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CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays

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based on:

Feng-Zhi Chen, Xin-Qiang Li, Shi-Can Peng, Ya-Dong Yang, Hong-Hao Zhang, JHEP 01 (2022) 108

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, JHEP 05 (2020) 151

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, Xin Zhang, PRD 100 (2019) 113006

Outline

- ☐ Introduction
- ☐ CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays within the SM
- ☐ CP asymmetries in $\tau \to K_S \pi \nu_\tau$ decays within a general EFT
- ☐ Summary

$$A_{\rm CP}^{\rm rate} \equiv \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}$$

BaBar Collaboration, PRD 85 (2012) 031102

➢ commonly discussed decay-rate asymmetry

➢ CP asymmetry in the angular distribution

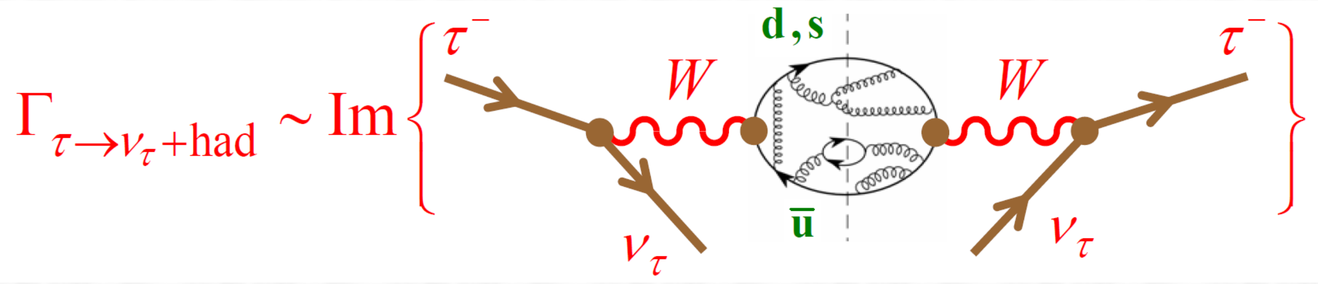
$$A_{CP}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d^2\Gamma(\tau^-\to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} - \frac{d^2\Gamma(\tau^+\to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d^2\Gamma(\tau^-\to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} + \frac{d^2\Gamma(\tau^+\to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

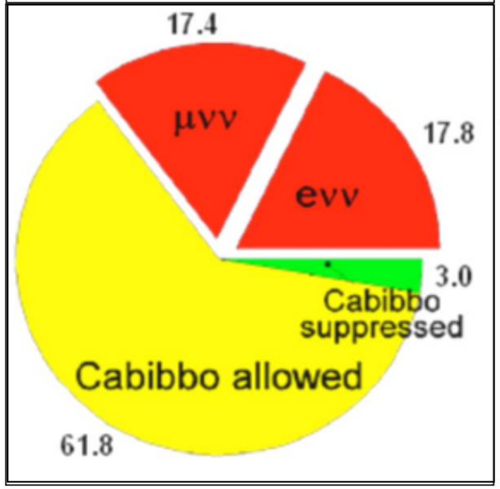
Semi-leptonic tau decays

Br $\simeq$ 66%!

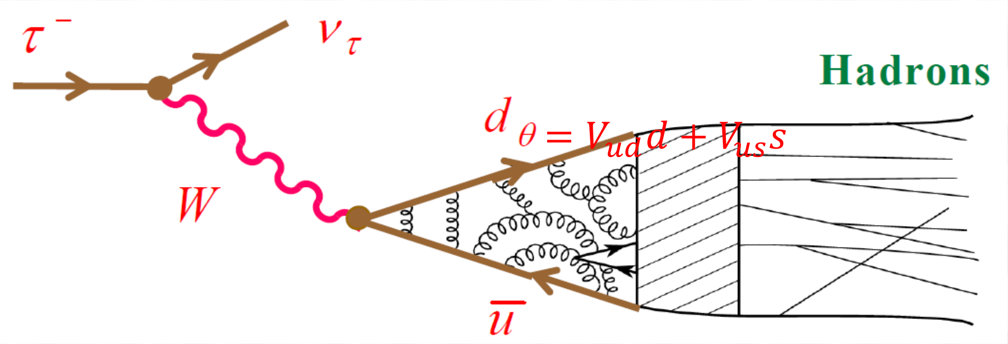
- **Only lepton heavy enough to decay into hadrons (really mesons):**
- **Can be used to extract the SM parameters: $\alpha_s(m_\tau)$, m_s, $|V_{us}|$, …**



Including QED & QCD corrections:



- **An ideal low-energy QCD-testing laboratory: how various QCD currents hadronized, information about various hadronic resonance parameters (M_R, Γ_R), …**



populated by resonances

Decay channel	Resonances
$\tau^- \to \pi^-\pi^0\nu_\tau$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K^-K_S\nu_\tau$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K_S\pi^-\nu_\tau$	$K^*(892), K^*(1410)$
$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$	$K^*(1410)$
$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$	$a_0(980)$

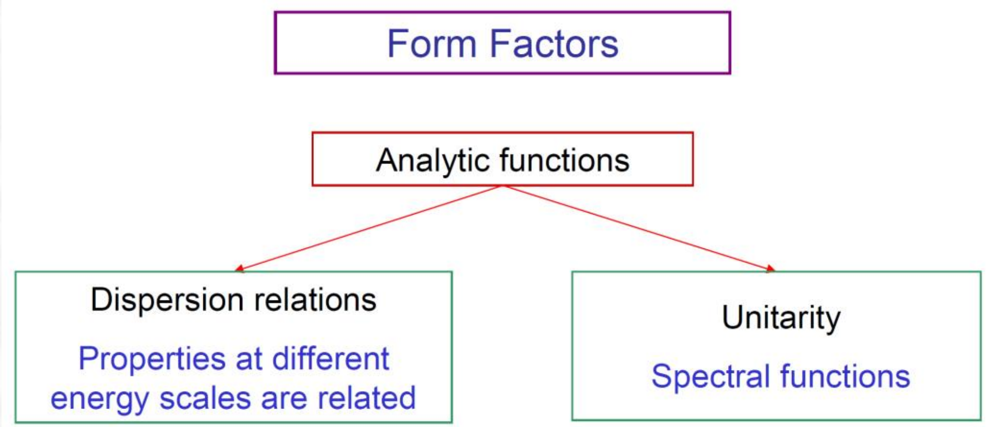
Semi-leptonic tau decays

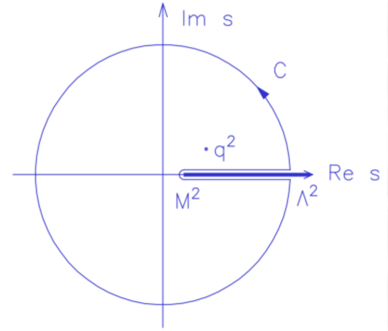
- **Decay amplitude:**

$$H_\mu = \langle H|(\mathcal{V}_\mu - \mathcal{A}_\mu)e^{i\mathcal{L}_{QCD}}|0\rangle = \sum_i \underbrace{(\ldots\ldots\ldots..\ldots)^i_\mu}_{\text{Lorentz structure}} \overbrace{F_i(q^2,\ldots)}^{\text{Form factor}}$$

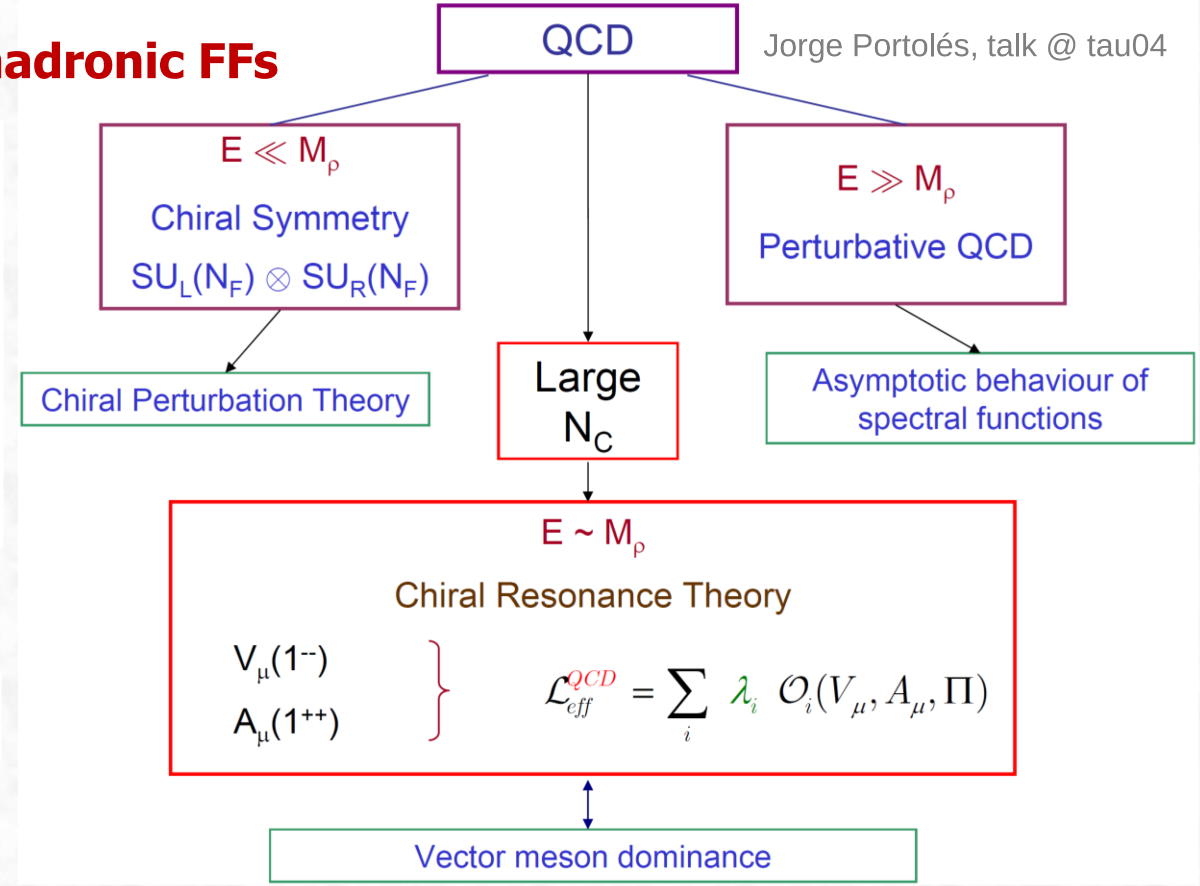
$$\mathcal{M}(\tau \to H\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{CKM}[\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma_5)u_\tau]H_\mu$$

- **Main tasks: determine accurately the hadronic FFs**



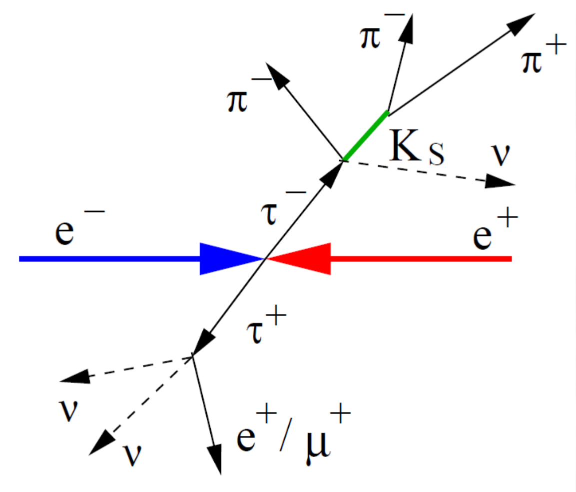


$$\mathrm{Im}\,\Pi_{\mu\nu} \propto \sum_n^\infty \int d\rho_n \langle 0|V_\mu|n\rangle\langle n|V_\nu^\dagger|0\rangle$$



Why $\tau \to K_S\pi\nu_\tau$ decays

- **Has largest Br among semi-leptonic decays with 1 kaon** [D. EPIFANOV *et al.* [Belle], PLB 654 (2007) 65]



$$\mathbf{Br}(\boldsymbol{\tau \to K_S\pi\nu_\tau}) = \mathbf{(0.404 \pm 0.002(stat.) \pm 0.013(syst.))\%}$$

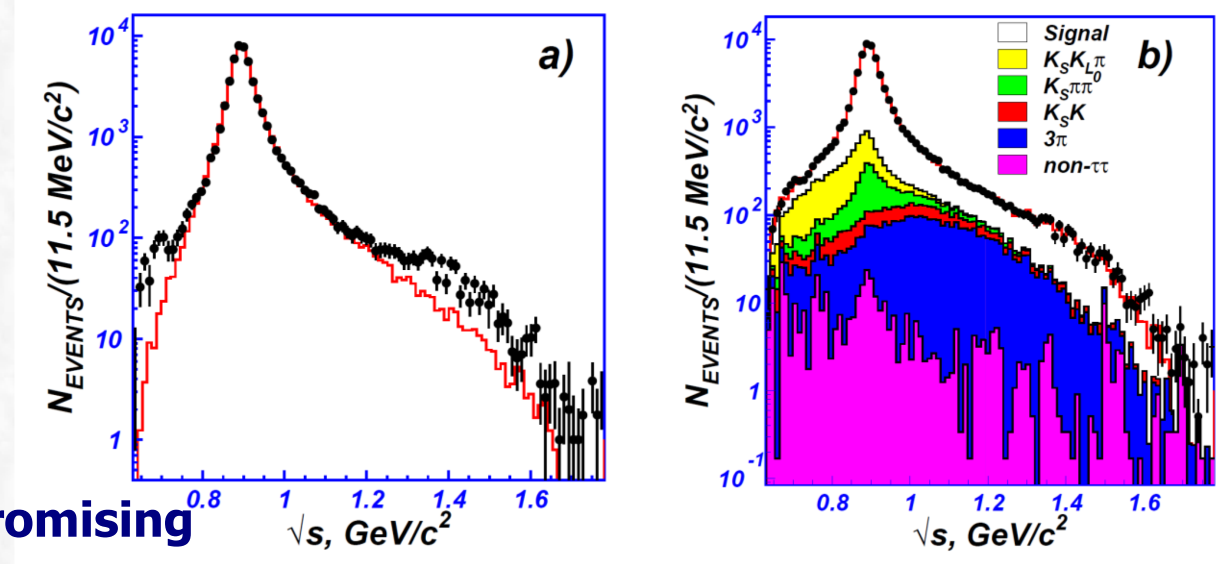
➢ The hadronic current parametrized by two form factors:

$$J^\mu = F_V(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)(q_1 - q_2)_\nu + F_S(q^2)q^\mu,\ q^\mu = q_1^\mu + q_2^\mu$$

