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BESIII上 $D_{(s)}$ 半轻衰变到矢量介子末态的实验研究

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第四届重味物理与量子色动力学研讨会

2022年7月29日@长沙

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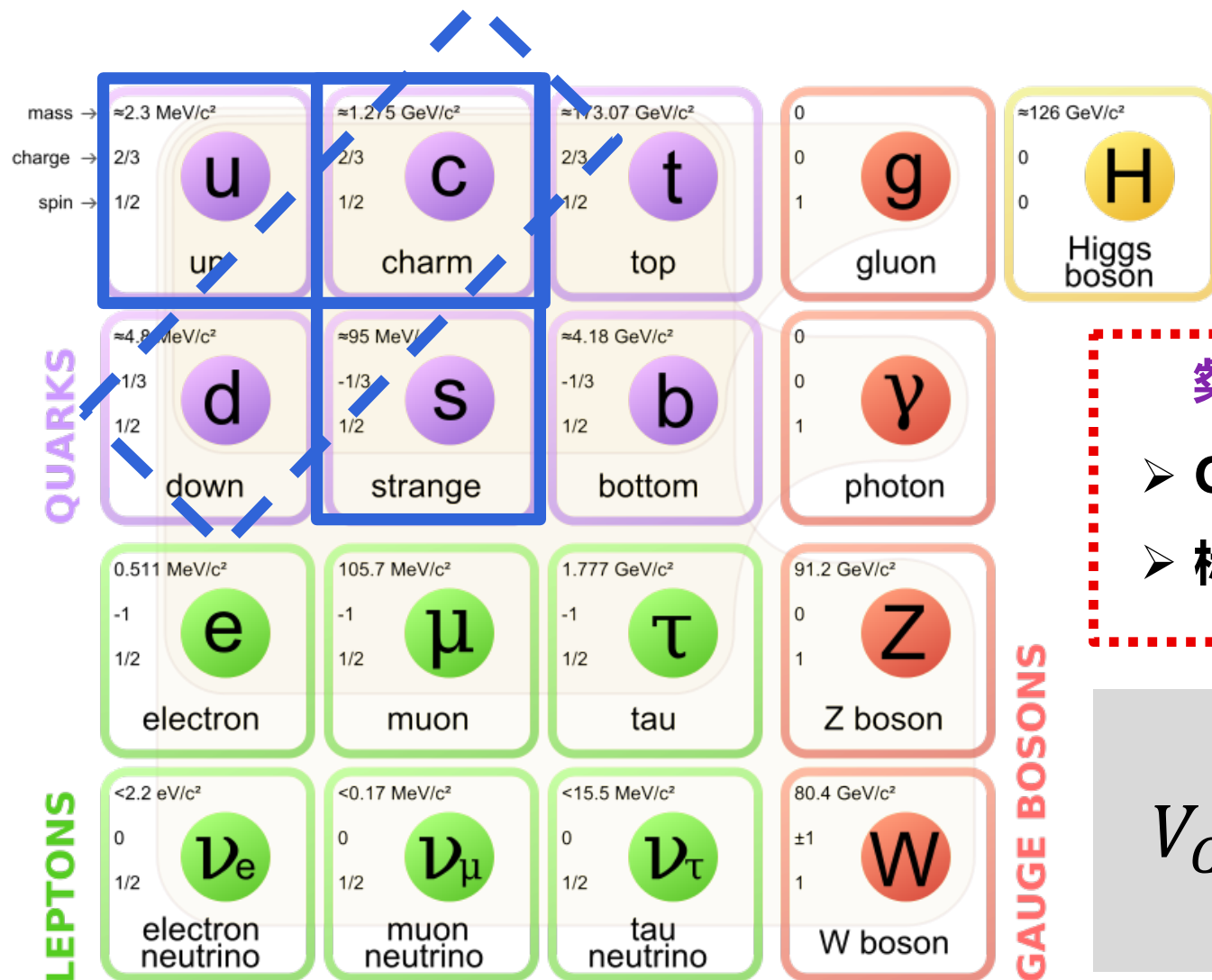
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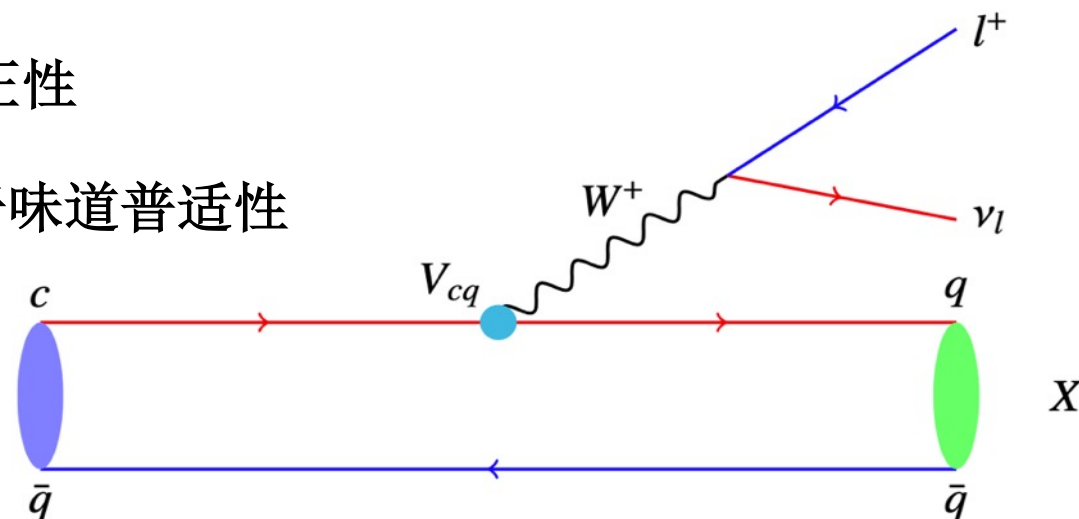
新物理

- QCD: 非微扰能区
- 标准模型检验：高精度前沿 -> 新物理

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



- 测量强子化形状因子 -> 检验和刻度 LQCD
- 测量CKM矩阵元素 $V_{cd(s)}$ -> 检验 CKM 矩阵幺正性
- $\mathcal{B}(D_{(s)} \rightarrow X\mu^+\nu_\mu)/\mathcal{B}(D_{(s)} \rightarrow Xe^+\nu_e)$ -> 检验轻子味道普适性
- 研究轻强子物理的良好场所



