Reformulating Jet Physics using Energy Flow Correlation

朱华星 (Hua Xing Zhu) 浙江大学

第四届重味物理与量子色动力学研讨会 长沙 2022年7月29日

Heavy flavor



Quark-gluon plasma



Jet in high energy scattering

Jet 4

Jet 5

Jet 3

High energy q/g evolution

Collimated spray of hadrons!



Jet 2

Jet 1

Run: 282712 Event: 474587238 2015-10-21 06:26:57 CEST



Jet in high energy scattering

High energy q/g evolution

Collimated spray of hadrons!

Jet 4

谢去病 山东大学



Jet 3

Hadronization



高能踫撞必先产生各种夸克。 夸克如何强子化成各种各样的强子?

强子化过程不仅出现于任何高能反应, 而且是与夸克禁闭、QCD真空结构等一系 列重大理论问题直接相关的非微扰QCD过 程。 ------ 其研究是当代物理中一个 基本又艰巨课题

Jet 1

let 2

Run: 282712 Event: 474587238 2015-10-21 06:26:57 CEST



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi Equation



 $\begin{cases} \mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} P_{ij}^{\text{spacelike}}(z) f_j(x/z,\mu^2) & \text{Relevant for PDFs, initial branching} \\ \mu^2 \frac{\partial D_i(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} P_{ji}^{\text{timelike}}(z) D_j(x/z,\mu^2) & \text{Relevant for FFs, jet fragmentation} \end{cases}$



- How to improve the accuracy of DGLAP?
- How to generalize DGLAP?





Two Questions









Three-loop corrections Moch, Vermaseren, Vogt, NPB 2004

Method: Optical theorem + OPE









Three-loop corrections Moch, Vermaseren, Vogt, NPB 2004

Method: Optical theorem + OPE







Establishing analytic continuation relation for DGLAP



Essentially local operator correlation

 $\int dy^{-} e^{ip^{+}zy^{-}} \langle P|\overline{\psi}(y^{-})\gamma^{+}\psi(0)|P\rangle$



Establishing analytic continuation relation for DGLAP



Essentially local operator correlation

$$\int dy^- e^{ip^+ zy^-} \langle P|\overline{\psi}(y^-)\gamma^+\psi(0)|P\rangle$$

$$\gamma_{\pm}^{\text{timelike}}(N) = \gamma_{\pm}^{\text{spa}}$$

H. Chen, T.Z. Yang,



 $\gamma_{\pm}^{
m celike}(N-\gamma_{\pm}^{
m timelike}(N))$



Generalizing DGLAP?



 $P(z, s_{12})$

Generalizing DGLAP?



Generalized DGLAP



 $P(z, s_{12})$

$P(z_1, z_2, z_3; s_{12}, s_{23}, s_{31})$



Generalizing DGLAP?



Generalized DGLAP







Two approaches to jet structure



Shape observables



Statistical correlation

Two approaches to jet structure



- Example: jet mass, jet broadening, thrust
- Advantage: close connection with new physics search
- Disadvantage: lack direct connection with field theory



Statistical correlation

Two approaches to jet structure



- Example: jet mass, jet broadening, thrust
- Advantage: close connection with new physics search
- Disadvantage: lack direct connection with field theory



Statistical correlation

 $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_{\Psi}, \quad \langle \mathcal{N}(n_1)\mathcal{N}(n_2)\rangle_{\Psi}, \cdots$

- Disadvantage: not yet suitable for new physics search
- Advantage:
 - Defined from most elementary field theory concept

Basham, Brown, Ellis, Love, PRL, 1978; H. Chen, I. Moult, X.Y. Zhang, HXZ, PRD, 2020







$\langle \hat{s}_i \hat{s}_j \rangle$ correlator of local operators



$\langle \hat{s}_i \hat{s}_j \rangle$ correlator of local operators



Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component







Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component







Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component

What information is encoded in energy flow correlation?