- F_V: $K^*(892)^\pm$, $K^*(1410)^\pm$, $K^*(1680)^\pm$;
- F_S: $K^*(800)^\pm(\kappa)$, $K^*(1430)^\pm$;

➢ The $K^*(892)$ alone not sufficient to describe the $K\pi$ spectrum

➢ Fitted result with $K^*(892) + K^*(800) + K^*(1410)$ model reproduces data well



- **Searches for CPV in $\tau \to K_S\pi\nu_\tau$ very promising**

Why $\tau \to K_S \pi \nu_\tau$ decays

Decay-rate asymmetry in $\tau \to K_S \pi \nu_\tau$ decays

$$A_{\rm CP}^{\rm rate} \equiv \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}$$

2.8 σ 偏差

$$\begin{cases} A_{\rm CP}^{\rm Exp} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3} \\ A_{\rm CP}^{\rm SM} = (3.6 \pm 0.1) \times 10^{-3} \end{cases}$$

BaBar Collaboration, PRD 85 (2012) 031102

I. Bigi and A. I. Sanda PLB 625 (2005) 47
Y. Grossman and Y. Nir, JHEP 04 (2012) 002

CP asymmetry in the angular distribution of $\tau \to K_S \pi \nu_\tau$ decays

$$A_{CP}^i = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} - \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} + \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}$$

Belle Collaboration, PRL 107 (2011) 131801

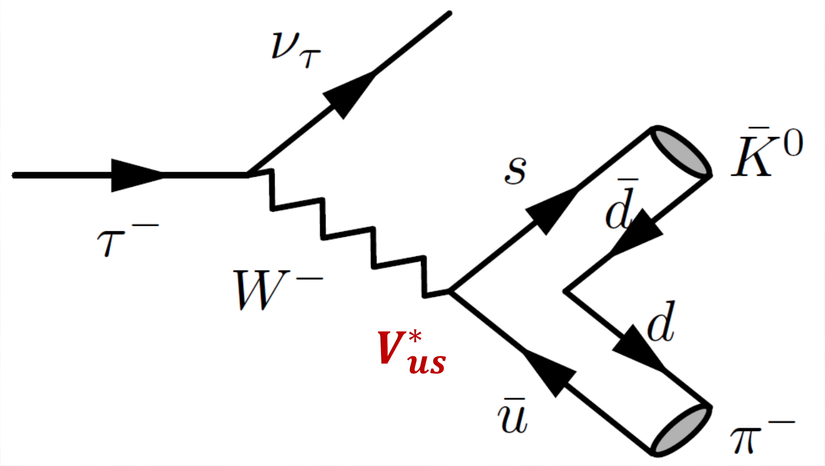
$\sqrt{s}$ [GeV]	$A_{{\rm SM},i}^{CP}$ [10^{-3}]	$A_{{\rm exp},i}^{CP}$ [10^{-3}]
$0.625 - 0.890$	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$
$0.890 - 1.110$	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$
$1.110 - 1.420$	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$
$1.420 - 1.775$	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$

compatible with zero with a sensitivity of $\mathcal{O}(10^{-3})$

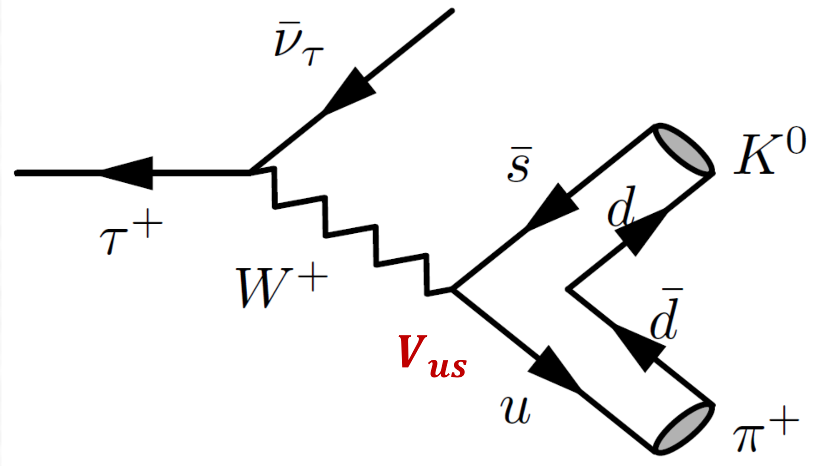
More precise measurements are expected from Belle II & SCTF, ……

$\tau \to K_S \pi \nu_\tau$ decays within the SM

- Feynman diagrams at the **tree level** in weak interaction within the SM:



CP



- According to the well-known **ΔS = ΔQ rule**, τ^- can only decay into $\bar{K}^0$, while τ^+ into K^0

- Within the SM, V_{us} **is real** (no weak phase) & the **same strong phase** between the two CP-related processes

$$\boxed{\mathcal{A}(\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0 \pi^- \nu_\tau)}$$

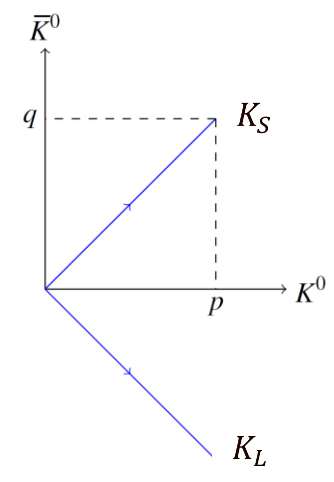
$\tau \to K_S \pi \nu_\tau$ decays within the SM

- **However, due to $K^0 - \bar{K}^0$ mixing, the exp. reconstructed kaons are the mass ($|K_S\rangle, |K_L\rangle$) rather than the flavor ($|K^0\rangle, |\bar{K}^0\rangle$) eigenstates**

$$|K_S^0\rangle = \frac{(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}, \qquad |K^0\rangle = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1+\epsilon)}\left[|K_S^0\rangle + |K_L^0\rangle\right],$$

$$|K_L^0\rangle = \frac{(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}}, \qquad |\bar{K}^0\rangle = \frac{\sqrt{2(1+|\epsilon|^2)}}{2(1-\epsilon)}\left[|K_S^0\rangle - |K_L^0\rangle\right].$$

ϵ characterizes the amount of CPV in neutral kaon system



- **In the absence of CPV, $\epsilon = 0$, we have then:**

$$\boxed{\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}$$

$$|K_S\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \qquad |K_L\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{d^2\Gamma(\tau^- \to K_{S,L}\pi^-\nu_\tau)}{ds\, d\cos\alpha} = \frac{1}{2}\frac{d^2\Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}{ds\, d\cos\alpha}$$

$$\frac{d^2\Gamma(\tau^+ \to K_{S,L}\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha} = \frac{1}{2}\frac{d^2\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}$$

$\Rightarrow$ no CP asymmetries in decay rate & angular distribution of $\tau \to K_S\pi\nu_\tau$ decays in the SM!

- **But the CPV in neutral kaon system well established: $\epsilon = 2.3 \times 10^{-3}$**

$\tau \to K_S \pi \nu_\tau$ decays within the SM

- **Once CPV in $K^0 - \bar{K}^0$ mixing included, non-zero CP asymmetries appear in the decays**

- **When $\langle K_{S,L}|$ intermediate states involved, the so-called reciprocal basis more efficient**

$$\begin{pmatrix} |K_L\rangle \\ |K_S\rangle \end{pmatrix} = \begin{pmatrix} p & -q \\ p & q \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix} = \boldsymbol{X}^T \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

$$\boldsymbol{X}^{-1}\boldsymbol{H}\boldsymbol{X} = \begin{pmatrix} \mu_L & 0 \\ 0 & \mu_S \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} p & p \\ -q & q \end{pmatrix}$$

  - $H = M - i/2\Gamma$ **is NOT a normal matrix**

$$[\boldsymbol{H}, \boldsymbol{H}^\dagger] = 0$$

  - X **is NOT unitary in the presence of CPV**

$$\boldsymbol{X}^\dagger \boldsymbol{X} = \boldsymbol{X}\boldsymbol{X}^\dagger = \begin{pmatrix} 1 & |p|^2 - |q|^2 \\ |p|^2 - |q|^2 & 1 \end{pmatrix}$$

  - **thus, we will have** $\langle \tilde{K}_{S,L}| \neq \langle K_{S,L}|$ **!**

- **The time evolution operator for the neutral kaon system:**

$$\boxed{\langle \tilde{K}_{S,L}| = \frac{1}{2}\left(p^{-1}\langle K^0| \pm q^{-1}\langle \bar{K}^0|\right)}$$

orthornormality

$$\langle \tilde{K}_S|K_S\rangle = \langle \tilde{K}_L|K_L\rangle = 1, \quad \langle \tilde{K}_S|K_L\rangle = \langle \tilde{K}_L|K_S\rangle = 0,$$

completeness $\quad |K_S\rangle\langle \tilde{K}_S| + |K_L\rangle\langle \tilde{K}_L| = 1$

J. P. Silva, PRD 62 (2000) 116008

$$\boxed{\exp(-i\boldsymbol{H}t) = e^{-i\mu_S t}|K_S\rangle\langle \tilde{K}_S| + e^{-i\mu_L t}|K_L\rangle\langle \tilde{K}_L|}$$

$\mu_{S,L} = M_{S,L} - i/2\Gamma_{S,L}$: two eigenvalues of H

$\tau \to K_S \pi \nu_\tau$ decays within the SM

- Experimentally, the K_S intermediate state is reconstructed via a $\pi^+\pi^-$ final state

- Due to CPV in $K^0 - \bar{K}^0$ mixing, $\pi^+\pi^-$ final state arises not only from K_S, but also from K_L

  The interference effect between the K_S & K_L amplitudes important for CPV!

- Complete time-dependent decay amplitudes of $\tau^\pm \to \pi^+\pi^-\pi^\pm\nu_\tau$ decays (omitting $\langle \pi\nu_\tau|$):

$$
\begin{aligned}
\mathcal{A}(\tau^- \to K_{S,L} \to \pi^+\pi^-) &= \langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle \tilde{K}_S|T|\tau^-\rangle + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle \tilde{K}_L|T|\tau^-\rangle \\
&= \frac{1}{2q}\left[\langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} - \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right] \underline{\langle \bar{K}^0|T|\tau^-\rangle}\,,
\end{aligned}
$$

$\Delta S = \Delta Q$ rule

$$
\begin{aligned}
\mathcal{A}(\tau^+ \to K_{S,L} \to \pi^+\pi^-) &= \langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle \tilde{K}_S|T|\tau^+\rangle + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle \tilde{K}_L|T|\tau^+\rangle \\
&= \frac{1}{2p}\left[\langle \pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} + \langle \pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right] \underline{\langle K^0|T|\tau^+\rangle}\,,
\end{aligned}
$$

- the kaon decays are independent of the τ decays

Time-dep. CPA in the angular distribution

□ Time-dependent double differential decay widths:

$$\frac{d^2\Gamma(\tau^- \to K_{S,L}\pi^-\nu_\tau \to [\pi^+\pi^-]\pi^-\nu_\tau)}{ds\,d\cos\alpha} = \frac{d^2\Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}{ds\,d\cos\alpha}\,\underline{\Gamma(\bar{K}^0(t) \to \pi^+\pi^-)}\,,$$

$$\frac{d^2\Gamma(\tau^+ \to K_{S,L}\pi^+\bar{\nu}_\tau \to [\pi^+\pi^-]\pi^+\bar{\nu}_\tau)}{ds\,d\cos\alpha} = \frac{d^2\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau)}{ds\,d\cos\alpha}\,\underline{\Gamma(K^0(t) \to \pi^+\pi^-)}\,,$$

s: the $K\pi$ invariant mass squared;

α: the angle between directions of K and τ seen in the $K\pi$ rest frame

□ Time-dependent $K(t) \to \pi^+\pi^-$ decay widths:

$$\Gamma(\bar{K}^0(t) \to \pi^+\pi^-) = \frac{|\langle\pi^+\pi^-|T|K_S\rangle|^2}{4|q|^2}\left[e^{-\Gamma_S t} + |\eta_{+-}|^2\,e^{-\Gamma_L t} - 2|\eta_{+-}|\,e^{-\Gamma t}\cos(\phi_{+-} - \Delta m t)\right]$$

$$\Gamma(K^0(t) \to \pi^+\pi^-) = \frac{|\langle\pi^+\pi^-|T|K_S\rangle|^2}{4|p|^2}\left[e^{-\Gamma_S t} + |\eta_{+-}|^2\,e^{-\Gamma_L t} + 2|\eta_{+-}|\,e^{-\Gamma t}\cos(\phi_{+-} - \Delta m t)\right]$$

$$\Delta m = M_L - M_S \qquad \Gamma = \frac{\Gamma_L + \Gamma_S}{2}$$

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle}$$

$$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}$$

$$\phi_{+-} = (43.51 \pm 0.05)^\circ$$

□ Time-dep. CP asymmetry in the angular distribution:

