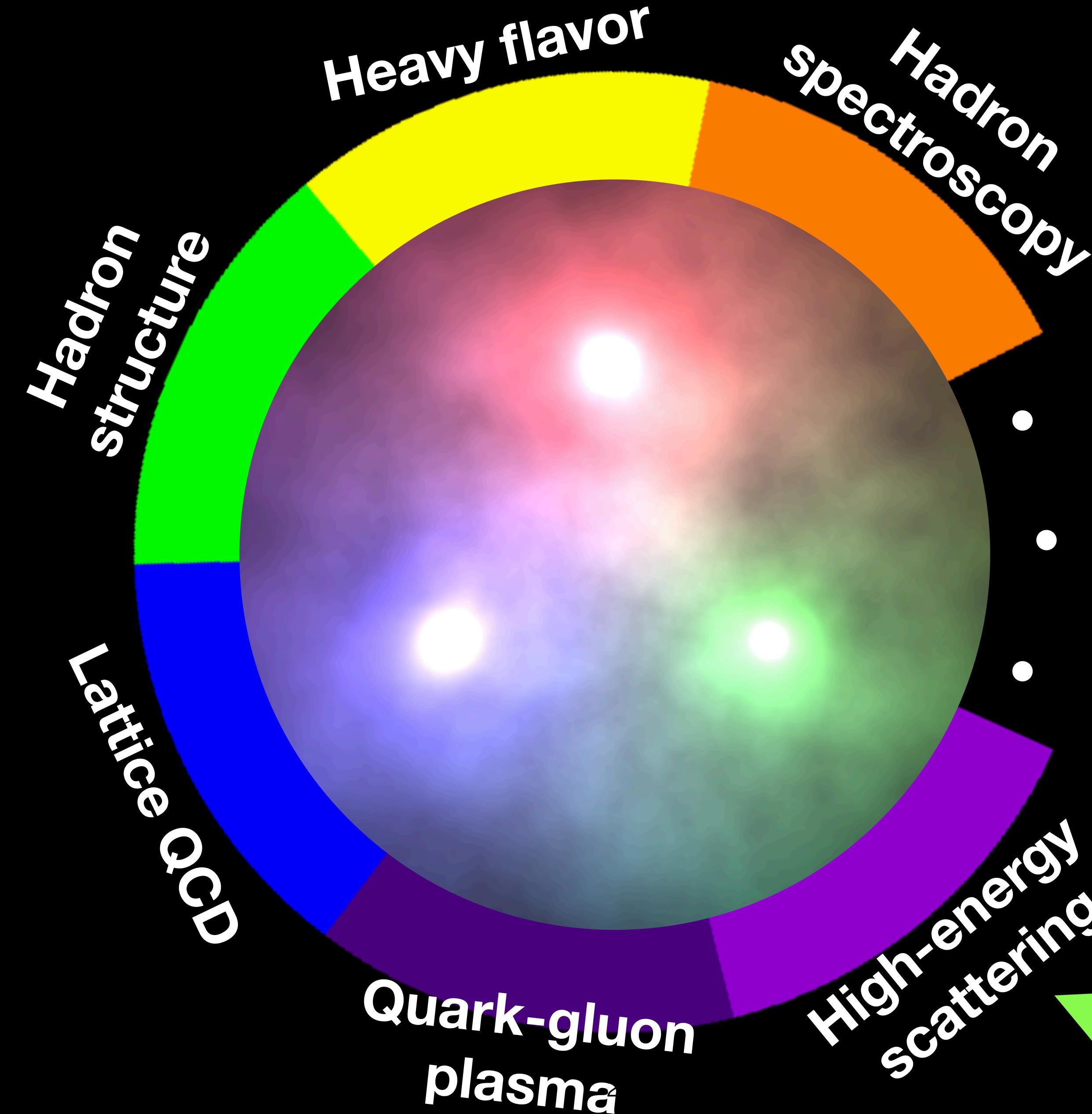


Reformulating Jet Physics using Energy Flow Correlation

朱华星 (Hua Xing Zhu)
浙江大学

第四届重味物理与量子色动力学研讨会
长沙 2022年7月29日

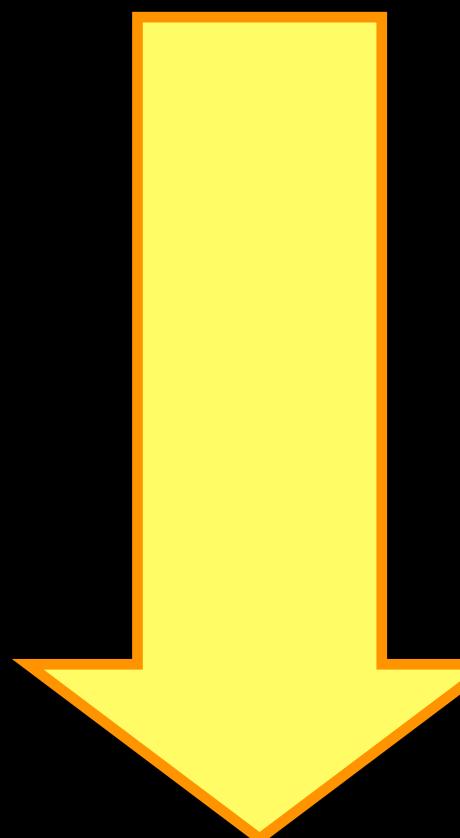
A Golden era of QCD



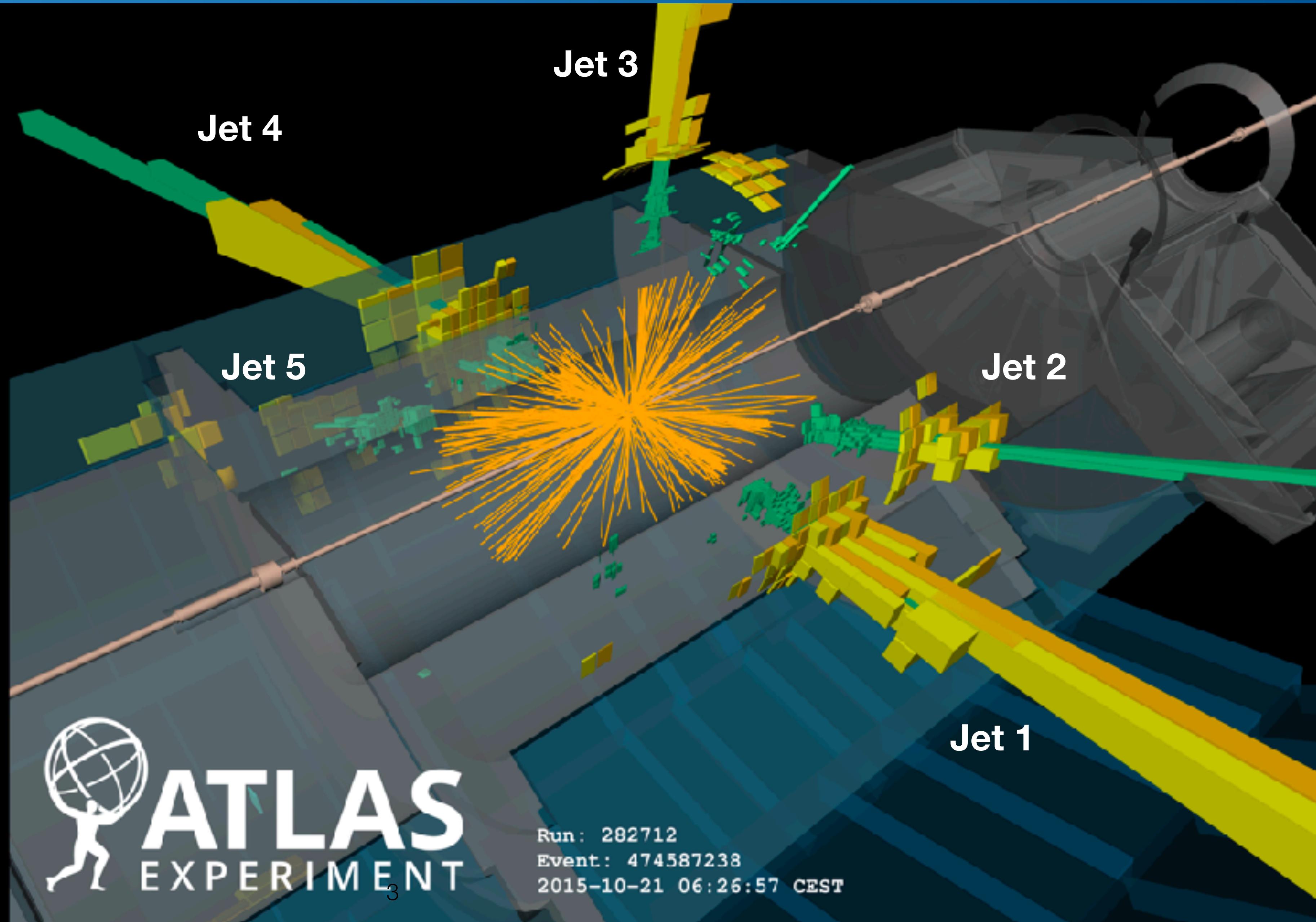
**Focus of
this talk!**

Jet in high energy scattering

High energy q/g evolution

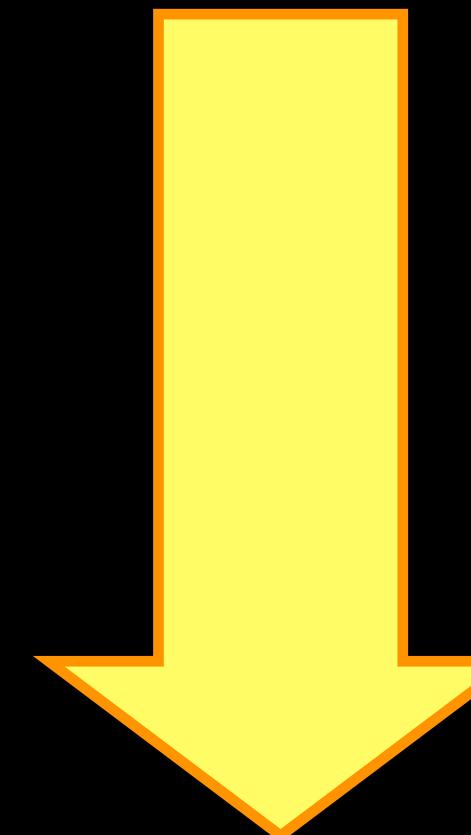


Collimated spray of hadrons!

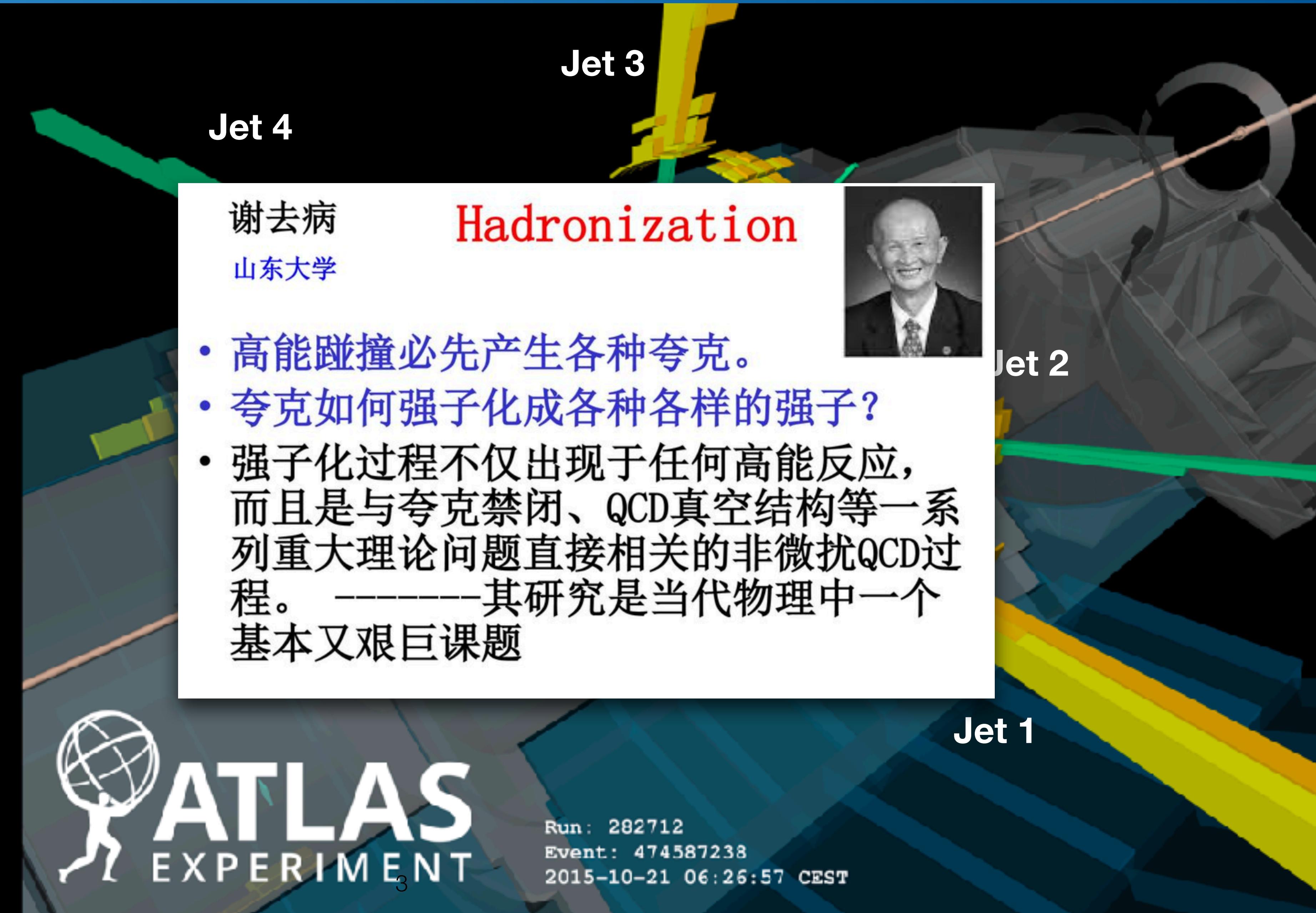


Jet in high energy scattering

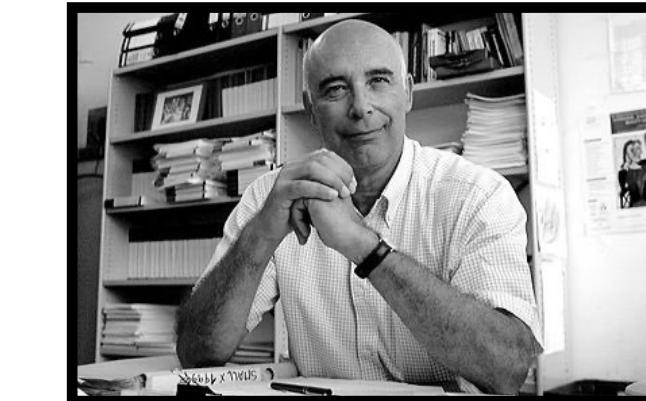
High energy q/g evolution



Collimated spray of hadrons!



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi Equation



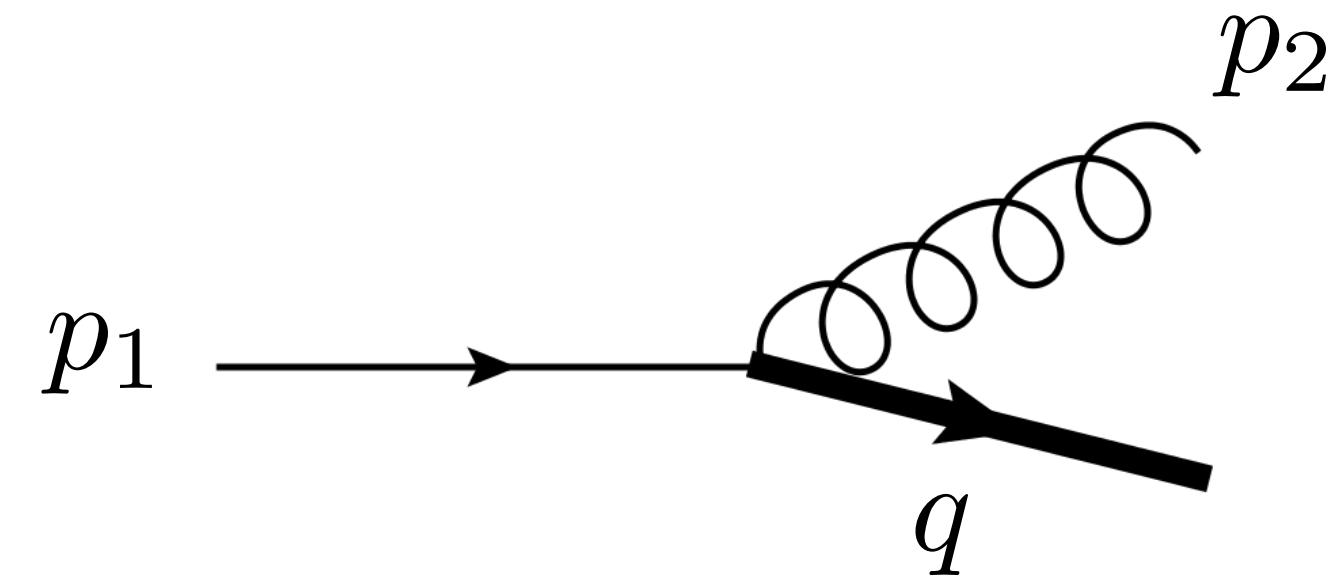
$$\left\{ \begin{array}{l} \mu^2 \frac{\partial f_i(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} P_{ij}^{\text{spacelike}}(z) f_j(x/z, \mu^2) \\ \\ \mu^2 \frac{\partial D_i(x, \mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} P_{ji}^{\text{timelike}}(z) D_j(x/z, \mu^2) \end{array} \right. \quad \begin{array}{l} \text{Relevant for PDFs, initial branching} \\ \\ \text{Relevant for FFs, jet fragmentation} \end{array}$$

Two Questions

- How to improve the accuracy of DGLAP?
- How to generalize DGLAP?

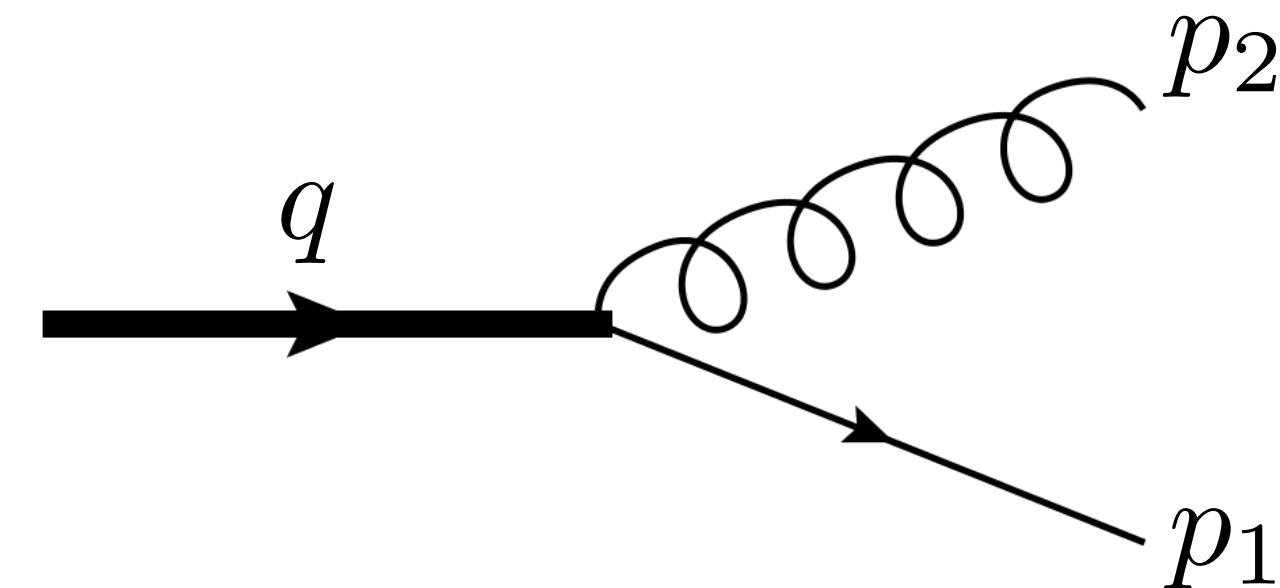
Completion of three-loop corrections to DGLAP

Space-like evolution



$$q^2 = (p_1 - p_2)^2 < 0$$

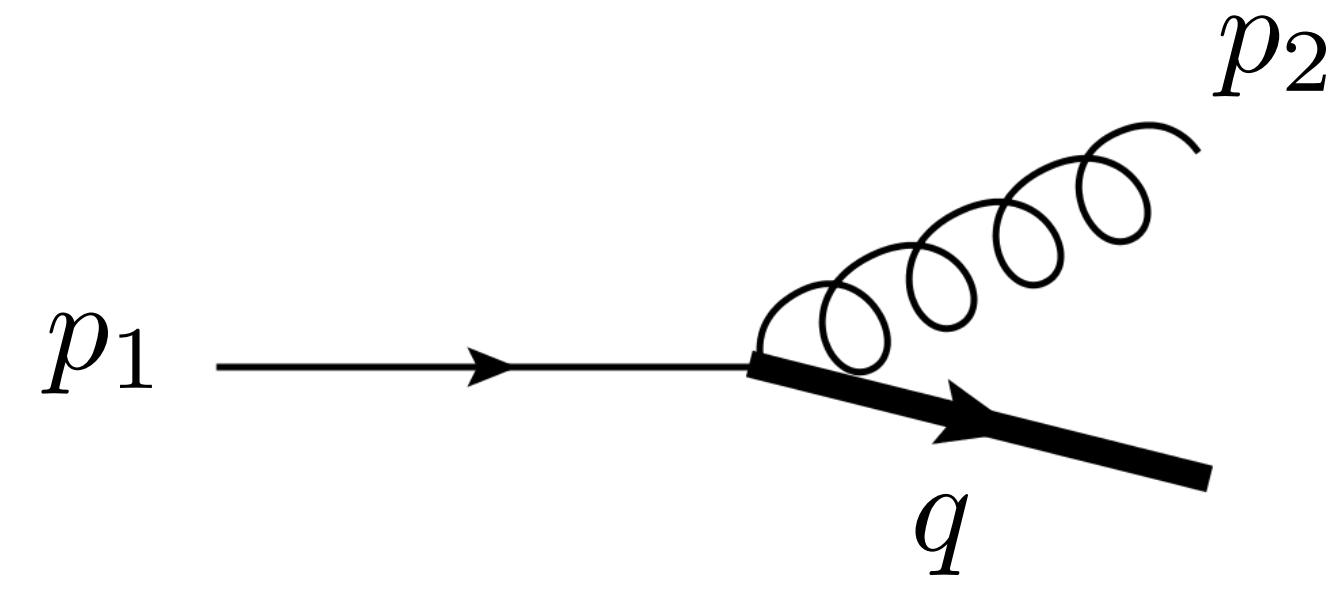
Time-like evolution



$$q^2 = (p_1 + p_2)^2 > 0$$

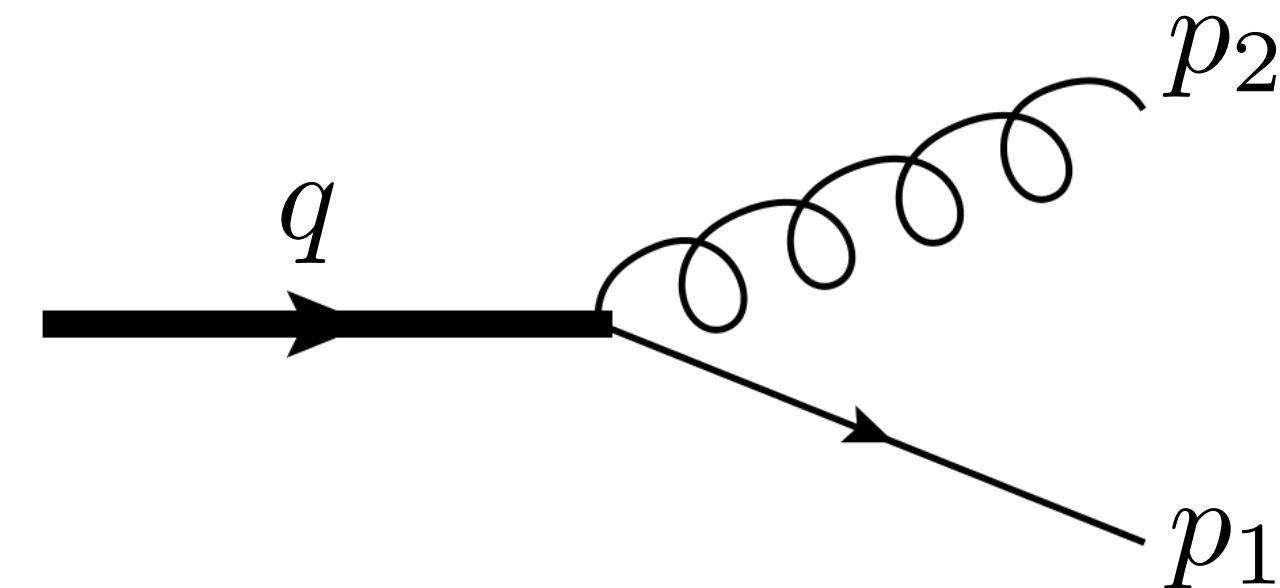
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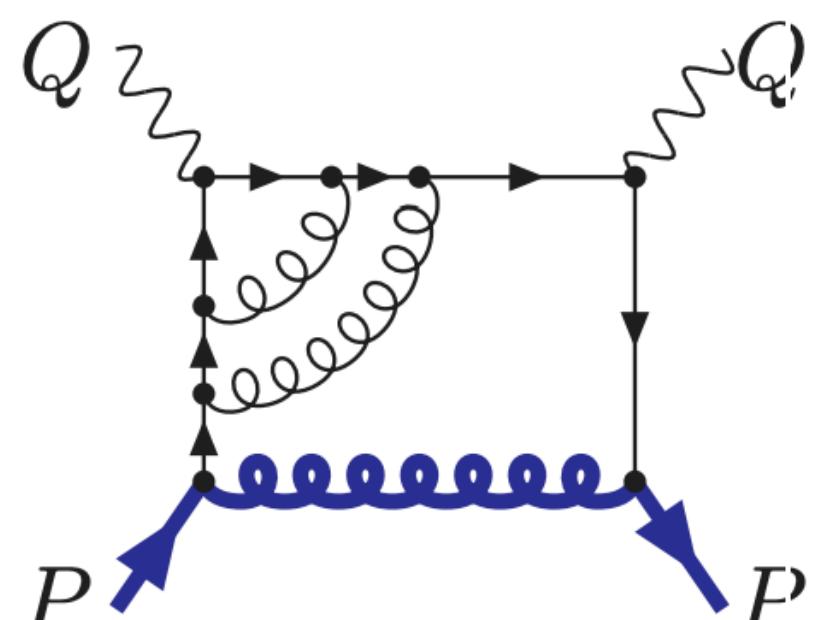