P. Komiske, I. Moult, J. Thaler, HXZ, 2201.07800

P. Komiske, I. Moult, J. Thaler, HXZ, 2201.07800

Higher point energy flow correlation

11

Three-point energy flow correlation

- Arbitrary select three energy detectors
- Measures their energy weighted cross section
- Iterate over all possible detector selection, and average over all scatterings

Simplest generalization beyond DGLAP

Application: gluon transverse spin

H. Chen, I. Moult, J. Thaler, HXZ, JHEP, 2022

lacksquare

 $G_{\bar{q}qq}^{(id)}(z) = (C_A - 2C_F)C_F \times \left\{\frac{1}{11520t^8(r-s+1)^4} \left[2952r^8(63s-58) - 24r^7\left(6148s^3 + 27246s^2 + 2726s^2\right)\right]\right\}$ $-62853s + 31810) + r^{6} (63582s^{5} + 494010s^{4} + 251712s^{3} - 3234312s^{2} + 3346592s - 518944)$ $-567392) + r^4 (720s^9 + 43740s^8 + 278667s^7 - 699957s^6 + 112182s^5 - 761156s^4 + 3562352s^3)$ $-3080832s^{2} + 963544s + 67568) + r^{3}(-2880s^{10} - 60480s^{9} - 37044s^{8} + 789742s^{7})$ $-1541669s^{6} + 1762726s^{5} - 2439712s^{4} + 2379488s^{3} - 1304096s^{2} + 272184s - 29040) + r^{2}s^{4}$ $\left(4320 s^{10}+30600 s^9-114070 s^8-77940 s^7+729201 s^6-1306849 s^5+1470360 s^4-1150278 s^3-114070 s^6-114070 s^6-11407$ $+621840s^2 - 180024s + 24720) + rs^3 \left(-2880s^9 + 1080s^8 + 40877s^7 - 101984s^6 + 69207s^5 - 101984s^6 - 101988s^6 - 101984s^6 - 101984s^6 - 101984s^6 - 101984s^6 - 101988s^6 - 101888s^6 - 101888s^6 - 101888s^6 - 10188855 - 101888555 - 101888555 - 1018855555 - 101885555 - 101885555$ $+59426s^{4} - 157328s^{3} + 136848s^{2} - 63996s + 12000) + s^{5}(720s^{8} - 3780s^{7} + 7415s^{6} - 6731s^{3})$

- 26

 $+r^{4}(-4158s^{6}+6468s^{5}-1795s^{4}-5375s^{3}+8200s^{2}-4912s+848)+2r^{3}(660s^{8}-1122s^{7})$ $+924 s^6-1187 s^5+2260 s^4-2885 s^3+1901 s^2-498 s+36\big)+r^2 \left(-242 s^8+440 s^7-396 s^6\right)$ $+ 198s^5 + 330s^4 - 1080s^3 + 1433s^2 - 923s + 216) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 - 22s^5 - 25s^3 + 216\right) s^2 + r \left(24s^8 - 46s^7 + 44s^6 + 216s^6 + 21$ $+76s^2 - 85s + 36) s^4 - \left(s^3 - 2s^2 + 2s - 1\right) s^{11} \bigg] g_2^{(1)} + \frac{1}{8} \bigg[-2r(s-1) + s^3 - 2s^2 + 2s - 1 \bigg] g_2^{(3)} \bigg\} \, .$

For $G_{\bar{q}qq}^{(id)}(z)$, we have

 $-62880s + 12360) + 2r^2 (10240s^7 - 7694s^6 + 17392s^5 - 75875s^4 + 119500s^3 - 97680s^2)$ $+12960) + s^{4} (120s^{7} - 120s^{6} + 140s^{5} + 73s^{4} + 380s^{3} - 2730s^{2} + 4020s - 2160) \left| g_{1}^{(2)} - \frac{1}{8t^{11}} \right|$ $1344r^7 + r^6 (-5628s^2 + 6552s - 3296) + 6r^5 (1232s^4 - 2114s^3 + 1803s^2 - 1006s + 380)$

 $+ s^{5} \left(120 s^{6} - 60 s^{5} + 100 s^{4} + 50 s^{3} + 34 s^{2} - 39 s + 72\right) \left] g_{1}^{(1)} + \frac{r}{960 t^{10}} \left[r^{5} \left(176896 - 136576 s\right) + 36576 s \right] \right] + r^{5} \left(176896 - 136576 s\right) \left[r^{5} \left(176896 - 136576 s\right) + 36576 s \right] + r^{5} \left(176896 - 136576 s\right) \right] + r^{5} \left(176896 - 136576 s\right) \left[r^{5} \left(176896 - 136576 s\right) + 100 s^{4} + 50 s^{3} + 34 s^{2} - 39 s + 72 \right) \right]$

