重味重子物理研究的机遇

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兰州大学

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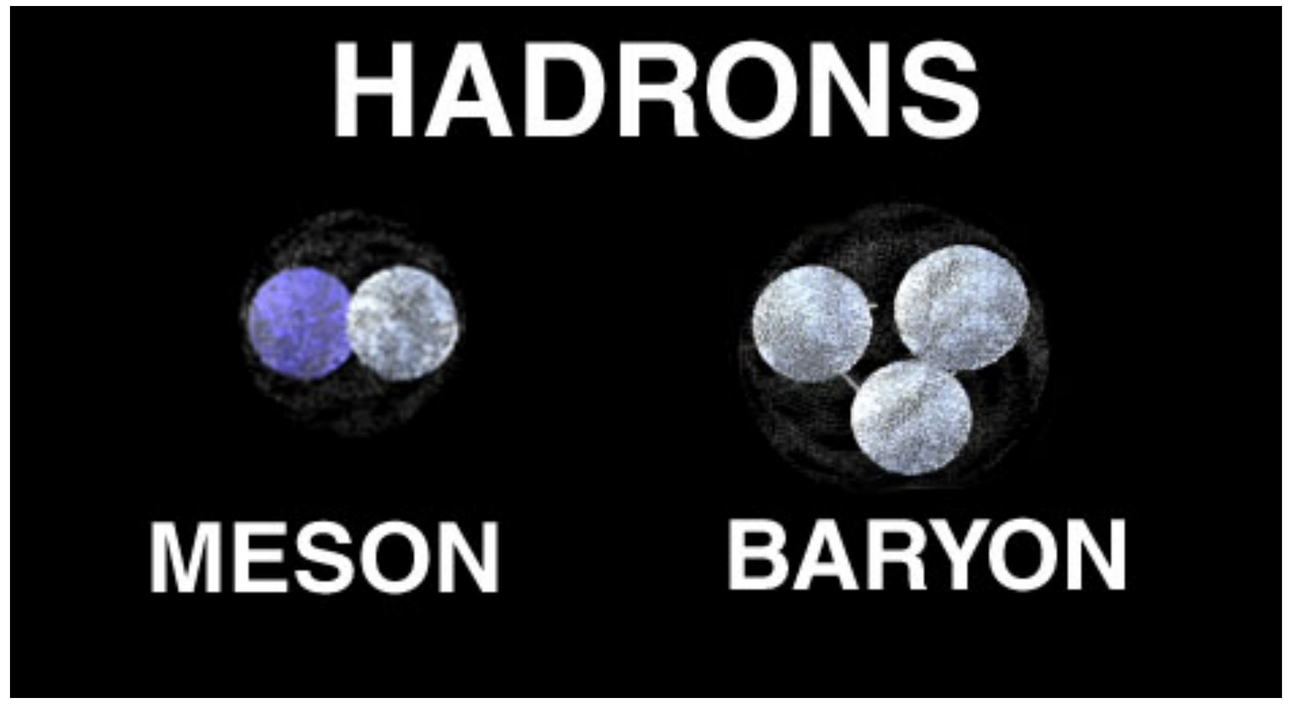
Outline

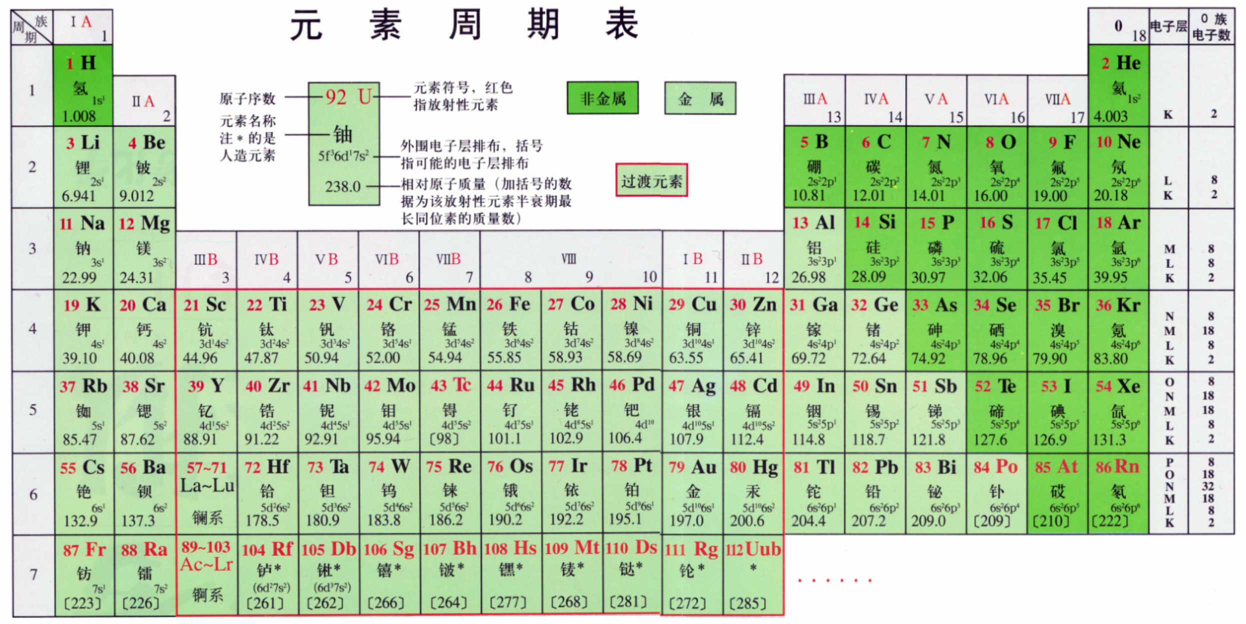
- Why baryon physics? Opportunities of baryon physics
- Recent progresses: PQCD, LCSR
- Prospects: LCDA and others
- Summary

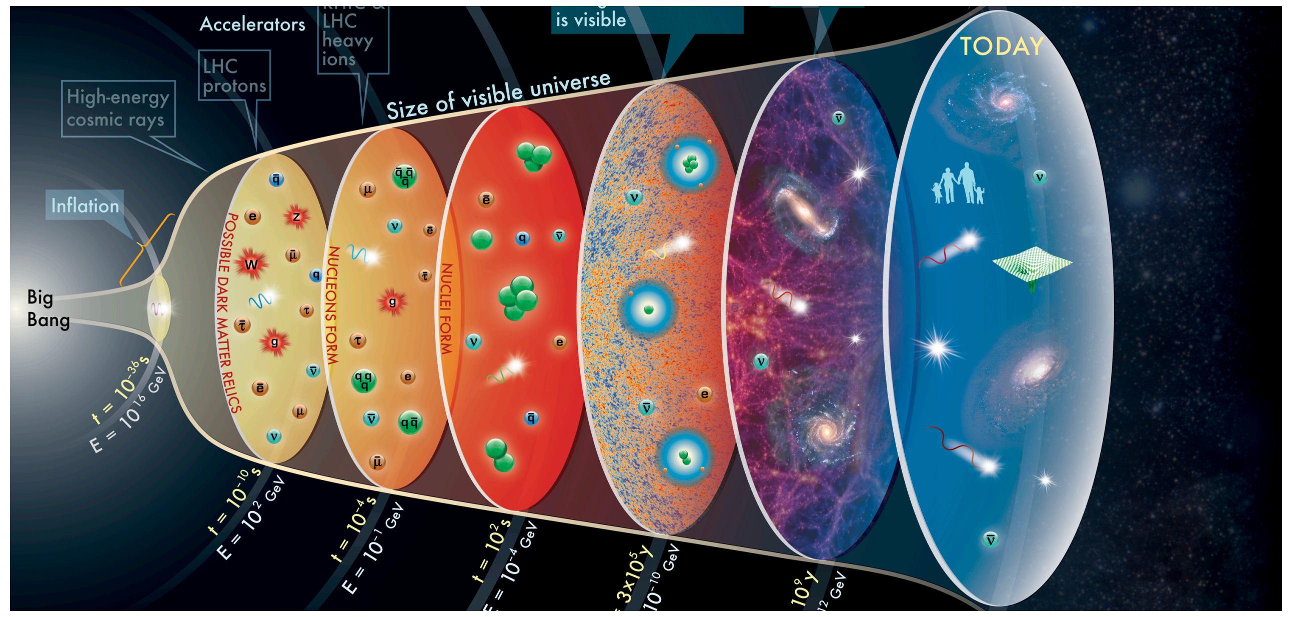
Sorry for not covering all the recent progresses due to the limited time.

Baryon physics



- The visible matter of the Universe is mainly made of baryons.
- Baryons play an important role in the evolution of the Universe, such as baryogenesis and big-bang nucleosythesis.





CP violation in baryons

- Sakharov conditions for **Baryogenesis:**

  1) **baryon** number violation
  2) C and **CP violation**
  3) out of thermal equilibrium

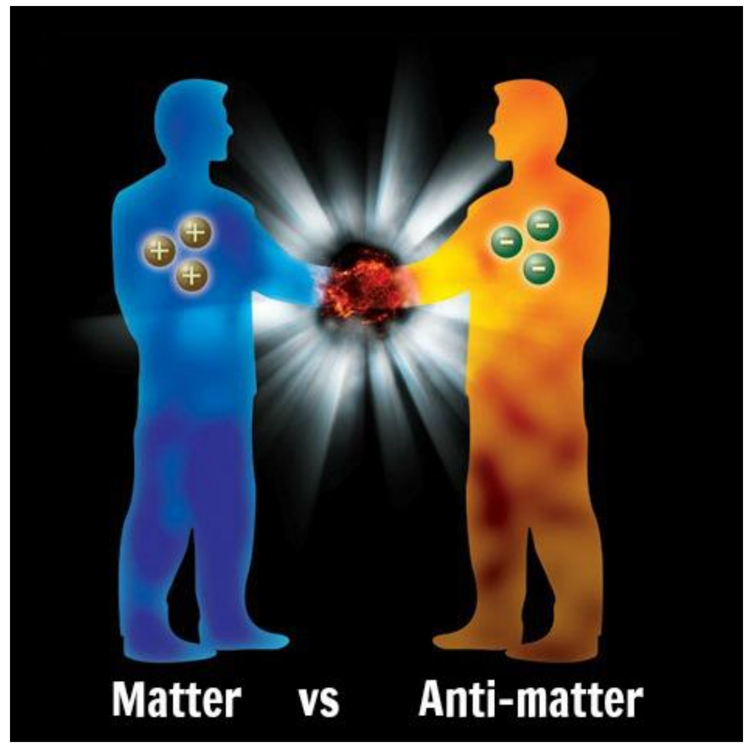
- **CPV: SM < BAU. => new source of CPV, NP**

- CPV is the most important issue in heavy flavor physics

- CPV well established in K, B and D mesons,

  but CPV never established in any baryon

- **Key goal is to predict and search for baryon CPV**



Experimental opportunities

- LHCb is a **baryon factory** !!

Large Production: $\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \rightarrow \frac{N_{\Lambda_b}}{N_{B^{0(-)}}} \sim 0.5$

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- CPV evidence: 3σ in $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$ [LHCb, Nature Physics 2017]
- Precision of baryon CPV measurements has reached to the order of **1%** [LHCb, PLB2018]

$$A_{CP}(\Lambda_b^0 \to p\pi^-) = (-3.5 \pm 1.7 \pm 2.0)\,\%\,, \quad A_{CP}(\Lambda_b^0 \to pK^-) = (-2.0 \pm 1.3 \pm 1.0)\,\%$$

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- CPV in some B-meson decays are as large as **10%**:

$$A_{CP}(\overline{B}^0 \to \pi^+\pi^-) = -0.32 \pm 0.04, \quad A_{CP}(\overline{B}^0 \to K^-\pi^+) = -0.084 \pm 0.004, \quad A_{CP}(\overline{B}_s^0 \to K^+\pi^-) = +0.213 \pm 0.017$$

- **It can be expected that CPV in b-baryons might be observed soon !!**

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- **It can be expected that CPV in b-baryons might be observed soon !!**
- **Theoretical precision is required to be improved**.

Theoretical opportunities

- **Baryons are very different from mesons!!**

- **Factorization:** Heavy-to-light form factor is factorizable at leading power in SCET. No end-point singularity! [Wei Wang, 1112.0237] Taking $\Lambda_b \to \Lambda$ as an example,

$$\xi_\Lambda = f_{\Lambda_b} \Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_\Lambda \Phi_\Lambda(y_i)$$

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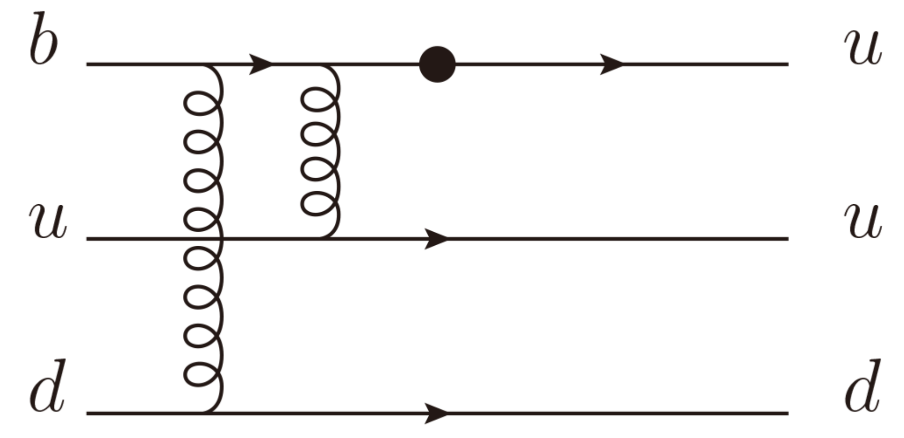
- However, the **leading-power result is one order of magnitude smaller** than the total one
    - Leading power: $\xi_\Lambda(0) = -0.012$ [W.Wang, 2011]
    - Total form factor: $\xi_\Lambda(0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]

- Two hard gluons suppressed by α_s^2 at the leading power. Compared to the soft contributions in the power corrections.



