Reformulating Jet Physics using Energy Flow Correlation

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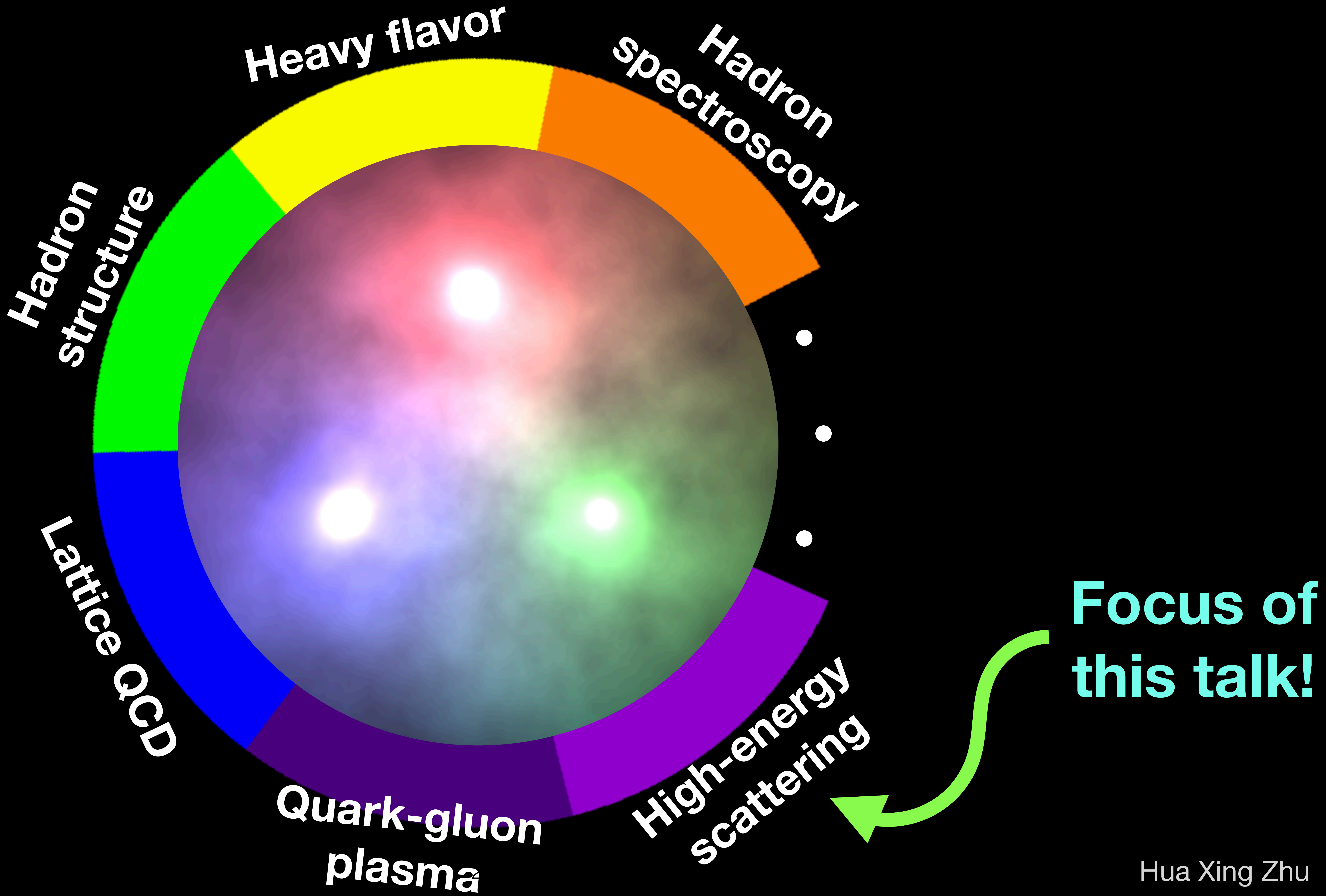
浙江大学

第四届重味物理与量子色动力学研讨会
长沙 2022年7月29日

A Golden era of QCD

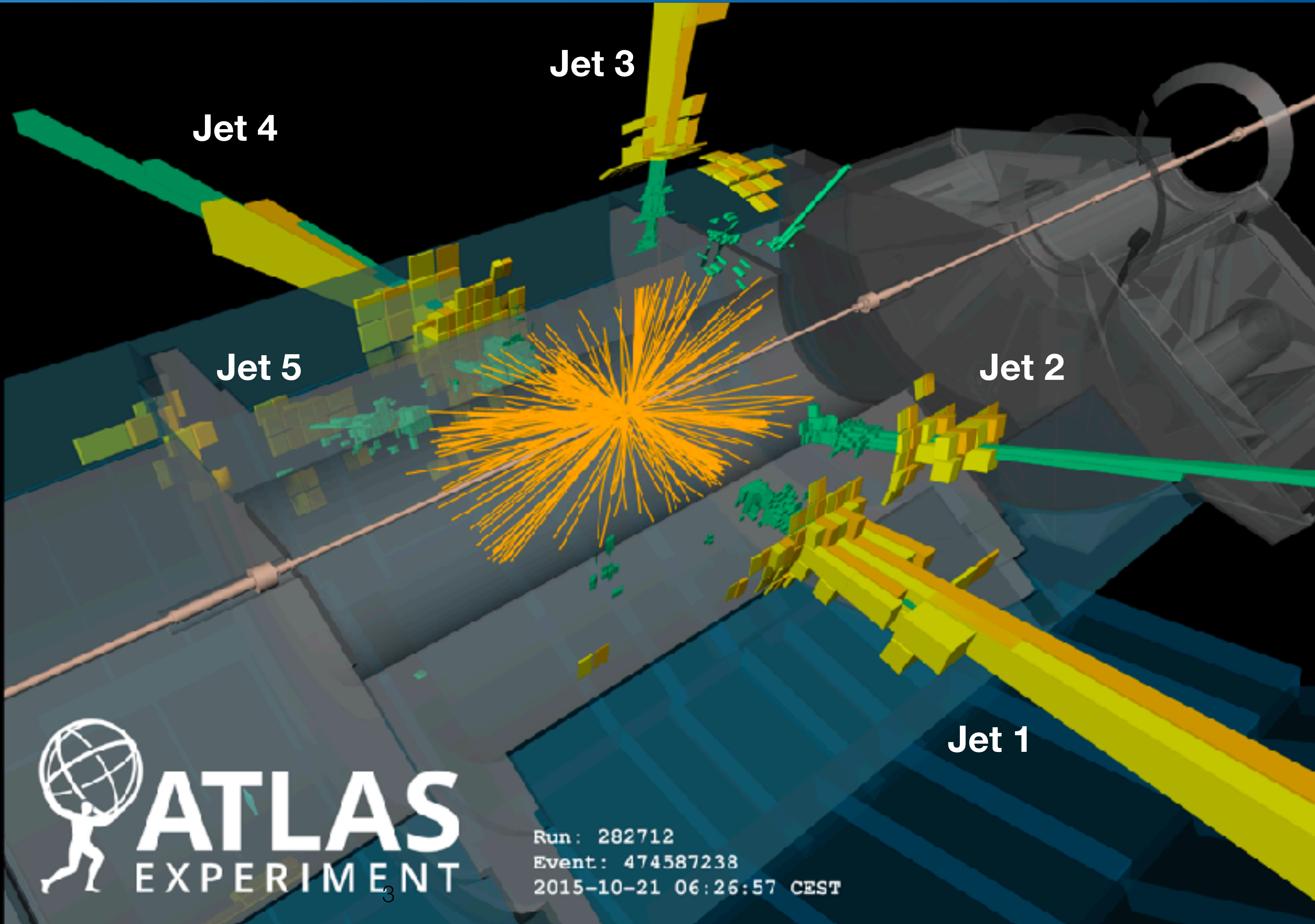


Hua Xing Zhu

Jet in high energy scattering

High energy q/g evolution

Collimated spray of hadrons!



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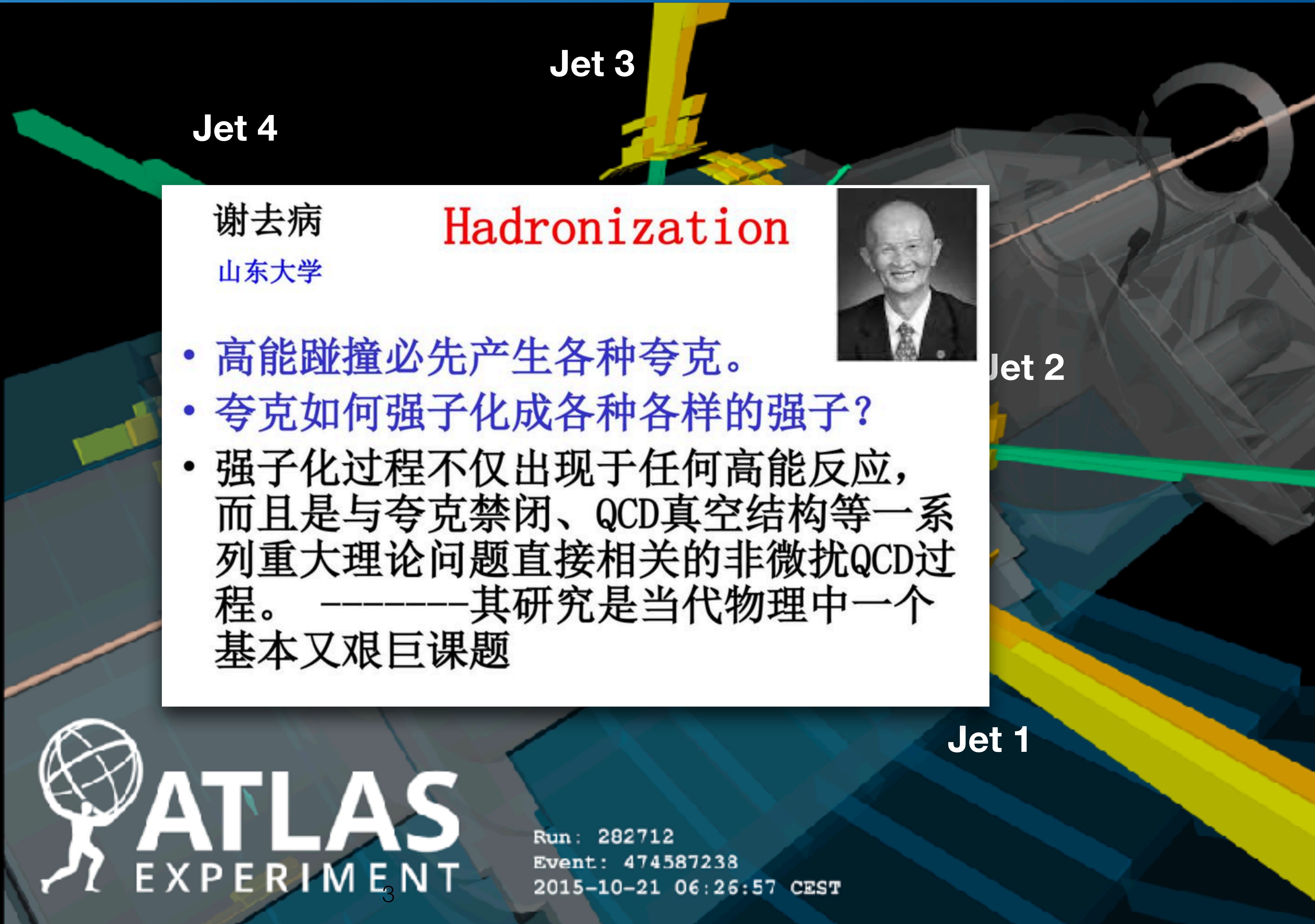
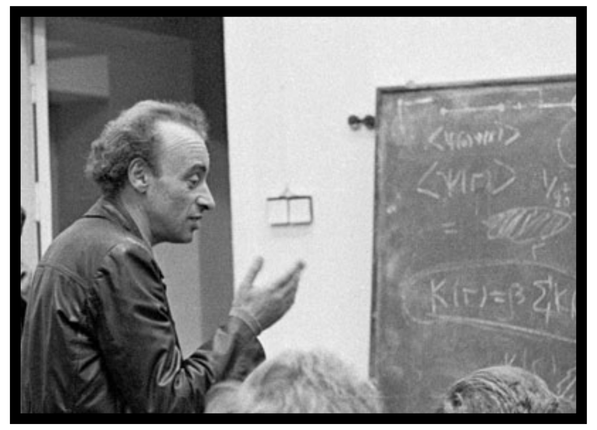
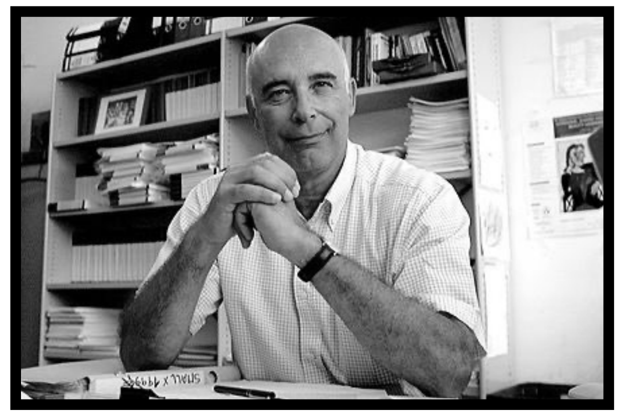


Dokshitzer-Gribov-Lipatov-Altarelli-Parisi Equation

$$\begin{cases} \mu^2 \dfrac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \displaystyle\int_x^1 \frac{dz}{z} P_{ij}^{\text{spacelike}}(z) f_j(x/z,\mu^2) \\ \mu^2 \dfrac{\partial D_i(x,\mu^2)}{\partial \mu^2} = \displaystyle\int_x^1 \frac{dz}{z} P_{ji}^{\text{timelike}}(z) D_j(x/z,\mu^2) \end{cases}$$

Relevant for PDFs, initial branching

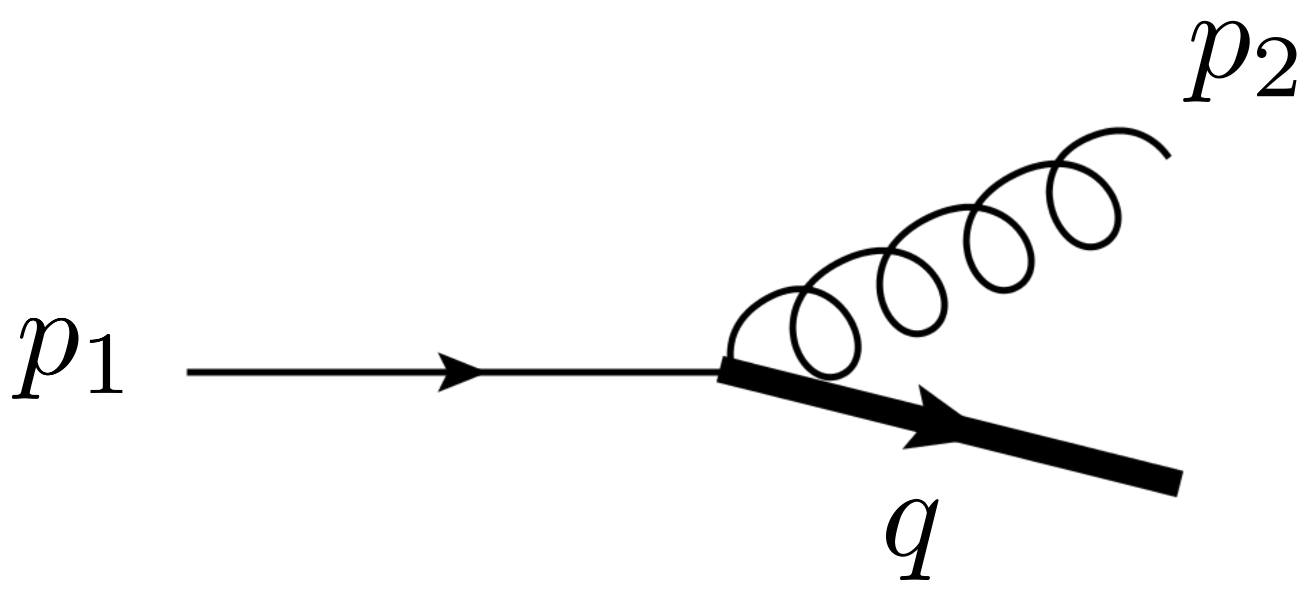
Relevant for FFs, jet fragmentation

Two Questions

- How to improve the accuracy of DGLAP?
- How to generalize DGLAP?

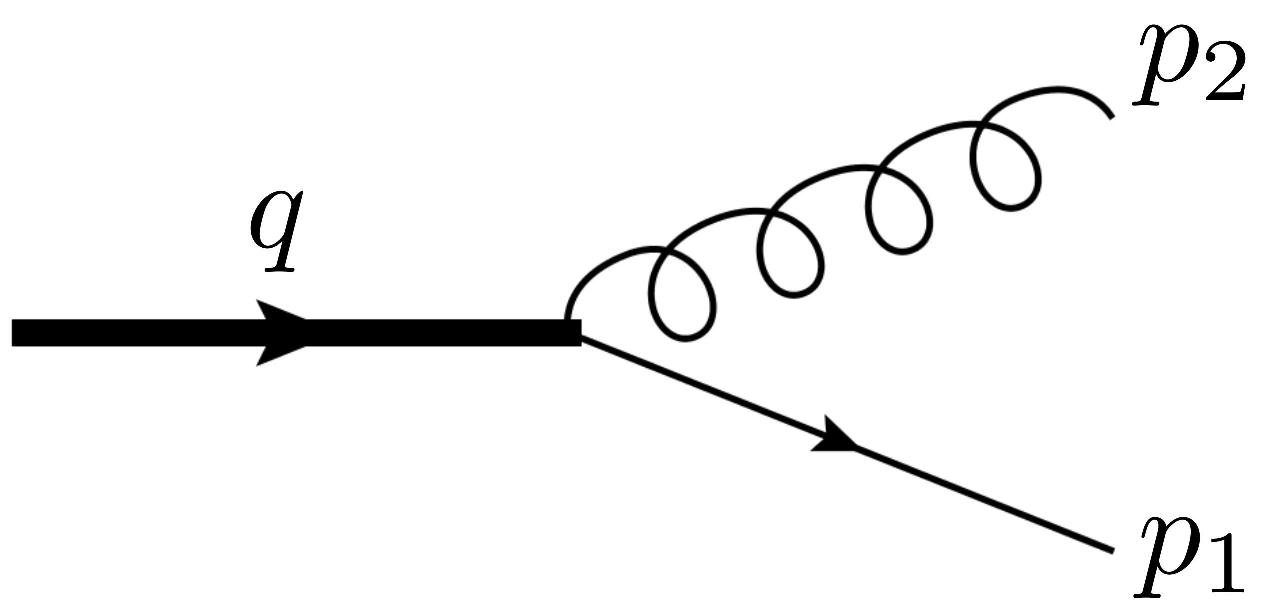
Completion of three-loop corrections to DGLAP

Space-like evolution



$$q^2 = (p_1 - p_2)^2 < 0$$

Time-like evolution



$$q^2 = (p_1 + p_2)^2 > 0$$

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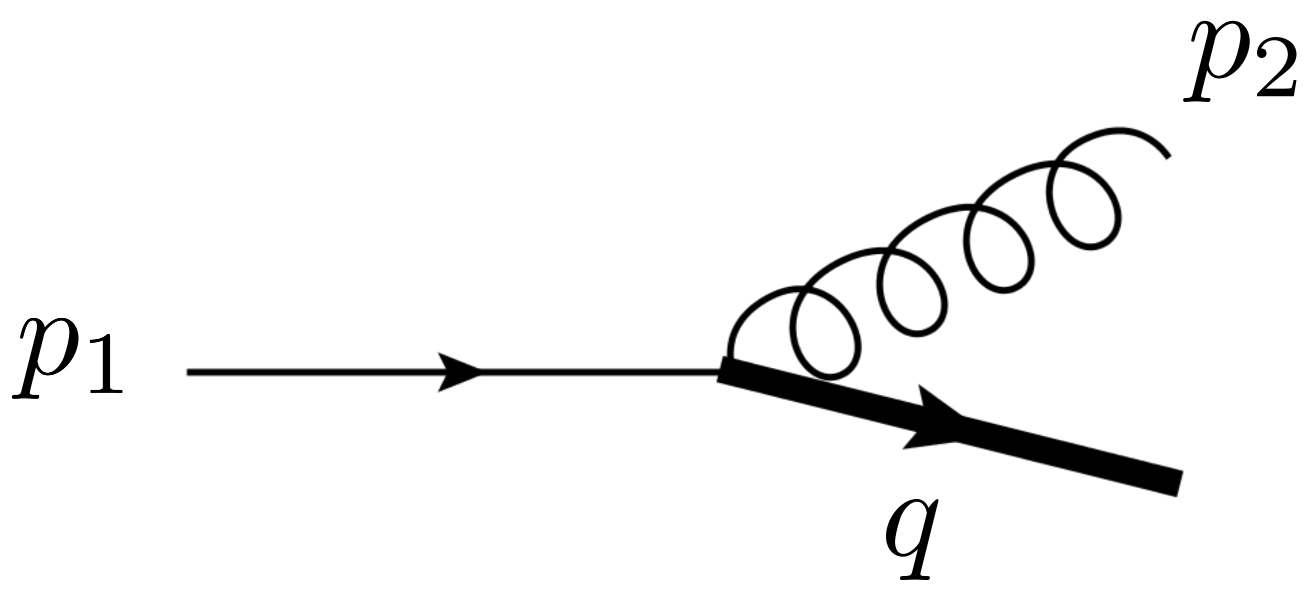

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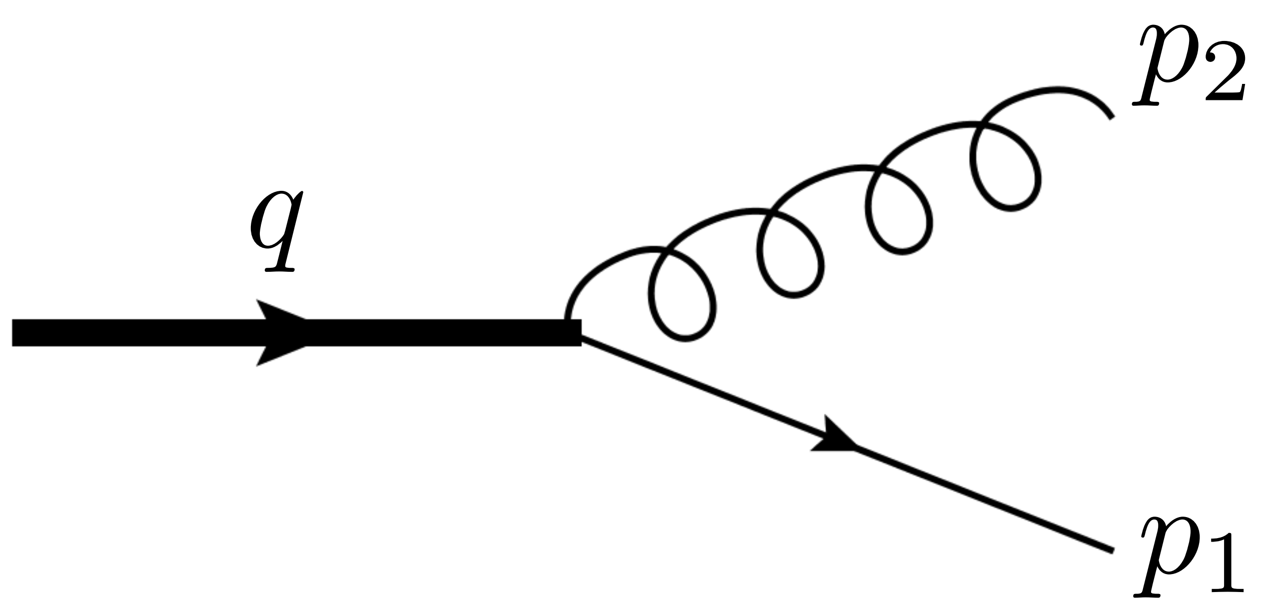

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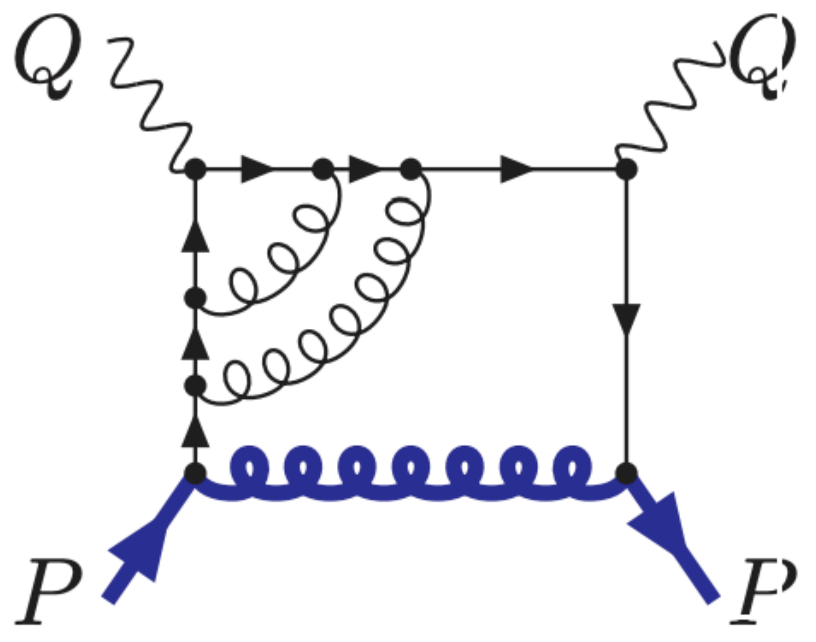
Three-loop corrections

Moch, Vermaseren, Vogt, NPB 2004

Method:
Optical theorem + OPE



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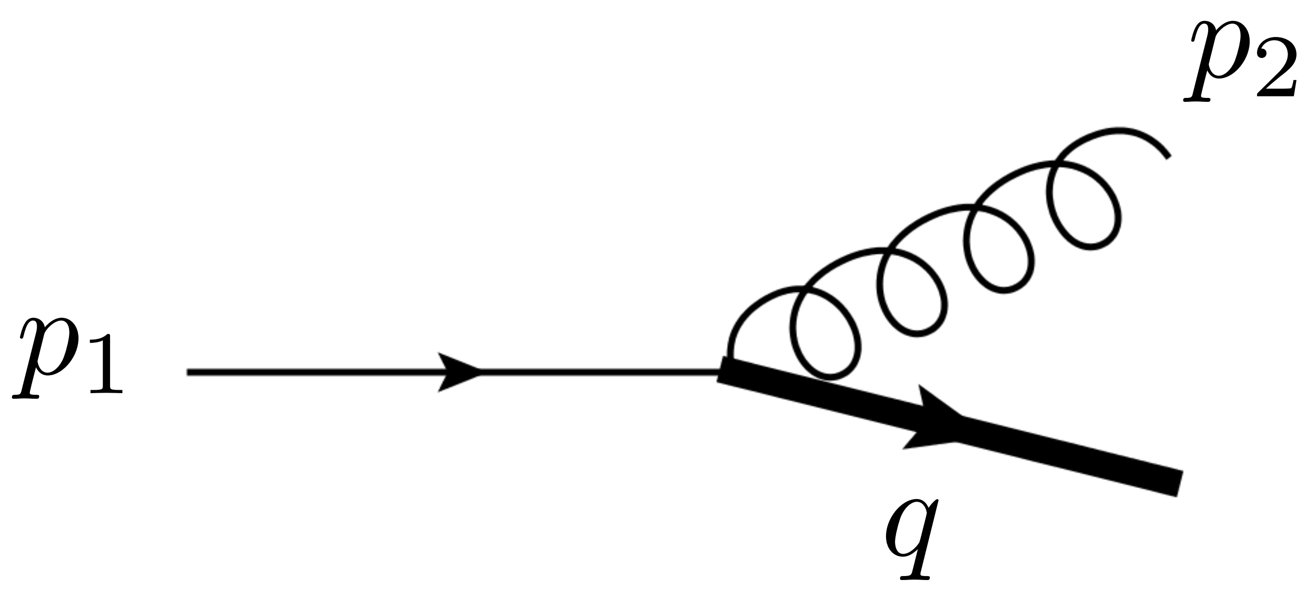

