinguish

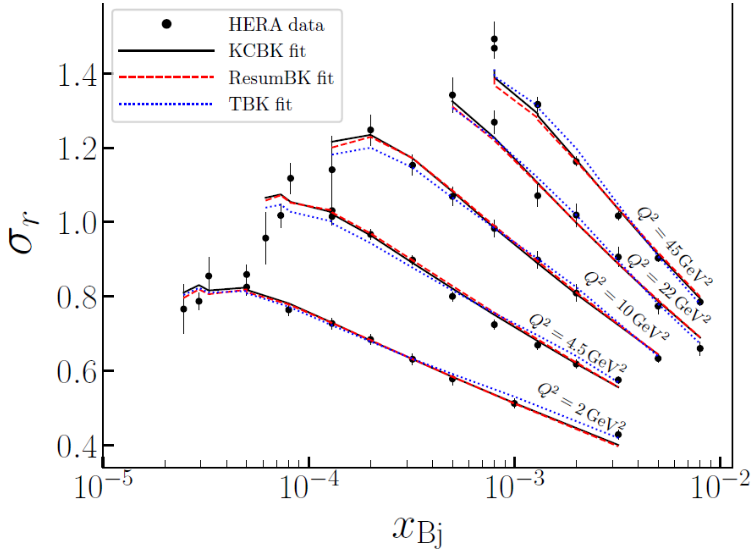
Reduce Error
Theory side
Higher order

More processes
Besides hadron

Searching for deterministic evidence of saturation

Efforts for higher precision in CGC

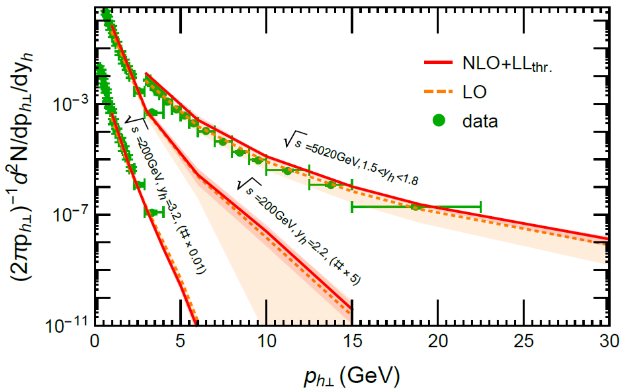




Inclusive DIS

[B. Ducloué, H. Hänninen, T. Lappi, Y. Zhu, 2017]

[G. Beuf, H. Hänninen, T. Lappi, H. Mäntysaari, 2020]



Single hadron production in pA

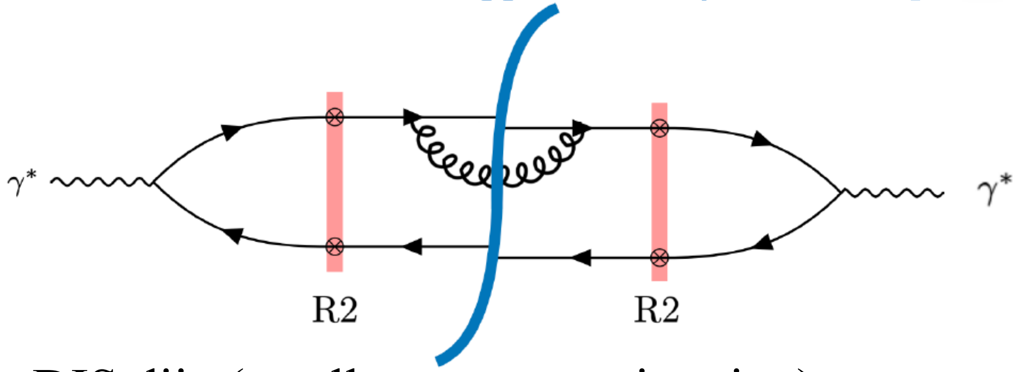
[G.Chirilli,B.Xiao,F.Yuan, 2012]

[Y.Shi, L.Wang, S.Wei, B.Xiao, 2021]

[Iancu, Mueller, Triantafyllopoulos, 2016]

[HL, Y.Ma, K.Chao, 2019]

[HL, Z.Kang, X.Liu, 2020]



DIS dijet(small cone approximation)

[P.Caucal, F.Salazar, R.Venugopalan, 2021]

[K.Roy, R.Venugopalan, 2019]

Motivation for NLO jet

Jet is a bunch of particles flying nearly in the same direction in high energy collider

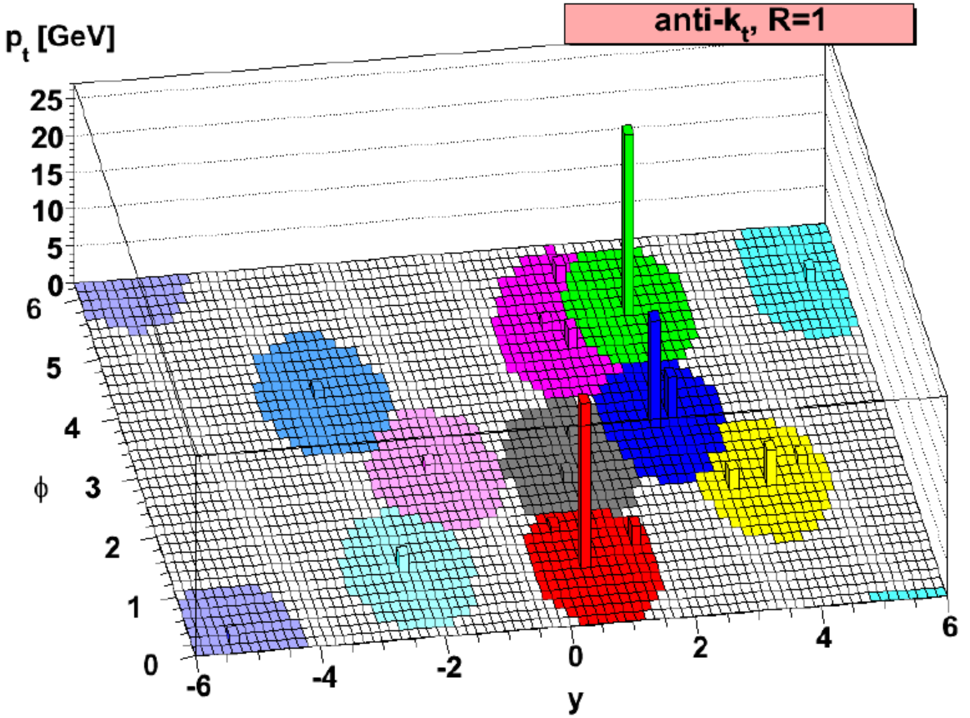
Jet algorithms are used to classify particles into jets

particles $\{p_i\}$ → $\{j_k\}$ jets

jet clustering algorithms

Comparing with hadron, jet is cleaner, in sense that is perturbatively calculable

R controls the extension of the jets



[Cacciari, Salam, and Soyez,JHEP,2008]

NLO attempts in small cone approximation:

