第四届重味物理与量子色动力学研讨会–湖南大学

利用光锥求和规则精确计算$\boldsymbol{B} \to \mathbf{D}$形状因子

南開大學 物理科学学院
NANKAI UNIVERSITY SCHOOL OF PHYSICS

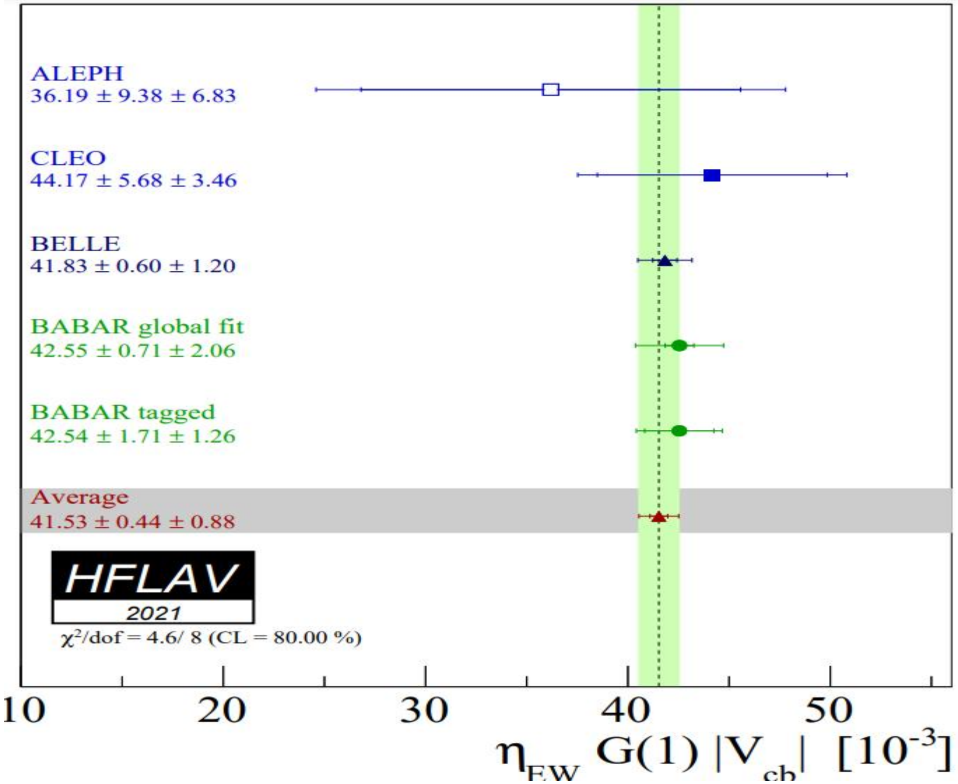
物理之约
MEET PHYSICS

报告人：高婧

合作导师：王玉明教授

Jing Gao et al. Physical Review D 101 (2020) 7,074035
Jing Gao et al. JHEP 05 (2022) 024

$|V_{cb}|$ Puzzle ???



遍举过程

2020 PDG $|V_{cb}| = (39.5 \pm 0.9) \times 10^{-3}$

单举过程

2020 PDG $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$

Particle Data Group collaboration, P. A. Zylaetal., PTEP (2020) 083C01

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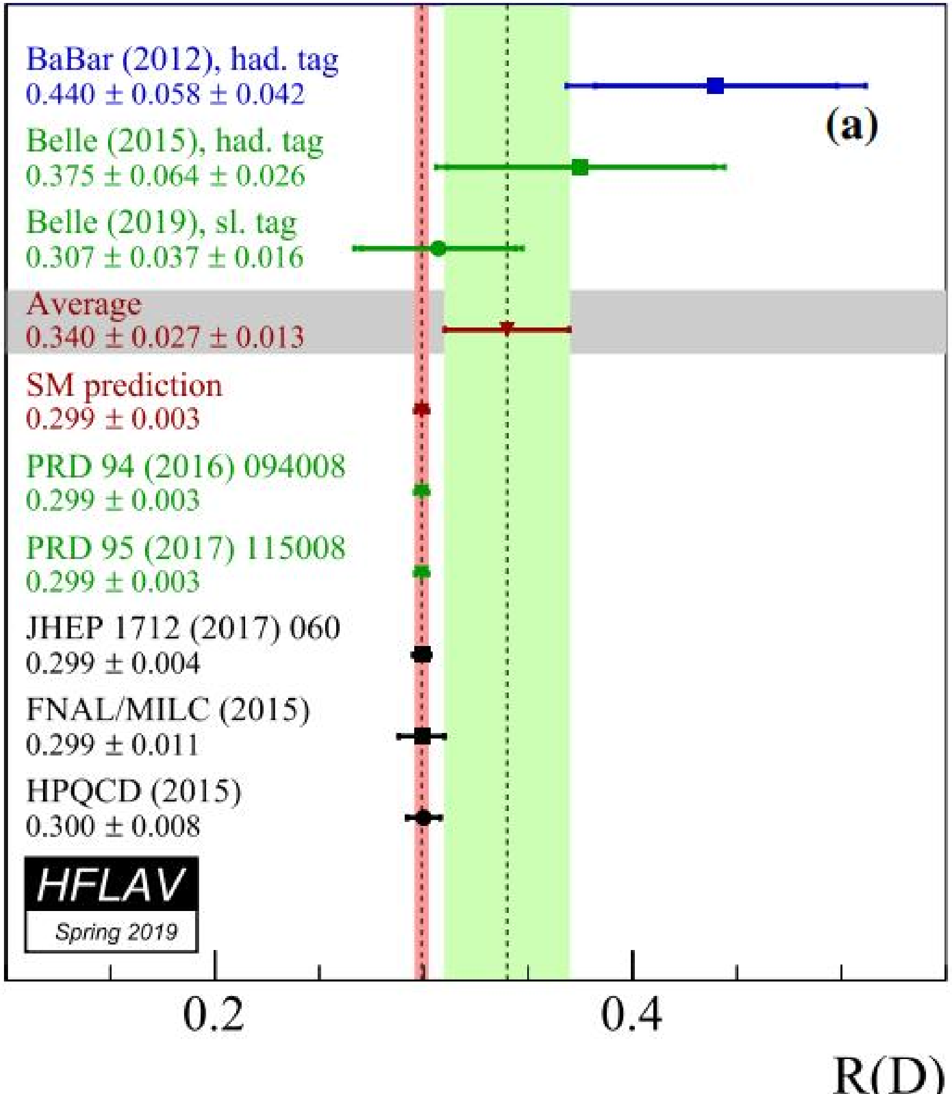
Can the Differences in the Determinations of V_{ub} and V_{cb} Be Explained by New Physics?