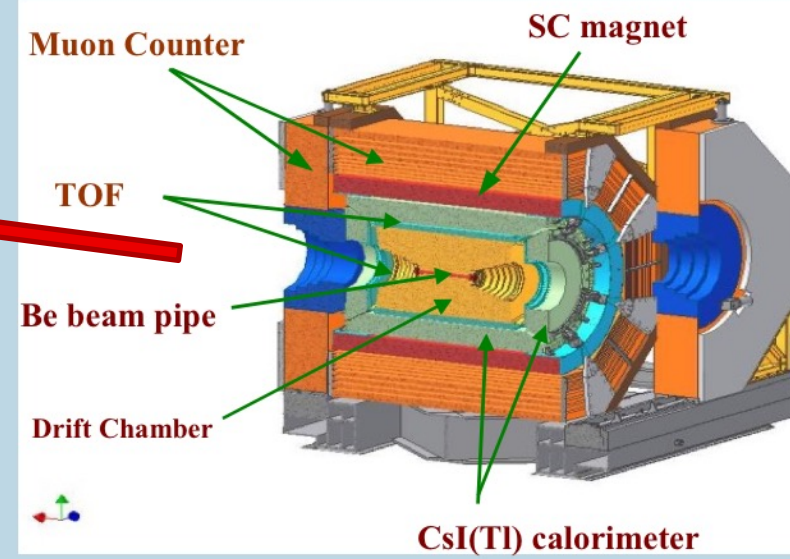
$$A(D \rightarrow X\ell\nu) = \frac{G_F}{\sqrt{2}} V_{cq}^* \bar{\nu} \gamma_\mu (1 - \gamma_5) \ell \langle X | \bar{q} \gamma^\mu (1 - \gamma_5) c | D_{(s)} \rangle$$

$$\Gamma(D_{(s)} \rightarrow P(S)\ell^+\nu_\ell) \propto |V_{cd(s)}|^2 |f_+(q^2)|^2 dq^2$$

$$\rightarrow \Gamma(D_{(s)} \rightarrow V\ell^+\nu_\ell) \propto |V_{cd(s)}|^2 \mathfrak{T}(A_1(q^2), A_2(q^2), V(q^2)) dq^2$$