[D. Ivanov, A.Papa, 2012]
[P.Caucal, F.Salazar, R.Venugopalan, 2021]
[E. Iancu and Y. Mulian ,2021]
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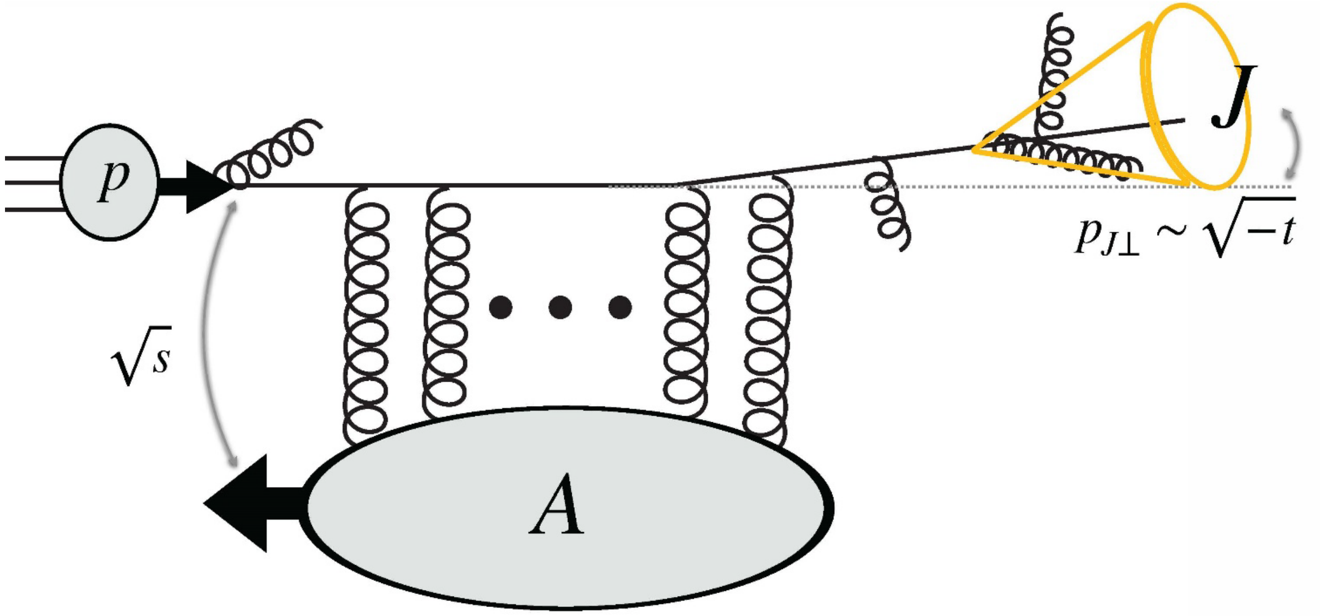
An apple-to-apple comparison of the CGC theory with the experimental results, including the jet clustering procedure that strictly follows the experimental analyses

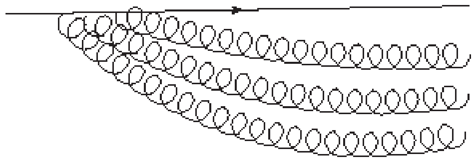
Forward single jet production: power counting



Power counting: $\lambda \sim \frac{p_{J\perp}}{\sqrt{s}} \ll 1$

By parameter λ , we have collinear modes and soft modes

- Collinear mode

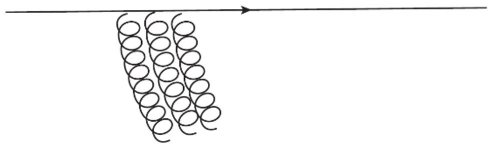


$\sqrt{s}(1, \lambda^2, \lambda)$

$(+, \perp, -)$

Energetic radiation along the z direction

- Soft mode



$\sqrt{s}(\lambda, \lambda, \lambda)$

Soft radiations with energy of order $p_{J\perp}$

Modes

⬇

Feynman Rules

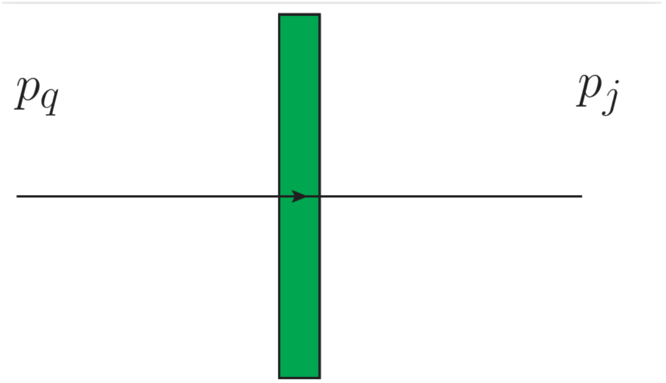
⬇

Amplitudes

See [Z.Kang,X.Liu, arXiv:1910.10166]

Forward single jet production: phase space

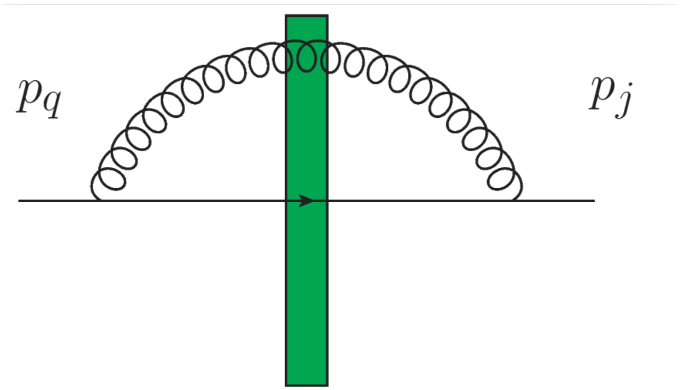
Leading Order



$$p_J = p_j$$

Trivial phase space

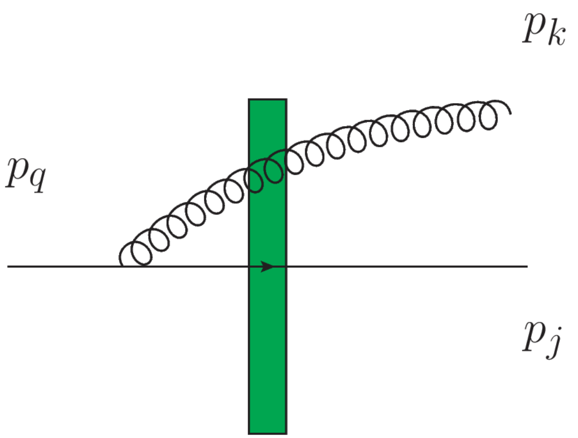
Virtual correction at NLO



$$p_J = p_j$$

Identical to LO

Real correction at NLO



Things are different for the real correction because of the jet algorithm

The same as single hardon case

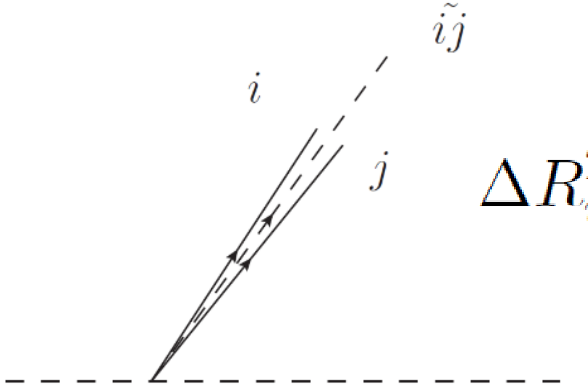
Anti-kT jet algorithm

The distances [Cacciari, Salam, and Soyez, JHEP,2008]

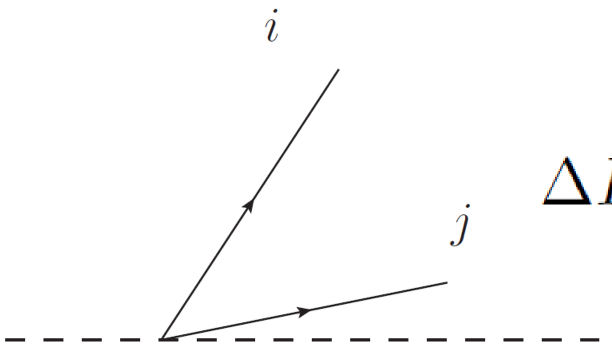
$$\rho_i = k_{T,i}^{-2} \qquad \rho_{ij} = \min(k_{T,i}^{-2}, k_{T,j}^{-2})\frac{\Delta R_{ij}^2}{R^2}$$

where $\Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (\eta_i - \eta_j)^2$