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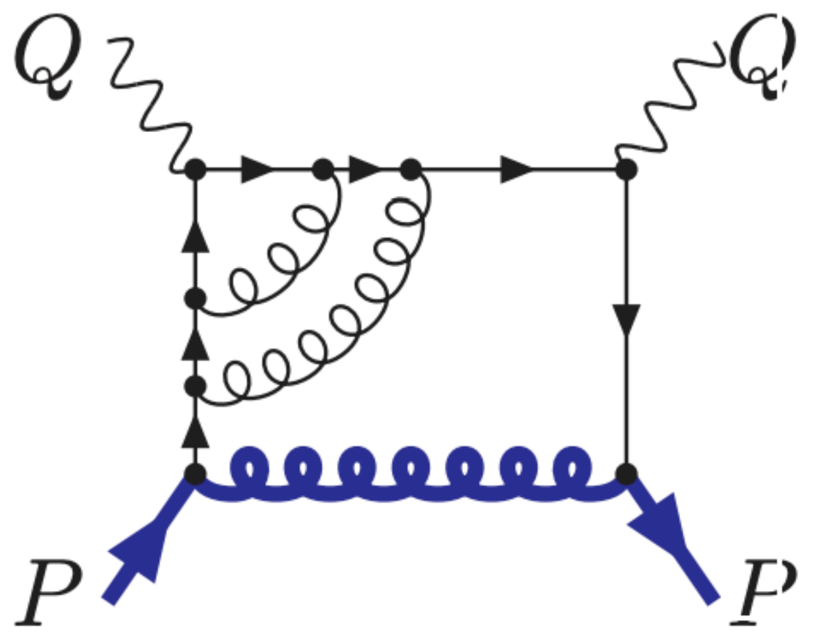

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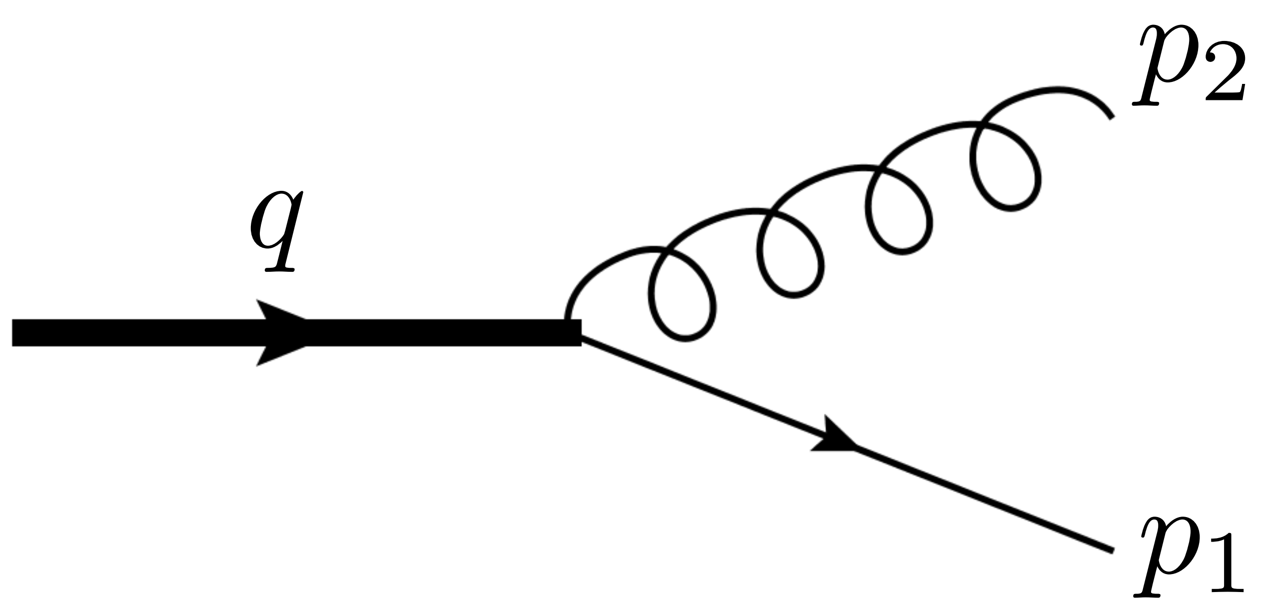

$$q^2 = (p_1 + p_2)^2 > 0$$

No complete 3-loop results until 2020

Cannot be fixed by previous method!

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} D_q \\ D_g \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \end{pmatrix}$$

H. Chen, T.Z. Yang, HXZ, Y.J. Zhu, CPC 2021

Completion of three-loop corrections to DGLAP

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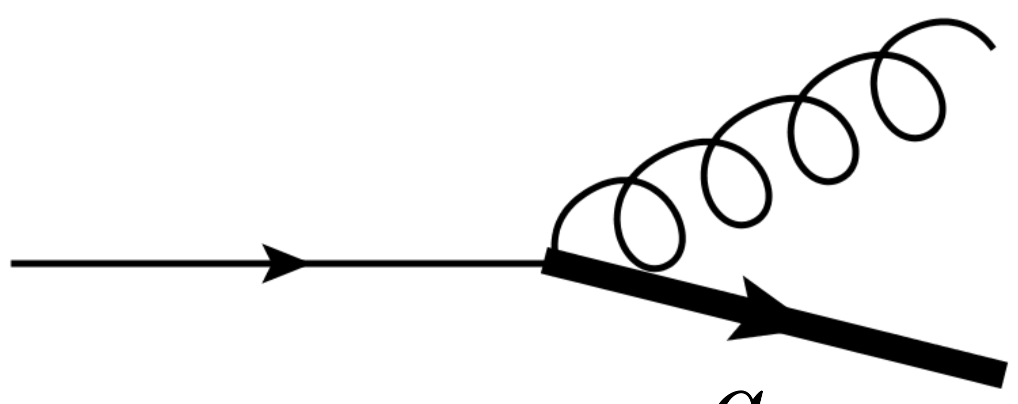

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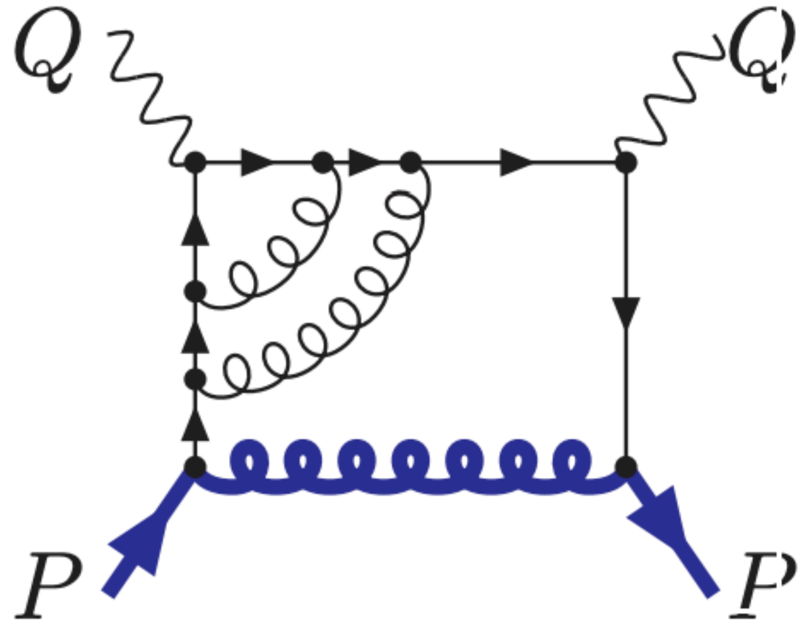
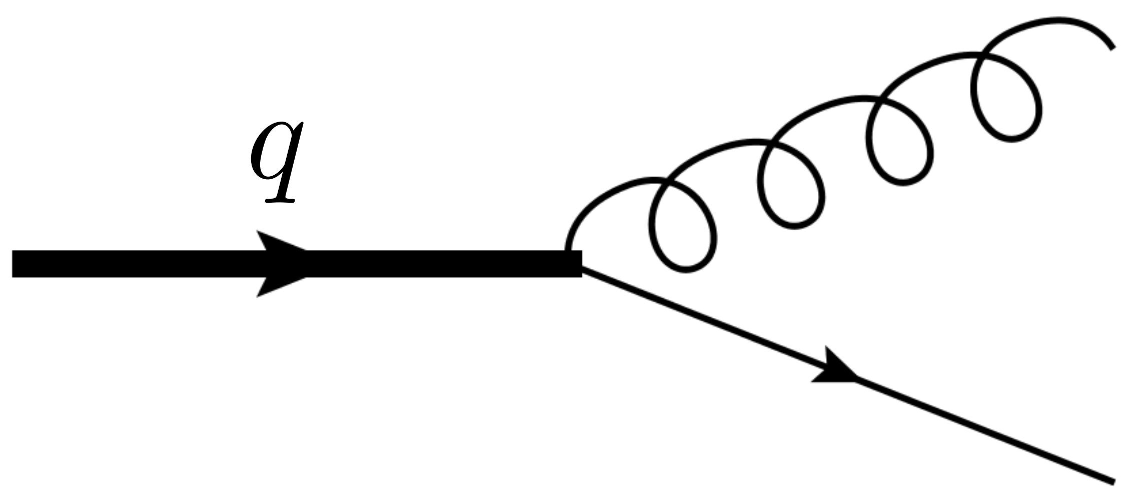


Analytical continuation?

Time-like evolution



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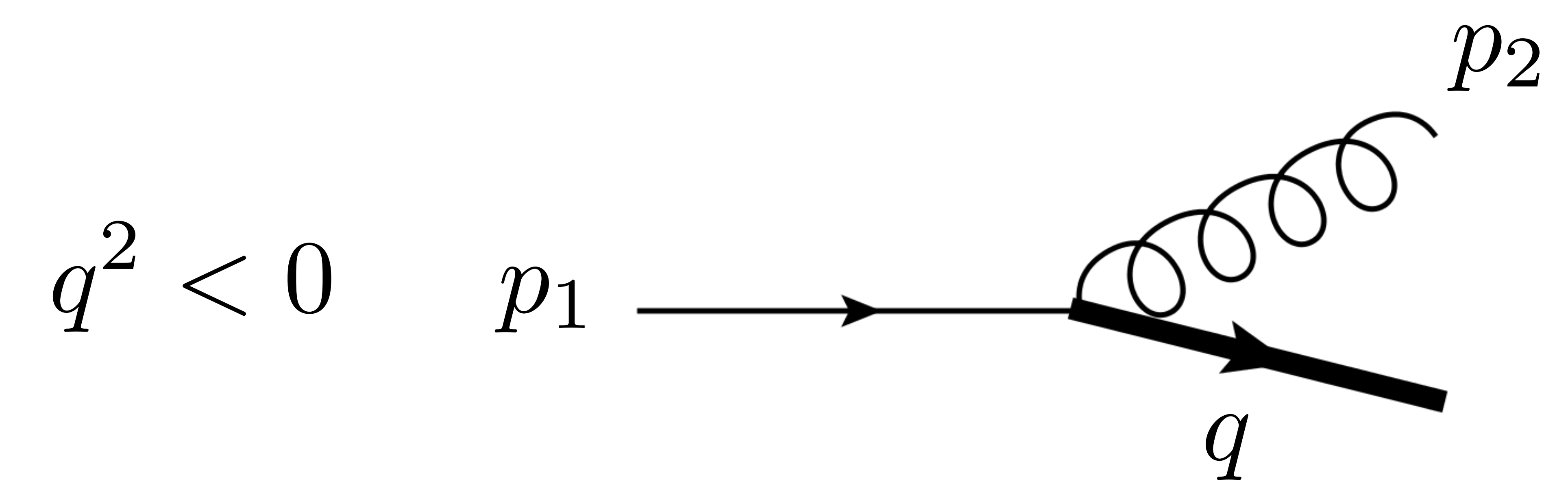
Establishing analytic continuation relation for DGLAP

Space-like evolution



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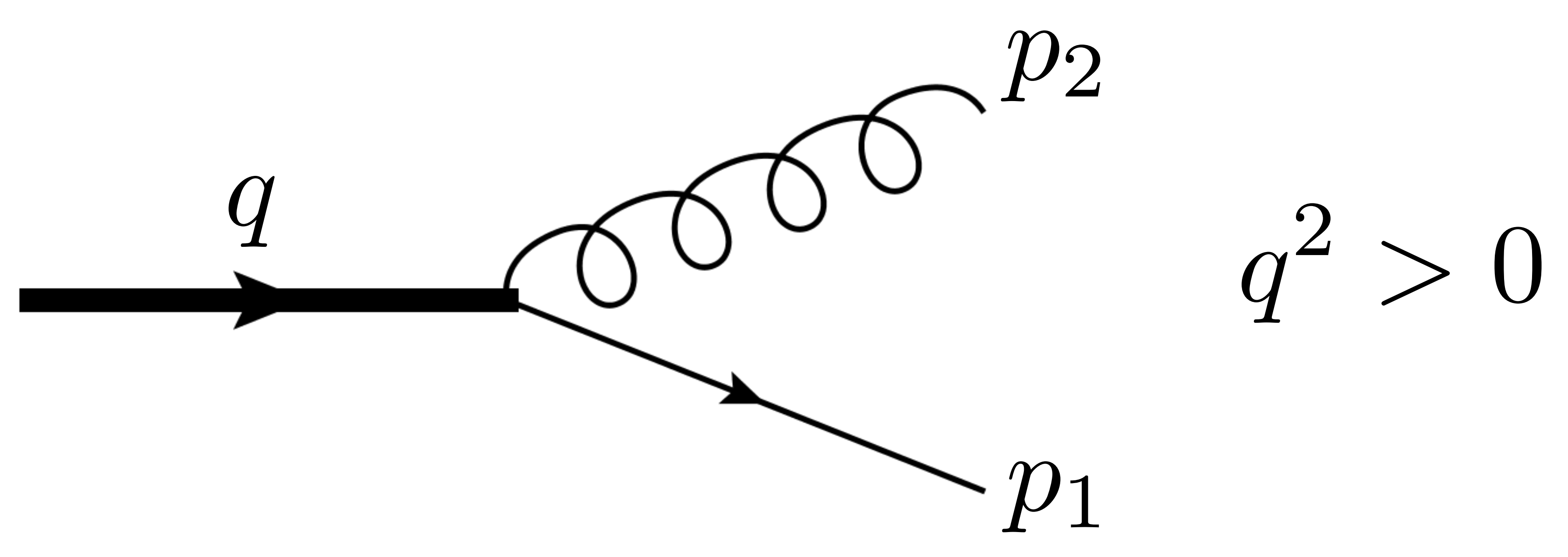


Essentially local operator correlation

$$\int dy^- \, e^{ip^+ z y^-} \langle P|\overline{\psi}(y^-)\gamma^+\psi(0)|P\rangle$$

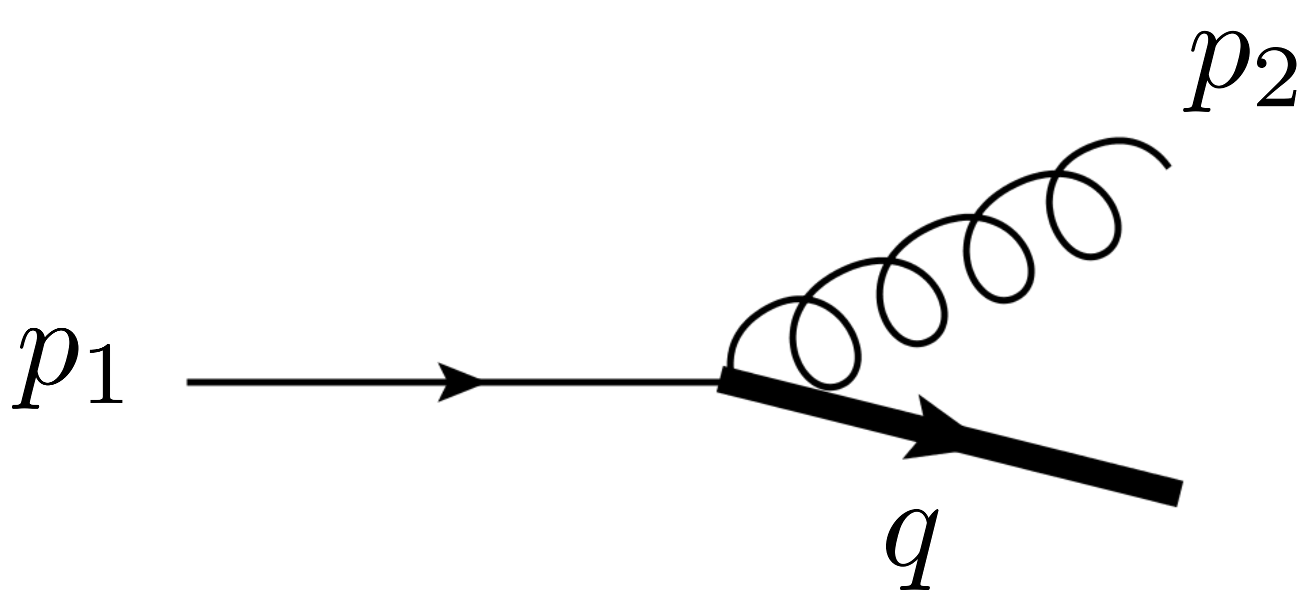
Intrinsic non-local operator correlation

$$\int dy^- \, e^{ip^+ y^-/z} \langle 0|\overline{\psi}(y^-)|hX\rangle\langle hX|\gamma^+\psi(0)|0\rangle$$

Establishing analytic continuation relation for DGLAP

Space-like evolution

$q^2 < 0$



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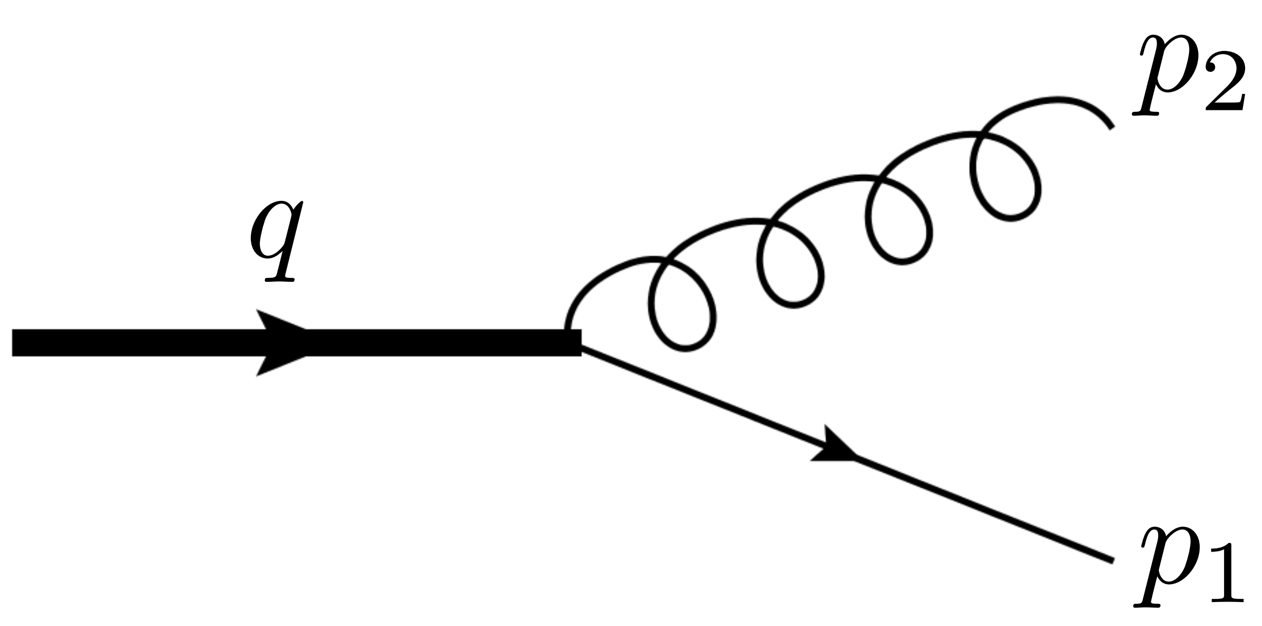

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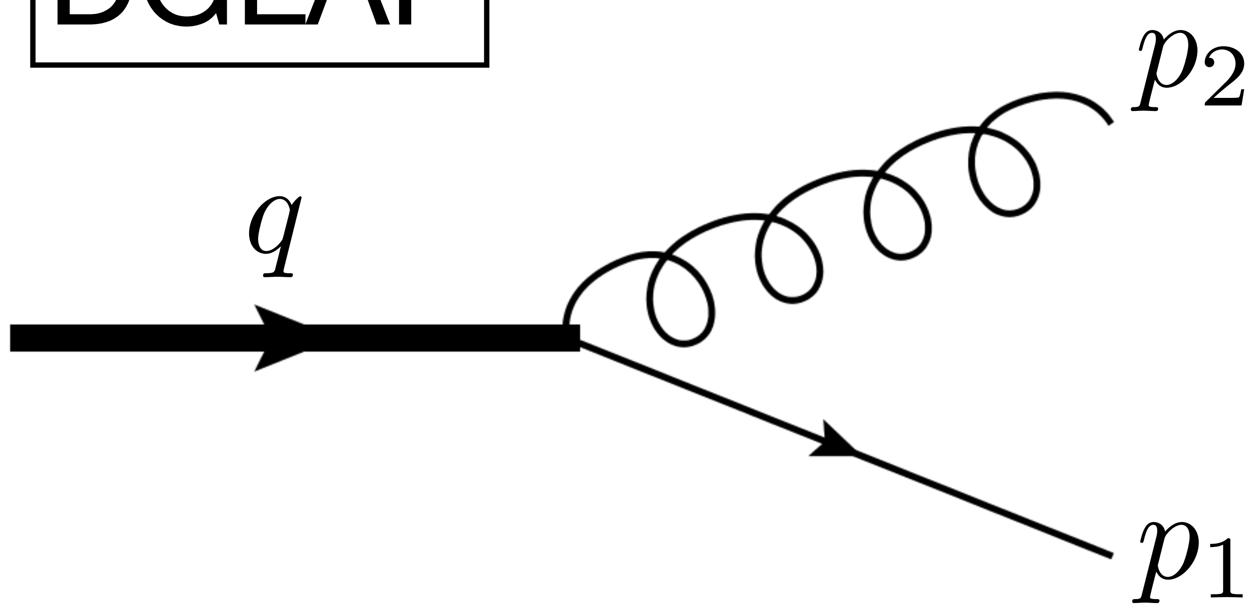
$$\gamma_{\pm}^{\text{timelike}}(N) = \gamma_{\pm}^{\text{spacelike}}(N - \gamma_{\pm}^{\text{timelike}}(N))$$

H. Chen, T.Z. Yang, HXZ, Y.J. Zhu, CPC 2021

Generalizing DGLAP?

DGLAP



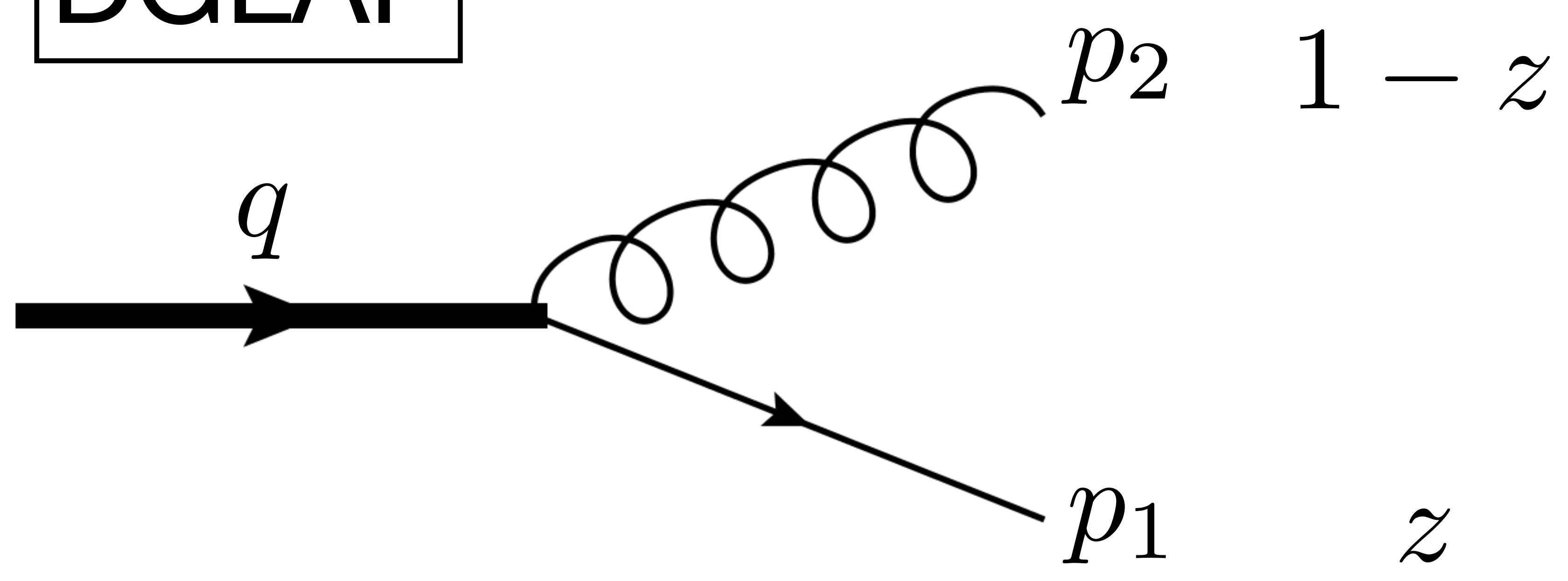
$1 - z$

z

$$P(z, s_{12})$$

Generalizing DGLAP?