 $+\,8r^4\left(25961s^3-30314s^2+26808s-14400\right)-4r^3\left(21989s^5-3502s^4-33955s^3+64966s^2\right)$ $-46404s + 7552) + 2r^{2}s \left(10240s^{6} - 3732s^{5} + 9214s^{4} - 29779s^{3} + 53618s^{2} - 40220s + 8136\right)$ $+ rs^{3} \left(-2460s^{6} + 1100s^{5} - 1963s^{4} - 1289s^{3} + 6814s^{2} - 11866s + 6984\right)$

 $+8860s - 1080) - 2rs^{2} (2100s^{6} - 2480s^{5} + 1835s^{4} - 1620s^{3} + 4810s^{2} - 6610s + 2952)$ $+ s^4 \left(240 s^6 - 300 s^5 + 260 s^4 - 67 s^3 - 83 s^2 + 150 s - 144\right) \bigg] - \frac{r - s + 1}{960 t^{10}} \bigg[r^5 (96256 - 136576 s) + 100 s^2 + 100 s^$

 $G_{q'q'q}(z) = C_F T_F n_F \times \left\{ \frac{1}{1920t^8} \left[-80640r^5 + 16r^4 \left(7897s^2 - 3194s + 2232 \right) - 8r^3 \left(11631s^4 + 2232 \right) \right] \right\} + \frac{1}{1920t^8} \left[-80640r^5 + 16r^4 \left(7897s^2 - 3194s + 2232 \right) - 8r^3 \left(11631s^4 + 2232 \right) \right] \right]$

 $-14936s^{3} + 19657s^{2} - 13290s + 4412) + 4r^{2} (7160s^{6} - 8067s^{5} + 1978s^{4} + 13063s^{3} - 18082s^{2})$

Here we present the results for each term separately. For $G_{q^{'}q^{'}q}(z)$, we have

Three-point energy flow correlation from perturbation theory

Need different organization to make underlying physics manifest H. Chen, M.X. Luo, I. Moult, T.Z. Yang, X.Y. Zhang, HXZ, JHEP, 2020

Large logarithms in perturbative theory $\alpha_s^n \log^n(\theta_{23}/\theta_{12})$

Analytic available perturbative prediction at leading order

- 27 -

 $+26656) + r^{6}(-2220s^{8} - 61957s^{7} - 221453s^{6} + 1556576s^{5} - 2797984s^{4} + 2347632s^{5})$ $-571352s^2 - 155776s + 141984) + r^5 (120s^{10} + 8760s^9 + 78553s^8 - 32546s^7 - 1397671s^6)$ $+4161836s^{5} - 5541244s^{4} + 3775936s^{3} - 1383964s^{2} + 221760s - 34320) + r^{4} \left(-480s^{11} + 280s^{11} +$ $-12360 s^{10}-24907 s^9+140321 s^8+408799 s^7-2358813 s^6+4396600 s^5-4238920 s^4$ $+2353060s^{3} - 733140s^{2} + 133920s - 12240) + 2r^{3}s(360s^{11} + 3360s^{10} - 8561s^{9} - 8786s^{8})$ $-26256s^7 + 293094s^6 - 710896s^5 + 853220s^4 - 589160s^3 + 234740s^2 - 52020s + 5400)$ $-10r^2s^3 (48s^{10} + 30s^9 - 813s^8 + 2248s^7 - 5386s^6 + 13816s^5 - 25504s^4 + 28729s^3 - 19312s^2$ $+7158s - 1140) + 20rs^5 \left(6s^9 - 30s^8 + 60s^7 - 43s^6 - 122s^5 + 480s^4 - 788s^3 + 692s^2 - 318s^2 + 692s^2 + 692s^2 - 318s^2 + 692s^2 - 318s^2 + 692s^2 + 692s^2 - 318s^2 + 692s^2 + 692s^2$ $+60)\bigg]g_{1}^{(2)}-\frac{1}{32t^{11}(r-s+1)^{5}}\bigg[r^{11}(2440-2604s)+r^{10}\left(4998s^{3}+5360s^{2}-21420s+10056\right)+r^{10}(4998s^{3}+5360s^{2}-21420s+10056)+r^{10}(4998s^{2}+2160s^{2}-2160s^{2$ $-4r^{9} \left(1155s^{5} + 3615s^{4} - 6755s^{3} - 6512s^{2} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s - 6530\right) + 2r^{8} \left(924s^{7} + 9702s^{6} - 12481s^{5} + 12573s^{7} + 125$ $-21516s^4 + 26037s^3 + 7588s^2 - 26702s + 3012) - 4r^7 \left(99s^9 + 2112s^8 + 4719s^7 - 31384s^6 + 1288s^7 + 1288s^6 + 12888s^6 + 1288s^6 + 1288s^6 + 1288s^6 + 1288s^6 + 1288s^6$ $+40666s^5 - 17272s^4 - 3317s^3 - 5698s^2 + 3375s - 2934) + 2r^6 (22s^{11} + 946s^{10} + 6292s^9)$ $-10890s^8 - 60868s^7 + 206316s^6 - 280018s^5 + 197026s^4 - 86265s^3 + 12864s^2 + 1150s^2 + 11505555 + 115055555 + 1150555555$ $-3636) - 2r^5 \left(s^{13} + 108s^{12} + 1651s^{11} + 1650s^{10} - 22858s^9 + 7666s^8 + 136020s^7 - 348348s^6 + 126020s^7 - 348388s^7 - 348388s^7 - 34838585 + 34838585 + 3483855 + 3483655 + 34838555 + 348565555 + 348555555 + 3485555555 + 3485555555$ $+419956s^{5} - 309054s^{4} + 139950s^{3} - 41416s^{2} + 5662s - 844) + 2r^{4} (5s^{14} + 205s^{13} + 1115s^{12} + 115s^{12} + 115s^{1$ $-3964s^{11} - 6468s^{10} + 28553s^9 + 1588s^8 - 129649s^7 + 260030s^6 - 272931s^5 + 174884s^4$