Andreas Crivellin and Stefan Pokorski

Phys. Rev. Lett. **114**, 011802 – Published 7 January 2015

R(D) 反常



- $R(D) \equiv \dfrac{B(\mathrm{B}\to D\tau\nu_\tau)}{B(B\to D\mu\nu_\mu)}$
- 实验拟合：

  $R(D) = 0.340 \pm 0.027 \pm 0.013$

  标准模型：

  $R(D) = 0.299 \pm 0.003$

  1.4σ
- 暗示了新物理信号的存在或是忽略了QCD动力学的修正效应，需要更精确的实验数据和理论计算

HFLAV collaboration, Y.S.Amhisetal., Eur.Phys.J.C81(2021)226

研究现状

- 零反冲区域:

HQET D.Bigi,P.GambinoandS.Schacht, JHEP11(2017) 061

LQCD MILCcollaboration,J.A.Baileyetal., Phys.Rev.D92(2015)034506

- 大反冲区域:

QCDF M. Beneke,G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl.Phys.B591(2000)313–418

SCET F.De Fazio, T.Feldmann and T.Hurth, JHEP02(2008)031

pQCD T.Kurimoto,H.-n.LiandA.I.Sanda, Phys.Rev.D67(2003)054028

QCD sum rules H.-B.Fu,X.-G.Wu,H.-Y.Han,Y.MaandT.Zhong, Nucl.Phys.B884(2014)172–192

- LCSR

  LP & LO S.Faller,A.Khodjamirian,C.KleinandT.Mannel, Eur.Phys.J.C60(2009)603–615

  LP & NLO Y.-M.Wang,Y.-B.Wei,Y.-L.ShenandC.-D.L¨u, JHEP06(2017)062

  partial NLP M.Bordone,M.JungandD.vanDyk, Eur.Phys.J.C80(2020)74

  Y.-M.Wang,Y.-B.Wei,Y.-L.ShenandC.-D.L¨u, JHEP06(2017)062

  S.Faller,A.Khodjamirian,C.KleinandT.Mannel, Eur.Phys.J.C60(2009)603–615

形状因子

- B → D 形状因子的定义:

$$\langle D(p)\,|\bar{c}\gamma_\mu b|\,\bar{B}(p_B)\rangle = \boxed{f_{BD}^{+}(q^2)}\left[2p_\mu + \left(1 - \frac{m_B^2 - m_D^2}{q^2}\right) q_\mu\right] + \boxed{f_{BD}^{0}(q^2)}\frac{m_B^2 - m_D^2}{q^2} q_\mu$$

Power counting:

- B → D 过程中D介子的动量:

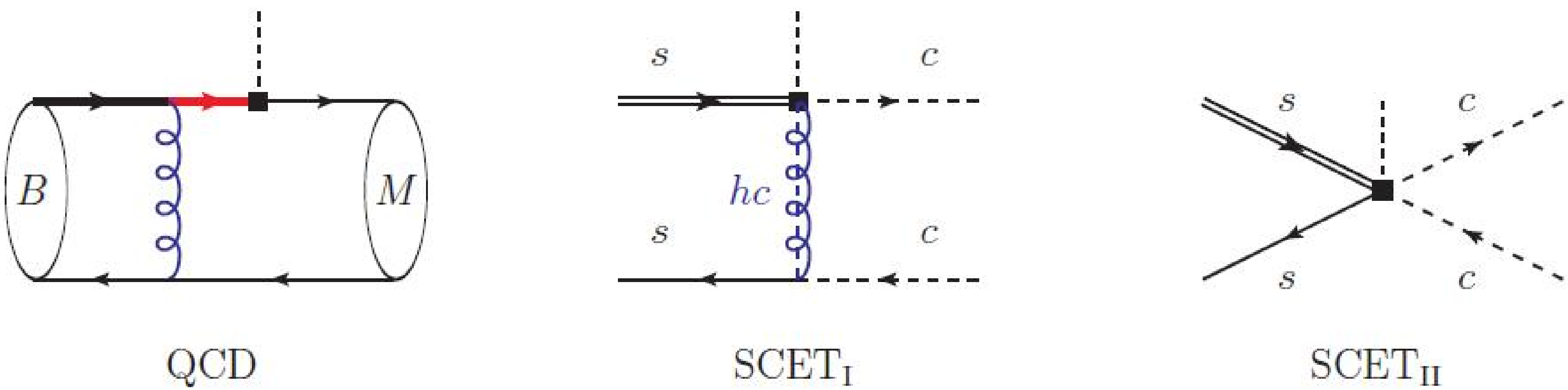
光锥坐标系 $p^\mu = n\cdot p\,\frac{\bar{n}^\mu}{2} + \bar{n}\cdot p\,\frac{n^\mu}{2} + p_\perp^\mu\,.$

- Power counting scheme: $\lambda = \frac{\Lambda_{QCD}}{m_b}$ $\quad m_c \sim \mathcal{O}(\sqrt{\Lambda_{\rm QCD} m_b})$

- 硬共线的动量: $(n\cdot p, \bar{n}\cdot p, p_\perp) \sim o(\lambda, 1, \sqrt{\lambda})$

形状因子的因子化

$$\text{QCD} \xrightarrow{\mathcal{O}(m_b)} \text{SCET}_{\text{I}} \xrightarrow{\mathcal{O}(\sqrt{\Lambda m_b})} \text{SCET}_{\text{II}}$$