$k_{T,i}$ transverse momenta η_i and ϕ_i the rapidity and azimuthal angle



$\Delta R_{ij}^2 \leq R^2$



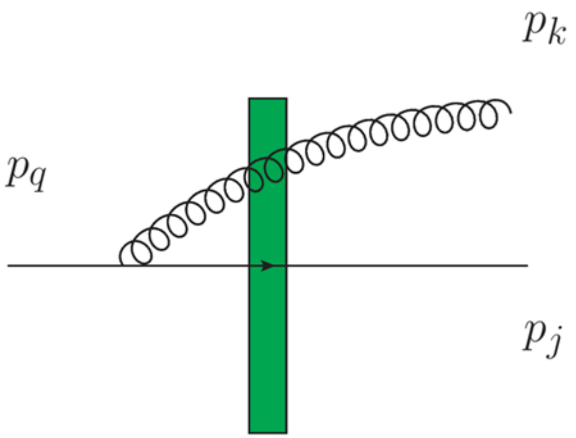
$\Delta R_{ij}^2 > R^2$

- If ρ_{ij} is the smallest, i and j will be clustered
- If ρ_i is the smallest, i will be a jet

Removed from the list

Until all the particles clustered

Real correction phase space



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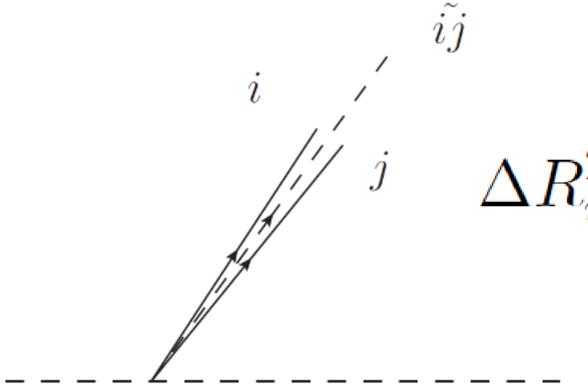

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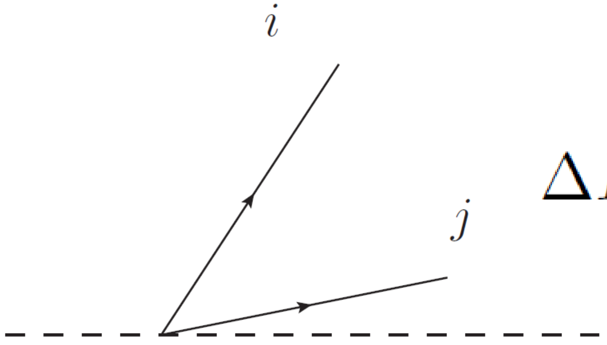

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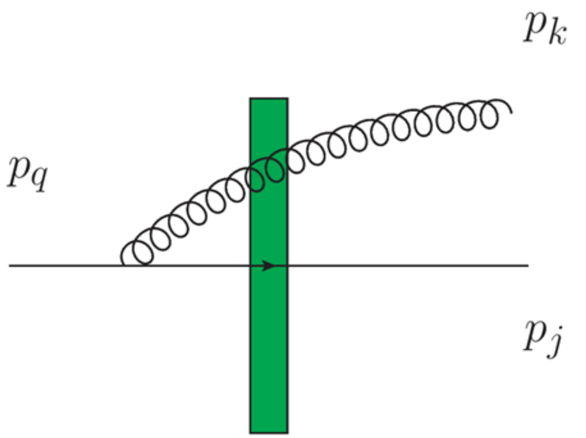

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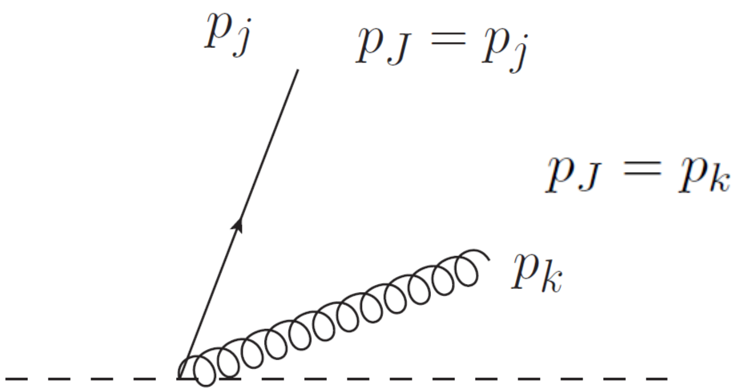
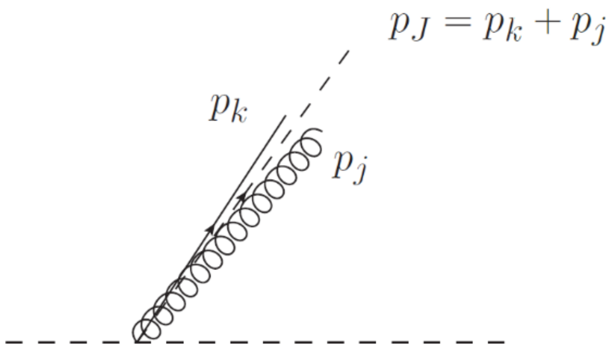


2 jets case

$$\Theta_2 = \Theta(\Delta R_{jk}^2 - R^2)$$



1 jet case

$$\Theta_1 = \Theta(R^2 - \Delta R_{jk}^2)$$

Constraint of phase space for jet

$$\int d\Phi \times \Theta_{alg}$$

No constraint for other process, e.g. hadron production

$$\int d\Phi \times 1$$

Difficulty for real correction for jet

Trivial example to highlight the difficulty

$$\int_0^1 dx \frac{f(x)}{x^{1+\epsilon}}$$ → Containing both jet algorithm dependence and divergence

Divergent → Can't calculate it numerically

Complicated → Can barely calculate it analytically

Construct subtraction term

The counter term

Can be calculated numerically

$$\int_0^1 dx \frac{f(x)}{x^{1+\epsilon}} = \int_0^1 dx \left(\frac{f(x)}{x^{1+\epsilon}} - \boxed{\frac{f(0)}{x^{1+\epsilon}}} \right) + \boxed{f(0) \int_0^1 \frac{dx}{x^{1+\epsilon}}} = \boxed{\int_0^1 \frac{f(x) - f(0)}{x}} - \frac{f(0)}{\epsilon}$$

No jet algorithm dependence

Can be calculated analytically

Shares exactly the same infrared behavior as the original integrand

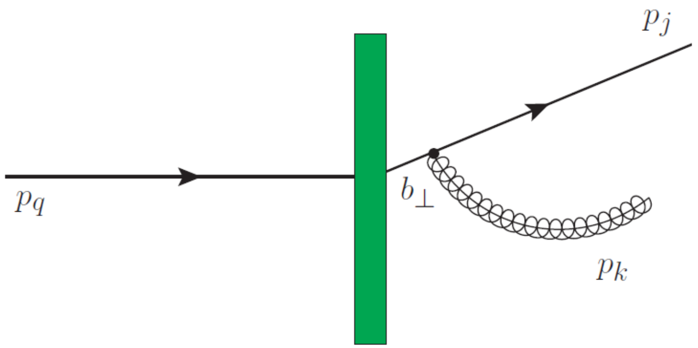
Added back

Finite

An example

The square of the matrix element of the final state radiation term is

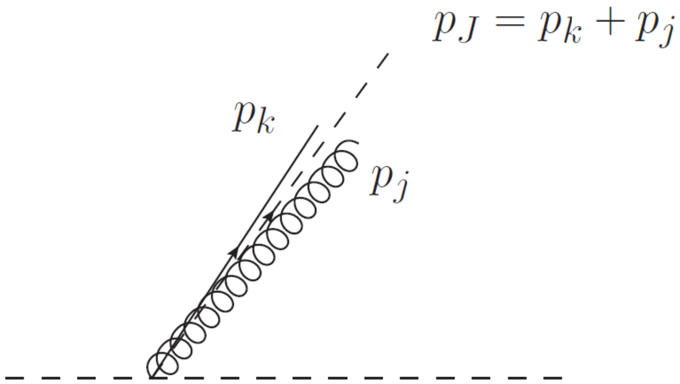
$$d\sigma_{fsr} \propto \quad xf(x)\frac{1+\xi^2}{(1-\xi)^{1+\eta}}\frac{\mathcal{F}_F(p_{k\perp}+p_{j\perp};X_f)}{[\xi p_{k\perp}-(1-\xi)p_{j\perp}]^2}(\Theta_1+\Theta_2)$$