- **More is different!!**

Theoretical challenges

- **Generalized factorization**: Not QCD-inspired. No W-exchange diagrams.
- **QCDF**: Diquark approximation. No hard spectator effects. No W-exchange diagrams
- **PQCD**: Not consistent with data.
- **Currently, no complete QCD-inspired method for non-leptonic b-baryon decays**

	EXP	GF	PQCD	QCDF
$Br(\Lambda_b \to p\pi)[\times 10^{-6}]$	4.3 ± 0.8	4.2+-0.7	$4.66^{+2.22}_{-1.81}$	4.11~4.57
$Br(\Lambda_b \to pK)[\times 10^{-6}]$	5.1 ± 0.9	4.8+-0.7	$1.82^{+0.97}_{-1.07}$	1.70~3.15
$A_{CP}(\Lambda_b \to p\pi)[\%]$	-2.5 ± 2.9	-3.9+-0.2	-32^{+49}_{-1}	-3.74~-3.08
$A_{CP}(\Lambda_b \to pK)[\%]$	-2.5 ± 2.2	5.8+-0.2	-3^{+25}_{-4}	8.1~11.4

Recent Progresses:

(1) PQCD and (2) LCSR

Theoretical progresses: PQCD

- PQCD successfully predicted correct CPV in B meson decays [Keum, Li, Sanda, 2000; Lu, Ukai, Yang, 2000].
- It is hopeful to predict correct CPV of b-baryons. W-exchange diagrams included.

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- The only prediction of b-baryon CPV by PQCD is given in [C.D.Lu, Y.M.Wang, H.Zou, Ali, Kramer, 2009]
- However, the form factors are two orders of magnitude smaller than Lattice or sum rules

Lattice [35]	0.22 ± 0.08
PQCD [67]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$

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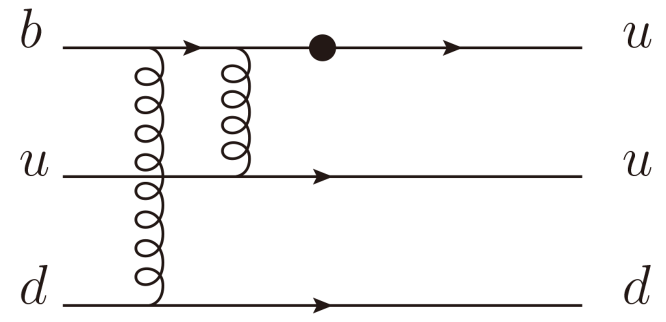
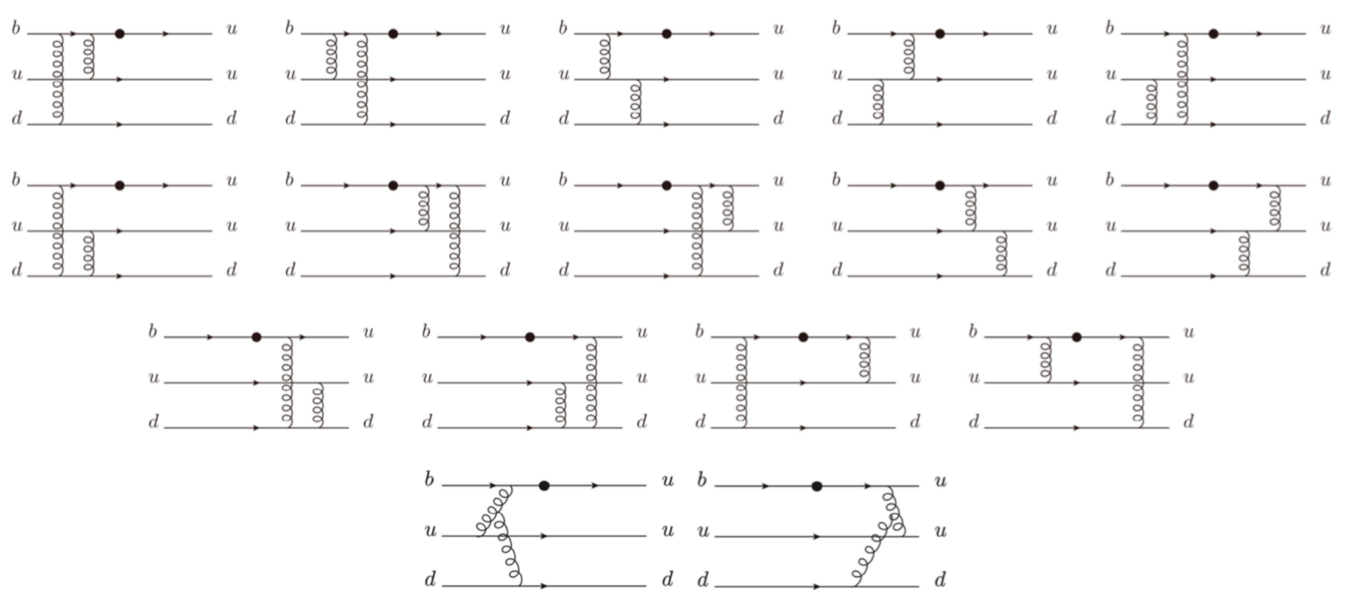
- Only the leading twist of light-cone distribution amplitudes (LCDAs) were considered.
- Recall that leading power is suppressed, so sub-leading power would be dominated.
- Consider higher twist LCDAs!!

$$\mathcal{A} = \Psi_{\Lambda_b}(x_i, b_i, \mu) \otimes H(x_i, b_i, x_i', b_i', \mu) \otimes \Psi_P(x_i', b_i', \mu)$$



Theoretical progresses: PQCD

	$f_1(0)$	$f_2(0)$	$g_1(0)$	$g_2(0)$
NRQM [76]	0.043			
heavy-LCSR [50]	$0.023^{+0.006}_{-0.005}$		$0.023^{+0.006}_{-0.005}$	
light-LCSR-$\mathcal{A}$ [77]	$0.14^{+0.03}_{-0.03}$	$-0.054^{+0.016}_{-0.013}$	$0.14^{+0.03}_{-0.03}$	$-0.028^{+0.012}_{-0.009}$
light-LCSR-$\mathcal{P}$ [77]	$0.12^{+0.03}_{-0.04}$	$-0.047^{+0.015}_{-0.013}$	$0.12^{+0.03}_{-0.03}$	$-0.016^{+0.007}_{-0.005}$
QCD-light-LCSR [78]	0.018	-0.028	0.018	-0.028
HQET-light-LCSR [78]	-0.002	-0.015		
3-point QSR [49]	0.22	0.0071		
lattice [47]	0.22 ± 0.08	0.04 ± 0.12	0.12 ± 0.14	0.04 ± 0.31
PQCD [32]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$			
this work (exponential)	0.27 ± 0.12	0.008 ± 0.005	0.31 ± 0.16	0.014 ± 0.008
this work (free parton)	0.24 ± 0.10	0.007 ± 0.004	0.27 ± 0.13	0.014 ± 0.010



- Higher twist LCDAs contribute to the correct order of form factors.

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2202.04804

Theoretical progresses: PQCD

proton

Λ_b

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	$\boxed{0.25}$	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

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- High-twist LCDA dominant: twist-5 of proton + twist-4 of Λ_b
- Consistent with the power analysis by SCET.

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- High-twist LCDA dominant: twist-5 of proton + twist-4 of Λ_b
- Consistent with the power analysis by SCET.
- Safely twist expansion. Twist-6 of proton is highly suppressed.
- Perturbation protected. Results are given with $\mu \geq$ 1GeV.

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2202.04804

Theoretical progresses: PQCD

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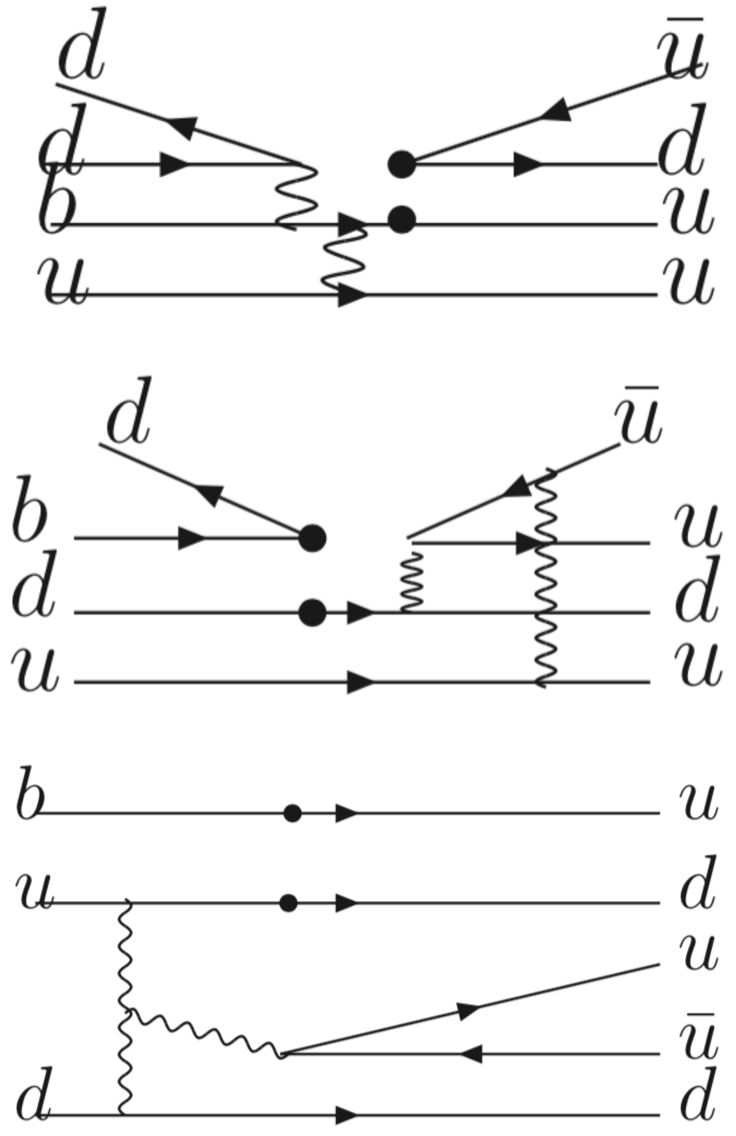
- The reports of referee of EPJC: “The calculation is a highly technical and serious task. **The study is comprehensive and original.** This research results not only provide a good and practicable explanation for the long-standing unresolved discrepancies in baryonic transition form factors between PQCD and Lattice QCD calculations, but also deepen our understandings on the specific contributions of higher twist hadronic LCDAs.”

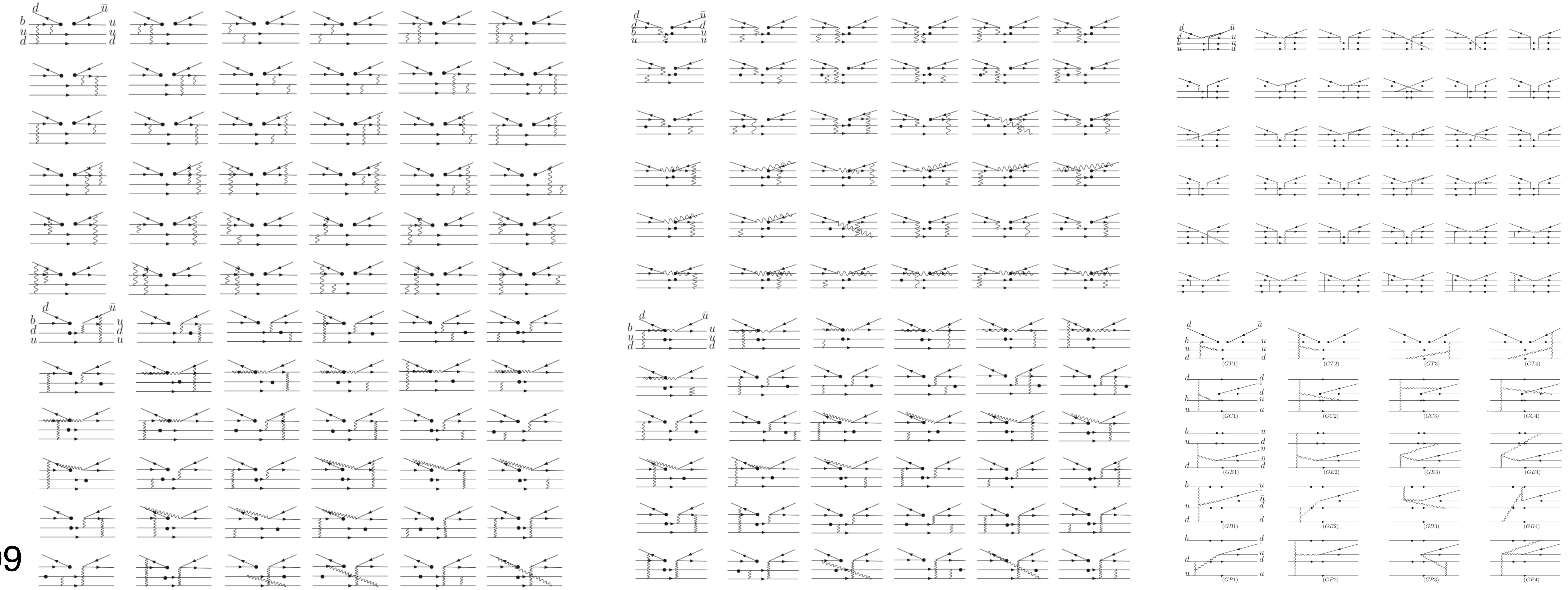
Non-leptonic baryon decays in PQCD

- **It can be expected that PQCD can predict CPV of b-baryons**

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, in preparation



Lu, Wang, Zou, Ali, Kramer, 2009



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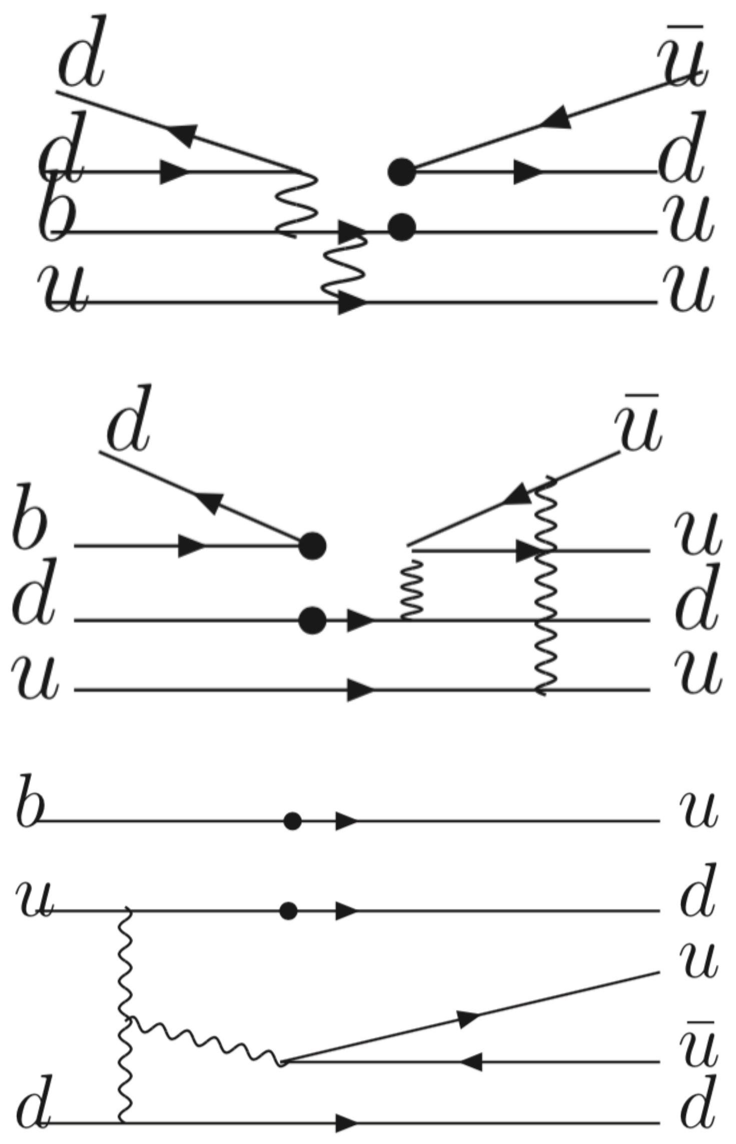