DGLAP



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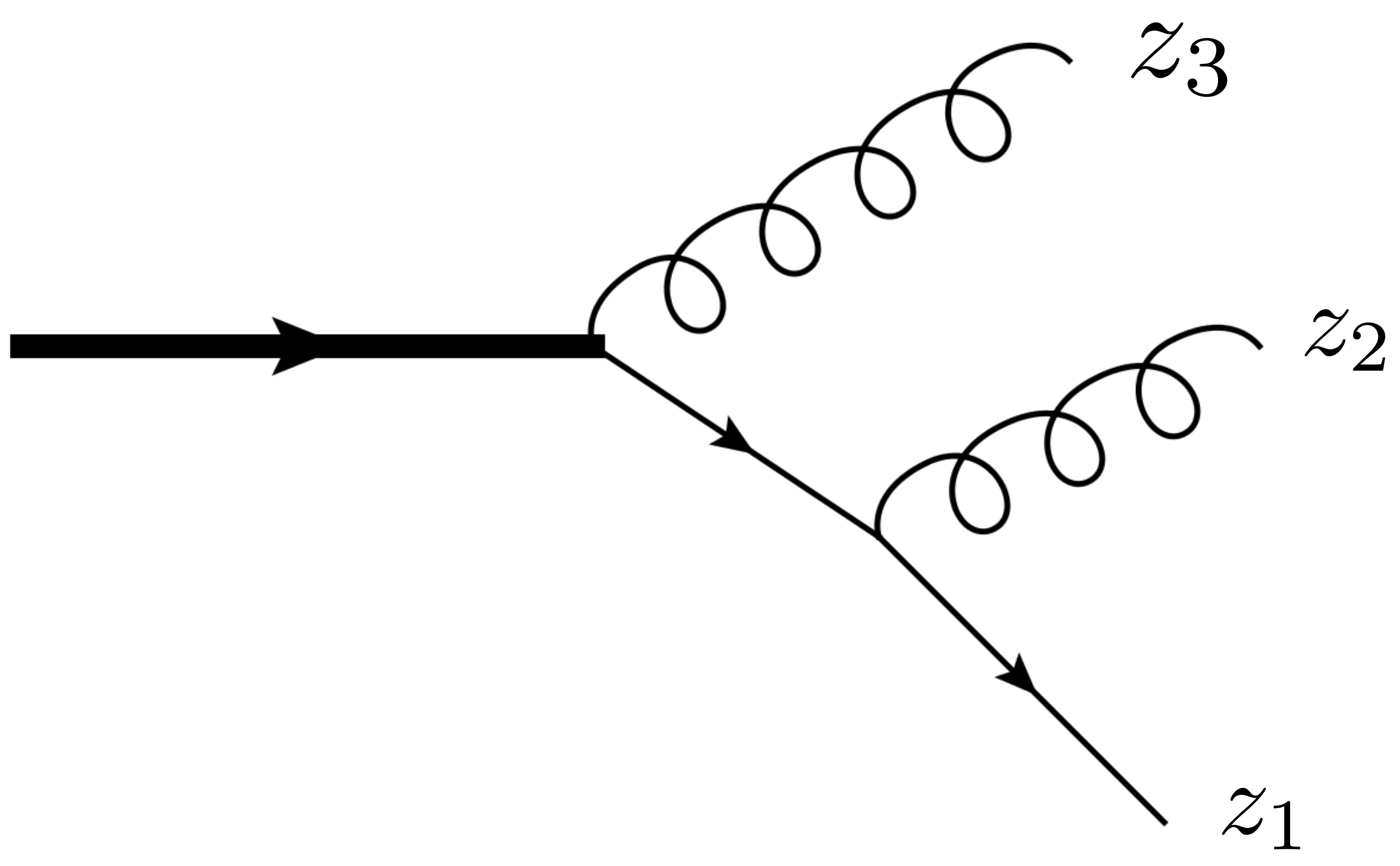
Generalized DGLAP



$$P(z_1, z_2, z_3; s_{12}, s_{23}, s_{31})$$

Generalizing DGLAP?

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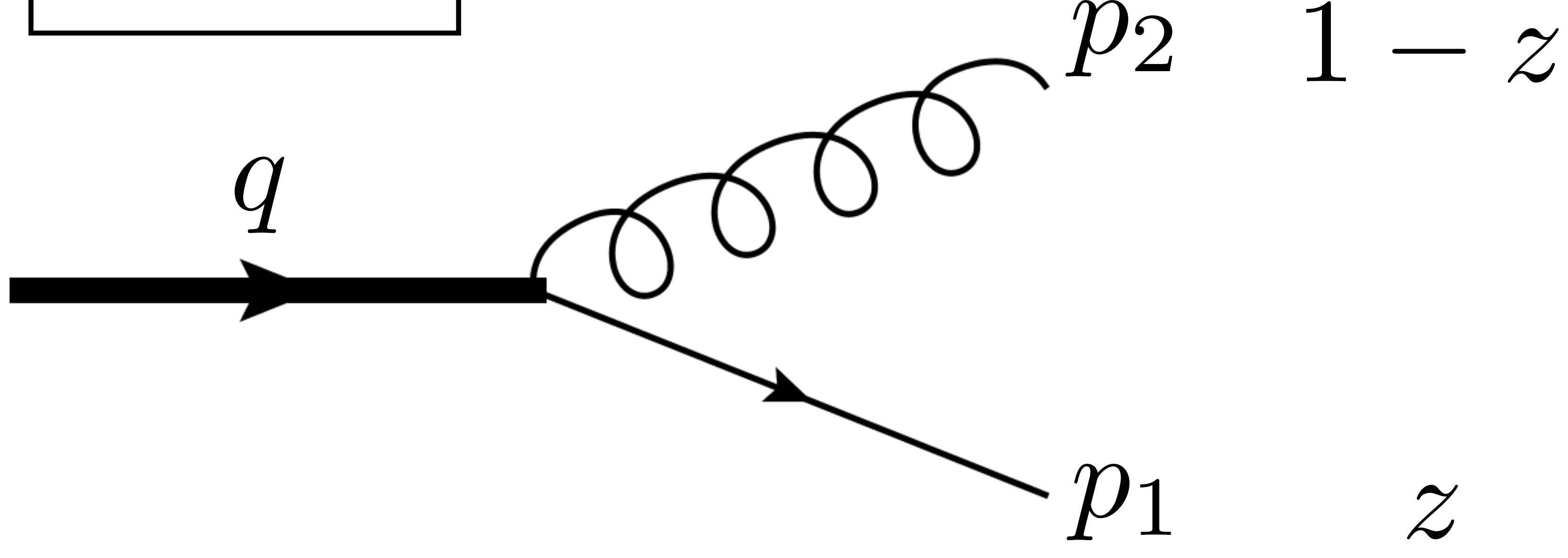

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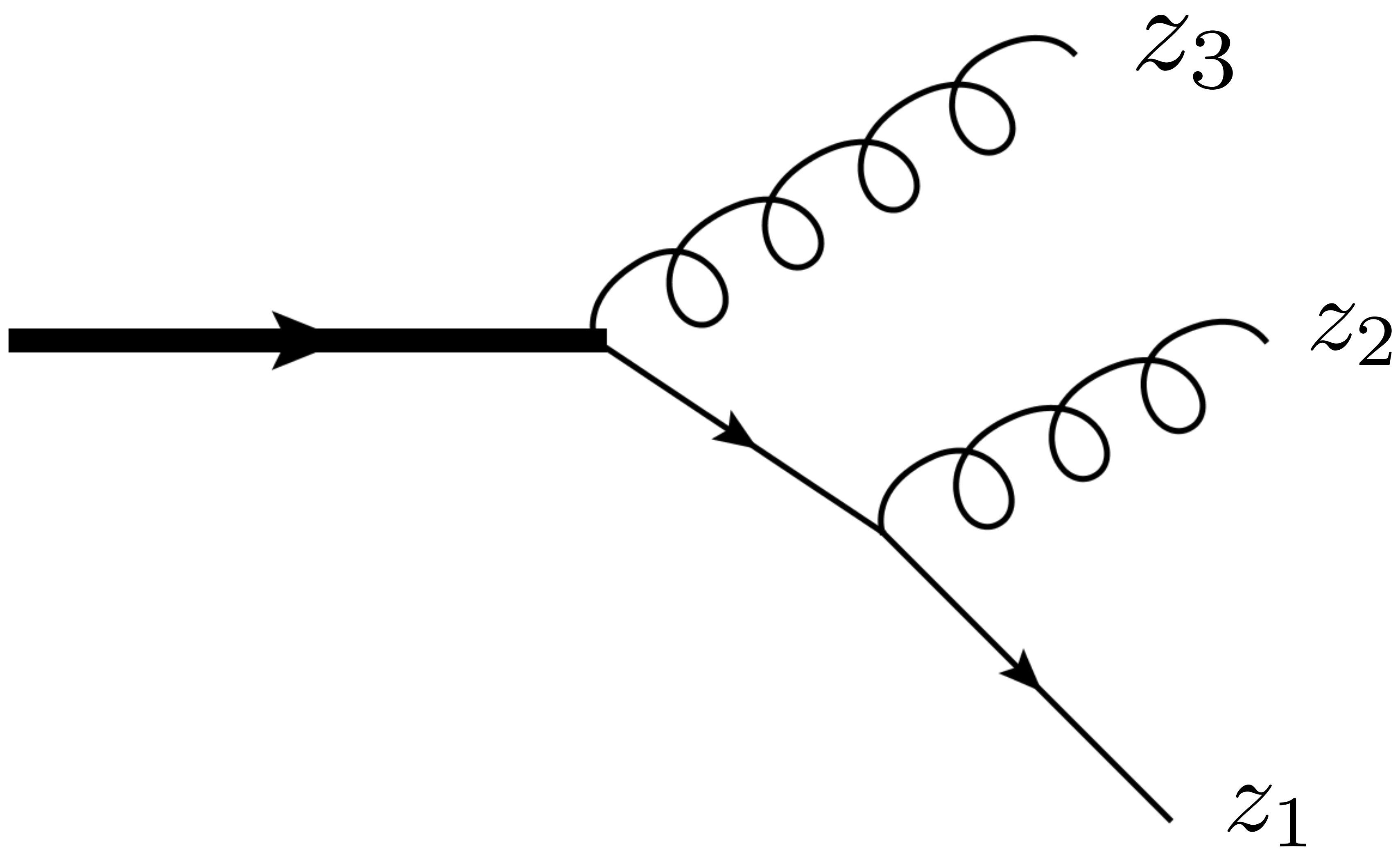
Generalized DGLAP



$$P(z_1, z_2, z_3; s_{12}, s_{23}, s_{31})$$

Integrate out z_i

Energy flow correlation

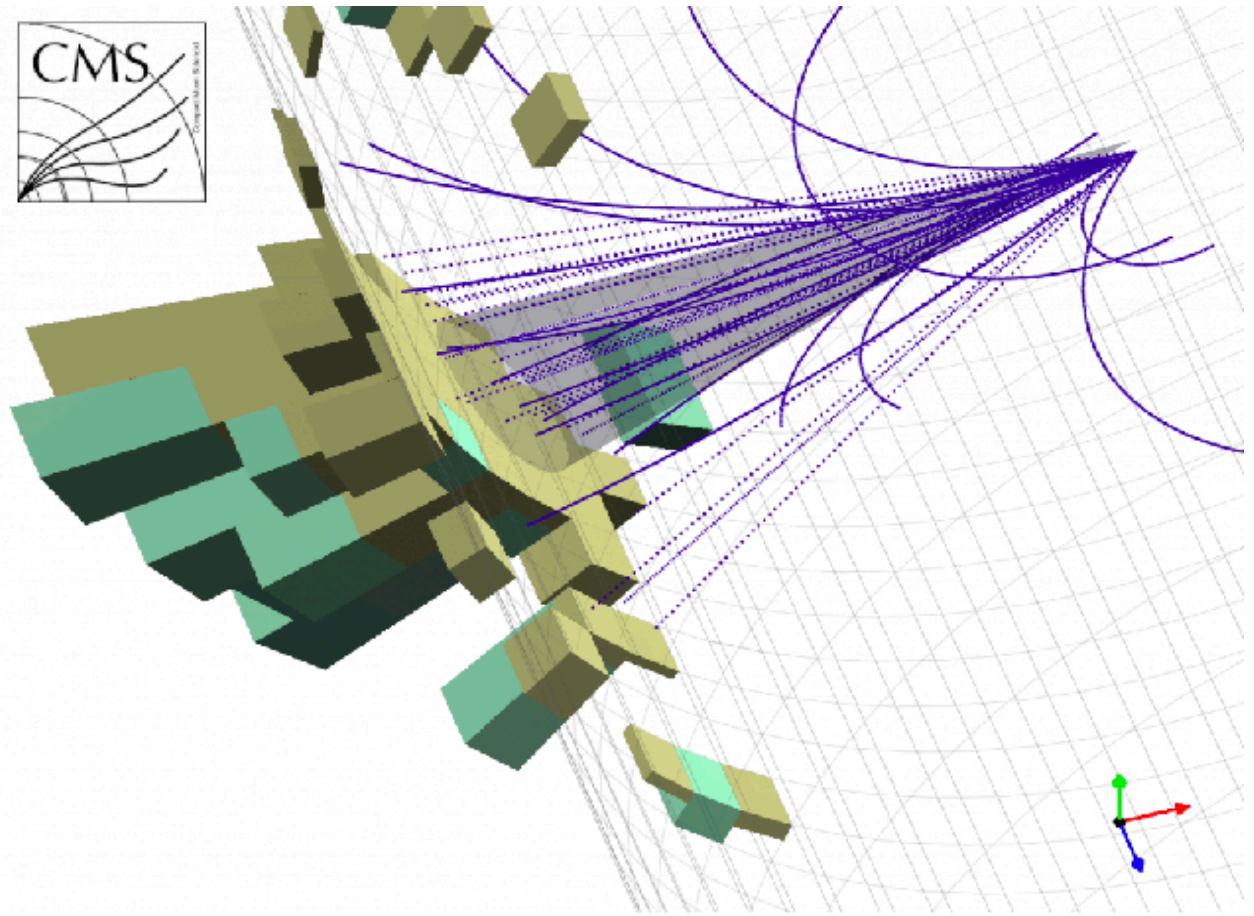
$G(\theta_{12}, \theta_{23}, \theta_{31})$

Integrate out s_{ij}

Multi-collinear splitting

$K(z_1, z_2, z_3)$

Two approaches to jet structure



$= \{p_1, p_2, \cdots p_n\}$

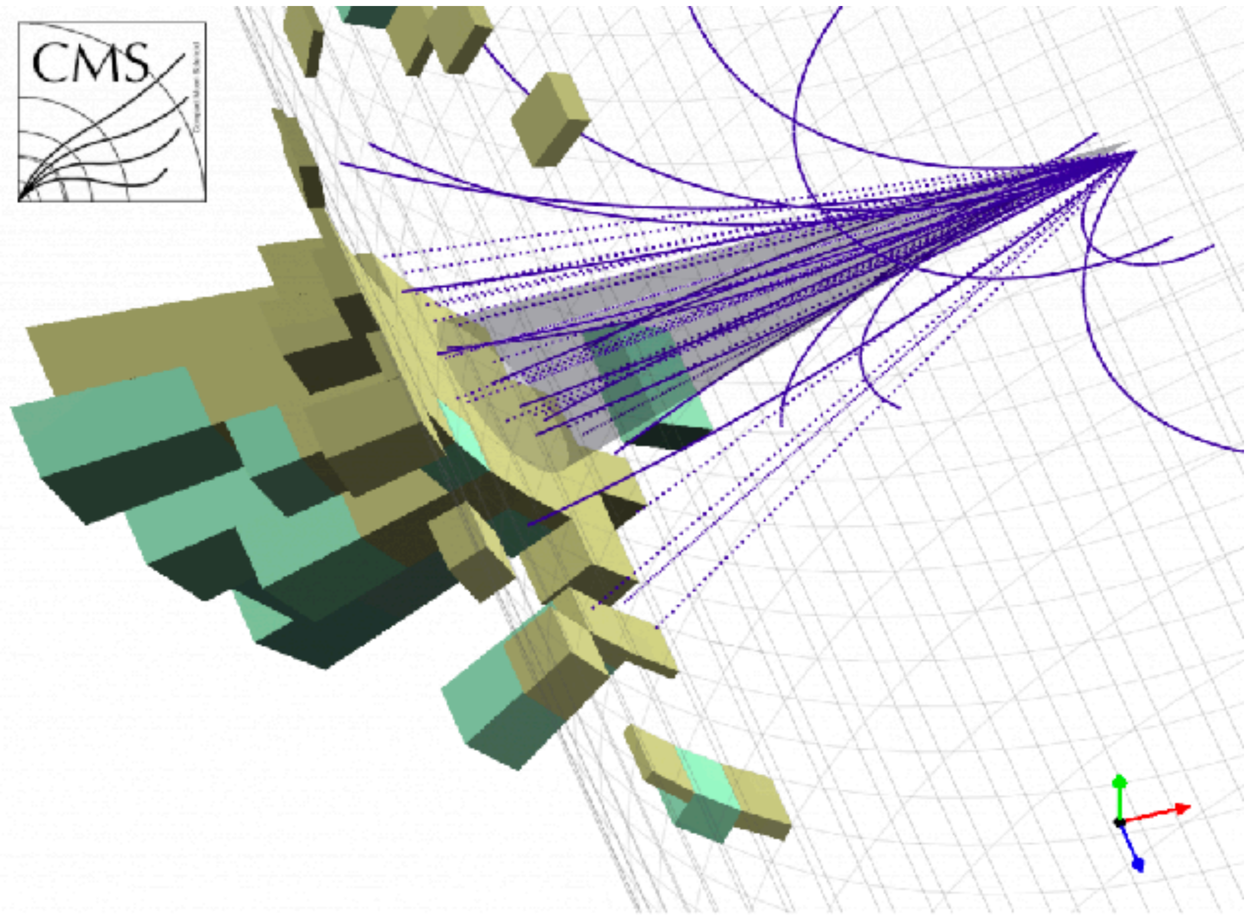
H

Shape observables

$\langle s_i s_j \rangle$

Statistical correlation

Two approaches to jet structure



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Statistical correlation

$$\text{Observable} = f(p_1, p_2, \cdots, p_n)$$

- Example: jet mass, jet broadening, thrust
- Advantage: close connection with new physics search
- Disadvantage: lack direct connection with field theory

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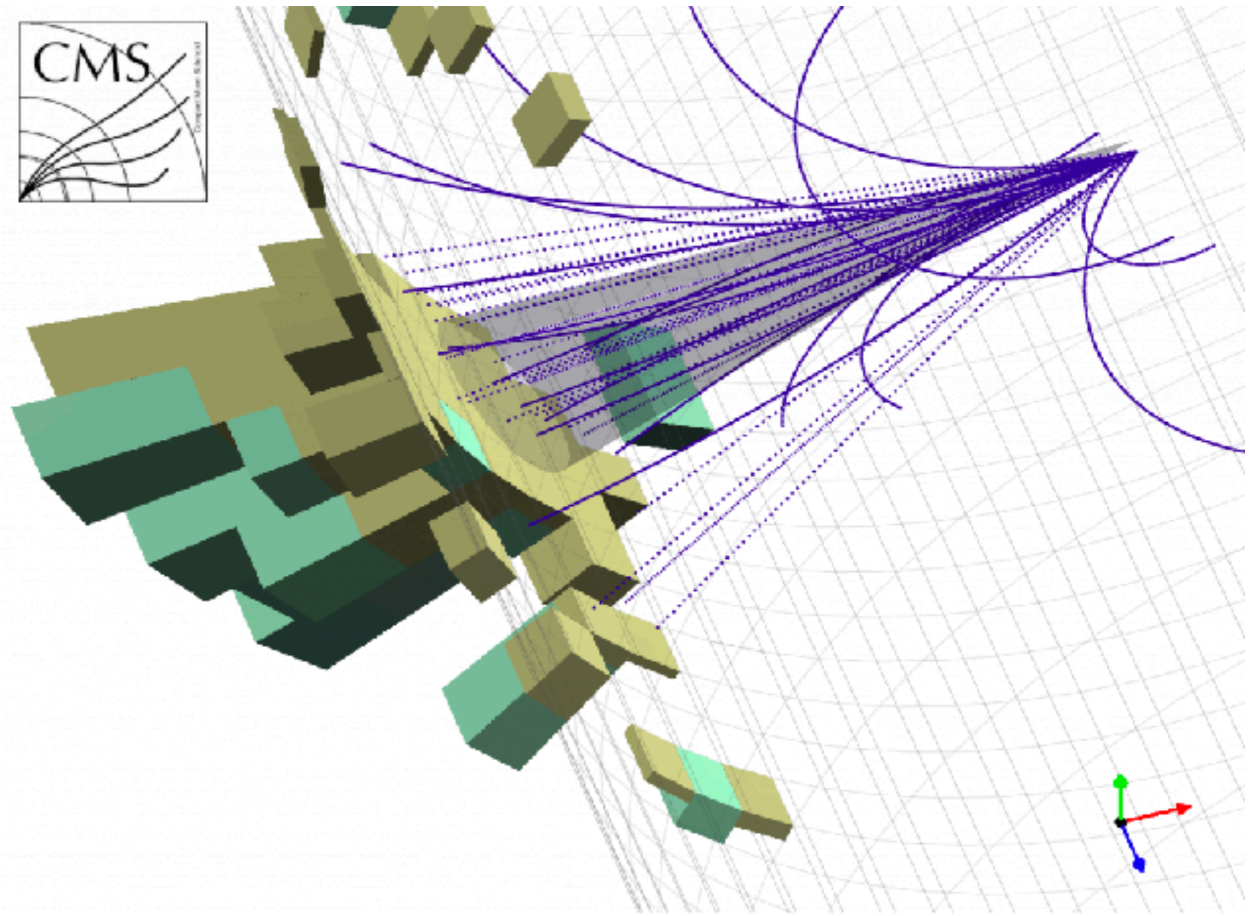

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Statistical correlation

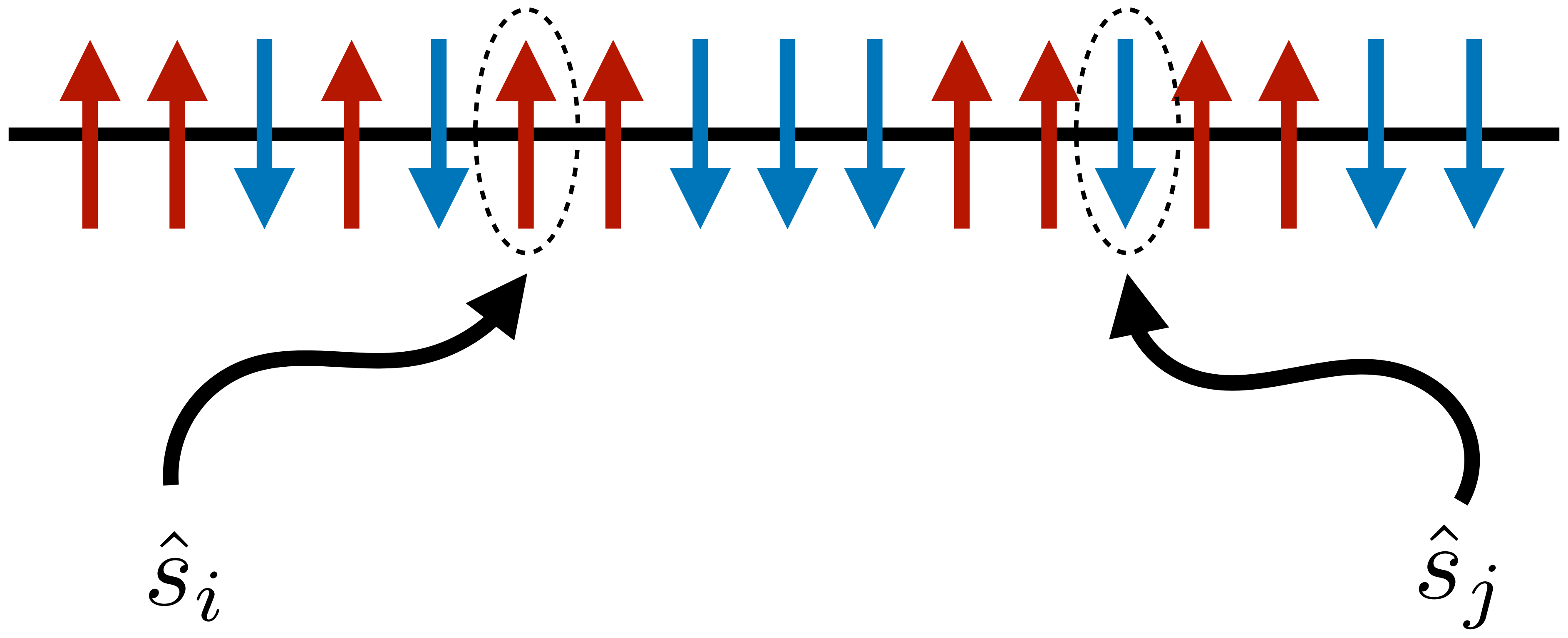
$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_\Psi , \quad \langle \mathcal{N}(n_1)\mathcal{N}(n_2)\rangle_\Psi , \cdots$$

- Disadvantage: not yet suitable for new physics search
- Advantage:
  - Defined from most elementary field theory concept

Basham, Brown, Ellis, Love, PRL, 1978;
H. Chen, I. Moult, X.Y. Zhang, HXZ, PRD, 2020

Spin correlation in Ising v.s. Energy flow correlation at the LHC

Ising model



$\langle \hat{s}_i \hat{s}_j \rangle$ correlator of local operators

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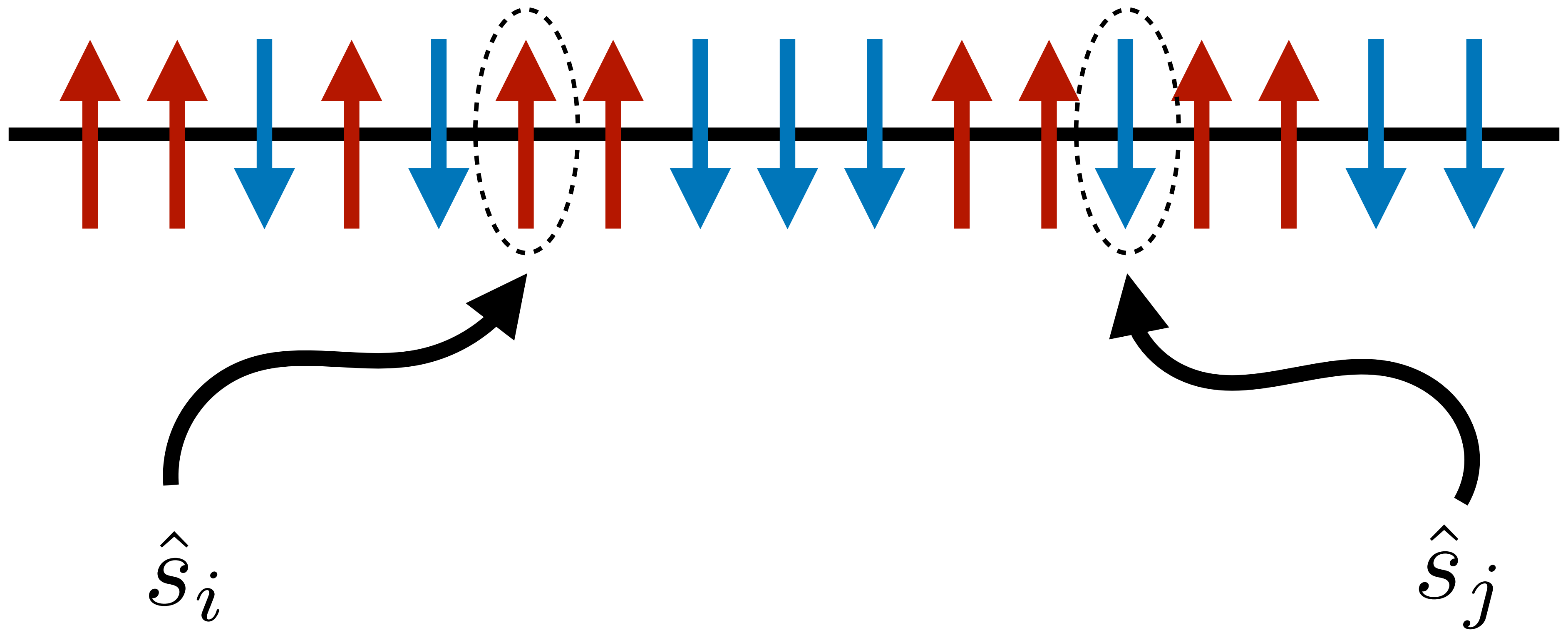