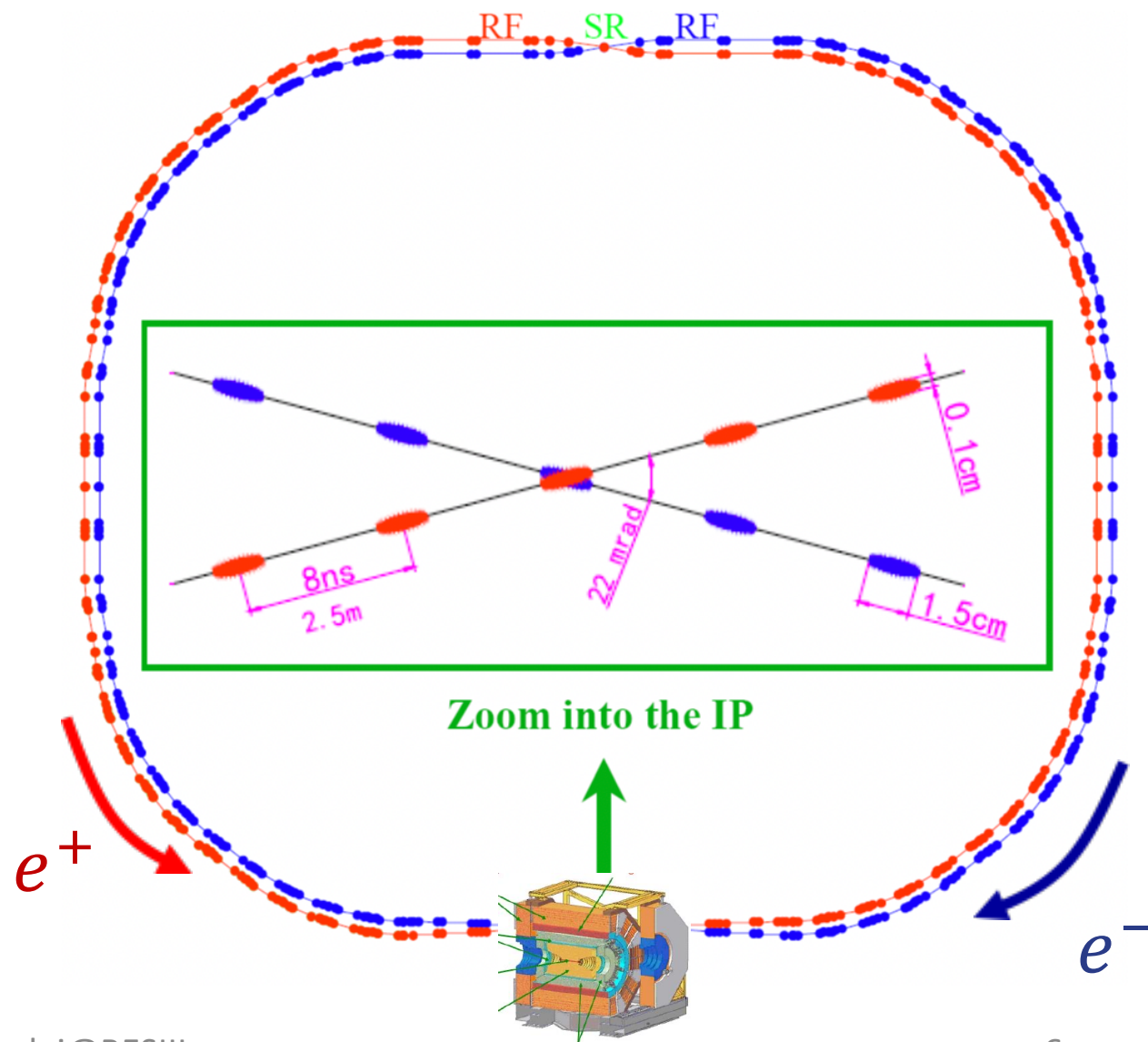


BESIII Detector

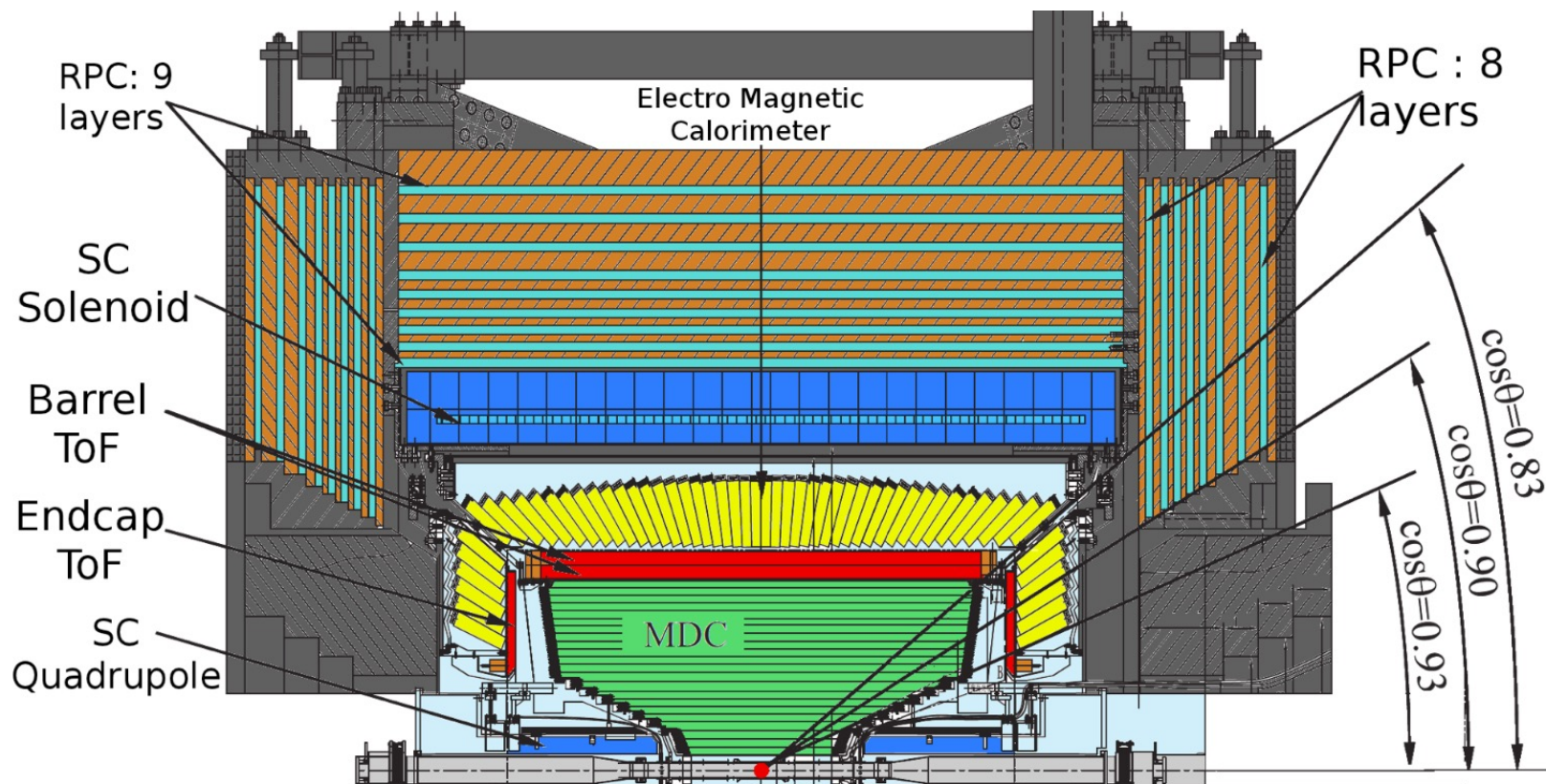


BEPCII 对撞机

- 对称型双环 e^+e^- 对撞机
- 环周长: 240 m
- 设计亮度: $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
- 实现时间: 2016年4月5日
- 质心系能量 E_{cm} : 2 – 5 GeV
- 束流夹角: 22 mrad