 $-274300s^{5} + 355114s^{4} - 384612s^{3} + 1346556s^{2} - 1134536s + 497664) + r^{6} \left(2220s^{8} + 78997s^{2} + 7897s^{2} + 7897s^{2} + 7897s^{2} + 7897s^{2} + 78997s^{2} + 7897s^{2} + 7897s^{$ $+442073s^{6} - 1802686s^{5} + 2690374s^{4} - 3239252s^{3} + 2476568s^{2} - 1303360s + 124416) + r^{5}$ $(-120s^{10} - 11040s^9 - 143233s^8 - 164574s^7 + 2504176s^6 - 5907364s^5 + 7687884s^4 - 5907364s^5 + 7687884s^4 - 5907364s^5 + 7687884s^4 - 5907364s^5 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907365 - 5907$ $-6057800s^{3} + 3293732s^{2} - 992024s + 217600) + r^{4} \left(600s^{11} + 21300s^{10} + 104147s^{9} - 239951s^{8} + 104147s^{10} + 10418s^{10} + 10418s^{10} + 10418s^{10} + 10885s^{10} + 10885s^{10} + 10885s^{10} + 10885s^{10} + 10885s^{10} + 108$ $-1238636s^7 + 5128180s^6 - 8773052s^5 + 8432296s^4 - 5077368s^3 + 1781128s^2 - 374016s^2 - 374016s^2$ $+20480) + r^{3}(-1200s^{12} - 19200s^{11} - 3678s^{10} + 206332s^{9} - 2900s^{8} - 1652064s^{7} + 4235853s^{6})$ $-5319852s^5 + 4040306s^4 - 1880724s^3 + 541556s^2 - 81944s + 5120) + r^2s^2$ (1200s¹¹) $+475910s^2 - 141380s + 18920) + rs^4 \left(-600s^{10} + 480s^9 + 7640s^8 - 24520s^7 + 38363s^6\right)$ $-49712s^5 + 71911s^4 - 84326s^3 + 63342s^2 - 25508s + 4280) + s^6 \left(120s^9 - 660s^8 + 1480s^7 + 1280s^8 + 1280s^8$ $-1710s^{6} + 913s^{5} + 93s^{4} - 557s^{3} + 393s^{2} - 162s + 30) \left[g_{1}^{(1)} + \frac{1}{1920t^{10}(r-s+1)^{4}} \right]$

 $-32256r^{10} + r^9(47636s^2 + 73184s - 99344) + r^8(-56620s^4 - 46364s^3 + 87148s^2)$

 $+69472s + 41264) + r^{7} \left(16043s^{6} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{3} - 135376s^{2} - 227408s^{2} + 212500s^{5} - 579364s^{4} + 463512s^{5} - 579364s^{4} + 463512s^{5} - 579364s^{2} + 212500s^{5} - 579364s^{5} + 212500s^{5} + 21$

 $+37826s - 43136) + 4r^8 (14155s^4 + 33716s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 35198s^2 - 170608s + 56064) + r^7 (-16043s^3 + 56066) + r^7 (-1606666) + r^7 (-1606666) + r^7 (-1606666) + r^7 (-$