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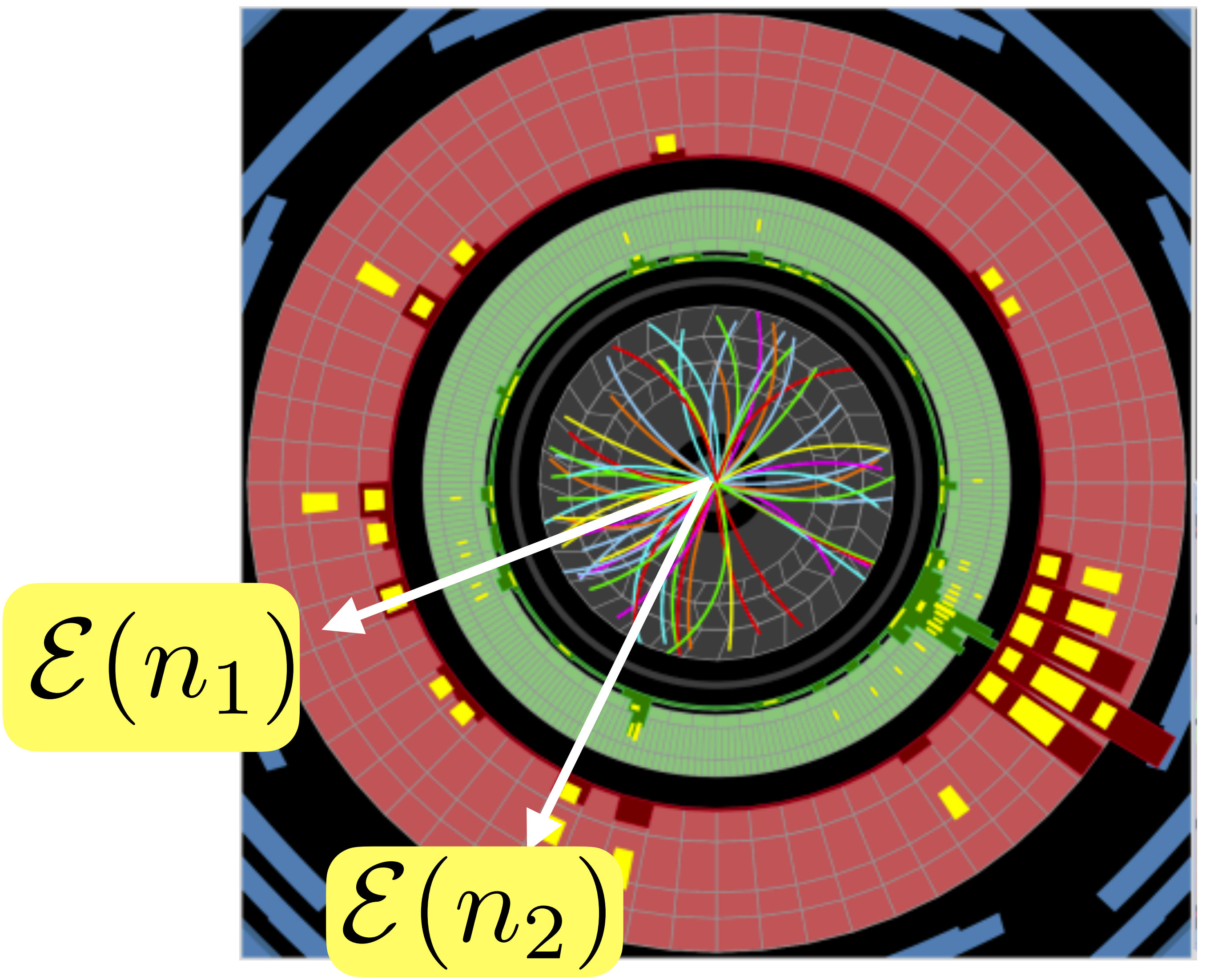
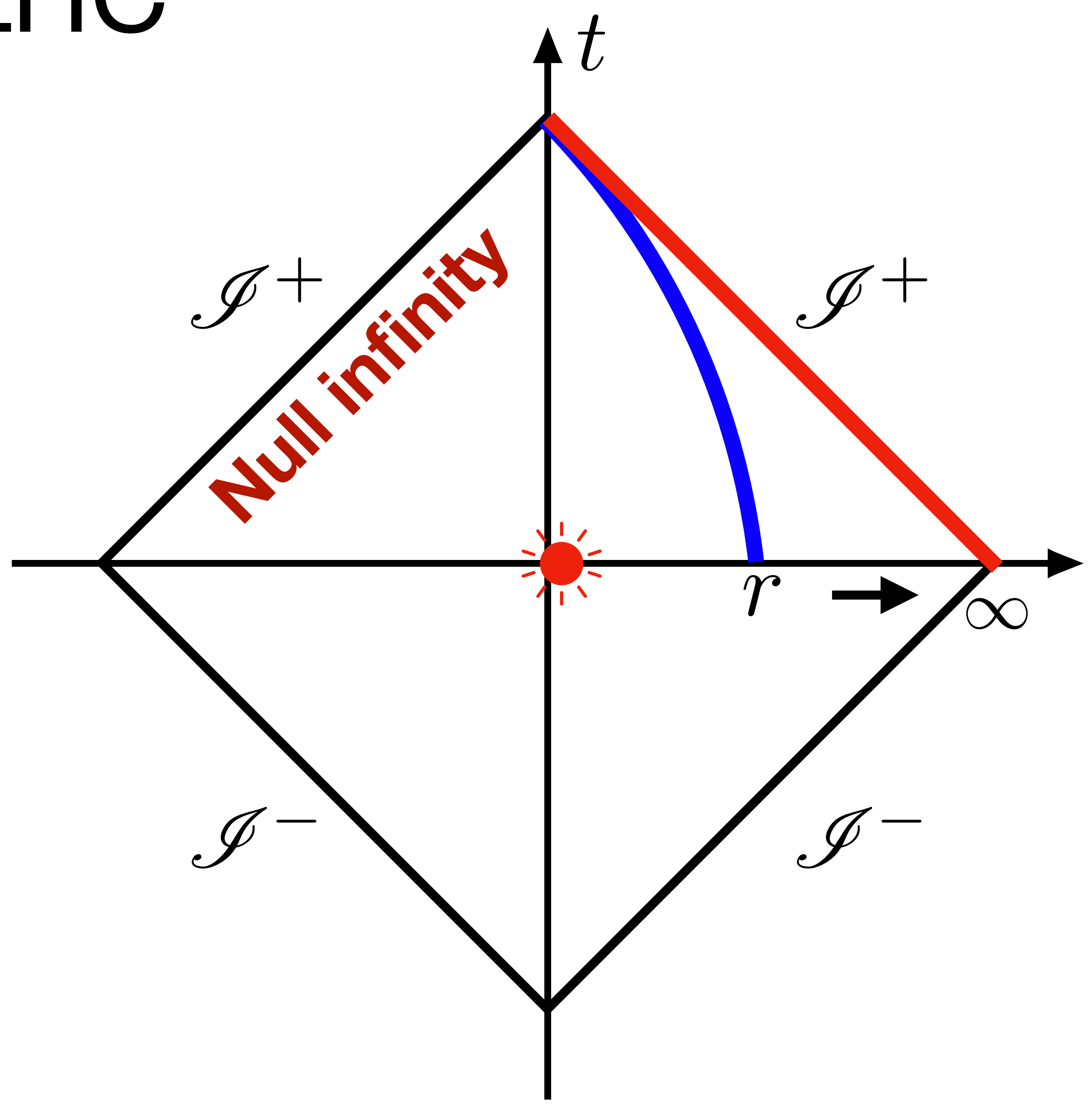
LHC





$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$

Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component

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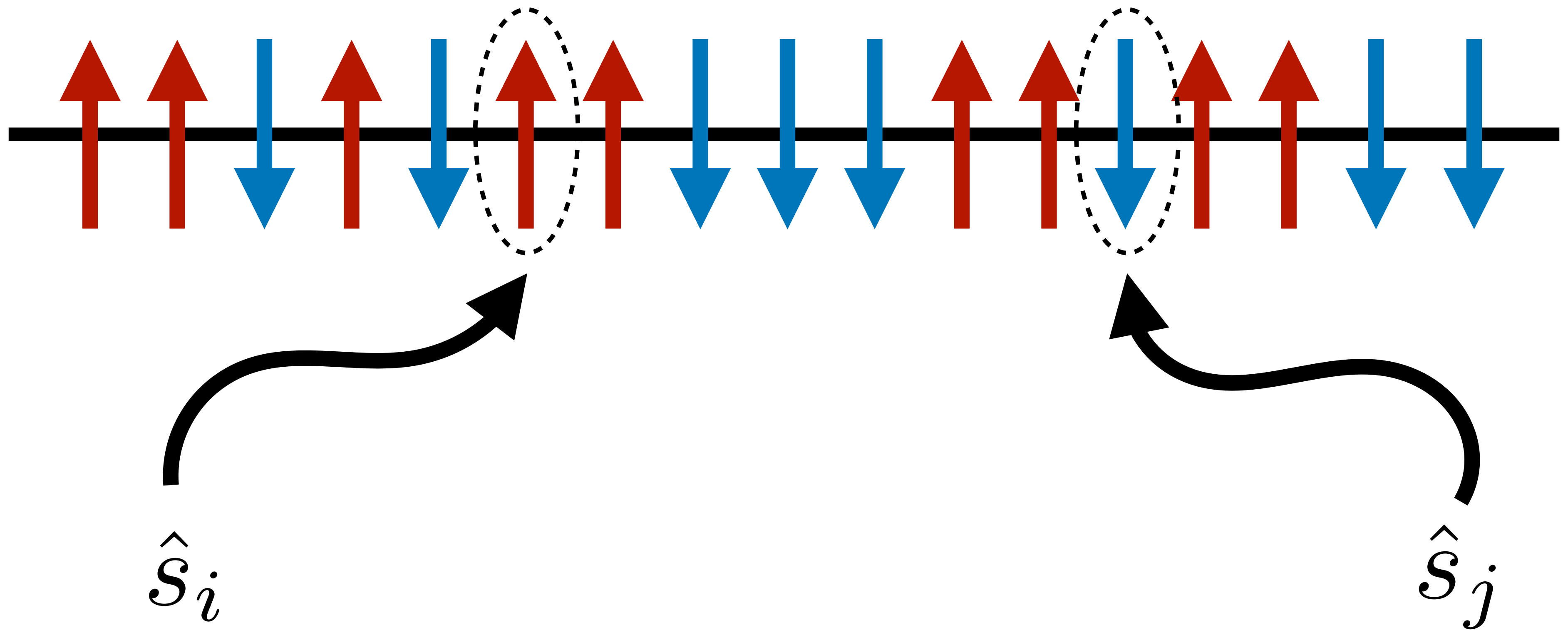

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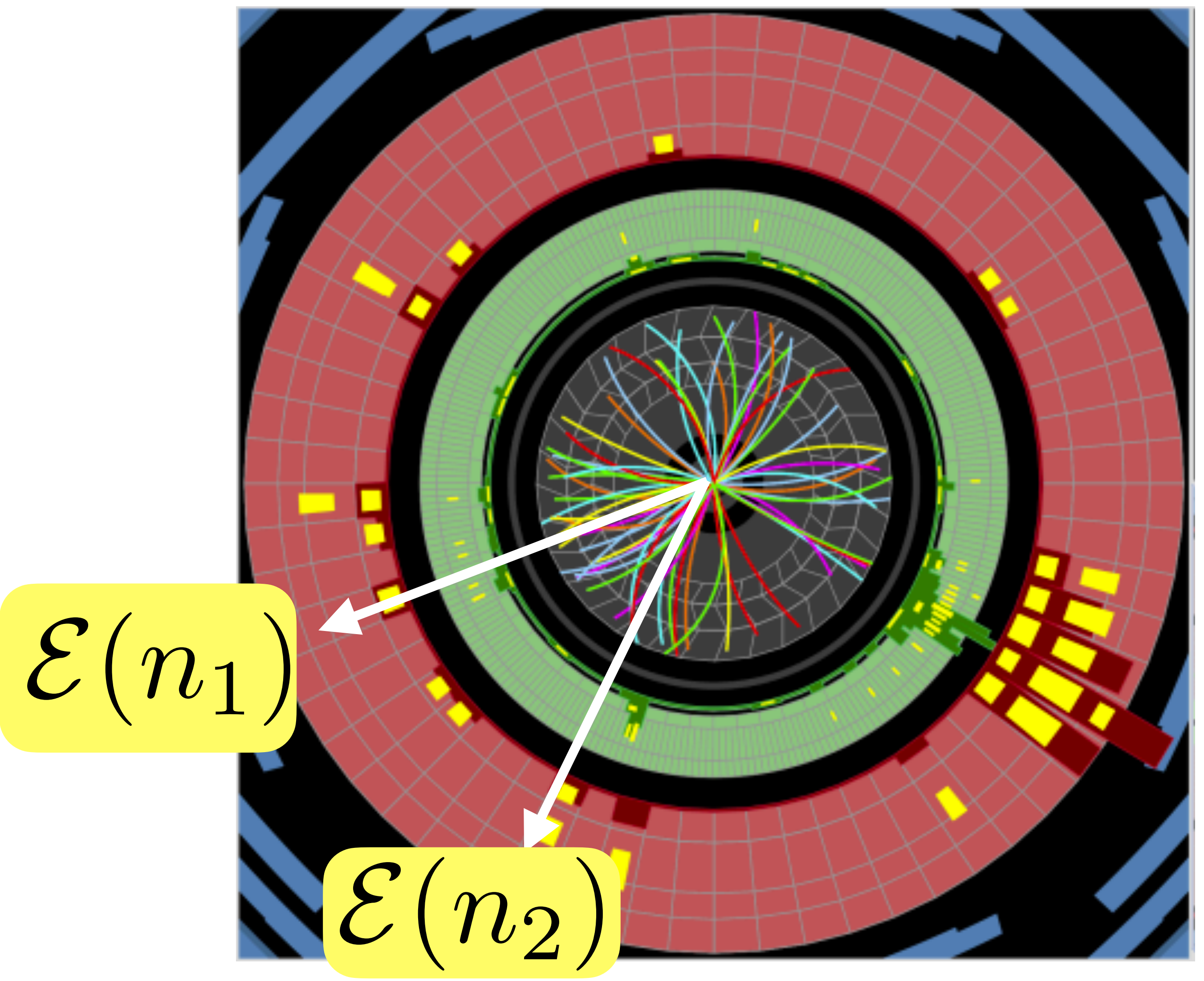
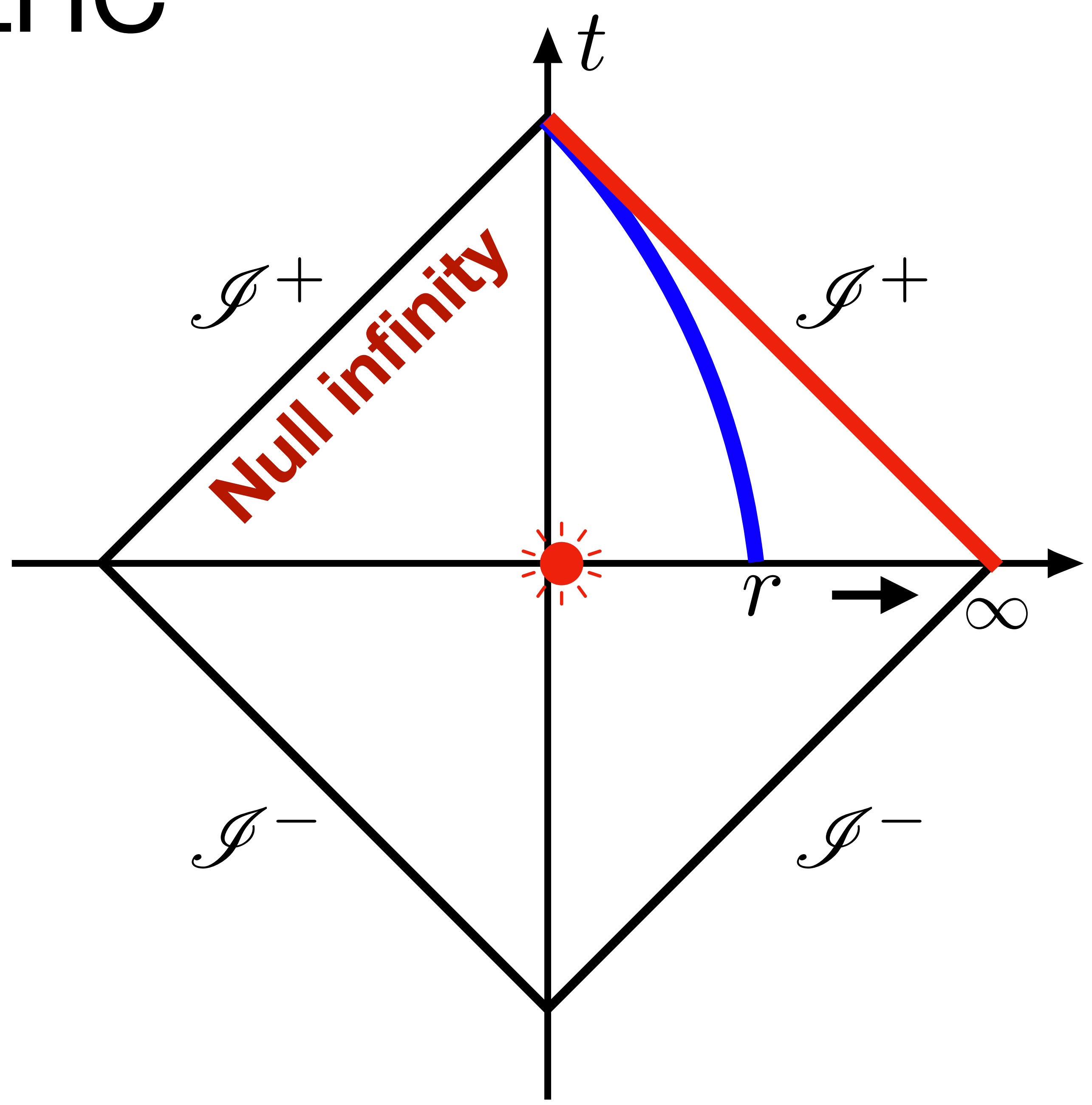
Phase transition and Scaling

$$\langle \hat{s}_i \hat{s}_j \rangle = \begin{cases} \frac{1}{r^\nu} \exp\left(-\frac{r}{\xi}\right) & T > T_c \\ \frac{1}{r^\nu} & T \sim T_c \end{cases}$$

LHC





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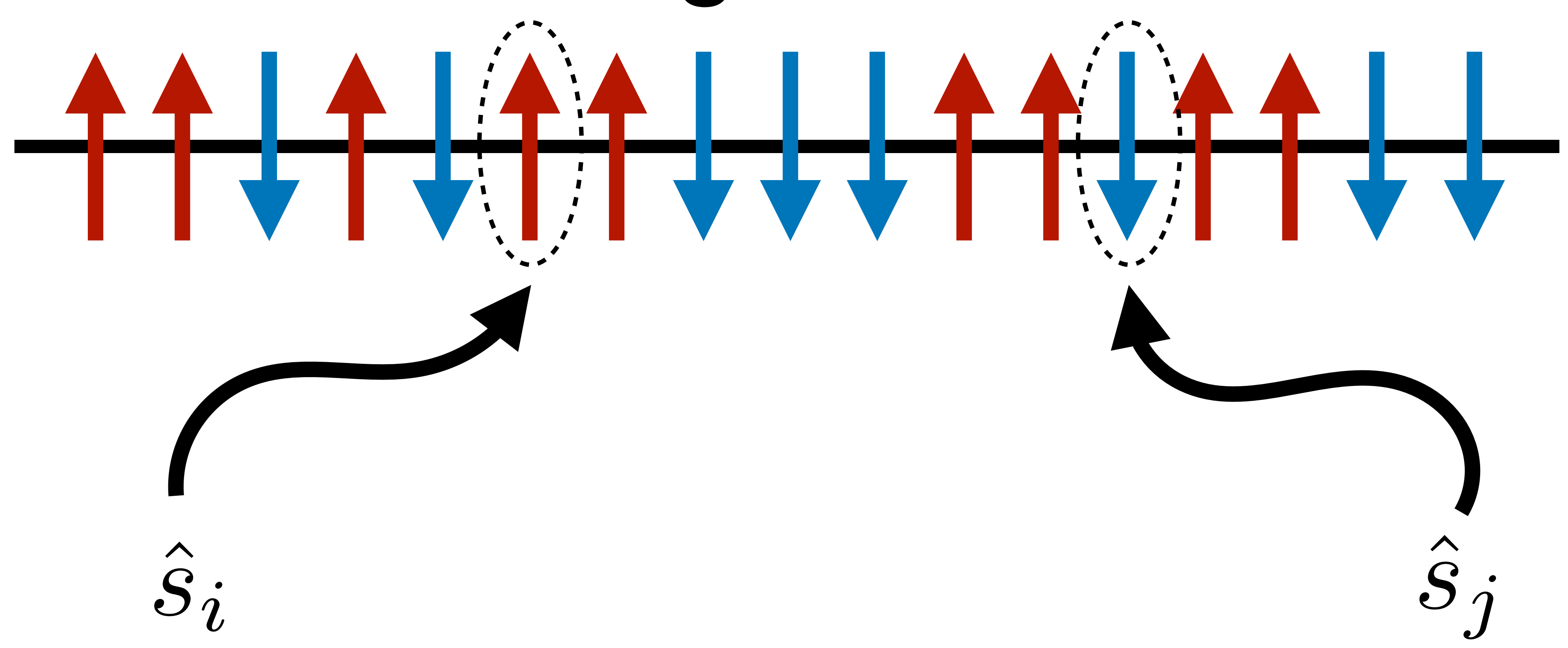

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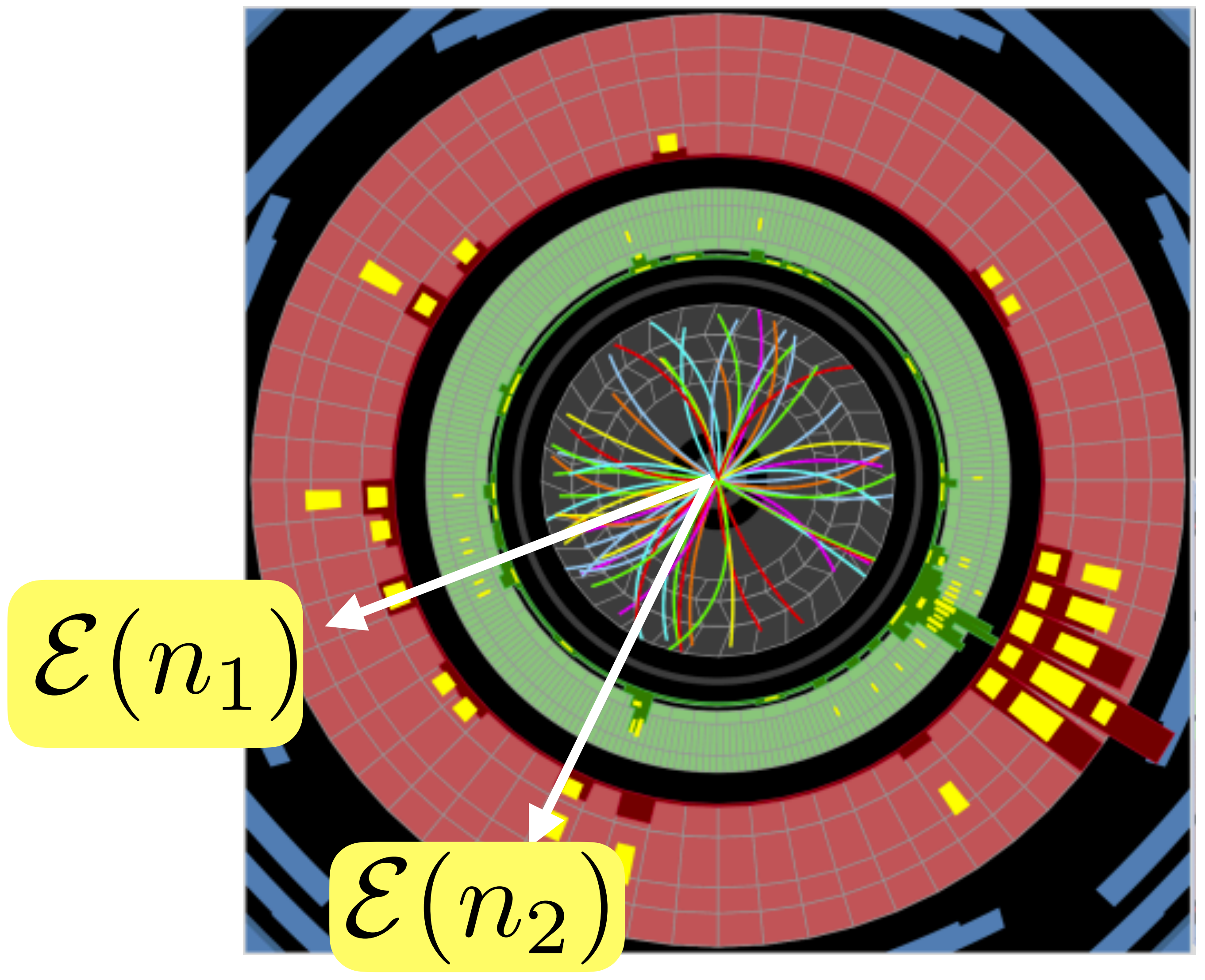