$$q^2 = (p_1 + p_2)^2 > 0$$

Three-loop corrections

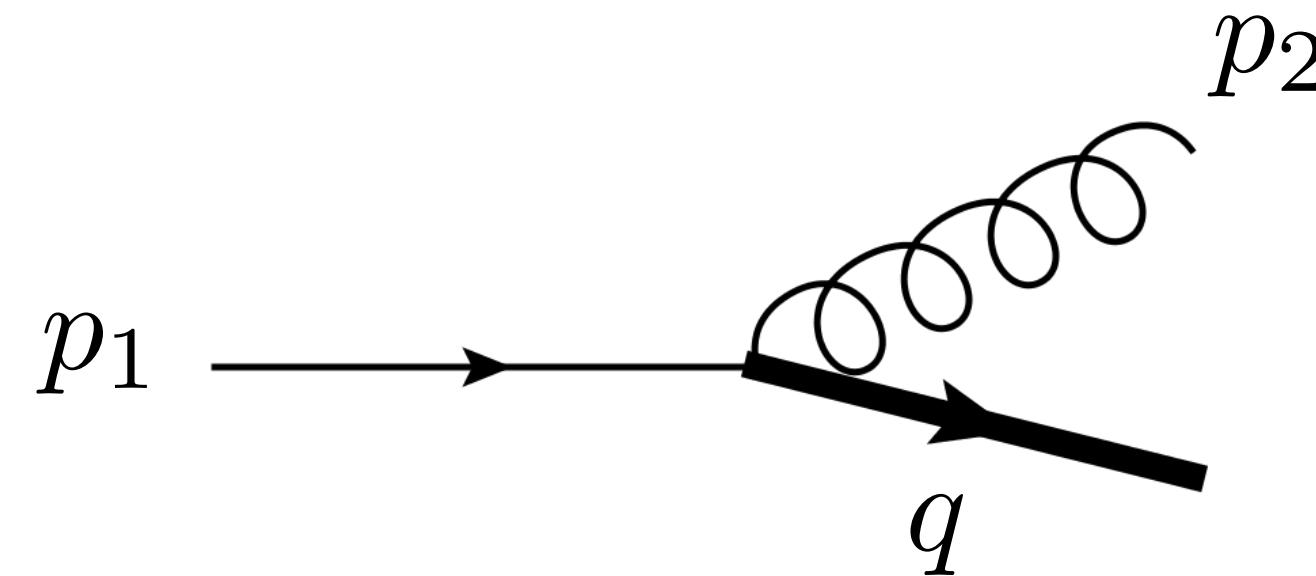
Moch, Vermaseren, Vogt, NPB 2004

Method:
Optical theorem + OPE



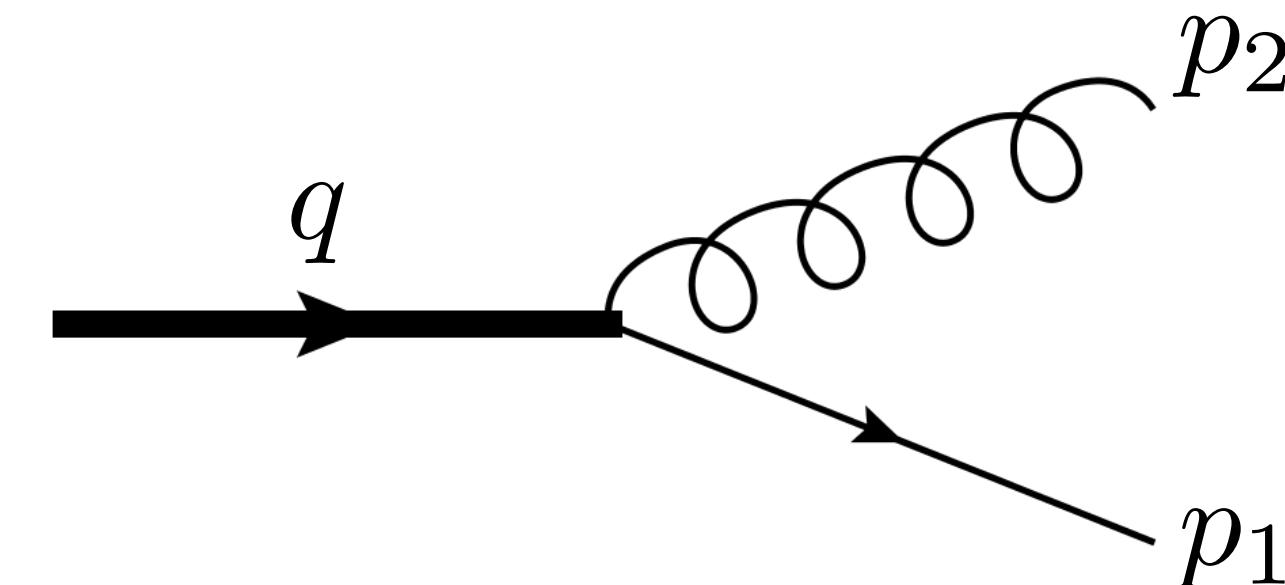
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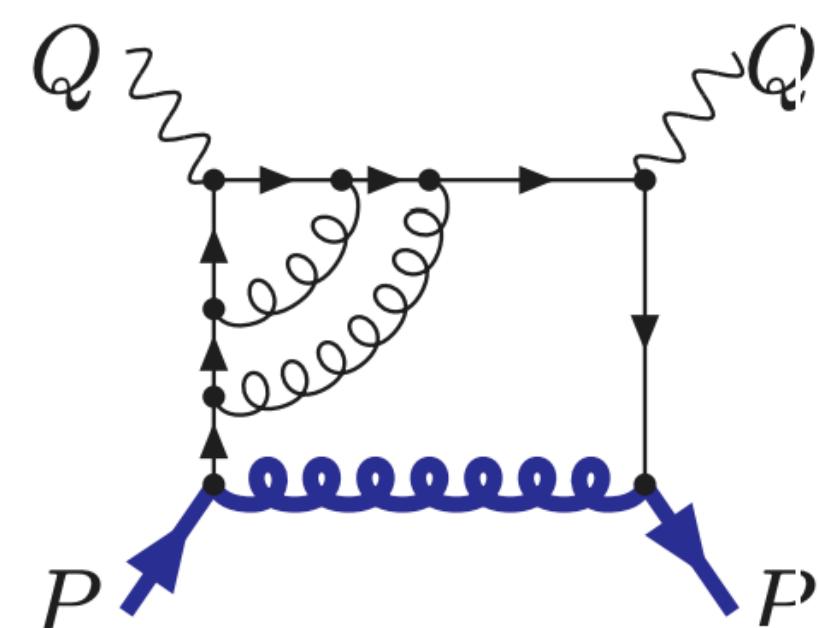
$$q^2 = (p_1 + p_2)^2 > 0$$

Cannot be fixed by previous method!

Three-loop corrections

Moch, Vermaseren, Vogt, NPB 2004

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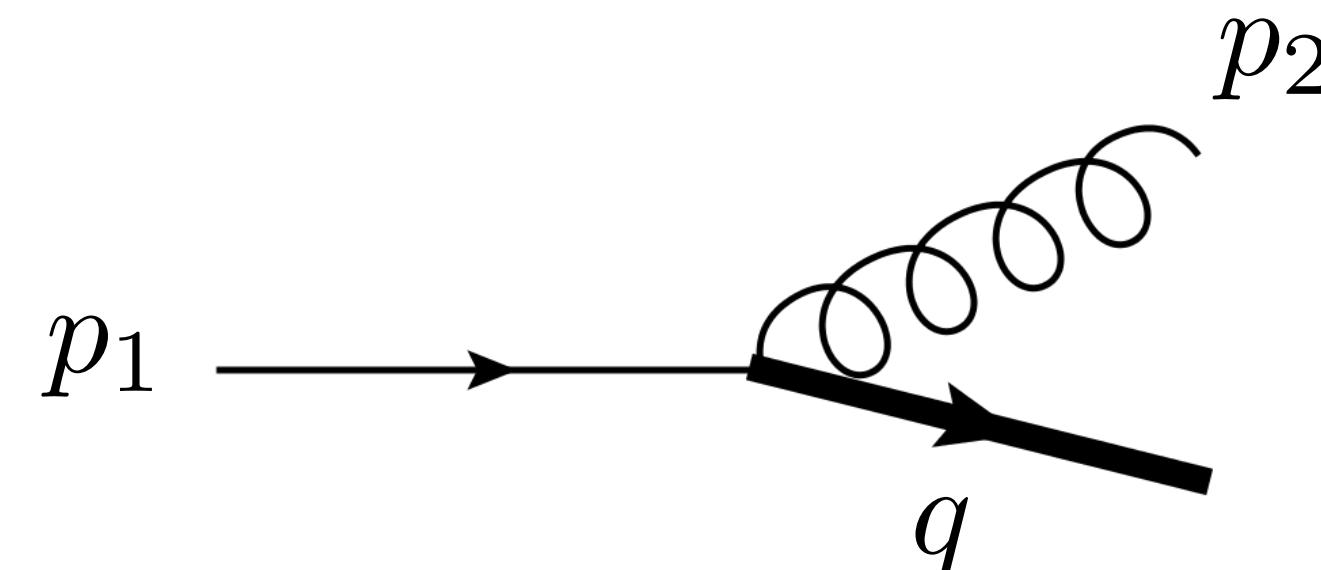


$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} D_q \\ D_g \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \end{pmatrix}$$

H. Chen, T.Z. Yang, HXZ, Y.J. Zhu,
CPC 2021

Completion of three-loop corrections to DGLAP

Space-like evolution

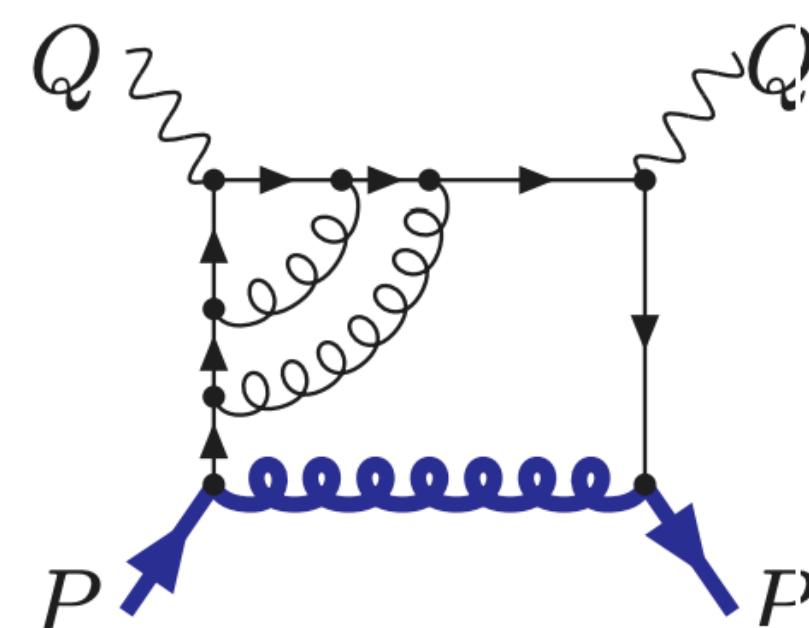


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Three-loop corrections

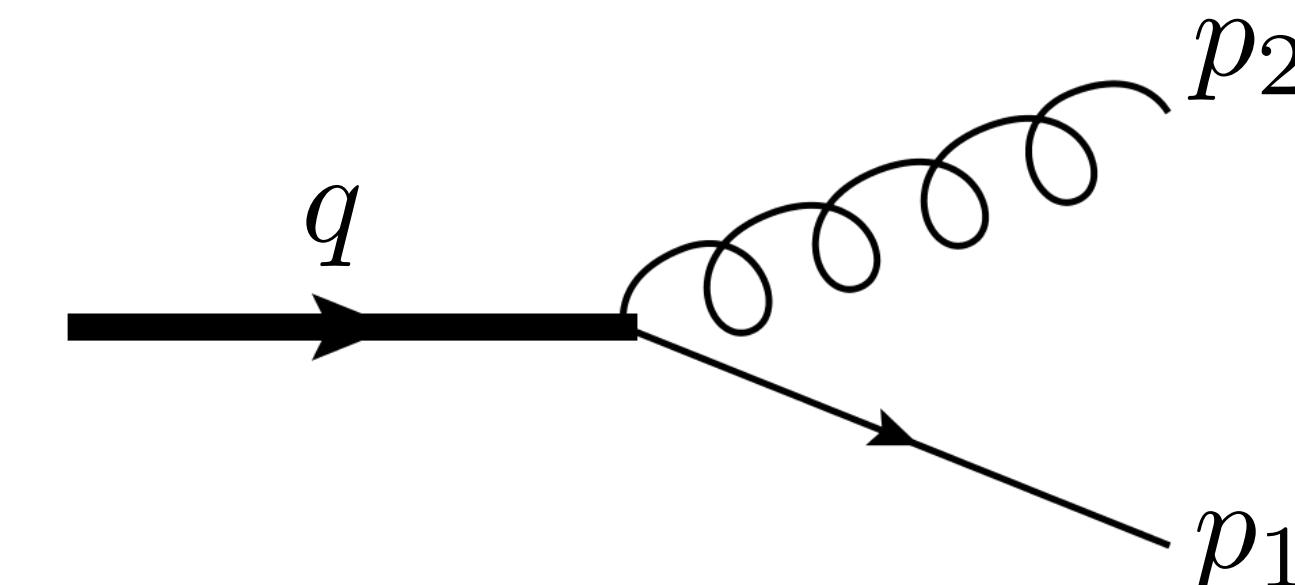
Moch, Vermaseren, Vogt, NPB 2004

Method:
Optical theorem + OPE



Analytical continuation?

Time-like evolution



$$q^2 = (p_1 + p_2)^2 > 0$$

No complete 3-loop results until 2020

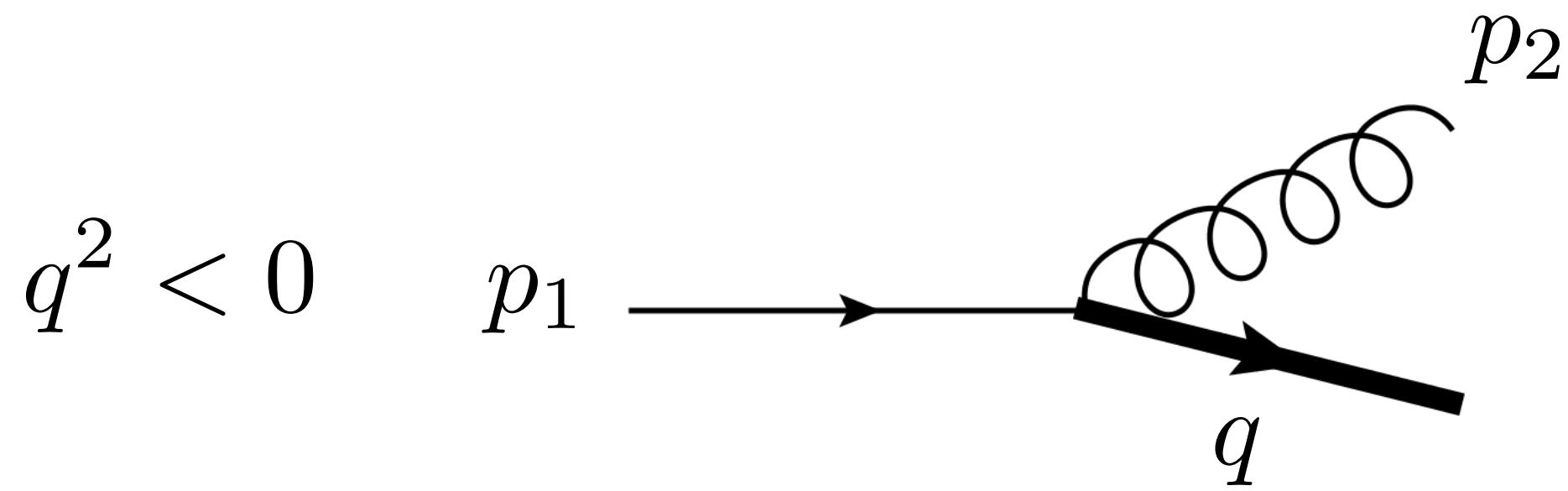
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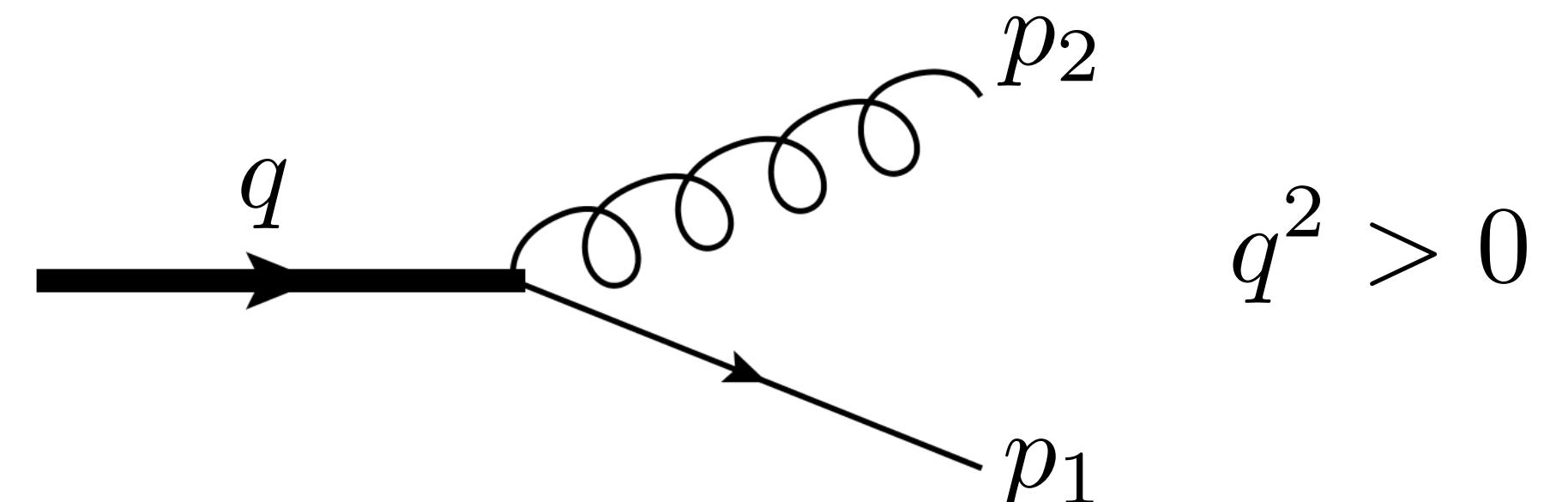
Establishing analytic continuation relation for DGLAP

Space-like evolution



$$q^2 < 0$$

Time-like evolution



$$q^2 > 0$$

Essentially local operator correlation

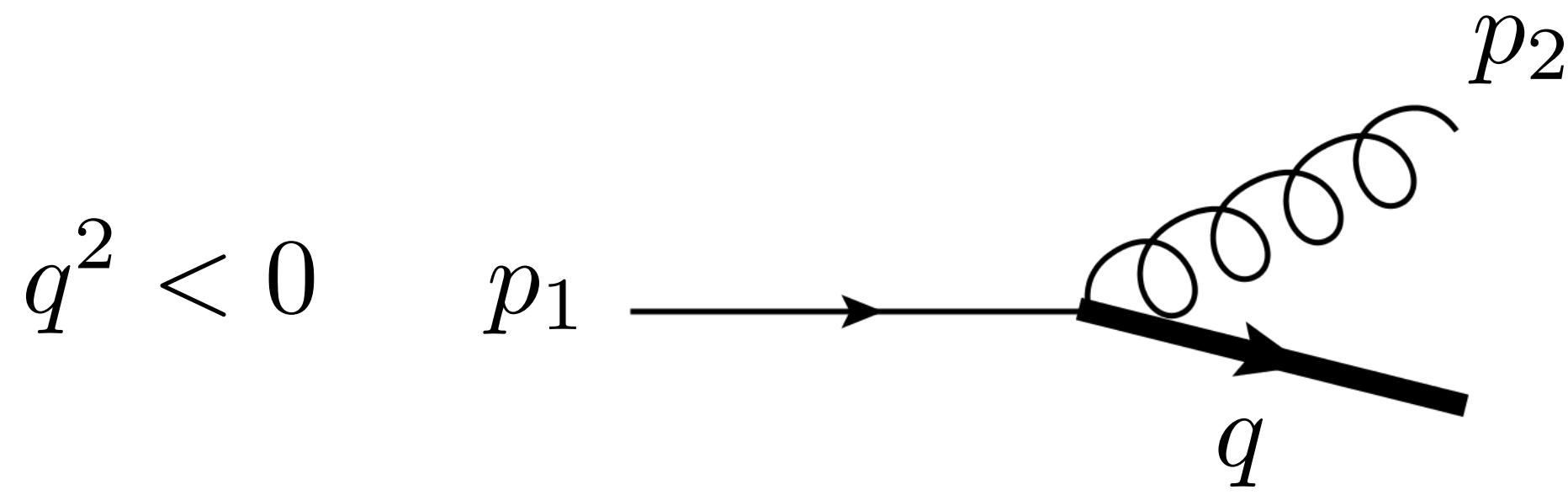
$$\int dy^- e^{ip^+ zy^-} \langle P | \bar{\psi}(y^-) \gamma^+ \psi(0) | P \rangle$$

Intrinsic non-local operator correlation

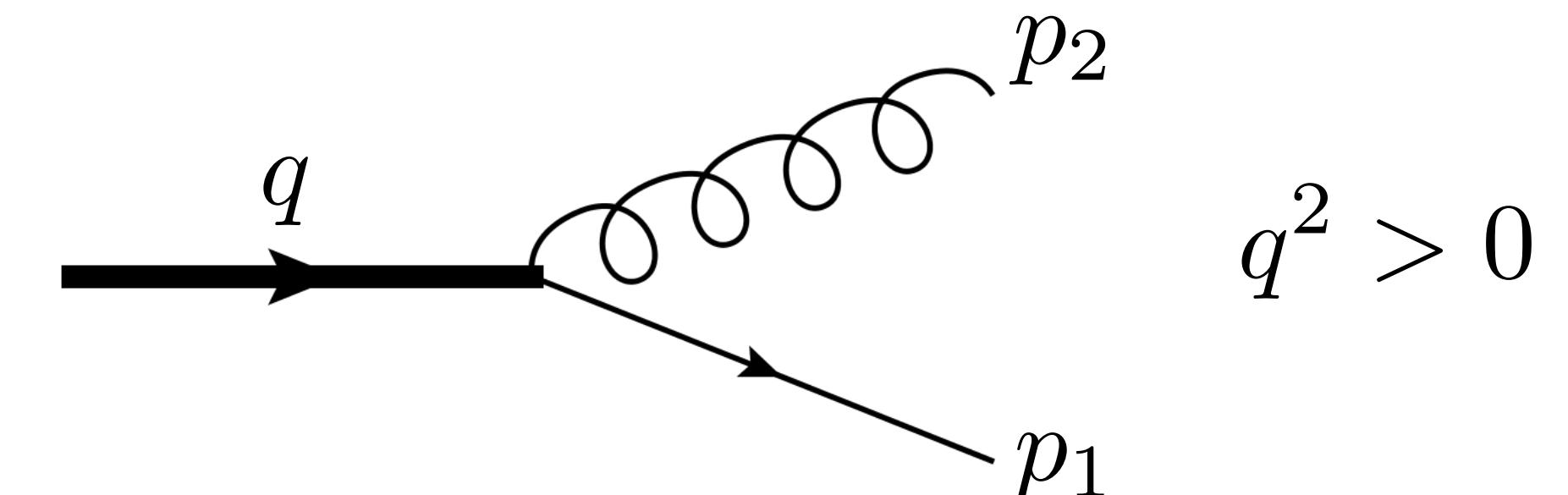
$$\int dy^- e^{ip^+ y^- / z} \langle 0 | \bar{\psi}(y^-) | hX \rangle \langle hX | \gamma^+ \psi(0) | 0 \rangle$$

Establishing analytic continuation relation for DGLAP

Space-like evolution



Time-like evolution



Essentially local operator correlation

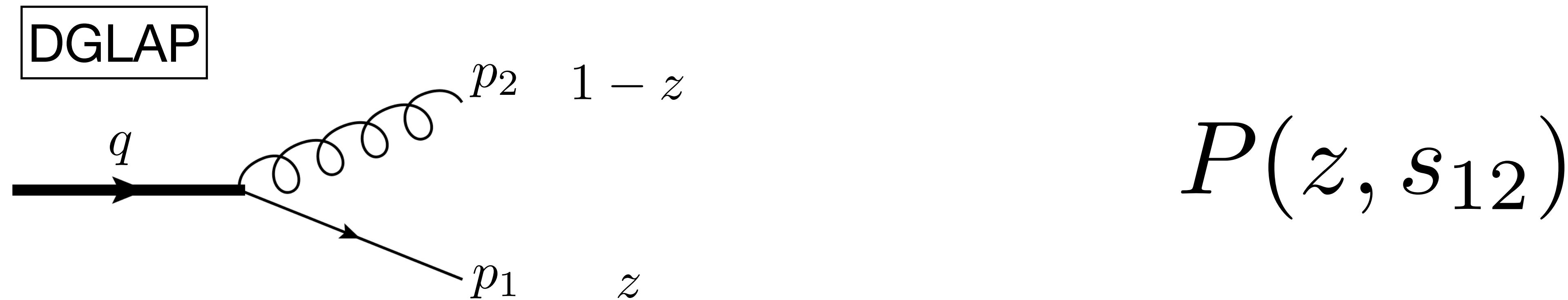
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Intrinsic non-local operator correlation

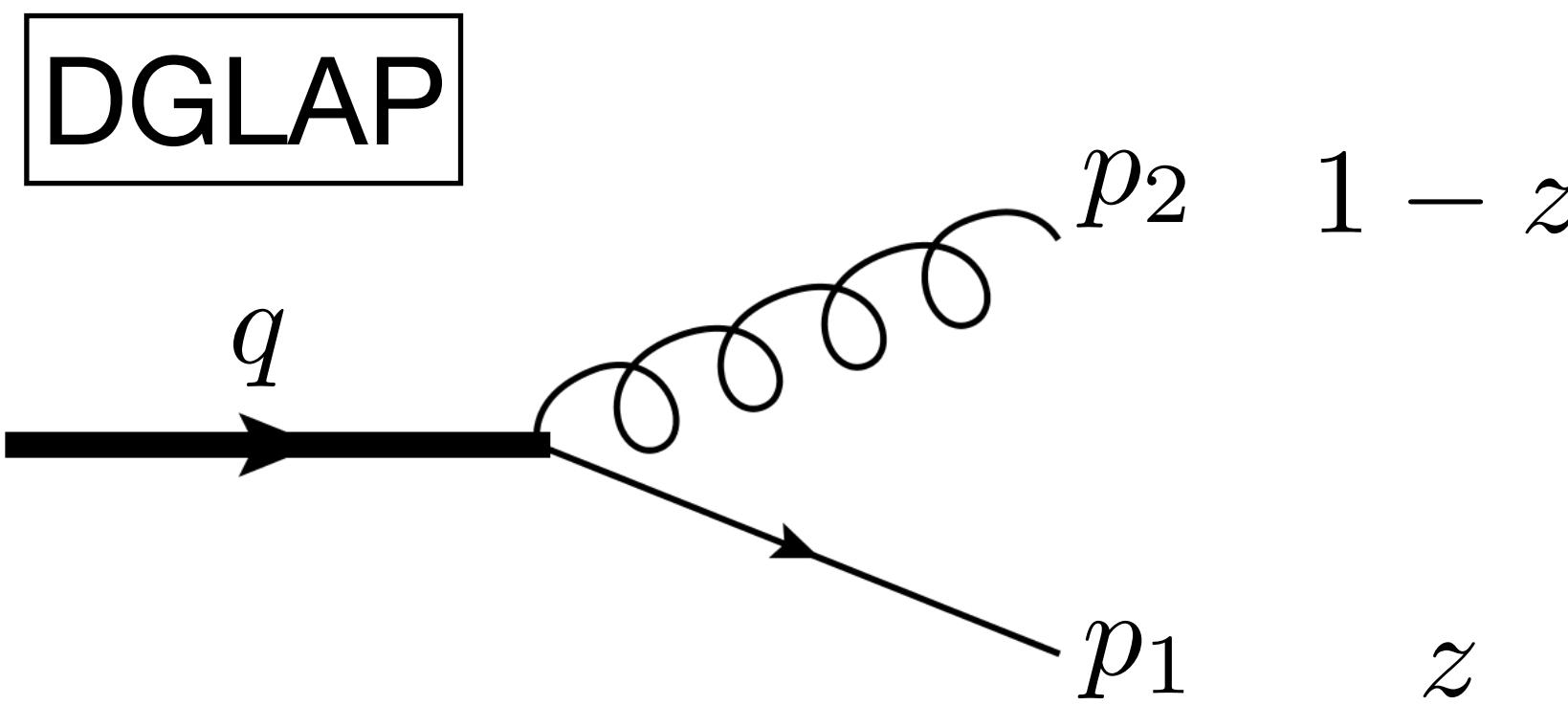
$$\int dy^- e^{ip^+ y^- / z} \langle 0 | \bar{\psi}(y^-) | hX \rangle \langle hX | \gamma^+ \psi(0) | 0 \rangle$$

$$\gamma_{\pm}^{\text{timelike}}(N) = \gamma_{\pm}^{\text{spacelike}}(N - \gamma_{\pm}^{\text{timelike}}(N))$$