+24616) $+ r^4 (-85920s^6 + 23472s^5 + 62208s^4 - 807932s^3 + 985072s^2 - 503784s + 42576)$ $+ 6r^3 (2100s^8 + 2800s^7 - 5925s^6 + 19077s^5 + 14154s^4 - 54072s^3 + 39244s^2 - 3664s - 464)$ $-2r^{2}(360s^{9} + 2340s^{8} - 5010s^{7} + 14083s^{6} - 11630s^{5} + 13365s^{4} - 23487s^{3} + 15924s^{2} + 2580s^{6})$ $(-1932) s + 2r (180s^8 - 405s^7 + 955s^6 - 530s^5 - 126s^4 + 336s^3 - 57s^2 - 213s + 462) s^3$

 $+2r\left(s^{4}-9s^{3}+10s^{2}-21s+10\right)s^{3}+s^{7}+s^{5}\left]g_{2}^{(1)}\right\}+C_{F}C_{A}\times\left\{\frac{1}{11520rt^{8}(r-s+1)}\right.$ $\left| 241920r^7 - 48r^6 \left(7897s^2 - 3389s + 8757 \right) + 8r^5 \left(34893s^4 - 43872s^3 + 114654s^2 - 55760s^2 + 114654s^2 + 55760s^2 + 55760s^2 + 114654s^2 + 55760s^2 + 55760s^$

 $-72r^{6} + r^{5} \left(-216s^{2} + 696s - 452\right) - 2r^{4} \left(18s^{4} - 309s^{3} + 896s^{2} - 995s + 344\right) + 2r^{3} \left(27s^{5} + 27355 + 27355\right) + 2r^{3} \left(27s^{5} + 27555\right) + 2r^{3} \left(27s^{5} + 2$ $-277s^4 + 726s^3 - 952s^2 + 462s - 42) + 2r^2 \left(-11s^5 + 87s^4 - 177s^3 + 246s^2 - 126s + 15\right)s$

 $+1132) + 2r^2 \left(37s^5 - 970s^4 + 2697s^3 - 3546s^2 + 3320s - 644 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 101s^2 + 1000565 \right)s + 2r \left(10s^4 - 37s^3 - 1000555 \right)s + 2r \left(10s^4 - 37s^3 - 1000555 \right)s + 2r \left(10s^4 - 37s^3 - 100555 \right)s + 2r \left(10s^4 - 37555 \right)s + 2r \left(10s^4 - 37555 \right)s + 2r \left(10s^4 - 100555 \right)s + 2r \left(10s^4 - 100555 \right)s + 2r \left(10s^4 - 100555 \right)s + 2r$ $+ 15741s^4 - 30520s^3 + 33116s^2 - 9492s + 512 \Big) + 2rs^2 \left(8s^5 - 509s^4 + 1810s^3 - 2534s^2 + 1810s^2 + 1805s^2 + 18055555 + 18055555 + 18055555 + 180555555555 + 1805555555555555555$ $- 339s) - 8r^5 \left(873s^3 - 8652s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13793s - 7642 \right) - 4r^4 \left(18s^5 - 4245s^4 + 27695s^3 - 52145s^2 + 13795s^2 + 13793s^2 + 13795s^2 + 137955s^2 + 13795s^2 + 13795s^2 + 13795s^2 + 137955s^2 + 13795s^2 + 1$ $+43770s - 11160) + 2r^3 (69s^6 - 6874s^5 + 37015s^4 - 75160s^3 + 75300s^2 - 29580s + 2520)$ $-2r^{2}s(41s^{6}-2172s^{5}+8771s^{4}-17455s^{3}+18790s^{2}-8010s+900)+rs^{3}(3s^{5}-664s^{4})$ $+1340s^3 - 2300s^2 + 3120s - 1200) + s^5(13s^4 + 35s^3 - 50s^2 + 30s - 60) \left| g_1^{(2)} - \frac{1}{8t^{11}} \right|$

 $G_{ggq}(z) = C_F^2 \times \left\{ \frac{1}{1920rt^8(r-s+1)} \left[4320r^6 + 48r^5 \left(123s^2 - 544s + 421 \right) + 8r^4 \left(18s^4 - 544s + 421 \right) \right] \right\}$ $-1602 s^3+6263 s^2-8495 s+3992\big)+4 r^3 \left(-51 s^5+2070 s^4-7280 s^3+11598 s^2-8480 s^2-8480$ $-8704) + 4r^3 \left(18s^5 - 3039s^4 + 20363s^3 - 40627s^2 + 37080s - 8448\right) - 2r^2 \left(33s^6 - 2920s^3 - 2920s$