$(d\omega = ds\,d\cos\alpha)$

$$A_i^{CP}(t_1,t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt - \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt + \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}$$

bin choice $[s_{1,i}, s_{2,i}]$, time interval $[t_1, t_2]$, exp.-dep. effects parametrized by $F(t)$

Time-dep. CPA in the angular distribution

- **The difference of the decay rates weighted by cosα:**

  α: the angle between directions of K and τ seen in the $K\pi$ rest frame

$$A_i^{CP}(t_1,t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt - \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt + \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}$$

$$= \frac{\left(\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+}\right)A_K^{CP}(t_1,t_2) + \left(\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+}\right)}{1 + A_K^{CP}(t_1,t_2)\cdot A_{\tau,i}^{CP}}$$

- **As the kaon decays independent of the τ decays, the 2nd line are obtained:**

$$\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}$$

$$\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}$$

$$A_K^{CP}(t_1,t_2) = \frac{\int_{t_1}^{t_2}F(t)\left[\bar{\Gamma}_{\pi^+\pi^-}(t) - \Gamma_{\pi^+\pi^-}(t)\right]dt}{\int_{t_1}^{t_2}F(t)\left[\bar{\Gamma}_{\pi^+\pi^-}(t) + \Gamma_{\pi^+\pi^-}(t)\right]dt},$$

$$A_{\tau,i}^{CP} = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega}{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega}\right]d\omega},$$

- **Within the SM:**

$$\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau)$$

$$\frac{d\Gamma^{\tau^\pm}}{d\omega} = \frac{d^2\Gamma(\tau^\pm \to K^0(\bar{K}^0)\pi^\pm\bar{\nu}_\tau(\nu_\tau))}{ds\,d\cos\alpha}$$

$$\frac{d\Gamma^{\tau^+}}{d\omega} = \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A_{\tau,i}^{CP} = 0\,, \quad \langle\cos\alpha\rangle_i^{\tau^-} = \langle\cos\alpha\rangle_i^{\tau^+}$$

$$\Downarrow$$

$$A_i^{CP}(t_1,t_2) = 2\,\langle\cos\alpha\rangle_i^{\tau^-} A_K^{CP}(t_1,t_2)$$

Time-dep. CPA in the angular distribution

◻ Results within the SM:

using the efficiency function $F(t)$ provided by the BaBar collaboration!

$$|\eta_{+-}| \approx \frac{2\Re e(\epsilon_K)}{\sqrt{2}},\ \phi_{+-} \approx 45^\circ,$$

$$\Gamma \approx \frac{\Gamma_S}{2},\ \text{and}\ \Delta m \approx \frac{\Gamma_S}{2}$$

- $A_K^{CP}(t_1, t_2)$: CPV in $K^0 - \bar{K}^0$ mixing

$$A_K^{\rm CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2{\rm Re}(\epsilon_K) = -(3.32 \pm 0.06) \times 10^{-3}$$

Thus easily reproduce Grossman-Nir result!

$$F(t) = \begin{cases} 1 & t_1 < t < t_2 \\ 0 & \text{otherwise.} \end{cases}$$

double-step form

Y. Grossman and Y. Nir, JHEP 04 (2012) 002

- $\langle \cos\alpha \rangle^{\tau^-} = \frac{2}{3} A_{\rm FB}^{\tau^-}$ L. Beldjoudi and T. N. Truong, PLB 351 (1995) 357368

$$A_{\rm FB}^{\tau^-}(s) = \frac{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha - \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha}{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha + \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha}$$

forward-backward asymmetry

Even in the SM, there is non-zero CPA in the angular distributions due to CPV in $K^0 - \bar{K}^0$ mixing

◻ Puzzle still exists:

2.8 σ 偏差

$$\begin{cases} A_{\rm CP}^{\rm Exp} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3} \\ A_{\rm CP}^{\rm SM} = (3.6 \pm 0.1) \times 10^{-3} \end{cases}$$

using the efficiency function $F(t)$ provided by the BaBar collaboration!

[BaBar, PRD 85 (2012) 031102]

Time-dep. CPA in the angular distribution

- **Results within the SM:** $\boxed{A_i^{CP}(t_1,t_2) = 2\,\langle\cos\alpha\rangle_i^{\tau^-} A_K^{CP}(t_1,t_2)}$

$$A_K^{\rm CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx -2{\rm Re}(\epsilon_K) = -(3.32 \pm 0.06) \times 10^{-3}$$

as input!

- **Angular observable $\langle\cos\alpha\rangle_i^{\tau^\pm}$:**

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{us} \left[\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau\right]\left[\bar{u}\gamma^\mu(1-\gamma_5)s\right] + {\rm h.c.}$$

$$\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha} = \frac{G_F^2|F_+(0)V_{us}|^2 m_\tau^3 S_{\rm EW}}{512\pi^3 s^3}\left(1-\frac{s}{m_\tau^2}\right)^2 \lambda^{1/2}(s, M_K^2, M_\pi^2)$$
$$\times\left\{\underline{\left|\tilde{F}_+(s)\right|^2}\left(\frac{s}{m_\tau^2} + \left(1-\frac{s}{m_\tau^2}\right)\cos^2\alpha\right)\lambda(s, M_K^2, M_\pi^2) + \Delta_{K\pi}^2\underline{\left|\tilde{F}_0(s)\right|^2}\right.$$
$$\left. - 2\Delta_{K\pi}\Re e\underline{\left[\tilde{F}_+(s)\tilde{F}_0^*(s)\right]}\lambda^{1/2}(s, M_K^2, M_\pi^2)\cos\alpha\right\},$$

normalized FFs

$$\boxed{\tilde{F}_{+,0}(s) = F_{+,0}(s)/F_+(0)}$$

- **Hadronic FFs:** $\langle \bar{K}^0(p_K)\pi^-(p_\pi)\,|\bar{s}\gamma^\mu u|\,0\rangle = \left[(p_K-p_\pi)^\mu - \frac{\Delta_{K\pi}}{s}q^\mu\right]F_+(s) + \frac{\Delta_{K\pi}}{s}q^\mu F_0(s)$

Time-dep. CPA in the angular distribution

◻ For $K\pi$ FFs, we adopt:

As the Breit-Wigner form violates Watson's theorem, and thus not physical and not applicable for CPV!

[Belle, PLB **654** (2007) 65]

- Vector form factor thrice-subtracted dispersion representation
  D.R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C59 (2009) 821

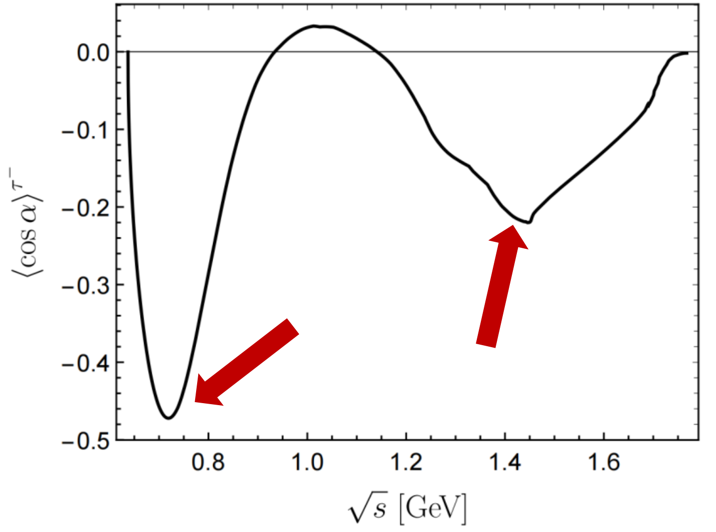
$$F_+(s) = \exp\left\{\lambda'_+ \frac{s}{M^2_{\pi^-}} + \frac{1}{2}(\lambda''_+ - \lambda'^2_+)\frac{s^2}{M^4_{\pi^-}} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta_+(s')}{(s')^3(s'-s-i\epsilon)}\right\},$$

- Scalar form factor coupled-channel dispersive representation
  M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B622 (2002) 279

$$F_0^1(s) = \frac{1}{\pi}\sum_{j=1}^{3}\int_{s_j}^{\infty} ds' \frac{\sigma_j(s')F_0^j(s')t_0^{1\to j}(s')^*}{s'-s-i\epsilon}, \ (1 \equiv K\pi,\ 2 \equiv K\eta, \text{ and } 3 \equiv K\eta')$$

◻ Angular observable: differential decay width weighted by cosα

$$\boxed{\begin{aligned}\langle\cos\alpha\rangle^{\tau^-}(s) &= \frac{\int_{-1}^{1}\cos\alpha\left(\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}\right)d\cos\alpha}{\int_{-1}^{1}\left(\frac{d^2\Gamma^{\tau^-}}{ds\,d\cos\alpha}\right)d\cos\alpha} \\ &= \frac{-2\Delta_{K\pi}\Re e[\tilde{F}_+(s)\tilde{F}_0^*(s)]\lambda^{1/2}(s,M_K^2,M_\pi^2)}{\left|\tilde{F}_+(s)\right|^2\left(1+\frac{2s}{m_\tau^2}\right)\lambda(s,M_K^2,M_\pi^2)+3\Delta_{K\pi}^2\left|\tilde{F}_0(s)\right|^2}\end{aligned}}$$



Time-dep. CPA in the angular distribution

◻ Results for $A_i^{CP}(t_1, t_2)$ in four mass bins:

Belle Collaboration, PRL 107 (2011) 131801

$\sqrt{s}$ [GeV]	$A^{CP}_{\mathrm{SM},i}$ $[10^{-3}]$	$A^{CP}_{\mathrm{exp},i}$ $[10^{-3}]$	n_i/N_s [%]
$0.625 - 0.890$	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$	36.53 ± 0.14
$0.890 - 1.110$	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$	57.85 ± 0.15
$1.110 - 1.420$	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$	4.87 ± 0.04
$1.420 - 1.775$	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$	0.75 ± 0.02

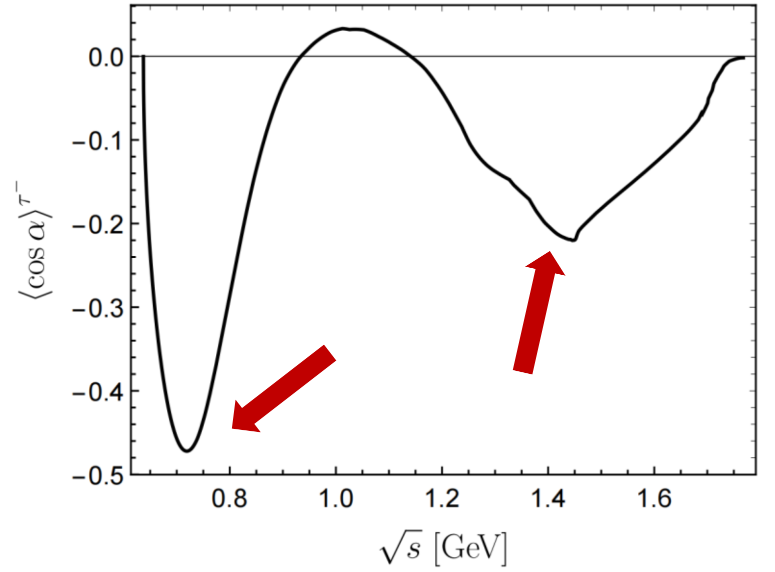
SM predictions still below Belle detection sensitivity of $\mathcal{O}(10^{-3})$, but expected to be detectable at Belle II, with $\sqrt{70}$ times more sensitive results!

◻ Two more optimal SM predictions:

as large as the SM prediction for the decay-rate asymmetry!

$$\begin{cases} A_{\mathrm{CP}}^{\mathrm{Exp}} = (-3.6 \pm 2.3 \pm 1.1) \times 10^{-3} \\ A_{\mathrm{CP}}^{\mathrm{SM}} = (3.6 \pm 0.1) \times 10^{-3} \end{cases}$$

$$\boxed{A_i^{CP}(t_1, t_2) = \begin{cases} (3.06 \pm 0.06) \times 10^{-3}, & 0.70\,\mathrm{GeV} < \sqrt{s} < 0.75\,\mathrm{GeV} \\ (1.38 \pm 0.18) \times 10^{-3}, & 1.40\,\mathrm{GeV} < \sqrt{s} < 1.50\,\mathrm{GeV} \end{cases}}$$



◻ Suggest to measure A_i^{CP} in these two mass intervals!