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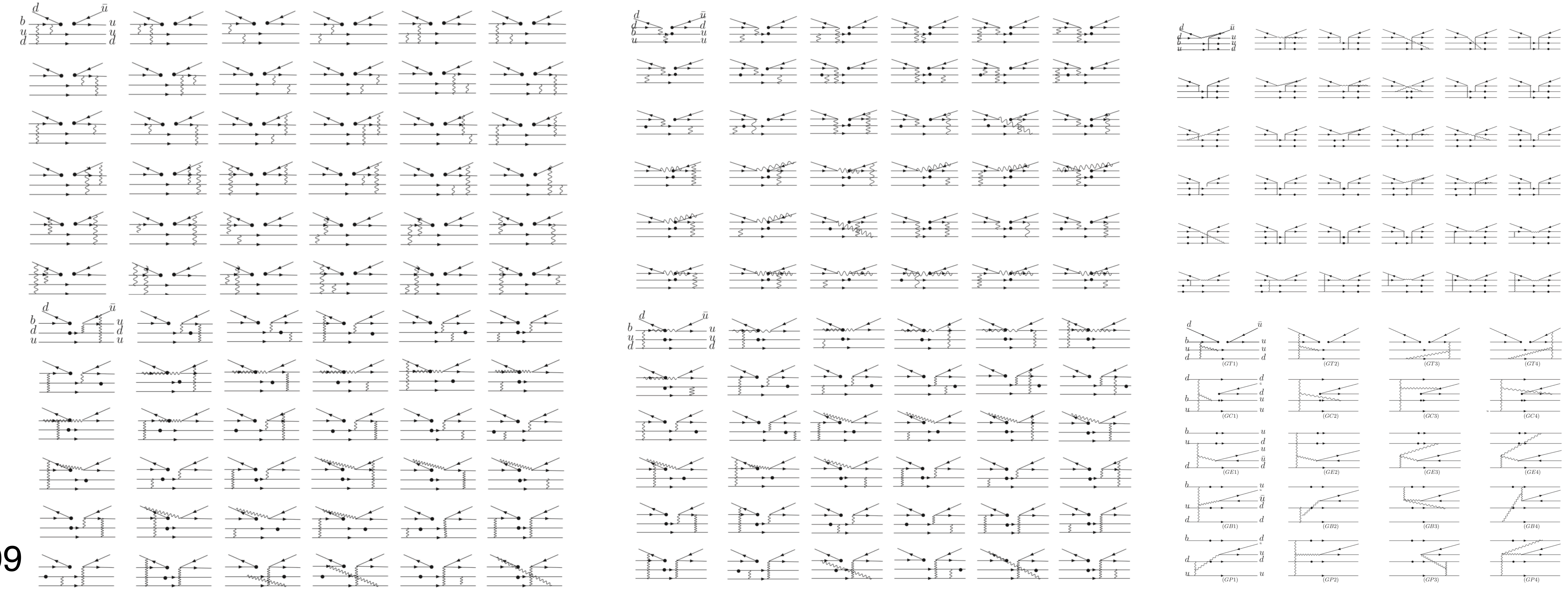
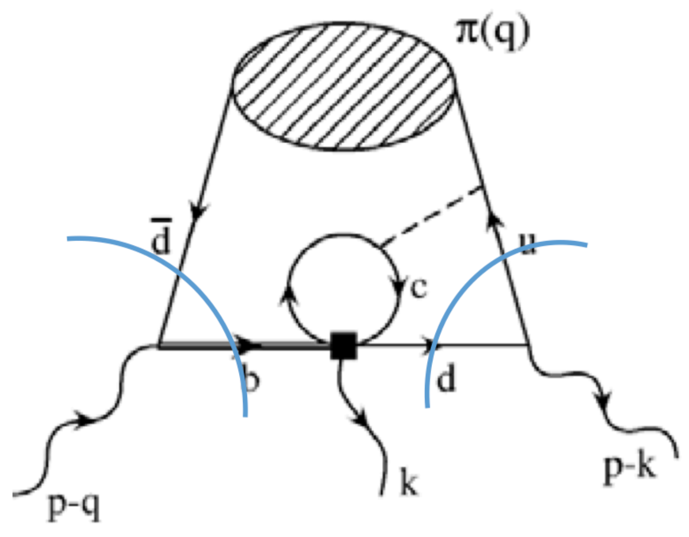


- $\Lambda_b \to \Lambda_c \pi, \Lambda_c K$ and $\Lambda_b \to \Lambda J/\Psi$ are recently studied in PQCD [Zhou Rui, C.Q.Zhang, J.M.Li, M.K.Jia, 2202.09181, 2206.04501]

Theoretical progresses: LCSR

- **Two-body hadronic decays of $\Lambda_b \to p\pi,\ pK$ are studied firstly in LCSRs** [H.Y.Jiang, Khodjamirian, FSY, S.Cheng, in preparation]
- LCSR has been studied in $B \to \pi\pi$ [Khodjamirian, 2001, 2003, 2005] and applied to predict CPV of D meson decays [Khodjamirian, 2017]



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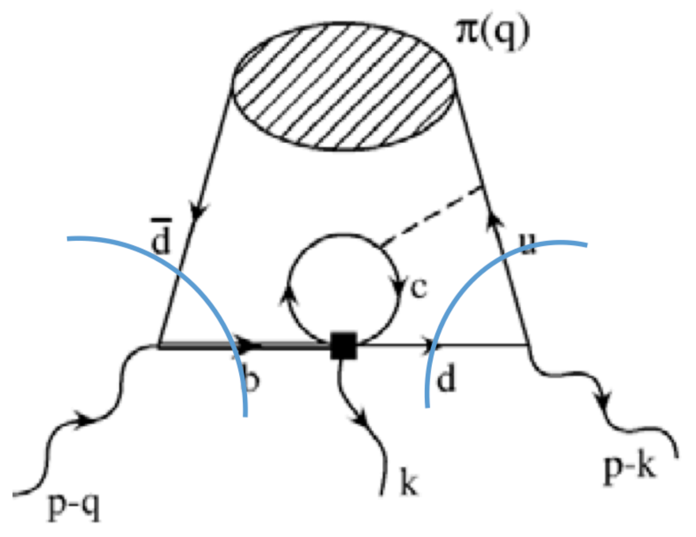

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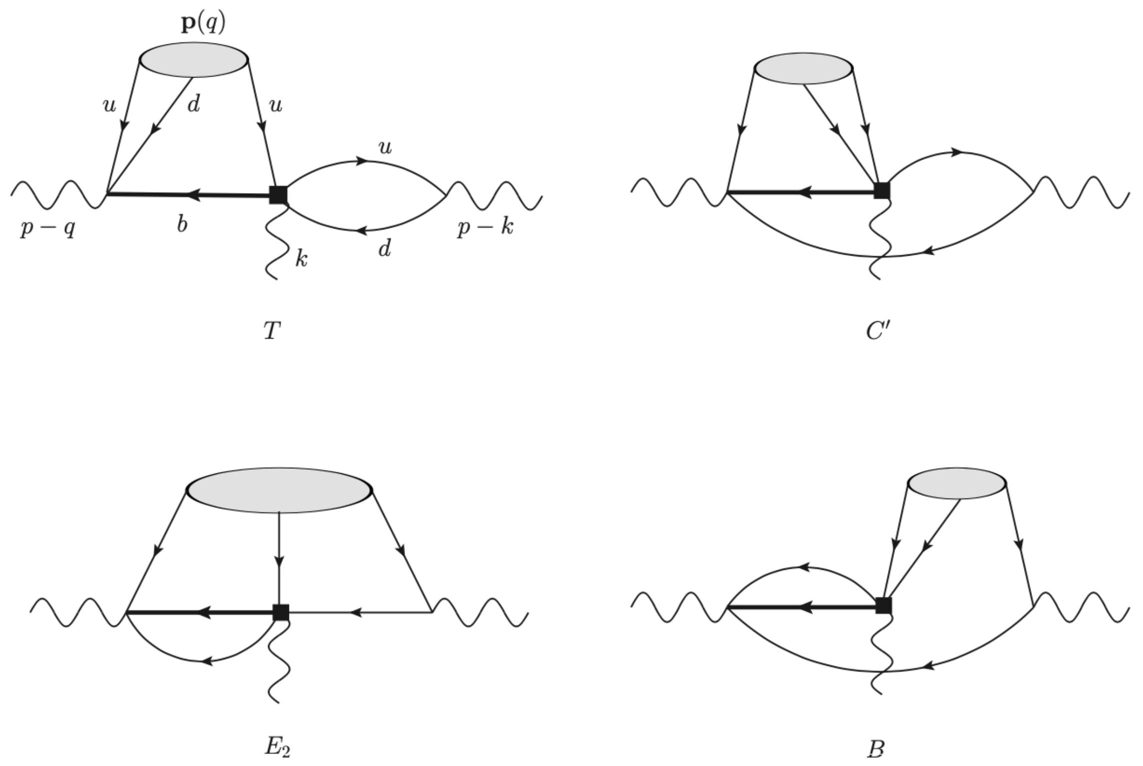
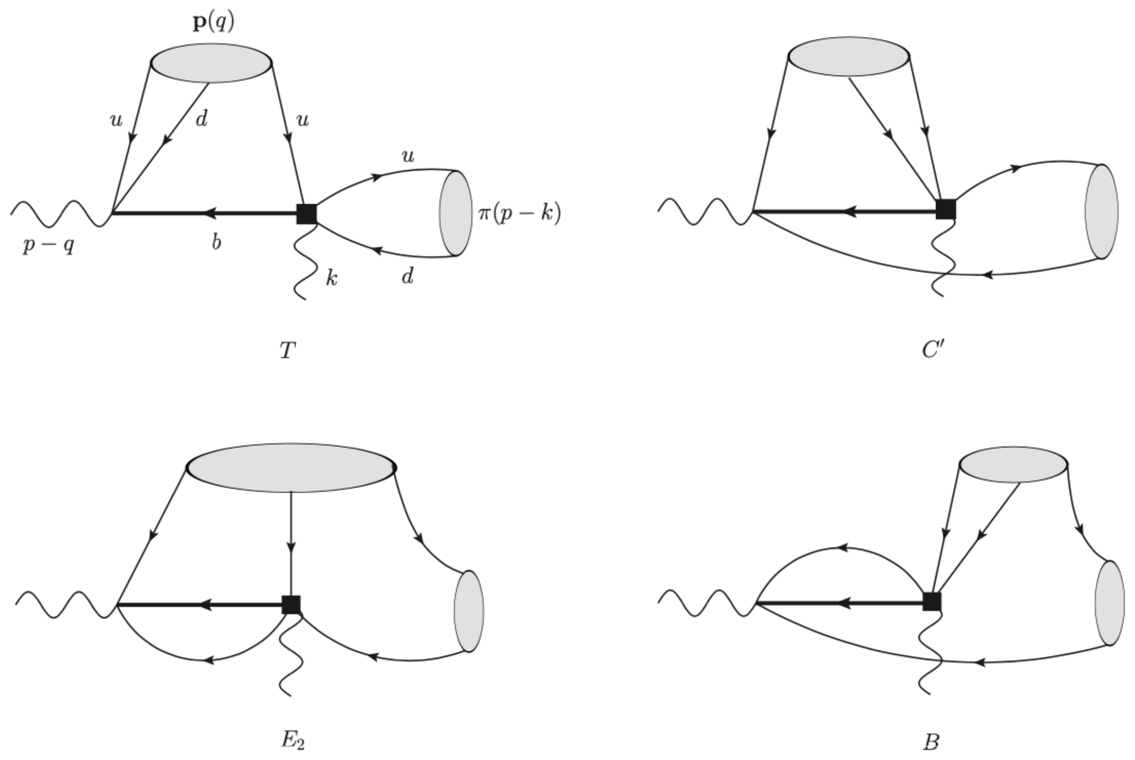
- It overcomes the difficulty of calculation on W-exchange diagrams in QCDF.



Three-point correlator scheme



Two-point correlator scheme

Theoretical progresses: LCSR

- **Two-body hadronic decays of b-baryons are studied firstly in LCSRs** [H.Y.Jiang, Khodjamirian, FSY, S.Cheng, in preparation]
- The full framework has been well established.
- Two-point correlators can be easily calculated, to cross check the 3-point results, and to be extended to NLO corrections.
- The preliminary numerical results are consistent with data.

Topology	3pt scheme	2pt scheme
$T(10^{-9})$	$(-1.57i, -1.51i)$	$(-1.79i, -1.80i)$
$C'(10^{-9})$	$(0.20i, 0.20i)$	$(0.26i, 0.25i)$

channel	$\Lambda_b \to p\pi$	$\Lambda_b \to pK$
topology	T,C', E_2, B P_C, $P_{C'}$	T,E_2 P_C
BR (10^{-6})	5.94	6.50
BR (PDG) (10^{-6})	4.5 ± 0.8	5.4 ± 1.0
A_{CP}	-0.018	-0.001
A_{CP} (PDG)	-0.025 ± 0.029	-0.025 ± 0.022

Prospects:

LCDA and others

Prospects: LCDA

- **Nucleons, hyperons, octet and decuplet states, excited states**
  - Motivation: Limited knowledge for nucleons. VERY very limited for all the others, especially for HIGH TWISTs.
  - Non-perturbative methods: LaMET and Lattice QCD, Dyson-Schwinger equation, Light-Front Quantization, QCD sum rules
- **b-baryon**
  - Motivation: Very model-dependent. Very large uncertainties of parameters.
  - Methods: QCD sum rules, phenomenologies.
  - Higher twists. Evolutions.