- 从 QCD 到 $SCET_I$ 算符：将硬的能标积分掉, 得到硬函数

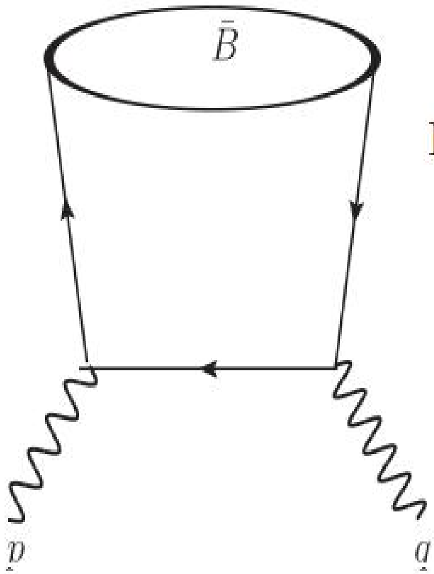
$$F_i^{B\to V}(n\cdot p) = \boxed{C_i^{(\mathrm{A0})}(n\cdot p)}\,\xi_a(n\cdot p) + \int d\tau\, \boxed{C_i^{(\mathrm{B1})}(\tau, n\cdot p)}\,\Xi_a(\tau, n\cdot p)\,, \quad (a = \parallel, \perp)\,,$$

- 从$SCET_I$算符到$SCET_{II}$算符:将硬共线的能标积分掉, 得到 jet function

$$\Xi_a \propto \boxed{J_a} \otimes \phi_M \otimes \phi_B$$

M. Beneke and D. S. Yang, Nucl. Phys. B 736 (2006)

夸克层次上的关联函数



两点关联函数

$$\Pi_\mu(n\cdot p,\bar{n}\cdot p) = i\int d^4x\, e^{ip\cdot x}\langle 0\,|T\,\{\bar{q}(x)\not{n}\gamma_5 c(x)\,,\bar{c}(0)\gamma_\mu b(0)\}|\,\bar{B}(p_B)\rangle$$

D介子内插流

重到轻弱流

- 在深欧空间下计算, $p^2 < 0$
- 在大反冲区域下计算 $q^2 = (p_B - p)^2$
- 沿着光锥方向展开 $x^2 = 0$

色散关系

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon}$$

- 将结果从深欧空间解析延拓到物理空间

PIETRO COLANGELO, ALEXANDER KHODJAMIRIAN, arXiv:hep-ph/0010175v1

强子层次上的关联函数

强子矩阵元定义:

$$\Pi_{\mu\nu}(p,q) = \frac{< 0 \mid \bar{q}(x)\gamma_\mu\gamma_5 c(x) \mid D(p) > \boxed{< D(p) \mid \bar{c}(0)\gamma_\mu b(0) \mid B(p+q) >}}{m_D^2 - p^2}$$

$$+ \int_{\omega_s}^{\infty} \frac{d\omega' \rho(n \cdot p, \omega')}{\omega' - \bar{n} \cdot p - i0}$$

- 插入一套完整的强子态
- 表示成基态和激发态以及共振态的贡献之和
- 矢量介子衰变常数 f_D

  重到轻的形状因子 $F_{B\to D}$

PIETRO COLANGELO, ALEXANDER KHODJAMIRIAN, arXiv:hep-ph/0010175v1

LCSR中的技术方法

夸克-强子对偶性假设

$$\int_{\boxed{s_0}}^{\infty} ds \frac{\rho^h(s)}{s-p^2} \simeq \frac{1}{\pi}\int_{\boxed{s_0}}^{\infty} ds \frac{Im\Pi^{(pert)}(s)}{s-p^2}$$

有效阈值

- 强子层次激发态和连续谱的贡献与夸克层次相互抵消

Borel 变换

$$\mathcal{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{\exp(-m^2/\boxed{M^2})}{M^{2(k-1)}} \rightarrow \text{Borel 质量}$$

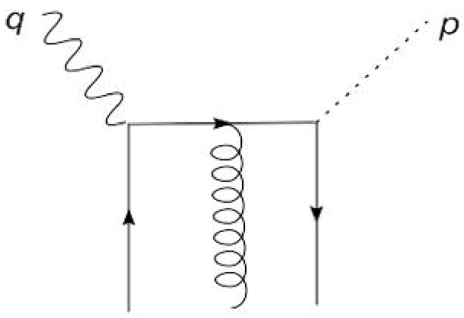
- 指数压低激发态和连续谱的贡献
- 减小对偶假设的敏感度

PIETRO COLANGELO, ALEXANDER KHODJAMIRIAN, arXiv:hep-ph/0010175v1

幂次修正贡献

来源一：高扭度B介子光锥分布振幅的贡献

➤ 三粒子B介子光锥分布振幅



$$\langle 0|\,\bar q(z_1\bar n)\,[z_1\bar n, z_2\bar n]\,g_s G_{\mu\nu}(z_2\bar n)\,\Gamma\,[z_2\bar n, 0]\,h_v(0)\,|\bar B_v\rangle$$
$$= \frac{1}{2}\tilde f_B(\mu) m_B\ \mathrm{Tr}\Big\{\gamma_5\Gamma P_+\Big[(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\big[\Psi_A - \Psi_V\big] - i\sigma_{\mu\nu}\Psi_V - (\bar n_\mu v_\nu - \bar n_\nu v_\mu)X_A$$
$$+ (\bar n_\mu\gamma_\nu - \bar n_\nu\gamma_\mu)\big[W + Y_A\big] - i\epsilon_{\mu\nu\alpha\beta}\bar n^\alpha v^\beta\gamma_5\tilde X_A + i\epsilon_{\mu\nu\alpha\beta}\bar n^\alpha\gamma^\beta\gamma_5\tilde Y_A$$
$$- (\bar n_\mu v_\nu - \bar n_\nu v_\mu)\not{\bar n}\, W + (\bar n_\mu\gamma_\nu - \bar n_\nu\gamma_\mu)\not{\bar n}\, Z\Big]\Big\}(z_1, z_2; \mu)$$