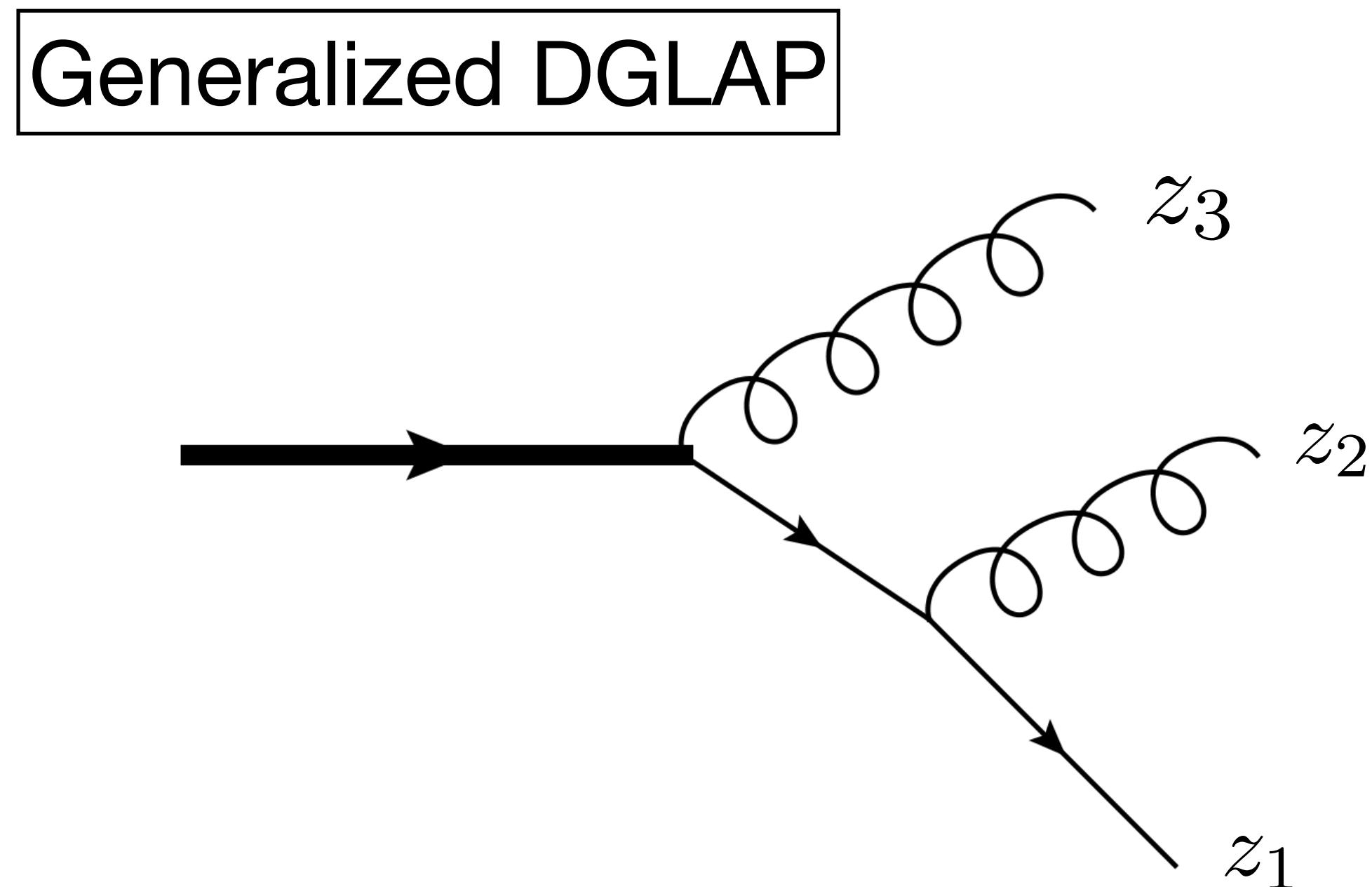
Generalizing DGLAP?



Generalizing DGLAP?

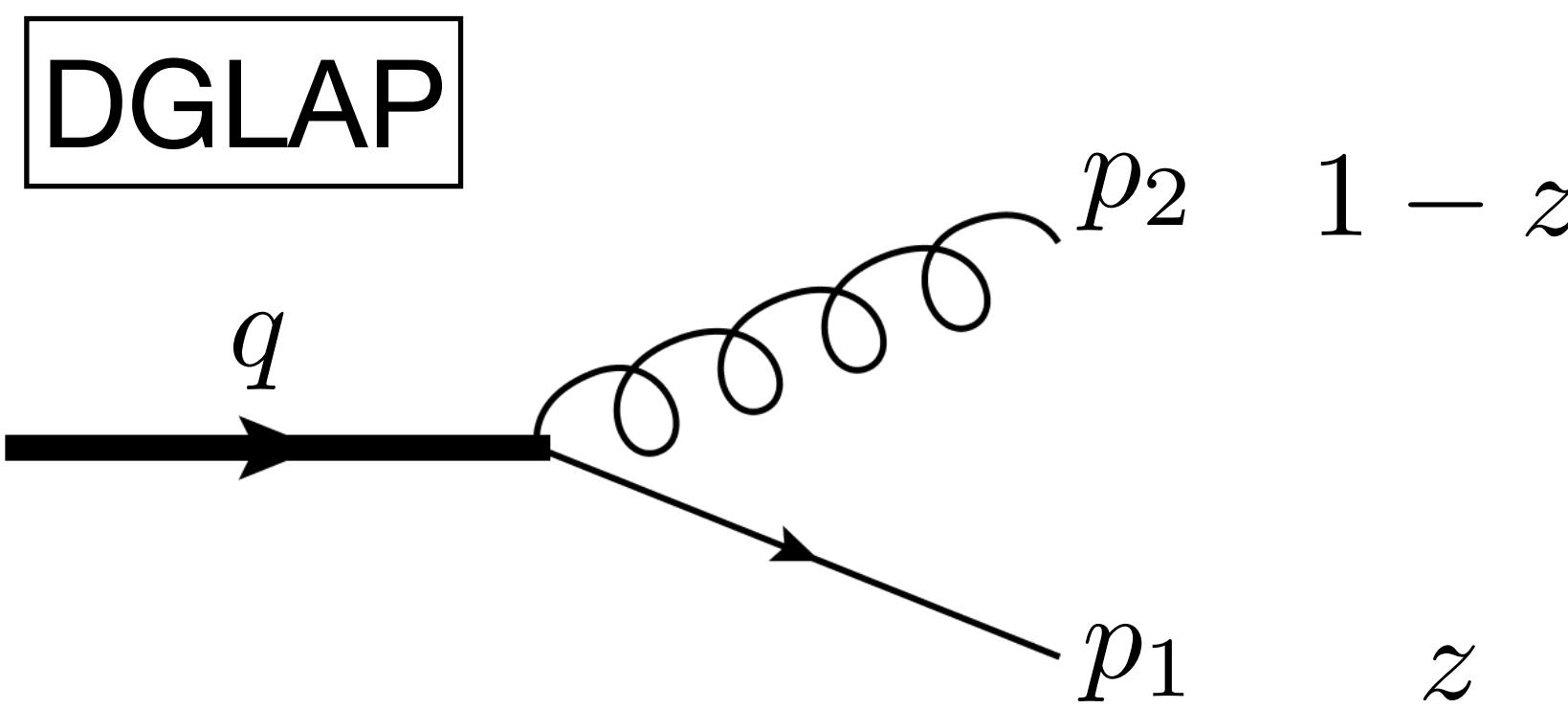


$$P(z, s_{12})$$



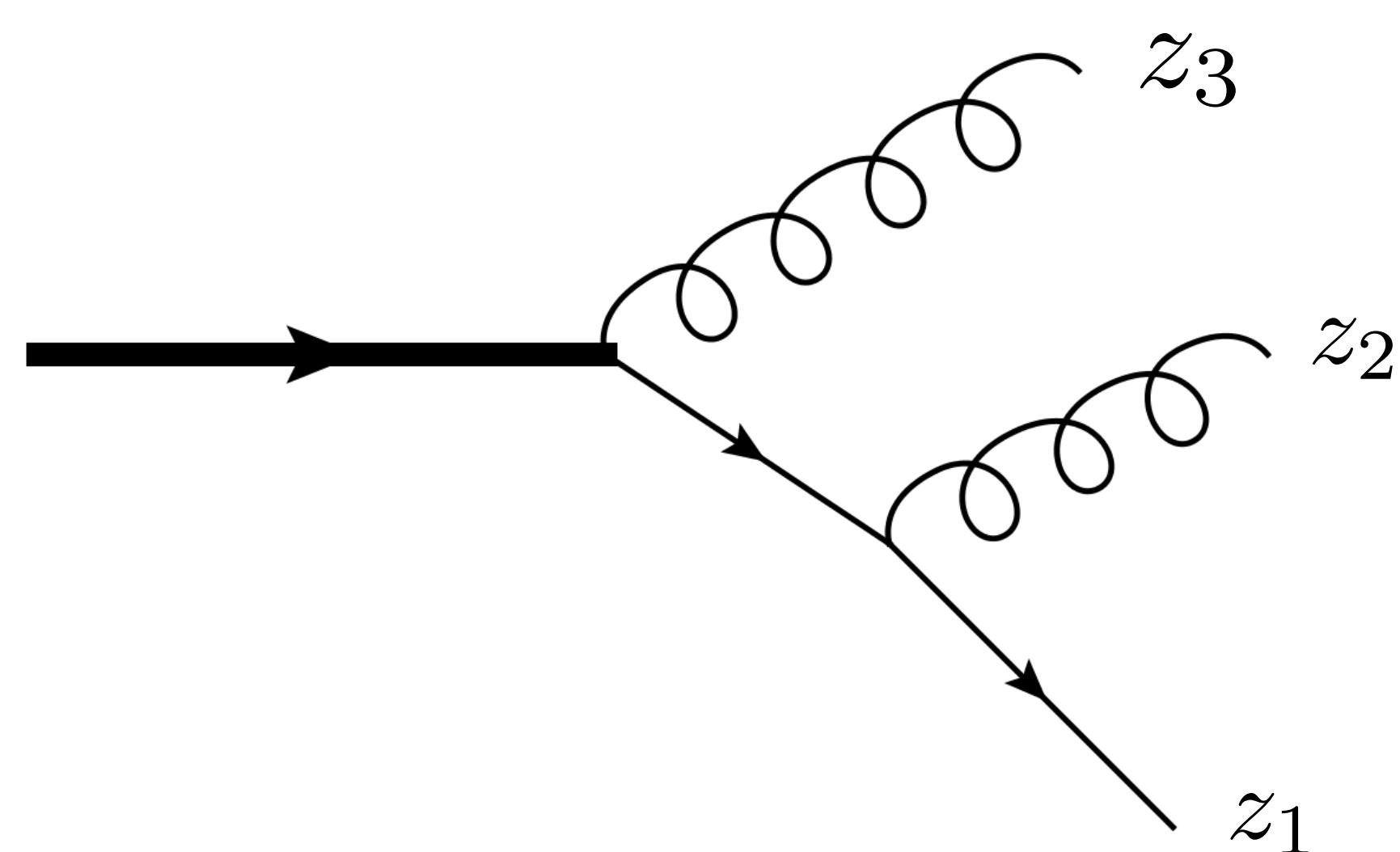
$$P(z_1, z_2, z_3; s_{12}, s_{23}, s_{31})$$

Generalizing DGLAP?



$$P(z, s_{12})$$

Generalized DGLAP



$$P(z_1, z_2, z_3; s_{12}, s_{23}, s_{31})$$

Integrate out z_i

Energy flow correlation
 $G(\theta_{12}, \theta_{23}, \theta_{31})$

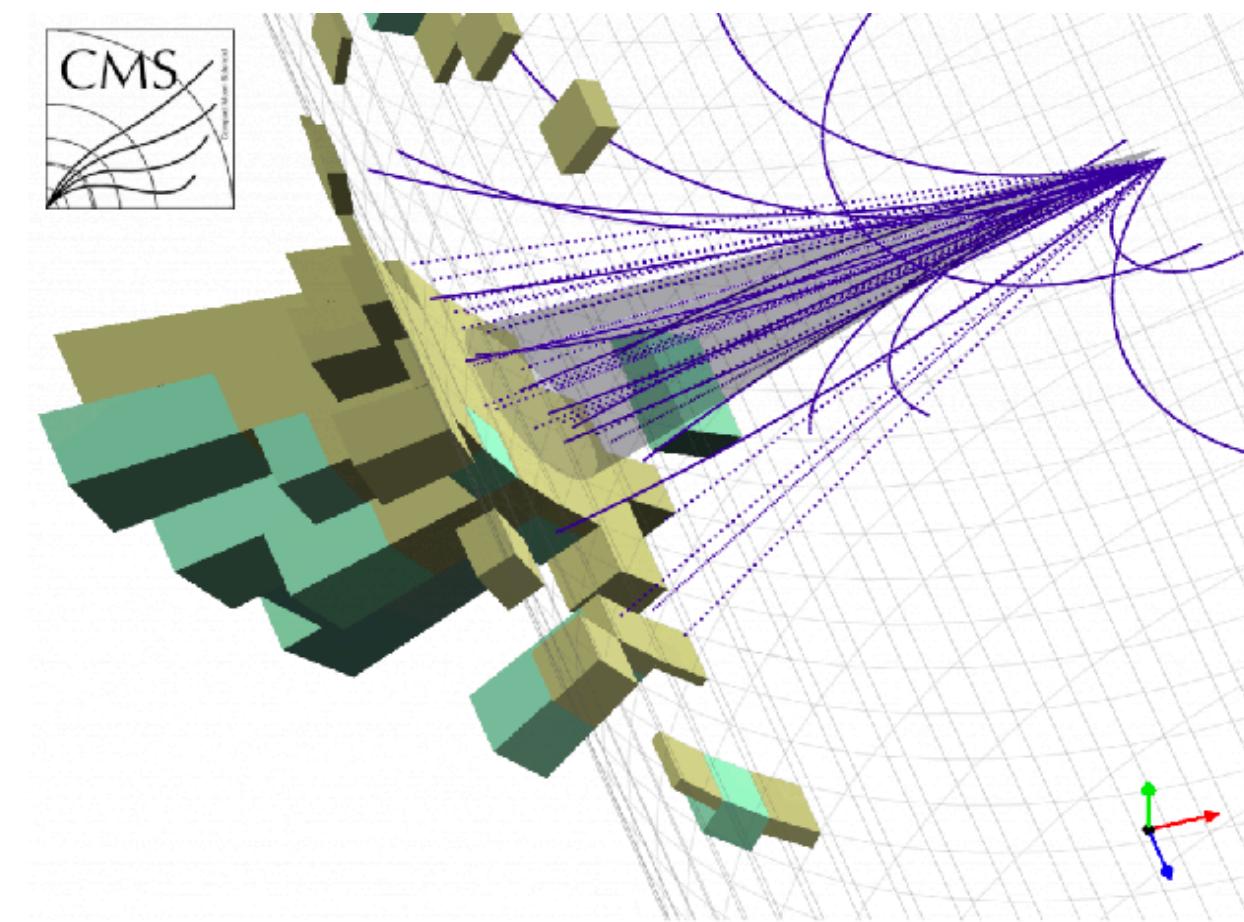
Integrate out s_{ij}

Multi-collinear splitting
 $K(z_1, z_2, z_3)$

Two approaches to jet structure

H ↘

Shape observables

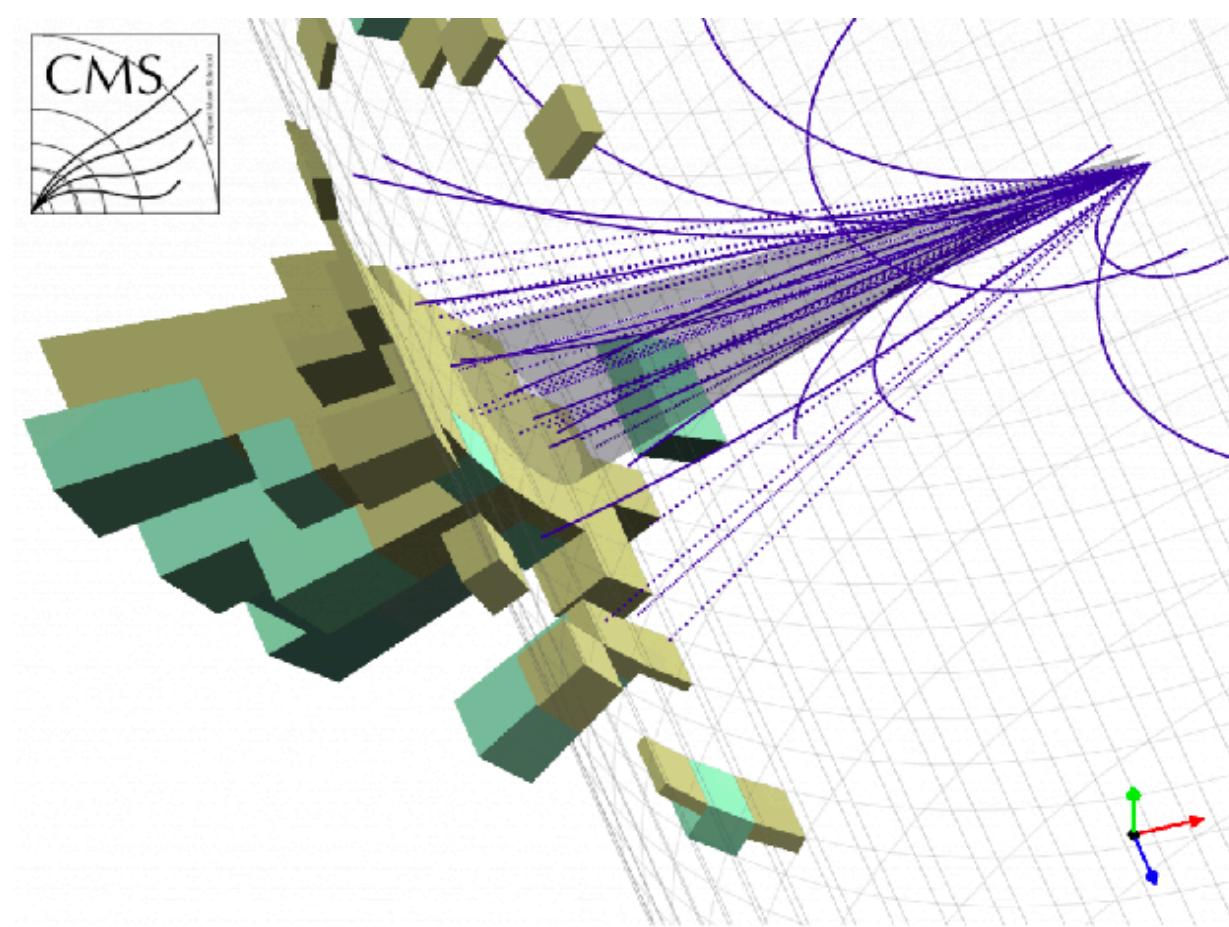


$$= \{p_1, p_2, \dots, p_n\}$$

↗ $\langle s_i s_j \rangle$

Statistical correlation

Two approaches to jet structure



$$= \{p_1, p_2, \dots, p_n\}$$

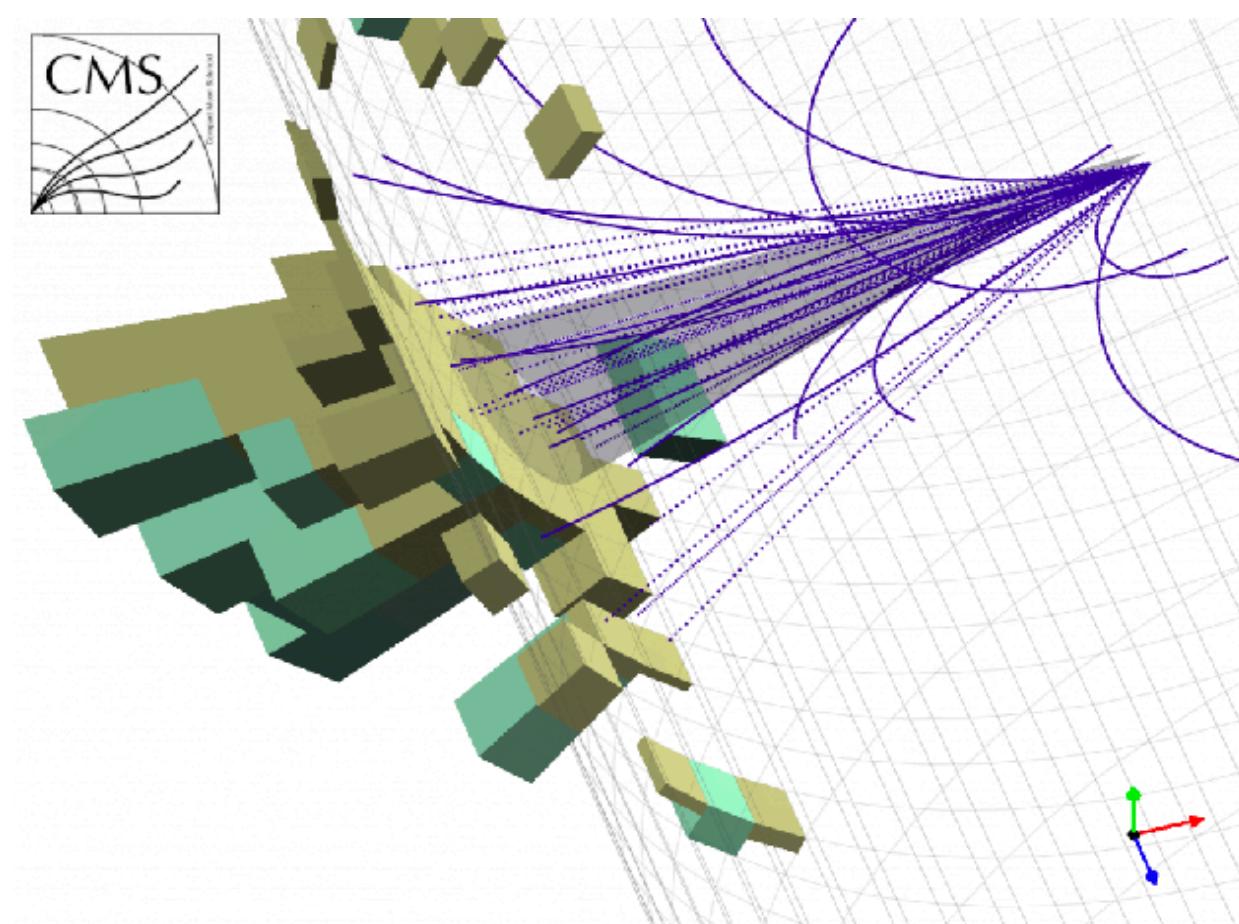
H
Shape observables

$\langle s_i s_j \rangle$
Statistical correlation

$$\text{Observable} = f(p_1, p_2, \dots, p_n)$$

- Example: jet mass, jet broadening, thrust
- Advantage: close connection with new physics search
- Disadvantage: lack direct connection with field theory

Two approaches to jet structure



$$= \{p_1, p_2, \dots, p_n\}$$

H ↘

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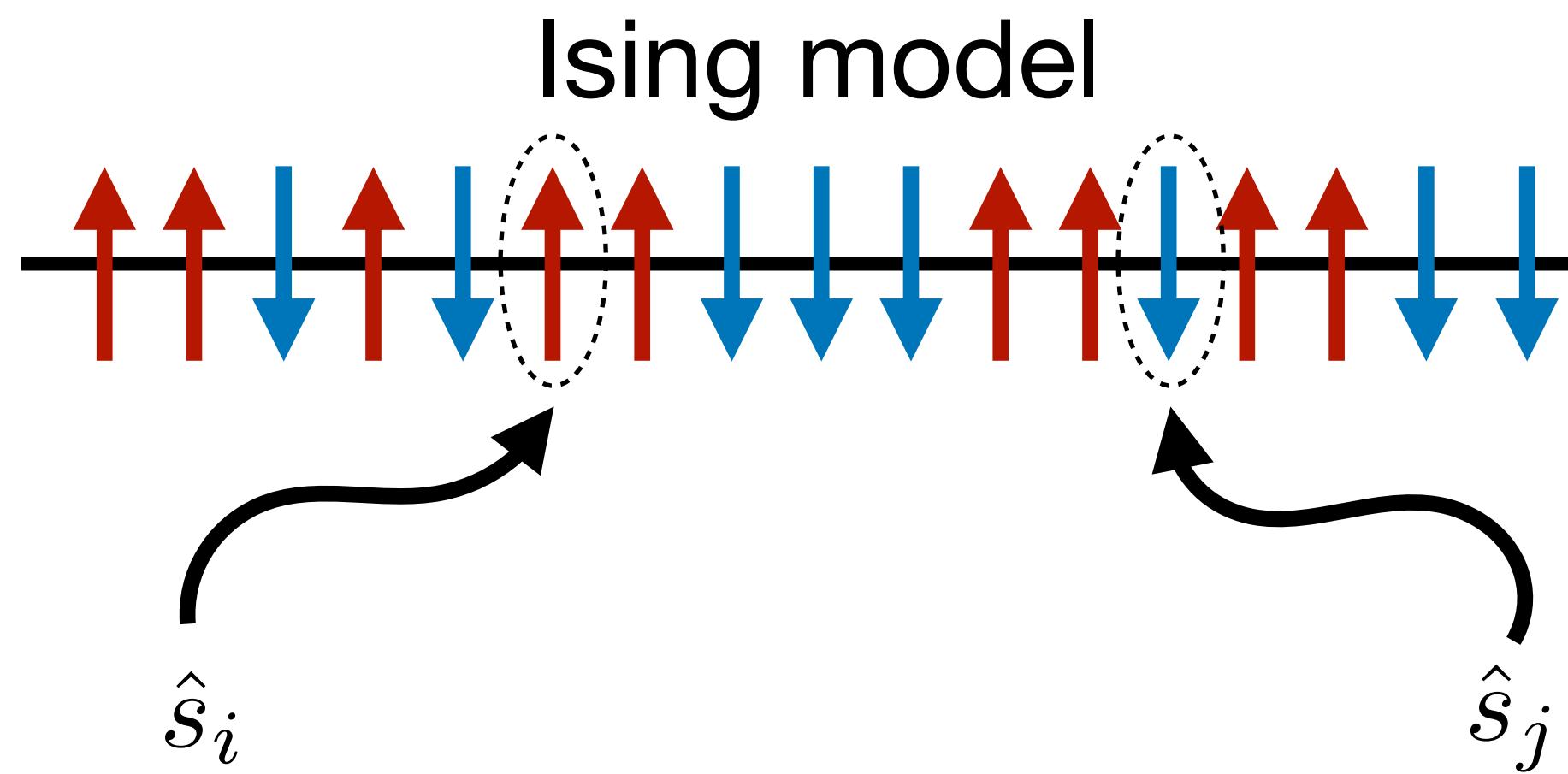
Statistical correlation