$\tau^{\pm} \to K^0(\bar{K}^0)\pi^{\pm}\bar{\nu}_\tau(\nu_\tau)$ withinin a general EFT

- **When NP presents in tau decays:**

$$\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) \neq \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau)$$

$$\frac{d\Gamma^{\tau^+}}{d\omega} \neq \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A^{CP}_{\tau,i} \neq 0\,, \quad \langle\cos\alpha\rangle_i^{\tau^-} \neq \langle\cos\alpha\rangle_i^{\tau^+}$$

- **CPA in angular distribution:**

$$\boxed{A_i^{\rm CP} \simeq \left(\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+}\right) A_K^{\rm CP} + \left(\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+}\right)}$$

- **The most general $SU(3)_C \otimes U(1)_{em}$-invariant low-energy effective Lagrangian:**

$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{us}}{\sqrt{2}}\Big\{\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\left[\gamma^\mu - (1-2\,\hat{\epsilon}_R)\gamma^\mu\gamma_5\right]s$$

$$+\,\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}\left[\hat{\epsilon}_S - \hat{\epsilon}_P\gamma_5\right]s + 2\,\hat{\epsilon}_T\,\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}s\Big\} + {\rm h.c.}$$

- **Decay amplitude for $\tau^- \to \bar{K}^0\pi^-\nu_\tau$:**

  scalar operator + tensor operator

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T$$

$$= \frac{G_F V_{us}}{\sqrt{2}}\left[L_\mu H^\mu + \hat{\epsilon}_S^* LH + 2\hat{\epsilon}_T^* L_{\mu\nu}H^{\mu\nu}\right]$$

Tensor form factors

- **Leptonic currents:**

$$L = \bar{u}(p_{\nu_\tau})(1+\gamma_5)u(p_\tau)\,,$$
$$L_\mu = \bar{u}(p_{\nu_\tau})\gamma_\mu(1-\gamma_5)u(p_\tau)\,,$$
$$L_{\mu\nu} = \bar{u}(p_{\nu_\tau})\sigma_{\mu\nu}(1+\gamma_5)u(p_\tau)\,,$$

- **Hadronic matrix elements:**

$$H = \langle \pi^- \overline{K}^0 \mid \bar{s}u \mid 0\rangle = F_S(s)$$
$$H^\mu = \langle \pi^- \overline{K}^0 \mid \bar{s}\gamma^\mu u \mid 0\rangle = Q^\mu F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s)$$
$$H^{\mu\nu} = \langle \pi^- \overline{K}^0 \mid \bar{s}\sigma^{\mu\nu} u \mid 0\rangle = iF_T(s)\,(p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu)$$

- **Tensor FF $F_T(s)$: due to lack of enough exp. data, has to be constructed theoretically**

$$\boxed{F_T(s) = F_T(0)\exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s'-s-i\epsilon)}\right\}}$$

➢ **for $F_T(0)$: obtained from the lowest chiral order of χPT with tensor source**

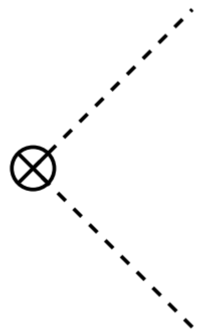
- $\mathcal{L}_4^{\chi\mathrm{PT}} = \Lambda_1 \langle t_+^{\mu\nu} f_{+\mu\nu}\rangle - \underline{i\Lambda_2 \langle t_+^{\mu\nu} u_\mu u_\nu\rangle} + \Lambda_3 \langle t_+^{\mu\nu} t^{+\mu\nu}\rangle + \Lambda_4 \langle t_+^{\mu\nu}\rangle^2$

⬇

$$\left\langle \bar{K}^0(p_K)\pi^-(p_\pi)\left|\frac{\delta L_4^{\chi\mathrm{PT}}}{\delta \bar{t}_{\mu\nu}}\right|0\right\rangle = i\frac{\Lambda_2}{F_\pi^2}(p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu) \Rightarrow \boxed{F_T(0) = \Lambda_2/F_\pi^2,\ \text{with } \Lambda_2 = (11.1 \pm 0.4)\mathrm{MeV}}$$

O. Cata and V. Mateu, JHEP 09 (2007) 078

I. Baum *et al.*, PRD 84 (2011) 074503



χPT

Tensor form factors

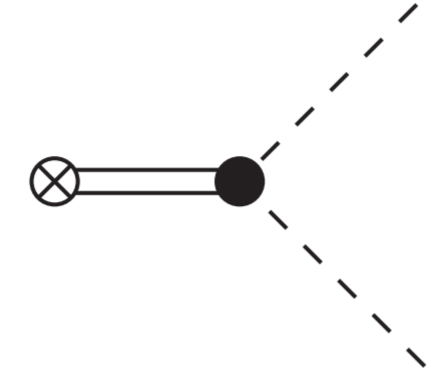
- With $s \in [(M_K + M_\pi)^2, m_\tau^2]$, **light resonances** to FFs must be included to give s-**dep.** of FFs

- As **spin-1 resonances** described equivalently by **vector** or **anti-symmetric tensor fields**, resonance contributions to $F_+(s)$ & $F_T(s)$ are both dominated by $K^*(892)$ & $K^*(1410)$

➢ $F_T(s)$ obtained with $\mathrm{R}\chi\mathrm{T}$ **including spin-1 resonances:**

- $\mathcal{L}_6^{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}_{kin}(\hat{V}_\mu) - \frac{1}{2\sqrt{2}}\left(f_V \langle \hat{V}_{\mu\nu} f_+^{\mu\nu}\rangle + \underline{i g_V \langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle) - f_V^T \langle \hat{V}_{\mu\nu} t_+^{\mu\nu}\rangle}\right)$

Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, PLB 223 (1989) 425



$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\left[1 + \frac{\sqrt{2} f_V^T g_V}{\Lambda_2}\frac{s}{M_{K^*}^2 - s} + \frac{\sqrt{2} f_V^{T\prime} g_V^\prime}{\Lambda_2}\frac{s}{M_{K^{*\prime}}^2 - s}\right]$$

$$= \frac{\Lambda_2}{F_\pi^2}\left[\frac{M_{K^*}^2 + \beta s}{M_{K^*}^2 - s} - \frac{\beta s}{M_{K^{*\prime}}^2 - s}\right]$$

energy-dep. width $\gamma_n(s)$

$$\boxed{\tilde{F}_T(s) = \frac{F_T(s)}{F_T(0)} = \frac{M_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \beta s}{D(M_{K^*}, \gamma_{K^*})} - \frac{\beta s}{D(M_{K^*}, \gamma_{K^*})}}$$

Feng-Zhi Chen, Xin-Qiang Li, Ya-Dong Yang, Xin Zhang, PRD 100 (2019) 113006

$\beta = \frac{\sqrt{2} f_V^T g_V}{\Lambda_2} - 1 \simeq \pm 0.75\gamma$: characterizes relative weight of the two resonances, and plays the same role as γ for $F_+(s)$

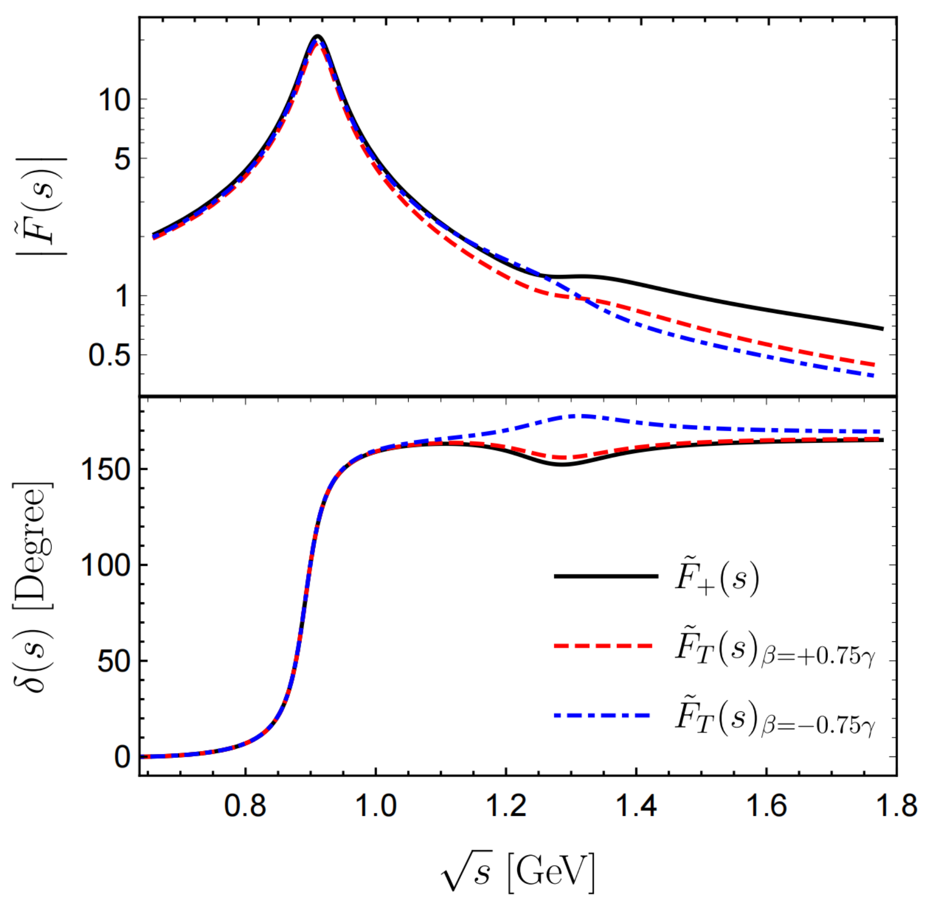
Tensor form factors

- **Combining χPT @ low s + RχT @ intermediate s + asymptotic behaviors @ high s, we obtain the once-subtracted dispersion representation:**

$$\boxed{F_T(s) = F_T(0)\exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s'-s-i\epsilon)}\right\}}$$

$$\delta_T(s) = \begin{cases} \arctan\left[\frac{\Im m\tilde{F}_T(s)}{\Re e\tilde{F}_T(s)}\right], & s_{K\pi} < s < s_{\text{cut}} \\ n_T\pi, & s \geq s_{\text{cut}} \end{cases}$$

asymptotic $1/s$ as dictated by pQCD



- in elastic region (below ~ 1.2 GeV), $\delta_T(s) = \delta_+(s)$ as required by Watson's FSI theorem [K. M. Watson, Phys. Rev. 95 (1954) 228]

- in inelastic region (above ~ 1.2 GeV), $\delta_T(s)$ and $\delta_+(s)$ start to behave differently due to the different relative weights of the two resonances $K^*(892)$ & $K^*(1410)$

CP-violating observables in general EFT

- **Decay-rate asymmetry:**

$$A_{\rm CP}^{\rm rate}(\tau \to K\pi\nu_\tau) = \frac{\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}{\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}$$
$$= \frac{{\rm Im}[\hat{\epsilon}_T]\, G_F^2 |V_{us}|^2 S_{\rm EW}}{128\,\pi^3\, m_\tau^2\, \Gamma(\tau \to K_S\pi\nu_\tau)} \int_{s_{K\pi}}^{m_\tau^2} ds \left(1 - \frac{m_\tau^2}{s}\right)^2 \lambda^{\frac{3}{2}}\left(s, M_K^2, M_\pi^2\right)$$
$$\times \underline{|F_T(s)|\,|F_+(s)|\,\sin\left[\delta_T(s) - \delta_+(s)\right]}\,,$$

Only vector-tensor interference as the only possible mechanism

- **CPA in angular distribution:**

$$A_{CP}^i \simeq \Delta_{K\pi}\, S_{\rm EW}\, \frac{N_s}{n_i} \int_{s_{1,i}}^{s_{2,i}} \Bigg\{ -\frac{{\rm Im}[\hat{\epsilon}_S]}{m_\tau(m_s - m_u)}\, \underline{{\rm Im}\left[F_+(s)F_0^*(s)\right]} - \frac{2{\rm Im}[\hat{\epsilon}_T]}{m_\tau}\, \underline{{\rm Im}\left[F_T(s)F_0^*(s)\right]}$$
$$+ \left[\left(\frac{1}{s} + \frac{{\rm Re}[\hat{\epsilon}_S]}{m_\tau(m_s - m_u)}\right) \underline{{\rm Re}\left[F_+(s)F_0^*(s)\right]} - \frac{2{\rm Re}[\hat{\epsilon}_T]}{m_\tau}\, \underline{{\rm Re}[F_T(s)F_0^*(s)]}\right] A_K^{CP} \Bigg\}\, C(s)\, ds\,.$$

Both from scalar-vector and vector-tensor interferences

- **Constraints on ${\rm Re}[\hat{\epsilon}_{S,T}]$: more stringent from decay rates of various exclusive τ decays**

$${\rm Re}[\hat{\epsilon}_S] = (0.8^{+0.8}_{-0.9} \pm 0.3)\%,\ {\rm Re}[\hat{\epsilon}_T] = (0.9 \pm 0.7 \pm 0.4)\%$$