For $G_{ggq}(z)$ we have

 $+r^{2}(s^{2}+2s+4)-2r(s^{2}-s+2)+s^{2}-2s+2\left]g_{2}^{(2)}+\frac{1}{16}(s-1)^{2}g_{2}^{(3)}-\frac{g_{2}^{(5)}}{199(r-s+1)}\right\}$

 $-72641s^3 + 18084s^2 - 2778s + 204 \Big) - 2r^3s \left(10s^{14} + 180s^{13} - 50s^{12} - 2855s^{11} + 6205s^{10} + 6205s^{10}$ $-854s^9 - 6854s^8 - 10256s^7 + 51400s^6 - 77864s^5 + 64878s^4 - 33808s^3 + 10754s^2 - 2004s^2 - 2004s^2$ $+180) + r^2 s^3 \left(20 s^{13} + 120 s^{12} - 724 s^{11} + 329 s^{10} + 4018 s^9 - 11098 s^8 + 15952 s^7 - 19508 s^6 + 15952 s^7 - 19508 s^7 + 19508 s^7 - 19508 s^7 + 19508 s^$ $+24576s^5 - 26846s^4 + 20520s^3 - 10228s^2 + 2956s - 380) - 2rs^5 (5s^{12} - 8s^{11} - 79s^{10})$ $+362s^9 - 694s^8 + 713s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s^2 + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s^2 + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s^2 + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^3 + 398s^2 - 136s^2 + 20) + s^{11} (2s^7 - 324s^6 - 224s^5 + 606s^4 - 652s^5 + 606s^5 + 606s^5$ $-14s^{6} + 42s^{5} - 70s^{4} + 70s^{3} - 43s^{2} + 16s - 4$ $g_{2}^{(1)} - \frac{1}{32(r-s+1)^{5}} \left[2r^{4} - 2r^{3}(s+2)\right]$

- 29

 $G_q(z) = G_{qq\bar{q}}^{(ab)}(z) + 2G_{qq\bar{q}}^{(nab)}(z) + G_{qqq}(z)$

 $\frac{1}{\sigma_{\rm tot}} \frac{d\Sigma_g}{dx_L d{\rm Re}(z) d{\rm Im}(z)} = \frac{g^4}{16\pi^5} \frac{1}{x_I} \bigg[G_g(z) + G_g(1-z) \bigg]$ $+\frac{1}{|1-z|^4}\left(G_g\left(\frac{z}{z-1}\right)+G_g\left(\frac{1}{1-z}\right)\right)+\frac{1}{|z|^4}\left(G_g\left(\frac{1}{z}\right)+G_g\left(\frac{z-1}{z}\right)\right)\right].$ (5.16)

For gluon jet we similarly write

 $-7s+2|g_{2}^{(3)}|$ 5.2.2 Gluon Jets

The color decomposition is

 $-120 s^8+320 s^7-47 s^6+367 s^5-2176 s^4+4300 s^3-3160 s^2-1320 s+1200\big)+s^5 \left(-13 s^4-120 s^2-1320 s^2-1320$ $-35s^3 + 50s^2 - 30s + 60) \left] g_1^{(2)} - \frac{1}{32t^{11}(r-s+1)} \left[-2688r^8 + 56r^7 \left(201s^2 - 245s + 189 \right) + 1088r^8 + 10$ $-4r^{6}(3696s^{4} - 6475s^{3} + 9443s^{2} - 7898s + 3522) + 4r^{5}(2079s^{6} - 2541s^{5} + 1190s^{4} + 6838s^{3})$ $-14437s^{2} + 10065s - 1866) - 2r^{4} (1320s^{8} - 1089s^{7} + 1155s^{6} + 474s^{5} + 4040s^{4} - 16766s^{3})$ $+16344s^{2} - 4874s + 268) + 4r^{3}(121s^{10} + 11s^{9} - 198s^{8} + 1221s^{7} - 2422s^{6} + 3195s^{5}$ $-4520s^4 + 3801s^3 - 1165s^2 + 21s + 12) + 2r^2s(-24s^{11} - 53s^{10} + 187s^9 - 605s^8 + 693s^2)$ $+124 s^6-1116 s^5+1866 s^4-1462 s^3+306 s^2+102 s-30\big)+2 r (s-1)^2 s^3 \left(s^9+11 s^8-7 s^7-100 s^2+100 s^2+10$ $+48 s^{6}+26 s^{5}+26 s^{4}+3 s^{3}+47 s^{2}-8 s-20\big)-s^{5} \left(s^{10}-3 s^{9}+7 s^{8}-7 s^{7}+2 s^{6}-2 s^{3}+2 s^{2}-2 s^{2}+2 s^{2}$ $-2s+2)\bigg]g_{2}^{(1)}+\frac{1}{32(r-s+1)}\bigg[4r^{2}(s-1)-2r\left(s^{3}-2s^{2}+5s-4\right)+s^{4}-3s^{3}+7s^{2}$