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle_{\Psi}, \quad \langle \mathcal{N}(n_1)\mathcal{N}(n_2) \rangle_{\Psi}, \dots$$

- Disadvantage: not yet suitable for new physics search
- Advantage:
 - Defined from most elementary field theory concept

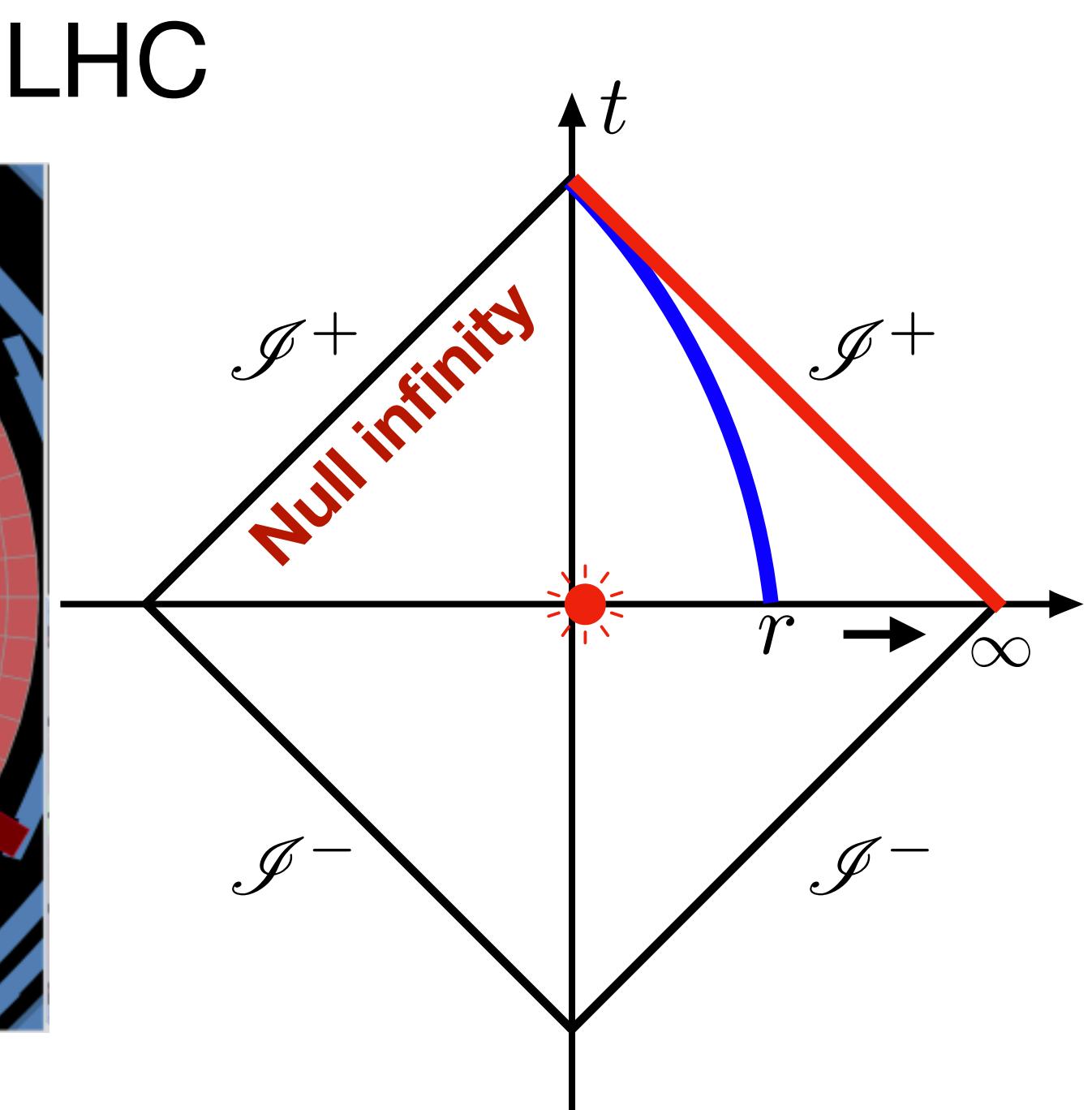
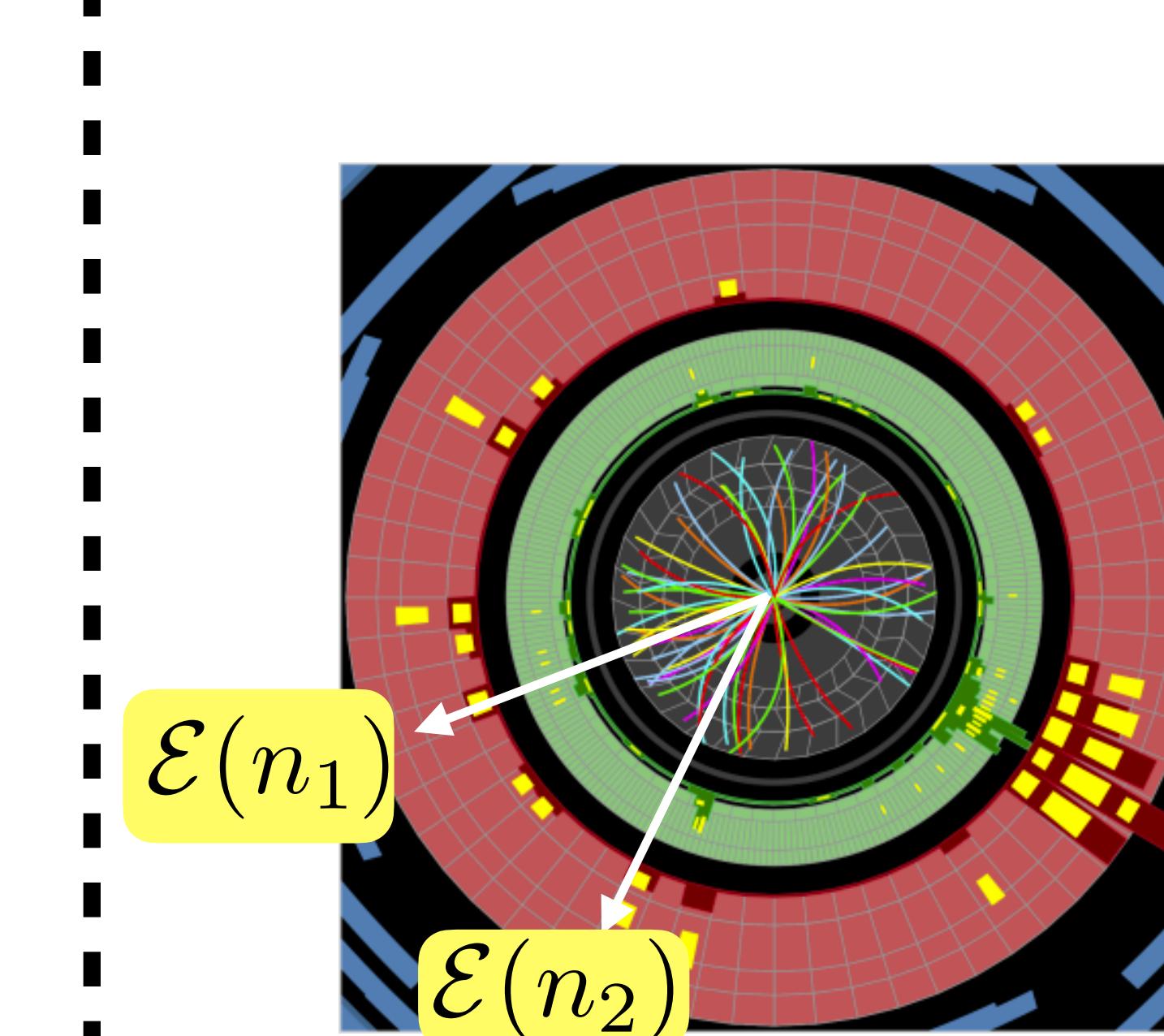
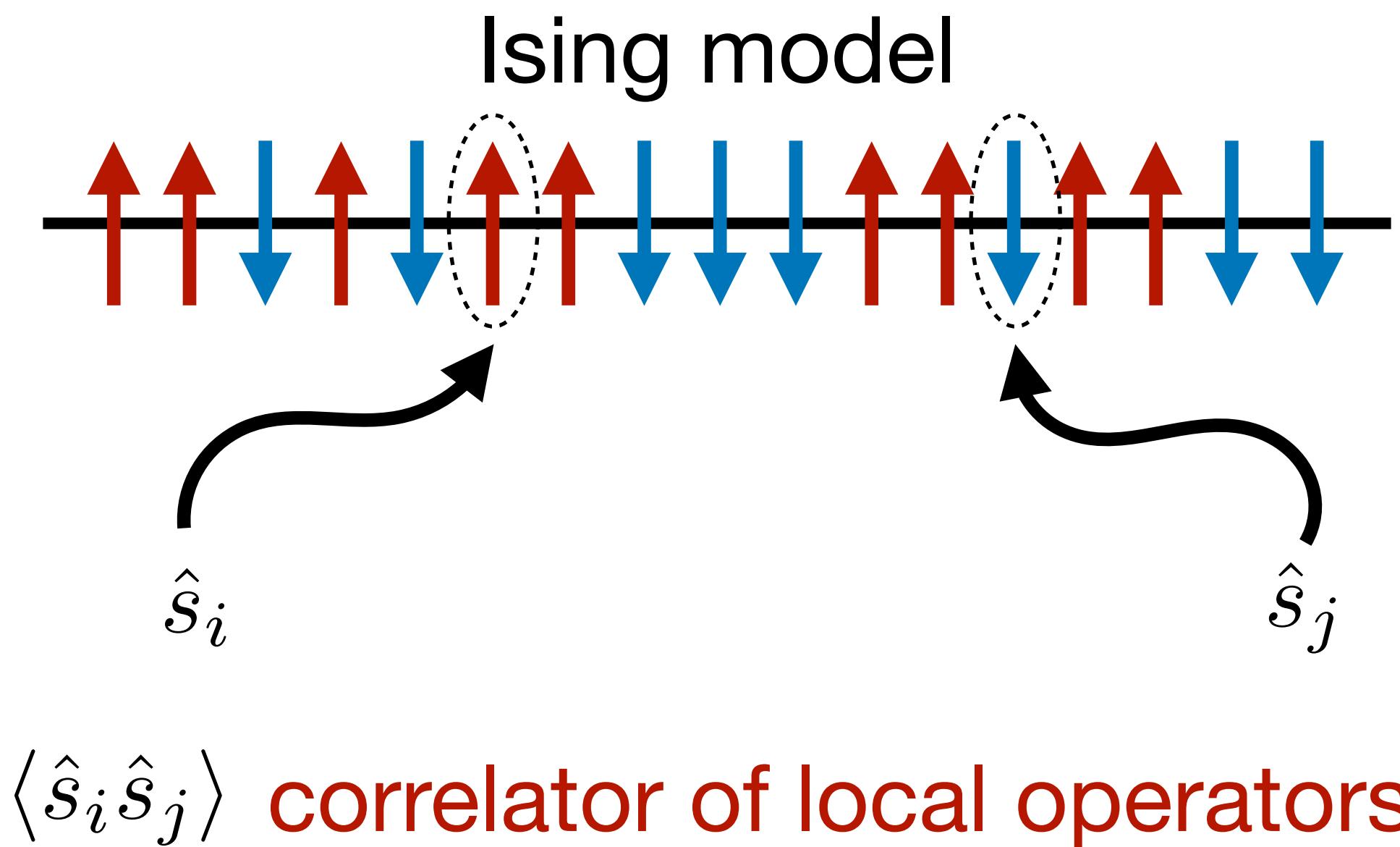
Basham, Brown, Ellis, Love, PRL, 1978;
H. Chen, I. Moult, X.Y. Zhang, HXZ, PRD, 2020

Spin correlation in Ising v.s. Energy flow correlation at the LHC



$\langle \hat{s}_i \hat{s}_j \rangle$ correlator of local operators

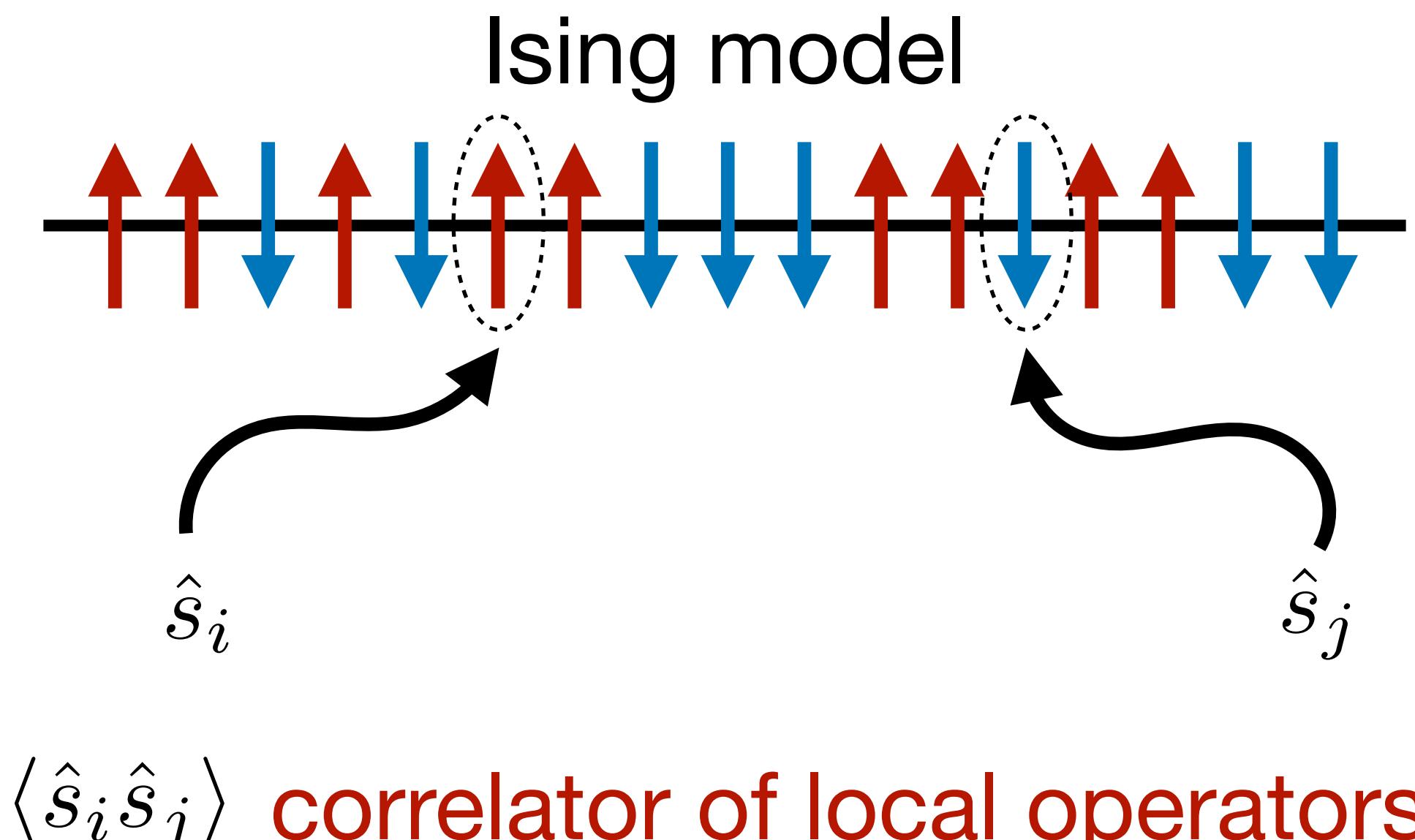
Spin correlation in Ising v.s. Energy flow correlation at the LHC



$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt \vec{n}_i T^{0i}(t, r\vec{n})$$

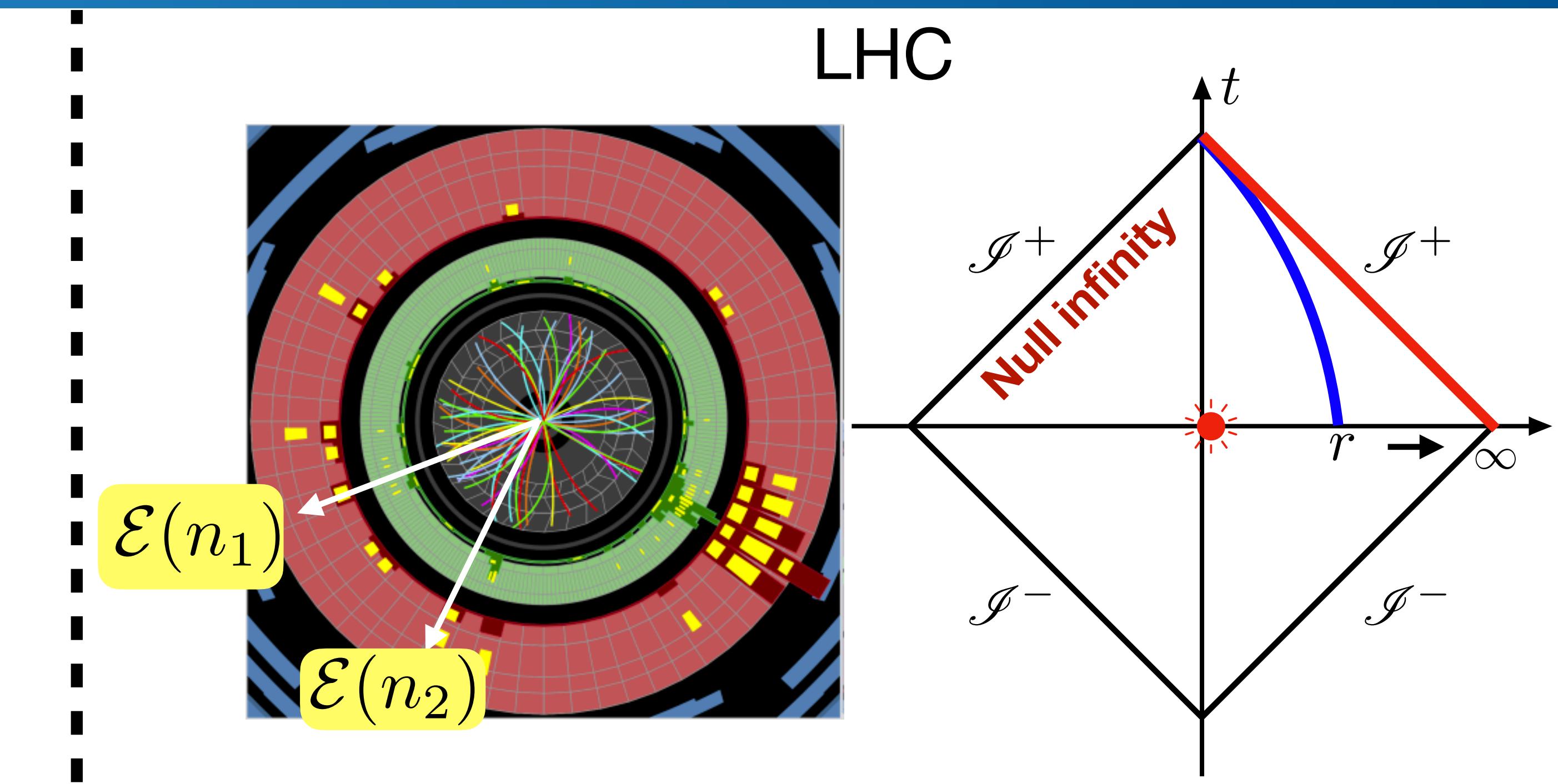
Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component

Spin correlation in Ising v.s. Energy flow correlation at the LHC



Phase transition and Scaling

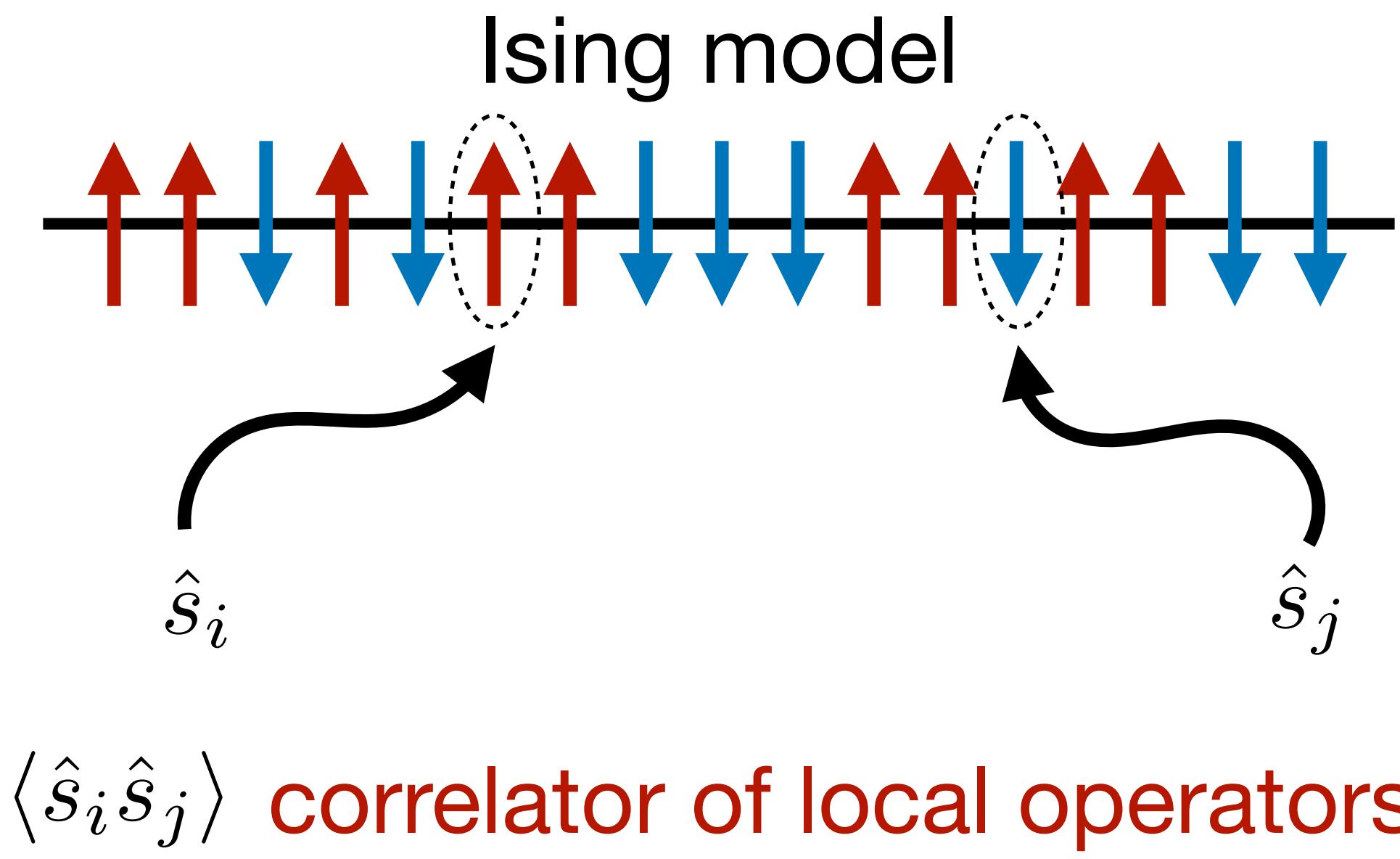
$$\langle \hat{s}_i \hat{s}_j \rangle = \begin{cases} \frac{1}{r^\nu} \exp\left(-\frac{r}{\xi}\right) & T > T_c \\ \frac{1}{r^\nu} & T \sim T_c \end{cases}$$



