Enhanced Next-to-Leading-Order Corrections to Weak Annihilation B-Meson Decays

沈月龙

中国海洋大学

In collaboration with Cai-Dian Lu（吕才典），Chao Wang（王超）and Yu-Ming Wang（王玉明）

Based on arXiv 2202.08073

Outline

- Nonleptonic B decays and factorization approach
- Annihilation diagrams in factorization approach
- Hard collinear contribution to the annihilation Diagrams
- Enhanced contribution at high order

Why nonleptonic B decays

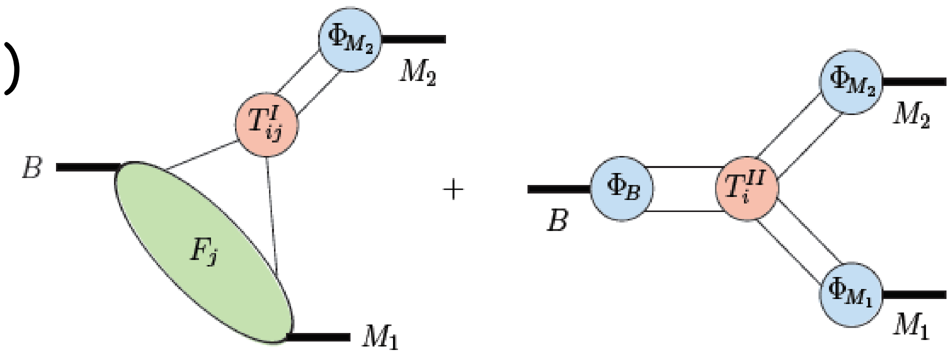
- Best place to study direct CP violation
- Best place to determine CKM phase angle
- Good place to test strong interaction theory
- Good place to search for the signal of new physics

Factorization approaches

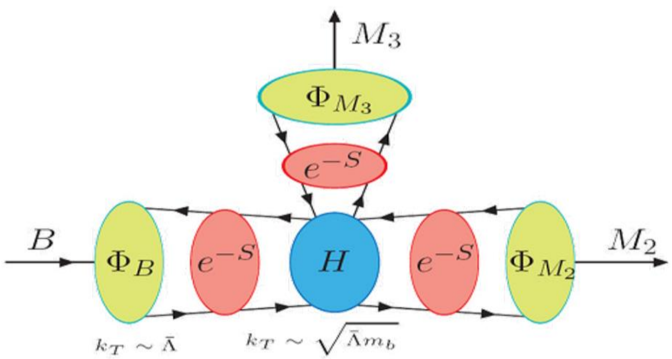
- Naïve(BSW) and generalized factorization approach(Cheng et al, Ali, Kramer, Lu)

$$
\begin{aligned}
< P_1 P_2|\mathcal{H}_{eff}|B > \; &= \; Z_1 < P_1|j^\mu|0 >< P_2|j_\mu|B > \\
&+ \; Z_2 < P_2|j'^\mu|0 >< P_1|j'_\mu|B >
\end{aligned}
$$

- QCD factorization(BBNS,1999)
- SCET(Bauer etal,2001)



- PQCD approach(Li, Lu, Xiao,…)



Nonleptonic B decays beyond leading power

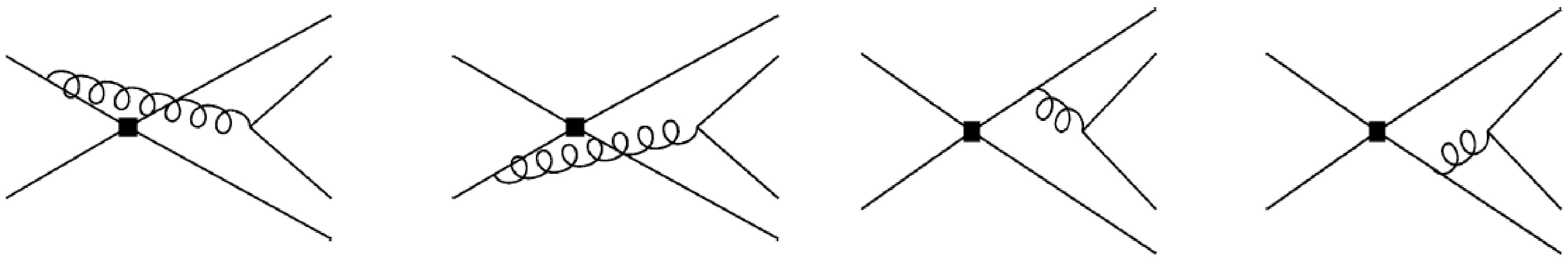
- Chiral enhanced contribution:

  - Chirally-enhanced twist-3 corrections (end-point divergences).
  - The scalar QCD penguin amplitude $r_\chi a_6$.