 $+525 s^8-625 s^7+2584 s^6+222 s^5-3078 s^4+6237 s^3-6078 s^2-1164 s+1214 \Big)-s^4 \left(60 s^8-60 s^2-1164 s+1214 \right)-s^4 \left(60 s^8-60 s^2-1164 s+1214 s+1214 \right)-s^4 \left(60 s^8-60 s^2-1164 s+1214 s+12$ $-90s^7 + 260s^6 + 5s^5 + 35s^4 + 10s^3 - 118s^2 + 24s - 274) \left] g_1^{(1)} + \frac{1}{1920t^{10}(r-s+1)} \left[128r^{-3} + 128r^{-3}$ $\left(1067 s - 1622\right) - 8 r^{6} \left(25961 s^{3} - 44511 s^{2} + 49583 s - 51754\right) + 4 r^{5} \left(21989 s^{5} + 4986 s^{4} + 4986$ $-77470s^3 + 193146s^2 - 237744s + 52776) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25479s^5 - 120500s^4 + 52776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25478s^5 + 25776\right) - 2r^4 \left(10240s^7 + 6535s^6 + 25478s^5 + 25776\right) - 2r^4 \left(10240s^7 + 2585s^5 + 25858s^5 + 258588s^5 + 258588s^5 + 258588s^5 + 258588s^5 + 258588s^5 + 258585858s^$ $+ 300490 s^3 - 401970 s^2 + 141240 s - 8280 \big) + 2 r^3 \left(1230 s^9 + 2910 s^8 - 2581 s^7 + 31058 s^6 \right) \\$

 $\left(21989s^5 + 10184s^4 - 54302s^3 + 187766s^2 - 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 181048s + 32256 \right) + r^3 \left(20480s^7 + 20994s^6 + 181048s + 1810488s + 181048s + 1810488s + 1810488s + 1810488s + 18$ $-1878s^{7} + 22026s^{6} - 33050s^{5} + 64563s^{4} - 71304s^{3} + 14688s^{2} + 2712s - 512) + 2rs^{2} \left(60s^{6} + 1288s^{2} + 1288s^{2}$

 $-39s^{8} + \frac{1}{1920t^{10}} \left[128r^{6}(992 - 1067s) + 8r^{5} \left(25961s^{3} - 34733s^{2} + 56757s - 39936 \right) - 4r^{4} \right]$

 $-93827 s^5 + 169085 s^4 - 189540 s^3 + 69600 s^2 - 1920 s - 720) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 - 1920 s - 720 \right) - 2 r^2 s \left(60 s^{10} + 495 s^9 - 845 s^8 + 69600 s^2 + 1920 s^2 +$

$G(\theta_{12}, z) =$

Power expansion of quantum field theory near the lightcone in general is a difficult problem

Power expansion in the squeeze limit $\theta_{23} \rightarrow 0$ (modulo logs)

$$\frac{1}{|z|^2}G^{(0)}(\theta_{12}) + G^{(1)}(\theta_{12}) + |z|^2G^{(2)}(\theta_{12}) +$$

• • •

Lightray OPE and symmetry

 $G(\theta_{12},z)$ $= \langle \Psi | \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{E}(n_3) | \Psi \rangle$ $= \sum \langle \Psi | \mathcal{E}(n_1) \mathbb{O}_{J=3}(n_2) | \Psi \rangle$ $\mathbb{O}_{J=3}$

Quantum number of the lightray operator

Boost \Leftrightarrow Dilation δ

Lightray operator labeled by dimension δ and spin j

Rotation \Leftrightarrow transverse spin j

Partial wave of Lorentz group SO(3,1)