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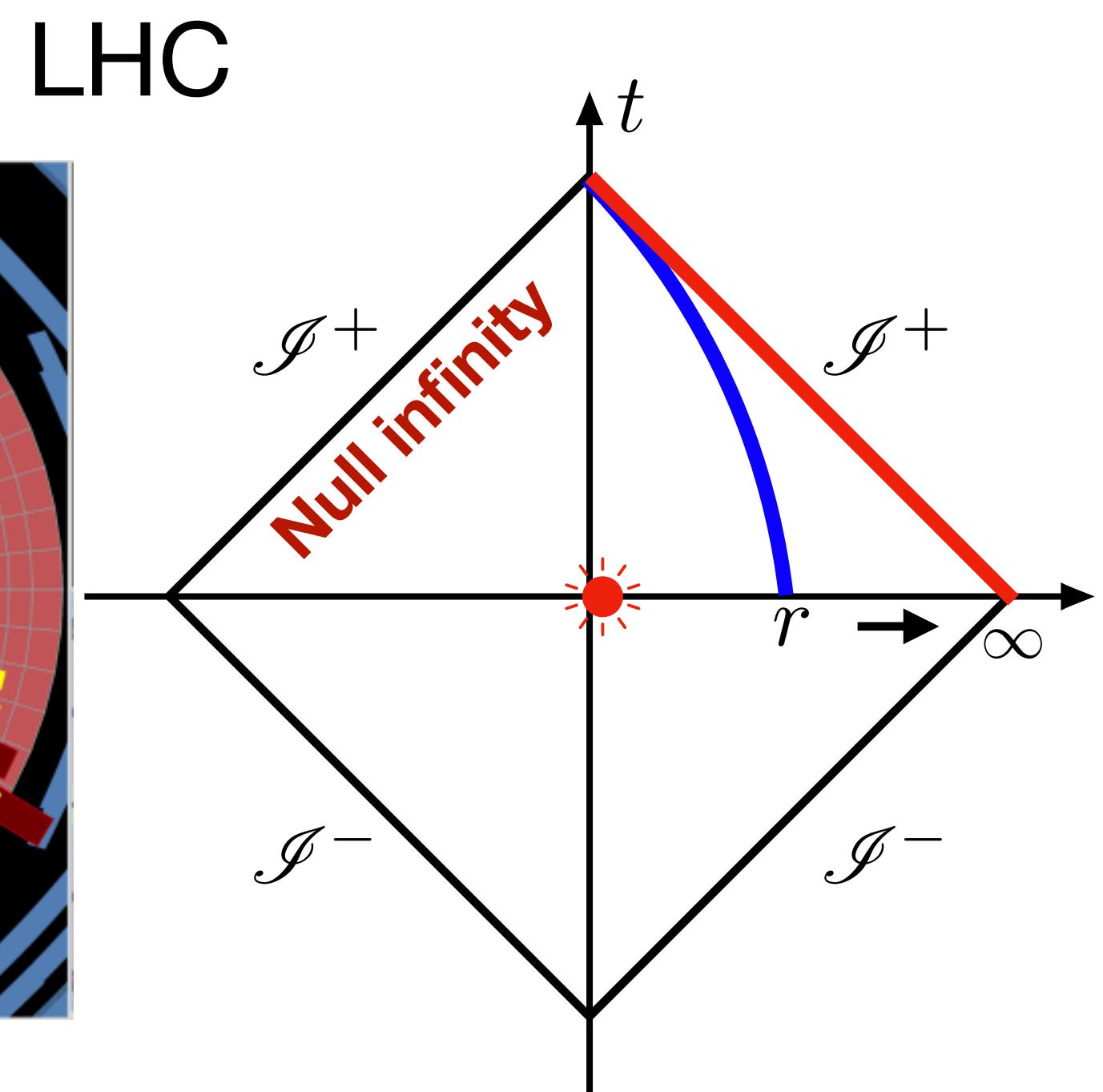
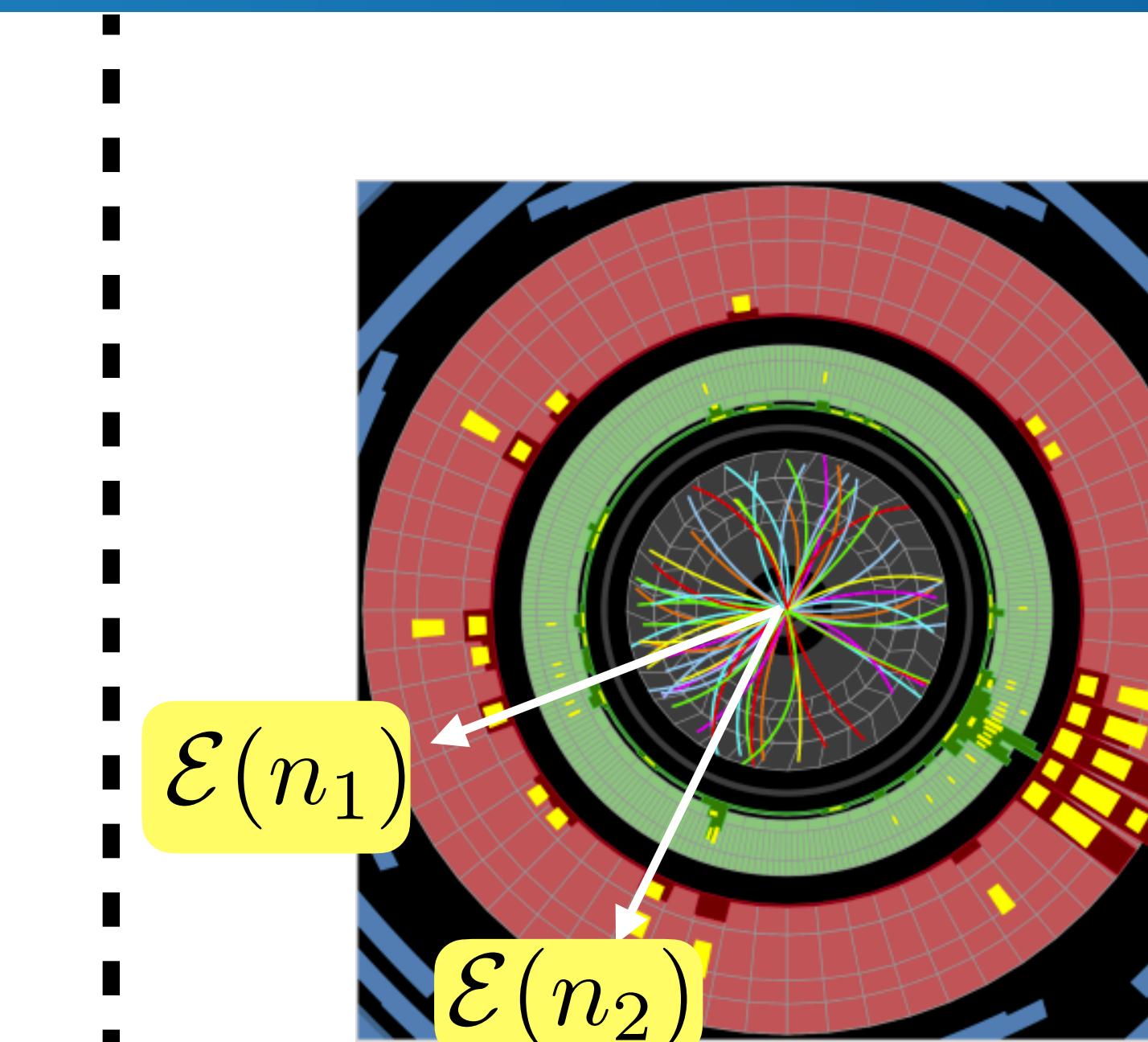
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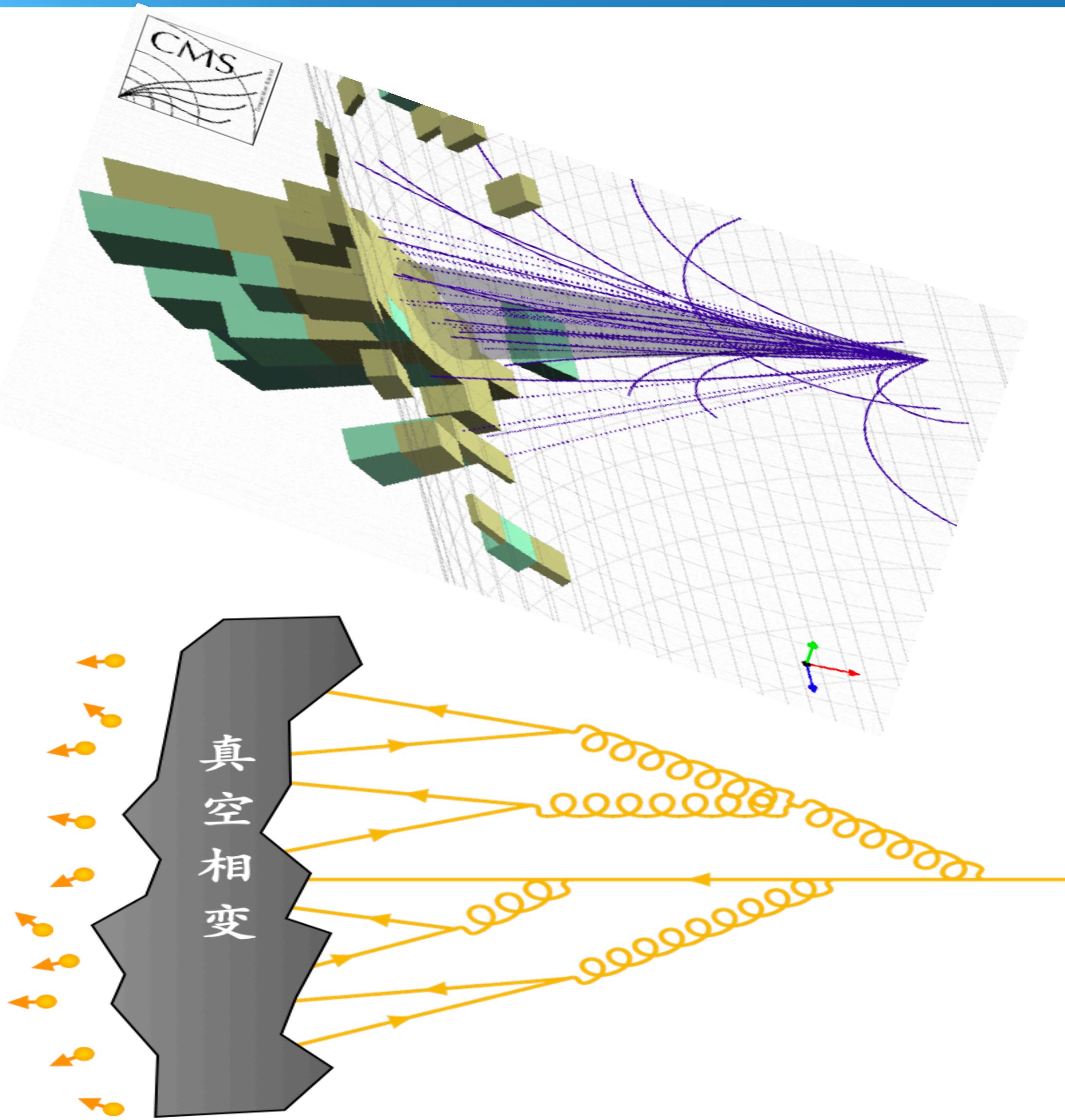


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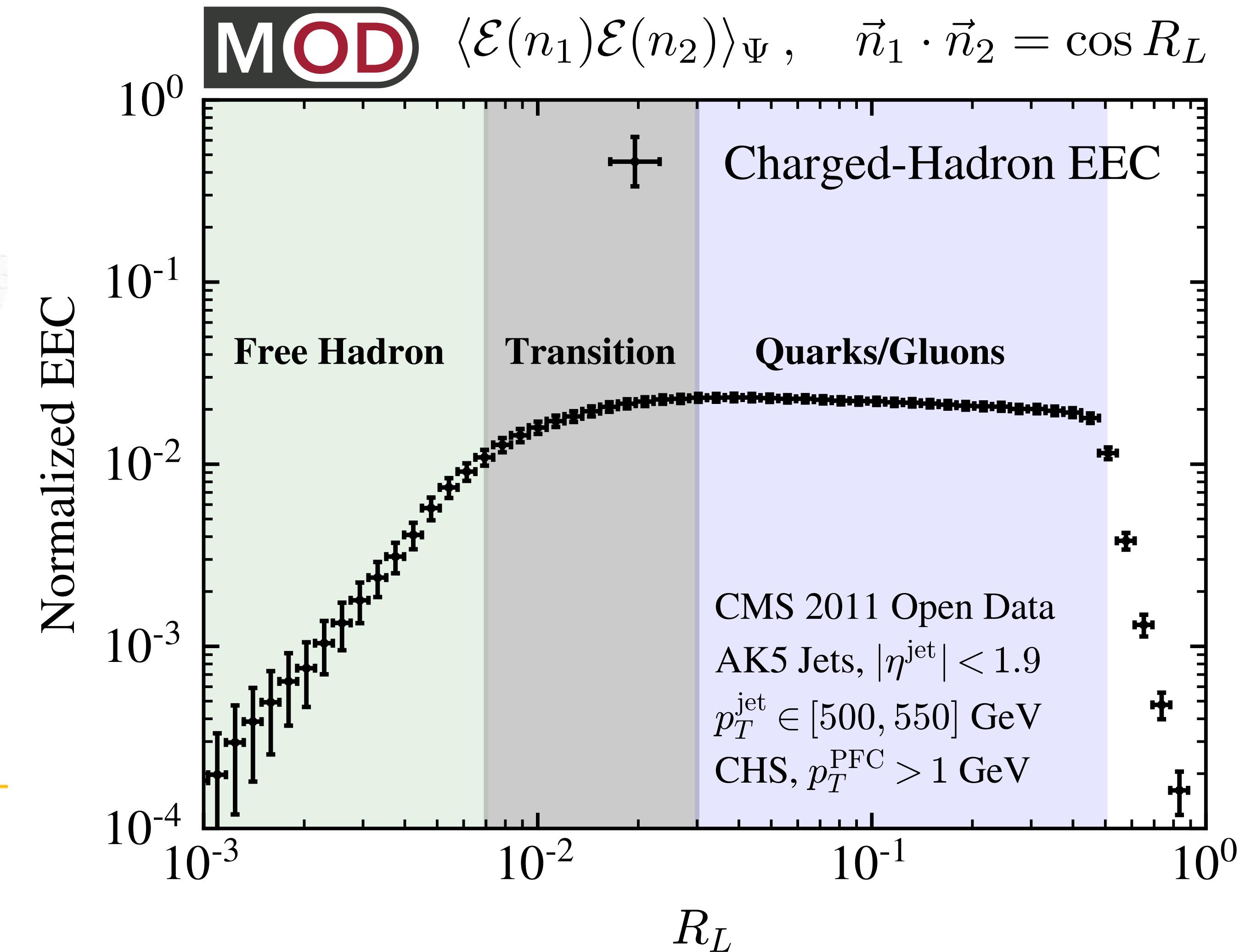
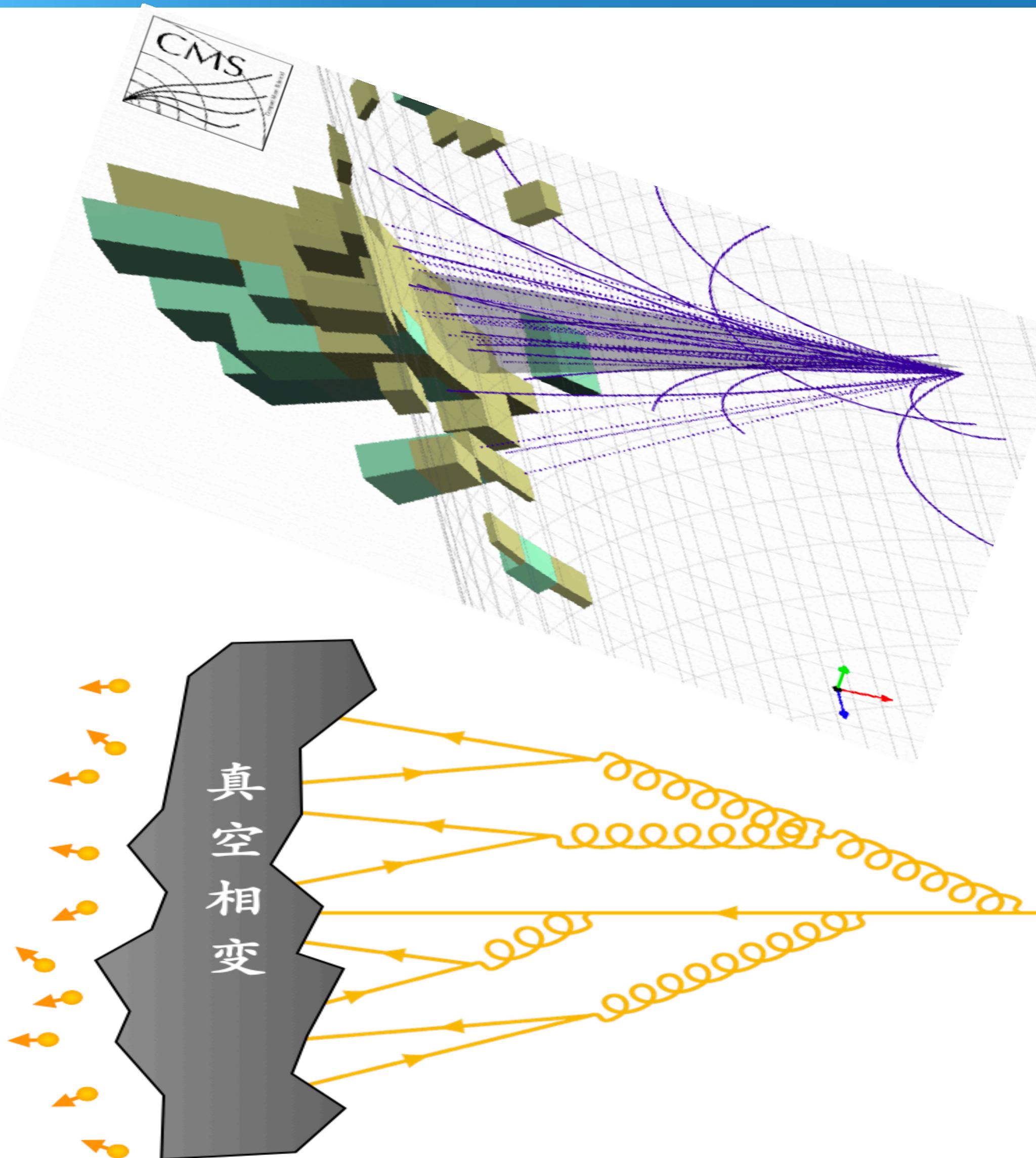
Energy weighted allow meaningful separate of perturbative and non-perturbative power suppressed component

What information is encoded in energy flow correlation?

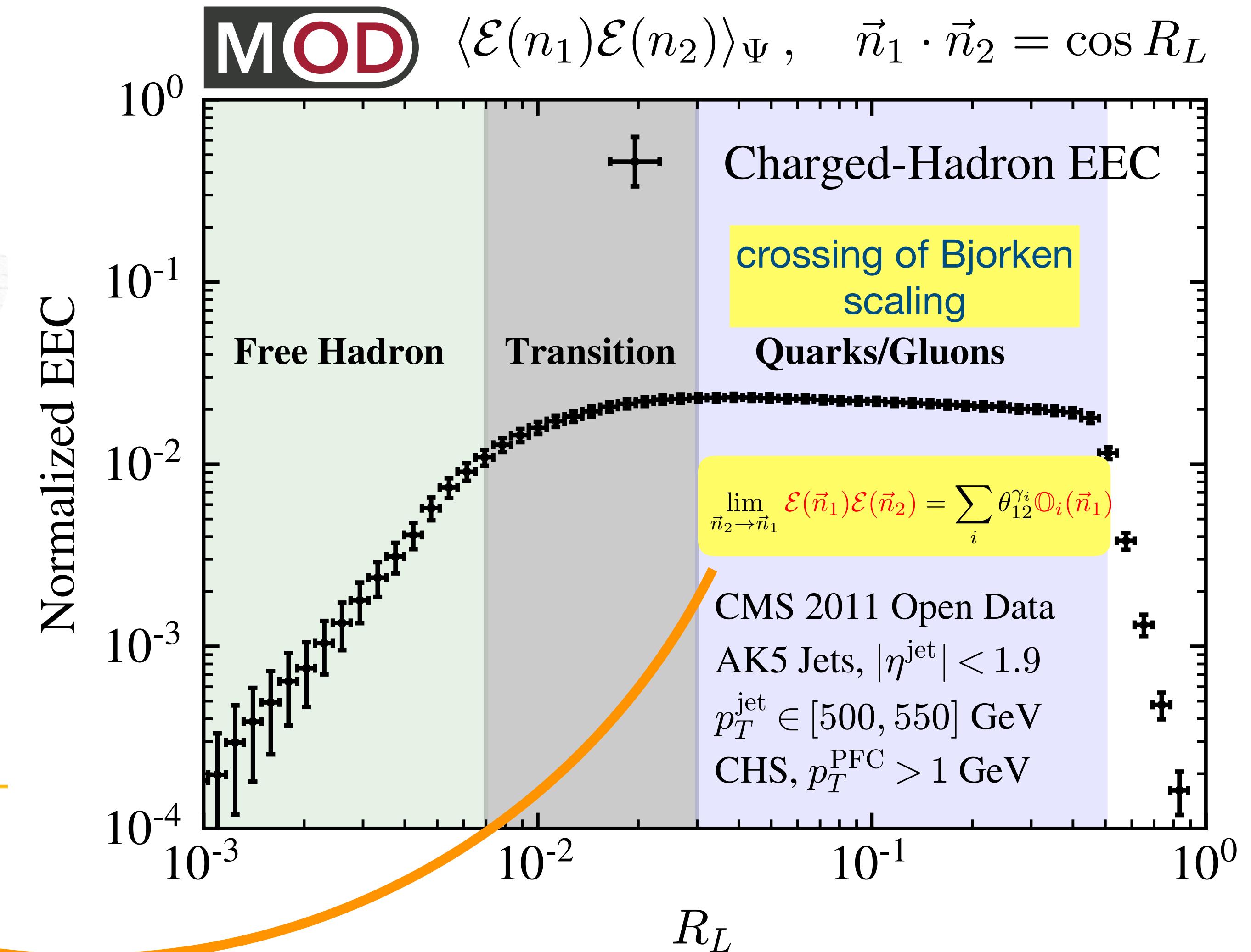
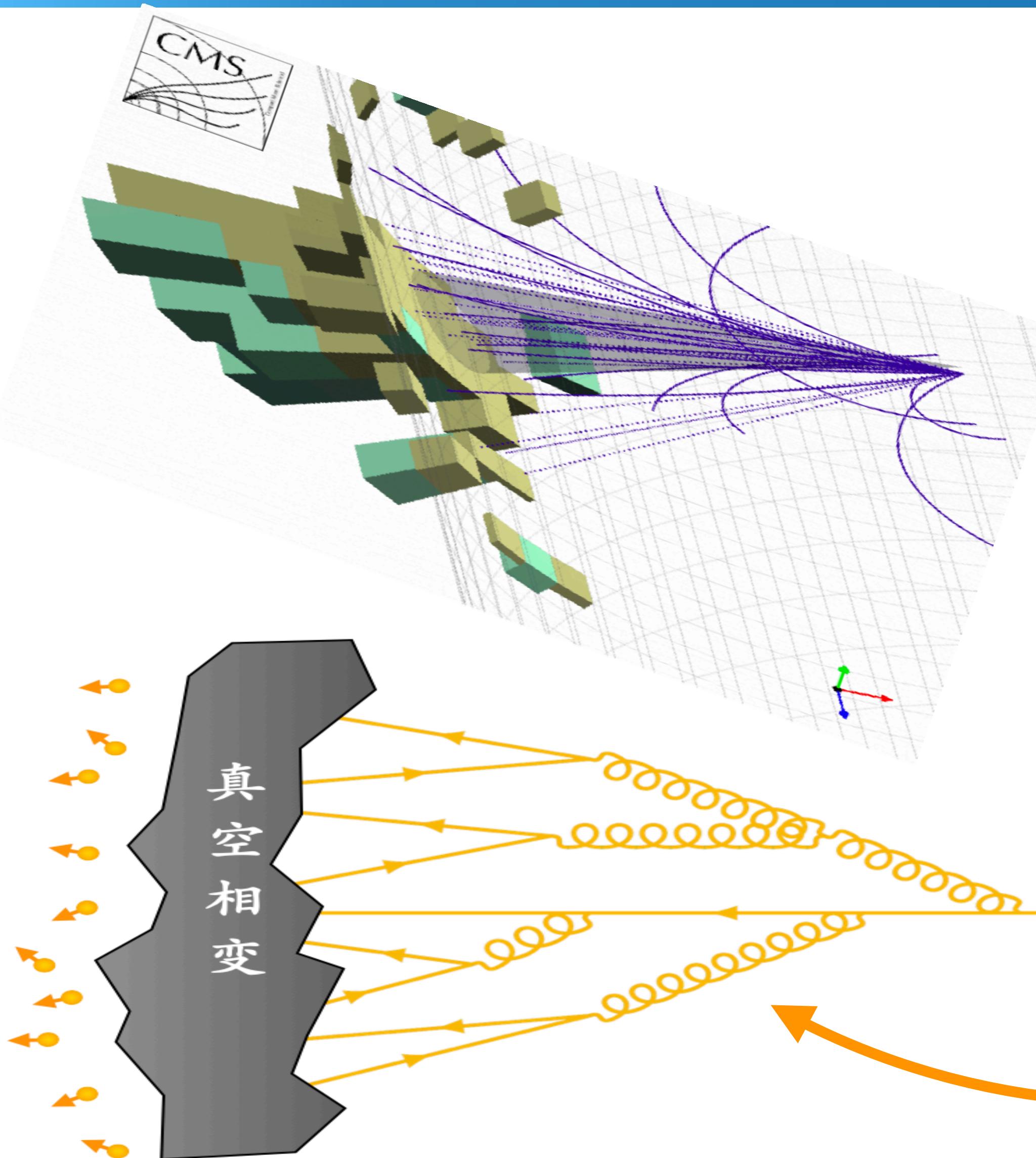
Seeing hadronization phase transition from EEC