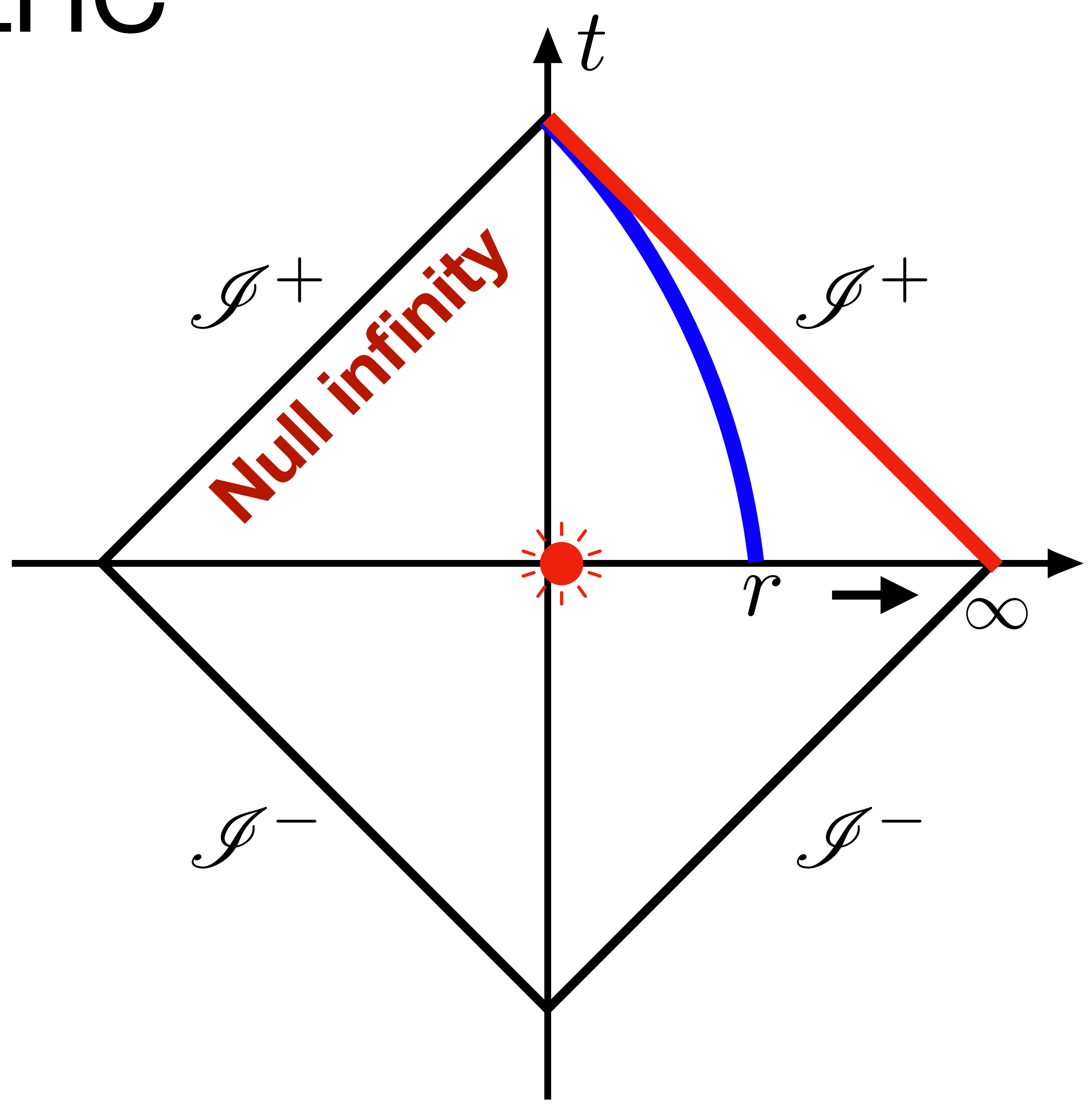

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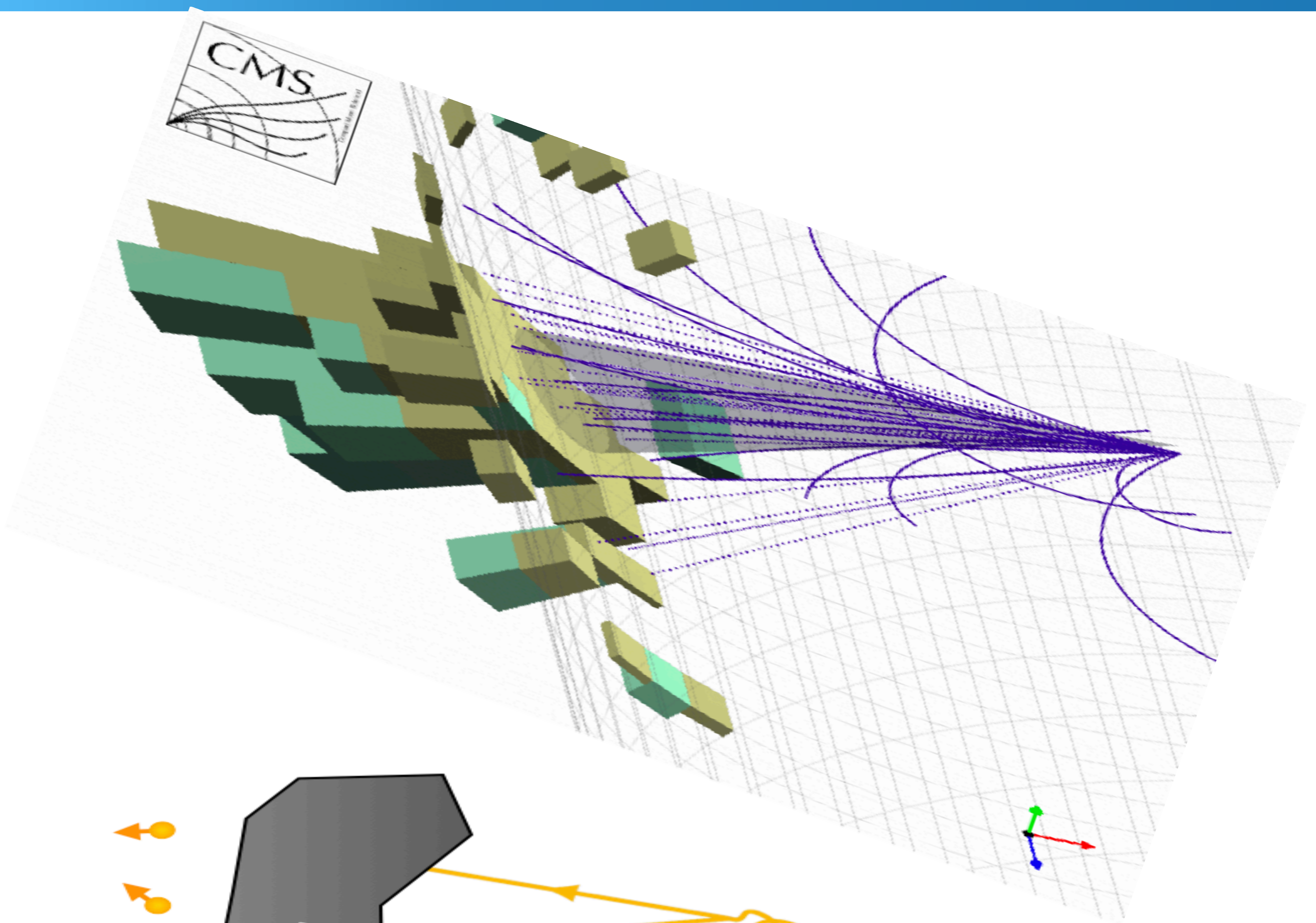
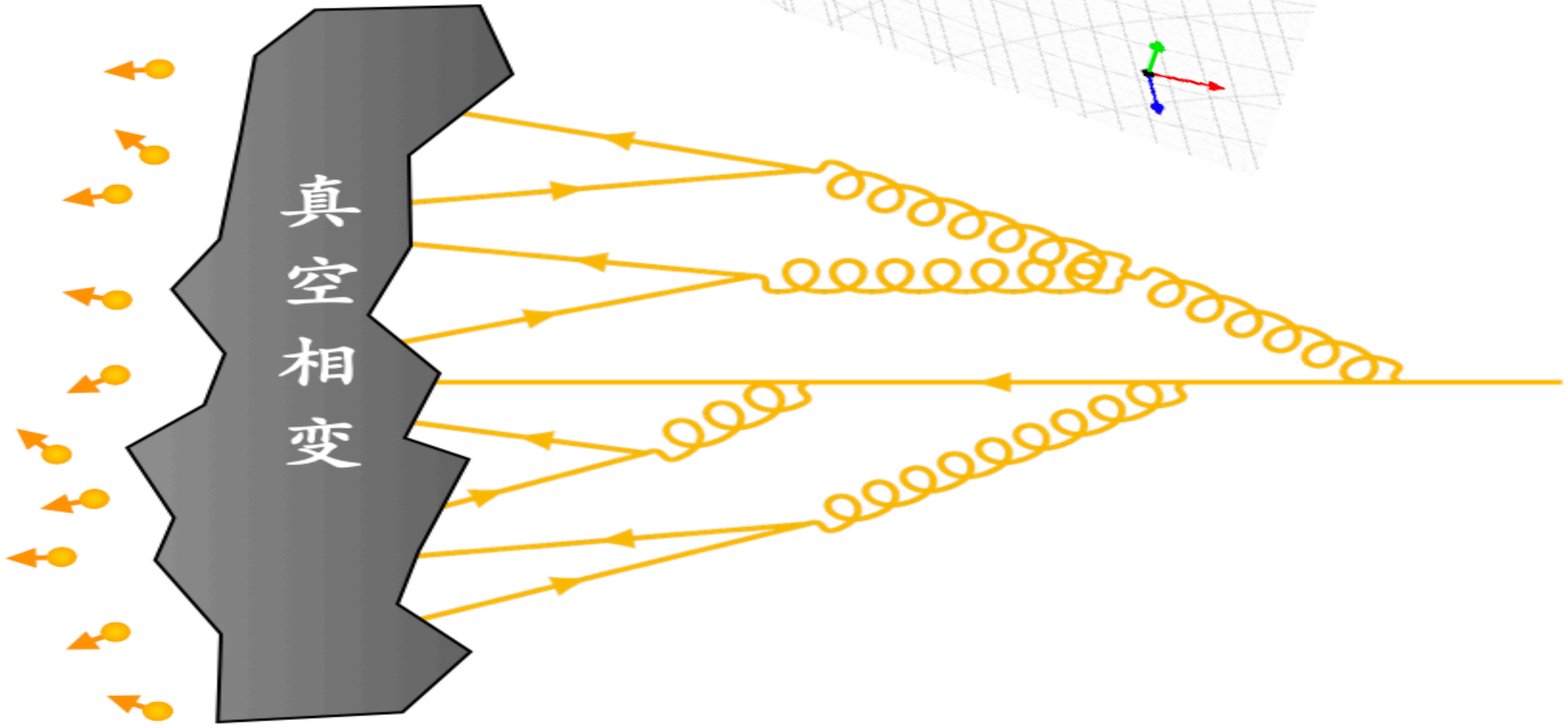
What information is encoded in energy flow correlation?

Seeing hadronization phase transition from EEC





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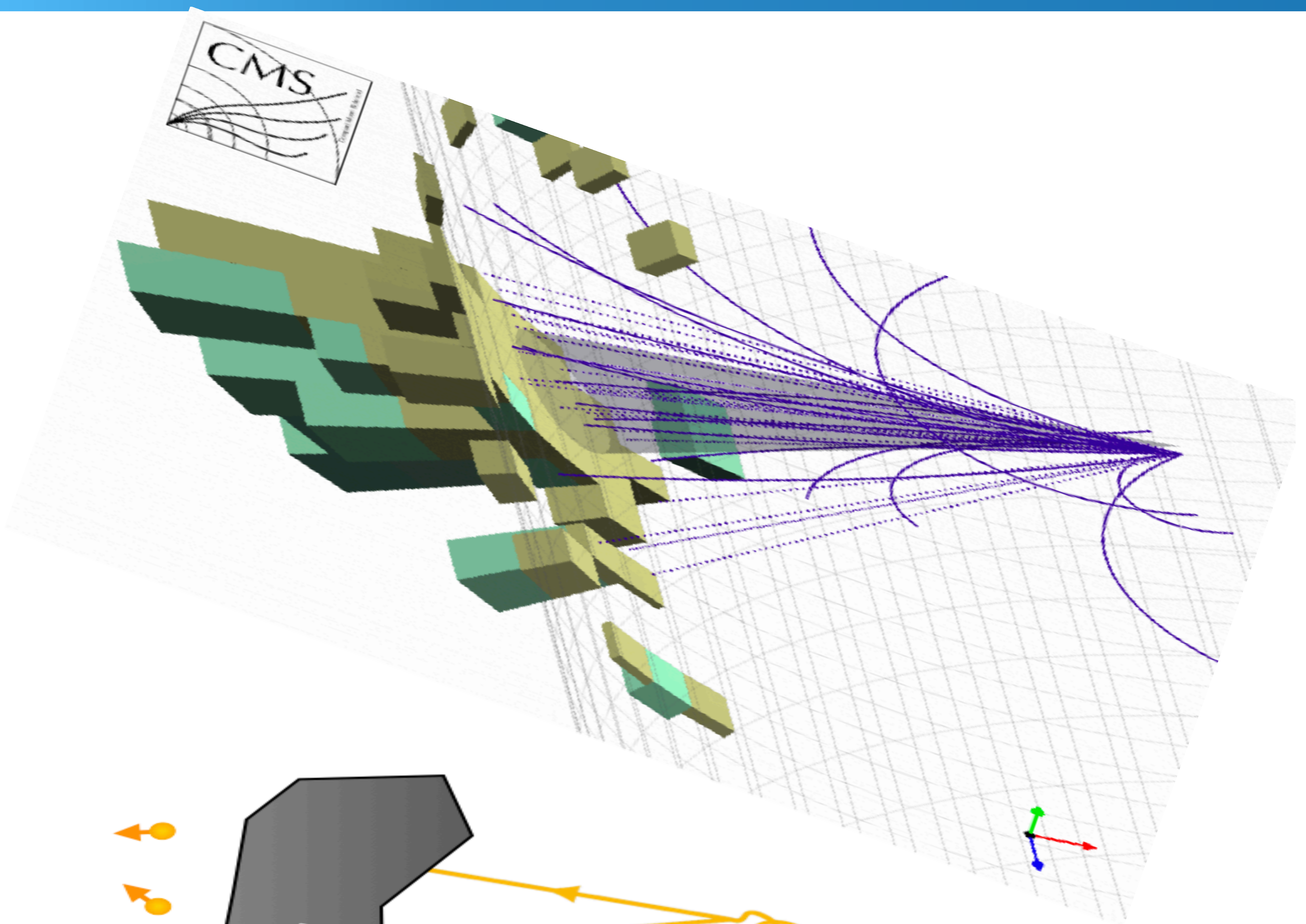

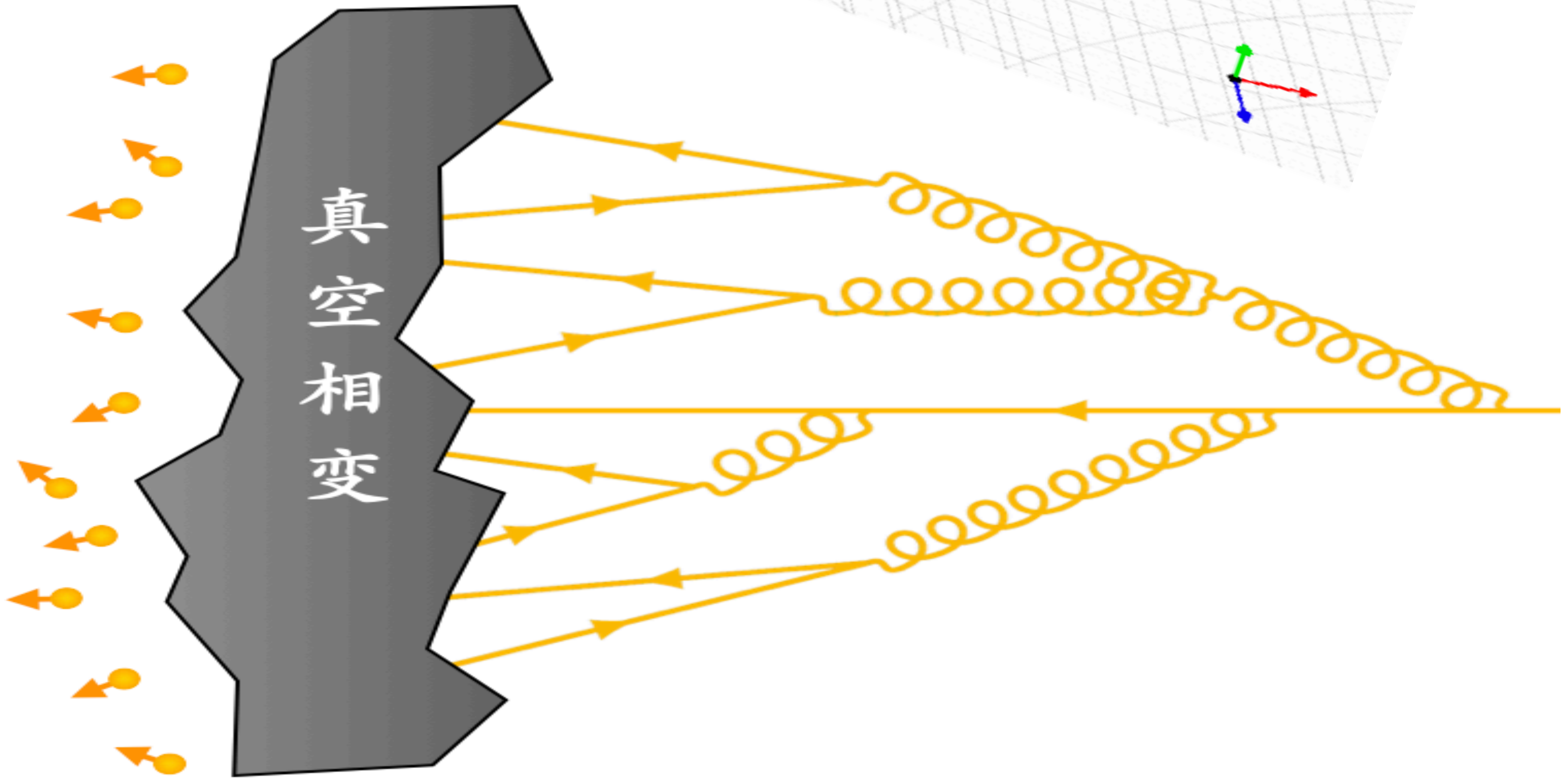

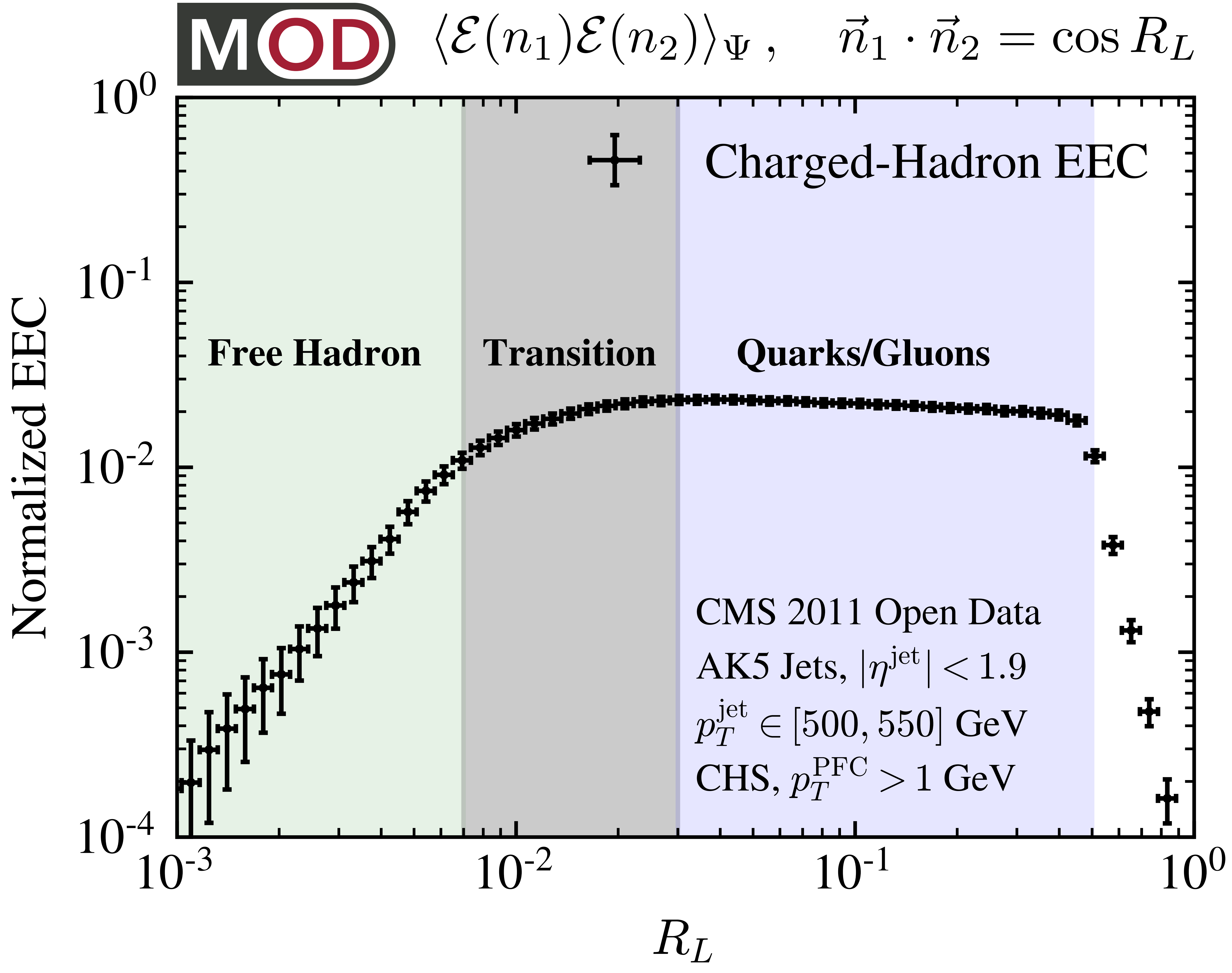

P. Komiske, I. Moult, J. Thaler, HXZ, 2201.07800

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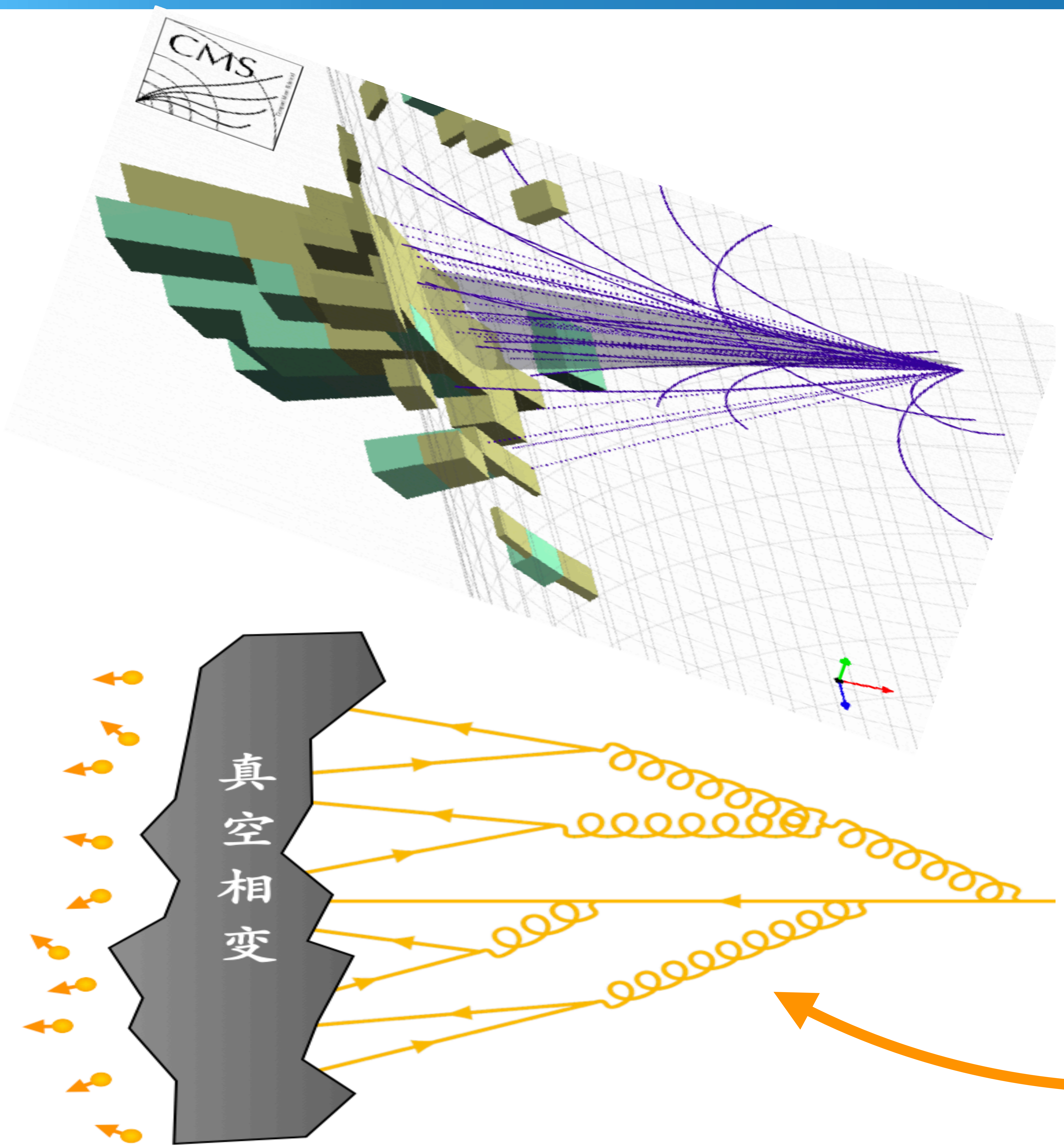