Final state radiation

final-final collinear limit

$$\xi p_{k\perp} \to (1-\xi)p_{j\perp}$$



1 jet case

$\Theta_1 = 1$ and $\Theta_2 = 0$

$$d\sigma^c_{fsr} \propto \tau f(\tau)\frac{1+\xi^2}{(1-\xi)^{1+\eta}}\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}$$

An example

Finite combination $d\sigma_{fsr} - d\sigma^c_{fsr}$

$$\frac{\alpha_s S_\perp}{2\pi^2}\frac{N_C}{2}\int_0^1 d\xi \frac{1+\xi^2}{1-\xi}\int d^2p_{k\perp}\left\{\Theta_2\, x f(x)\frac{\mathcal{F}_F(p_{k\perp}+p_{J\perp};X_f)}{[\xi p_{k\perp}-(1-\xi)p_{J\perp}]^2}+\Theta_1\,\tau f(\tau)\frac{\mathcal{F}_F(p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}-\tau f(\tau)\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}\right\}$$

Free of divergence → Numerically calculable

The counter term $d\sigma^c_{fsr}$

$$\frac{\alpha_s S_\perp}{2\pi^2}\frac{N_C}{2}\tau f(\tau)\int_0^1 d\xi \frac{1+\xi^2-\epsilon(1-\xi)^2}{(1-\xi)^{1+\eta}}\left(\frac{\nu}{p_q^+}\right)^\eta \int d^{D-2}p_{k\perp}\frac{\mathcal{F}_F(p_{k\perp}+\xi p_{J\perp};X_f)}{[p_{k\perp}-(1-\xi)p_{J\perp}]^2}$$

No dependence on jet algorithm → Analytically calculable

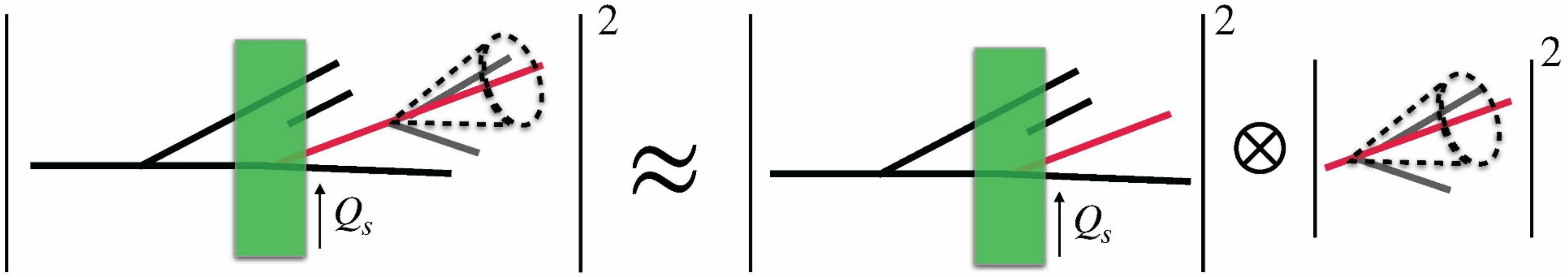
Small-R approximation

$$d\sigma_{fsr} \propto \quad xf(x)\frac{1+\xi^2}{(1-\xi)^{1+\eta}}\frac{\mathcal{F}_F(p_{k\perp}+p_{j\perp};X_f)}{[\xi p_{k\perp}-(1-\xi)p_{j\perp}]^2}(\Theta_1+\Theta_2)$$

$\Theta_1 \to \Theta_{1,R}$, $\Theta_2 \to \Theta_{2,R}$ → Analytical calculable

Drop other $\mathcal{O}(R^2)$ contribution → Factorization under the Approximation

$$d\sigma_R = \int d\xi \frac{d\zeta}{\zeta^2} xf(x) d\hat{\sigma}_{q\to q}(\xi, p_J/\zeta)\, J_q(\zeta)$$



$d\hat{\sigma}_{q\to q}$ partonic single hadron production result

$J_q(\zeta)$ semi-inclusive quark jet function in the large Nc limit

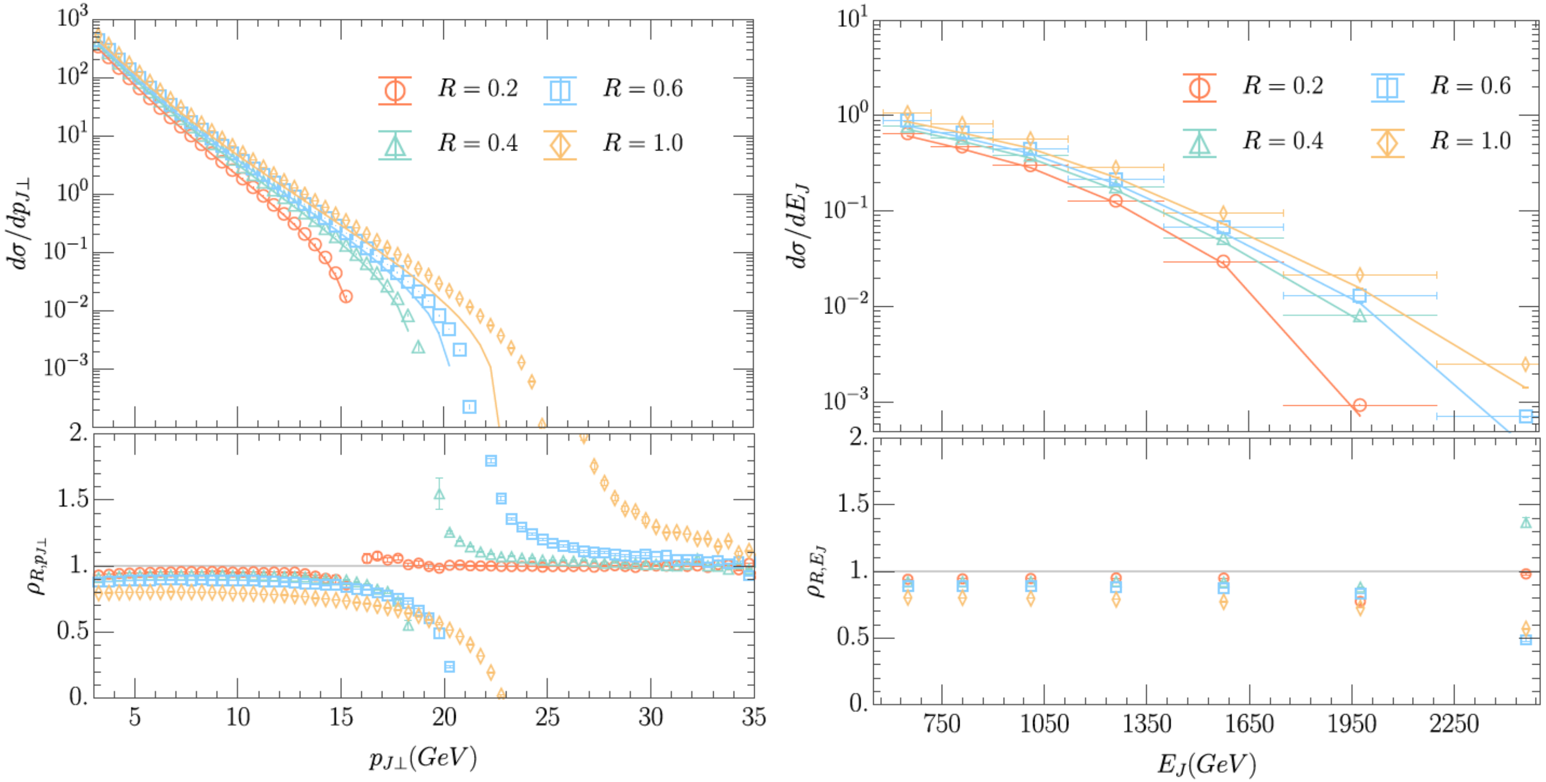
Similar factorization exists for collinear case with formally identical to the siJF [Kang, Ringer and Vitev, 2016]