Prospects: others

- **Non-leptonic decays**
  - PQCD: threshold Sudakov factor, factorization
  - Power counting under SCET
  - More processes for predictions on baryon CPV
- **Form factors: tree and FCNC** See Wei Wang's, Yu-Shuai Li's talk
  - More precise: many methods like LQCD, LCSR, SCET, DSE, LCQM.
  - More processes: higher excited states.
- **CPV observables, polarizations and angular distributions**
  - T-odd observables, Lee-Yang parameters α, β, γ
- **Inclusive decays and Lifetimes**
  - b-baryons, charmed baryons

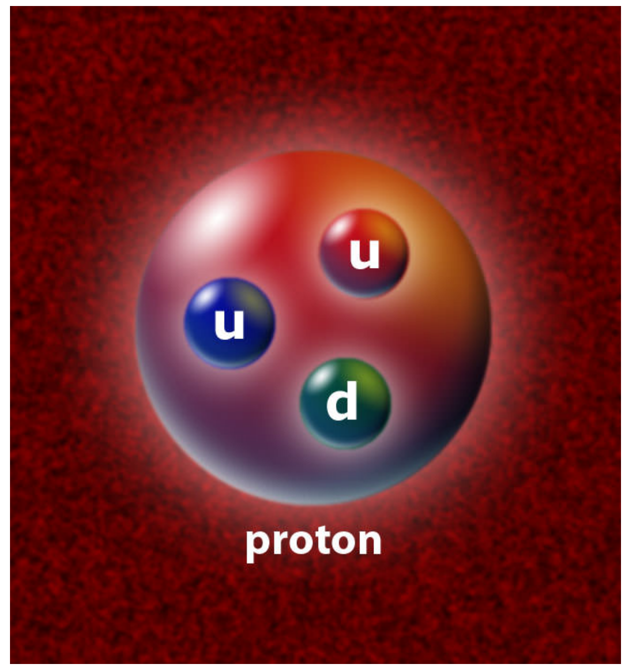
Summary and outlook

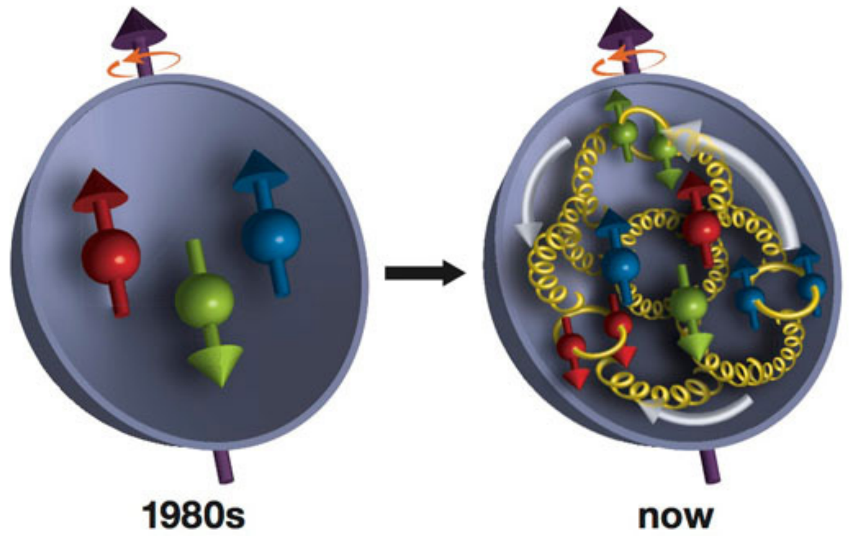
- Baryon physics is an opportunity of heavy flavor physics at the current stage.

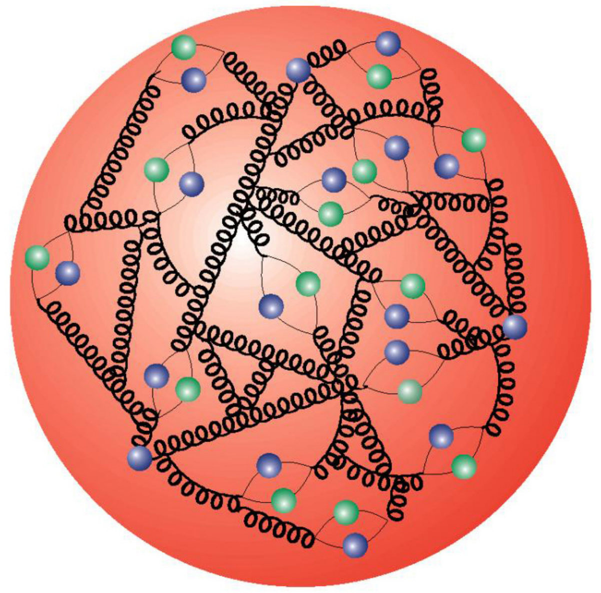
Backups

Baryon physics

- However, our knowledge on the basic nucleon are even limited.
- The mass and spin puzzles of proton are among the most important problems in physics.
- Related to the inner structures and perturbative and non-perturbative QCD dynamics.







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Large Production: $\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \rightarrow \frac{N_{\Lambda_b}}{N_{B^{0(-)}}} \sim 0.5$

Machine	CEPC (10^{12} Z)	Belle II (50 ab^{-1} + 5 ab^{-1} at $\Upsilon(5S)$)	LHCb (50 fb^{-1})
Data taking	2030-2040	$\rightarrow$ 2025	$\rightarrow$ 2030
B^+	6×10^{10}	3×10^{10}	3×10^{13}
B^0	6×10^{10}	3×10^{10}	3×10^{13}
B_s	2×10^{10}	3×10^{8}	8×10^{12}
B_c	6×10^{7}	–	6×10^{10}
b baryons	10^{10}	–	10^{13}

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B_c	6×10^{7}	–	6×10^{10}
b baryons	10^{10}	–	10^{13}

	2011	2012	2018	2023	2029	2035
LHCb	Run I		Run II	Run III	Run IV	Run V
Integrated luminosity	1 fb^{-1}	3 fb^{-1}	9 fb^{-1}	23 fb^{-1}	50 fb^{-1}	300 fb^{-1}

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Machine	CEPC (10^{12} Z)	Belle II (50 ab^{-1} + 5 ab^{-1} at $\Upsilon(5S)$)	LHCb (50 fb^{-1})
Data taking	2030-2040	$\to$ 2025	$\to$ 2030
B^+	6×10^{10}	3×10^{10}	3×10^{13}
B^0	6×10^{10}	3×10^{10}	3×10^{13}
B_s	2×10^{10}	3×10^{8}	8×10^{12}
B_c	6×10^{7}	–	6×10^{10}
b baryons	10^{10}	–	10^{13}

	2011	2012	2018	2023	2029	2035
LHCb	Run I		Run II	Run III	Run IV	Run V
Integrated luminosity	1 fb^{-1}	3 fb^{-1}	9 fb^{-1}	23 fb^{-1}	50 fb^{-1}	300 fb^{-1}

- BESIII and Belle II have fruitful results on charmed baryons and hyperons

Theoretical challenges

Theoretical challenges

- Motivation is to predict CPV. **QCD studies on non-leptonic baryon decays are limited**
- **Generalized factorization** [Y.K.Hsiao, C.Q.Geng, 2015; Liu, C.Q.Geng, 2021]:

  Advantages: Easily extended to more decay channels, and multi-body decays.

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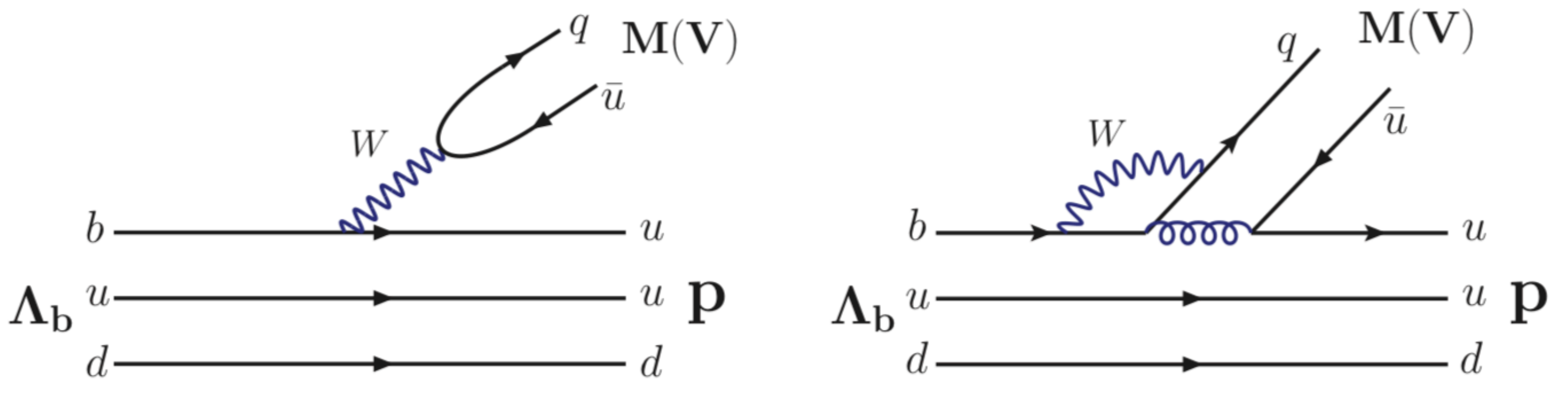

$$\mathcal{A}(\Lambda_b \to pM) = i\frac{G_F}{\sqrt{2}} m_b f_M \left[\alpha_M \langle p|\bar{u}b|\Lambda_b\rangle + \beta_M \langle p|\bar{u}\gamma_5 b|\Lambda_b\rangle\right]$$

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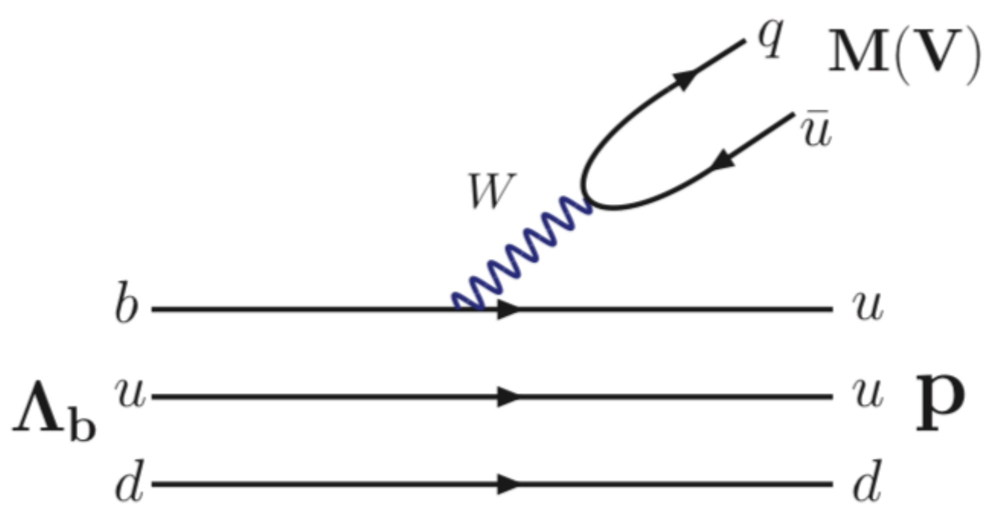

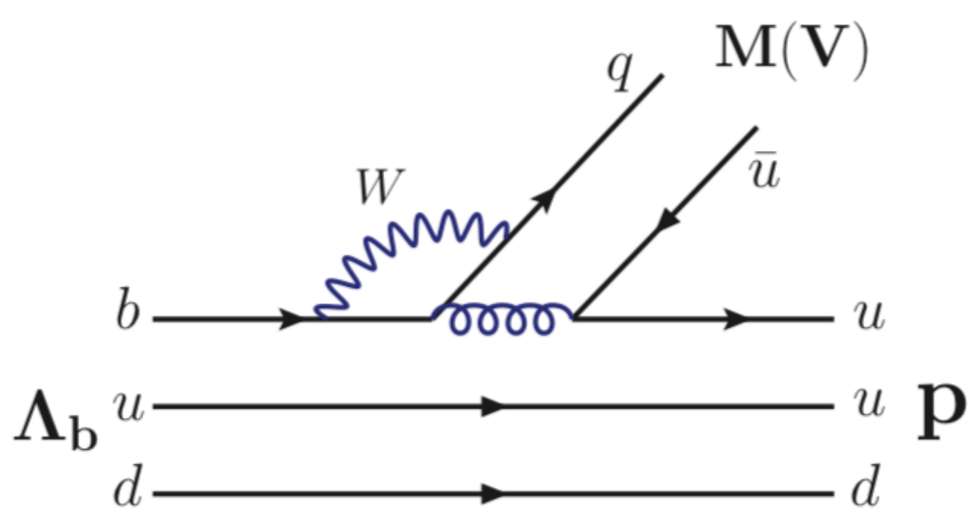

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LHCb: $\Lambda_b^0 \to p\pi^+\pi^-\pi^-$, 3σ CPV, Nature Physics 2017

Decays of the Λ_b^0 (*bud*) baryon to final states consisting of hadrons with no charm quarks are predicted to have non-negligible *CP* asymmetries in the SM, as large as 20% for certain three-body decay modes[13]. It is important to measure the size and nature of these *CP* asymmetries in as many decay modes as possible, to determine