➤ 两粒子高扭度B介子光锥分布振幅

$$\langle 0|\,\bar q(x)\,\Gamma\,[x, 0]\,h_v(0)\,|\bar B_v\rangle$$
$$= -\frac{i}{2}\tilde f_B(\mu) m_B\left\{\mathrm{Tr}\left[\gamma_5\Gamma P_+\right]\left(\Phi_B^+ + x^2 G_B^+\right)\right.$$
$$\left. - \frac{1}{2}\mathrm{Tr}\left[\gamma_5\Gamma P_+\not{x}\right]\frac{1}{v\cdot x}\left[\left(\Phi_B^+ - \Phi_B^-\right) + x^2\left(G_B^+ - G_B^-\right)\right]\right\}(v\cdot x, \mu)$$

V.M.Braun,Y.JiandA.N.Manashov, JHEP05(2017)022

幂次修正贡献

来源二：粲夸克传播子展开的贡献

$$\frac{(\not{p}-\not{k})+m_c}{(p-k)^2-m_c^2} = \underbrace{\frac{1}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}}_{\text{LP}} + \underbrace{\frac{1}{n\cdot p\bar{n}\cdot\hat{p}}\left[\bar{n}\cdot p\frac{\not{n}}{2}-\not{k}+\frac{n\cdot k\bar{n}\cdot p}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}\right]}_{\text{NLP}} + \underbrace{\frac{m_c}{n\cdot p}\frac{1}{\bar{n}\cdot\hat{p}}\left[1+\frac{n\cdot k\bar{n}\cdot p}{n\cdot p\bar{n}\cdot\hat{p}}\right]}_{m_c\ \text{NLP}}$$

来源三：重夸克场展开的贡献

$$\bar{q}\gamma_\mu b = \bar{q}\gamma_\mu h_v + \frac{1}{2m_b}\bar{q}\gamma_\mu i\not{D}_\perp h_v + \cdots$$

形状因子的次领先幂次总贡献：

$$\boxed{f_{BD}^{\ell,\text{NLP}}(q^2) = \sum_{\text{C}=\text{HT},\text{CE}} f_{BD}^{\ell,\text{C}}(q^2) + f_{BD}^{\ell,\text{HQE}}(q^2)}$$

数值分析

总的形状因子:

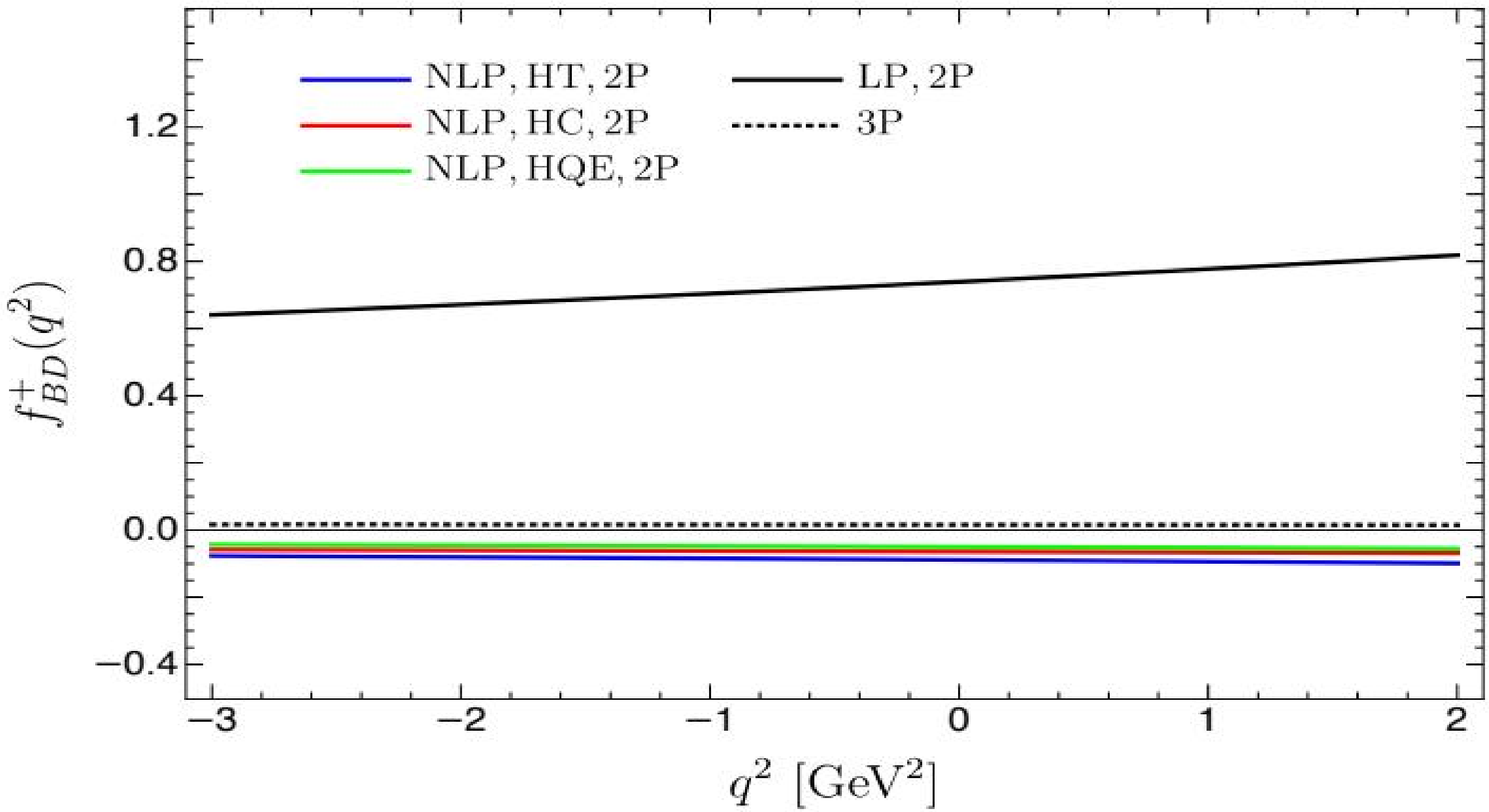
$$f_{BD}^{(+,0)}(q^2) = \underbrace{f_{BD}^{(+,0),LP}(q^2)}_{LP@LO+NLO} + \underbrace{f_{BD}^{(+,0),NLP}(q^2)}_{NLP@LO}$$