- Annihilation diagrams: sizable, large strong phase…

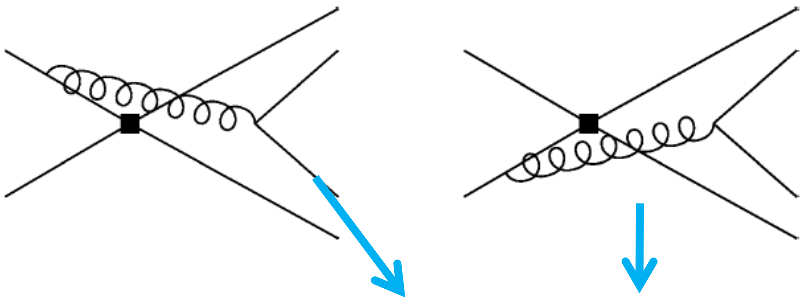
- Other power suppressed contributions

Annihilation diagrams in factorization approaches



	QCDF	PQCD	Zero-bin subtraction
Leading region	hard	hard	hard
factorization	NO	Yes	Yes
strong phase	nonzero	nonzero	zero

Parameterization of endpoint singularity in QCDF



$$A_1^i = \pi\alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_2}(x)\, \Phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_2}(x) \Phi_{m_1}(y) \frac{2}{\bar{x}y} \right\},$$

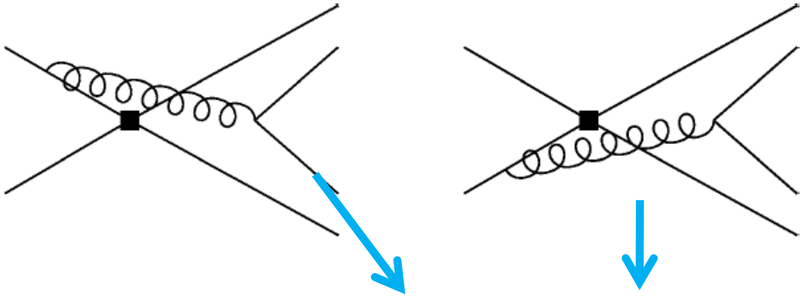
- Parameterization of logarithmic divergence

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

- The parameter X_A contains strong phase, which is assumed to be caused by the soft rescattering

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right) \ln \frac{m_B}{\Lambda_h}$$

Parameterization of endpoint singularity in QCDF



$$A_1^i = \pi\alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_2}(x)\, \Phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_2}(x) \Phi_{m_1}(y) \frac{2}{\bar{x}y} \right\},$$

$$\Phi_P(x,\mu) = 6x(1-x)$$

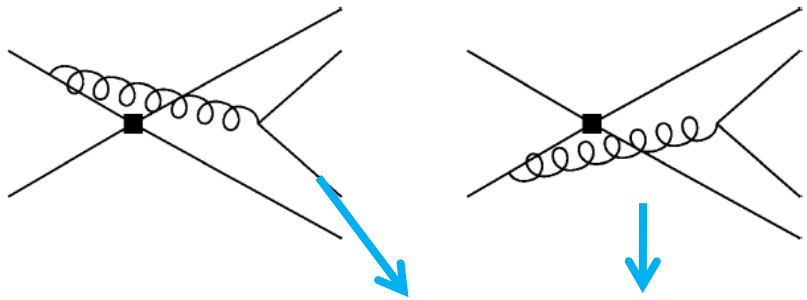
- Parameterization of logarithmic divergence

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

- The parameter X_A contains strong phase, which is assumed to be caused by the soft rescattering

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right) \ln \frac{m_B}{\Lambda_h}$$