13. Hsiao, Y. K. & Geng, C. Q. Direct *CP* violation in Λ_b^0 decays. *Phys. Rev. D* **91**, 116007 (2015).

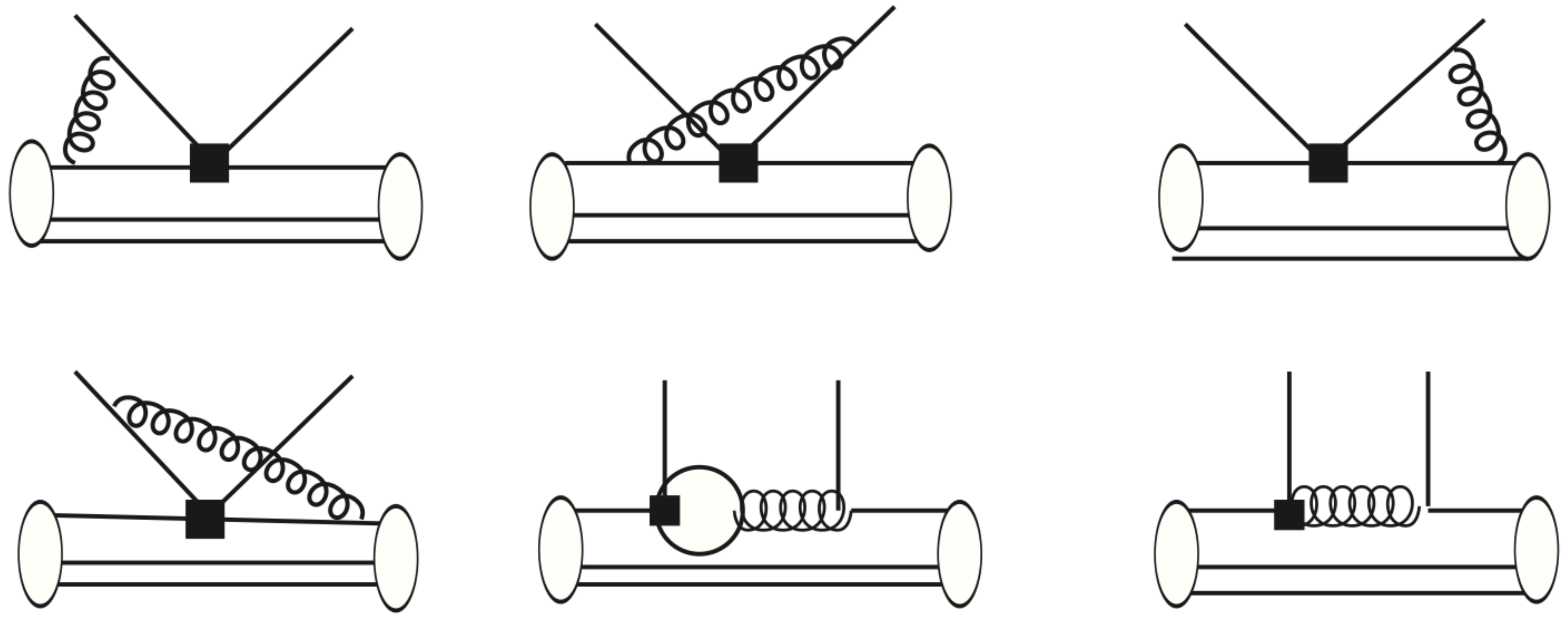
Theoretical challenges

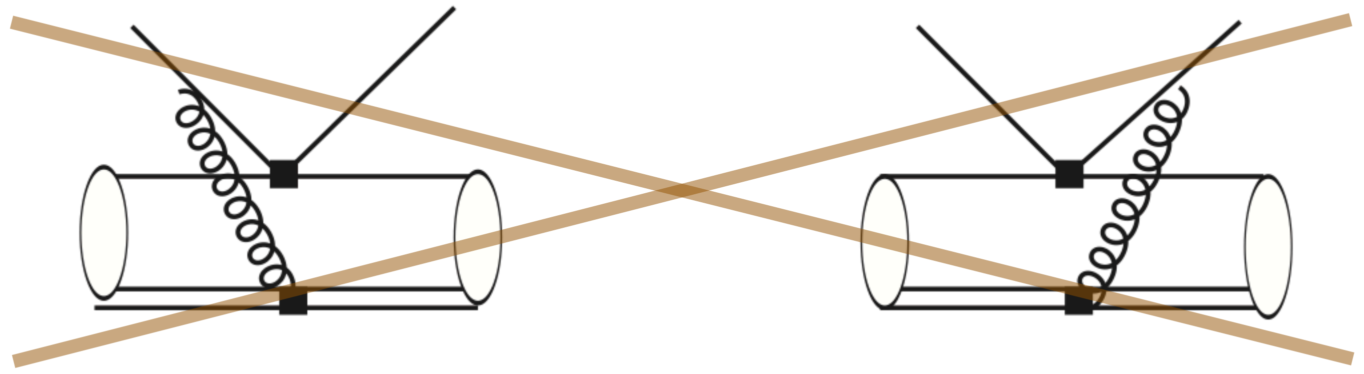
- **Generalized factorization**: Not QCD-inspired. No W-exchange diagrams.

- **QCDF** [J.Zhu, H.W.Ke, Z.T.Wei, 2016, 2018]:

  Advantages: QCD inspired. Collinear factorization.

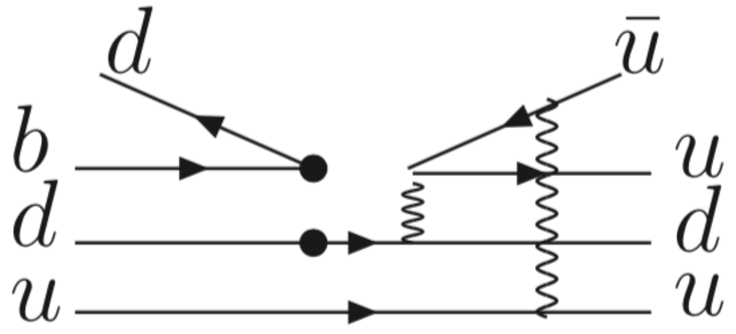
  Disadvantages: Diquark approximation. No hard spectator effects and W-exchange diagrams



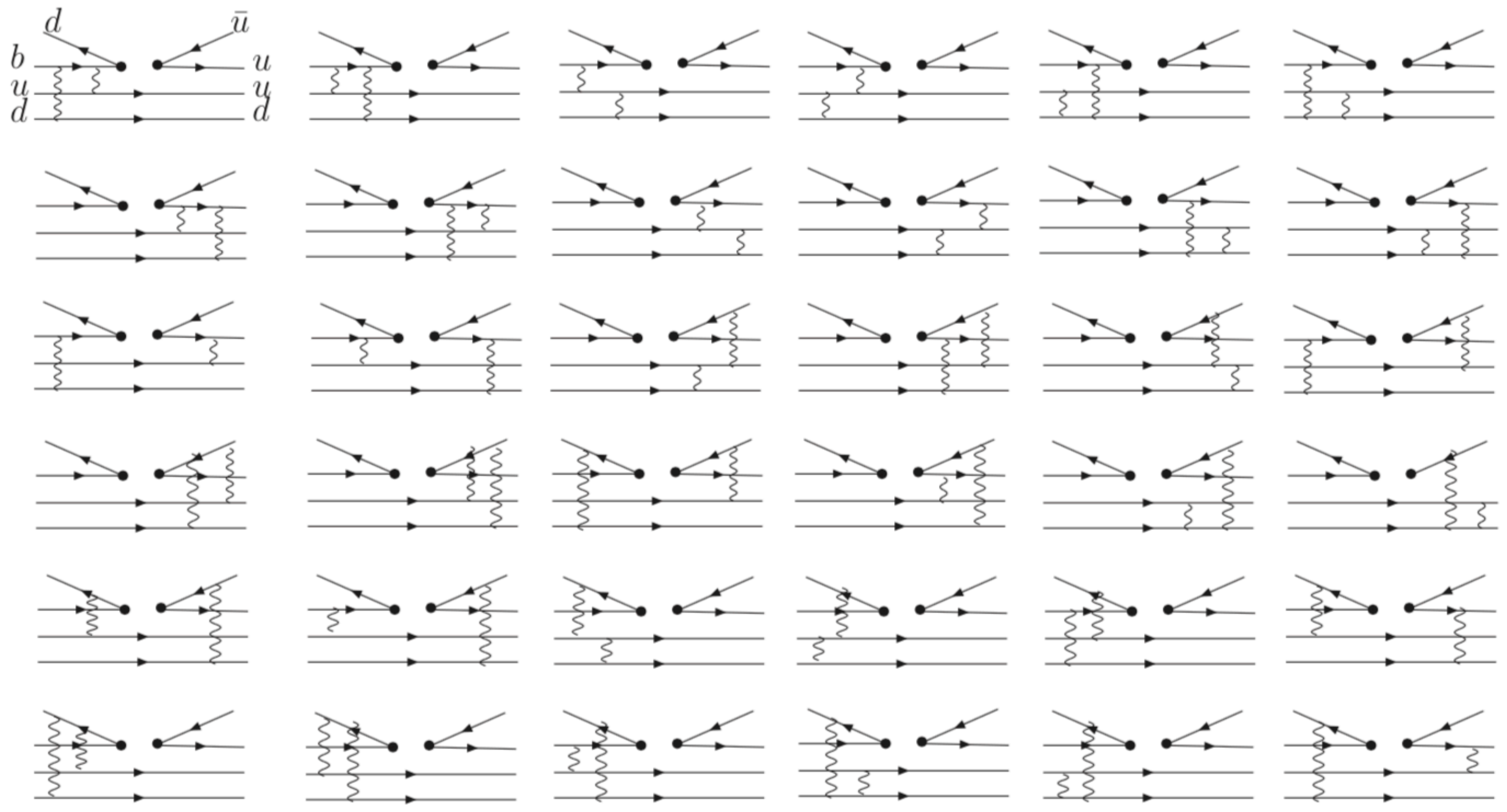


$$\langle HM|O_i|\Lambda_b^0\rangle = \sum_j F_j^{\Lambda_b H}(M^2)\int_0^1 dx\, T_{ij}^I(x)\Phi_M(x) + \cancel{\int_0^1 d\xi dx dy\, T^{II}(\xi,x,y)\Phi_B(\xi)\Phi_{M_1}(y)\Phi_{M_2}(x)}$$

Theoretical challenges



Two hard gluons.



More than 200

Feynman diagrams

Theoretical challenges

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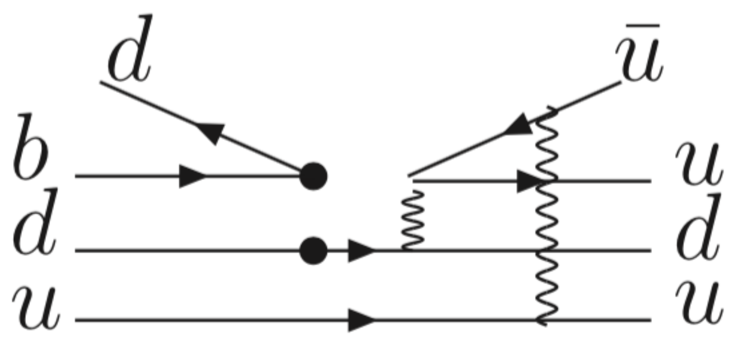
  - Advantages: QCD inspired. k_T factorization. All contributions included in principle.



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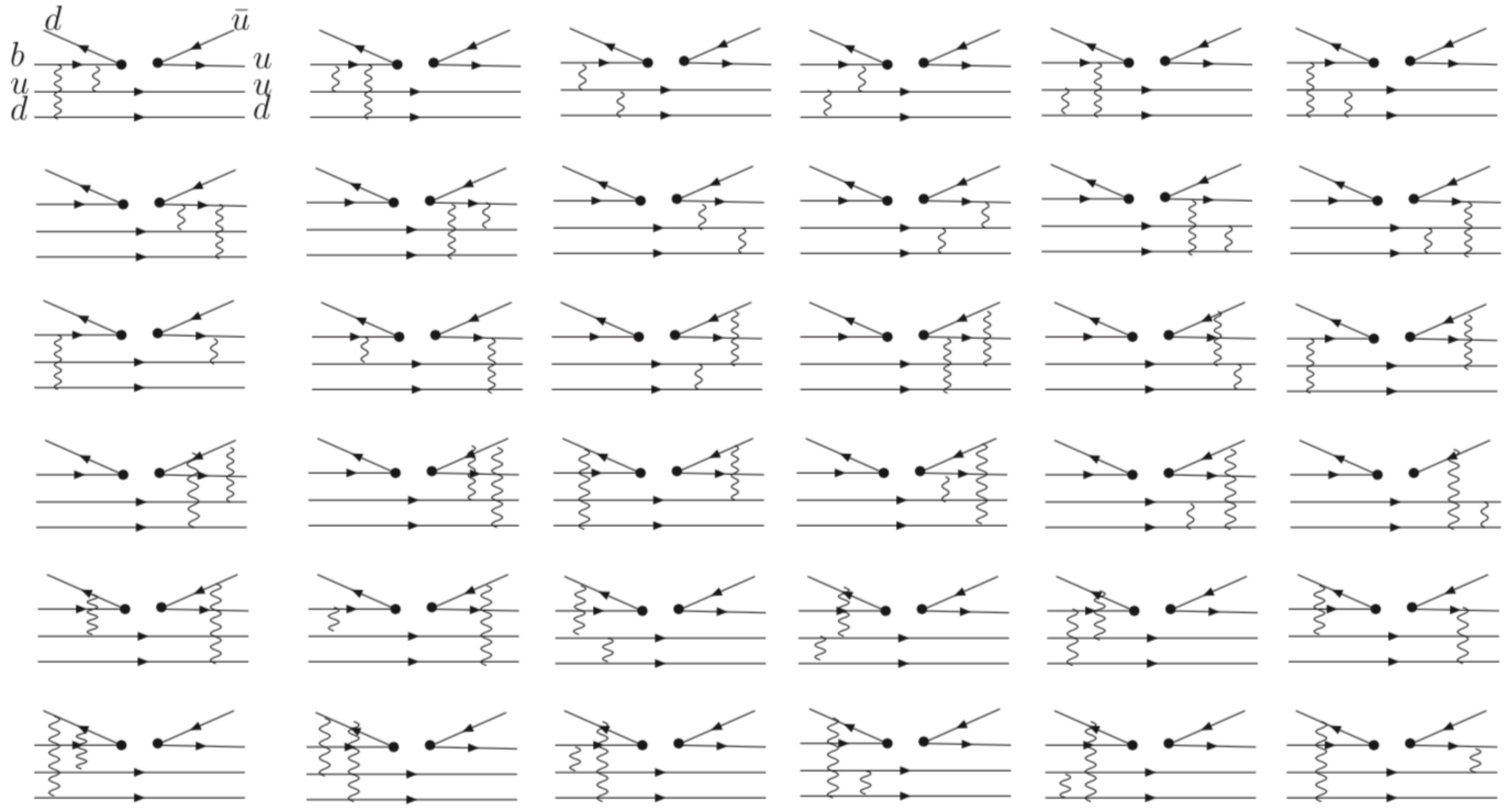

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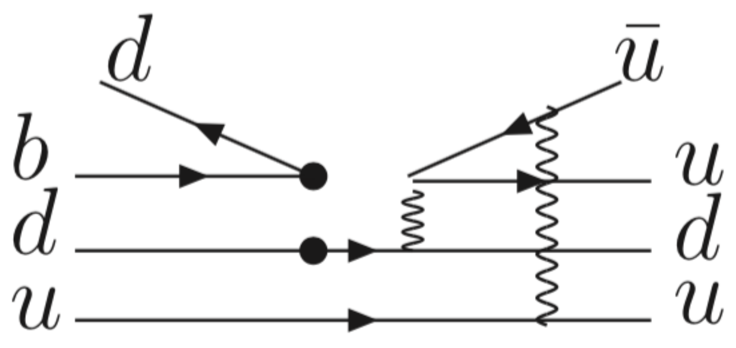
    - Disadvantages: not consistent with data.