理论输入参数:

Parameters	values	Parameters	values
m_{B^-}	5.27933 GeV	m_{D^0}	1.86483 GeV
$m_{B^{*-}}$	5.32470 GeV	$m_{D^{*0}}$	2.00685 GeV
$m_b(m_b)$	$4.193^{+0.022}_{-0.035}$ GeV	$m_c(m_c)$	$1.288(20)$ GeV
f_B	(190.0 ± 1.3) MeV	f_D	$212.0(7)$ MeV
μ_{h_1}	$[m_b/2,\ 2\,m_b]$	μ_{h_2}	$[m_b/2,\ 2\,m_b]$
$\mu = \mu_{hc}$	$1.5(5)$ GeV	μ_0	1.0 GeV
M^2	(4.5 ± 1.0) GeV^2	s_0	$6.0(5)$ GeV^2
λ_B	$0.35(15)$ GeV		$\{0.7\,, 6.0\}$
$(\lambda_E^2/\lambda_H^2)$	$0.5(1)$	$\{\widehat{\sigma}_1, \widehat{\sigma}_2\}$	$\{0.0\,, \pi^2/6\}$
$(2\lambda_E^2 + \lambda_H^2)$	$0.25(15)$ GeV^2		$\{-0.7\,, -6.0\}$

数值分析

大反冲动量区域形状因子中不同来源的幂次修正

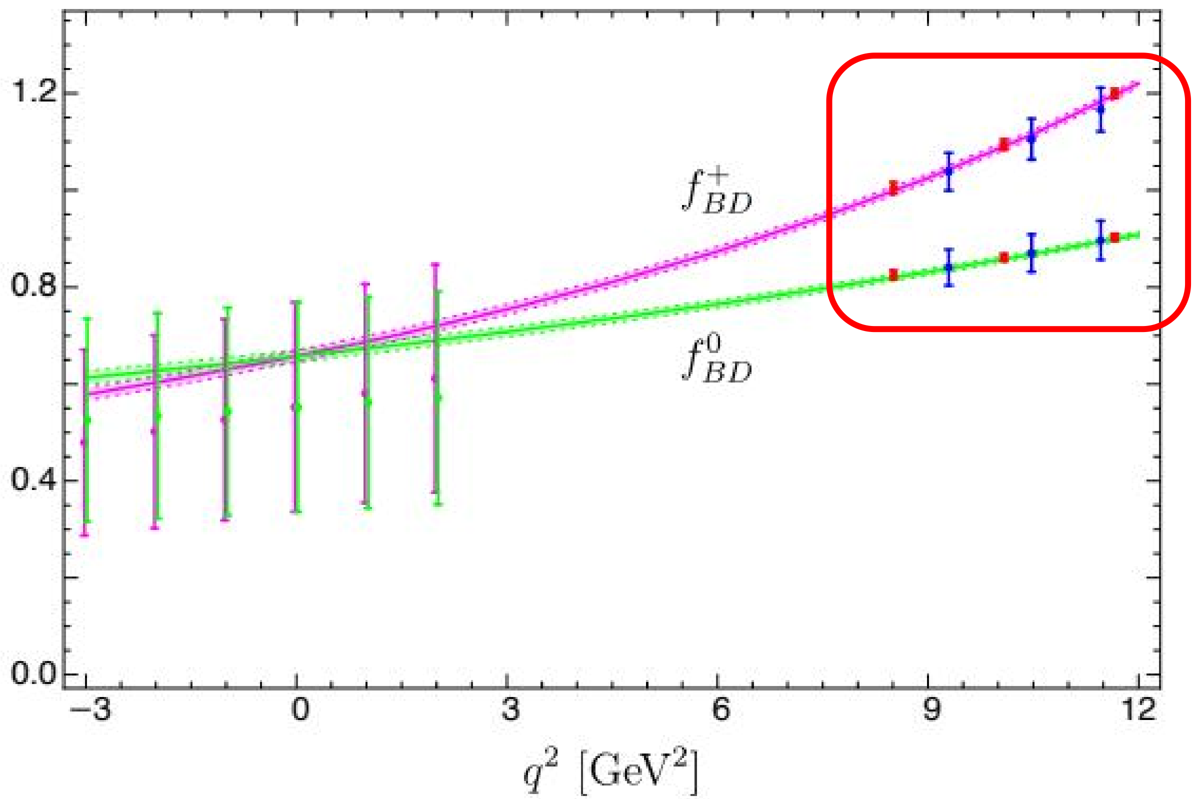


- ✓ 三粒子的幂次修正贡献为10%
- ✓ 其它高幂次贡献与领头幂次贡献符号相反

A.Khodjamirian,C.Klein,T.MannelandN.Offen, Phys.Rev.D80(2009)114005

数值分析

全动量区域总的形状因子

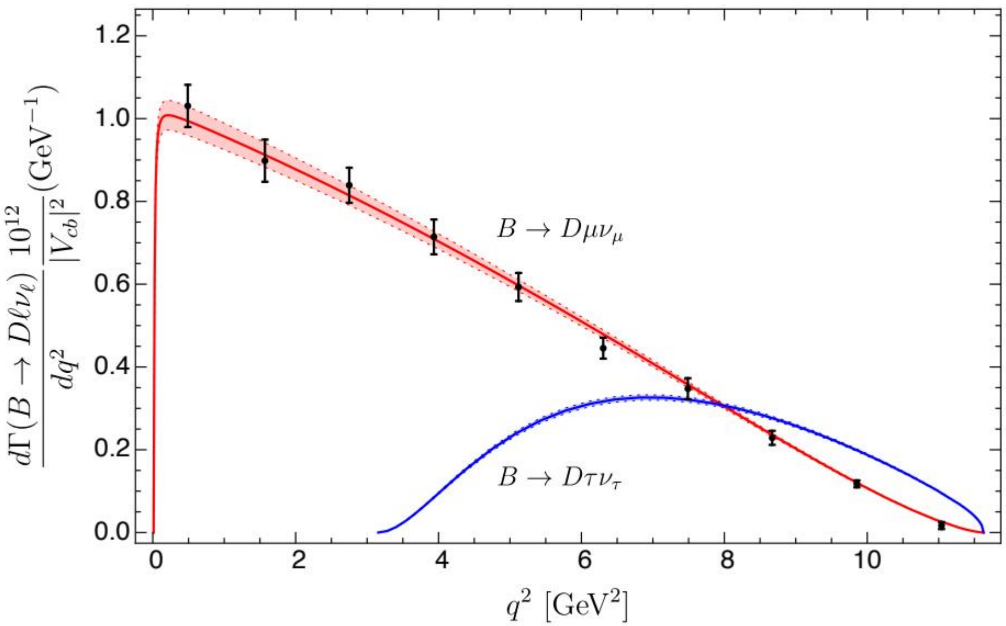


格点结果:
红色-FNAL/MILC
蓝色-HPQCD

- ✓ BGL参数化方法
- ✓ 大反冲区域 LCSR结果+零反冲区域LQCD结果

数值分析

唯象学研究：$|V_{cb}|$与R(D)



$$\frac{d\Gamma(B\to D\ell\nu_\ell)}{dq^2} = \frac{\eta_{\rm EW}^2 G_F^2 |V_{cb}|^2}{24\pi^3 m_B^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |\vec{p}_D| \left[\left(1+\frac{m_\ell^2}{2q^2}\right) m_B^2 |\vec{p}_D|^2 \, |f_{BD}^+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_B^2-m_D^2)^2 \, |f_{BD}^0(q^2)|^2\right]$$