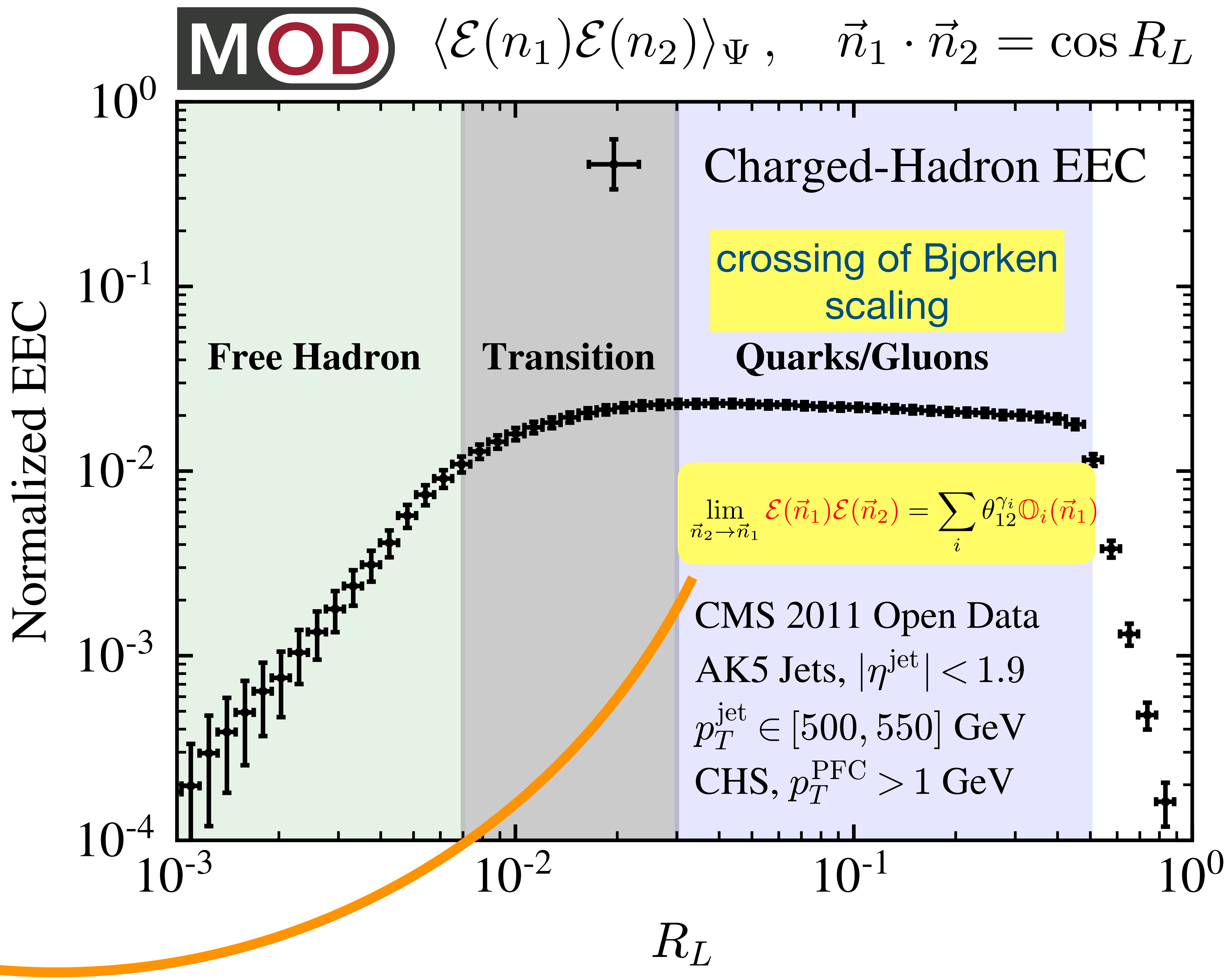

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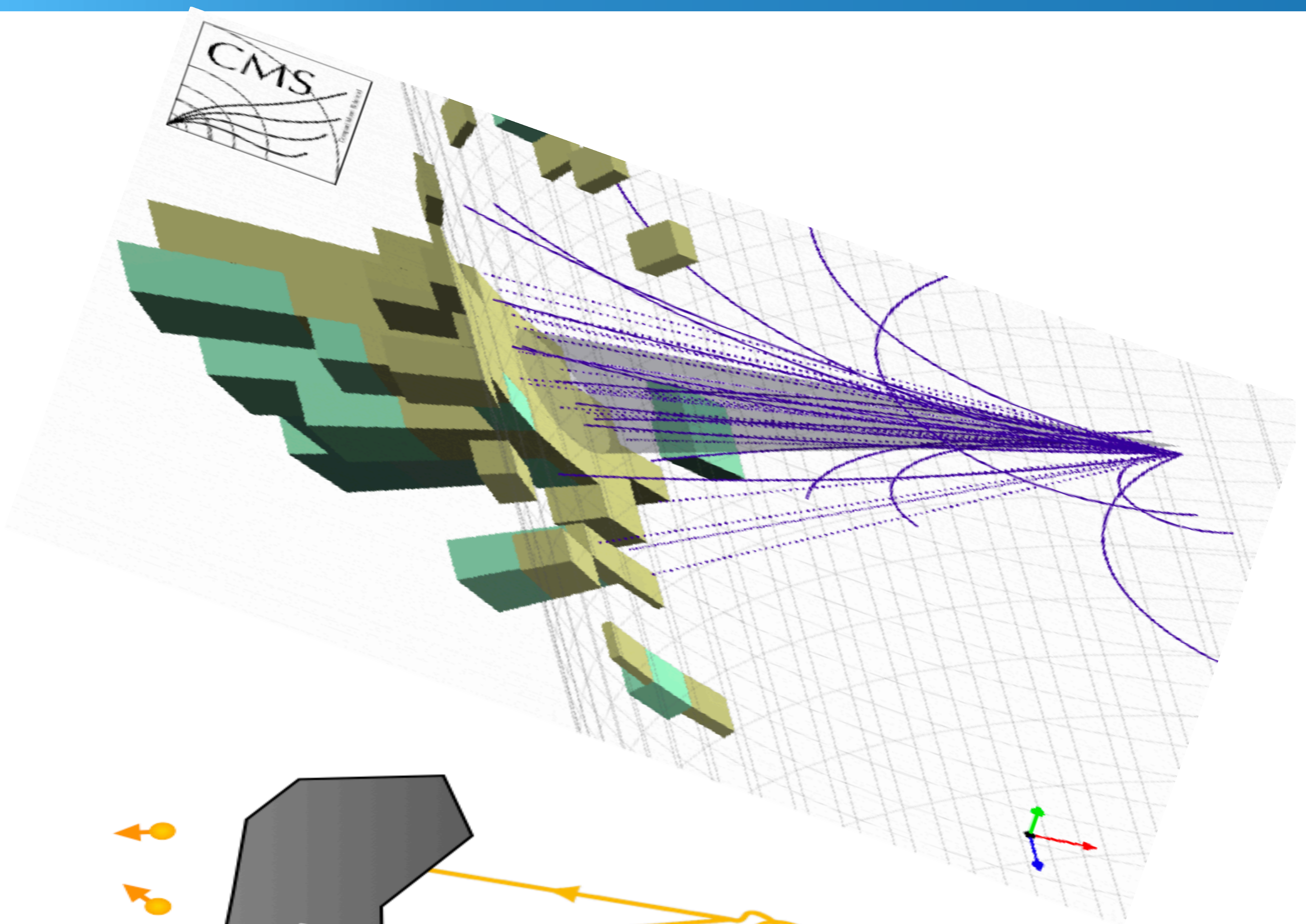

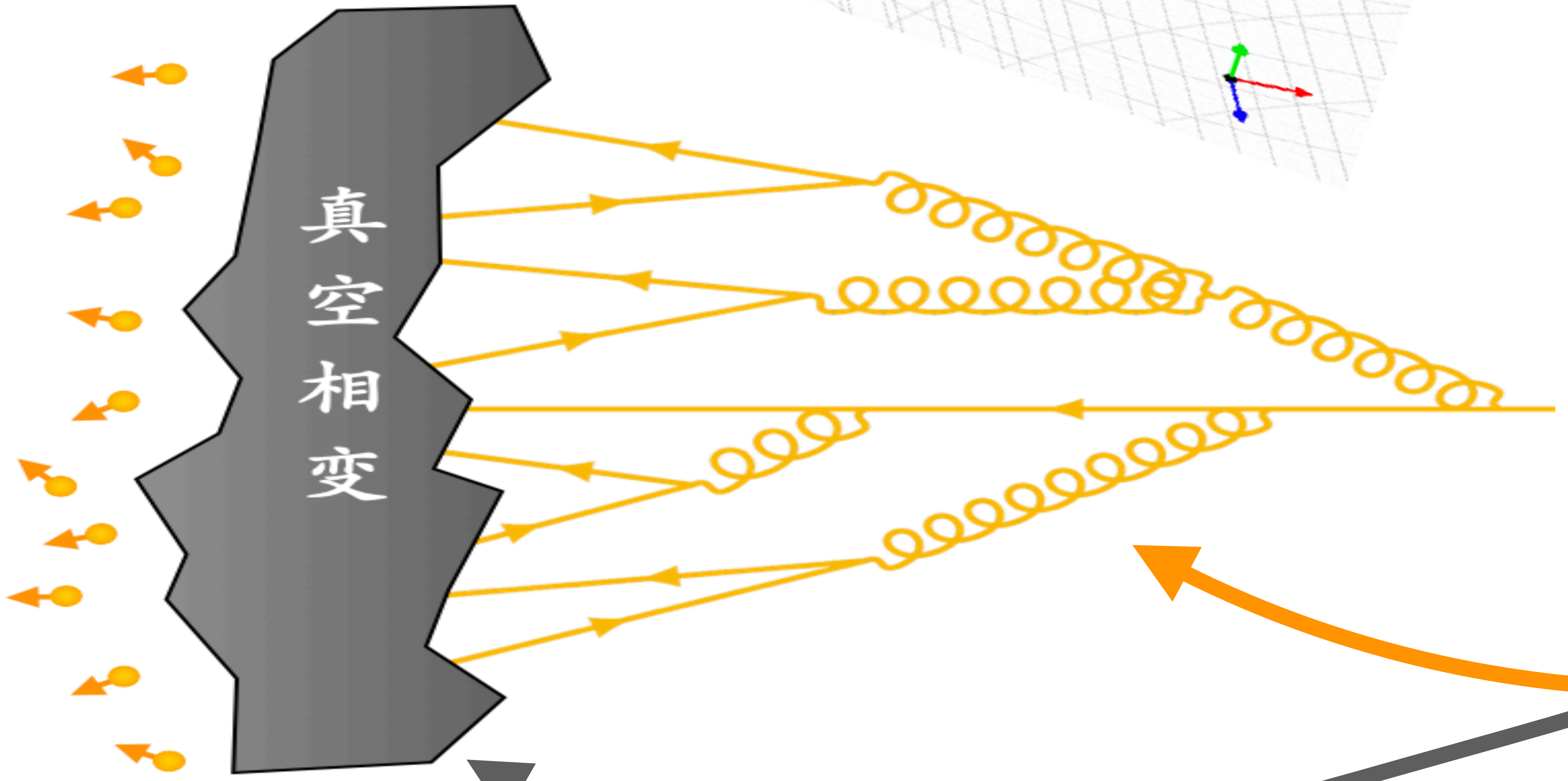

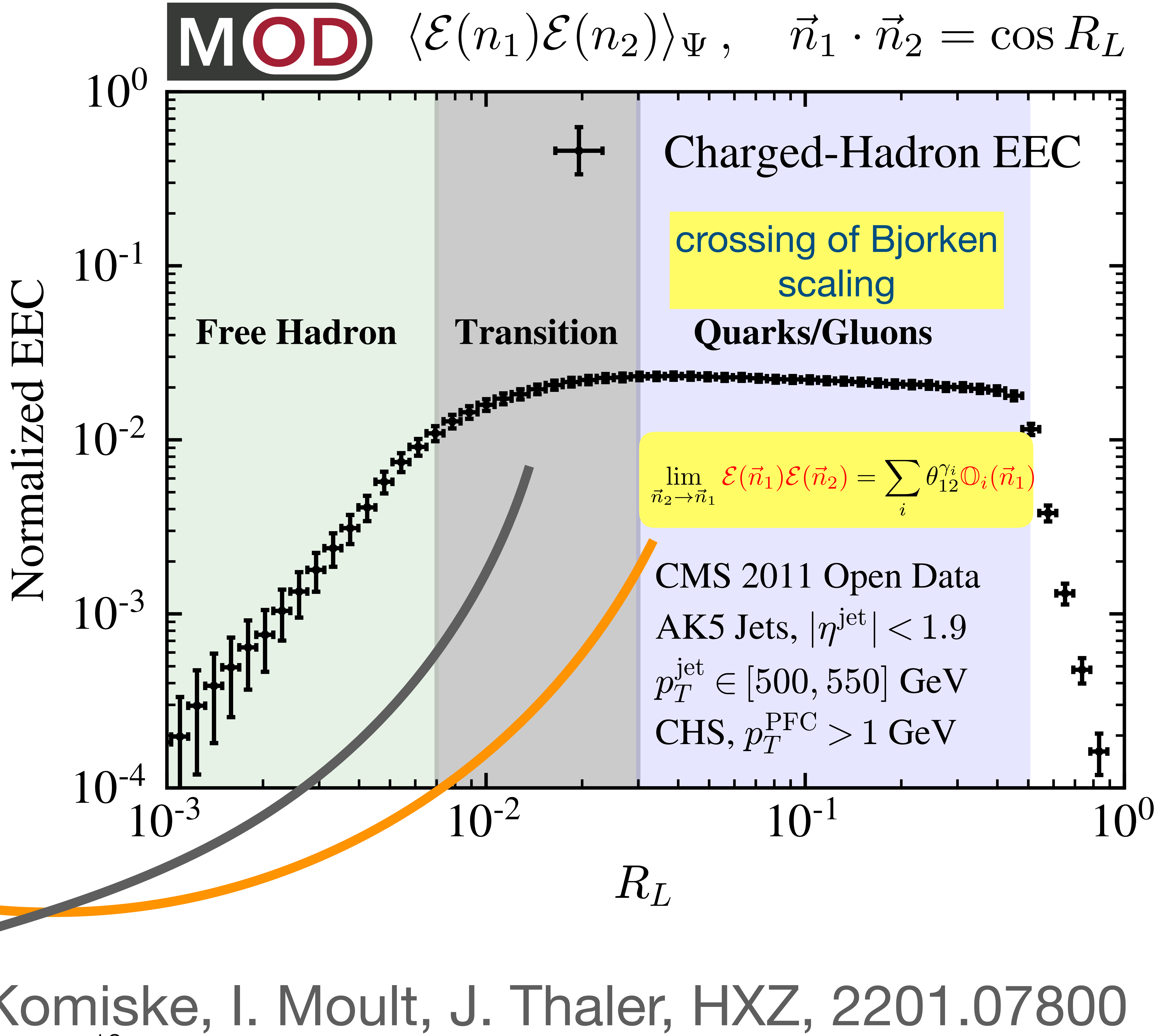

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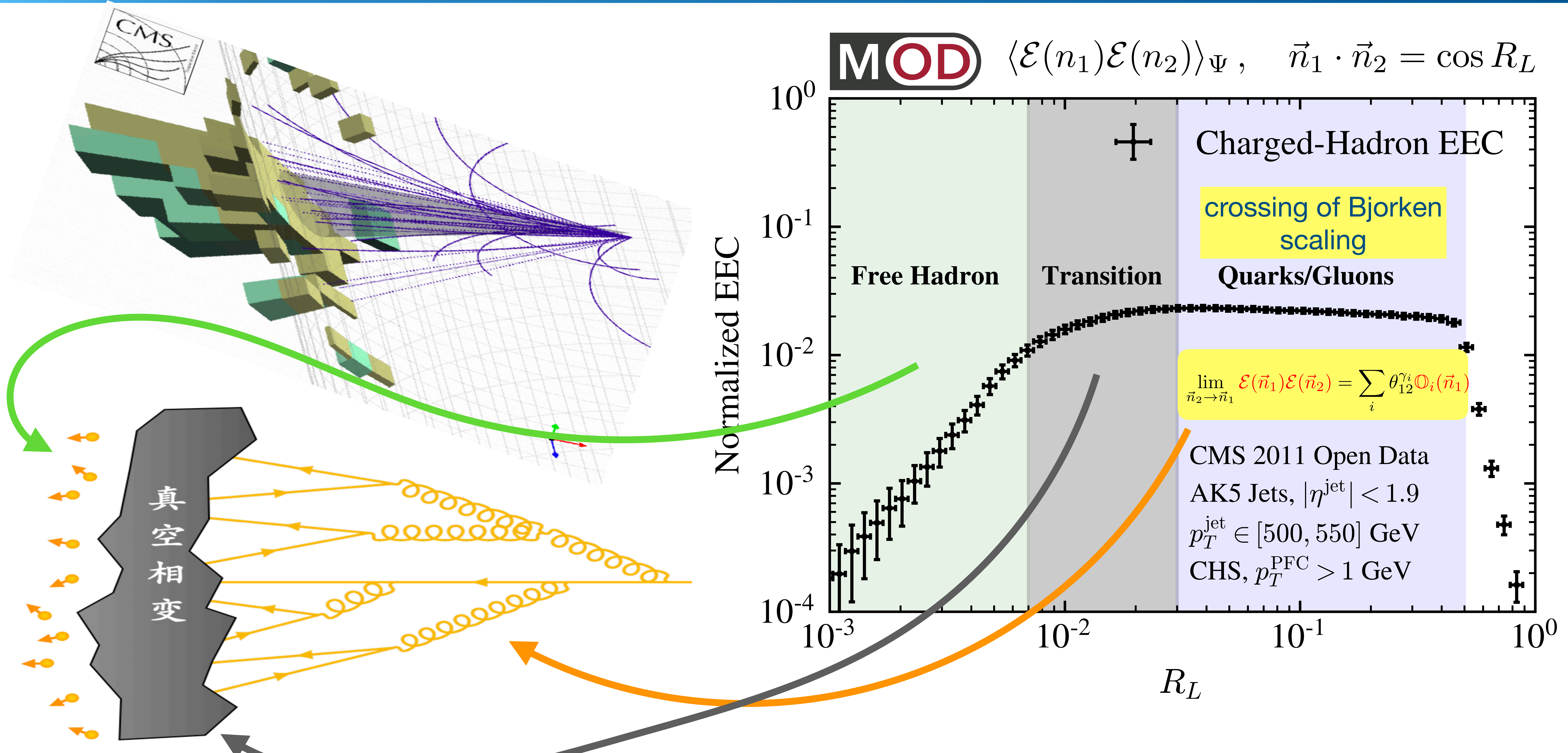

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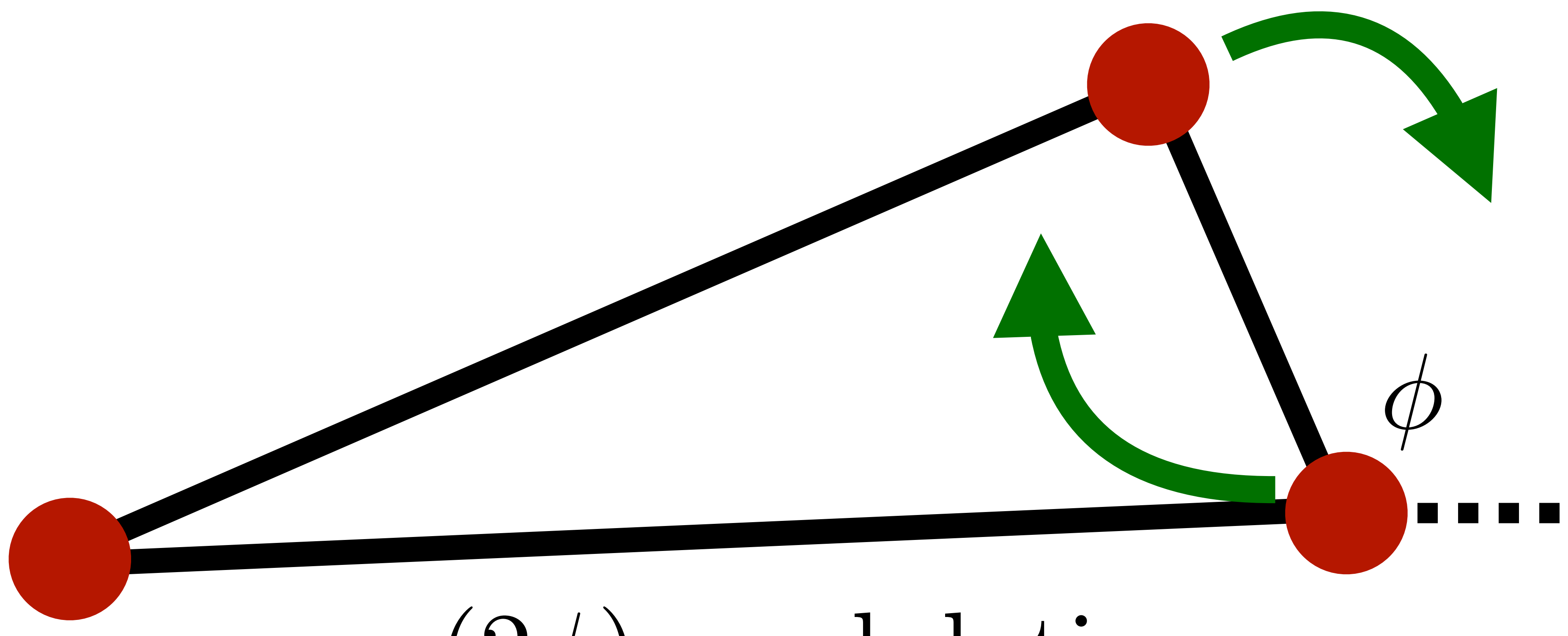
Higher point energy flow correlation

Three-point energy flow correlation

- Arbitrary select three energy detectors
- Measures their energy weighted cross section
- Iterate over all possible detector selection, and average over all scatterings