Generalized to other jet observables

Comparison between full and small-R

$$\rho_{R,p_{J\perp}} \equiv \frac{d\sigma_{\rm full}/dp_{J,\perp}}{d\sigma_{{\rm small}\ R}/dp_{J,\perp}}$$

- The approximation works well , but can break down if strong cancellation exists



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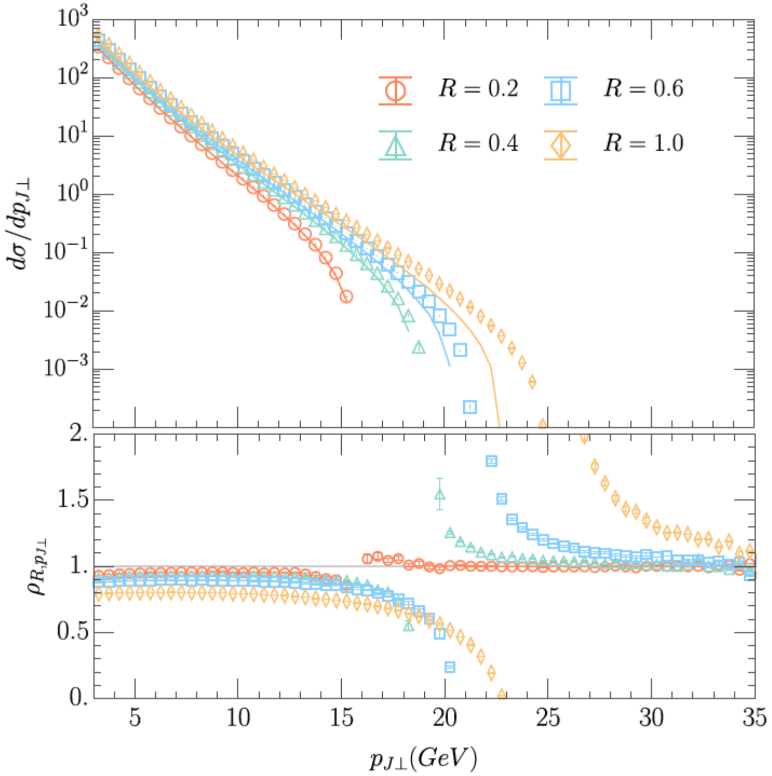
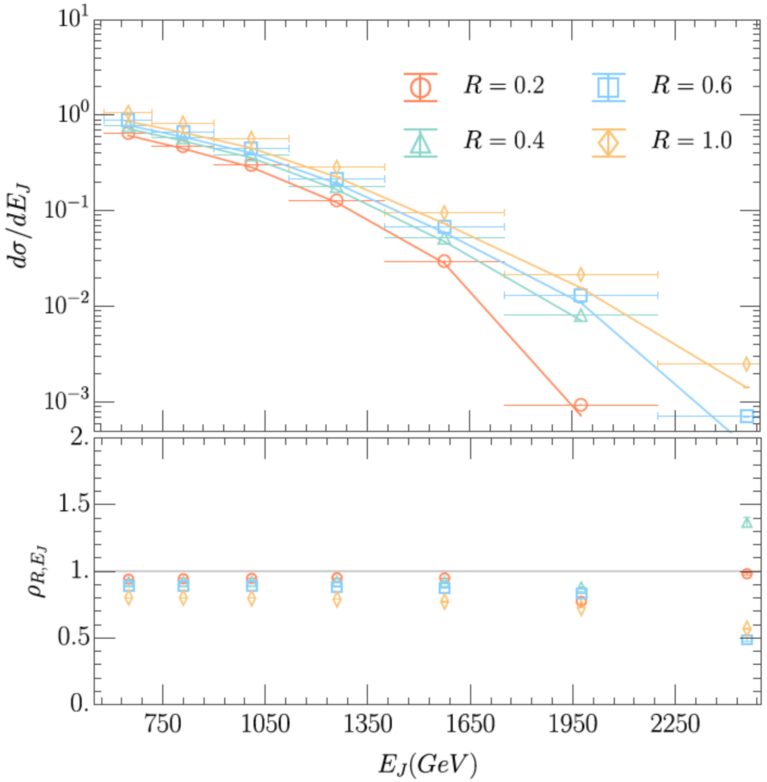
- E_J Spectrum

  The division of the energy bins follows [CMS Collaboration,Sirunyan et al.,2019]

  Distribution of other observables can be generated by histogram





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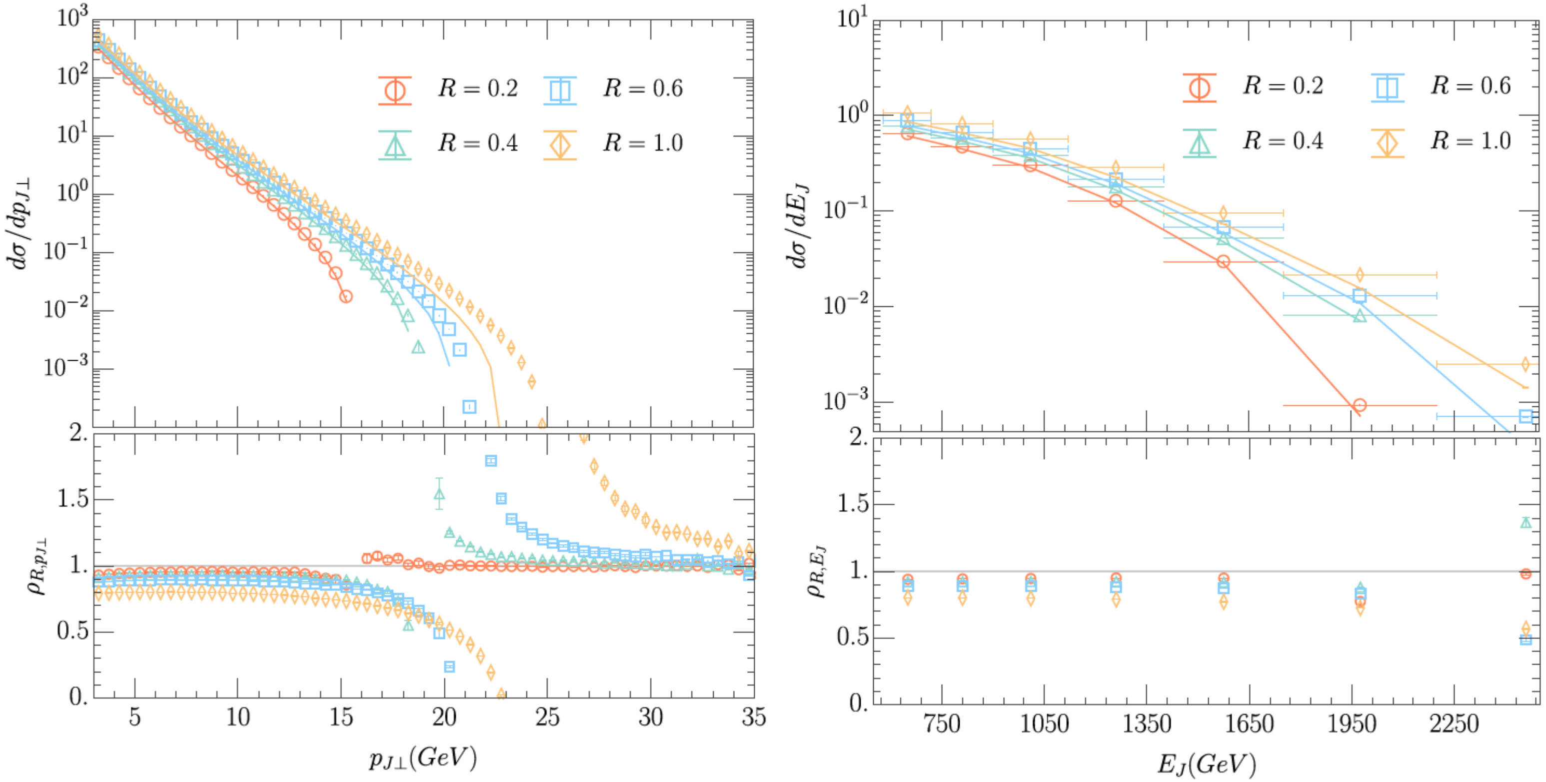