Parameterization of endpoint singularity in QCDF



$$A_1^i = \pi\alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_2}(x)\, \Phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_2}(x) \Phi_{m_1}(y) \frac{2}{\bar{x} y} \right\},$$

$$\Phi_P(x,\mu) = 6x(1-x)$$

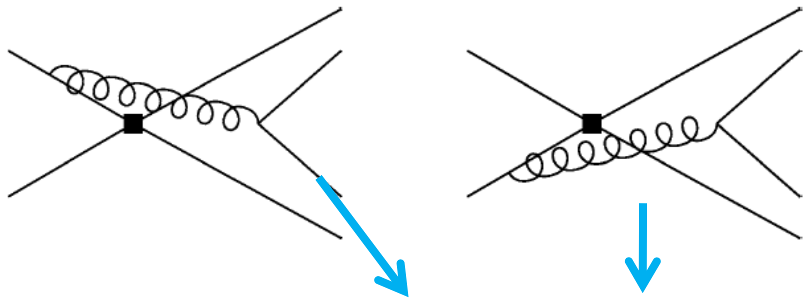
- Parameterization of logarithmic divergence

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

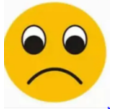
- The parameter X_A contains strong phase, which is assumed to be caused by the soft rescattering

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right) \ln \frac{m_B}{\Lambda_h}$$

Parameterization of endpoint singularity in QCDF



$$A_1^i = \pi\alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_2}(x)\, \Phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_2}(x) \Phi_{m_1}(y) \frac{2}{\bar{x} y} \right\},$$

$$\Phi_P(x,\mu) = 6x(1-x)$$

- Parameterization of logarithmic divergence

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

- The parameter X_A contains strong phase, which is assumed to be caused by the soft rescattering

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right) \ln \frac{m_B}{\Lambda_h}$$

PQCD approach and zero-bin subtraction

- TMD factorization [Keum, Li, Sanda, 2001; Lü, Ukai, Yang, 2001]:

$$\frac{1}{xm_B^2 - k_T^2 + i\varepsilon} = \mathrm{P}\frac{1}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2)\,.$$

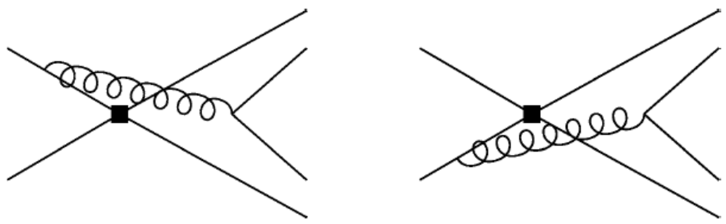
  - Including the partonic transverse momentum generates the non-vanishing strong phase.
  - In the QCD factorization approach the TMD effect is part of the higher-twist contribution.

- Zero-bin subtraction [Arnesen, Ligeti, Rothstein, Stewart, 2008]:

$$\int_0^1 dx \frac{\phi_M(x,\mu)}{\bar{x}^2} = \int_0^1 dx \frac{\phi_M(x,\mu) + \bar{x}\phi_M'(1,\mu)}{\bar{x}^2} - \phi_M'(1,\mu)\ \ln\left(\frac{m_B}{\mu_-}\right).$$

  - The factorization scale μ_- taken at the m_b scale. Cancellation of the μ_--scale dependence?
  - SCET definition of $\phi_M'(1,\mu)$? Weak annihilation is real? [Beneke, 2007]

Power counting of different regions



$$A_1^i = \pi \alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_2}(x)\, \Phi_{M_1}(y) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2} \Phi_{m_2}(x) \Phi_{m_1}(y) \frac{2}{\bar{x} y} \right\},$$