BESIII 探测器



MDC

$$\frac{\delta p}{p} < 0.5\% \text{ @1 GeV}$$

$$\frac{\delta(dE/dx)}{dE/dx} < 6\%$$

TOF

$$\delta t \text{ 80 ps Barrel}$$

$$\delta t \text{ 110 ps Endcap}$$

EMC

$$\frac{\delta E}{E} < 2.5\% \text{ @1 GeV}$$

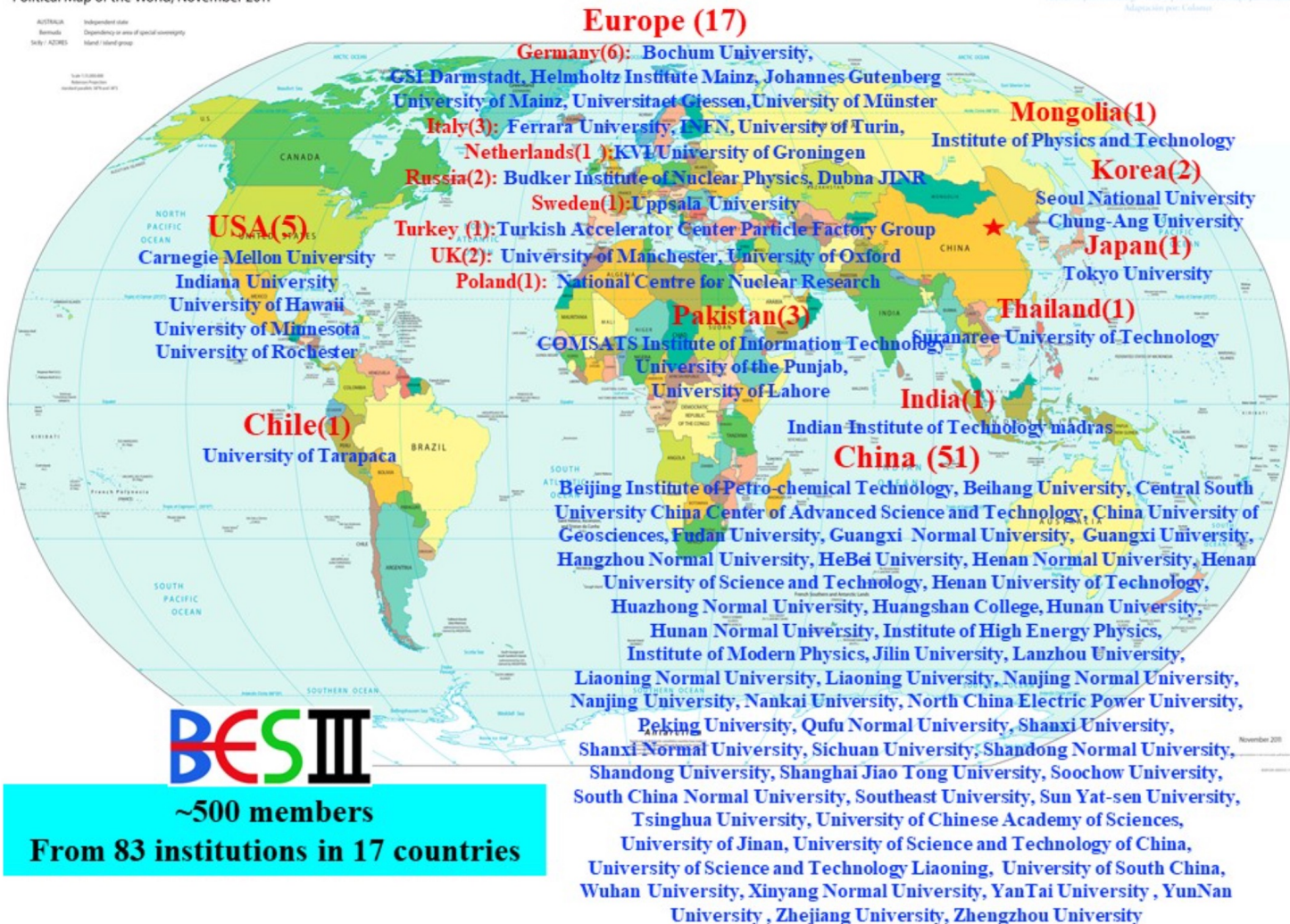
$$\delta z = 0.6/\sqrt{E}$$

MUC

$$\delta(xy) < 2 \text{ cm}$$

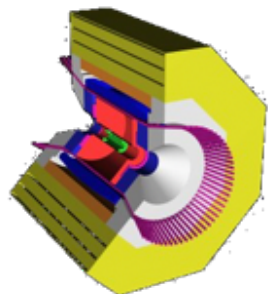
BESIII 合作组

Political Map of the World, November 2011

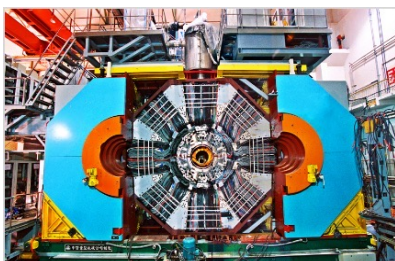




CLEO-c



BESIII



- 对称型 e^+e^- 对撞机
- E_{cm} : 2 – 5 GeV
- 阈值附近产生粲介子对



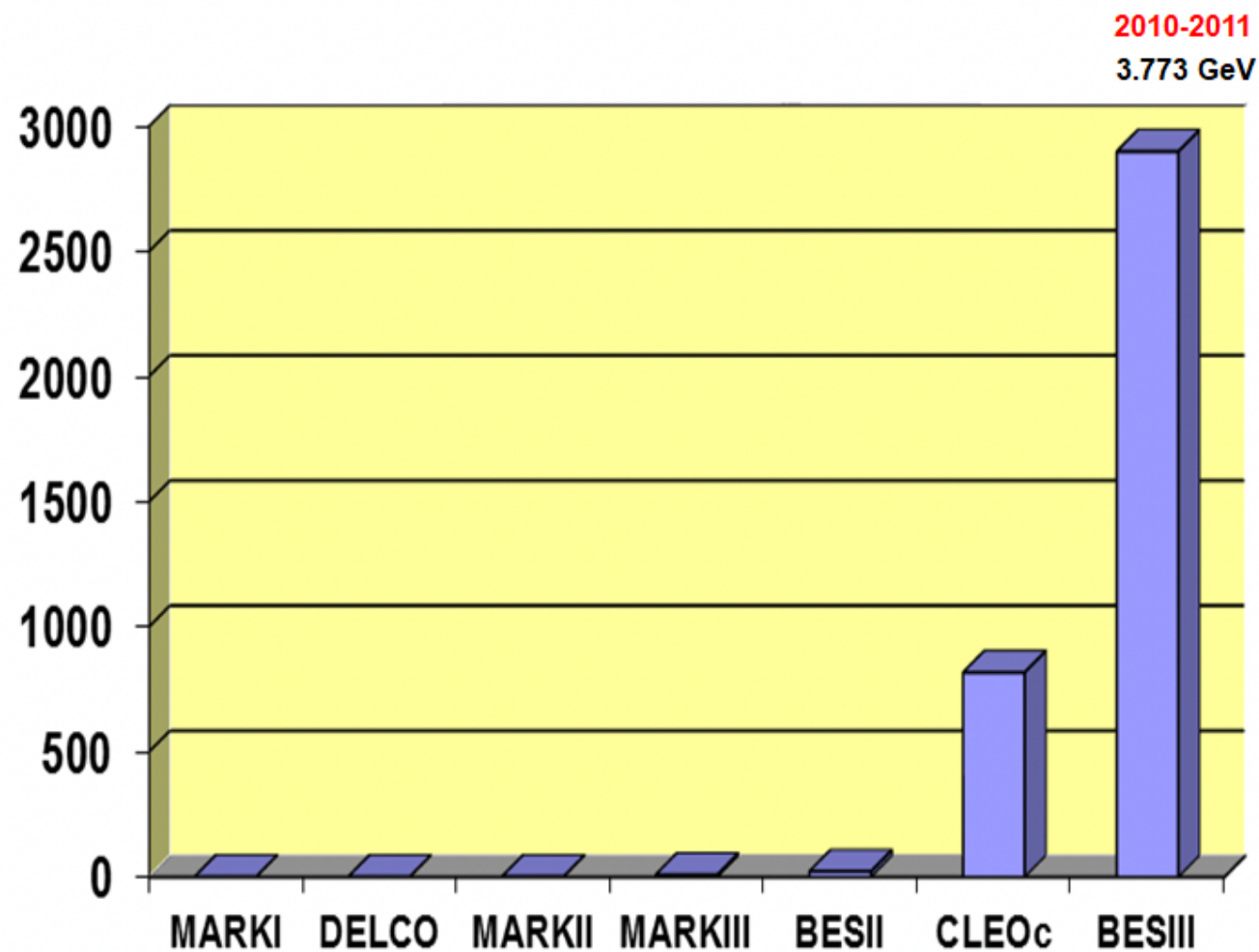
Belle, Belle-II

- 不对称型 e^+e^- 对撞机
- E_{cm} : 10.8 GeV
- $b\bar{b}$ 和 $c\bar{c}$ 衰变产生粲介子

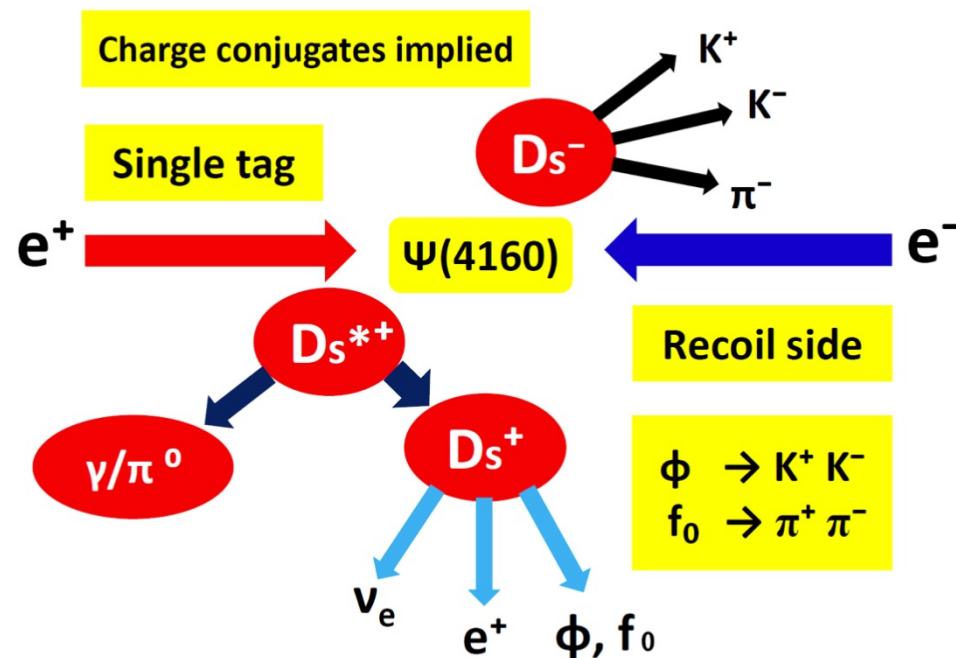


实验组	数据大小	所在能区	时间
BESIII	$D^{+(0)}: 2.93 \text{ fb}^{-1}$ $D_s^+: 7.33 \text{ fb}^{-1}$	3.773 GeV 4.128-4.223 GeV	2010-2011 2013-2017
CLEO-c	$D^{+(0)}: 0.82 \text{ fb}^{-1}$ $D_s^+: 0.6 \text{ fb}^{-1}$	3.770 GeV 4.170 GeV	至2008
BABAR	468 fb^{-1}	$\Upsilon(4S)$ 附近	至2008
Belle	976 fb^{-1}	$\Upsilon(4S)$ 附近	至2010

$D^{0(+)}$ samples at $\psi(3770)$



以 D_s 衰变为例(复杂情况)



$$\mathcal{B}_\gamma(D_S^* \rightarrow \gamma D_S)$$

$$N_{tag} = 2N_{D_S^+ D_S^-} \mathcal{B}_{tag} \epsilon_{tag}$$