$$\boxed{|V_{cb}|} = \begin{cases} \left(40.2^{+0.6}_{-0.5}\big|_{\rm th} \; {}^{+1.4}_{-1.4}\big|_{\rm exp}\right)\times 10^{-3}, & \text{[BaBar 2010} \\ \left(40.9^{+0.6}_{-0.5}\big|_{\rm th} \; {}^{+1.0}_{-1.0}\big|_{\rm exp}\right)\times 10^{-3}, & \text{[Belle 2016 [} \end{cases}$$

$$\boxed{R(D)} \equiv \frac{\mathcal{B}(B\to D\tau\nu_\tau)}{\mathcal{B}(B\to D\mu\nu_\mu)} = 0.302\pm 0.003\,.$$

总结与展望

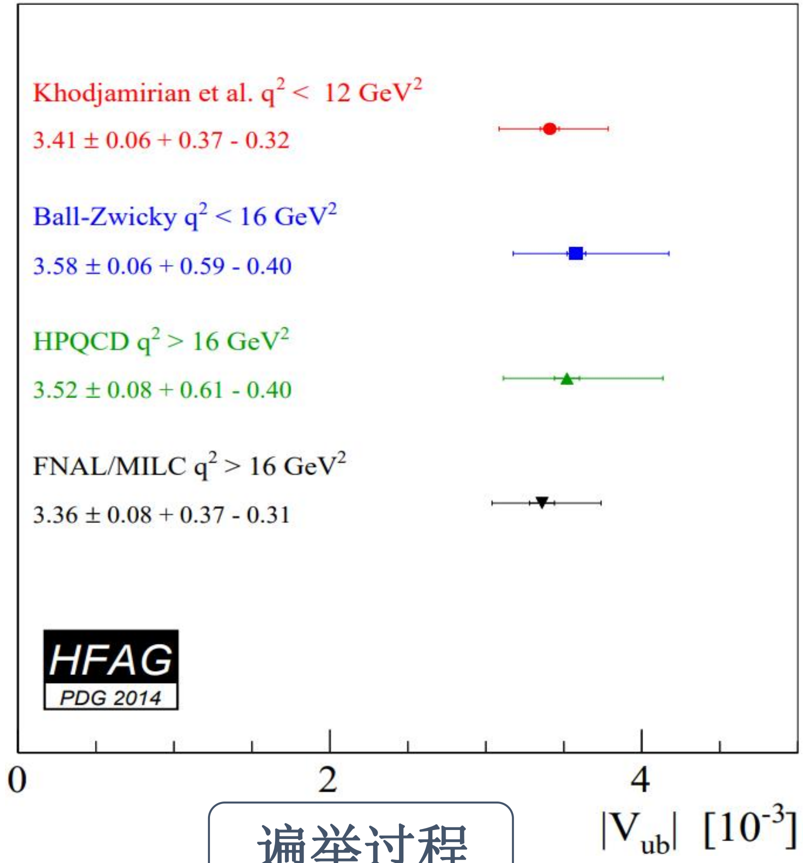
本工作结果

	LCSR[8]	Lattice	HQET[9]	This work
$\lvert V_{cb}\rvert \times 10^3$	$41.3(1.4)$[15], $40.6^{+1.2}_{-1.3}$[15] $39.2^{+3.5}_{-3.4}$[33], $41.4(4.0)$[33] $40.3(0.8)$[38]	$39.36(68)$[80] $38.40(0.7)$[139] $43.0(2.7)$[140]	$39.3(1.0)$[14]	$40.2^{+0.6}_{-0.5}\big\rvert_{\rm th}\ {}^{+1.4}_{-1.4}\big\rvert_{\rm BaBar}$ $40.9^{+0.6}_{-0.5}\big\rvert_{\rm th}\ {}^{+1.0}_{-1.0}\big\rvert_{\rm Belle}$
$R(D)$	$0.305^{+0.022}_{-0.025}$[33] $0.296(6)$[36] $0.297(3)$[38]	$0.299(11)$[18] $0.300(8)$[19] $0.301(6)$[141]	$0.299(3)$[14]	$0.302(3)$