- Asymptotic form of DAs of light mesons

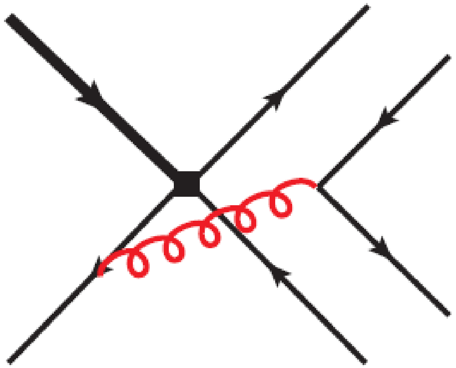
$$\Phi_P(x,\mu) = 6x(1-x)$$

	Momentum fractions	b-emission diagram	Light-emission diagram
Hard gluon	$x\sim 1, \bar{x}\sim 1, dx\sim 1$ $y\sim 1, \bar{y}\sim 1, dy\sim 1$	1	1
Hard-collinear gluon	$x\sim 1, \bar{x}\sim \lambda, dx\sim \lambda$ $y\sim 1, \bar{y}\sim 1, dy\sim 1$	λ	1
Soft gluon	$x\sim 1, \bar{x}\sim \lambda, dx\sim \lambda$ $y\sim \lambda, \bar{y}\sim 1, dy\sim \lambda$	λ^3	λ

Factorization of pure annihilation decays at tree level

- The denominator of the propagators

$$\frac{1}{y\, m_B^2\, [\bar{x} - \omega/m_B + i\epsilon]}\ \frac{1}{y\, \bar{x}\, m_B^2 + i\epsilon}$$



Regularized the endpoint singularity

- Factorization at tree level(leading twist)

$$\mathcal{G}_{\mathcal{B}_1} \equiv \int_0^\infty d\omega\, \phi_B^+(\omega) \int_0^1 dx\, \phi_{M_2}(x) \int_0^1 dy\, \phi_{M_1}(y)\ \frac{1}{\bar{x}\, y\ (\bar{x} - \omega/m_B + i\epsilon)}$$

The Strong Phase

$$\mathcal{G}_{\mathcal{B}_1} \equiv \int_0^\infty d\omega\, \phi_B^+(\omega) \int_0^1 dx\, \phi_{M_2}(x) \int_0^1 dy\, \phi_{M_1}(y)\, \frac{1}{\bar{x}\, y\, (\bar{x} - \omega/m_B + i\epsilon)}$$

- Inserting asymptotic form of light meson DAs and GN model for B meson, we can obtain an approximate result

$$18\left[\left(\ln\frac{m_B}{\lambda_B} + \gamma_{\rm E} - 2\right) - i\,\pi\right]$$

- A perturbative strong phase is generated

- In comparison with QCDF

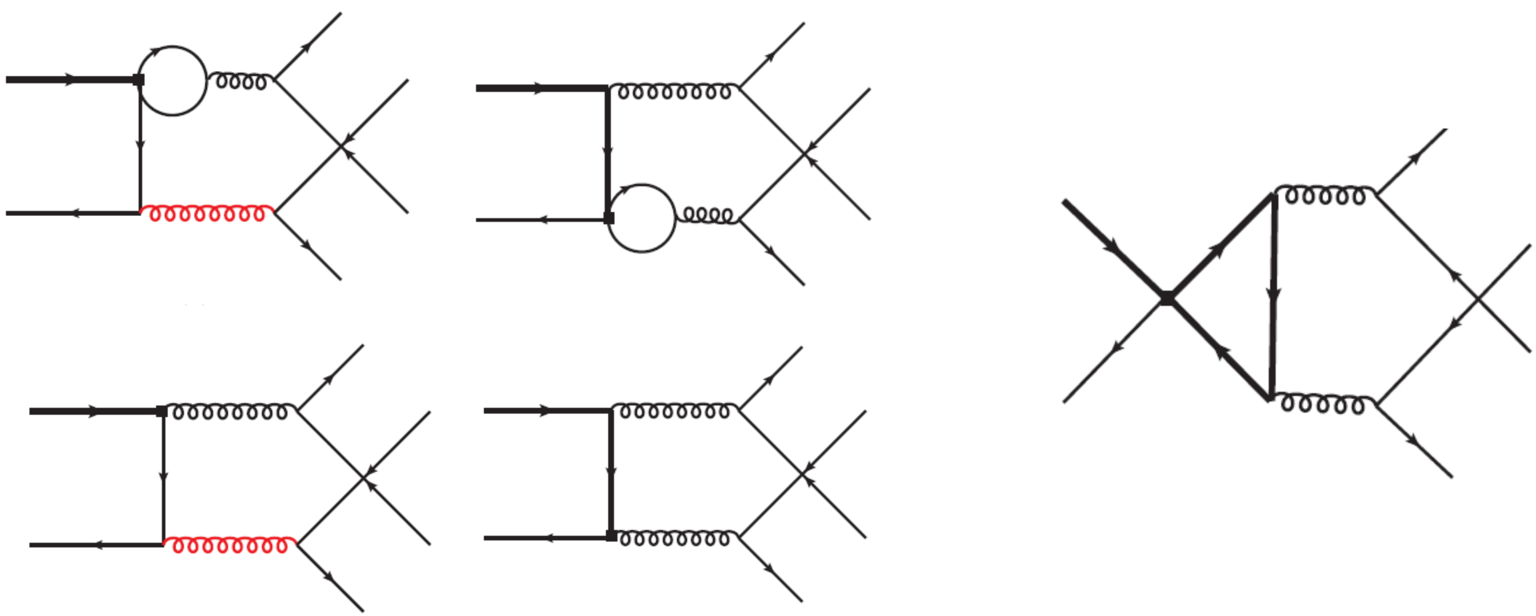
$$X_A^i \approx \left[1 + \frac{(\gamma_{\rm E}-1) - i\,\pi}{\ln\frac{m_B}{\lambda_B}}\right] \ln\frac{m_B}{\lambda_B}$$