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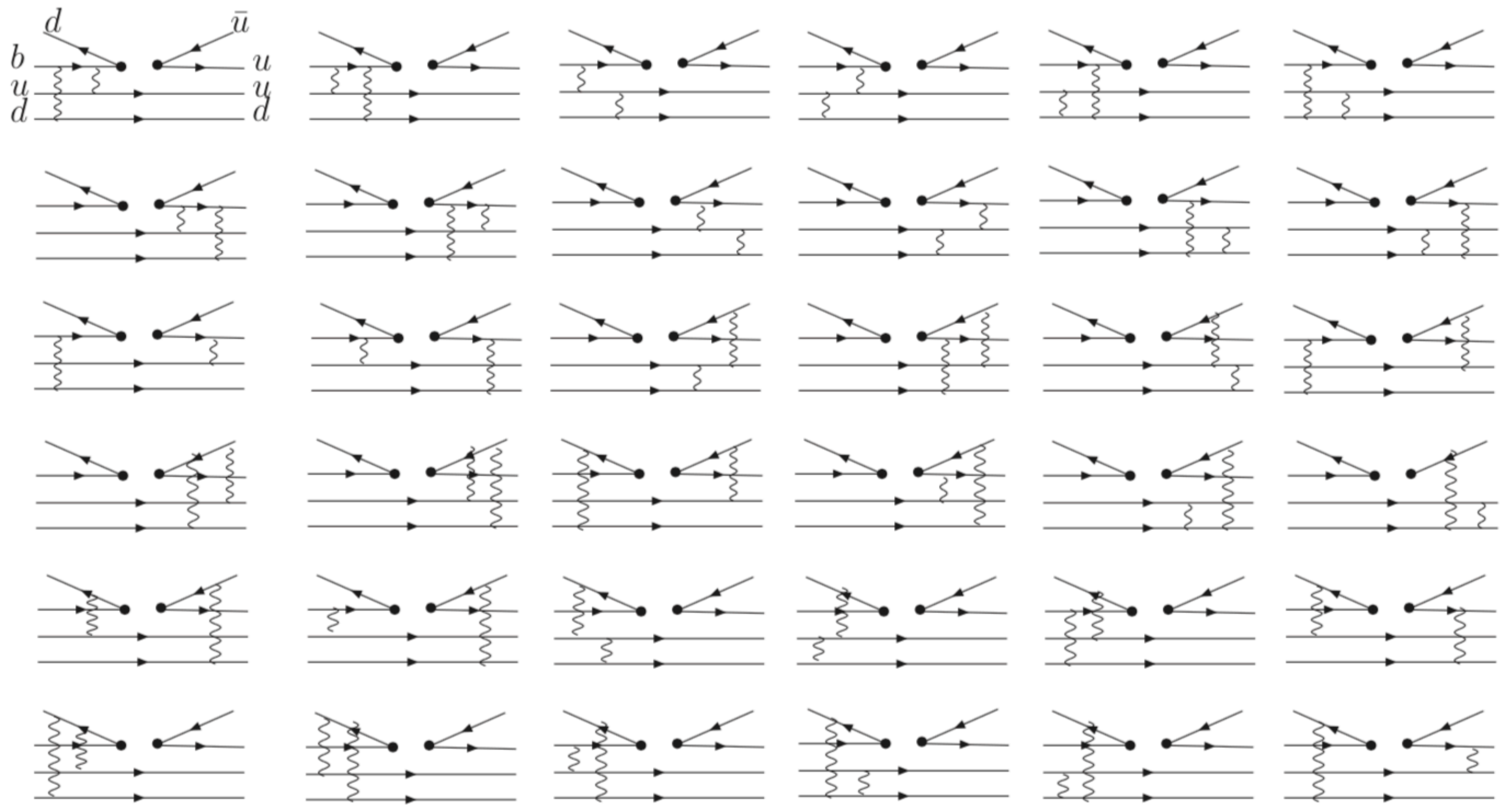
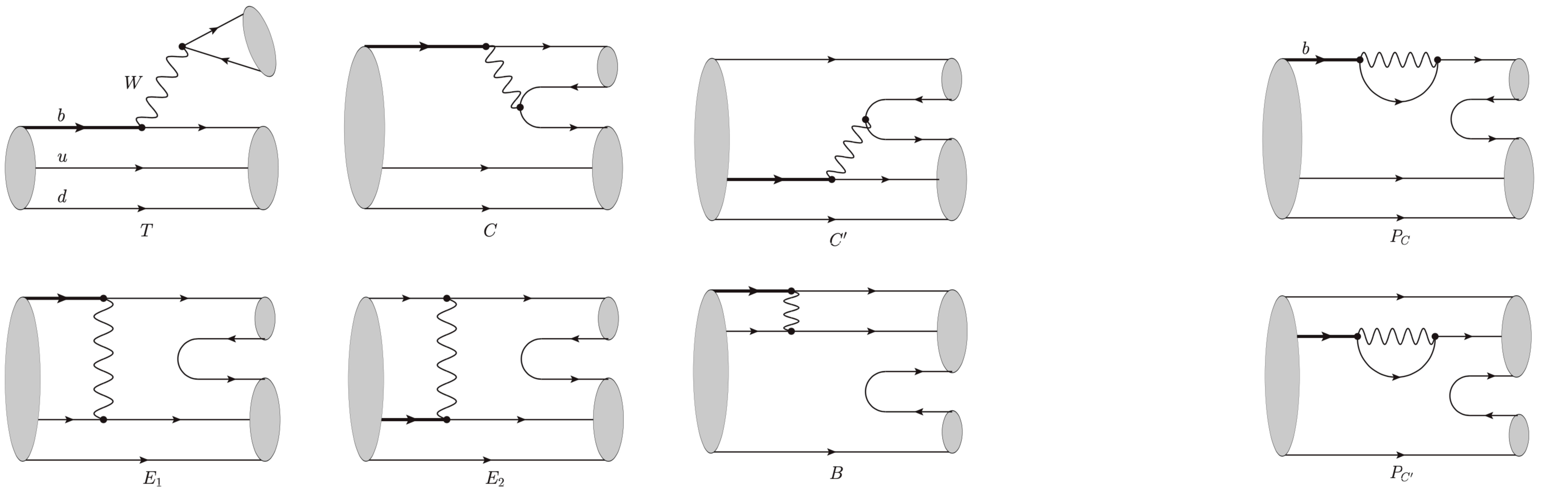


	EXP	PQCD
$Br(\Lambda_b \to p\pi)[\times 10^{-6}]$	4.3 ± 0.8	$4.66^{+2.22}_{-1.81}$
$Br(\Lambda_b \to pK)[\times 10^{-6}]$	5.1 ± 0.9	$1.82^{+0.97}_{-1.07}$
$A_{CP}(\Lambda_b \to p\pi)[\%]$	-2.5 ± 2.9	-32^{+49}_{-1}
$A_{CP}(\Lambda_b \to pK)[\%]$	-2.5 ± 2.2	-3^{+25}_{-4}

Theoretical challenges

- Topological diagrams of non-leptonic decays: more diagrams than mesons.



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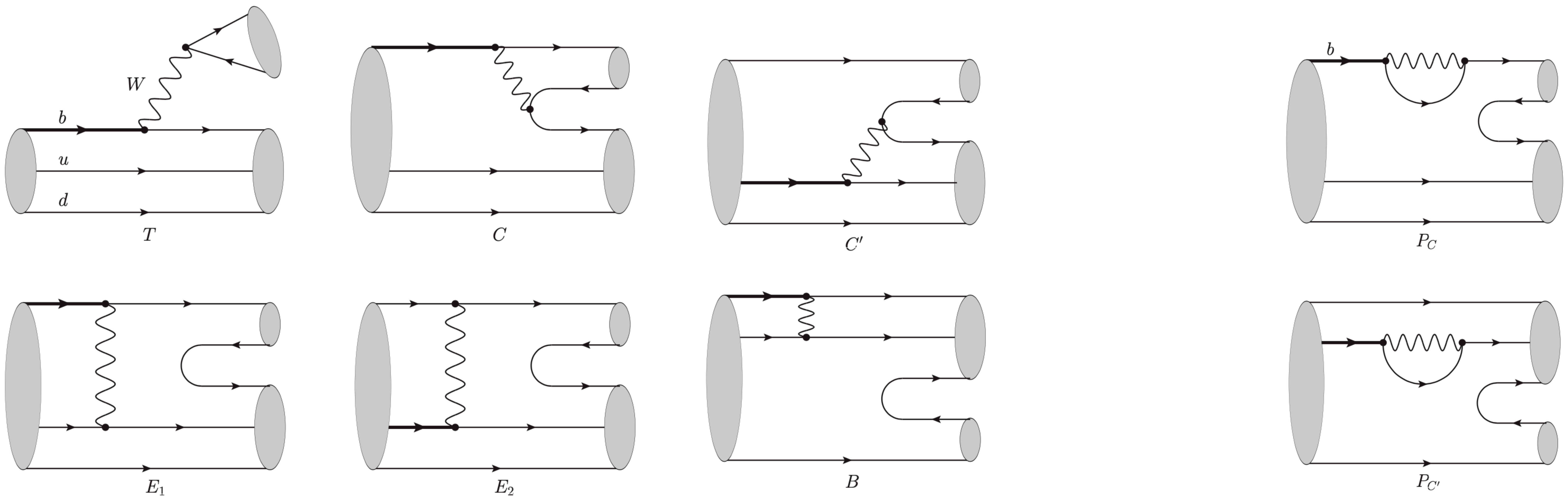

- **Annihilation-type diagrams contribute significantly to strong phases of CPV in B meson decays. To precisely predict CPV, W-exchange diagrams have to be calculated**

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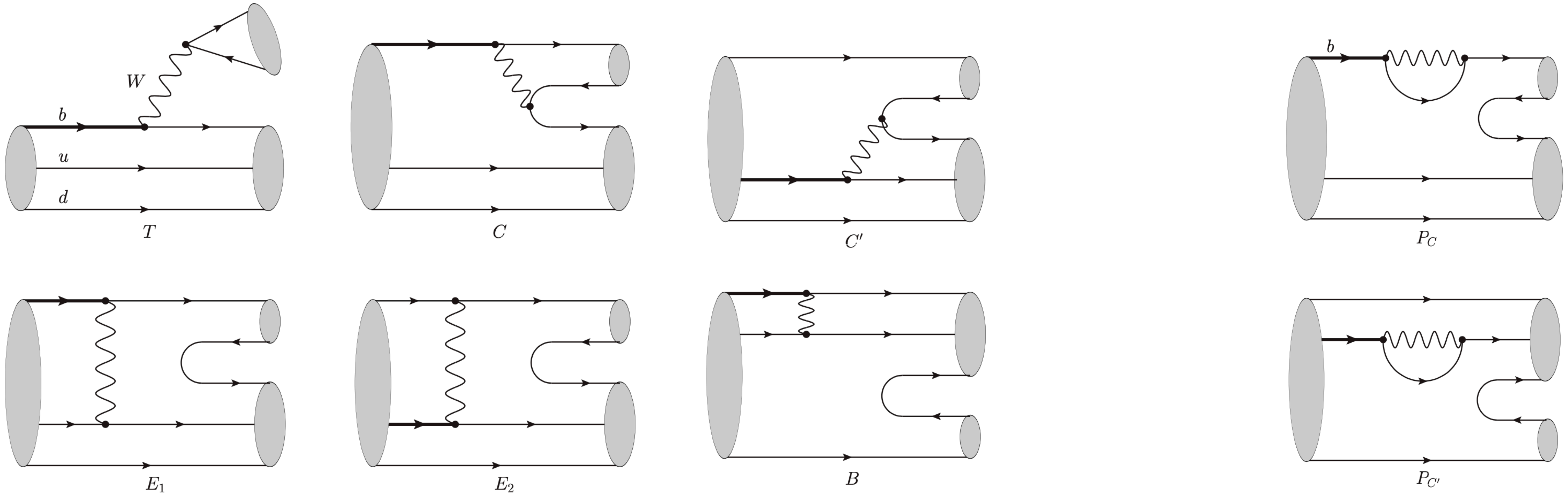

- **Annihilation-type diagrams contribute significantly to strong phases of CPV in B meson decays. To precisely predict CPV, W-exchange diagrams have to be calculated**

- More non-factorizable diagrams: C’, E1, E2, B. They are challenging in theory.

$$\frac{|C|}{|T|} \sim \frac{|C'|}{|C|} \sim \frac{|E_1|}{|C|} \sim \frac{|E_2|}{|C|} \sim O\left(\frac{\Lambda^h_{\rm QCD}}{m_Q}\right) \qquad \Lambda_b \to \Lambda_c \pi$$

Leibovich, Ligeti, Stewart, Wise, 2004

Theoretical progresses: PQCD

- PQCD successfully predicted correct CPV in B meson decays [Keum, Li, Sanda, 2000; Lu, Ukai, Yang, 2000].