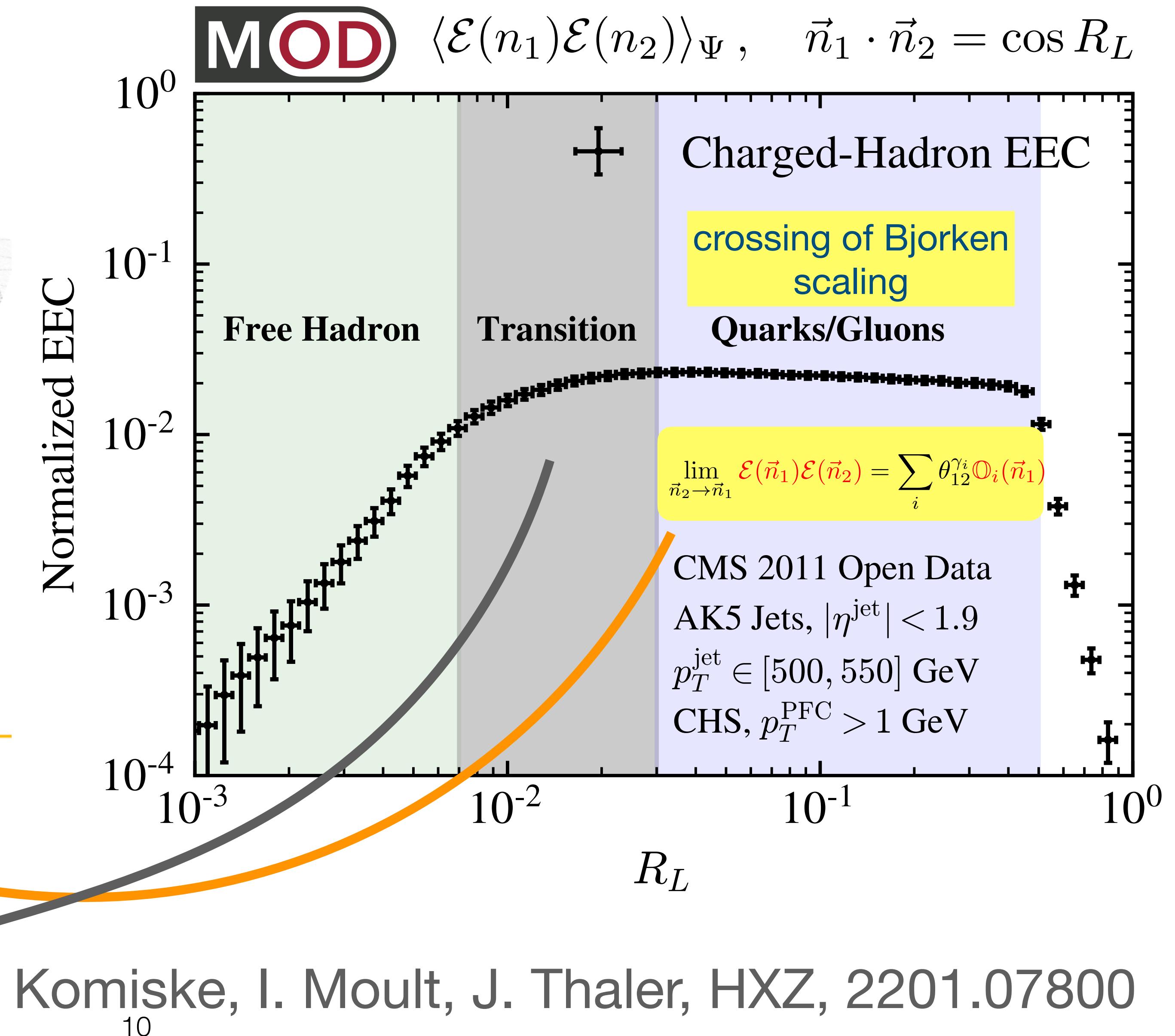
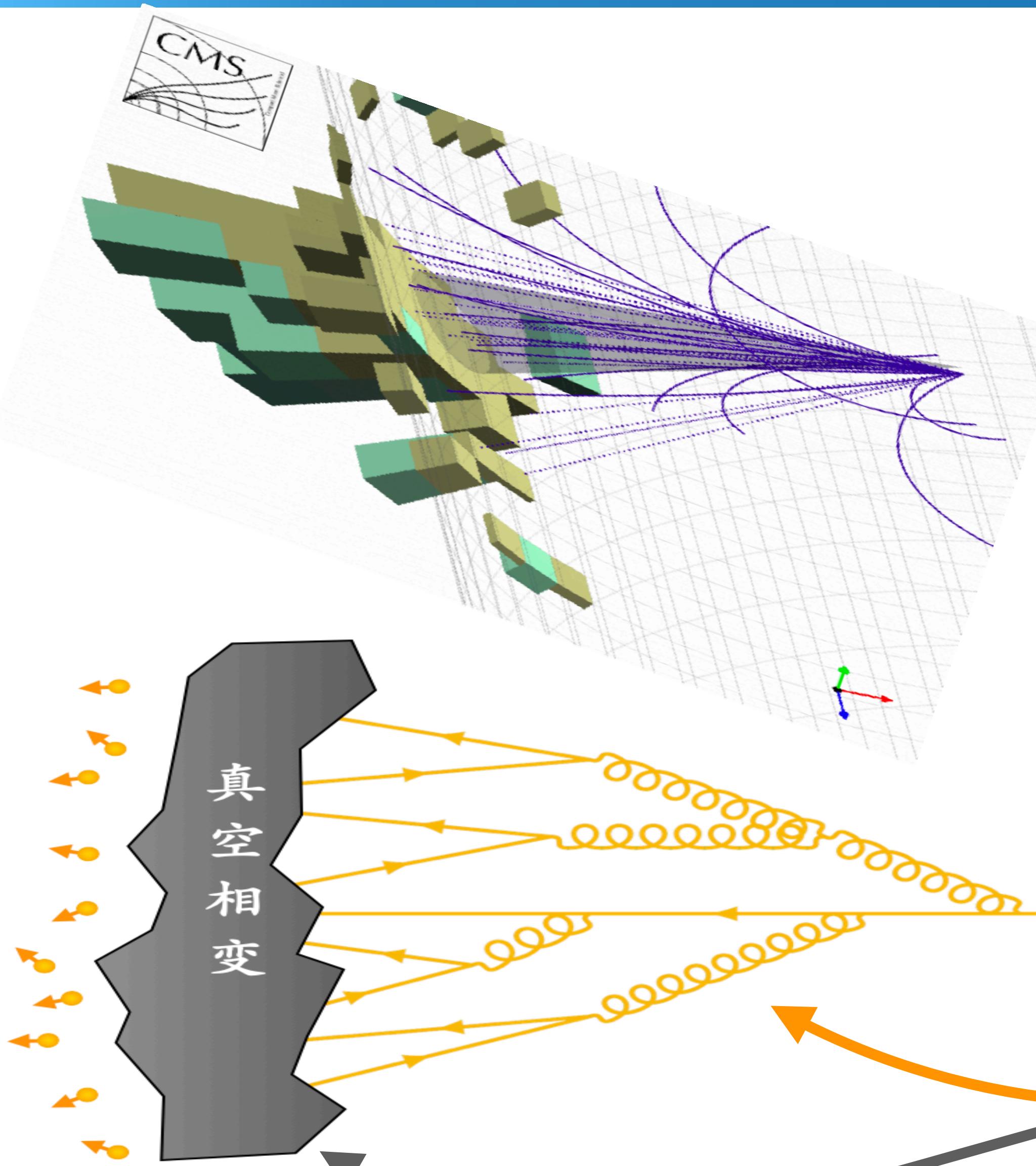
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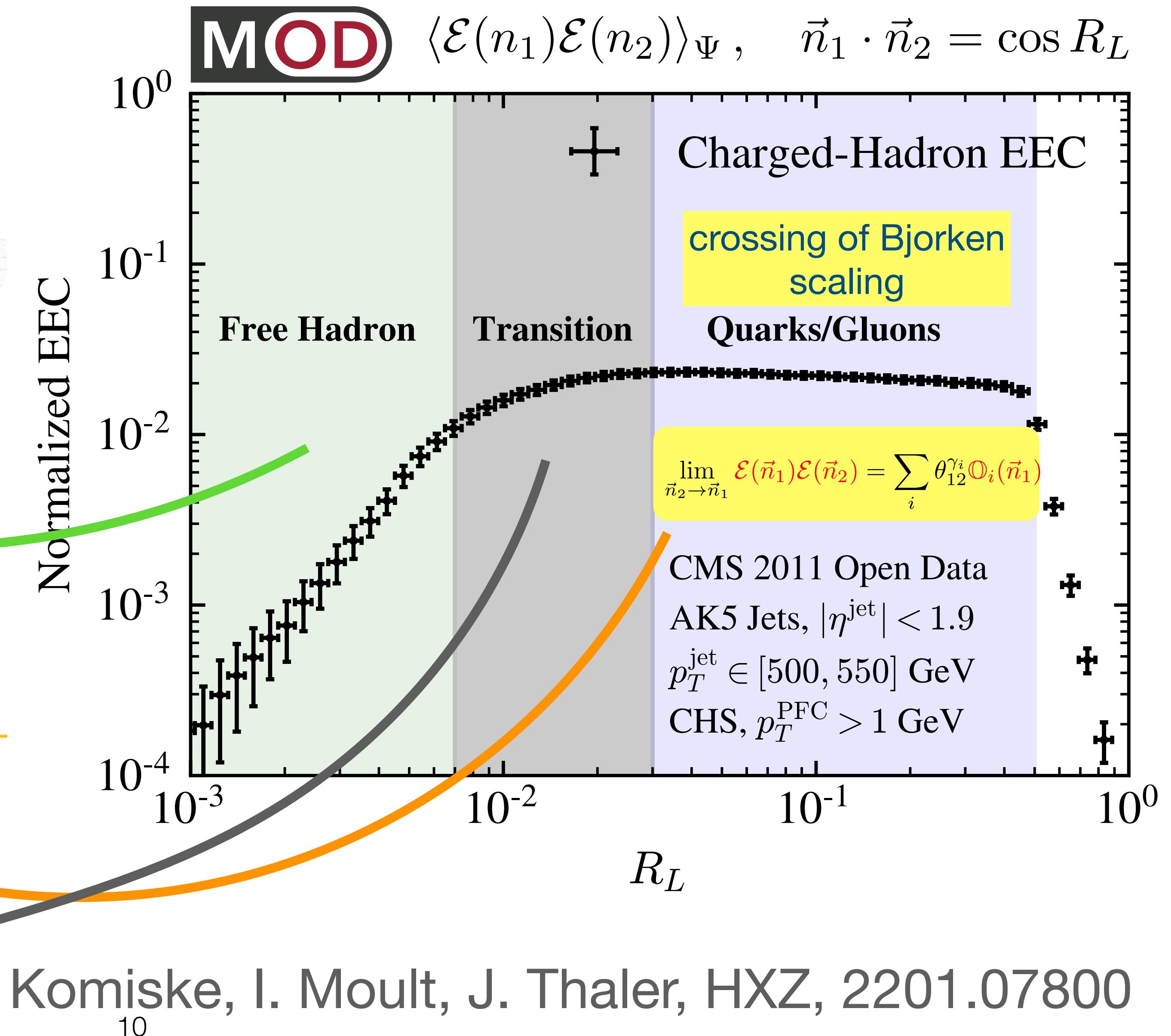
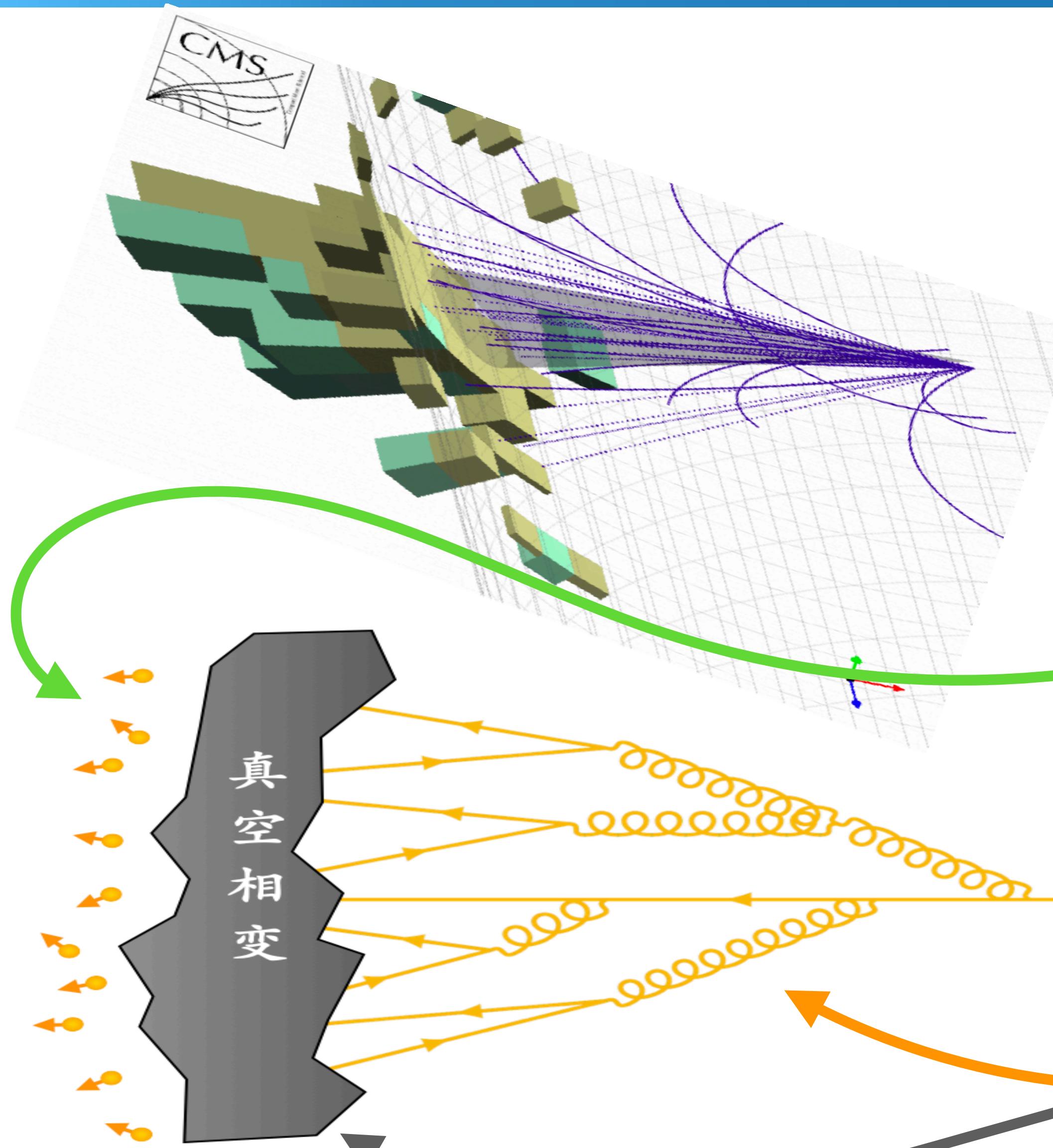
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Seeing hadronization phase transition from EEC



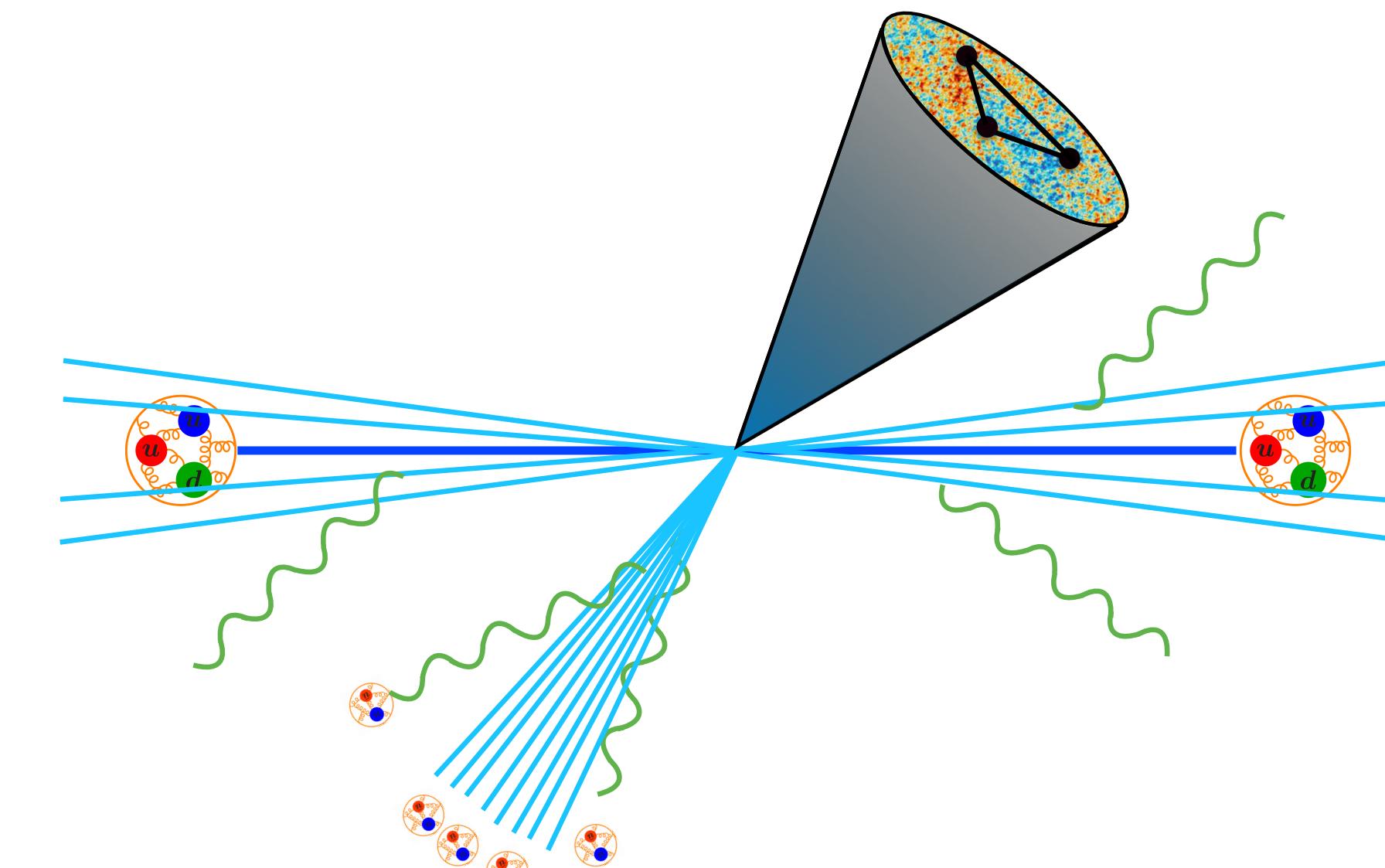
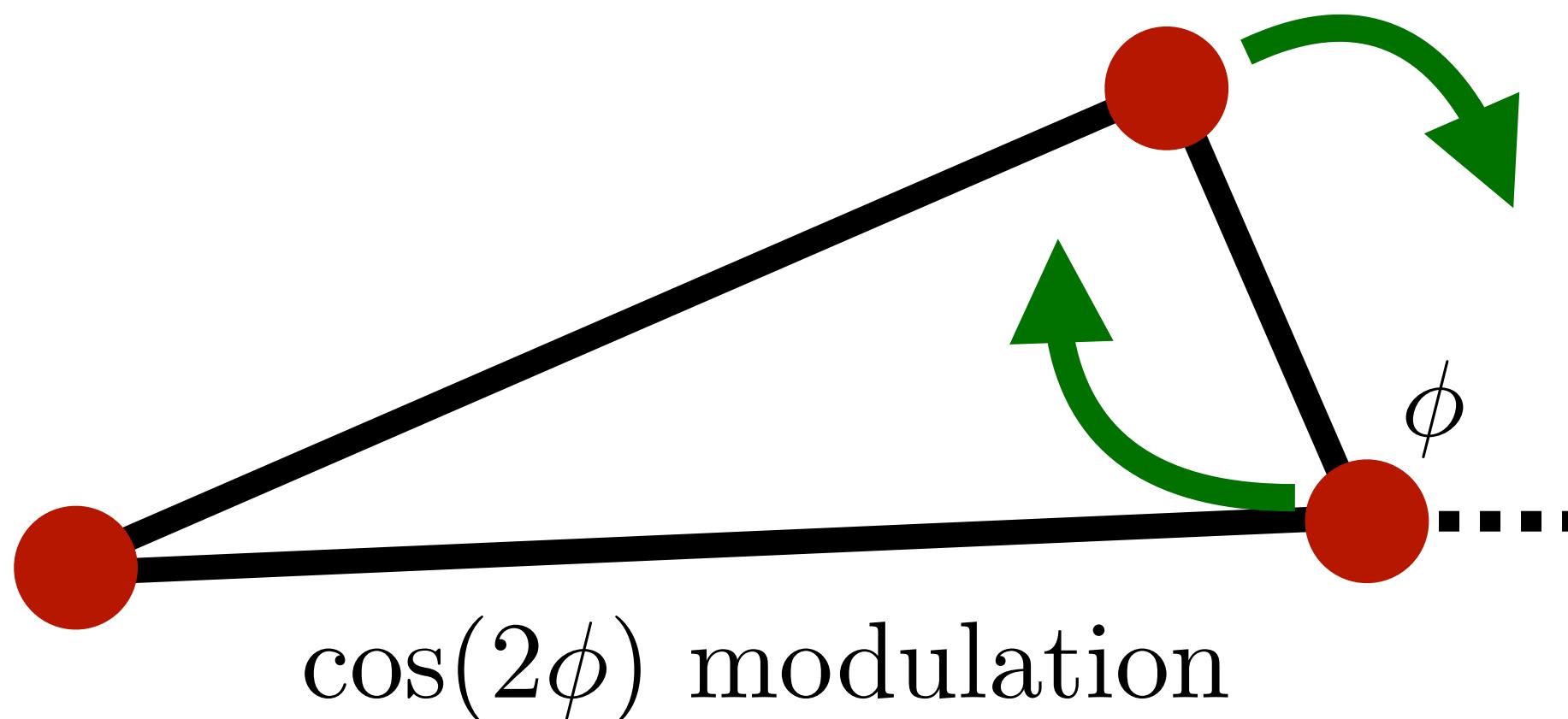
Higher point energy flow correlation

Three-point energy flow correlation

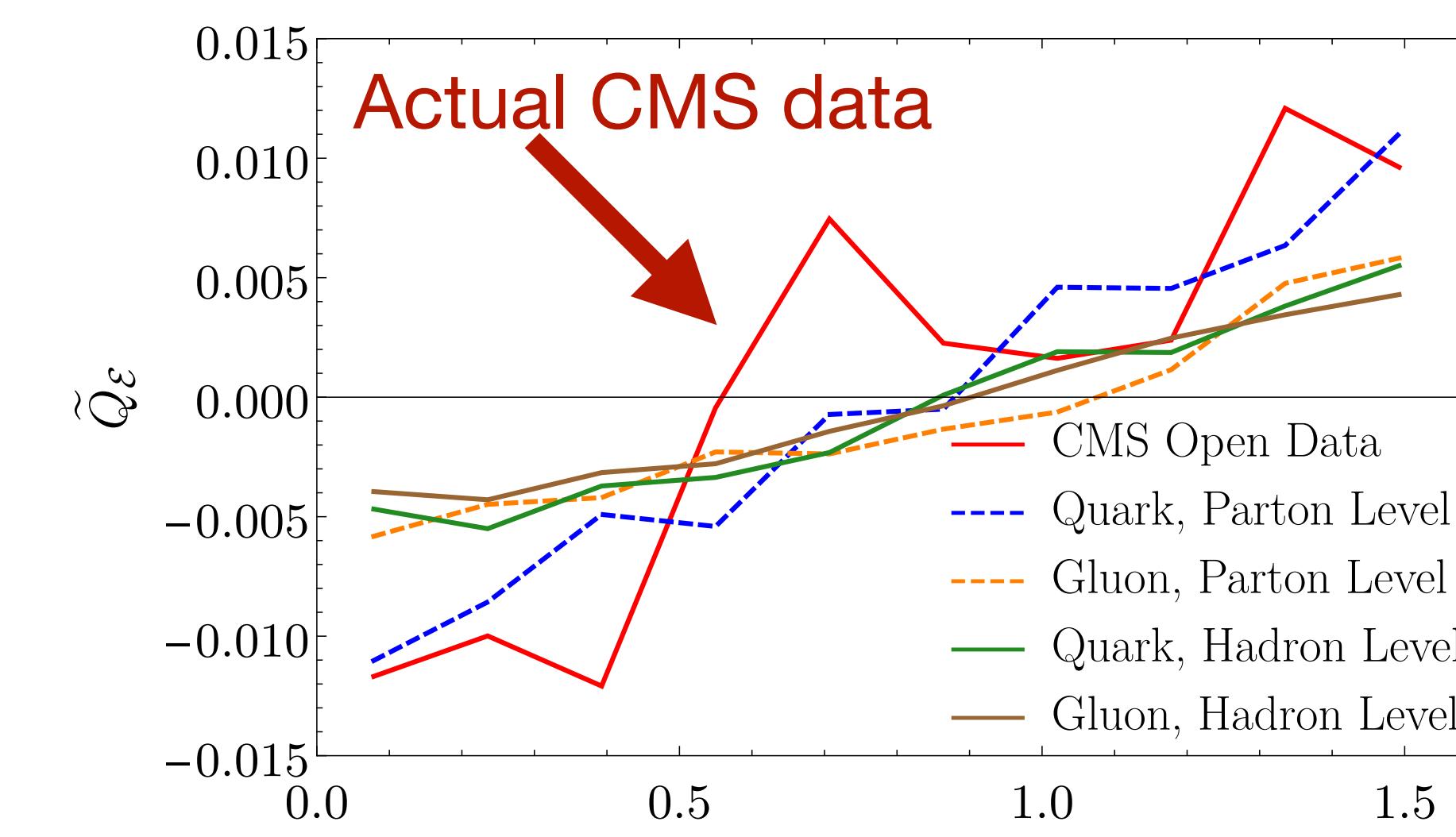
- Arbitrarily select three energy detectors
- Measures their energy weighted cross section
- Iterate over all possible detector selection, and average over all scatterings

Simplest generalization beyond DGLAP

Application: gluon transverse spin



Azimuthal Dependence, $\xi \in (0.1, 0.2)$



Three-point energy flow correlation from perturbation theory

$$G(\theta_{12}, \theta_{23}, \theta_{31}) = G(\theta_{12}, z)$$

$$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \quad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$$

Here we present the results for each term separately. For $G_{q'q'q}(z)$, we have

$$G_{q'q'q}(z) = C_F T_F n_F \times \left\{ \frac{1}{1920 t^4} \left[-80640 s^5 + 16 r^4 (7897 s^2 - 3194 s + 2232) - 8 r^3 (11631 s^4 - 14936 s^3 + 19657 s^2 - 13290 s + 4412) + 4 r^2 (2160 s^6 - 8067 s^5 + 1978 s^4 + 13063 s^3 - 18082 s^2 + 8860 s - 1080) - 2 r^3 (2100 s^6 - 2480 s^5 + 1835 s^4 - 1620 s^3 + 4810 s^2 - 6610 s + 2952) + s^4 (240 s^6 - 300 s^5 + 260 s^4 - 67 s^3 - 83 s^2 + 150 s - 144) \right] + \frac{1}{1920 t^{10}} (r-s+1)^4 \left[32256 s^{10} - 4 r^3 (11909 s^2 - 374300 s^5 + 355114 s^4 - 384612 s^3 + 1346556 s^2 - 1134536 s + 497664) + r^7 (2220 s^8 + 78997 s^7 + 44207 s^6 - 180268 s^5 + 269037 s^4 - 323925 s^3 + 247656 s^2 - 1303360 s + 124416) + r^5 (-120 s^{10} - 11404 s^9 - 143233 s^8 - 164574 s^7 + 2504176 s^6 - 590736 s^5 + 768788 s^4 - 6057800 s^3 + 3293732 s^2 - 992024 s + 217600) + r^4 (600 s^{11} + 21300 s^{10} + 104147 s^9 - 239951 s^8 + 8 r^4 (25961 s^3 - 30314 s^2 + 2608 s^1 - 14400) - r^3 (21989 s^5 - 35024 s^4 - 33955 s^3 + 64966 s^2 - 46404 s + 7552) + 2 r^2 (10240 s^6 - 3732 s^5 + 9214 s^4 - 29779 s^3 + 53618 s^2 - 40220 s + 8136) + r s^3 (-2460 s^6 + 1100 s^5 - 1963 s^4 - 1289 s^3 + 6814 s^2 - 11866 s + 6984) + s^5 (120 s^8 - 60 s^7 + 100 s^6 + 50 s^5 + 34 s^4 - 39 s^3 + 72) \right] g_1^{(1)} + \frac{r}{960 t^{10}} \left[r^5 (176896 s^2 - 32256 s^1 - 18084 s^0 - 2778 s + 204) - 2 r^3 s (10 s^{14} + 180 s^{13} - 50 s^{12} - 285 s^{11} + 6205 s^{10} - 854 s^9 - 6854 s^8 - 10256 s^7 + 51400 s^6 - 77864 s^5 + 64878 s^4 - 33808 s^3 + 10754 s^2 - 2004 s + 180) + r^2 s^3 (20 s^{13} + 120 s^{12} - 724 s^{11} + 329 s^{10} + 4018 s^9 - 11598 s^8 + 15952 s^7 - 19508 s^6 + 24576 s^5 - 26846 s^4 + 20520 s^3 - 10228 s^2 + 2956 s - 380) - 2 r s^5 (5 s^{12} - 8 s^{11} - 79 s^{10} + 362 s^9 - 625 s^8 + 606 s^7 + 398 s^6 - 136 s^5 + 20) + s^{11} (2 s^7 - 1878 s^6 + 22026 s^5 - 33505 s^4 + 64563 s^3 - 71304 s^2 + 14688 s^1 + 2712 s - 512) + 2 r s^2 (60 s^9 + 525 s^8 - 625 s^7 + 2584 s^6 + 222 s^5 - 3078 s^4 + 6237 s^3 - 6078 s^2 - 1164 s + 1214) - s^8 (60 s^8 - 904 s^7 + 260 s^6 + 5 s^5 + 35 s^4 + 10 s^3 - 118 s^2 + 24 s - 274) \right] g_2^{(1)} + \frac{1}{32(r-s+1)^3} \left[2 r^4 - 2 r^3 (s+2) - 14 s^6 + 42 s^5 - 70 s^4 + 70 s^3 - 43 s^2 + 16 s - 4 \right] g_2^{(2)} - \frac{1}{32(r-s+1)^2} \left[2 r^4 - 2 r^3 (s+2) + r^2 (s^2 + 2 s + 4) - 2 r (s^2 - s + 2) + s^2 - 2 s + 2 \right] g_2^{(3)} + \frac{1}{16} (s-1)^2 g_2^{(3)} - \frac{g_2^{(5)}}{192(r-s+1)} \right].$$

For $G_{qqq}(z)$ we have

$$G_{qqq}(z) = C_F^2 \times \left\{ \frac{1}{1920 t^{10}} (r-s+1)^4 \left[4320 s^6 + 48 s^5 (123 s^2 - 544 s + 421) + 8 r^4 (18 s^4 - 1602 s^3 + 6263 s^2 - 8495 s + 3992) + 4 r^3 (-51 s^5 + 2070 s^4 - 7280 s^3 + 11598 s^2 - 8408 s + 1132) + 2 r^2 (37 s^6 - 970 s^5 + 26974 s^4 - 3546 s^3 + 3320 s + 644) + 2 r (10 s^4 - 37 s^3 - 101 s^2 - 1710 s + 193 s^5 + 93 s^4 - 557 s^3 + 393 s^2 - 162 s + 30) \right] g_1^{(1)} + \frac{1}{1920 t^{10}} (r-s+1)^4 \left[-32256 s^10 + 46364 s^9 - 46364 s^8 + 87148 s^7 + 47636 s^6 + 73184 s - 93344 \right] g_1^{(2)} - \frac{1}{8 t^{11}} \left[1344 r^7 + r^6 (-5628 s^2 + 6552 s - 3296) + 6 r^5 (1232 s^3 - 2114 s^2 + 1803 s^1 - 1006 s + 380) + r^4 (-4158 s^5 + 6468 s^4 - 1795 s^3 - 5375 s^2 + 8200 s^1 - 4912 s + 848) + 2 r^3 (660 s^6 - 1122 s^5 + 924 s^4 - 1187 s^3 + 2260 s^2 - 2885 s^1 + 1901 s^0 - 498 s + 36) + r^2 (-242 s^8 + 440 s^7 - 396 s^6 + 198 s^5 - 1080 s^4 + 1433 s^3 - 923 s^2 + 216) s^2 + r (248 s^6 - 46 s^5 - 22 s^4 - 25 s^3 - 25 s^2 + 76 s^2 - 85 s + 36) s^4 - (s^3 - 2 s^2 + 2 s - 1) s^5 \right] g_2^{(1)} + \frac{1}{8} \left[-2 r (s-1) + s^3 - 2 s^2 + 2 s - 1 \right] g_2^{(3)} \right\}.$$