Lorentz invariance of individual partial wave

Quadratic Casimir

 $2z^2(z-1)\partial_z^2 + 2\bar{z}^2(\bar{z}-1)\partial_{\bar{z}}^2$

Pöschl–Teller potential, 1933

$$\begin{array}{l} ,z)\\ n_{1})\mathcal{E}(n_{2})\mathcal{E}(n_{3})|\Psi\rangle\\ \Psi|\mathcal{E}(n_{1})\mathbb{O}_{J=3}(n_{2})|\Psi\rangle = G(\theta_{12})\sum_{\delta,j}g_{\delta,j}(z) \end{array}$$

Identical to Casimir equation for 2D Euclidean conformal block! Dolan, Osborn, NPB, 2003 15

Comment on SO(3,1) partial wave expansion

Three-point energy flow correlation is completely determined by OPE coefficients and dimension of and spin of (lightray) operators appeared

Partial wave expansion provides a systematic solution to power expansion and resummation problem for energy flow correlation

Partial wave expansion of three-point energy flow correlation

$$g_{\delta,j}(z) = G(\theta_{12}) \sum_{\delta,j} c_{\delta,j} g_{\delta,j}(z)$$

$$k_h(z) = z^j {}_2F_1(h, h-z)$$

$$g_{\delta,j}(z) = \frac{1}{1+\delta_{j,0}} \left(\frac{k_{\frac{\delta-j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z}) + (z+z)k_{\frac{\delta+j}{2}}(\bar{z}) +$$

H. Chen, I. Moult, J. Sandor, HXZ, 2202.04085 C.-H. Chang, D. Simmons-Duffin, 2202.04090

Concrete example of partial wave expansion

$$g_{\delta,j}(z) = \frac{1}{1+\delta_{j,0}} \left(\frac{k_{\frac{\delta-j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta-j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta-j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(z)k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} + \frac{k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}}(\bar{z})}{1+\delta_{j,0}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}} \right) + \frac{1}{2} \left(\frac{k_{\frac{\delta+j}{2}}(\bar{z})k_{\frac{\delta+j}{2}} \right) + \frac{$$

$$G(\theta_{12}, z) = \frac{1}{\theta_{12}^2} \frac{\alpha_s^2}{(4\pi)^2} C_A^2 \left[\frac{1}{720} g_{4,2} + \frac{49}{200} g_{4,0} + g_{4,2} = 2\cos(2\phi) |z|^4 + 2\cos(3\phi) |z|^5 + \cdots \right]$$

$$g_{4,0} = 2|z|^4 + 2\cos(\phi) |z|^5 + \cdots + g_{6,2} = 2\cos(2\phi) |z|^6 + (\cos\phi + 3\cos(3\phi)) |z|^6 + (\cos\phi + 3\cos(3\phi)$$

- Partial wave expansion is an efficient organization of power expansion \bullet
 - Each partial wave resum infinite power of kinematical power corrections
 - Logarithmic resummation achieved by including anomalous dimension in δ

17

OPE coefficients from inversion formula

 $G(z, \alpha_s) = \sum_{\delta, j} c_{\delta, j} g_{\delta, j}(z)$

Given perturbative data G(z, α_s), how to we reconstruct the OPE data?

Lorentzian inversion formula S. Caron-hunt, 2017; D. Simmons-Duffin, D. Stanford, E. Witten, 2017

H. Chen, I. Moult, J. Sandor, HXZ, 2202.04085

Even Spin Odd Spin

RG improved perturbation theory v.s. CMS open data

Data/theory agree at the level of ± 20% !

H. Chen, I. Moult, J. Thaler, HXZ, JHEP, 2022

- Succesful of high energy LHC program relies on accurate description of QCD evolution
 - Generalized DGLAP: multi-branching kernel $P(\{\theta_{ij}\}, \{z_i\})$ depending on angles and momentum fractions
- Introduced statistical correlation approach to jet structure based on energy flow correlation
- Provided a data visualization of q/g hadronization phase transition. Very interesting to compare difference between in vacuum and in medium
- Power expansion by Lorentzian partial wave decomposition
 - Clean organization of kinematical and dynamical power corrections
 - Automatically lead to resummation of large logarithms beyond leading power

Summary

Speculation: 2D EFT of QCD on the light shell?

- Regge limit: Lipatov, 1988; Verlinde^2, 1993 \bullet
- e+e- scattering: H. Georgi, Kestin, Sajjad, 2010

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

• Energy flow correlation provides much concrete theory data to explore this idea