$$N_{sig} = 2N_{D_S^+ D_S^-} \mathcal{B}_{tag} \mathcal{B}_{sig} \mathcal{B}_\gamma \epsilon_{sig}$$

$$\mathcal{B}_{sig} = \frac{N_{sig}}{\mathcal{B}_\gamma N_{tag} \epsilon_{sig} / \epsilon_{tag}}$$

$$\mathcal{B}_{sig} = \frac{N_{sig}}{\mathcal{B}_\gamma \sum_\alpha N_{tag}^\alpha \epsilon_{sig}^\alpha / \epsilon_{tag}^\alpha}$$

$$U_{miss} = E_{miss} - |\vec{p}_{miss}|$$

$$M_{miss}^2 = E_{miss}^2 - |\vec{p}_{miss}|^2$$

$$\Gamma(D_{(s)} \rightarrow V \ell^+ \nu_\ell) \propto |V_{cd(s)}|^2 \mathfrak{T}(A_1(q^2), A_2(q^2), V(q^2), \dots) dm^2 dq^2 d\cos(\theta_h) d\cos(\theta_\ell) d\chi$$

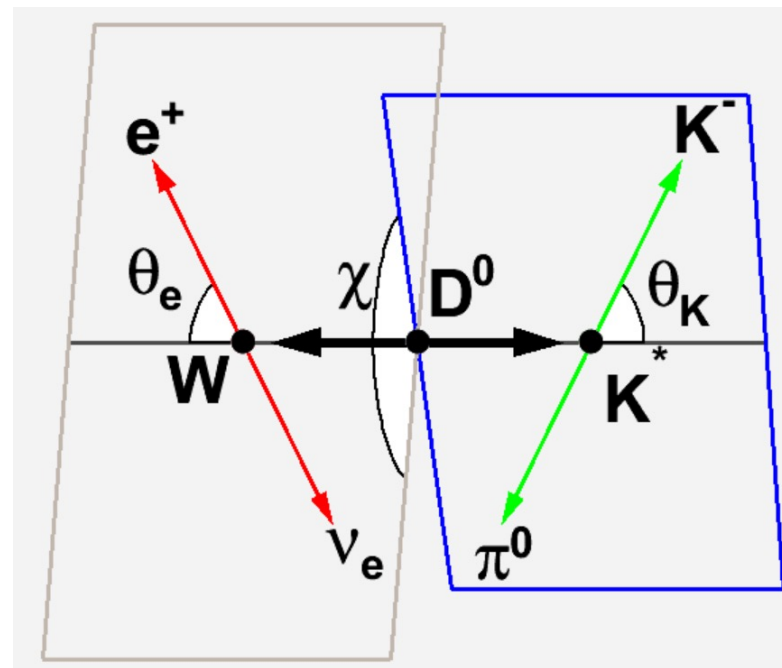
$V: \rho, \omega, K^*, \phi$

理论: Phys. Rev. **137**, B438(1965), Phys.Rev.D 46,5040(1992)

- 衰变强度 $\mathfrak{T}$ 可以包含 S, P, D 波过程的成分
- 不分bin的最大似然方法 (基于 RooFit 框架来实现)
- 形状因子 (单极点参数化, 双极点参数化 ...)

$$A_i(q^2) = \frac{A_i(0)}{1 - q^2/M_A^2} \quad V(q^2) = \frac{V(0)}{1 - q^2/M_V^2}$$

$$r_V = \frac{V(0)}{A_1(0)} \quad r_2 = \frac{A_2(0)}{A_1(0)}$$



- 对负的log似然函数求极小:

$$NLL = - \sum_{i=1}^N \ln \frac{\omega(\xi_i, \eta)}{\sigma_s}$$

$\omega(\xi_i, \eta)$ 为衰变强度, σ_s 为归一化因子, 采用信号蒙特卡洛样本:

$$\sigma_s = \int d\xi \omega(\xi, \eta) \epsilon(\xi) \propto \frac{1}{N_{selected}} \sum_{k=1}^{N_{selected}} \frac{\omega(\xi_k, \eta)}{\omega(\xi_k, \eta_0)}$$