S. Gonzàlez-Solís, A. Miranda, J. Rendón and P. Roig, Phys. Lett. B 804 (2020) 135371

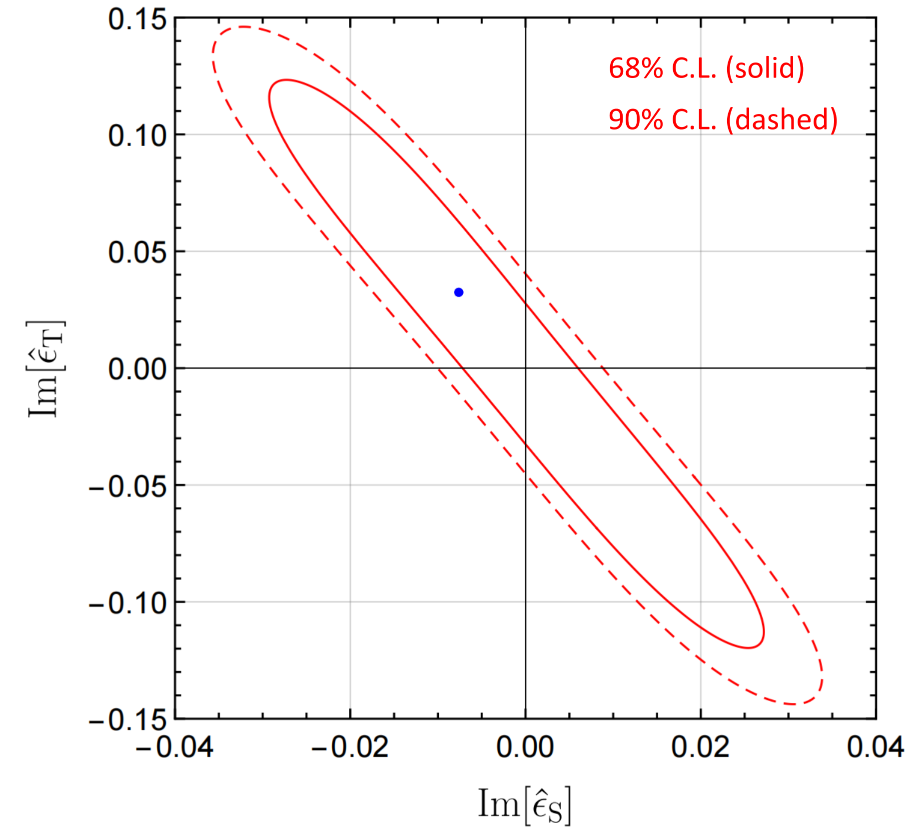
- **Constraints on ${\rm Im}[\hat{\epsilon}_{S,T}]$: more sensitive to these CP-violating observables**

CP-violating observables in general EFT

- **Fit results on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$ from $\mathcal{B}^{\tau^-}_{\mathrm{exp}}$ and four bins of $A^{CP}_{\mathrm{exp},i}$:**

$$\chi^2 = \sum_{i=1}^{4}\left(\frac{A^{CP}_{\mathrm{exp},i} - A^{CP}_{\mathrm{th},i}}{\sigma_i}\right)^2 + \left(\frac{\mathcal{B}^{\tau^-}_{\mathrm{exp}} - \mathcal{B}^{\tau^-}_{\mathrm{th}}}{\sigma_{\mathcal{B}}}\right)^2$$

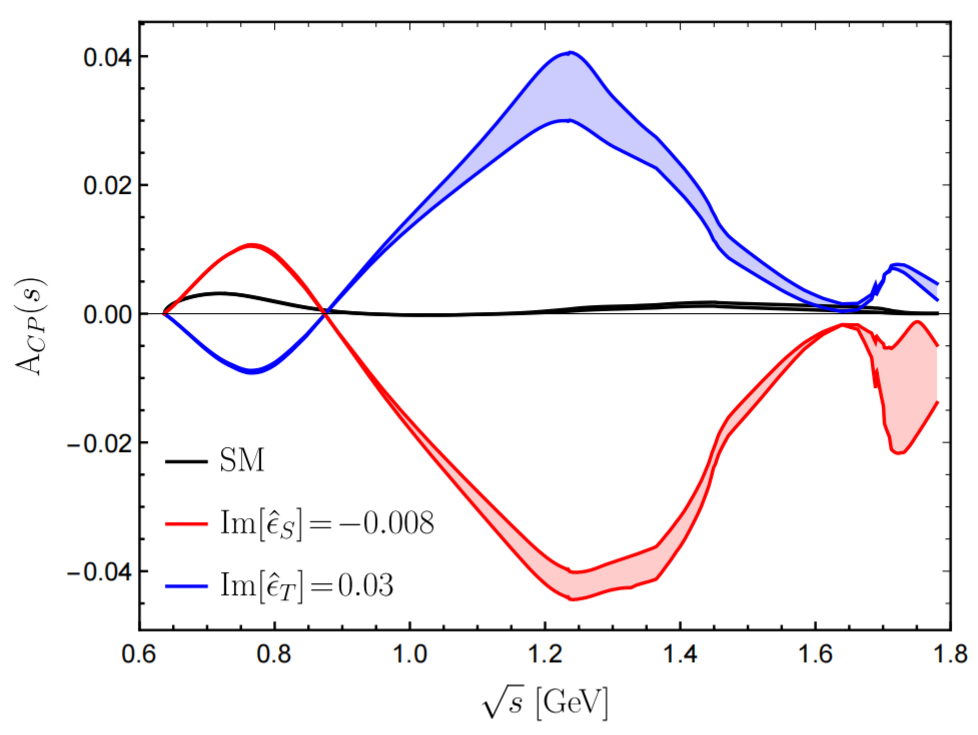
$$\chi^2_{\mathrm{min}} = 4.20, \quad \mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027, \quad \mathrm{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12$$



- remarkably negative correlation between $\mathrm{Im}[\hat{\epsilon}_S]$ & $\mathrm{Im}[\hat{\epsilon}_T]$, as both vector & tensor FFs dominated by $K^*(892)$ and $K^*(1410)$, and thus have almost the same phases, especially in elastic region
- bound on $\mathrm{Im}[\hat{\epsilon}_S]$ consistent with $|\mathrm{Im}(\eta_S)| < 0.026$ @ 90% C.L. obtained by Belle [PRL 107 (2011) 131801]
- upper bound on $\mathrm{Im}[\hat{\epsilon}_T]$ only of $\mathcal{O}(10^{-1})$, much weaker than $2|\mathrm{Im}[\hat{\epsilon}_T]| \le 10^{-5}$ from neutron EDM & $D^0 - \bar{D}^0$ mixing [V. Cirigliano *et al.*, PRL 120 (2018) 141803]

CP-violating observables in general EFT

- A_i^{CP} **in the presence of non-standard scalar & tensor interactions**



- with best-fit values of $\mathrm{Im}[\hat{\epsilon}_S]$ and $\mathrm{Im}[\hat{\epsilon}_T]$, the CPA distributions have almost the same magnitude but opposite in sign in whole $K\pi$ invariant-mass region
- the maximum absolute values are reached at around $\sqrt{s} = 1.2\ \mathrm{GeV}$ for both cases
- the non-standard scalar & tensor contributions about one order of magnitude larger than SM prediction

- **We strongly recommend to make more precise studies of CP asymmetry in the angular distributions, especially at Belle II & SCTF, ...**

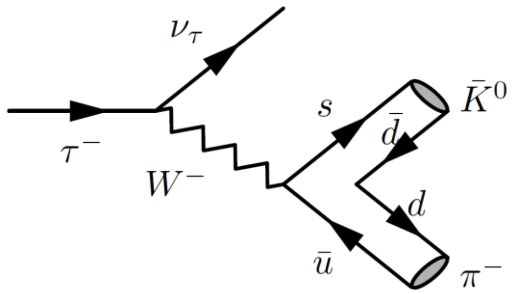
Belle-II, *PTEP* **2019** (2019) 123C01; H. Sang, X. Shi, X. Zhou, X. Kang and J. Liu, *CPC* **45** (2021) 053003

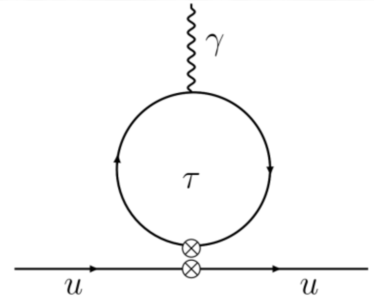
Summary

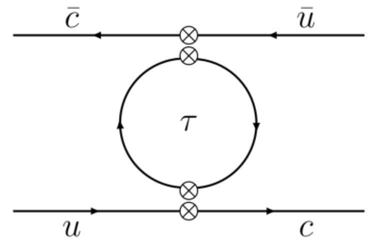
- **$\tau \to K_S \pi \nu_\tau$ decays: very promising for searching CPV effects**
  - in the SM, there exist non-zero decay-rate asymmetry & CP asymmetry in angular distribution due to CPV in $K^0 - \bar{K}^0$ mixing, with results of $\mathcal{O}(10^{-3})$ and detectable @ Belle II & STCF, …
  - With a general EFT, only vector-tensor interference produces a direct decay-rate asymmetry, while both scalar-vector & vector-tensor interferences possible for CPAs in angular distribution

- **Bounds on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$:**

only $\tau \to K_S \pi \nu_\tau$	SMEFT
$\mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027,$	$\mathrm{Im}[\hat{\epsilon}_S] \in [-3.1, 1.6] \times 10^{-4}$ @ 2σ
$\mathrm{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12,$	$\lvert\mathrm{Im}[\hat{\epsilon}_T]\rvert \lesssim 4 \times 10^{-6}$





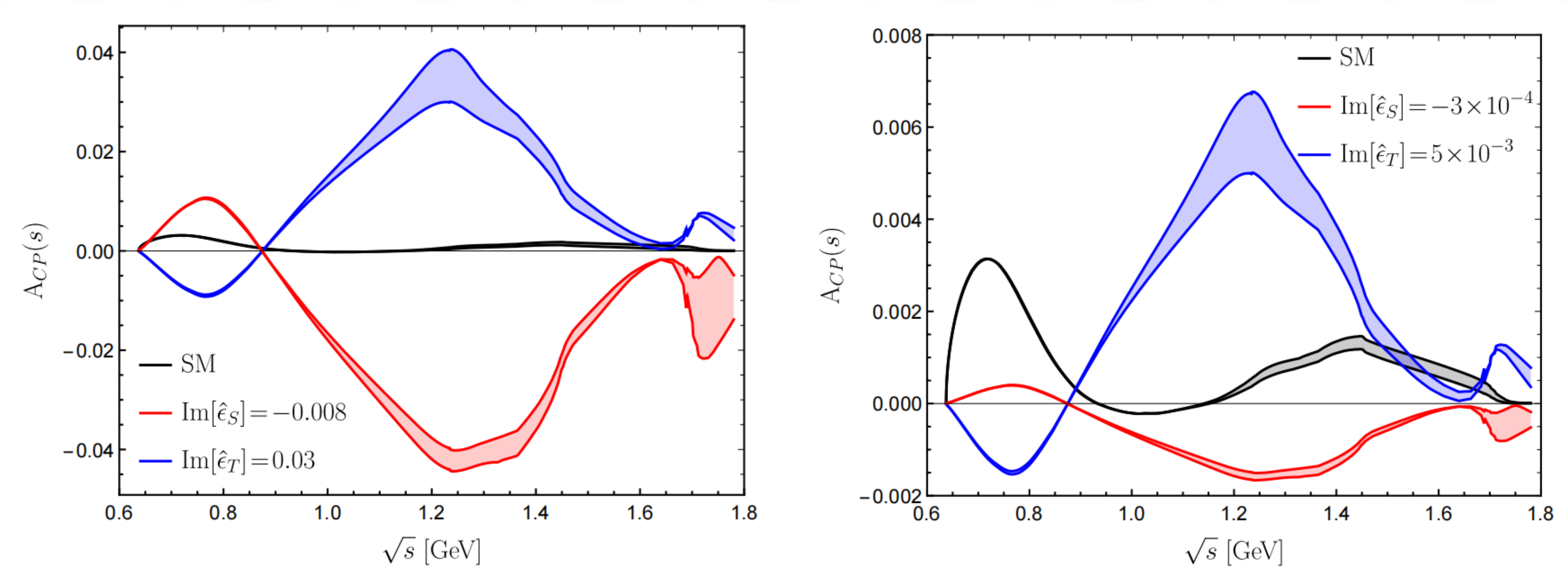


For NP well above v_{EW}, constraints from $\tau \to K_S \pi \nu_\tau$ much less stringent than from d_n & $D^0 - \bar{D}^0$ mixing

- **The BaBar 2.8σ deviation for A_{CP}^{rate} not easily explained by heavy NP !**

Summary

- **Predictions for CP asymmetry in the angular distribution in three different cases:**



sensitive to the non-standard scalar & vector interactions!

constraint only from tau decays

constraints from d_n and $D^0-\bar{D}^0$ mixing

- **We strongly suggest to measure these observables more precisely @ Belle II & STCF!**

Thank you for your attention!