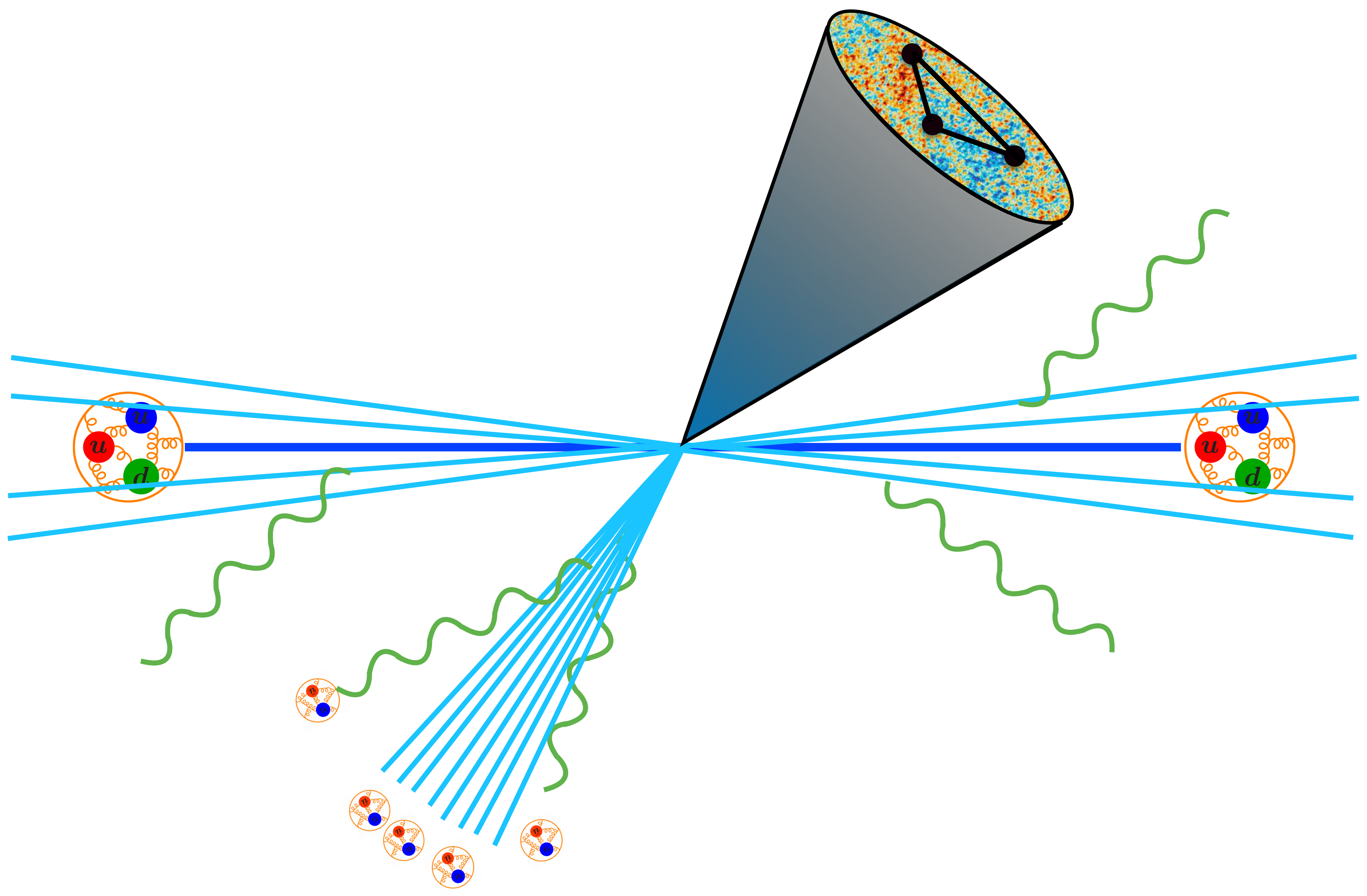
Simplest generalization beyond DGLAP

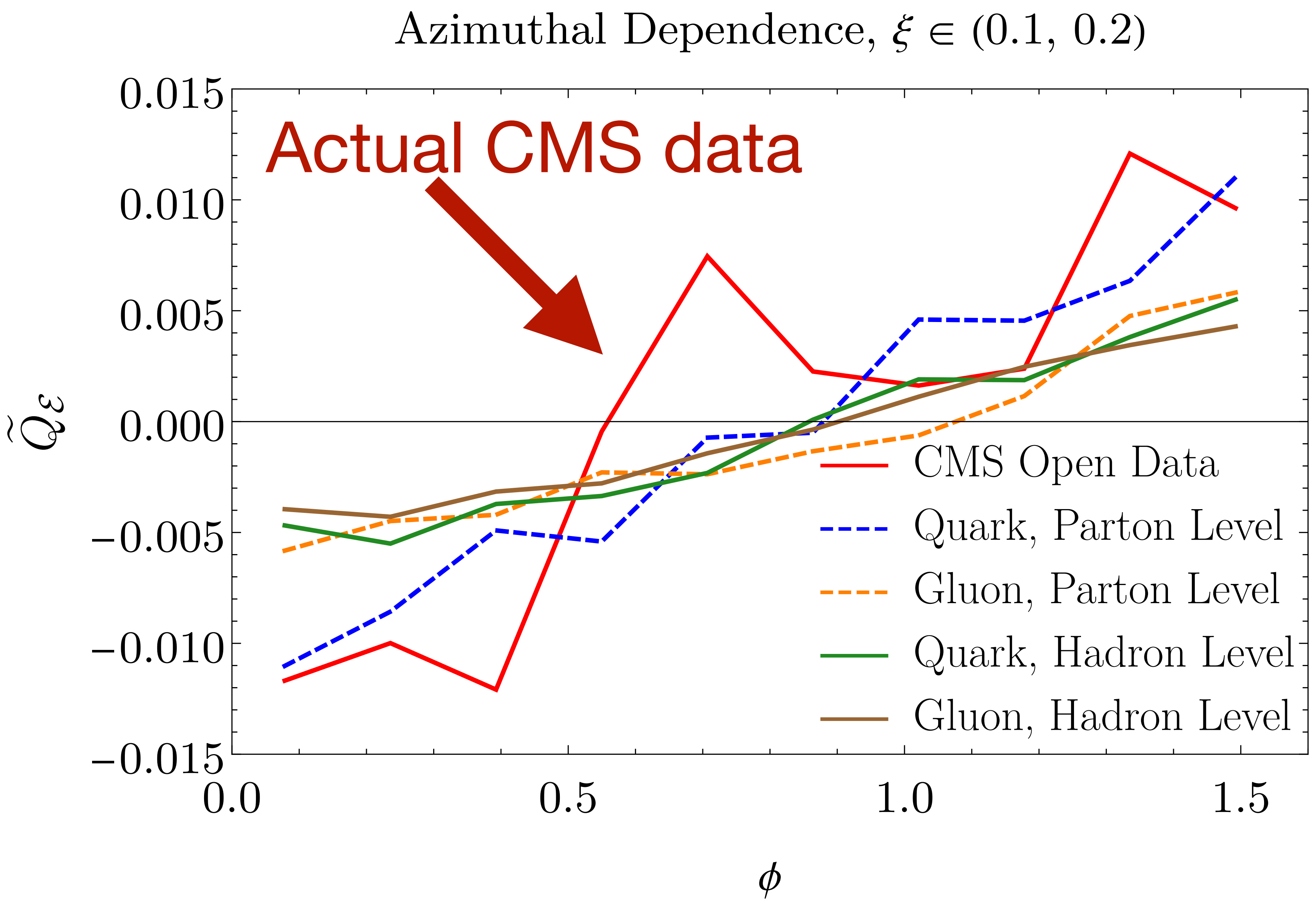
Application: gluon transverse spin



$\cos(2\phi)$ modulation

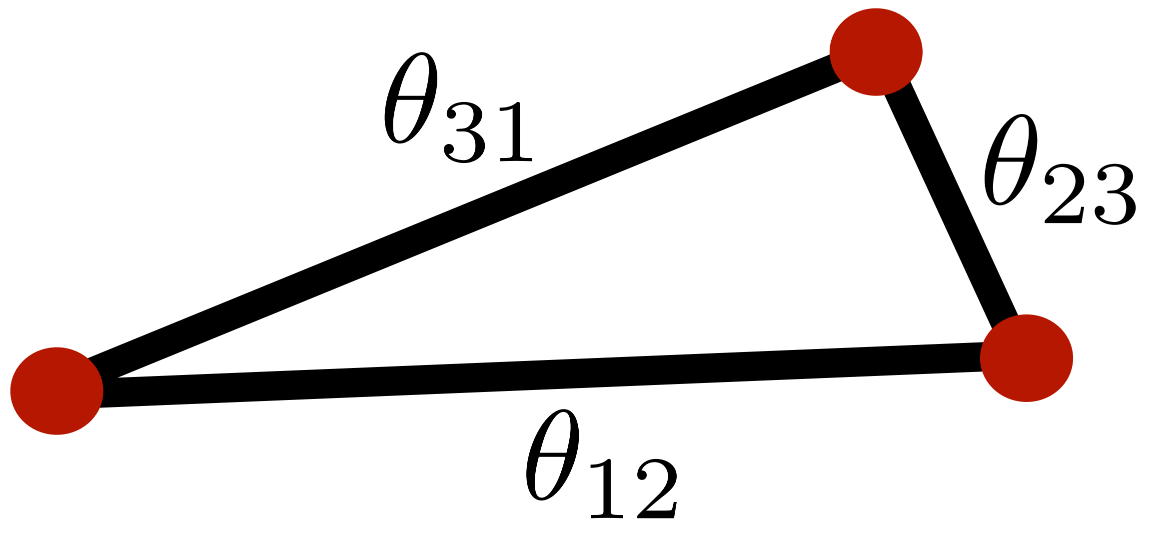
H. Chen, I. Moult, HXZ, PRL, 2021





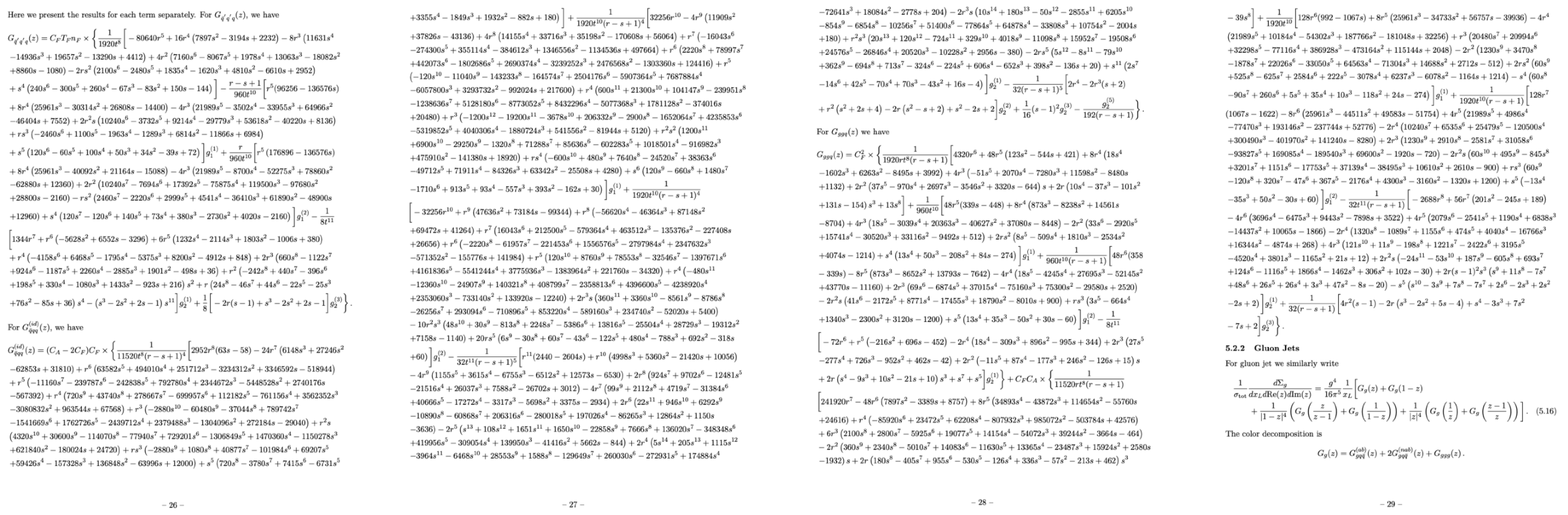
H. Chen, I. Moult, J. Thaler, HXZ, JHEP, 2022

Three-point energy flow correlation from perturbation theory



$$G(\theta_{12}, \theta_{23}, \theta_{31})$$
$$= G(\theta_{12}, z)$$

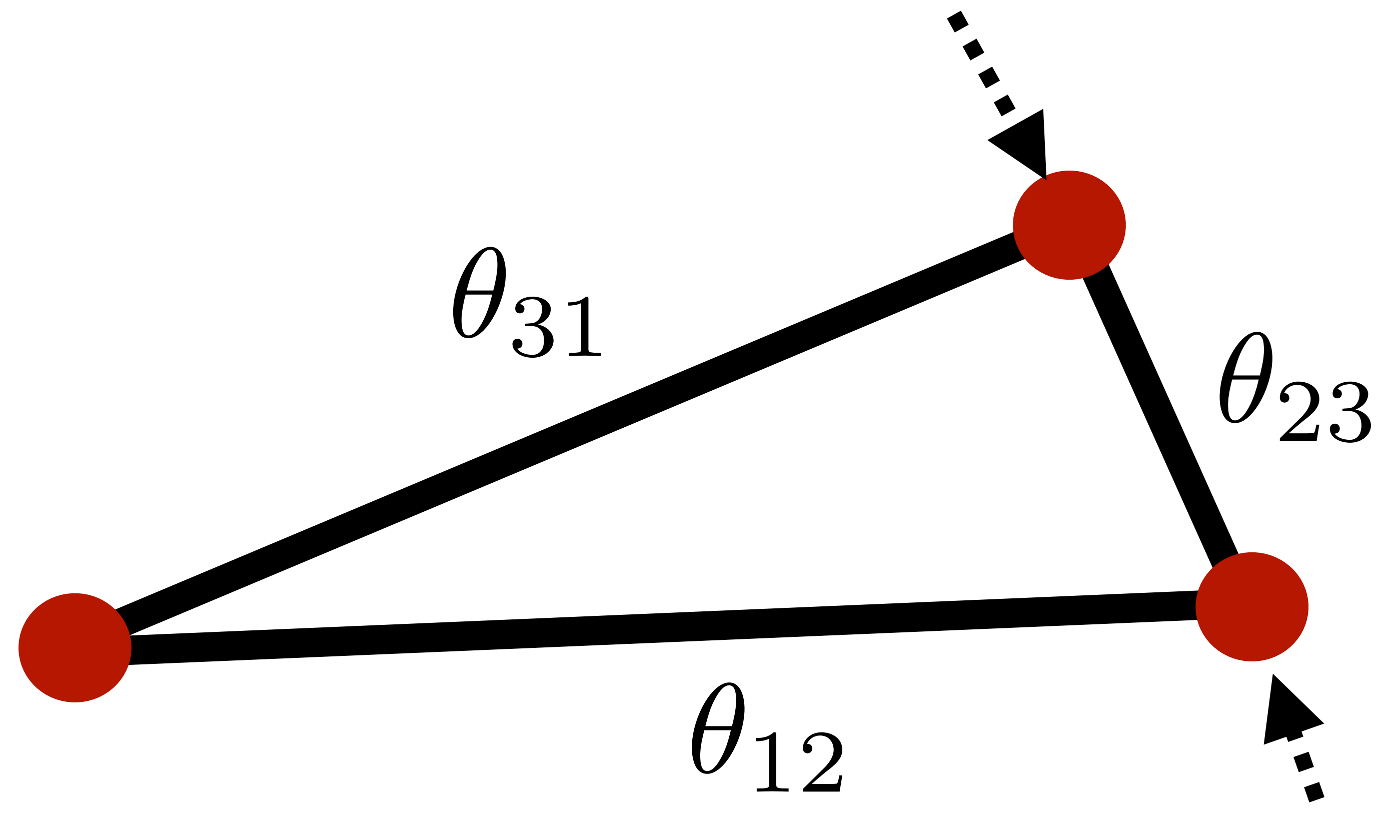
$$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \quad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$$



- Analytic available perturbative prediction at leading order
- Large logarithms in perturbative theory $\alpha_s^n \log^n(\theta_{23}/\theta_{12})$
- Need different organization to make underlying physics manifest

H. Chen, M.X. Luo, I. Moult, T.Z. Yang, X.Y. Zhang, HXZ, JHEP, 2020

Multi-pole expansion



$$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \qquad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$$

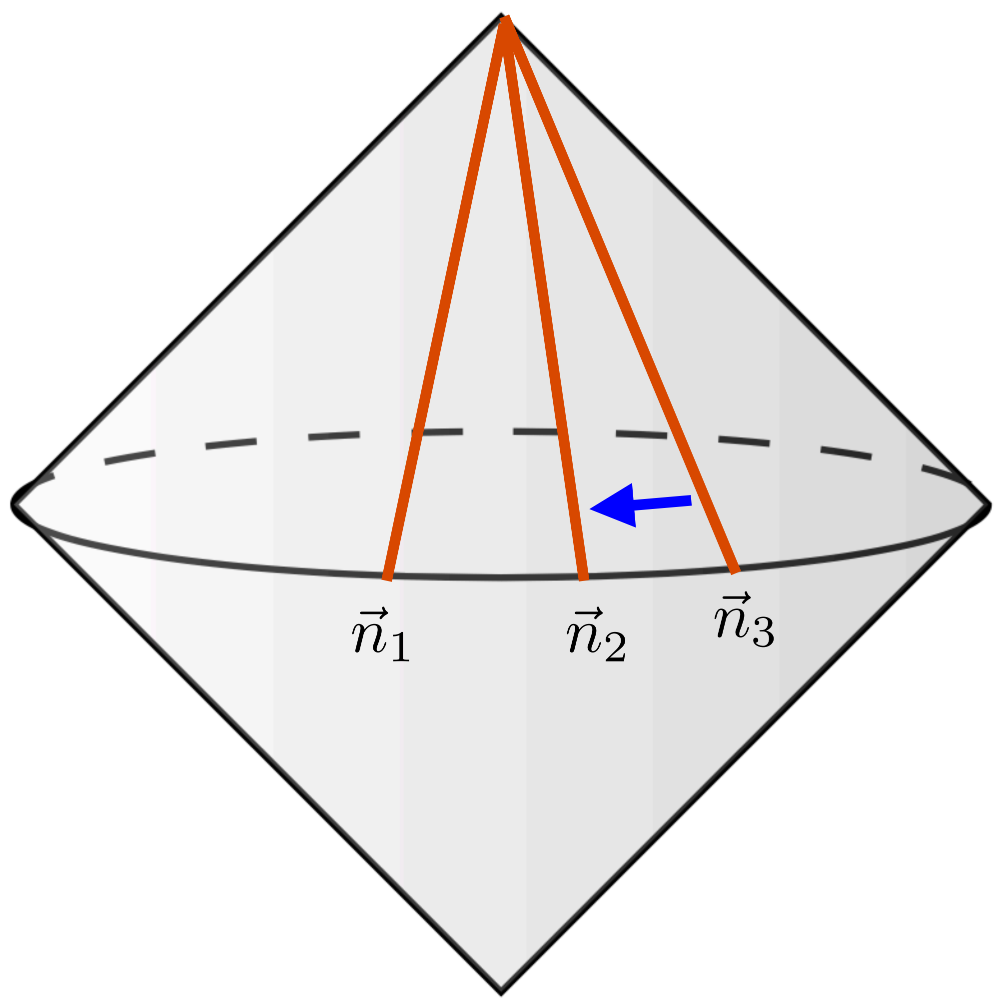
Power expansion in the squeeze limit $\theta_{23} \to 0$ (modulo logs)

$$G(\theta_{12}, z) = \frac{1}{|z|^2} G^{(0)}(\theta_{12}) + G^{(1)}(\theta_{12}) + |z|^2 G^{(2)}(\theta_{12}) + \cdots$$

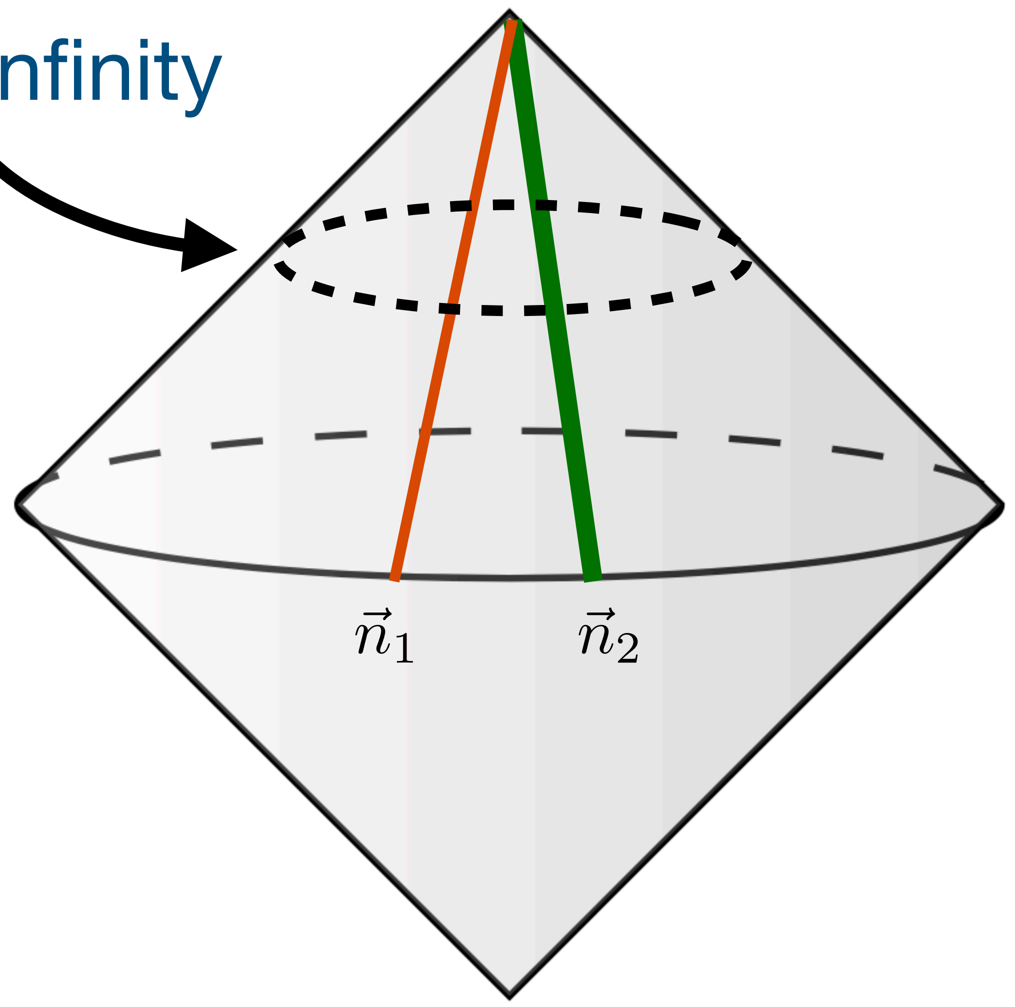
Power expansion of quantum field theory near the lightcone in general is a difficult problem

Lightray OPE and symmetry

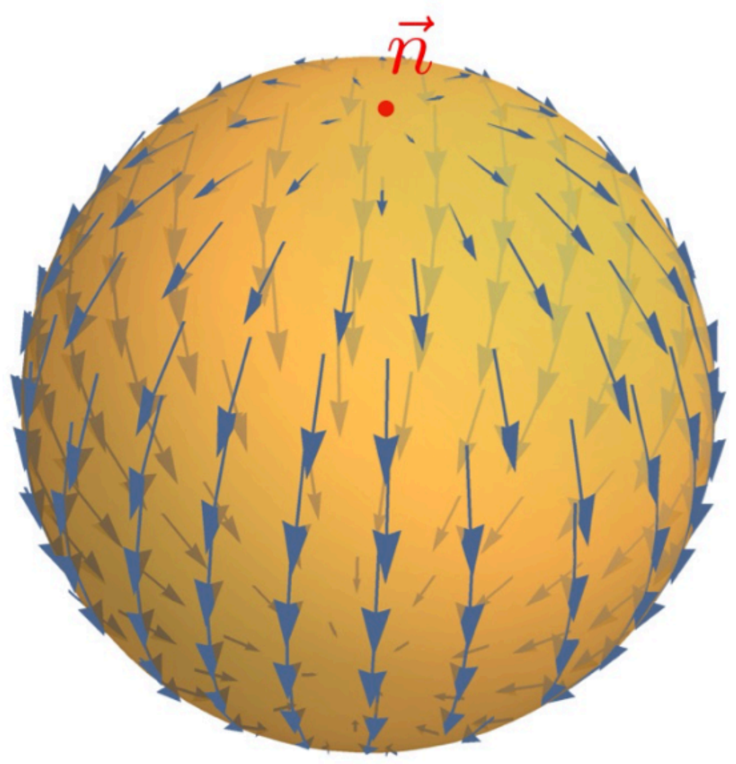
$$G(\theta_{12}, z) = \langle \Psi | \mathcal{E}(n_1)\mathcal{E}(n_2)\mathcal{E}(n_3) | \Psi \rangle = \sum_{\mathbb{O}_{J=3}} \langle \Psi | \mathcal{E}(n_1)\mathbb{O}_{J=3}(n_2) | \Psi \rangle$$



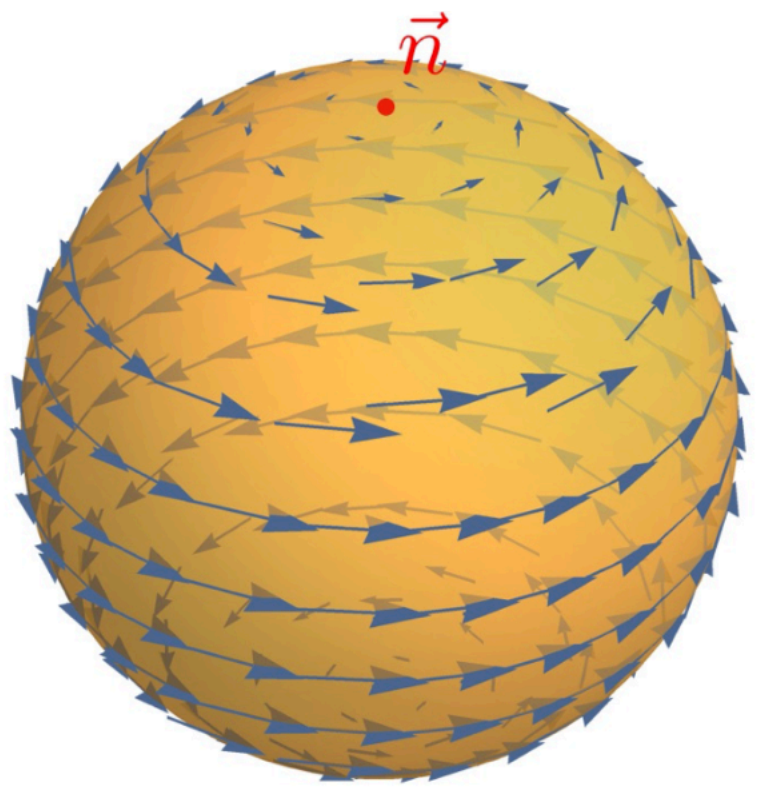
$$= \sum_{\mathbb{O}_{J=3}}$$



Quantum number of the lightray operator



Boost ⇔ Dilation δ

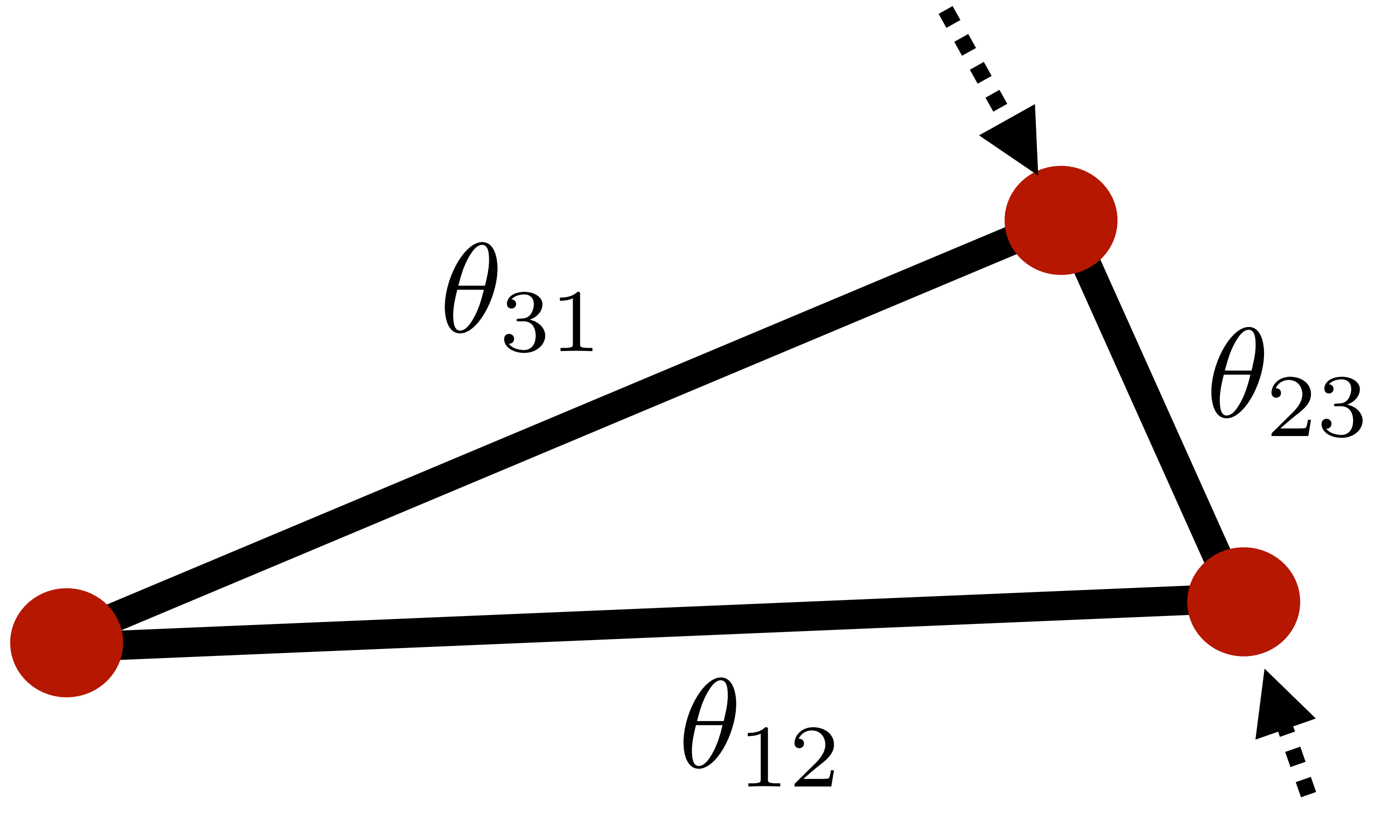


Rotation ⇔ transverse spin j

$$\mathbb{O}^{J=3}_{\delta,j}$$

Lightray operator labeled by dimension δ and spin j

Partial wave of Lorentz group SO(3,1)



$$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \qquad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$$

$$\begin{aligned} & G(\theta_{12}, z) \\ =& \langle \Psi | \mathcal{E}(n_1)\mathcal{E}(n_2)\mathcal{E}(n_3)|\Psi\rangle \\ =& \sum_{\mathbb{O}_{J=3}} \langle \Psi | \mathcal{E}(n_1)\mathbb{O}_{J=3}(n_2)|\Psi\rangle = G(\theta_{12}) \sum_{\delta,j} g_{\delta,j}(z) \end{aligned}$$

Lorentz invariance of individual partial wave

$$\mathcal{C}_2(\partial_z, \partial_{\bar{z}}) g_{\delta,j}(z) = \lambda_{\delta,j} g_{\delta,j}(z)$$

Quadratic Casimir

Eigenvalue

$$2z^2(z-1)\partial_z^2 + 2\bar{z}^2(\bar{z}-1)\partial_{\bar{z}}^2$$

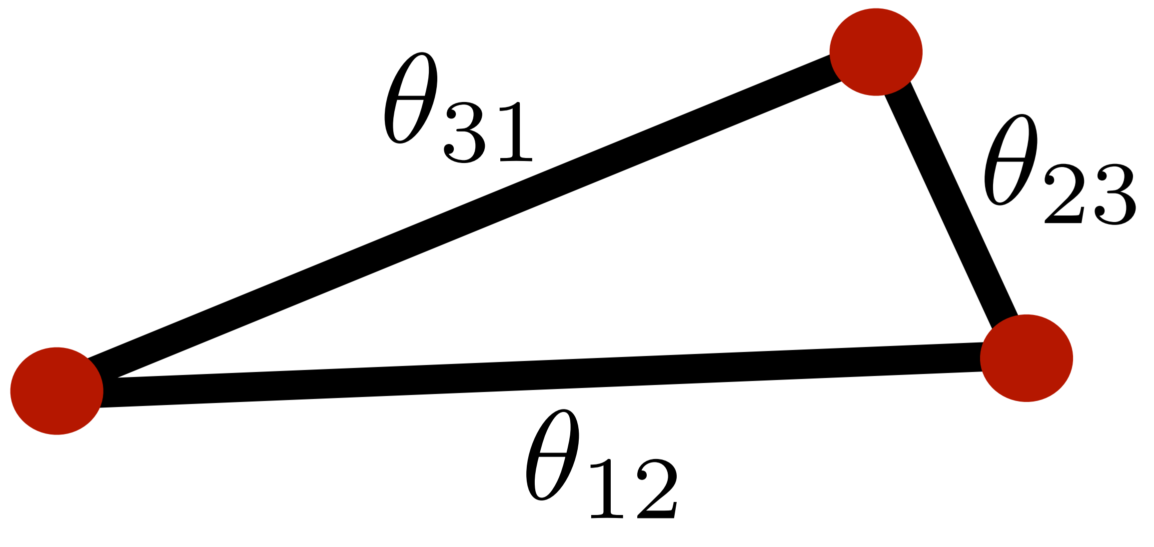
$$\delta(\delta-2) + j^2$$

Identical to Casimir equation for 2D Euclidean conformal block!