- 本底较低时可直接在NLL中扣除:

$$NLL = (-\ln L_{data}) - (-\ln L_{bkg})$$

- 本底较高时:
- $$- \sum_{i=1}^N \ln \left((1-f_b) \frac{\omega(\xi_i, \eta)}{\int d\xi_i \omega(\xi_i, \eta) \epsilon(\xi_i)} + f_b \frac{B_\epsilon(\xi_i)}{\int d\xi_i B_\epsilon(\xi_i) \epsilon(\xi_i)} \right)$$

- 本底的问题
 - 本底较高，尤其是缪子道
 - 本底如何参数化
- 物理公式的问题
 - 轻子质量是否可以忽略 (e/μ)
 - 考虑轻子质量项的完整振幅公式（包含S，P和D波）



Phys. Rev. D 94, 032001(2016)

- $N_{tot} = 18262$ (本底水平: 0.8%)
- $\mathcal{B}(D^+ \rightarrow K^- \pi^+ e^+ \nu_e) = (3.77 \pm 0.03 \pm 0.08)\%$

$$f_{S\text{-wave}} = (6.05 \pm 0.22 \pm 0.18)\%$$

- 测量形状因子:

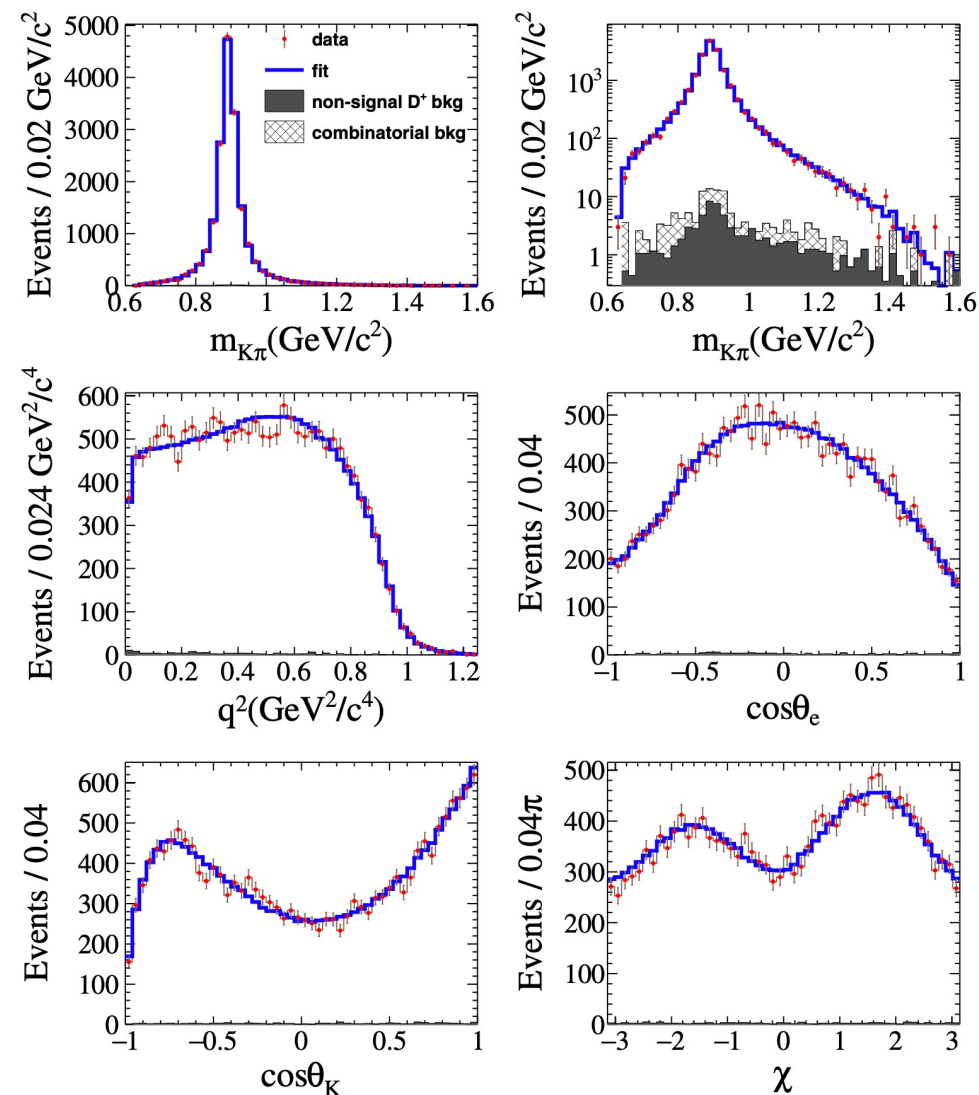
$$r_V = 1.411 \pm 0.058 \pm 0.007,$$

$$r_2 = 0.788 \pm 0.042 \pm 0.008$$

输入 $G_f, \tau_{D^+}, |V_{cs}| \rightarrow$

$$A_1(0) = 0.589 \pm 0.010 \pm 0.012 \text{ (零宽度假设)}$$

$$A_1(0) = 0.619 \pm 0.011 \pm 0.013 \text{ (考虑宽度)}$$

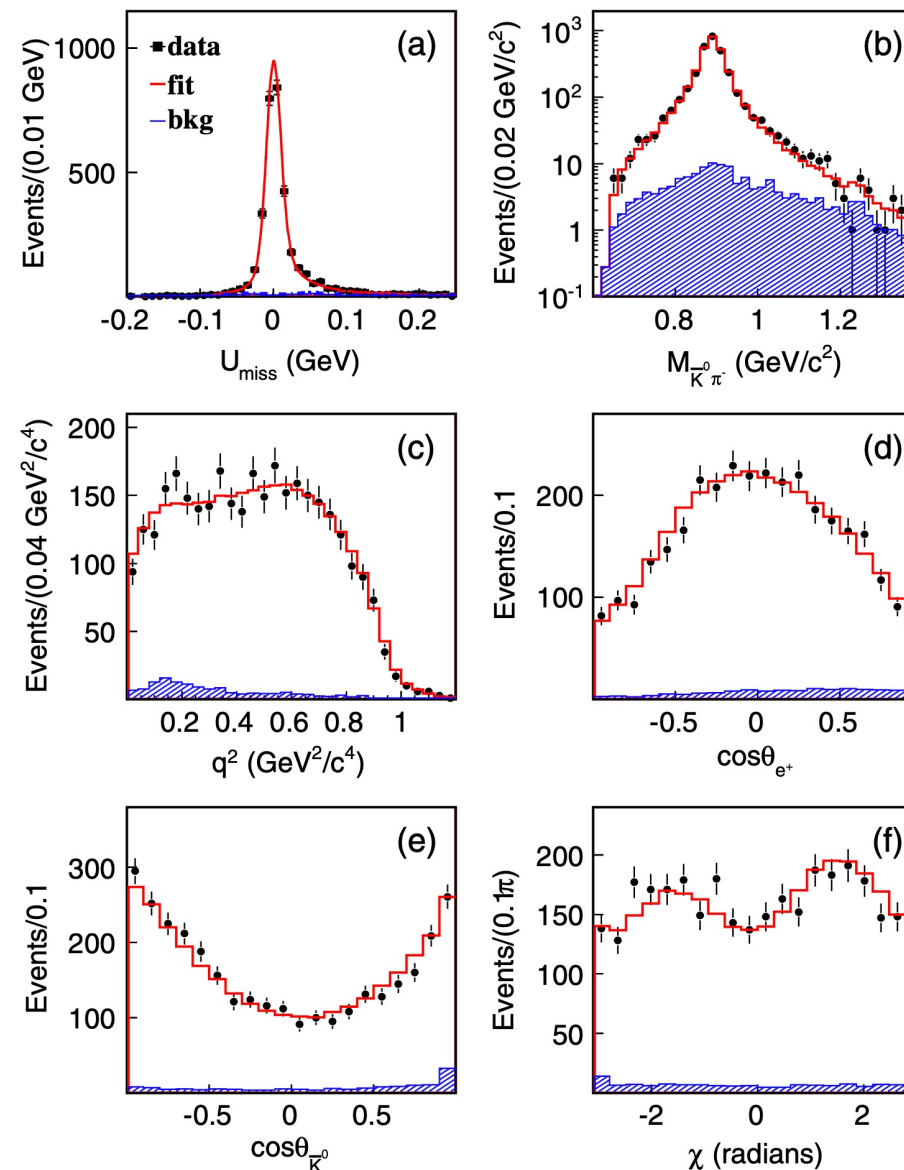




c -> s 类型半轻衰变: $D^0 \rightarrow K^{*-} e^+ \nu_e$

Phys. Rev. D 99, 0111003(R)(2019)

- $N_{sig} = 3112 \pm 64$ (本底水平: 0.6%)
- $\mathcal{B}(D^0 \rightarrow \bar{K}^0 \pi^+ e^+ \nu_e) = (1.434 \pm 0.029 \pm 0.032)\%$
- $f_{S\text{-wave}} = (5.51 \pm 0.97 \pm 0.62)\%$
- 首次测量形状因子:
 $r_V = 1.46 \pm 0.07 \pm 0.02,$
 $r_2 = 0.67 \pm 0.06 \pm 0.01$