直接CP破坏(%)	GFA	QCDF	PQCD	exp.
$B \to \pi^+\pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 20$	$+32 \pm 4$
$B \to K^+\pi^-$	$+10 \pm 3$	$+5 \pm 9$	-17 ± 5	-8.3 ± 0.4

- It is hopeful to predict correct CPV of b-baryons. W-exchange diagrams included.
- The only prediction of b-baryon CPV by PQCD is given for $\Lambda_b \to p\pi, pK$ in [C.D.Lu, Y.M.Wang, H.Zou, Ali, Kramer, 2009]

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- The only prediction of b-baryon CPV by PQCD is given for $\Lambda_b \to p\pi, pK$ in [C.D.Lu, Y.M.Wang, H.Zou, Ali, Kramer, 2009]
- However, the form factors are two orders of magnitude smaller than Lattice or sum rules

Lattice [35]	0.22 ± 0.08	Detmold, Lehner, Meinel, 2015
PQCD [67]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$	H.n.Li, 1999; C.D.Lu, Y.M.Wang, H.Zou, Ali, Kramer, 2009

	factorizable	non-factorizable
$f_1(\Lambda_b \to p\pi)$	$1.47 \times 10^{-11} - i1.97 \times 10^{-11}$	$-2.43 \times 10^{-9} - i2.05 \times 10^{-9}$
$f_2(\Lambda_b \to p\pi)$	$1.26 \times 10^{-11} - i1.94 \times 10^{-11}$	$-1.75 \times 10^{-9} - i1.20 \times 10^{-9}$
$f_1(\Lambda_b \to pK)$	$-1.52 \times 10^{-11} - i0.62 \times 10^{-11}$	$-0.88 \times 10^{-9} + i0.54 \times 10^{-10}$
$f_2(\Lambda_b \to pK)$	$0.17 \times 10^{-11} - i0.60 \times 10^{-11}$	$-1.06 \times 10^{-9} + i1.67 \times 10^{-9}$

Form Factors

	$f_1(0)$	$f_2(0)$	$g_1(0)$	$g_2(0)$
NRQM [76]	0.043			
heavy-LCSR [50]	$0.023^{+0.006}_{-0.005}$		$0.023^{+0.006}_{-0.005}$	
light-LCSR-$\mathcal{A}$ [77]	$0.14^{+0.03}_{-0.03}$	$-0.054^{+0.016}_{-0.013}$	$0.14^{+0.03}_{-0.03}$	$-0.028^{+0.012}_{-0.009}$
light-LCSR-$\mathcal{P}$ [77]	$0.12^{+0.03}_{-0.04}$	$-0.047^{+0.015}_{-0.013}$	$0.12^{+0.03}_{-0.03}$	$-0.016^{+0.007}_{-0.005}$
QCD-light-LCSR [78]	0.018	-0.028	0.018	-0.028
HQET-light-LCSR [78]	-0.002	-0.015		
3-point QSR [49]	0.22	0.0071		
lattice [47]	0.22 ± 0.08	0.04 ± 0.12	0.12 ± 0.14	0.04 ± 0.31
PQCD [32]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$			

[32] C. D. Lu, Y. M. Wang, H. Zou, A. Ali and G. Kramer, "Anatomy of the pQCD approach to the baryonic decays $\Lambda_b \to p\pi, pK$," Phys. Rev. D **80**, 034011 (2009) [arXiv:0906.1479 [hep-ph]].

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Theoretical progresses: PQCD

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2202.04804

proton

Λ_b

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	$\boxed{0.25}$	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

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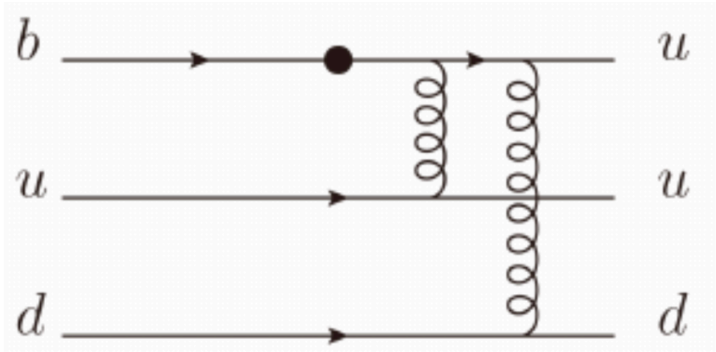


$$= \int [dx][db] \cdot \phi_{\Lambda_b} \cdot T_H \cdot \phi_p$$

$$\boxed{r = \frac{m_p}{M_{\Lambda_b}}}$$

D_7	twist-3	twist-4	twist-5	twist-6
twist-2	~ 0	$r \cdot 2\sqrt{2}(1-x_1)x_3$	$r^2 \cdot 2\sqrt{2}x_3$	$r^3 \cdot 4\sqrt{2}(1-x_1)(1-x_2')$
twist-3^{+-}	$x_3(1-x_1)$	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	~ 0
twist-3^{-+}	~ 0	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	$r^3 \cdot (1-x_2')$
twist-4	$4\sqrt{2}x_3$	$\boxed{r \cdot 2\sqrt{2}(1-x_1)(1-x_2')}$	$\boxed{r^2 \cdot 2\sqrt{2}(1-x_2')}$	~ 0

Λ_b LCDAs

$x_1 \to 1,\ x_{2,3} \to 0$

Prospects: LCDAs

- Light-cone distribution amplitudes (LCDA) are fundamental structures of hadrons.
- LCDAs are important in the factorization—PQCD, LCSR, SCET, QCDF…
- LCDAs are non-perturbative quantities, thus difficult for predictions.
- LCDAs of b-baryons and light baryons are much less known
- They are however important inputs in the calculations.
- So they dominate the theoretical uncertainties.
- The errors in each method are large.
- The differences between different methods are large.
- Theoretical efforts are urgently required.

Theoretical progresses: LCSR

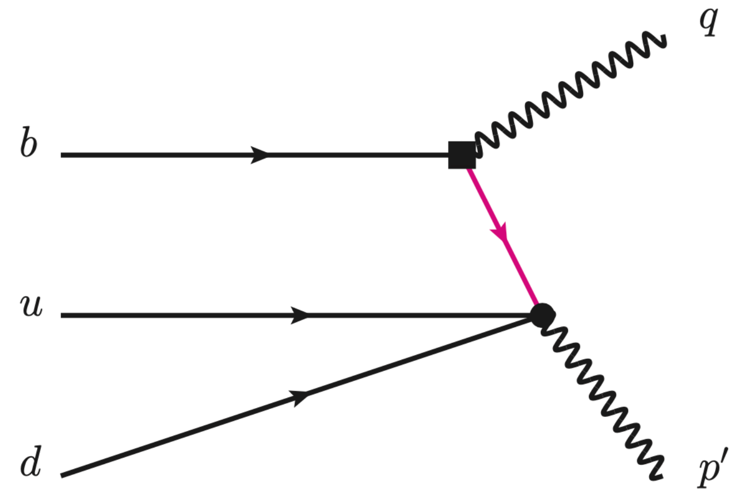
- Heavy-to-light form factors have been systematically studied in the light-cone sum rules (LCSR)

✓Next-to-leading order corrections [Y.M.Wang, Y.L.Shen, 2016]

✓LCDAs of heavy baryons [Bell, Feldman, Y.M.Wang, Yip, 2013]

✓$\Lambda_b \to p, N^*$ transitions [K.S.Huang, W.Liu, Y.L.Shen, FSY, 2205.06095]

$$\Pi_{\mu,a}(p,q) = \frac{1}{m_p^2 - p^2}\sum_{s'}\langle 0\,|\eta_i(0)|\, p\,(p,s')\rangle\,\langle p\,(p,s')\,|j_{\mu,a}(0)|\,\Lambda_b(p+q)\rangle$$
$$+\frac{1}{m_{N^*}^2 - p^2}\sum_{s'}\langle 0\,|\eta_i(0)|\, N^*\,(p,s')\rangle\,\langle N^*\,(p,s')\,|j_{\mu,a}(0)|\,\Lambda_b(p+q)\rangle + \ldots$$



- Test three interpolating currents
- Test five models of Λ_b LCDAs

$$\eta_{\mathrm{IO}}(x) = \varepsilon^{abc}\left[u^{aT}(x)C\gamma^\rho u^b(x)\right]\gamma_5\gamma_\rho d^c(x)$$
$$\eta_{\mathrm{TE}}(x) = \varepsilon^{abc}\left[u^{aT}(x)C\sigma^{\rho\sigma} u^b(x)\right]\gamma_5\sigma_{\rho\sigma} d^c(x)$$
$$\eta_{\mathrm{LP}}(x) = \varepsilon^{abc}\left[u^{aT}(x)C\,\not{n}\, u^b(x)\right]\gamma_5\,\not{n}\, d^c(x)$$

Theoretical progresses: LCSR

$\Lambda_b \to p$

	f_1	f_2	g_1	g_2
Ioffe current				
Gegenbauer-1	0.53 ± 0.39	-0.12 ± 0.099	0.53 ± 0.39	-0.12 ± 0.099
Gegenbauer-2	0.50 ± 0.077	-0.11 ± 0.021	0.50 ± 0.077	-0.11 ± 0.021
QCDSR	0.13 ± 0.023	-0.023 ± 0.004	0.13 ± 0.023	-0.023 ± 0.004
Exponential	0.14 ± 0.087	-0.026 ± 0.017	0.14 ± 0.087	-0.026 ± 0.017
Free-parton	0.17 ± 0.11	-0.031 ± 0.022	0.17 ± 0.11	-0.031 ± 0.022
Tensor current				
Gegenbauer-1	0.37 ± 0.34	-0.070 ± 0.058	0.37 ± 0.34	-0.070 ± 0.058
Gegenbauer-2	0.36 ± 0.079	-0.068 ± 0.015	0.36 ± 0.079	-0.068 ± 0.015
QCDSR	0.11 ± 0.023	-0.023 ± 0.005	0.11 ± 0.023	-0.023 ± 0.005
Exponential	0.12 ± 0.071	-0.024 ± 0.014	0.12 ± 0.071	-0.024 ± 0.014
Free-parton	0.16 ± 0.10	-0.033 ± 0.021	0.16 ± 0.10	-0.033 ± 0.021
LP current				
Gegenbauer-1	0.29 ± 0.062	-0.050 ± 0.011	0.29 ± 0.062	-0.050 ± 0.011
Gegenbauer-2	0.31 ± 0.071	-0.050 ± 0.013	0.31 ± 0.071	-0.050 ± 0.013
QCDSR	0.29 ± 0.061	-0.050 ± 0.010	0.29 ± 0.061	-0.050 ± 0.010
Exponential	0.27 ± 0.11	-0.045 ± 0.017	0.27 ± 0.11	-0.045 ± 0.017
Free-parton	0.38 ± 0.15	-0.063 ± 0.024	0.38 ± 0.15	-0.063 ± 0.024
heavy-LCSR[13]	$0.023^{+0.006}_{-0.005}$	$-0.039^{+0.009}_{-0.009}$	$0.023^{+0.006}_{-0.005}$	$-0.039^{+0.009}_{-0.009}$
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QCD-light-LCSR[16]	0.018	-0.028	0.018	-0.028
HQET-light-LCSR[16]	-0.002	-0.015	-0.002	-0.015
PQCD-Exponential[32]	0.27 ± 0.12	0.008 ± 0.005	0.31 ± 0.13	0.014 ± 0.010
PQCD-Free-parton[32]	0.24 ± 0.10	0.007 ± 0.004	0.27 ± 0.16	0.014 ± 0.008
CCQM[26]	0.080	-0.036	0.007	-0.001
RQM[27]	0.169	-0.050	0.196	-0.0002
LFQM[28]	0.1131	-0.0356	0.1112	-0.0097
LQCD[29]	0.22 ± 0.08	0.04 ± 0.12	0.12 ± 0.14	0.04 ± 0.31

- Ioffe and tensor currents are preferred for proton
- Λ_b LCDA models of QCDSR, exponential, and free-parton are preferred.
- $\Lambda_b \to N^*$ are helpful to distinguish them.

[K.S.Huang, W.Liu, Y.L.Shen, FSY, 2205.06095]

$\Lambda_b \to N^*$

	F_1	F_2	G_1	G_2
Ioffe Current				
Gegenbauer-1	0.23 ± 0.57	0.002 ± 0.12	0.23 ± 0.57	0.002 ± 0.12
Gegenbauer-2	0.20 ± 0.15	0.009 ± 0.029	0.20 ± 0.15	0.009 ± 0.029
QCDSR	0.015 ± 0.021	0.019 ± 0.005	0.015 ± 0.021	0.019 ± 0.005
Exponential	0.029 ± 0.031	0.017 ± 0.008	0.029 ± 0.031	0.017 ± 0.008
Free-parton	0.006 ± 0.026	0.028 ± 0.015	0.006 ± 0.026	0.028 ± 0.015
Tensor Current				
Gegenbauer-1	0.32 ± 0.29	-0.11 ± 0.087	0.32 ± 0.29	-0.11 ± 0.087
Gegenbauer-2	0.32 ± 0.069	-0.10 ± 0.023	0.32 ± 0.069	-0.10 ± 0.023
QCDSR	0.10 ± 0.019	-0.035 ± 0.007	0.10 ± 0.019	-0.035 ± 0.007
Exponential	0.10 ± 0.062	-0.036 ± 0.021	0.10 ± 0.062	-0.036 ± 0.021
Free-parton	0.14 ± 0.089	-0.050 ± 0.032	0.14 ± 0.089	-0.050 ± 0.032
LP Current				
Gegenbauer-1	1.16 ± 0.22	-0.34 ± 0.065	1.16 ± 0.22	-0.34 ± 0.065
Gegenbauer-2	1.22 ± 0.24	-0.34 ± 0.075	1.22 ± 0.24	-0.34 ± 0.075
QCDSR	1.16 ± 0.22	-0.34 ± 0.064	1.16 ± 0.22	-0.34 ± 0.064
Exponential	1.07 ± 0.42	-0.30 ± 0.11	1.07 ± 0.42	-0.30 ± 0.11
Free-parton	1.48 ± 0.60	-0.43 ± 0.16	1.48 ± 0.60	-0.43 ± 0.16
LCSR(1)[30]	-0.562 ± 0.015	0.451 ± 0.0133	0.523 ± 0.014	-0.454 ± 0.013
LCSR(2)[30]	-0.185 ± 0.005	0.184 ± 0.006	0.143 ± 0.004	-0.093 ± 0.003
LCSR-1[31]	-0.297 ± 0.080	-0.213 ± 0.064	-0.028 ± 0.084	0.106 ± 0.031
LCSR-2[31]	-0.202 ± 0.060	-0.0640 ± 0.0018	-0.144 ± 0.043	0.062 ± 0.002

Light-Cone Distribution Amplitudes: Λ_b

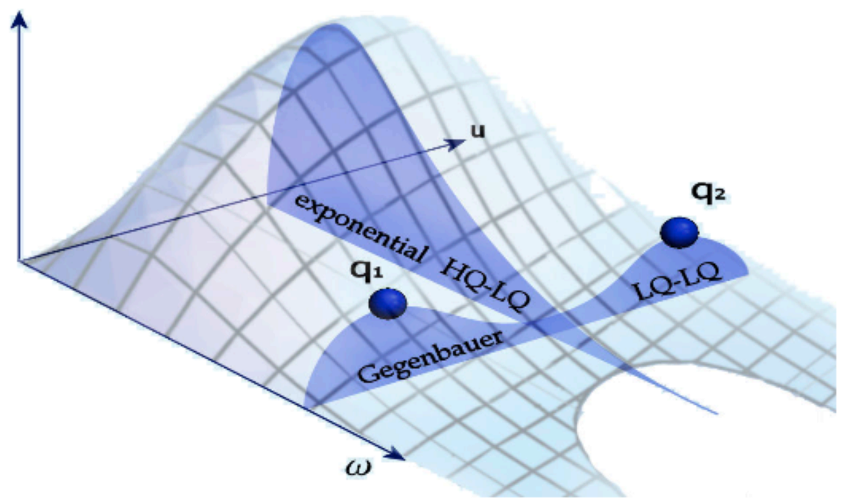
$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{1}{8\sqrt{2}N_c}\left\{f^{(1)}_{\Lambda_b}(\mu)[M_1(x_2,x_3)\gamma_5 C^T]_{\gamma\beta} + f^{(2)}_{\Lambda_b}(\mu)[M_2(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}\right\}[\Lambda_b(p)]_\alpha$$

$$M_1(x_2,x_3) = \frac{\bar{n}\!\!\!/ n\!\!\!/}{4}\psi_3^{+-}(x_2,x_3) + \frac{n\!\!\!/ \bar{n}\!\!\!/}{4}\psi_3^{-+}(x_2,x_3),$$
$$M_2(x_2,x_3) = \frac{n\!\!\!/}{\sqrt{2}}\psi_2(x_2,x_3) + \frac{\bar{n}\!\!\!/}{\sqrt{2}}\psi_4(x_2,x_3),$$