- Negative cross section for large $p_{J\perp}$

$$1 + \alpha_s\# + \boxed{\alpha_s L} + \ldots + \alpha_s^n L^n + \ldots$$

large threshold log → threshold resummation → $1 + \alpha_s\# + e^{\alpha_s L}$

For hadron case, see

[Y.Shi, L.Wang, S.Wei, B.Xiao,2021]

[HL, Z.Kang, X.Liu,2020]

Summary and Outlook

- We applied the power counting method to derive the complete NLO single jet production in pA collisions using CGC and show that the CGC factorization is valid at the NLO level for jet production.
- Our calculation is fully differential over the final state physical kinematics. The calculation is thus not limited to the jet spectra predictions but is able to predict any distribution of infrared safe quantities.
- We showed that in the small jet radius limit, the single inclusive jet cross section can be further factorized into the same short-distance cross section as the single inclusive hadron production, with only the fragmentation functions replaced by the siJFs.
- We look forward to do a full comparison to the experimental data, for instance, to [CMS Collaboration,Sirunyan et al.,**2019**].
- The method in the small-R approximation can be generalized to other jet related observables or other processes depend on constraint, for instance, the isolated photon production [B. Ducloue, T. Lappi, and H. Mantysaari, 2018].

Thank You!

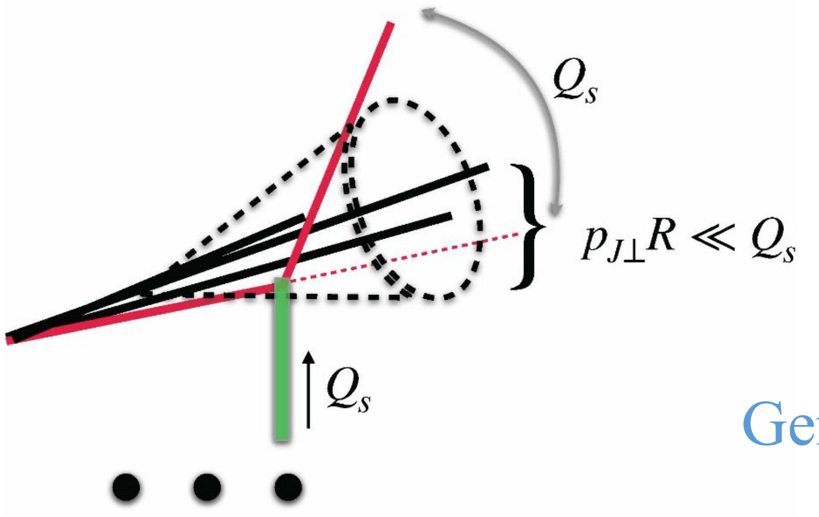
The Back up

Small-R approximation

The parton inside the jet has a typical transverse momentum scale

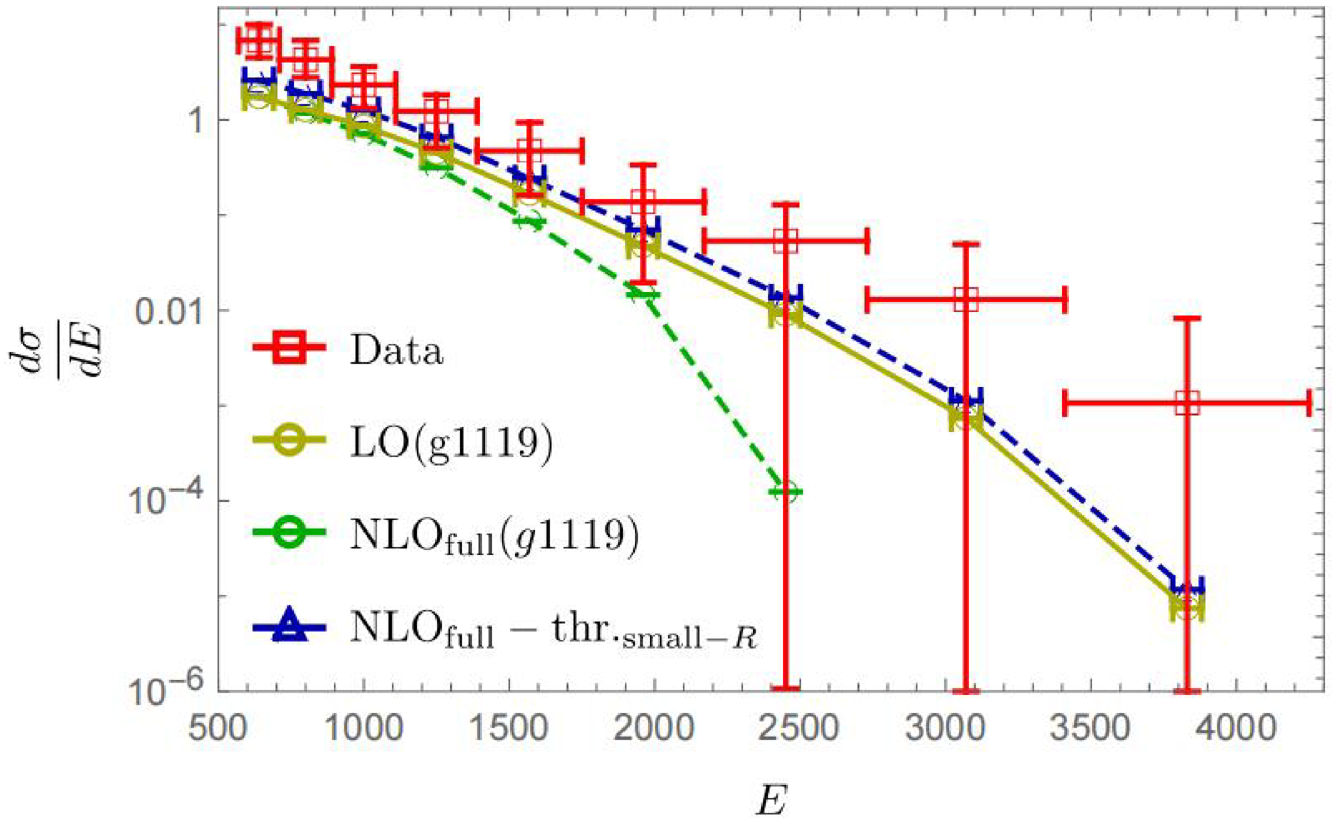
$p_{J\perp}R$ $\qquad$ $p_{J\perp}R \ll p_{J\perp} \sim Q_s$

The parton interacted with the shock wave will be knocked out to the jet because of obtaining an $p_\perp \sim Q_s$