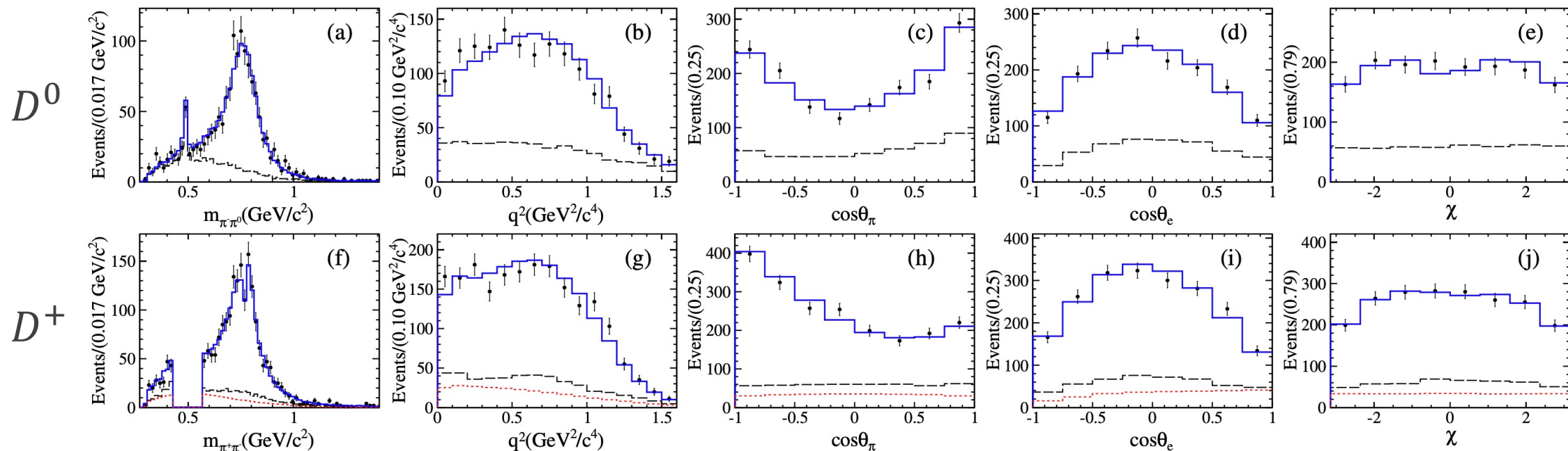




c -> s 类型半轻衰变到矢量介子末态

衰变过程	实验/年份/信号数	分支比/形状因子/极化比例	BESIII研究现状
$D_s^+ \rightarrow \phi(K^+K^-)e^+\nu_e$	BESIII/2018/26	Y/N/N	内审中@4.13 ~ 4.23GeV
	CLEO/2015/207	Y/N/N	
	BABAR/2008/25k	Y/Y/N	
	CLEO-II/1994/308	N/Y/Y	
$D_s^+ \rightarrow \phi(K^+K^-)\mu^+\nu_\mu$	BESIII/2018/22	Y/N/N	内审中@4.13 ~ 4.23GeV
	FOCUS/2004/793	N/Y/N	
	E687/1994/90	N/Y/Y	
$D^+ \rightarrow \bar{K}^{*0}(K^-\pi^+)e^+\nu_e$	BESIII/2016/18k	Y/Y/N	✓
	BABAR/2011/70k	Y/Y/N	
	CLEO/2010/5k	Y/Y/N	
$D^+ \rightarrow \bar{K}^{*0}(K^-\pi^+)\mu^+\nu_\mu$	CLEO/2010/5k	Y/Y/N	进行中
	FOCUS/2002/15k	N/Y/N	
$D^+ \rightarrow \bar{K}^{*0}(\bar{K}^0\pi^0)e^+\nu_e$	N	N/N/N	进行中
$D^+ \rightarrow \bar{K}^{*0}(\bar{K}^0\pi^0)\mu^+\nu_\mu$	N	N/N/N	进行中
$D^0 \rightarrow K^{*-}(K^-\pi^0)e^+\nu_e$	CLEO/2005/94	Y/N/N	进行中
$D^0 \rightarrow K^{*-}(K^-\pi^0)\mu^+\nu_\mu$	N	N/N/N	进行中
$D^0 \rightarrow K^{*-}(\bar{K}^0\pi^-)e^+\nu_e$	BESIII/2019/3k	Y/Y/N	✓
	CLEO/2005/125	Y/N/N	
$D^0 \rightarrow K^{*-}(\bar{K}^0\pi^-)\mu^+\nu_\mu$	FOCUS/2005/175	Y/Y/N	进行中

Phys. Rev. Lett. 122, 062001 (2019)



- $R = \frac{\mathcal{B}(D^+ \rightarrow f_0(500)e^+\nu_e) + \mathcal{B}(D^+ \rightarrow f_0(980)e^+\nu_e)}{\mathcal{B}(D^+ \rightarrow a_0(980)e^+\nu_e)} > 2.7 @ 90\% CL$
- 支持 f_0 和 a_0 为四夸克态 ($R = 3$, PRD82, 034016(2010))