For $G_{qqq}^{(id)}(z)$, we have

$$G_{qqq}^{(id)}(z) = (C_A - 2C_F)C_F \times \left\{ \frac{1}{1520 t^8 (r-s+1)^4} \left[2952 r^8 (63 s - 58) - 24 r^7 (6148 s^3 + 27246 s^2 - 62853 s + 31810) + r^6 (35382 s^5 + 494010 s^4 + 251712 s^3 - 3234312 s^2 + 3346592 s - 518944) + r^5 (-1160 s^7 - 239787 s^6 - 242638 s^5 + 92786 s^4 + 2344672 s^3 - 5448528 s^2 + 2740176 s - 567392) + r^4 (720 s^9 + 43740 s^8 + 27867 s^7 - 99957 s^6 + 112182 s^5 - 761156 s^4 + 3562352 s^3 + r^3 (-2880 s^{10} - 60480 s^9 + 37044 s^8 + 789742 s^7 - 1541669 s^6 + 1762726 s^5 - 2439712 s^4 + 2379488 s^3 - 1304962 s^2 - 1306849 s^1 + 1470360 s^0 - 1150278 s^3 + 43204 s^10 + 306009 s^9 - 114070 s^8 - 77940 s^7 + 729201 s^6 - 1306849 s^5 + 1470360 s^4 - 1150278 s^3 + 621840 s^2 - 18024 s + 24720) + r^2 s^8 (-2880 s^9 + 1080 s^8 + 40877 s^7 - 101984 s^6 + 69207 s^5 + 59426 s^4 - 157328 s^3 + 136848 s^2 - 6396 s + 12000) + s^5 (720 s^8 - 3780 s^7 + 7415 s^6 - 6731 s^5) \right] g_1^{(1)} + \frac{1}{1520 t^{10}} (r-s+1)^4 \left[-854 s^8 - 6854 s^7 - 10256 s^6 + 51400 s^5 - 77864 s^4 + 64878 s^3 - 33808 s^2 + 10754 s^1 - 2004 s + 180) + r^2 s^3 (20 s^{13} + 120 s^{12} - 724 s^{11} + 329 s^{10} + 4018 s^9 - 11598 s^8 + 15952 s^7 - 19508 s^6 + 24576 s^5 - 26846 s^4 + 20520 s^3 - 10228 s^2 + 2956 s - 380) - 2 r s^5 (5 s^{12} - 8 s^{11} - 79 s^{10} + 362 s^9 - 625 s^8 + 606 s^7 + 398 s^6 - 136 s^5 + 20) + s^{11} (2 s^7 - 1878 s^6 + 22026 s^5 - 33505 s^4 + 64563 s^3 - 71304 s^2 + 14688 s^1 + 2712 s - 512) + 2 r s^2 (60 s^9 + 525 s^8 - 625 s^7 + 2584 s^6 + 222 s^5 - 3078 s^4 + 6237 s^3 - 6078 s^2 - 1164 s + 1214) - s^8 (60 s^8 - 904 s^7 + 260 s^6 + 5 s^5 + 35 s^4 + 10 s^3 - 118 s^2 + 24 s - 274) \right] g_1^{(2)} + \frac{1}{32(r-s+1)^3} \left[2 r^4 - 2 r^3 (s+2) - 14 s^6 + 42 s^5 - 70 s^4 + 70 s^3 - 43 s^2 + 16 s - 4 \right] g_2^{(1)} - \frac{1}{32(r-s+1)^2} \left[2 r^4 - 2 r^3 (s+2) + r^2 (s^2 + 2 s + 4) - 2 r (s^2 - s + 2) + s^2 - 2 s + 2 \right] g_2^{(3)} + \frac{1}{16} (s-1)^2 g_2^{(3)} - \frac{g_2^{(5)}}{192(r-s+1)} \right\}.$$

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- Analytic available perturbative prediction at leading order
- Large logarithms in perturbative theory $a_s^n \log^n(\theta_{23}/\theta_{12})$
- Need different organization to make underlying physics manifest

H. Chen, M.X. Luo, I. Moult, T.Z. Yang, X.Y. Zhang, HXZ, JHEP, 2020

$$\begin{aligned} & -39 s^8 \Big] + \frac{1}{1920 t^{10}} \left[128 s^6 (992 - 1067 s) + 8 r^5 (25961 s^3 - 34733 s^2 + 56757 s - 39936) - 4 r^4 (21989 s^5 - 10184 s^4 - 54302 s^3 + 181048 s^2 - 32256 s^1 + r^3 (20480 s^7 + 20994 s^6 + 32298 s^5 - 7116 s^4 + 386928 s^3 - 473164 s^2 + 115144 s + 2048) - 2 r^2 (1230 s^9 + 3470 s^8 - 1878 s^7 + 22026 s^6 - 33505 s^5 + 64563 s^4 - 71304 s^3 + 14688 s^2 + 2712 s - 512) + 2 r s^2 (60 s^9 + 525 s^8 - 625 s^7 + 2584 s^6 + 222 s^5 - 3078 s^4 + 6237 s^3 - 6078 s^2 - 1164 s + 1214) - s^8 (60 s^8 - 904 s^7 + 260 s^6 + 5 s^5 + 35 s^4 + 10 s^3 - 118 s^2 + 24 s - 274) \right] g_1^{(1)} + \frac{1}{1920 t^{10}} (r-s+1)^4 \left[128 s^6 (992 - 1067 s) + 8 r^5 (25961 s^3 - 34733 s^2 + 56757 s - 39936) - 4 r^4 (21989 s^5 - 10184 s^4 - 54302 s^3 + 181048 s^2 - 32256 s^1 + r^3 (20480 s^7 + 20994 s^6 + 32298 s^5 - 7116 s^4 + 386928 s^3 - 473164 s^2 + 115144 s + 2048) - 2 r^2 (1230 s^9 + 3470 s^8 - 1878 s^7 + 22026 s^6 - 33505 s^5 + 64563 s^4 - 71304 s^3 + 14688 s^2 + 2712 s - 512) + 2 r s^2 (60 s^9 + 525 s^8 - 625 s^7 + 2584 s^6 + 222 s^5 - 3078 s^4 + 6237 s^3 - 6078 s^2 - 1164 s + 1214) - s^8 (60 s^8 - 904 s^7 + 260 s^6 + 5 s^5 + 35 s^4 + 10 s^3 - 118 s^2 + 24 s - 274) \right] g_1^{(2)} + \frac{1}{32(r-s+1)^3} \left[2 r^4 - 2 r^3 (s+2) - 14 s^6 + 42 s^5 - 70 s^4 + 70 s^3 - 43 s^2 + 16 s - 4 \right] g_2^{(1)} - \frac{1}{32(r-s+1)^2} \left[2 r^4 - 2 r^3 (s+2) + r^2 (s^2 + 2 s + 4) - 2 r (s^2 - s + 2) + s^2 - 2 s + 2 \right] g_2^{(3)} + \frac{1}{16} (s-1)^2 g_2^{(3)} - \frac{g_2^{(5)}}{192(r-s+1)} \right]. \end{aligned}$$

5.2.2 Gluon Jets

For gluon jet we similarly write

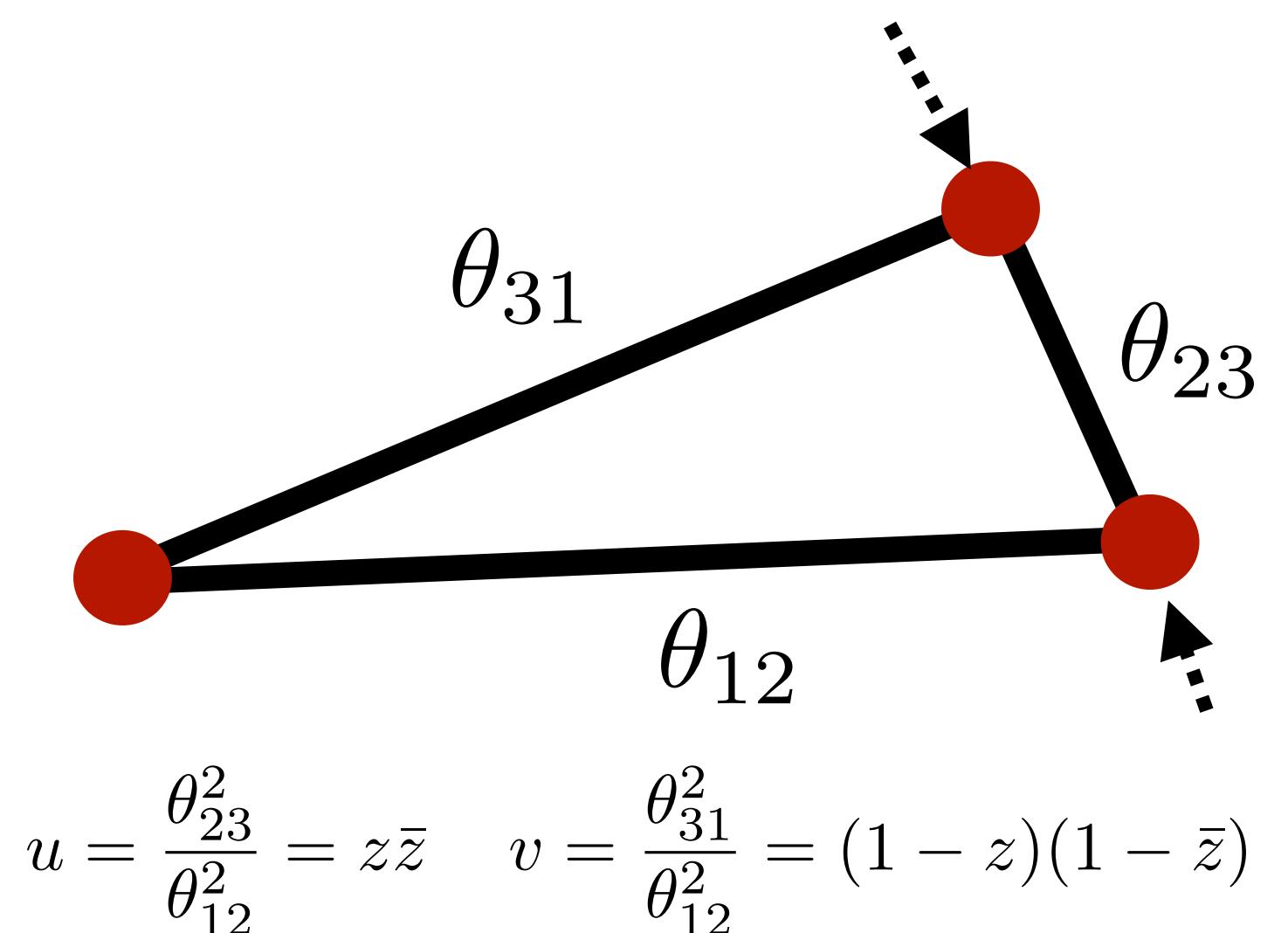
$$\begin{aligned} & \frac{1}{\sigma_{\text{tot}} dx dz d\text{Re}(z) dh(z)} = \frac{g^4}{16\pi^2 x L} \left[G_g(z) + G_g(1-z) + \frac{1}{|1-z|^2} \left(G_g \left(\frac{z}{z-1} \right) + G_g \left(\frac{1}{z} \right) \right) + \frac{1}{|z|^4} \left(G_g \left(\frac{1}{z} \right) + G_g \left(\frac{z-1}{z} \right) \right) \right]. \end{aligned}$$

The color decomposition is

$$G_g(z) = G_{ggg}^{(ab)}(z) + 2G_{gqg}^{(nab)}(z) + G_{gpg}(z).$$

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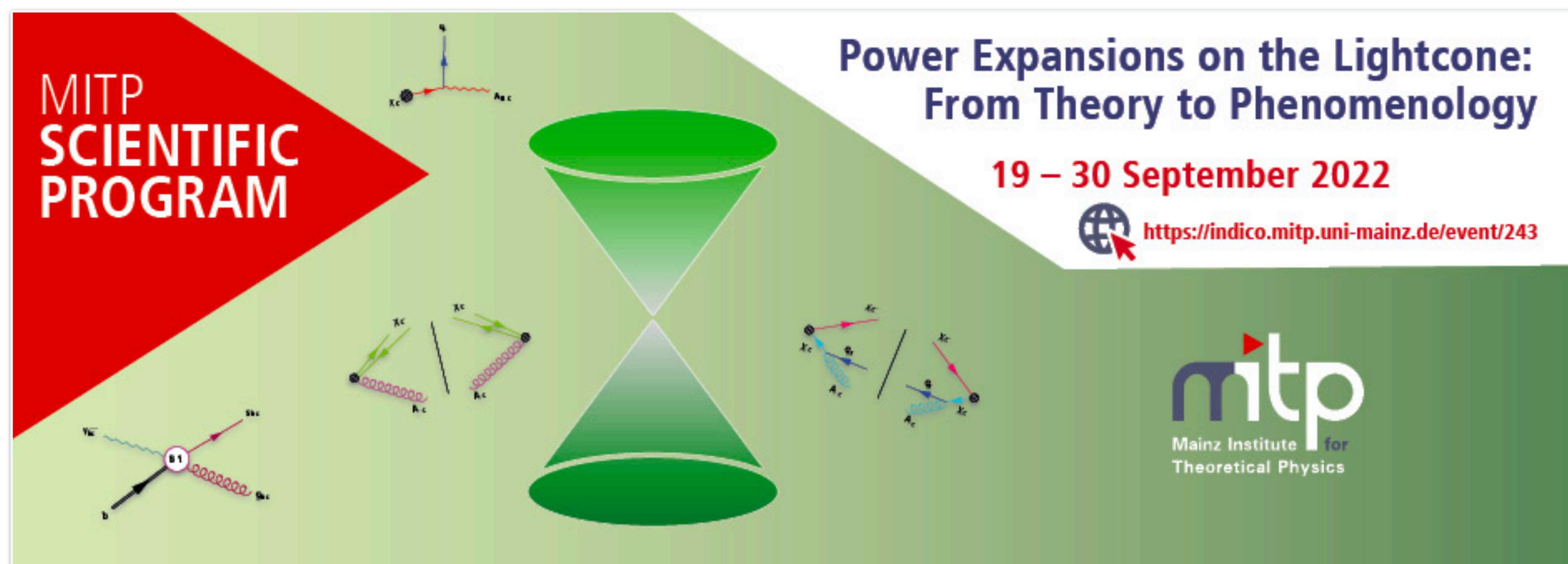
Multi-pole expansion



Power expansion in the squeeze limit $\theta_{23} \rightarrow 0$ (modulo logs)

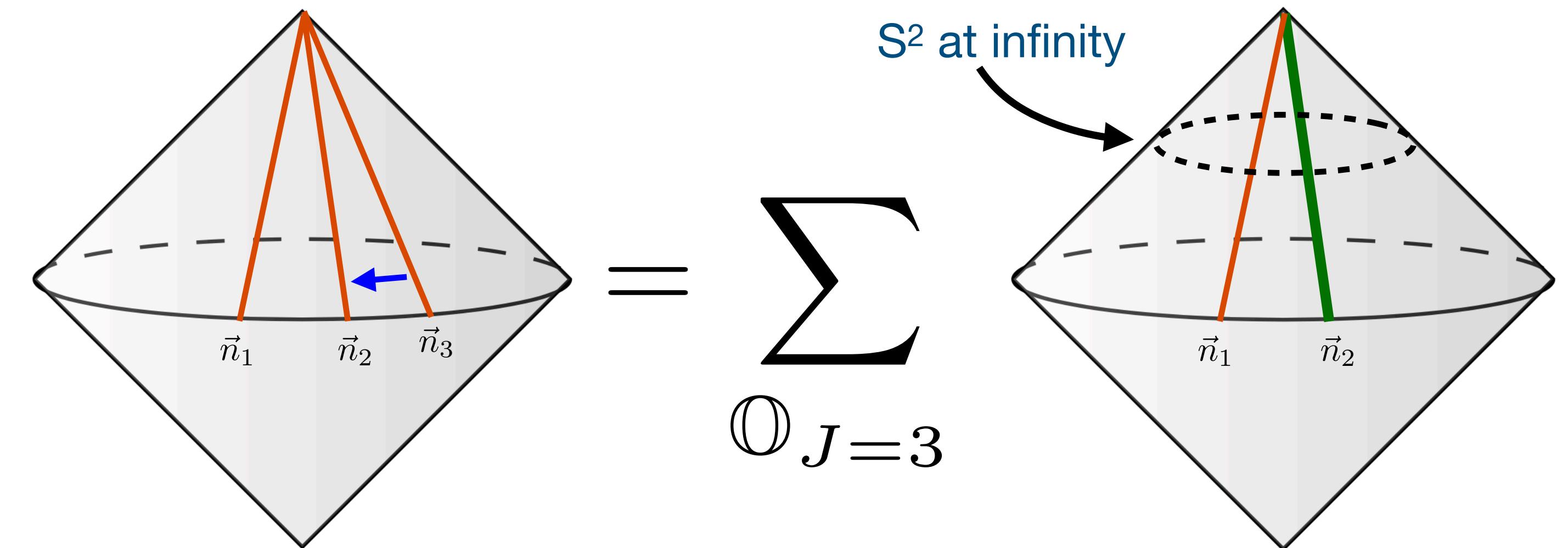
$$G(\theta_{12}, z) = \frac{1}{|z|^2} G^{(0)}(\theta_{12}) + G^{(1)}(\theta_{12}) + |z|^2 G^{(2)}(\theta_{12}) + \dots$$

Power expansion of quantum field theory near the lightcone in general is a difficult problem

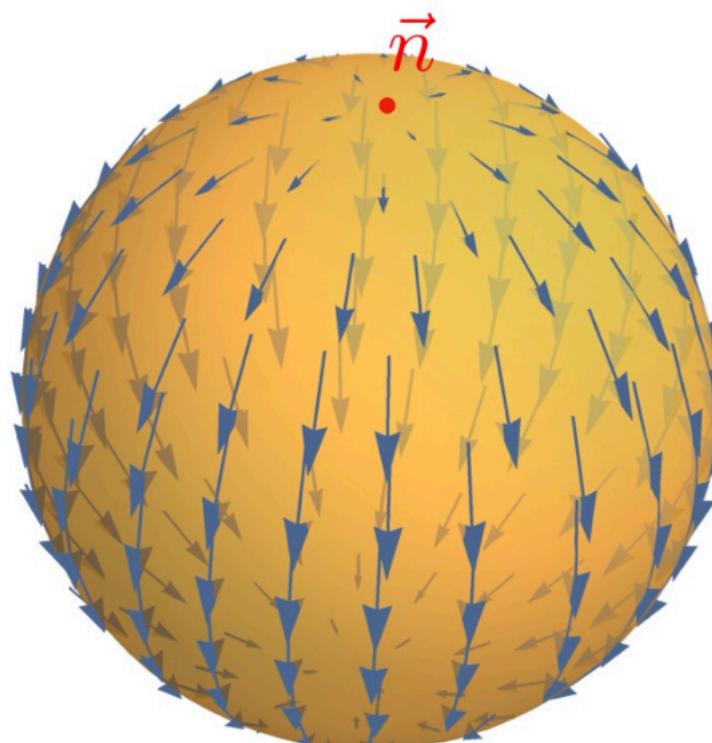


Lightray OPE and symmetry

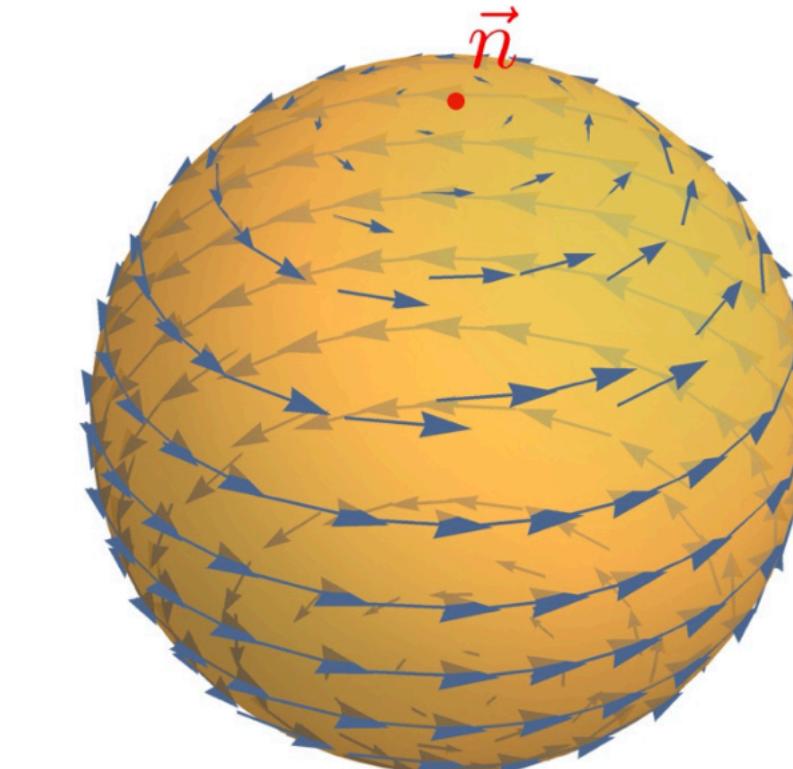
$$\begin{aligned}
 G(\theta_{12}, z) &= \langle \Psi | \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{E}(n_3) | \Psi \rangle \\
 &= \sum_{\mathbb{O}_{J=3}} \langle \Psi | \mathcal{E}(n_1) \mathbb{O}_{J=3}(n_2) | \Psi \rangle
 \end{aligned}$$



Quantum number of the lightray operator



Boost \Leftrightarrow Dilation δ

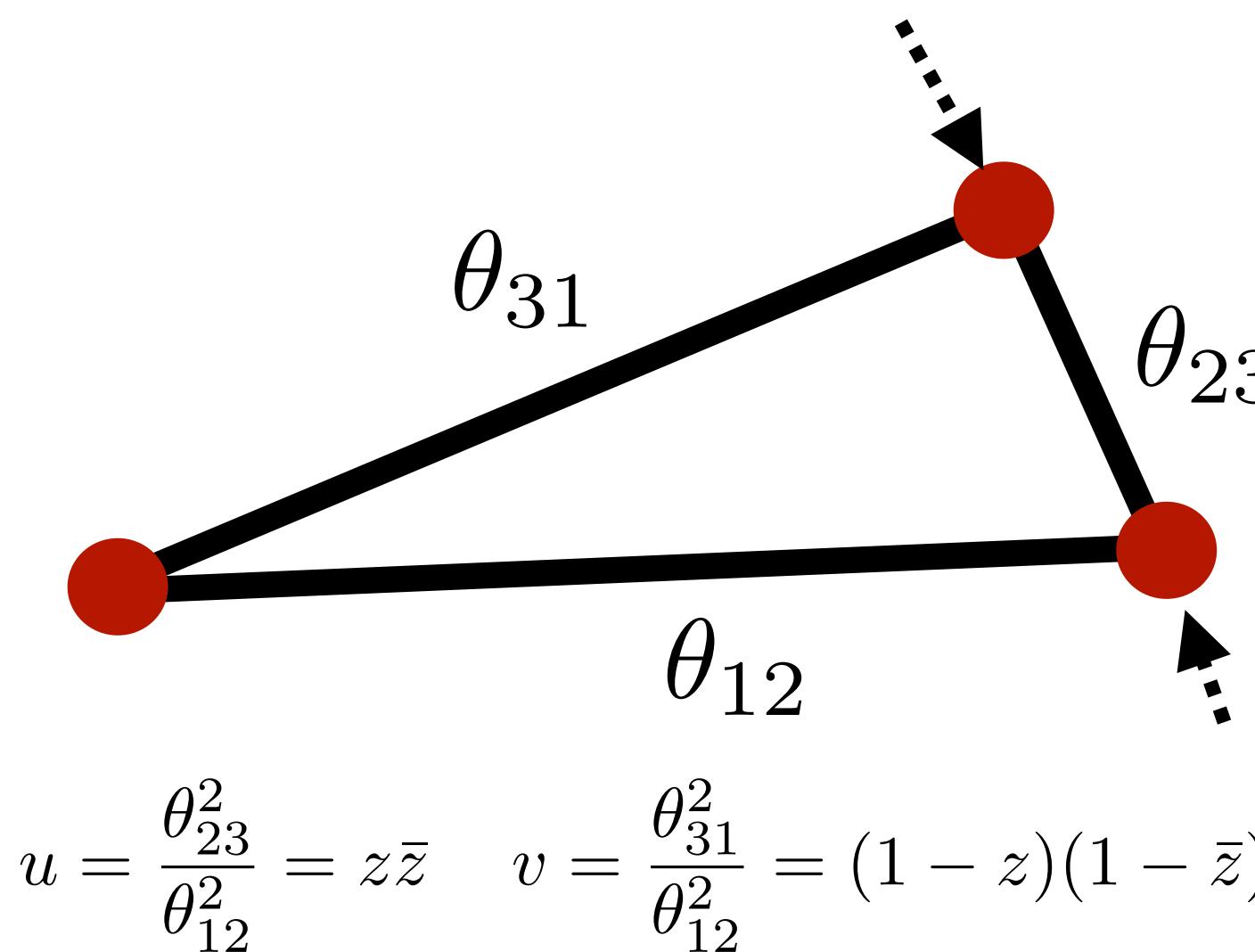


Rotation \Leftrightarrow transverse spin j

$\mathbb{O}_{\delta,j}^{J=3}$

Lightray operator labeled by dimension δ and spin j

Partial wave of Lorentz group $SO(3,1)$



$$\begin{aligned} G(\theta_{12}, z) &= \langle \Psi | \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{E}(n_3) | \Psi \rangle \\ &= \sum_{\mathbb{O}_{J=3}} \langle \Psi | \mathcal{E}(n_1) \mathbb{O}_{J=3}(n_2) | \Psi \rangle = G(\theta_{12}) \sum_{\delta,j} g_{\delta,j}(z) \end{aligned}$$

Lorentz invariance of individual partial wave

$$\mathcal{C}_2(\partial_z, \partial_{\bar{z}})g_{\delta,j}(z) = \lambda_{\delta,j}g_{\delta,j}(z)$$

Quadratic Casimir

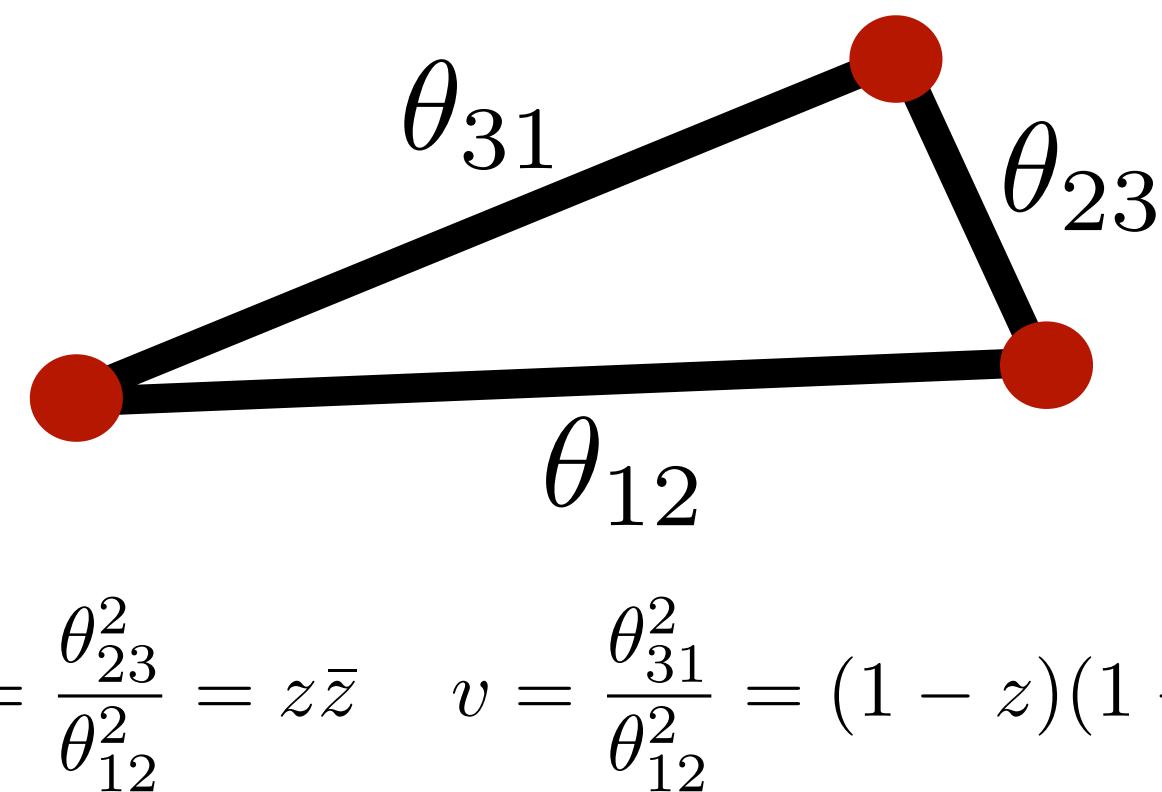
$$[2z^2(z-1)\partial_z^2 + 2\bar{z}^2(\bar{z}-1)\partial_{\bar{z}}^2]$$

Eigenvalue

$$\delta(\delta-2) + j^2$$

Identical to Casimir equation for 2D Euclidean conformal block!