Generalized to other jet observables

Result comparing to Data



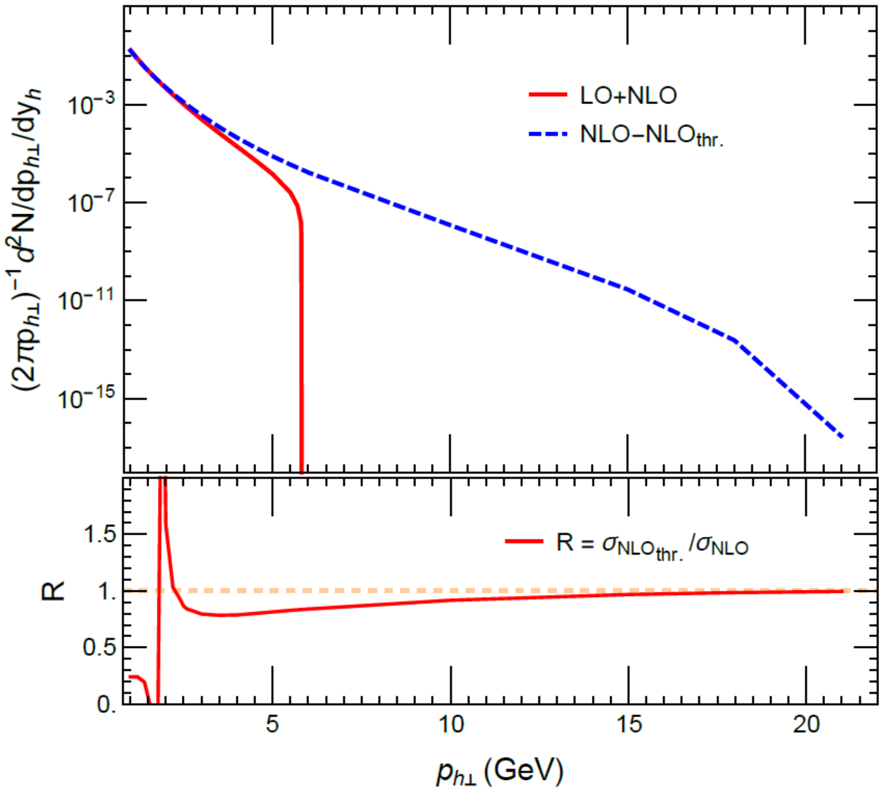
The division of the energy bins follows
[CMS Collaboration,Sirunyan et al.,2019]

Negative cross section

[HL, Kang, Liu, Phys. Rev. D.2020(rapid communication)]

[Chirilli,Xiao,Yuan, Phys.Rev.Lett.2012]
[Stasto et al,Phys.Rev.Lett,2014]

Even with kinematic constraint/soft term, there is still negative cross section for large $p_{h\perp}$



Threshold logs:

$$z \sim \frac{p_{h\perp} e^y}{\sqrt{s}} \sim 1$$

$$\tilde{P}^{(1)}_{i\to i}(z) \to \frac{1}{(1-z)_+} \sim \log(1-z)$$

The main negative contribution

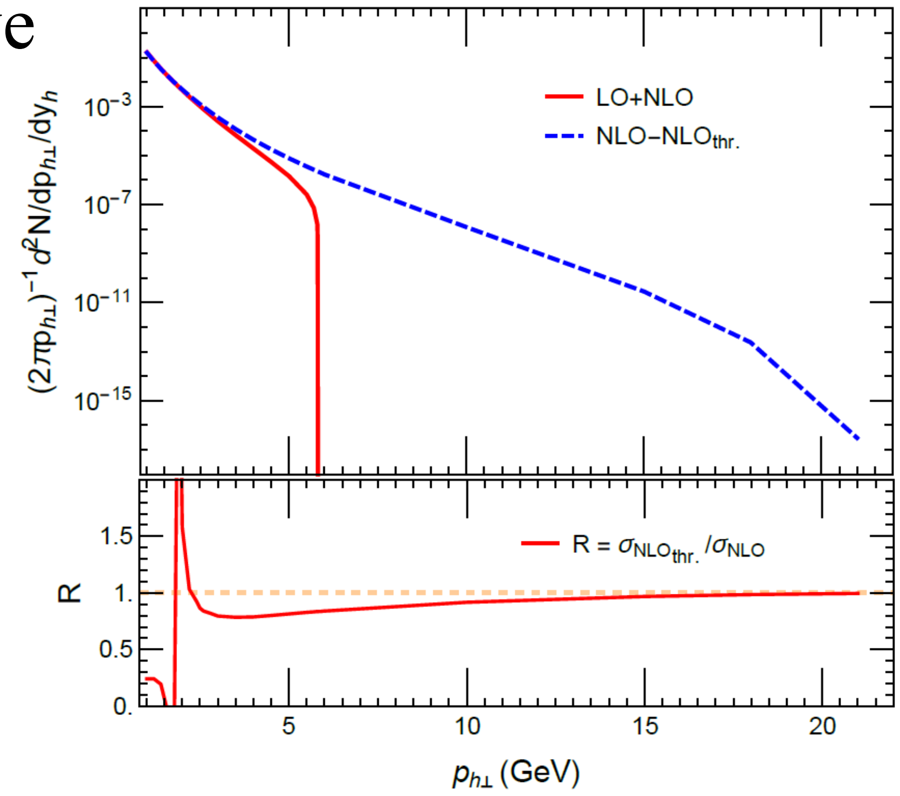
Evaluate the threshold contribution

[HL, Kang, Liu, Phys. Rev. D.2020(rapid communication)]

The cross section remains positive after threshold part is subtracted

The threshold part dominate the NLO part at large $p_{h,\perp}$

Strong evidence that the negative cross section is driven by the threshold contribution



Resummation of threshold logs can solve the negative cross section problem

Threshold resummation

Mellin transformation $M_N(f(\xi)) = \int_0^1 d\xi \xi^{N-1} f(\xi)$

$$M_N(1) \to 0 \qquad M_N\left(\frac{1}{(1-\xi)_+}\right) \to -\ln \bar{N} \qquad M_N\left(\left[\frac{\ln(1-\xi)}{1-\xi}\right]_+\right) \to \frac{1}{2}\ln^2 \bar{N} + \frac{\pi^2}{12}$$

The small-R limit result becomes

$$d\hat{\sigma}^{(1)}_{q\to q,thr.} = \langle \mathcal{M}_0 | \frac{\alpha_s}{\pi} \left(\mathbf{T}_i^2 + \mathbf{T}_j^2\right) \ln \bar{N} \ln \frac{\mu^2}{p_{J\perp}^2}$$

$$-\frac{\alpha_s}{\pi} \int \frac{dr_\perp}{\pi} \left[-2\ln \bar{N} \left(\frac{x_\perp \cdot y_\perp}{x_\perp^2 y_\perp^2}\right)_+ + \ln \frac{X_f}{X_A} \left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2}\right)_+ \right] \mathbf{T}_j^{a'} W_{a'a}(r_\perp) \mathbf{T}_i^a | \mathcal{M}_0 \rangle$$

$\mathbf{T}_i^a$ Catani operator

Threshold resummation

$$\frac{\alpha_s}{\pi}\left(\mathbf{T}_i^2+\mathbf{T}_j^2\right)\ln\bar{N}\ln\frac{\mu^2}{p_{J\perp}^2}$$

Terms proportional to $\mathbf{T}_j^2$ can be resummed by the techniques the Sudakov logarithms resummation

$$-\frac{\alpha_s}{\pi}\int\frac{dr_\perp}{\pi}\left[-2\ln\bar{N}\left(\frac{x_\perp\cdot y_\perp}{x_\perp^2 y_\perp^2}\right)_+ +\ln\frac{X_f}{X_A}\left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2}\right)_+\right]\mathbf{T}_j^{a'}W_{a'a}(r_\perp)\mathbf{T}_i^a$$

This term can not be resummed by the Sudakov log resummation techniques, shares the same color structure as the BK evolution.

At higher orders, every additional ISR will generate an additional Wilson line that complicates the color structures.