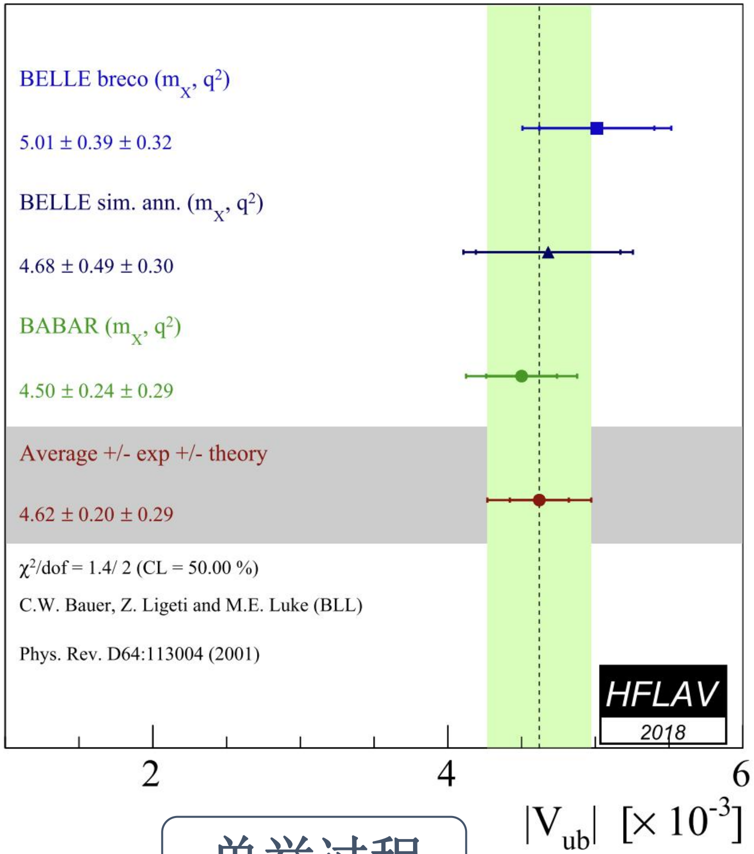
- ✓ 完整地计算了$B \to D$形状因子的幂次修正效应
- ✓ 在大反冲区域形状因子的幂次修正效应为20%
- ✓ $\lvert V_{cb}\rvert$与R(D)的不确定性更小，前者检验标准模型，后者检验轻子普适性

Thanks for your attention !

$|V_{ub}|$ Puzzle ???



遍举过程

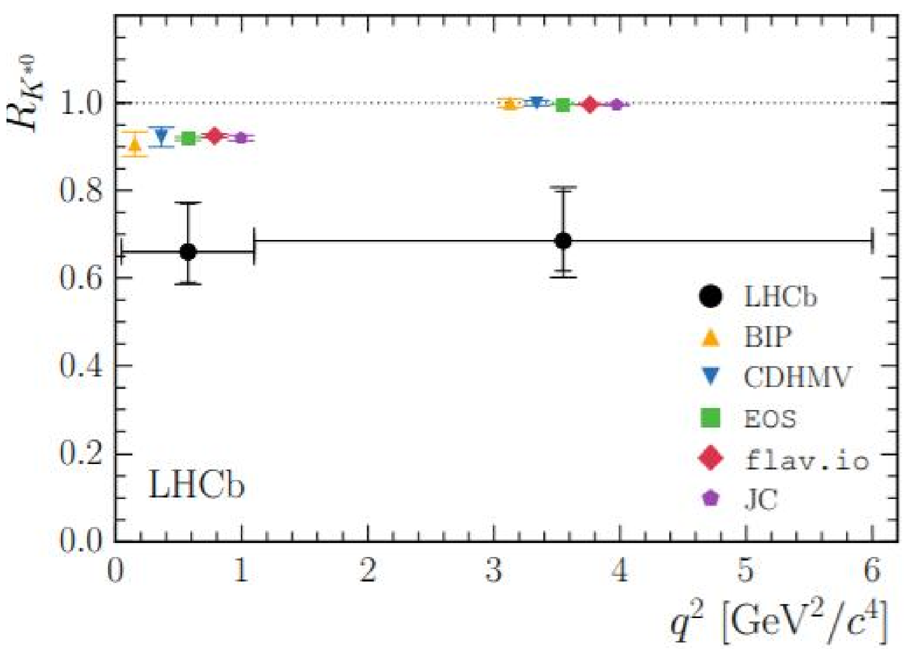


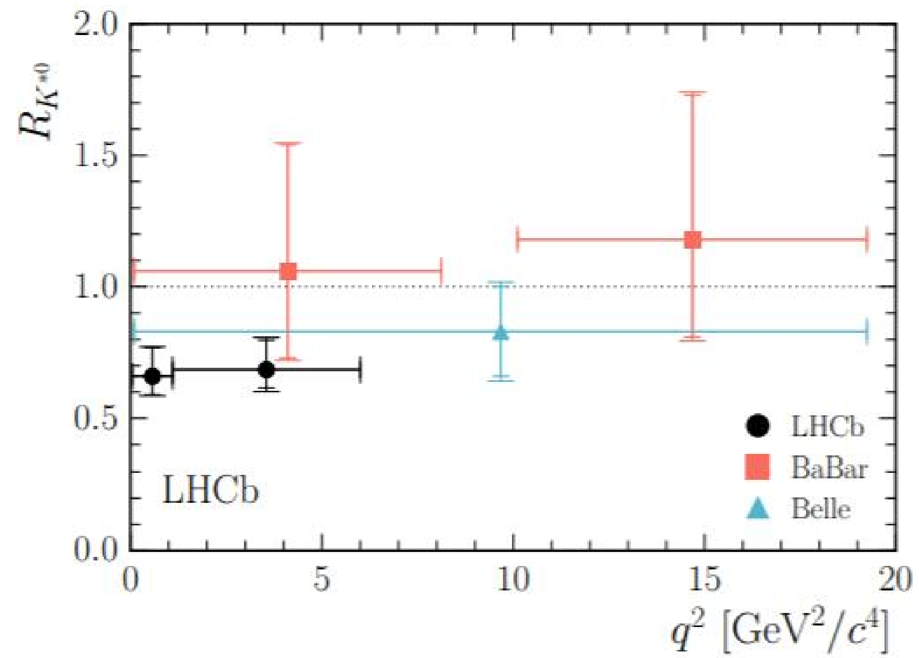
单举过程

$$\begin{cases} V_{ub} = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3} & LQCD \\ V_{ub} = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3} & LCSR \end{cases}$$

$$|V_{ub}| = (4.62 \pm 0.20 \pm 0.29) \times 10^{-3}$$

FCNC过程中反常现象





- $R_K^* = \dfrac{\mathcal{B}(B \to K^* \mu^+ \mu^-)}{\mathcal{B}(B \to K^* e^+ e^-)}$ 用来检验轻子的普适性
- 暗示了新物理信号的存在或者忽略了QCD动力学的修正效应
- 需要更多的实验数据和理论计算