Pöschl–Teller potential, 1933

Dolan, Osborn, NPB, 2003

Comment on SO(3,1) partial wave expansion



$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \quad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$

Partial wave expansion of three-point energy flow correlation

$$G(\theta_{12}, z) = G(\theta_{12}) \sum_{\delta,j} c_{\delta,j} g_{\delta,j}(z)$$

Partial wave coefficient
(OPE coefficient)

$$g_{\delta,j}(z) = \frac{1}{1+\delta_{j,0}} \left(k_{\frac{\delta-j}{2}}(z) k_{\frac{\delta+j}{2}}(\bar{z}) + (z \leftrightarrow \bar{z}) \right)$$

$$k_h(z) = z^j \, {}_2F_1(h, h-1, 2h, z)$$

Three-point energy flow correlation is completely determined by OPE coefficients and dimension of and spin of (lightray) operators appeared

Partial wave expansion provides a systematic solution to power expansion and resummation problem for energy flow correlation

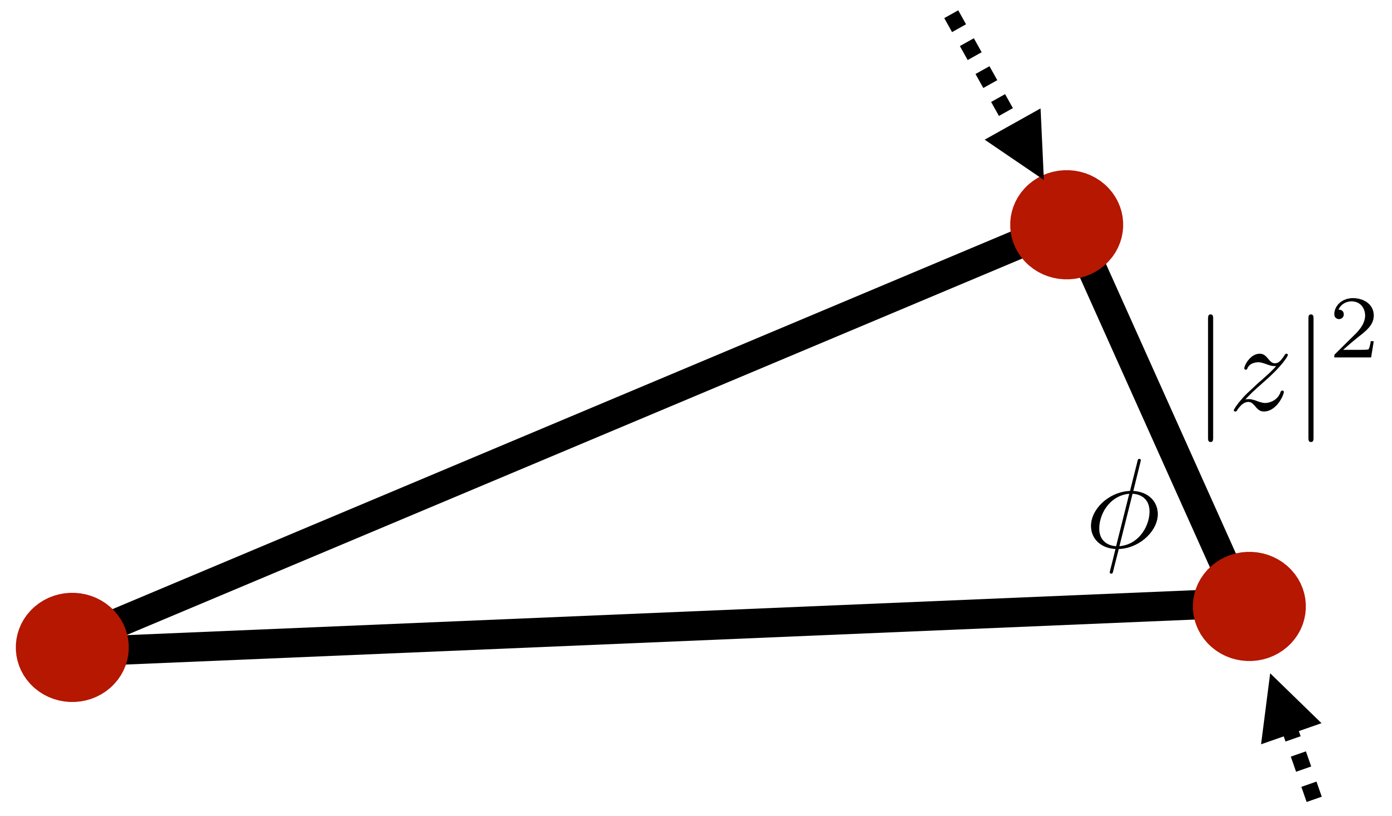
H. Chen, I. Moult, J. Sandor, HXZ, 2202.04085
C.-H. Chang, D. Simmons-Duffin, 2202.04090

Concrete example of partial wave expansion

$$g_{\delta,j}(z) = \frac{1}{1+\delta_{j,0}} \left(k_{\frac{\delta-j}{2}}(z) k_{\frac{\delta+j}{2}}(\bar{z}) + (z \leftrightarrow \bar{z}) \right) \qquad k_h(z) = z^j \, {}_2F_1(h, h-1, 2h, z)$$

$$G(\theta_{12}, z) = \frac{1}{\theta_{12}^2} \frac{\alpha_s^2}{(4\pi)^2} C_A^2 \left[\frac{1}{720} g_{4,2} + \frac{49}{200} g_{4,0} + \left(\frac{834469}{126000} - \frac{2\pi^2}{3} \right) g_{6,2} + \left(\frac{318193}{50400} - \frac{5\pi^2}{8} \right) g_{6,0} + \cdots \right]$$

$$g_{4,2} = 2\cos(2\phi)|z|^4 + 2\cos(3\phi)|z|^5 + \cdots$$
$$g_{4,0} = 2|z|^4 + 2\cos(\phi)|z|^5 + \cdots$$
$$g_{6,2} = 2\cos(2\phi)|z|^6 + (\cos\phi + 3\cos(3\phi))|z|^7 + \cdots$$
$$g_{6,0} = 2|z|^6 + 4\cos\phi|z|^7 + \cdots$$



- Partial wave expansion is an efficient organization of power expansion
  - Each partial wave resum infinite power of kinematical power corrections
  - Logarithmic resummation achieved by including anomalous dimension in δ

OPE coefficients from inversion formula

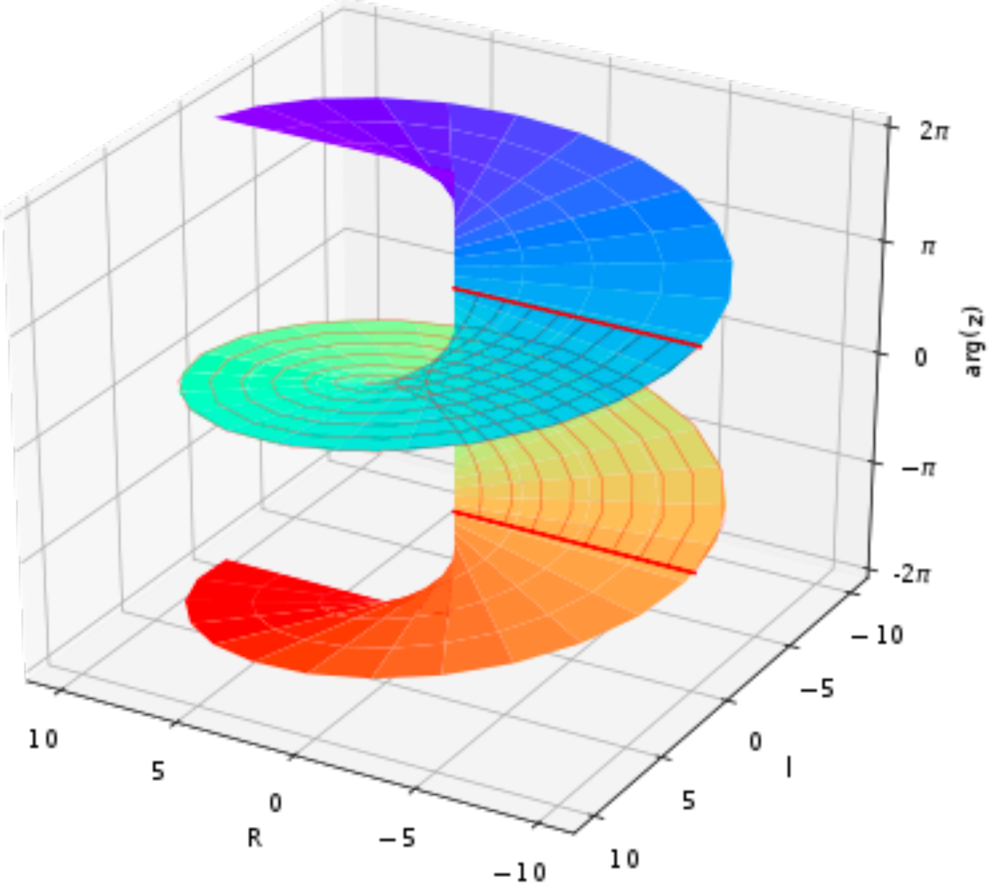
$$G(z,\alpha_s) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z)$$

Given perturbative data G(z, α_s), how to we reconstruct the OPE data?

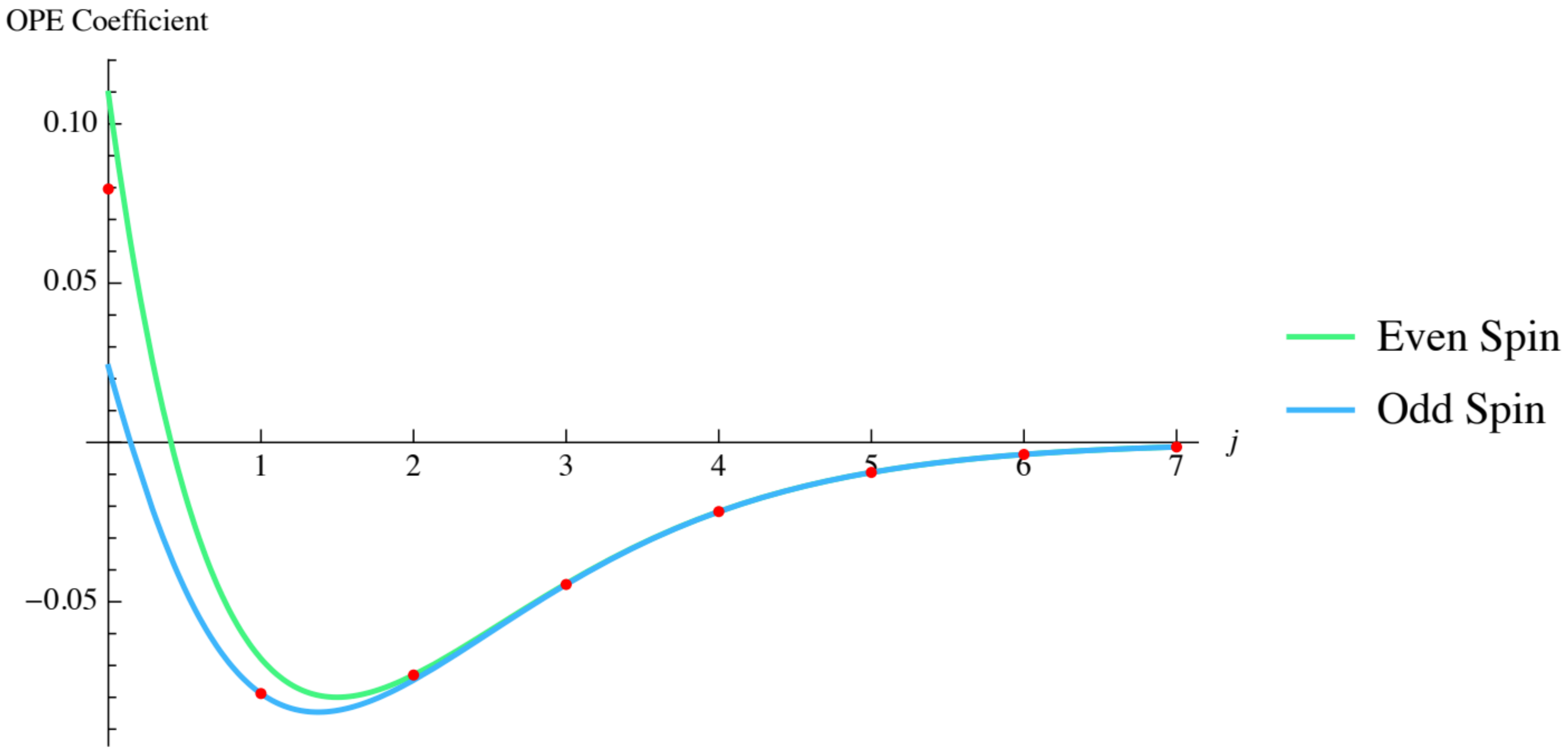
Lorentzian inversion formula S. Caron-hunt, 2017; D. Simmons-Duffin, D. Stanford, E. Witten, 2017

$$G(z,\alpha_s) = \sum_{\delta,j} (c^t_{\delta,j} + (-1)^j c^u_{\delta,j})\, g_{\delta,j}(z)$$

$$c^t_{\delta,j} = \int_0^1 dz\, d\bar{z}\, \mu(z,\bar{z})\, g_{\delta-1,j+1}(z)\, \mathrm{dDisc}[G(z)]$$

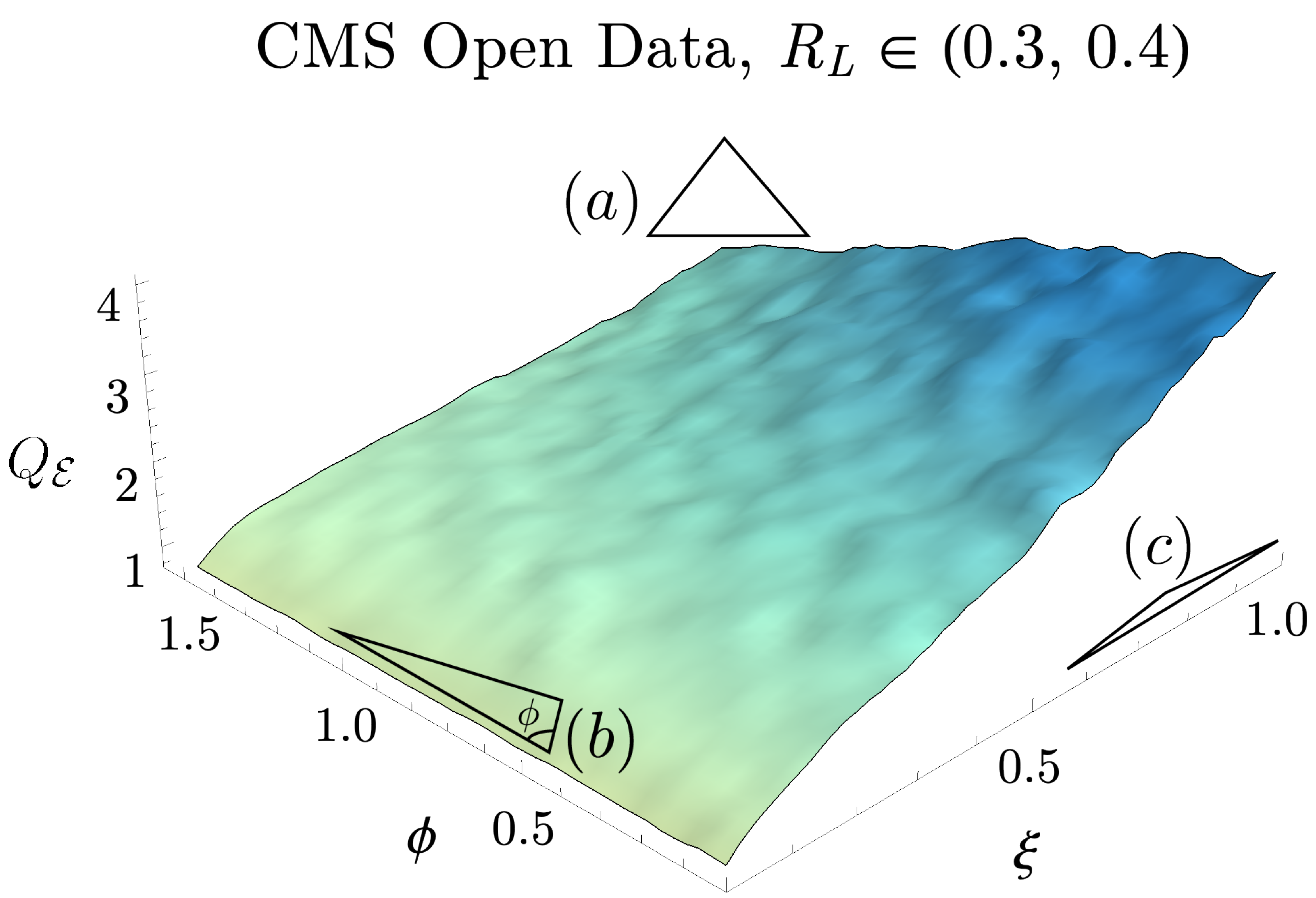


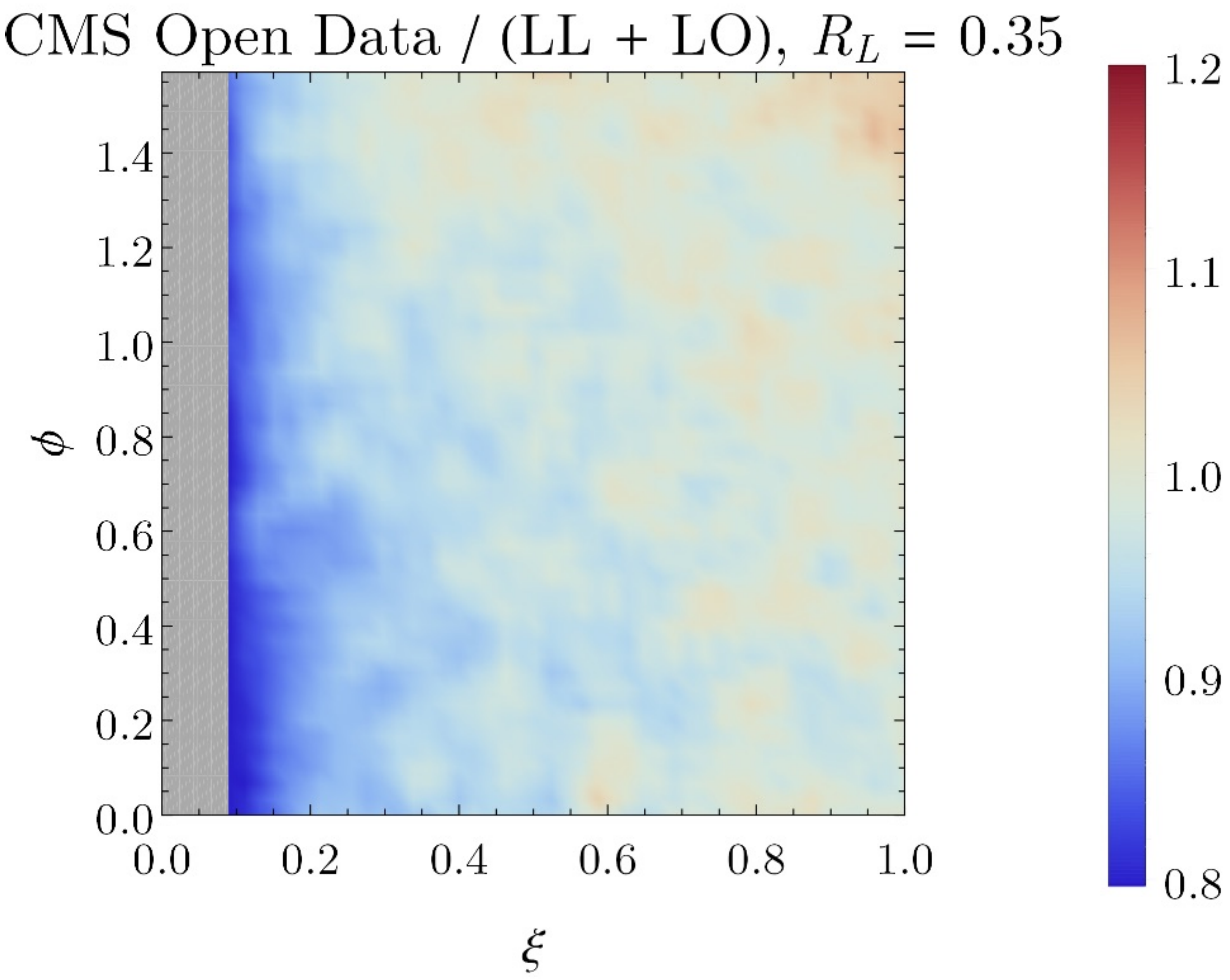
$$G(z,\bar{z}) - \frac{1}{2} G(z,\bar{z}^{\circlearrowright}) - \frac{1}{2} G(z,\bar{z}^{\circlearrowleft})$$



H. Chen, I. Moult, J. Sandor, HXZ, 2202.04085

RG improved perturbation theory v.s. CMS open data





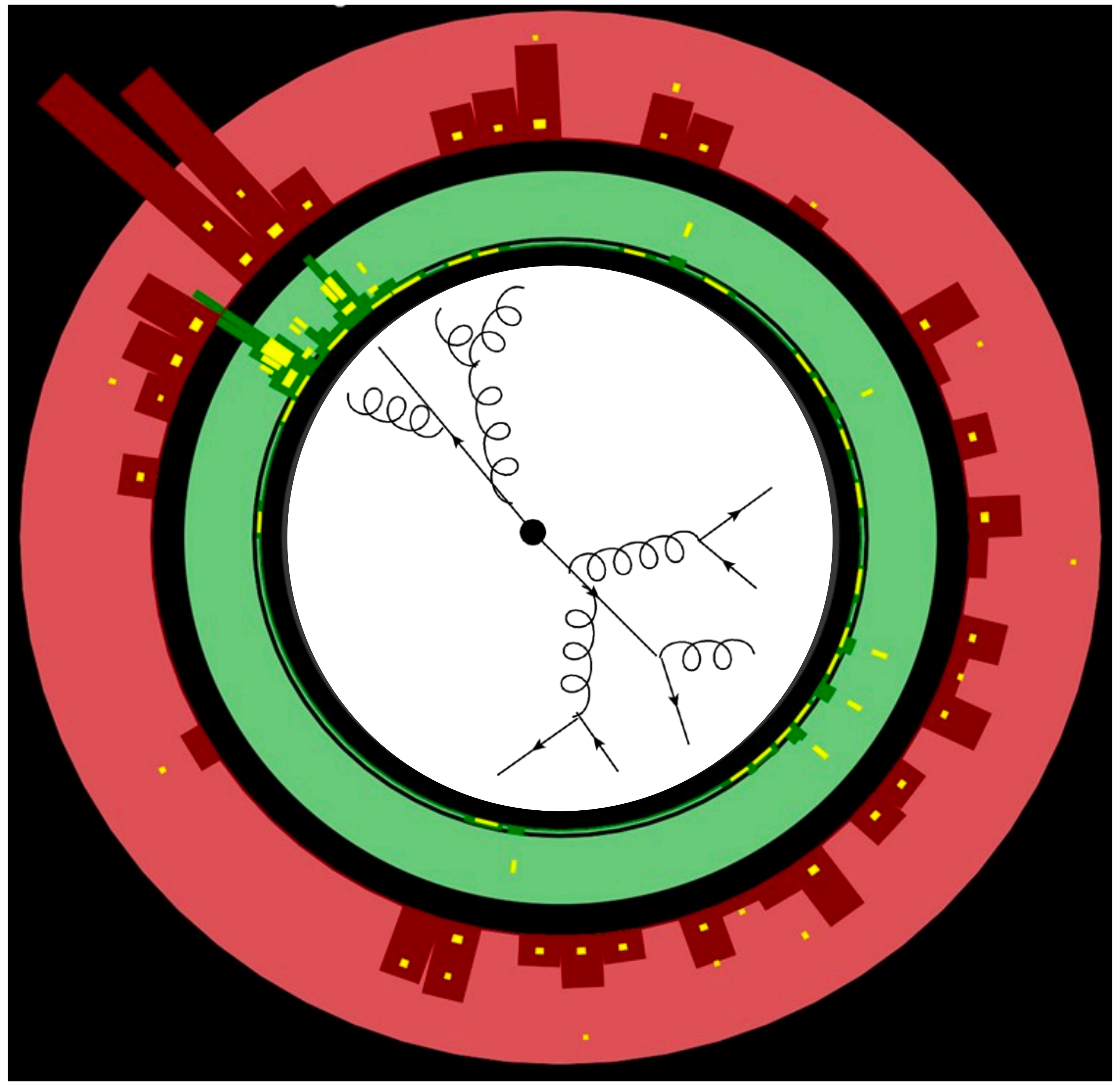
Data/theory agree at the level of ± 20% !

H. Chen, I. Moult, J. Thaler, HXZ, JHEP, 2022

Summary

- Succesful of high energy LHC program relies on accurate description of QCD evolution
  - Generalized DGLAP: multi-branching kernel $P(\{\theta_{ij}\},\{z_i\})$ depending on angles and momentum fractions
- Introduced statistical correlation approach to jet structure based on energy flow correlation
- Provided a data visualization of q/g hadronization phase transition. Very interesting to compare difference between in vacuum and in medium
- Power expansion by Lorentzian partial wave decomposition
  - Clean organization of kinematical and dynamical power corrections
  - Automatically lead to resummation of large logarithms beyond leading power

Speculation: 2D EFT of QCD on the light shell?



If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

- Regge limit: Lipatov, 1988; Verlinde^2, 1993
- e+e- scattering: H. Georgi, Kestin, Sajjad, 2010
- Energy flow correlation provides much concrete theory data to explore this idea