Backup

Experimental facilities for tau physics

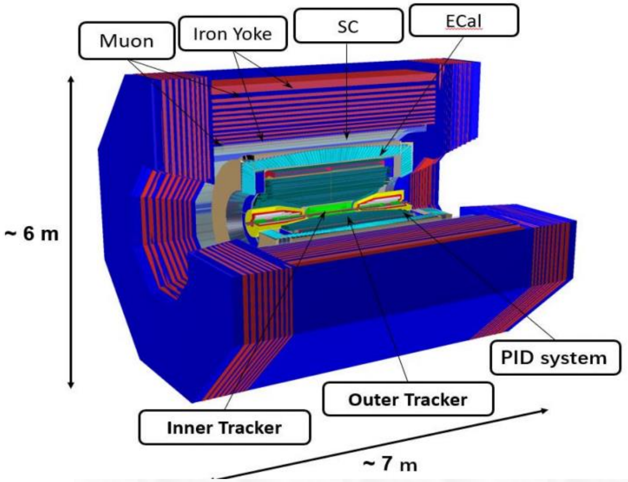
- **Many dedicated facilities, with large tau samples** [C. Z. Yuan, talk @ IAS Program on HEP 2021]

Experiment	Integrated luminosity (fb^{-1})	Cross section (nb)	Number of produced τ pairs (10^9)	Typical tag efficiency	Tagged τ pairs (10^9)	Fraction of Non-τ background
BESIII	50	0 ~ 3.6	~ 0.15	10%	0.015	<1%
BaBar+Belle	1,500	0.9	1.35	33%	0.45	8%
LEP (ALEPH, DELPHI, L3, OPAL)	0.20×4	1.5	0.0012	79% (ALEPH), 92% within $\lvert\cos\theta\rvert<0.90$	0.0007	1.2% (ALEPH)
STCF/SCT	10,000	2.5	25	10%=BESIII	1.5	<1%=BESIII
Belle II	50,000	0.9	45	33%=Belle	15	8%=Belle
CEPC	45,000	1.5	70	87% (↑10% over ALEPH)	60	<1.2%@ALEPH
FCC-ee	115,000	1.5	170	87% (↑10% over ALEPH)	150	<1.2%@ALEPH

- **Lots of tau physics programs with these large tau samples: see *biennial tau workshop!***

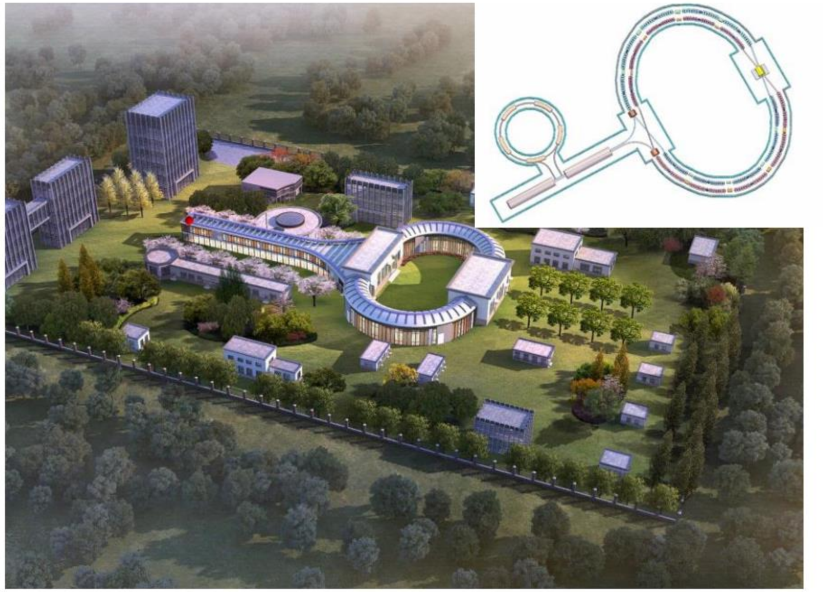
Super Tau-Charm Facility (STCF) in China

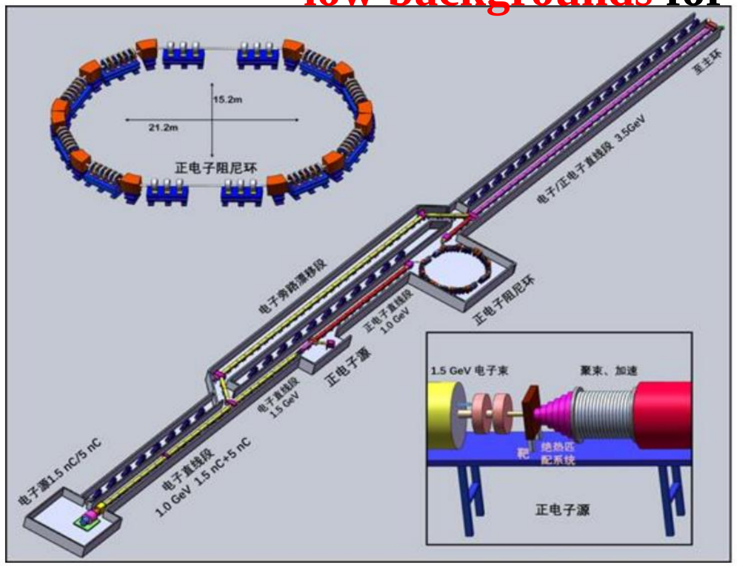
- Peaking luminosity >0.5×10^{35} $cm^{-2}s^{-1}$ at 4 GeV
- Energy range E_{cm} = 2-7 GeV
- Potential to increase luminosity and realize beam polarization
- A nature extension and a viable option for China accelerator project in the post BEPCII/BESIII era



expected to have higher **detection efficiency** and **low backgrounds** for productions at **threshold**

excellent resolution, kinematic constraining





1 ab^{-1} data expected per year

Xiaoru Zhou, talk @ charm 2020

The CPV sensitivity with 1ab^{-1} @ 4.26 GeV[1]:

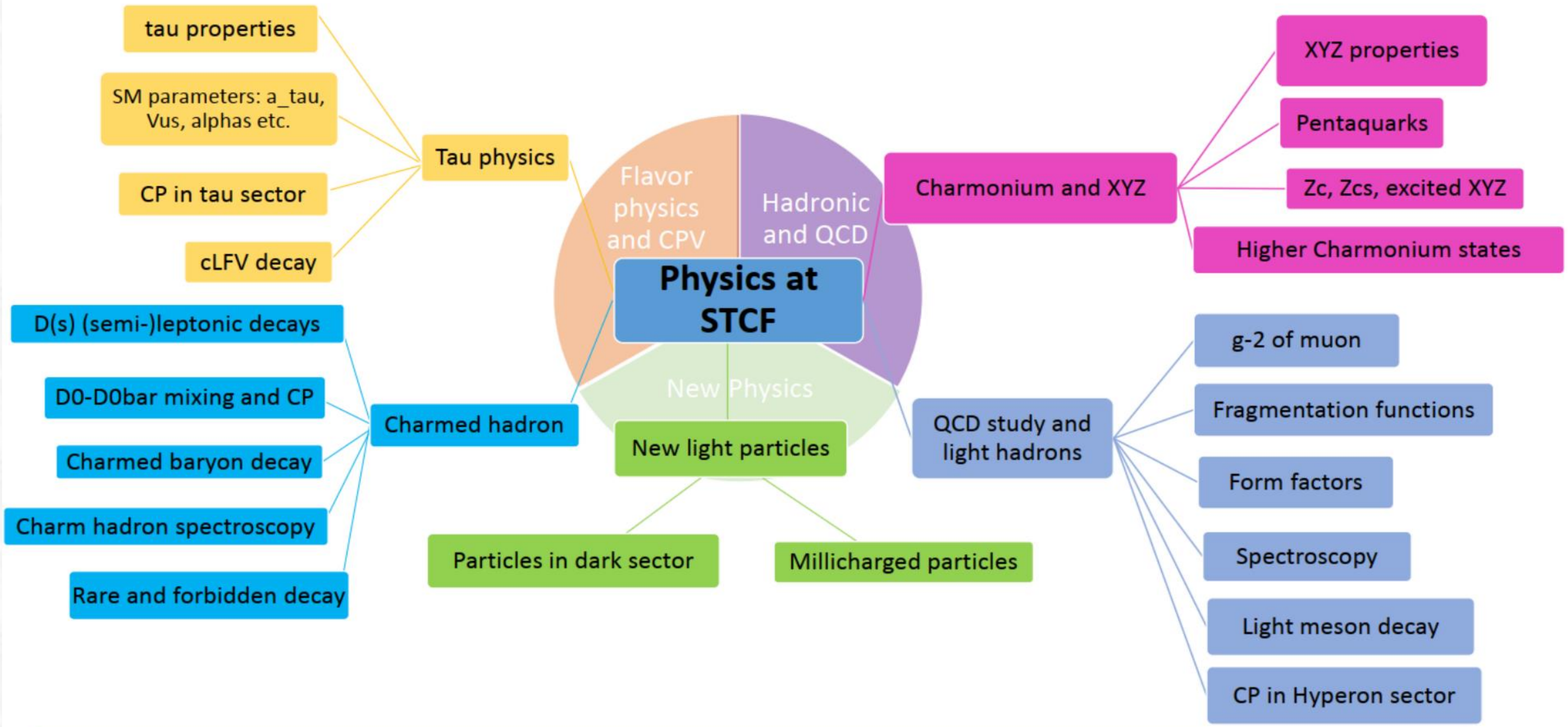
$A_{STCF} \sim 9.7 \times 10^{-4}$

With 10 ab^{-1} data:

$A_{STCF} \sim 3.1 \times 10^{-4}$

Physics at STCF

Xiaoru Zhou, talk @ charm 2020



- **rich of physics program, unique for physics with c quark and τ leptons,**
- **important playground for study of QCD, exotic hadrons, flavor and search for new physics.**

Physics @ a STCF in a nutshell

V. Vorobyev, talk @ charm 2021

Input for B meson studies at LHCb and Belle II

- ✓ Measurement of the strong phases of D decay amplitudes
- ✓ Measurement of absolute branching fractions
- ✓ Searches for rare and forbidden decays of the charm quark
- ✓ CP violation in charm
- ✓ …

charm

QCD

- ✓ Physics of highly-excited quarkonium
- ✓ Molecular states
- ✓ Baryon interaction at threshold
- ✓ Search for glueballs in decays of J/ψ and $\psi(2S)$
- ✓ …

tau

Test of the electroweak sector of the SM

- ✓ Precision measurement of the τ lepton properties
- ✓ Michel parameters, tests of lepton universality
- ✓ Precision measurement of hadronic τ decays
- ✓ Search for CP and T violation in τ decays
- ✓ …

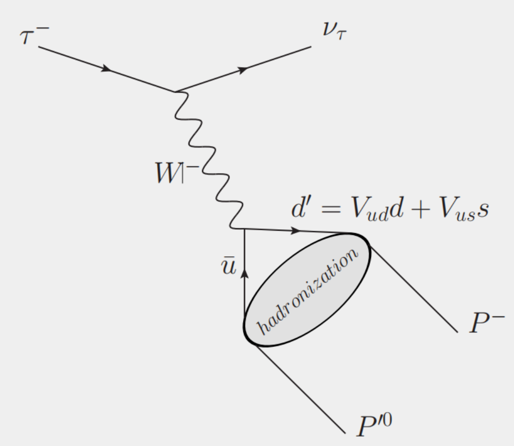
QCD, α_s, V_{us}. Test of the electroweak model, searches for non-standard contributions

6

Two-meson semi-leptonic tau decays

- **Invariant mass distribution of $\tau^- \to P^- P^0 \nu_\tau$ decay:**

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2 \times \left\{\left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^- P^0}^{3/2}(s) |F_V^{P^- P^0}(s)|^2 + 3 \frac{\Delta_{P^- P^0}^2}{s^2} \lambda_{P^- P^0}^{1/2}(s) |F_S^{P^- P^0}(s)|^2 \right\}$$



- **What do we know theoretically about these FFs?**
  - **their low-energy behaviors: given by χPT** [Gasser and Leutwyler, Annals Phys. 158 (1984) 142]
  - **their high-energy behaviors ($\sim 1/s$): given by pQCD** [Brodsky and Lepage, PLB 87 (1979) 359]
  - **for the intermediate energy region: models**

- **For vector FF: thrice-subtracted dispersion representation**

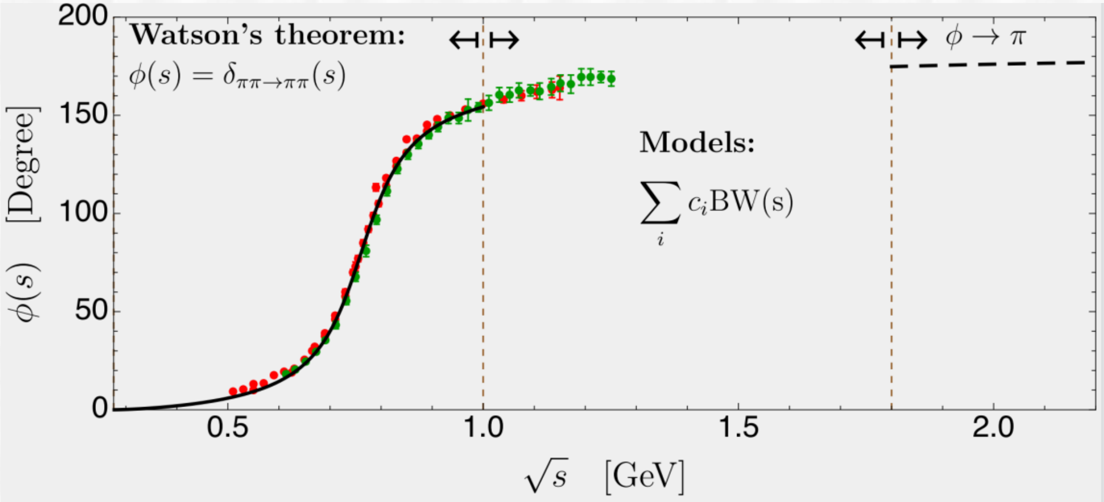
$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\phi(s')}{(s')^3 (s' - s - i\varepsilon)}\right]$$