The factorization formula

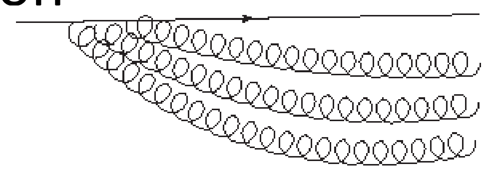
We firstly reexamine the factorization formula by power counting

$$\frac{d\sigma}{dy_h d^2 p_{h\perp}} = \sum_{i,j=g,q} \frac{1}{4\pi^2} \int \frac{d\xi}{\xi^2} \frac{dx}{x} z x f_{i/P}(x,\mu) D_{h/j}(\xi,\mu)$$

$$\times \int d^2 b_\perp d^2 b'_\perp \, e^{i p'_\perp \cdot r_\perp} \Big\langle \langle \mathcal{M}_0(b'_\perp) | \boldsymbol{\mathcal{J}}(z,\mu,\nu,b_\perp,b'_\perp) \boldsymbol{\mathcal{S}}(\mu,\nu,b_\perp,b'_\perp) | \mathcal{M}_0(b_\perp) \rangle \Big\rangle_\nu$$

$|\mathcal{M}_0(b_\perp)\rangle$ Standard color space notation [Catani et al.NPB, 2000]

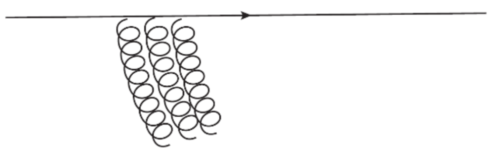
$\boldsymbol{\mathcal{J}}$ Jet function Contribution from Collinear radiation



Gluon in forward direction with momentum

$\sqrt{s}(1, \lambda^2, \lambda)$ $\lambda \sim p_{h,\perp}/\sqrt{s} \ll 1$

$\boldsymbol{\mathcal{S}}$ Soft function Contribution from soft radiation



Gluon in central direction with momentum

$\sqrt{s}(\lambda, \lambda, \lambda)$

Large log and evolution

For the threshold region $z \to 1$ $\quad \bar{n}\cdot k = \bar{n}\cdot p(1-z) \sim p'_\perp$

real emitted gluon $(\bar{n}\cdot k, n\cdot k, k_\perp) \sim \sqrt{s}(\lambda,\lambda,\lambda)$ soft

$\mathcal{J}$ Contains only virtual correction contribution

$\mathcal{S}$ Contains real correction contribution

$\mathcal{J}$ and $\mathcal{S}$ can be calculated perturbatively

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots$$

$$S^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_s}\right) + \alpha_s D_s(\nu_s) + \ldots$$

We reproduce the full fixed order results with $z \to 1$

$D_s(\nu_s)$ contains $\ln(\nu_s/\bar{n}\cdot p)$, $\ln(\nu_s/p'_\perp)$ and $\frac{1}{(1-z)_+}$

$\nu_J = \bar{n}\cdot p \qquad \nu_s \sim p'_\perp \qquad p'_\perp \ll \bar{n}\cdot p$

So the evolution equation is $\nu\frac{d}{d\nu}\mathcal{F}(\nu) = \gamma_{\mathcal{F}}\mathcal{F}(\nu) \qquad \mathcal{F} = \mathcal{J} \text{ or } \mathcal{S}$

Leading log result

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots$$

$$J^{(0)} + J^{(1)} \propto (1 + \alpha_s \ln\left(\frac{\nu}{\nu_J}\right))(1 + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots)$$

All order

Evolution kernel $\boldsymbol{U}_{\mathcal{F}}(\nu, \nu_{\mathcal{F}})$

$\boldsymbol{\mathcal{F}}(\nu_{\mathcal{F}})$ Initial condition

$$\boldsymbol{\mathcal{F}}(\nu) = \boldsymbol{U}_{\mathcal{F}}(\nu, \nu_{\mathcal{F}})\, \boldsymbol{\mathcal{F}}(\nu_{\mathcal{F}})$$

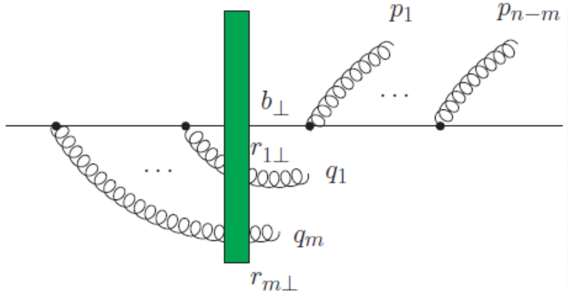
$$\boldsymbol{U}_J\, \boldsymbol{U}_S = \exp\left[-\frac{\alpha_s}{\pi}\int \frac{\mathrm{d}x_\perp}{\pi}\left(\ln\frac{\nu_S}{\nu_J} I_{BK,r} + \ln\frac{X_f}{X_A} I_{BK}\right)\mathbf{T}_i^a \mathbf{T}_j^{a'}\, W_{aa'}(x_\perp)\right]$$

Proof under strong ordering limit

For the leading log ,considering the independent n-multiple soft gluon strong ordering emission at $\mathrm{N}^{(n)}\mathrm{LO}$, in which

$$q_1^- \gg q_2^- \gg \cdots \gg q_m^- \qquad p_1^- \gg p_2^- \gg \ldots p_{n-m}^-$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}\left|\sum_{m=0}^{n}\right.$$

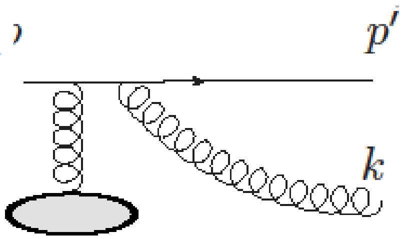


$$\left.\right|^2$$

$$= \langle \mathcal{M}_0 | \exp\left\{ -\frac{\alpha_s}{\pi}\int \frac{dr_\perp}{\pi}\left[-2\ln \bar{N}\left(\frac{x_\perp \cdot y_\perp}{x_\perp^2 y_\perp^2}\right)_+ + \ln\frac{X_f}{X_A}\left(\frac{z_\perp^2}{x_\perp^2 y_\perp^2}\right)_+ \right] \mathbf{T}_j^{a'} W_{a'a}(r_\perp)\mathbf{T}_i^a \right\} |\mathcal{M}_0\rangle$$

Our resummation formula hold in this limit.

Dominate terms for large $p_{h,\perp}$



$$\frac{d^2\hat{\sigma}^{(1)}}{dz d^2 p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z) \ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{dx_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\, e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{{r'_\perp}^2{r''_\perp}^2}\right.$$

$$\left.+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{{r'_\perp}^2{r''_\perp}^2}\right]_+\right\}W_{aa'}(x_\perp)+\ldots$$

$$\frac{\bar{n}\cdot p'}{\bar{n}\cdot p}=z \quad \frac{\bar{n}\cdot k}{\bar{n}\cdot p}=1-z$$

$$\tilde{P}_{i\to i}(z)\to\frac{2}{(1-z)_+}$$

$$x_p = p_{h,\perp}e^{y_h}/\xi\sqrt{s}\to 1$$

$$x_p < z < 1$$

The threshold contribution is proportional to $\dfrac{f(x_p/z)-f(x_p)}{1-z}$

Because the PDF decreases rapidly when x_p is large, $f(x_p/z)\ll f(x_p)$ even when z is not far from 1

$\dfrac{f(x_p/z)-f(x_p)}{1-z}\to-\dfrac{f(x_p)}{1-z}$ and becomes a large log and is negative

Generating Histogram

With form of $d\sigma$ and information of p_j and p_k

Distribution of any observable is available by histogram

We take E_J as example, the steps are as follow

1. Divide the observable spectrum into N different bins
   $$(E_{J,0}, E_{J,1})\,,\ (E_{J,1}, E_{J,2})\,,\ \ldots\ ,(E_{J,i}, E_{J,i+1})\,,\ \ldots\ ,(E_{J,N-1}, E_{J,N})$$
2. Generate the momenta p_j and p_k out of the free variables p_J^+ and $p_{J\perp}$ according to whether it is a 1-jet or 2-jets case the event is kept is the momenta satisfies the jet clustering algorithm , otherwise vetoed
3. Get E_J by p_j and p_k according to 1-jet or 2-jets case,
   if $E_J \in (E_{J,i}, E_{J,i+1})$, fill the event into this bin with weight $d\sigma$
4. Repeat step 2 and 3