LHCb-PAPER-2017-013, arXiv:1705.05802v2

幂次修正贡献

定义一个小参数:

$$\lambda_{sc} \equiv \overline{\omega}_{sc}/\Lambda_{\rm QCD} \qquad \overline{\omega}_{sc} = \omega_s - \omega_c \sim \mathcal{O}(\lambda\,\Lambda_{\rm QCD}) \ll \Lambda_{\rm QCD}$$

$$\omega_M \sim \omega_s \gg \overline{\omega}_{sc}$$

不同函数的power counting:

$$\mathcal{F}_{2,k}(\phi) \sim \mathcal{O}\left(\overline{\omega}_{sc}^{2-k}\,\phi(\overline{\omega}_{sc})\right), \qquad k = 1,2,3$$

$$\mathcal{F}_{3,k}(\phi) \sim \mathcal{O}\left(\overline{\omega}_{sc}^{3-k}\,\phi(\lambda_{sc}, \overline{\omega}_{sc}, \Lambda_{\rm QCD})\right), \quad k = 2,3$$

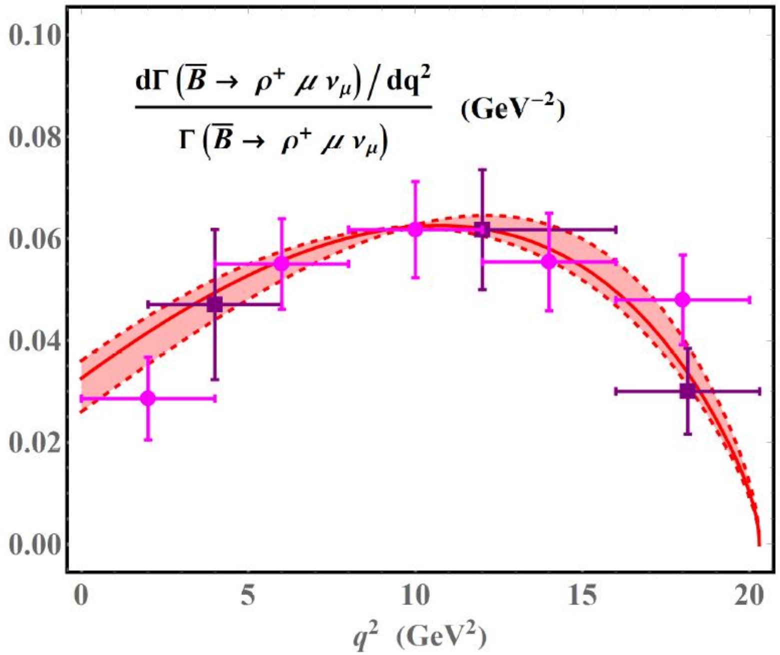
形状因子的power counting:

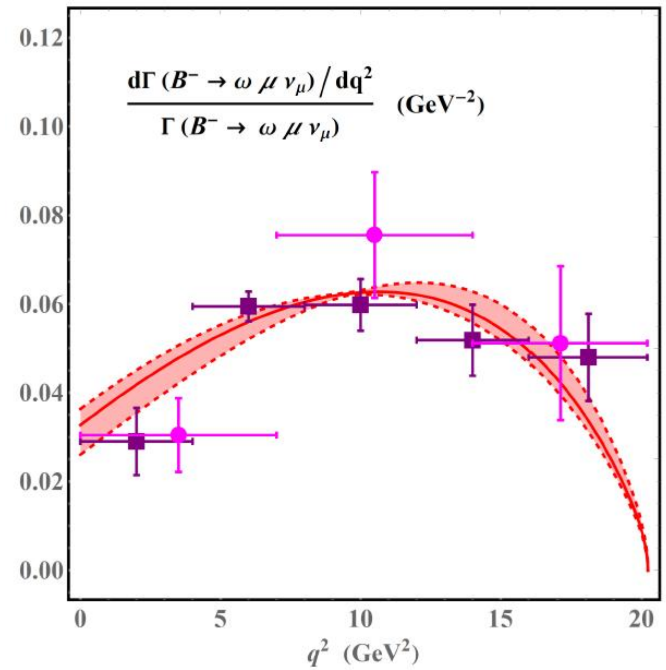
$$\boxed{f_{BD}^{(+,0),{\rm HT}} \sim \mathcal{O}(\lambda^4\,\lambda_{sc}^{-1}), \quad f_{BD}^{(+,0),{\rm CE}} \sim \mathcal{O}(\lambda^3\,\lambda_{sc}^{1}), \quad f_{BD}^{(+,0),{\rm HQE}} \sim \mathcal{O}(\lambda^4\,\lambda_{sc}^{1})}$$

半轻衰变 $B \to (\rho, \omega) l \bar{\nu}_l$

抽取 CKM 矩阵元$|V_{ub}|$

$$\frac{d^2\Gamma(B \to V \ell \bar{\nu}_\ell)}{dq^2\, d\cos\theta} = \frac{G_F^2\, |V_{ub}|^2}{256\, \pi^3\, m_B^3} \frac{q^2}{c_V^2} \lambda^{1/2}(m_B^2, m_V^2, q^2) \left\{ \sin^2\theta\, \boxed{|H_0(q^2)|^2} + (1-\cos\theta)^2 \frac{\boxed{|H_+(q^2)|^2}}{2} + (1+\cos\theta)^2 \frac{\boxed{|H_-(q^2)|^2}}{2} \right\}$$





$$|V_{ub}| = \left(3.05\,{}^{+0.67}_{-0.52}\Big|_{\rm th.}\,{}^{+0.19}_{-0.20}\Big|_{\rm exp.}\right) \times 10^{-3}$$

$$|V_{ub}| = \left(2.54\,{}^{+0.56}_{-0.40}\Big|_{\rm th.}\,{}^{+0.18}_{-0.19}\Big|_{\rm exp.}\right) \times 10^{-3}$$