$\tau^- \to \pi^-\pi^0\nu_\tau$ decay as an example

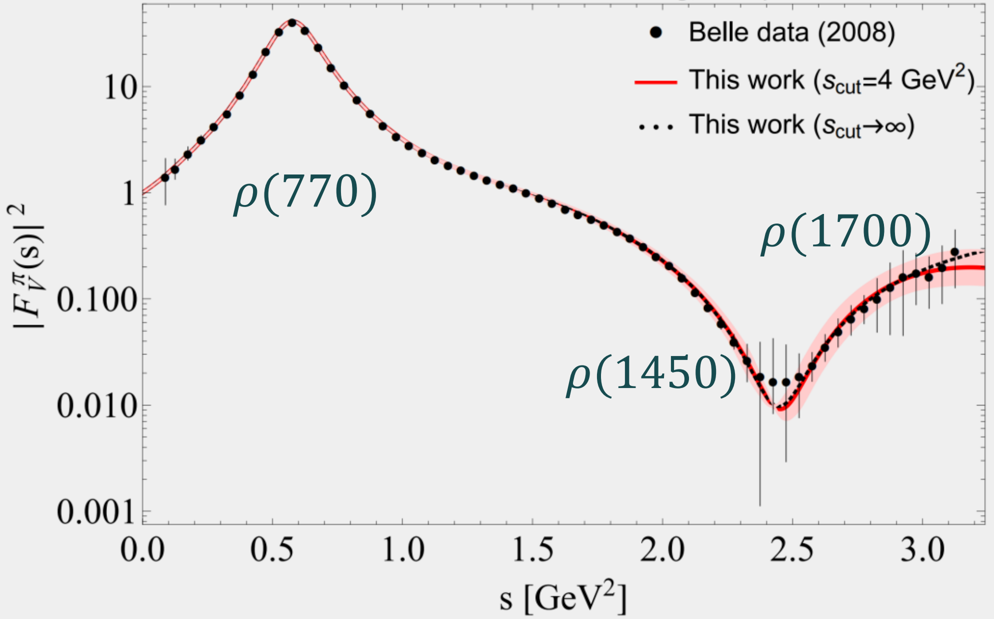
- **For vector FF: thrice-subtracted dispersion representation:**

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right]$$

- **Form-factor phase $\phi(s)$:**



González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436



- **Form-factor modulus squared $|F_V^\pi(s)|^2$:**

- **Information on the resonances:**

Resonance	Model parameter (M,Γ) [MeV]	Pole position (M,Γ) [MeV]
$\rho(1450)$	1376(6), 603(22)	1289(8), 540(16)
$\rho(1700)$	1718(4), 465(9)	1673(4), 445(8)

Constraints on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$ from other processes

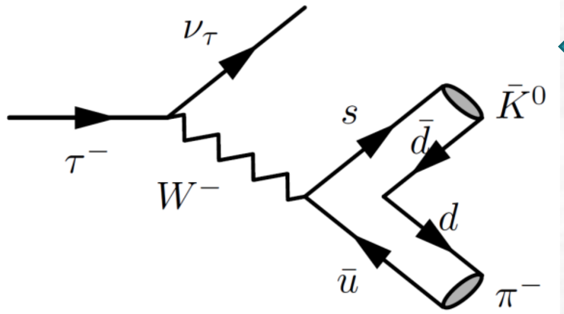
- If BSM interactions originate from a **weakly-coupled heavy NP well above the EW scale**

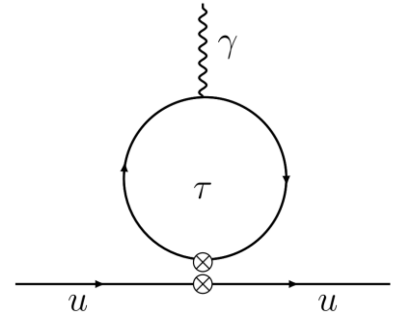
  → $SU(2)_L$ **invariance** of $\mathcal{L}_{eff}$ implies other processes to put further limits on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$

- For **tensor operator:**

$$\mathcal{L}_{\mathrm{SMEFT}} \supset [C^{(3)}_{\ell equ}]_{klmn} (\bar{\ell}^i_{Lk}\sigma_{\mu\nu}e_{Rl})\epsilon^{ij}(\bar{q}^j_{Lm}\sigma^{\mu\nu}u_{Rn}) + \mathrm{h.c.}$$

$$[C^{(3)}_{\ell equ}]_{klmn}[(\bar{\nu}_{Lk}\sigma_{\mu\nu}e_{Rl})(\bar{d}_{Lm}\sigma^{\mu\nu}u_{Rn}) - V_{am}(\bar{e}_{Lk}\sigma_{\mu\nu}e_{Rl})(\bar{u}_{La}\sigma^{\mu\nu}u_{Rn})] + \mathrm{h.c.}$$





$\tau \to K_S\pi\nu_\tau$ **& neutron EDM share the same WC:**

$$d_u(\mu) = -2\sqrt{2}G_F \frac{em_\tau}{\pi^2} V_{us}^2 \,\mathrm{Im}[\hat{\epsilon}_T(\mu)] \log\frac{\Lambda}{\mu}$$

$$\boxed{[C^{(3)}_{\ell equ}]_{3321} = -2\sqrt{2}G_F V_{us}\hat{\epsilon}_T^*}$$

- **Assuming the neutron EDM receives contribution only from $\hat{\epsilon}_T$, we get:**

$$|d_n = g_T^u(\mu)d_u(\mu)| < 1.8\times 10^{-26} e\ \mathrm{cm} \Longrightarrow |\mathrm{Im}[\hat{\epsilon}_T]| \le \frac{1.5\times 10^{-5}}{\ln\left(\frac{\Lambda}{\mu_\tau}\right)} \lesssim 4\times 10^{-6} \text{ for } \Lambda \gtrsim 100\mathrm{GeV}\ \&\ \mu_\tau = 2\mathrm{GeV}$$

nEDM collaboration, PRL 124 (2020) 081803

$A^i_{CP}|_{\mathrm{Max}} \sim \mathcal{O}(10^{-6})$ ⟵ **strongest limit obtained so far!**

Constraints on Im$[\hat{\epsilon}_{S,T}]$ from other processes

- **Caution: tensor operator $(\bar{\tau}\sigma_{\mu\nu}\tau_R)(\bar{u}_L\sigma^{\mu\nu}u_R)$ can also associate with the WC combination**

$$V_{ud}\,\mathrm{Im}[C^{(3)}_{\ell equ}]_{3311} + V_{us}\,\mathrm{Im}[C^{(3)}_{\ell equ}]_{3321} = 2\sqrt{2}G_F\left(V_{ud}^2\,\mathrm{Im}[\epsilon_T]_{3311} + V_{us}^2\,\mathrm{Im}[\epsilon_T]_{3321}\right)$$

$$d_u(\mu) = -2\sqrt{2}G_F\,\frac{e m_\tau}{\pi^2}\left(V_{ud}^2\,\mathrm{Im}[\epsilon_T]_{3311} + V_{us}^2\,\mathrm{Im}[\epsilon_T]_{3321}\right)\log\frac{\Lambda}{\mu}$$

- when delicate cancellation exists between them, stringent limit on $\mathrm{Im}[\hat{\epsilon}_T]$ can be diluted

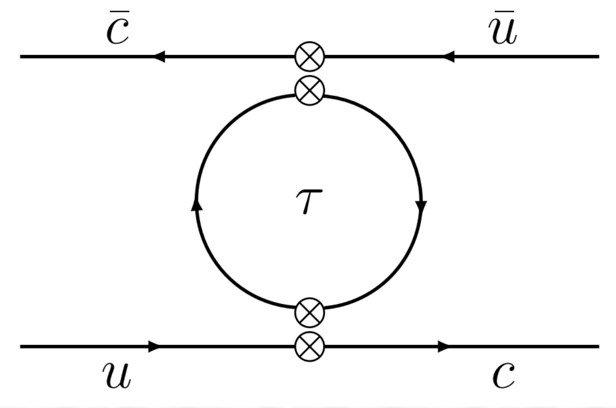
- **In this case, another interesting constraint on $(\bar{\tau}\sigma_{\mu\nu}\tau_R)(\bar{c}_L\sigma^{\mu\nu}u_R)$ from $D^0-\bar{D}^0$ mixing:**

$$V_{cd}[C^{(3)}_{\ell equ}]_{3311} + V_{cs}[C^{(3)}_{\ell equ}]_{3321} = V_{ud}V_{cd}[\epsilon_T]_{3311} + V_{us}V_{cs}[\epsilon_T]_{3321}$$

double insertion

$$M_{12}^{\mathrm{NP}} = \frac{1}{2M_D}\left[C_2'(\mu)\langle D^0|(\bar{c}_L^\alpha u_R^\alpha)(\bar{c}_L^\beta u_R^\beta)|\bar{D}^0\rangle(\mu) + C_3'(\mu)\langle D^0|(\bar{c}_L^\alpha u_R^\beta)(\bar{c}_L^\beta u_R^\alpha)|\bar{D}^0\rangle(\mu)\right]$$

$$C_2' = \frac{1}{2}C_3' = 16G_F^2\,\frac{m_\tau^2}{\pi^2}\left(V_{ud}V_{cd}\,[\epsilon_T]_{3311} + V_{us}V_{cs}\,[\epsilon_T]_{3321}\right)^2\log\frac{\Lambda}{\mu_\tau}$$



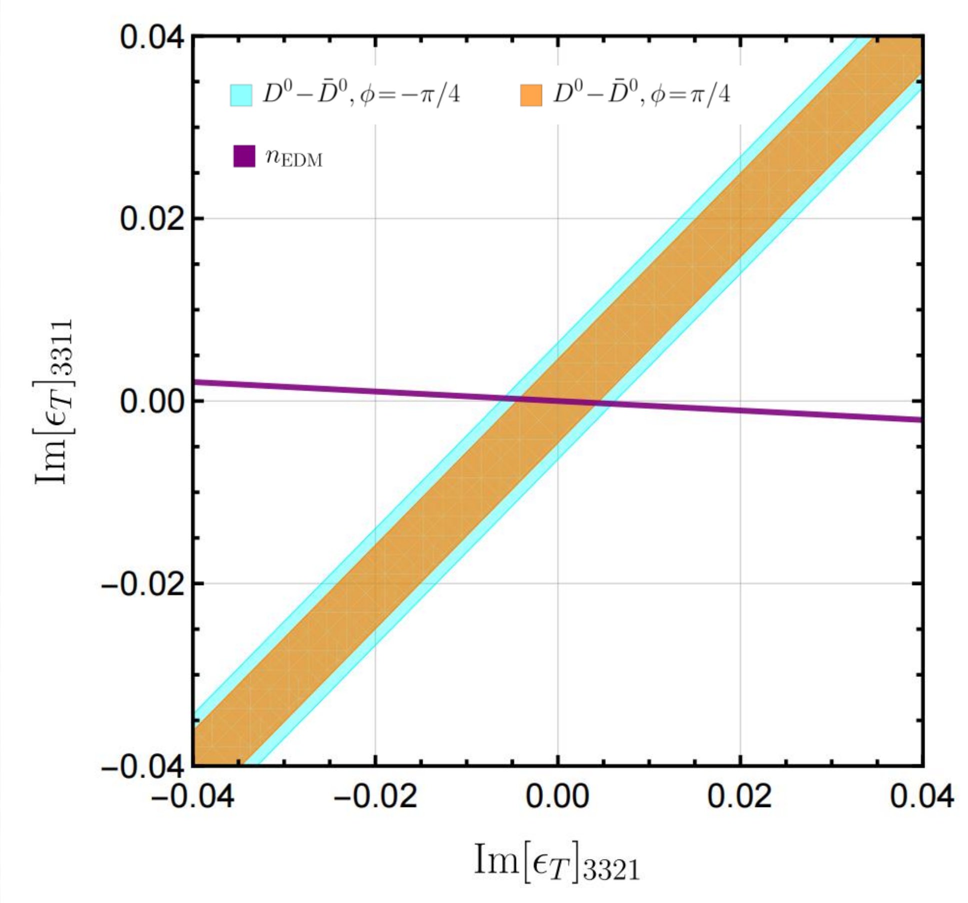
$$x_{12} = \frac{2|M_{12}|}{\Gamma_D} = (0.409 \pm 0.048)\%, \qquad \phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = (0.58^{+0.91}_{-0.90})^{\mathrm{o}}$$

we can get further constraint on $\mathrm{Im}[\hat{\epsilon}_T]$!