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{f'_{\Lambda_b}}{8\sqrt{2}N_c}[(p\!\!\!/ + m_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b(p)]_\alpha \psi(x_i,\mu),$$

$$\psi(x_i) = N x_1 x_2 x_3\ exp\left(-\frac{m^2_{\Lambda_b}}{2\beta^2 x_1} - \frac{m_l^2}{2\beta^2 x_2} - \frac{m_l^2}{2\beta^2 x_3}\right),$$

Light-Cone Distribution Amplitudes: Λ_b



Model-I: Gegenbauer-1

$$\psi_2(x_2,x_3) = m^4_{\Lambda_b} x_2 x_3 \left[\frac{1}{\epsilon_0^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_0} + a_2 C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right)\frac{1}{\epsilon_1^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_1}\right]$$

$$\psi_3^{+-}(x_2,x_3) = \frac{2m^3_{\Lambda_b} x_2}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$

$$\psi_3^{-+}(x_2,x_3) = \frac{2m^3_{\Lambda_b} x_3}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$

$$\psi_4(x_2,x_3) = \frac{5}{\mathcal{N}} m^2_{\Lambda_b} \int_{m_{\Lambda_b}(x_2+x_3)/2}^{s_0} ds\, e^{-s/\tau}(s - m_{\Lambda_b}(x_2+x_3)/2)^3,$$

with the Gegenbauer moment $a_2 = 0.333^{0.250}_{-0.333}$, the Gegenbauer polynomial $C_2^{3/2}(x) = 3(5x^2-1)/2$, the parameters $\epsilon_0 = 200^{+130}_{-60}$ MeV, $\epsilon_1 = 650^{+650}_{-300}$ MeV and $\epsilon_3 = 230 \pm 60$

Ball, Braun, Gardi, 0804.2424, PLB 2008

Model-II: Gegenbauer-2

$$\psi_2(x_2,x_3) = m^4_{\Lambda_b} x_2 x_3 \frac{a_2^{(2)}}{{\epsilon_2^{(2)}}^4} C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}/(x_2+x_3)/\epsilon_2^{(2)}},$$

$$\psi_3^{+-}(x_2,x_3) = m^3_{\Lambda_b}(x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} + \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$

$$\psi_3^{-+}(x_2,x_3) = m^3_{\Lambda_b}(x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} - \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$

$$\psi_4(x2,x3) = m^2_{\Lambda_b} \frac{a_2^{(4)}}{{\epsilon_2^{(4)}}^2} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(4)}},$$

$a_2^{(2)} = 0.391 \pm 0.279$, $a_2^{(3)} = -0.161^{+0.108}_{-0.207}$, $a_2^{(4)} = -0.541^{+0.173}_{-0.09}$, $b_3^{(3)} = -0.24^{+0.24}_{-0.147}$, $\epsilon_2^{(2)} = 0.551^{+\infty}_{-0.356}$ GeV, $\epsilon_2^{(3)} = 0.055^{+0.01}_{-0.02}$ GeV, $\epsilon_2^{(4)} = 0.262^{+0.116}_{-0.132}$ GeV and $\eta_3^{(3)} = 0.633 \pm 0.099$ GeV.

Ali, Hambrock, Parkhomenko, W.Wang, 2012

Light-Cone Distribution Amplitudes: Λ_b

Model-III: Exponential

$$\psi_2(x_2, x_3) = \frac{x_2 x_3}{\omega_0^4} m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_3^{+-}(x_2, x_3) = \frac{2x_2}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_3^{-+}(x_2, x_3) = \frac{2x_3}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$
$$\psi_4(x_2, x_3) = \frac{1}{\omega_0^2} m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$\omega_0 = 0.4$ GeV

Model-IV: Free Parton

$$\psi_2(x_2, x_3) = \frac{15 x_2 x_3 m_{\Lambda_b}^4 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})$$
$$\psi_3^{+-}(x_2, x_3) = \frac{15 x_2 m_{\Lambda_b}^3 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}),$$
$$\psi_3^{-+}(x_2, x_3) = \frac{15 x_3 m_{\Lambda_b}^3 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}),$$
$$\psi_4(x_2, x_3) = \frac{5 m_{\Lambda_b}^2 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^3}{8\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}),$$

$\bar{\Lambda} \equiv (m_{\Lambda_b} - m_b)/2 \approx 0.8$ GeV

Bell, Feldmann, Y.M.Wang, Yip, 1308.6114, JHEP2013

Light-Cone Distribution Amplitudes: proton

$$\langle 0|\, \varepsilon^{ijk} u_\alpha^{i'}(a_1 z)\, [a_1 z, a_0 z]_{i',i}\, u_\beta^{j'}(a_2 z)\, [a_2 z, a_0 z]_{j',j}\, d_\gamma^{k'}(a_3 z)\, [a_3 z, a_0 z]_{k',k}\, |P(P,\lambda)\rangle$$

$$\begin{aligned}
&4\,\langle 0|\, \varepsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z)\,|P\rangle = \\
&= S_1 M C_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + S_2 M C_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + P_1 M\,(\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + P_2 M\,(\gamma_5 C)_{\alpha\beta}\, N_\gamma^- \\
&\quad + V_1\,(\not{p} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + V_2\,(\not{p} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + \frac{V_3}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma \\
&\quad + \frac{V_4}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + V_5 \frac{M^2}{2pz}\,(\not{z} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} V_6\,(\not{z} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma \\
&\quad + A_1\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + A_2\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N_\gamma^- + \frac{A_3}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^+\right)_\gamma \\
&\quad + \frac{A_4}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^-\right)_\gamma + A_5 \frac{M^2}{2pz}\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N_\gamma^+ + \frac{M^2}{2pz} A_6\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N_\gamma^- \\
&\quad + T_1\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + T_2\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + T_3 \frac{M}{pz}\,(i\sigma_{pz} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma \\
&\quad + T_4 \frac{M}{pz}\,(i\sigma_{zp} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + T_5 \frac{M^2}{2pz}\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} T_6\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma \\
&\quad + M \frac{T_7}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^+\right)_\gamma + M \frac{T_8}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^-\right)_\gamma\,,
\end{aligned} \tag{2.9}$$

Braun, Fries, Mahnke, Stein, hep-ph/0007279, NPB 2000

Light-Cone Distribution Amplitudes: proton

- Twist-3 LCDAs

$$
\begin{aligned}
V_1(x_i) =& 120x_1x_2x_3[\phi_3^0 + \phi_3^+(1-3x_3)],\\
A_1(x_i) =& 120x_1x_2x_3(x_2-x_1)\phi_3^-,\\
T_1(x_i) =& 120x_1x_2x_3[\phi_3^0 + \frac{1}{2}(\phi_3^- - \phi_3^+)(1-3x_3)].
\end{aligned}
$$

- Twist-4 LCDAs

$$
\begin{aligned}
V_2(x_i) =& 24x_1x_2[\phi_4^0 + \phi_4^+(1-5x_3)],\\
V_3(x_i) =& 12x_3[\psi_4^0(1-x_3) + \psi_4^-(x_1^2+x_2^2-x_3(1-x_3)) + \psi_4^+(1-x_3-10x_1x_2)],\\
A_2(x_i) =& 24x_1x_2(x_2-x_1)\phi_4^-,\\
A_3(x_i) =& 12x_3(x_2-x_1)[(psi_4^0 + \psi_4^+) + \psi_4^-(1-2x_3)],\\
T_2(x_i) =& 24x_1x_2[\xi_4^0 + \xi_4^+(1-5x_3)],\\
T_3(x_i) =& 6x_3[(\xi_4^0 + \phi_4^0 + \psi_4^0)(1-x_3) + (\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3))\\
&+ (\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],\\
T_7(x_i) =& 6x_3[(-\xi_4^0 + \phi_4^0 + \psi_4^0)(1-x_3) + (-\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3))\\
&+ (-\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],\\
S_1(x_i) =& 6x_3(x_2-x_1)[(\xi_4^0 + \phi_4^0 + \psi_4^0 + \xi_4^+ + \phi_4^+ + \psi_4^+) + (\xi_4^- + \phi_4^- - \psi_4^-)(1-2x_3)],\\
P_1(x_i) =& 6x_3(x_2-x_1)[(\xi_4^0 - \phi_4^0 - \psi_4^0 + \xi_4^+ - \phi_4^+ - \psi_4^+) + (\xi_4^- - \phi_4^- + \psi_4^-)(1-2x_3)].
\end{aligned}
$$

- Twist-5 LCDAs

$$
\begin{aligned}
V_4(x_i) =& 3[\psi_5^0(1-x_3) + \psi_5^-(2x_1x_2 - x_3(1-x_3)) + \psi_5^+(1-x_3-2(x_1^2+x_2^2))],\\
V_5(x_i) =& 6x_3[\phi_5^0 + \phi_5^+(1-2x_3)],\\
A_4(x_i) =& 3(x_2-x_1)[-\psi_5^0 + \psi_5^- x_3 + \psi_5^+(1-2x_3)],\\
A_5(x_i) =& 6x_3(x_2-x_1)\phi_5^-,\\
T_4(x_i) =& \frac{3}{2}[(\xi_5^0 + \psi_5^0 + \phi_5^0)(1-x_3) + (\xi_5^- + \phi_5^- - \psi_5^-)(2x_1x_2 - x_3(1-x_3))\\
&+ (\xi_5^+ + \phi_5^+ + \psi_5^+)(1-x_3-2(x_1^2+x_2^2))],\\
T_5(x_i) =& 6x_3[\xi_5^0 + \xi_5^+(1-2x_3)],\\
T_8(x_i) =& \frac{3}{2}[(\psi_5^0 + \phi_5^0 - \xi_5^0)(1-x_3) + (\phi_5^- - \phi_5^- - \xi_5^-)(2x_1x_2 - x_3(1-x_3))\\
&+ (\phi_5^+ + \phi_5^+ - \xi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))],\\
S_2(x_i) =& \frac{3}{2}(x_2-x_1)[-(\psi_5^0 + \phi_5^0 + \xi_5^0) + (\xi_5^- + \phi_5^- - \psi_5^0)x_3 + (\xi_5^+ + \phi_5^+ + \psi_5^0)(1-2x_3)],\\
P_2(x_i) =& \frac{3}{2}(x_2-x_1)[(\psi_5^0 + \phi_5^0 - \xi_5^0) + (\xi_5^- - \phi_5^- + \psi_5^0)x_3 + (\xi_5^+ - \phi_5^+ - \psi_5^0)(1-2x_3)].
\end{aligned}
$$

- Twist-6 LCDAs

$$
\begin{aligned}
V_6(x_i) =& 2[\phi_6^0 + \phi_6^+(1-3x_3)],\\
A_6(x_i) =& 2(x_2-x_1)\phi_6^-,\\
T_6(x_i) =& 2[\phi_6^0 + \frac{1}{2}(\phi_6^- - \phi_6^+)(1-3x_3)],
\end{aligned}
$$

Light-Cone Distribution Amplitudes: proton

Table 2: Parameters in the proton LCDAs in units of 10^{-2} GeV^2 [73]. The accuracy of those parameters without uncertainties is of order of 50%.

	ϕ_i^0	ϕ_i^-	ϕ_i^+	ψ_i^0	ψ_i^-	ψ_i^+	ξ_i^0	ξ_i^-	ξ_i^+
twist-3 $(i=3)$	0.53 ± 0.05	2.11	0.57						
twist-4 $(i=4)$	-1.08 ± 0.47	3.22	2.12	1.61 ± 0.47	-6.13	0.99	0.85 ± 0.31	2.79	0.56
twist-5 $(i=5)$	-1.08 ± 0.47	-2.01	1.42	$1.61 \pm .047$	-0.98	-0.99	0.85 ± 0.31	-0.95	0.46
twist-6 $(i=6)$	0.53 ± 0.05	3.09	-0.25						

Parameters of LCDAs of proton

Model	Method	$f_N \cdot 10^3$ Gev^2	$\lambda_1 \cdot 10^3$ Gev^2	$\lambda_2 \cdot 10^3$ Gev^2	A_1^u	V_1^d	f_1^u	f_1^d	f_2^d	Ref.
	QCDSR	5.0(5)	-27(9)	54(19)						
ASY		-	-	-	0	1/3	1/10	3/10	4/15	
CZ	QCDSR	5.3(5)	-	-	0.47	0.22	-	-	-	[1]
KS	QCDSR	5.1(3)	-	-	0.34	0.24	-	-	-	[2]
COZ	QCDSR	5.0(3)	-	-	0.39	0.23	-	-	-	[3]
SB	QCDSR	-	-	-	0.38	0.24	-	-	-	[4]
BK	PQCD	6.64	-	-	0.08	0.31	-	-	-	[5]
BLW	QCDSR	-	-	-	0.38(15)	0.23(3)	0.07(5)	0.40(20)	0.22(5)	[6]
BLW	LCSR（LO）	-	-	-	0.13	0.30	0.09	0.33	0.25	[6]
ABO1	LCSR（NLO）	-	-	-	0.11	0.30	0.11	0.27	-	[7]
ABO2	LCSR（NLO）				0.11	0.30	0.11	0.29	-	[7]
LAT09	LATTICE	3.23 (63)	-35.57 (65)	70.02 (13)	0.19 (2)	0.20 (1)	-	-	-	[8]
LAT14	LATTICE	3.07 (36)	-38.77 (18)	77.64 (37)	0.07 (4)	0.31 (2)	-	-	-	[9]
LAT19	LATTICE	3.54（6）	-44.9 (42)	93.4 (48)	0.30 (32)	0.192 (22)	-	-	-	[10]

thanks to K.S.Huang

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