Comment on SO(3,1) partial wave expansion



$$u = \frac{\theta_{23}^2}{\theta_{12}^2} = z\bar{z} \quad v = \frac{\theta_{31}^2}{\theta_{12}^2} = (1-z)(1-\bar{z})$$

Partial wave expansion of three-point energy flow correlation

$$G(\theta_{12}, z) = G(\theta_{12}) \sum_{\delta,j} c_{\delta,j} g_{\delta,j}(z)$$

Partial wave coefficient
(OPE coefficient)

$$g_{\delta,j}(z) = \frac{1}{1 + \delta_{j,0}} \left(k_{\frac{\delta-j}{2}}(z) k_{\frac{\delta+j}{2}}(\bar{z}) + (z \leftrightarrow \bar{z}) \right)$$

Three-point energy flow correlation is completely determined by OPE coefficients and dimension of and spin of (lightray) operators appeared

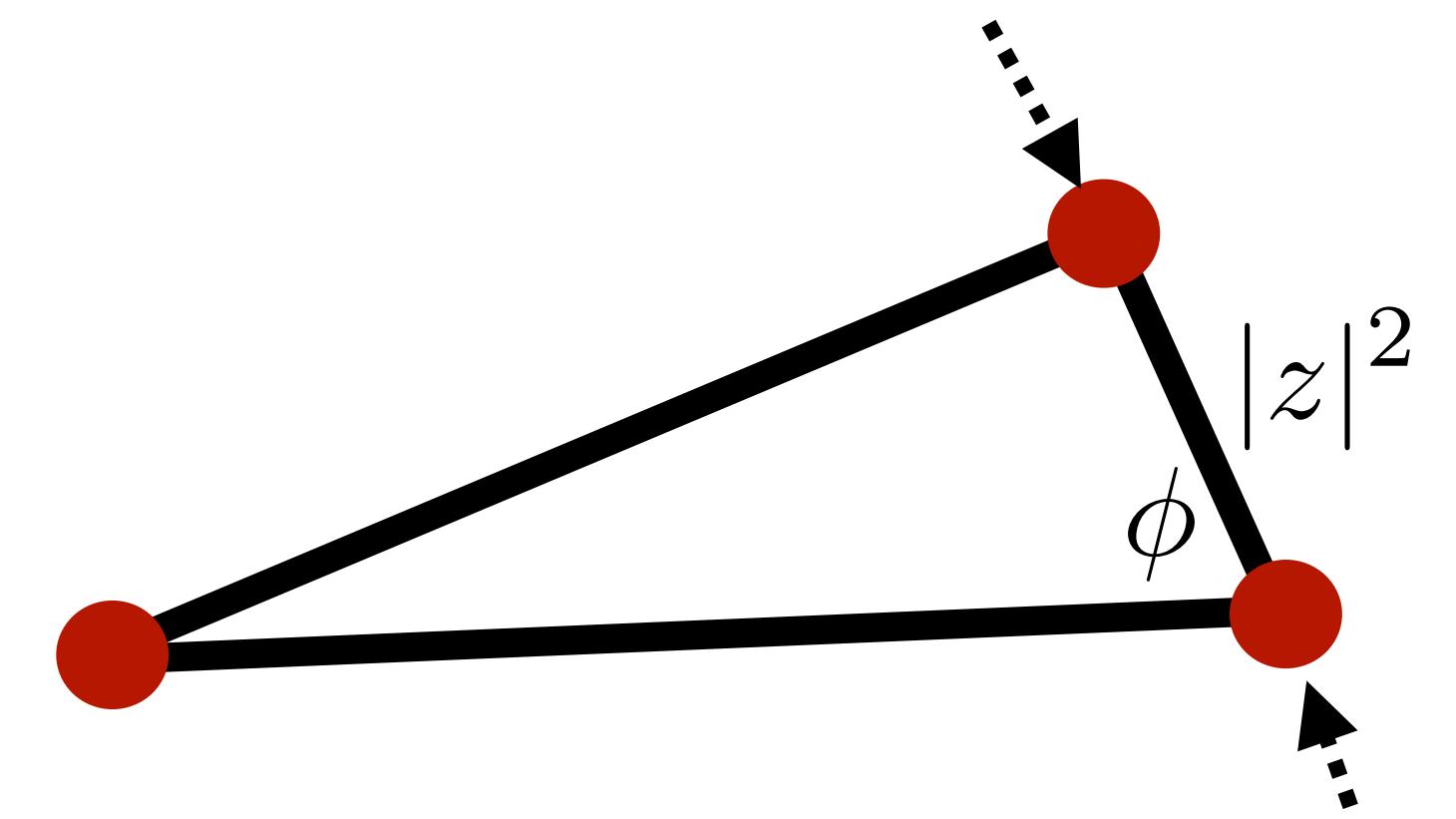
Partial wave expansion provides a systematic solution to power expansion and resummation problem for energy flow correlation

Concrete example of partial wave expansion

$$g_{\delta,j}(z) = \frac{1}{1 + \delta_{j,0}} \left(k_{\frac{\delta-j}{2}}(z) k_{\frac{\delta+j}{2}}(\bar{z}) + (z \leftrightarrow \bar{z}) \right) \quad k_h(z) = z^j {}_2F_1(h, h-1, 2h, z)$$

$$G(\theta_{12}, z) = \frac{1}{\theta_{12}^2} \frac{\alpha_s^2}{(4\pi)^2} C_A^2 \left[\frac{1}{720} g_{4,2} + \frac{49}{200} g_{4,0} + \left(\frac{834469}{126000} - \frac{2\pi^2}{3} \right) g_{6,2} + \left(\frac{318193}{50400} - \frac{5\pi^2}{8} \right) g_{6,0} + \dots \right]$$

$ $	$g_{4,2} = 2 \cos(2\phi) z ^4 + 2 \cos(3\phi) z ^5 + \dots$	$ $
$ $	$g_{4,0} = 2 z ^4 + 2 \cos(\phi) z ^5 + \dots$	$ $
$ $	$g_{6,2} = 2 \cos(2\phi) z ^6 + (\cos \phi + 3 \cos(3\phi)) z ^7 + \dots$	$ $
$ $	$g_{6,0} = 2 z ^6 + 4 \cos \phi z ^7 + \dots$	$ $



- Partial wave expansion is an efficient organization of power expansion
- Each partial wave resum infinite power of kinematical power corrections
- Logarithmic resummation achieved by including anomalous dimension in δ

OPE coefficients from inversion formula

$$G(z, \alpha_s) = \sum_{\delta, j} c_{\delta, j} g_{\delta, j}(z)$$

Given perturbative data $G(z, \alpha_s)$, how to we reconstruct the OPE data?

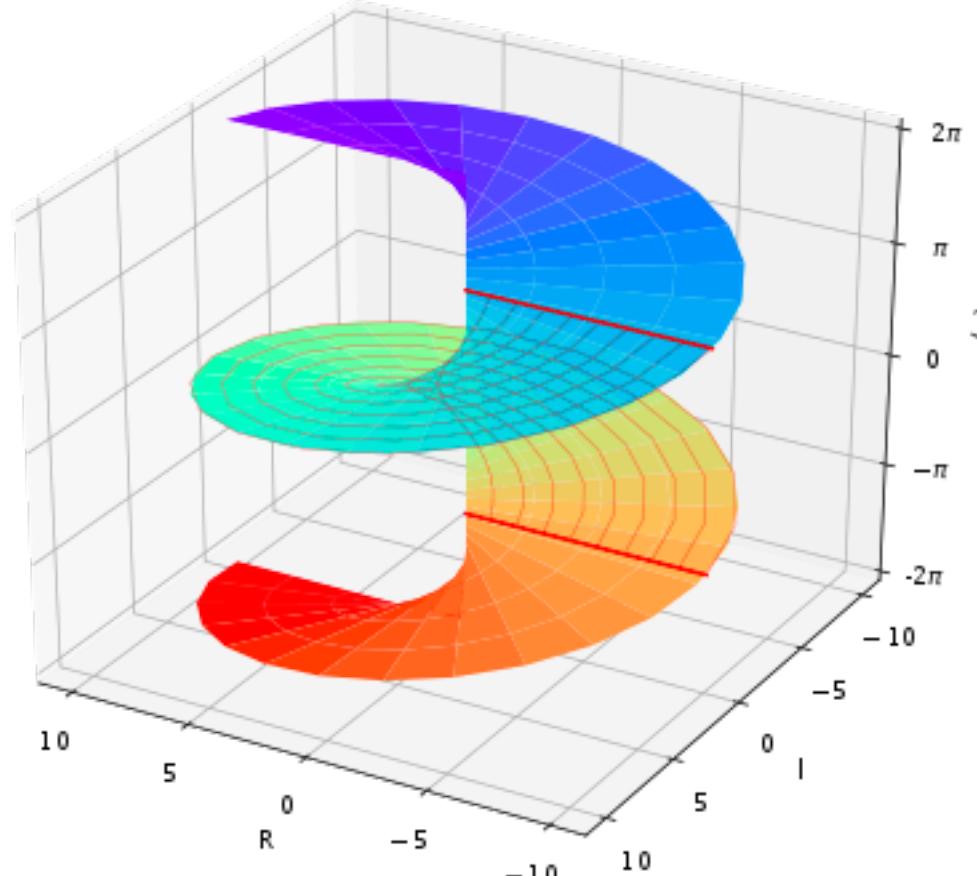
Lorentzian inversion formula

S. Caron-hunt, 2017;

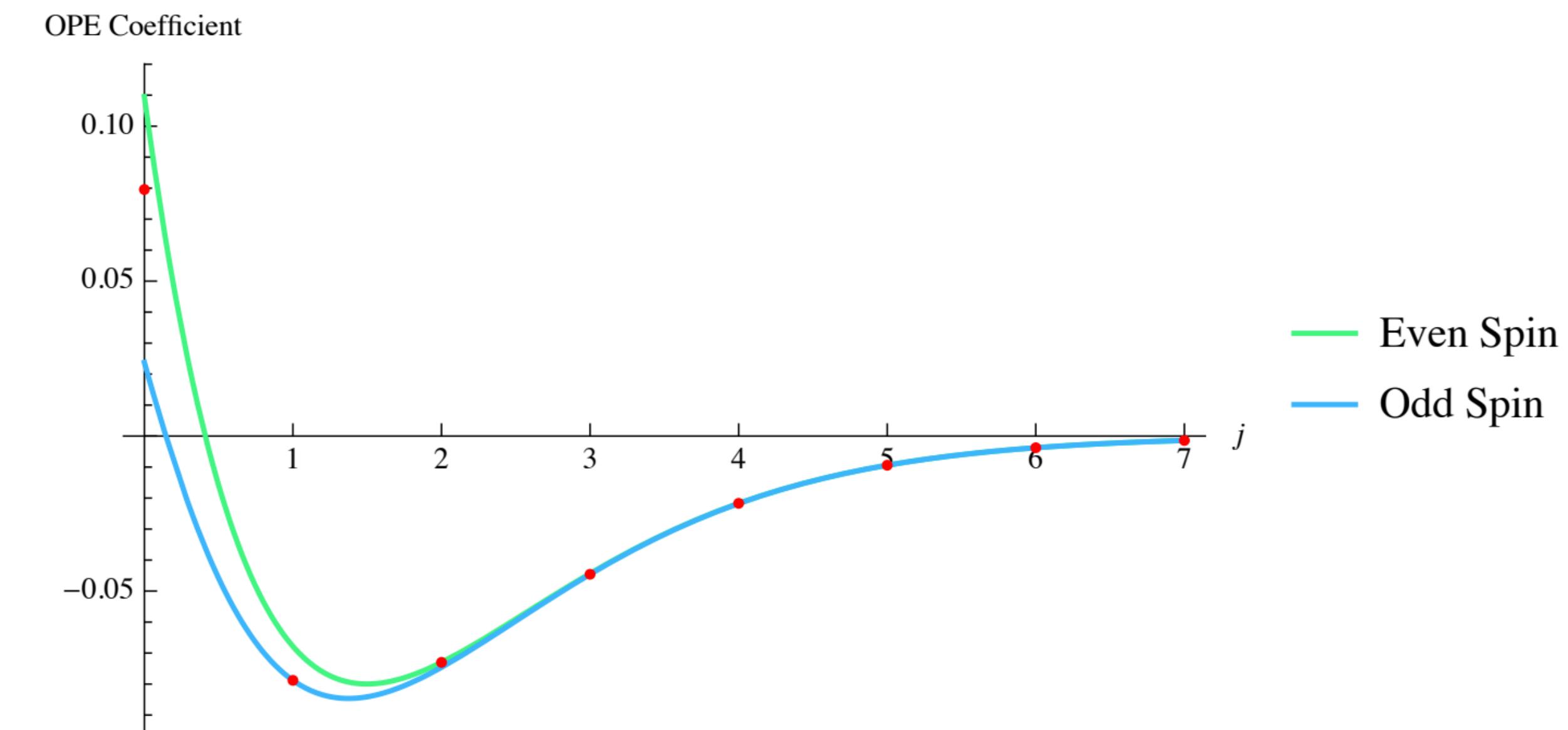
D. Simmons-Duffin, D. Stanford, E. Witten, 2017

$$G(z, \alpha_s) = \sum_{\delta, j} (c_{\delta, j}^t + (-1)^j c_{\delta, j}^u) g_{\delta, j}(z)$$

$$c_{\delta, j}^t = \int_0^1 dz d\bar{z} \mu(z, \bar{z}) g_{\delta-1, j+1}(z) d\text{Disc}[G(z)]$$

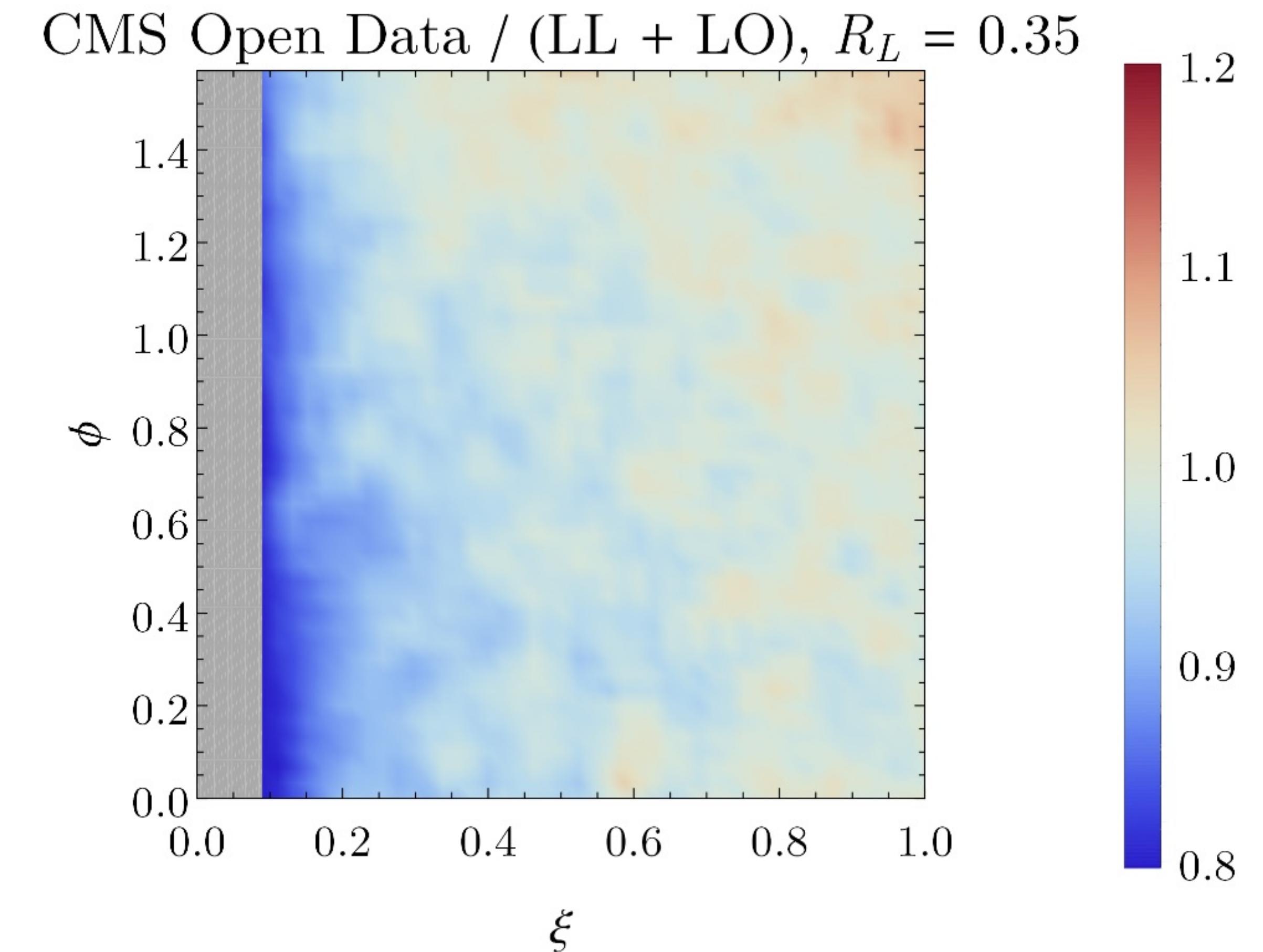
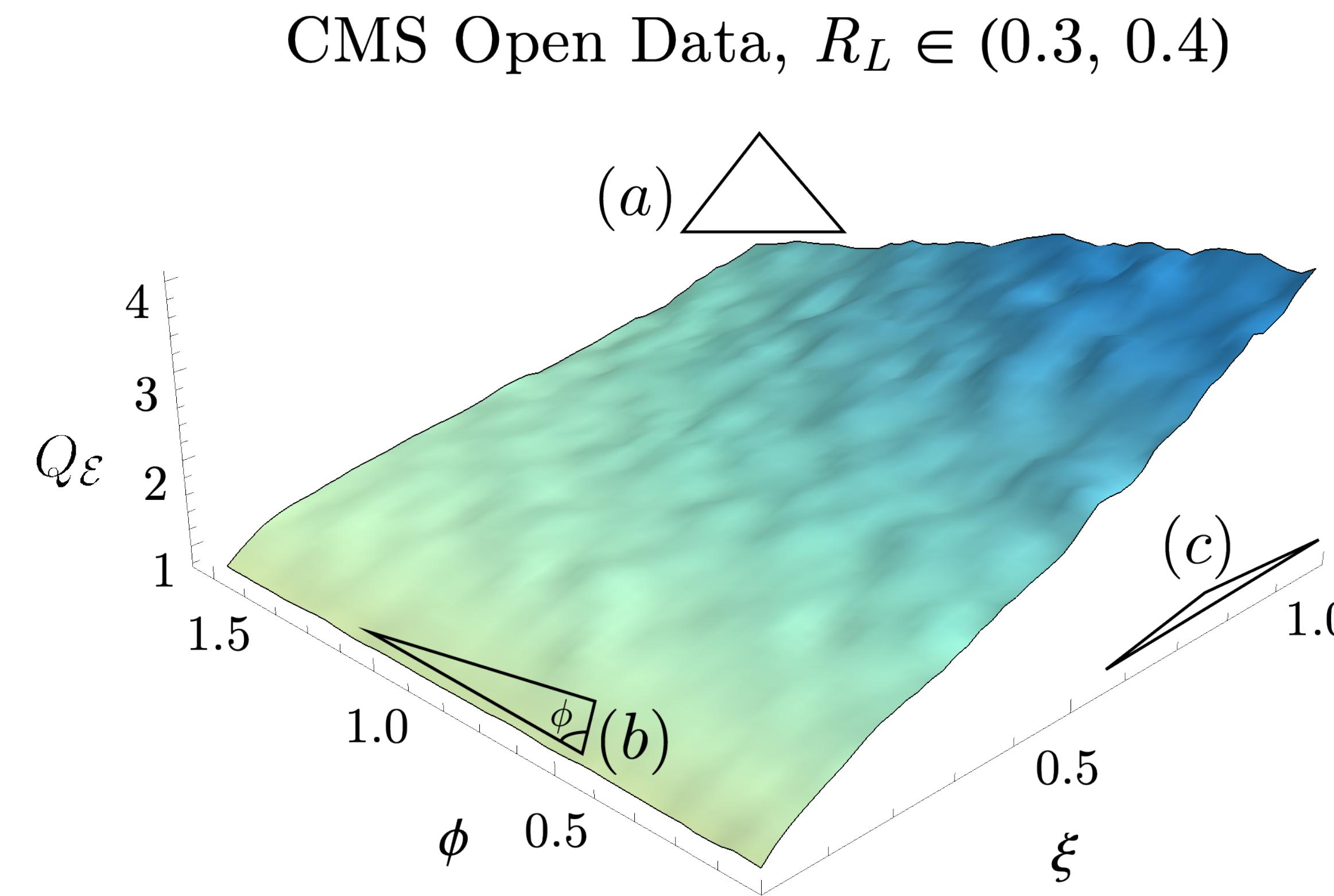


$$G(z, \bar{z}) - \frac{1}{2}G(z, \bar{z}^\circlearrowleft) - \frac{1}{2}G(z, \bar{z}^\circlearrowright)$$



H. Chen, I. Moult, J. Sandor, HXZ, 2202.04085

RG improved perturbation theory v.s. CMS open data



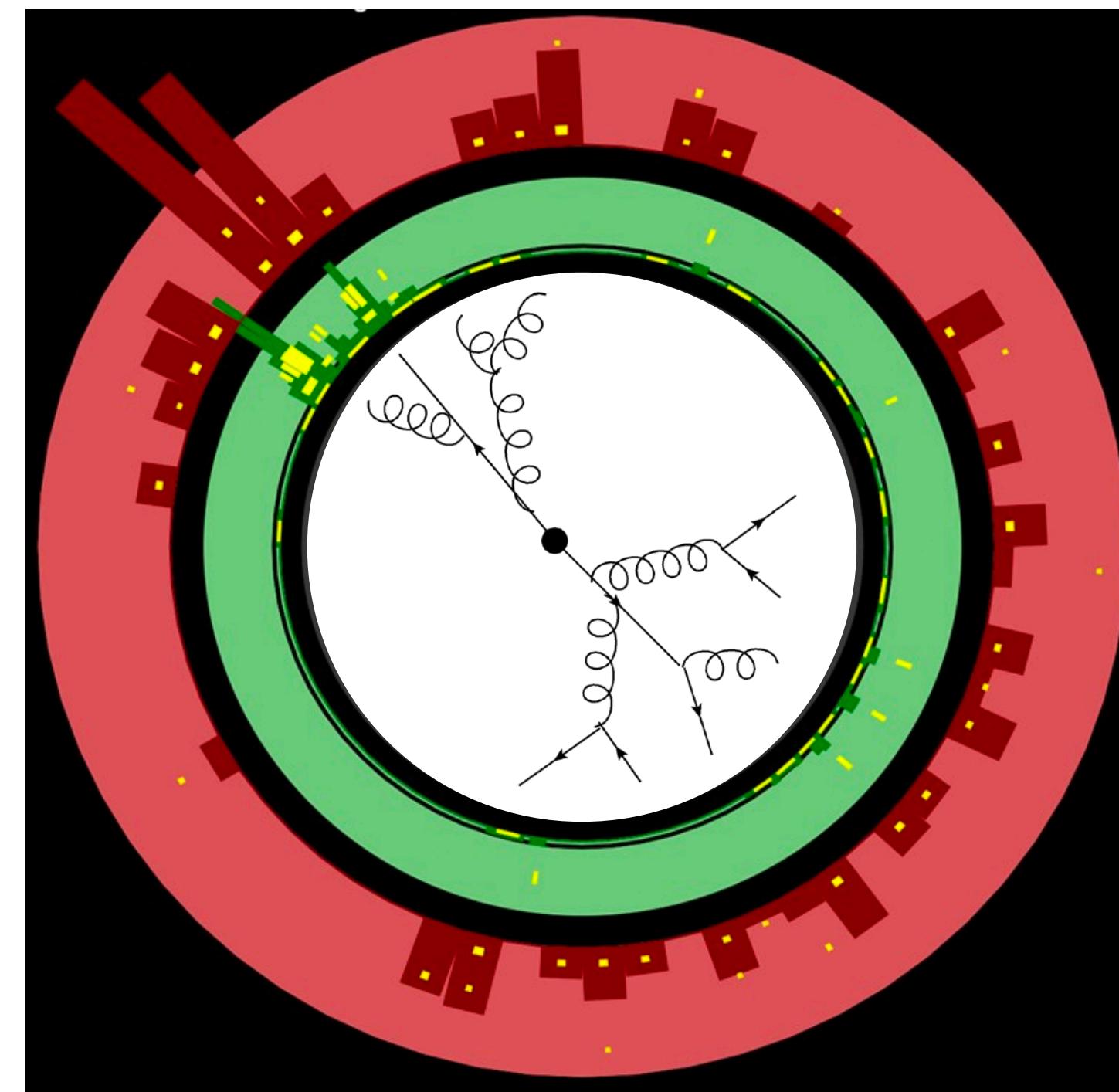
Data/theory agree at the level of $\pm 20\%$!

H. Chen, I. Moult, J. Thaler, HXZ, JHEP, 2022

Summary

- Successful of high energy LHC program relies on accurate description of QCD evolution
 - Generalized DGLAP: multi-branching kernel $P(\{\theta_{ij}\}, \{z_i\})$ depending on angles and momentum fractions
- Introduced statistical correlation approach to jet structure based on energy flow correlation
- Provided a data visualization of q/g hadronization phase transition. Very interesting to compare difference between in vacuum and in medium
- Power expansion by Lorentzian partial wave decomposition
 - Clean organization of kinematical and dynamical power corrections
 - Automatically lead to resummation of large logarithms beyond leading power

Speculation: 2D EFT of QCD on the light shell?



If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

- Regge limit: Lipatov, 1988; Verlinde², 1993
- e+e- scattering: H. Georgi, Kestin, Sajjad, 2010
- Energy flow correlation provides much concrete theory data to explore this idea