$$\left(1 + \varrho_A\, e^{i\,\varphi_A}\right) \ln\frac{m_B}{\Lambda_h}$$

The enhanced higher order contribution

- The pure annihilation decays such as $\bar{B}_s \to \pi^+\pi^-$ are penguin dominated

- The tree operators can provide enhancement at higher order of coupling constant



Weak annihilation amplitudes at NLO

- The amplitude of $B \to g^* g^* \to M_1 M_2$

$$\langle M_1(p) M_2(q) | g^*(p_g, \alpha)\, g^*(\tilde{p}_g, \beta) \rangle = \begin{cases} g^{\perp}_{\alpha\beta}\, \mathscr{S}_{\parallel}(M_1 M_2) & \text{for } M_1 M_2 = PP,\, V_L V_L \\ i\,\varepsilon_{\alpha\beta pq}\, \mathscr{S}_{\perp}(M_1 M_2) & \text{for } M_1 M_2 = PV,\, VP\,. \end{cases}$$

$$\langle g^*(p_g, \alpha)\, g^*(\tilde{p}_g, \beta) | \mathscr{H}_{\text{eff}} | \bar{B}_q \rangle = i\,\varepsilon^{\perp}_{\alpha\beta}\, F_V(p_g^2, \tilde{p}_g^2) + g^{\perp}_{\alpha\beta}\, F_A(p_g^2, \tilde{p}_g^2)$$

Symmetry relations of the two transition form factors:

$$F_V(p_g^2, \tilde{p}_g^2) = -F_V(\tilde{p}_g^2, p_g^2)\,, \qquad F_A(p_g^2, \tilde{p}_g^2) = F_A(\tilde{p}_g^2, p_g^2).$$

- Symmetry relation indicates

  - The displayed NLO diagrams will NOT contribute to $B \to PV, VP$ decays.
  - Only the axial-vector form factor F_A will be relevant due to the symmetry properties

Weak annihilation amplitudes at NLO

- The NLO amplitude

$$\mathcal{T}^{p,(1)} \supset \sum_{i=1}^{6} C_i\, \mathcal{P}_i^{p,(1)}(M_1 M_2) + C_8^{\rm eff}\, \mathcal{P}_{8g}^{(1)}(M_1 M_2) + \sum_{i=1}^{6} C_i\, \mathcal{I}_i^{p,(1)}(M_1 M_2)\,,$$

$$\sum_{i=1}^{6} C_i\, \mathcal{P}_i^{p,(1)} = \left(C_2 - \frac{C_1}{2\,N_c}\right) H_1(m_p) + \left[(C_3 + 16\,C_5) - \frac{1}{2\,N_c}\,(C_4 + 16\,C_6)\right] [H_1(m_b) + H_1(0)] + (C_4 + 10\,C_6)\ [H_1(m_b) + H_1(m_c) + 3\,H_1(0)] - \left[5\,C_4 - 8\,C_5 + 4\left(\frac{1}{N_c} + 5\right) C_6\right] H_2$$

$$C_8^{\rm eff}\, \mathcal{P}_{8g}^{(1)} = C_8^{\rm eff}\, H_3\,, \qquad \sum_{i=1}^{6} C_i\, \mathcal{I}_i^{p,(1)} = [(C_3 + 4\,C_5) + C_F\,(C_4 + 4\,C_6)]\ H_4\,,$$