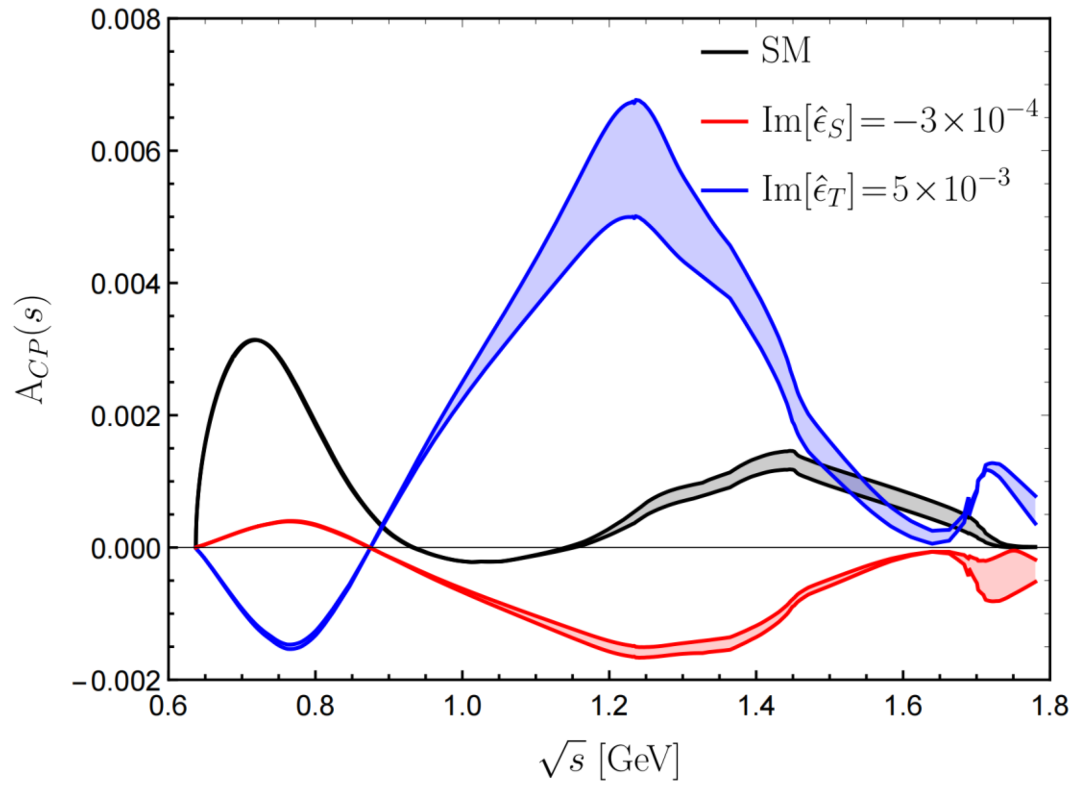
Constraints on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$ from other processes

- **Combined constraints from d_n and $D^0 - \bar{D}^0$ mixing:**

$|\mathrm{Im}[\hat{\epsilon}_T]| \lesssim 4 \times 10^{-6}$



- upper limit: $|\mathrm{Im}[\epsilon_T]| \sim 5 \times 10^{-3}$, significantly diluted



- **Prediction for CPA with $|\mathrm{Im}[\epsilon_T]| = 5 \times 10^{-3}$:** ***still has a significant impact on the CPA!***

Constraints on Im[$\hat{\epsilon}_{S,T}$] from other processes

- **For scalar operator: originate from the following two SMEFT operators**

$$\mathcal{L}_{\rm SMEFT} \supset [C^{(1)}_{\ell equ}]_{klmn}(\bar{\ell}^i_{Lk} e_{Rl})\epsilon^{ij}(\bar{q}^j_{Lm} u_{Rn}) + [C_{\ell edq}]_{klmn}(\bar{\ell}^i_{Lk} e_{Rl})(\bar{d}_{Rm} q^i_{Ln}) + \text{h.c.}$$
$$\supset [C^{(1)}_{\ell equ}]_{klmn}\left[(\bar{\nu}_{Lk} e_{Rl})(\bar{d}_{Lm} u_{Rn}) - V_{am}(\bar{e}_{Lk} e_{Rl})(\bar{u}_{La} u_{Rn})\right] \longrightarrow (\bar{\nu}_\tau \tau_R)(\bar{s}_L u_R)$$
$$+ [C_{\ell edq}]_{klmn}\left[V^*_{an}(\bar{\nu}_{Lk} e_{Rl})(\bar{d}_{Rm} u_{La}) + (\bar{e}_{Lk} e_{Rl})(\bar{d}_{Rm} d_{Ln})\right] + \text{h.c.} \longrightarrow (\bar{\nu}_\tau \tau_R)(\bar{s}_R u_L)$$

scalar operator

$(\bar{\nu}_\tau \tau_R)(\bar{s} u)$

$$\boxed{-2\sqrt{2}G_F V^*_{us}\hat{\epsilon}^*_S = \left[C^{(1)}_{\ell equ}\right]_{3321} + V^*_{ud}\left[C_{\ell edq}\right]_{3321} + V^*_{us}\left[C_{\ell edq}\right]_{3322} + V^*_{ub}\left[C_{\ell edq}\right]_{3323}}$$

- constraint on $\hat{\epsilon}_S$ can be obtained from other processes;
- when potential cancellations exist between $C^{(1)}_{\ell equ}$ & $C_{\ell edq}$, allowed values of $\hat{\epsilon}_S$ can be diluted

- **Mixing between scalar and tensor operators:**

$$\hat{\epsilon}_S = -\frac{[C^{(1)}_{\ell equ}]^*_{3321}}{2\sqrt{2}G_F V_{us}}, \qquad \hat{\epsilon}_T = -\frac{[C^{(3)}_{\ell equ}]^*_{3321}}{2\sqrt{2}G_F V_{us}}$$

$$\begin{pmatrix}\hat{\epsilon}_S\\ \hat{\epsilon}_T\end{pmatrix}_{(\mu=2\ \text{GeV})} = \begin{pmatrix}1.72 & -0.0242\\ -2.17\times10^{-4} & 0.825\end{pmatrix}\begin{pmatrix}\hat{\epsilon}_S\\ \hat{\epsilon}_T\end{pmatrix}_{(\mu=m_Z)}, \quad \begin{pmatrix}C^{(1)}_{\ell equ}\\ C^{(3)}_{\ell equ}\end{pmatrix}_{(\mu=m_Z)} = \begin{pmatrix}1.20 & -0.185\\ -0.00381 & 0.959\end{pmatrix}\begin{pmatrix}C^{(1)}_{\ell equ}\\ C^{(3)}_{\ell equ}\end{pmatrix}_{(\mu=1\ \text{TeV})}$$

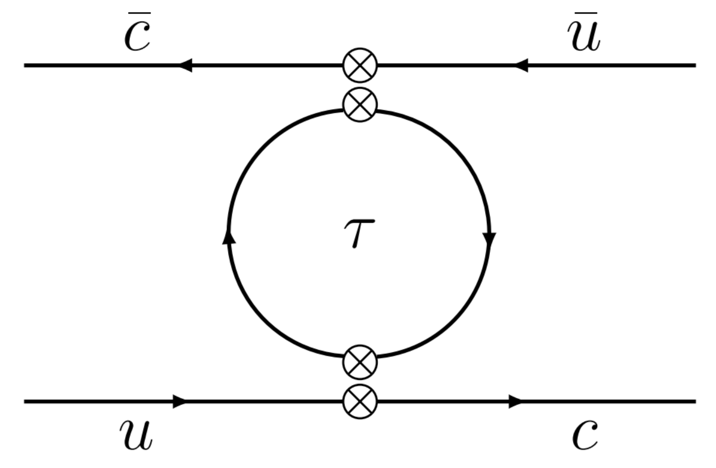
Constraints on Im[$\hat{\epsilon}_{S,T}$] from other processes

- **For scalar operator: no direct contribution to d_n, but can mix into tensor operator via RGE**

$|\mathrm{Im}[\hat{\epsilon}_T]| \lesssim 4 \times 10^{-6} \longrightarrow |\mathrm{Im}[\hat{\epsilon}_S(\mu_\tau)]| \lesssim 2.3 \times 10^{-3}$, comparable to $\mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027$

- **The scalar operator $(\bar{\tau}_L \tau_R)(\bar{c}_L u_R)$ contributes to $D^0 - \bar{D}^0$ mixing:**

double insertion



$$M_{12}^{\mathrm{NP}} = \frac{1}{2M_D}\left[C_2'(\mu)\langle D^0|(\bar{c}_L^\alpha u_R^\alpha)(\bar{c}_L^\beta u_R^\beta)|\bar{D}^0\rangle(\mu)\right]$$

$$C_2' = -G_F^2 \frac{m_\tau^2}{\pi^2}(V_{us}V_{cs}\,\hat{\epsilon}_S)^2 \log\frac{\Lambda}{\mu_\tau}$$

$$x_{12} = \frac{2|M_{12}|}{\Gamma_D} = (0.409 \pm 0.048)\%,$$

$$\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = (0.58^{+0.91}_{-0.90})^{\mathrm{o}}$$

HFLAV collaboration, EPJC 81 (2021) 226

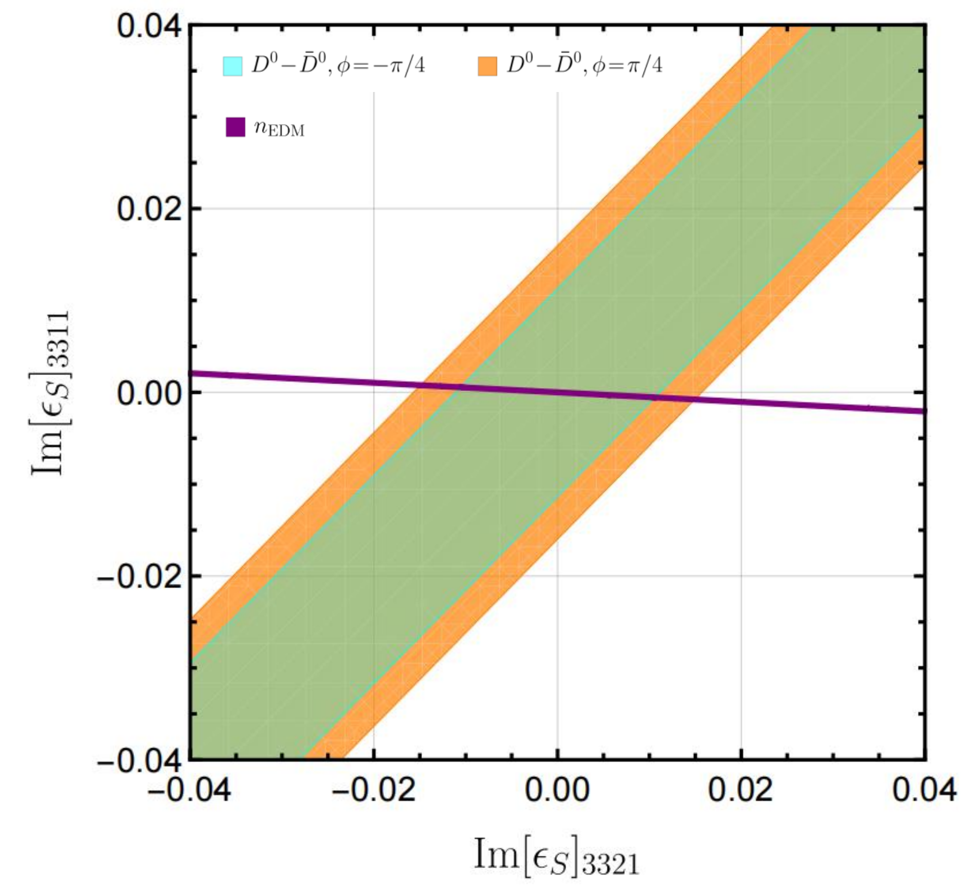
$\mathrm{Im}[\hat{\epsilon}_S(\mu_\tau)] \in [-3.1, 1.6] \times 10^{-4}$ @ 2σ

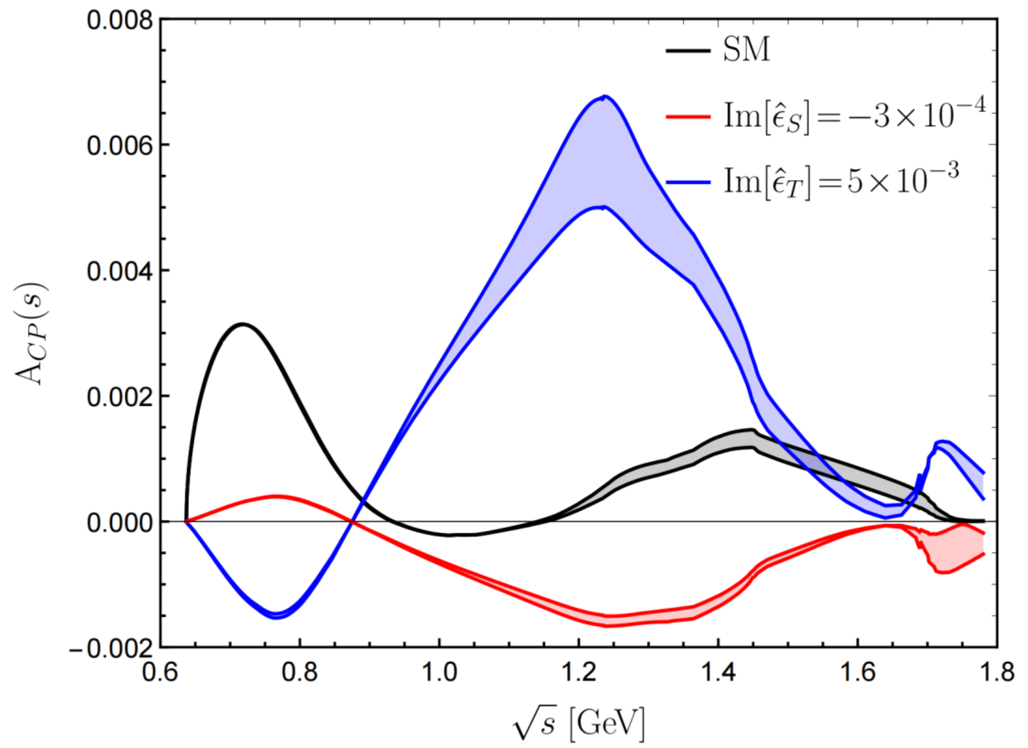
- Constraint from $D^0 - \bar{D}^0$ mixing is one order of magnitude stronger than from neutron EDM
- $A_{CP}^i|_{\mathrm{Max}} \sim \mathcal{O}(10^{-3})$: of the same order as A_{SM}^{CP}

Constraints on $\mathrm{Im}[\hat{\epsilon}_{S,T}]$ from other processes

- **Caution: once cancellations occur, bounds diluted:** $V_{ud}\mathrm{Im}[C^{(1)}_{\ell equ}]_{3311} + V_{us}[C^{(1)}_{\ell equ}]_{3321}$ (for d_n),

$V_{cd}[C^{(1)}_{\ell equ}]_{3311} + V_{cs}[C^{(1)}_{\ell equ}]_{3321}$ (for $D^0 - \bar{D}^0$ mixing)





- **Predictions for CPA with $\mathrm{Im}[\hat{\epsilon}_S(\mu_\tau)] = -3\times10^{-4}$: slightly smaller than the SM prediction**