- The triangle diagrams is vanishing for tree operator
- The charming loop with large Wilson coefficient or CKM matrix element enhanced NLO contribution

The predicted amplitudes

	$\nu = m_b/2$	$\nu = m_b$	$\nu = 2\,m_b$
$\mathcal{T}^{u,\,(0)}$	$2.03 - 17.7\,i$	$2.00 - 17.4\,i$	$1.97 - 17.2\,i$
$\mathcal{T}^{c,\,(0)}$	$-0.38 + 3.35\,i$	$-0.25 + 2.16\,i$	$-0.16 + 1.42\,i$
$\alpha_s/(4\,\pi)\ \mathcal{T}^{u,\,(1)}$	$-2.29 - 3.63\,i$	$-2.41 - 3.61\,i$	$-2.47 - 3.57\,i$
$\alpha_s/(4\,\pi)\ \mathcal{T}^{c,\,(1)}$	$-1.65 - 2.34\,i$	$-1.82 - 2.40\,i$	$-1.91 - 2.43\,i$

TABLE I. Numerical predictions of the dynamical quantities $\mathcal{T}^{p,\,(0)}$ and $\mathcal{T}^{p,\,(1)}$ $(p = u,\, c)$ for $\bar{B}_s \to \pi^+\pi^-$ with three distinct renormalization scales ν.

The predicted CP asymmetries

	$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}$	$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}$
$\bar{B}_s \to \pi^+ \pi^-,\ \pi^0 \pi^0$	$-36.3^{+8.2}_{-1.3}$ (0.0 ± 0.0)	$-4.2^{+21.4}_{-9.0}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \rho_L^+ \rho_L^-,\ \rho_L^0 \rho_L^0$	$-36.3^{+8.3}_{-1.8}$ (0.0 ± 0.0)	$-4.3^{+21.5}_{-9.0}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \omega_L \omega_L$	$-36.3^{+8.3}_{-3.1}$ (0.0 ± 0.0)	$-3.8^{+21.8}_{-9.7}$ $(35.9^{+15.6}_{-11.2})$
$\bar{B}_s \to \rho_L \omega_L$	0.0 ± 0.0 $(0.0 \pm 0.0\ )$	$-71.0^{+6.3}_{-5.4}$ $(-71.0^{+6.3}_{-5.4})$
$\bar{B}_d \to K^+ K^-$	$39.0^{+3.2}_{-5.6}$ (0.0 ± 0.0)	$-2.2^{+19.1}_{-26.4}$ $(-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \to K_L^{*+} K_L^{*-}$	$39.6^{+4.9}_{-6.7}$ (0.0 ± 0.0)	$-1.4^{+19.7}_{-26.9}$ $(-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \to \phi_L \phi_L$	$38.3^{+11.4}_{-15.8}$ (0.0 ± 0.0)	$27.8^{+5.7}_{-25.9}$ (0.0 ± 0.0)

Summary

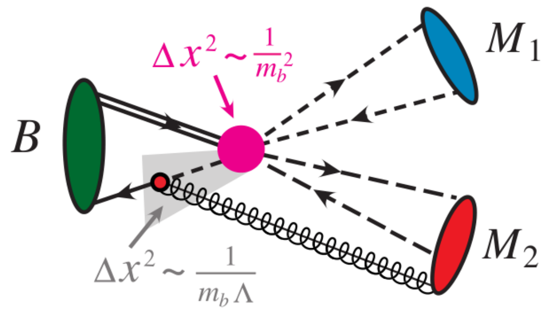
- We found the collinear gluon exchange can contribution at leading power of annihilation diagram which is missed in the previous studies, and they can be formally factorized at tree level.

- The higher order contribution from quark loop diagrams can significantly modify the CP violation of pure annihilation B decays.

Discussions

- We have not discussed the factorization property beyond tree level: LCDA at endpoint region? Soft gluon exchange?

- We have not included the leading power multi-particle contributions



- A more comprehensive study with soft-collinear effective theory is needed