- $r_V = 1.695 \pm 0.083 \pm 0.051$, $r_2 = 0.845 \pm 0.056 \pm 0.039$

Signal mode	This analysis ($\times 10^{-3}$)
$D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e$	$1.445 \pm 0.058 \pm 0.039$
$D^0 \rightarrow \rho^- e^+ \nu_e$	$1.445 \pm 0.058 \pm 0.039$
$D^+ \rightarrow \pi^- \pi^+ e^+ \nu_e$	$2.449 \pm 0.074 \pm 0.073$
$D^+ \rightarrow \rho^0 e^+ \nu_e$	$1.860 \pm 0.070 \pm 0.061$
$D^+ \rightarrow \omega e^+ \nu_e$	$2.05 \pm 0.66 \pm 0.30$
$D^+ \rightarrow f_0(500)e^+\nu_e, f_0(500) \rightarrow \pi^+\pi^-$	$0.630 \pm 0.043 \pm 0.032$
$D^+ \rightarrow f_0(980)e^+\nu_e, f_0(980) \rightarrow \pi^+\pi^-$	< 0.028

Phys. Rev. D 104, L091103 (2021)

- $N_{sig} = 570 \pm 40$
- $\mathcal{B}(D^0 \rightarrow \rho^- \mu^+ \nu_\mu) = (1.35 \pm 0.09 \pm 0.09) \times 10^{-3}$
与多个理论计算结果一致
(LFQM, CCQM, and LCSR methods).
- $\mathcal{R}_{\mu/e} = 0.90 \pm 0.11$ **vs** SM: 0.93-0.96 -> No LFUV

Y. L. Wu, M. Zhong, and Y. B. Zuo, Int. J. Mod. Phys. A 21, 6125 (2006)

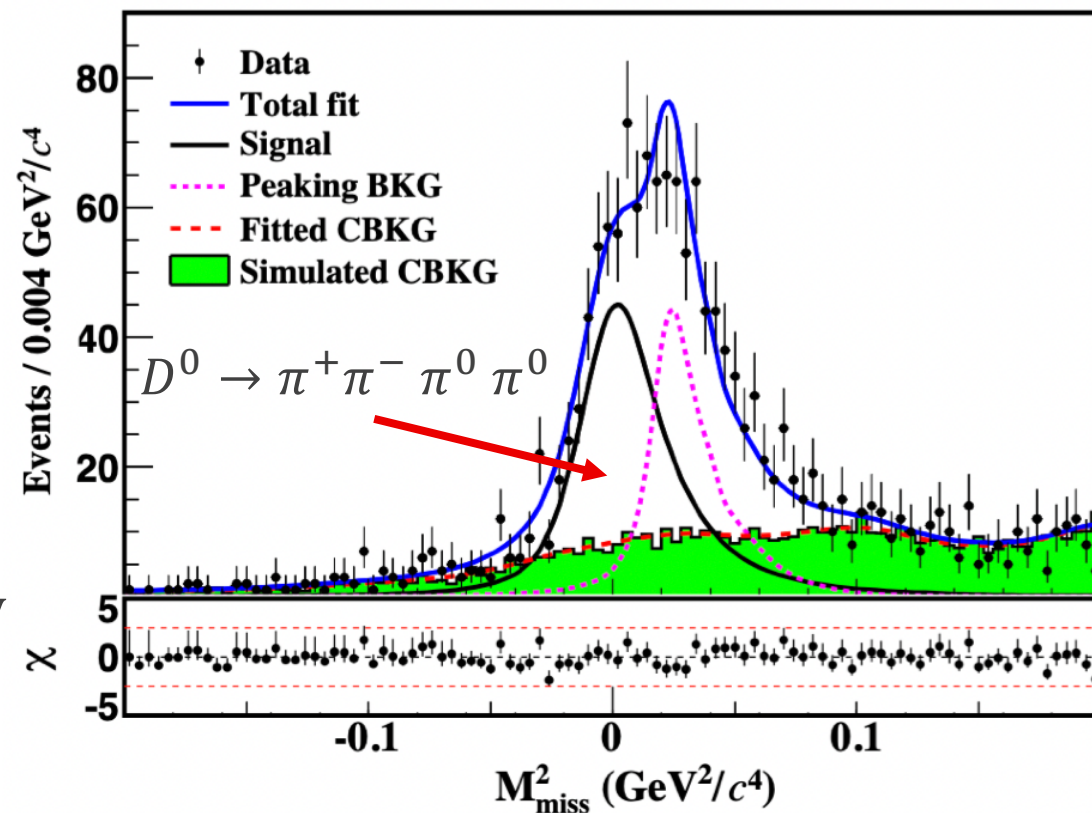
T. Sekihara and E. Oset, Phys. Rev. D92, 054038 (2015)

N. R. Soni, M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, and C. T. Tran, Phys. Rev. D98, 114031 (2018)

M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C. T. Tran, Front. Phys.14, 64401 (2019)

H. Y. Cheng and X. W. Kang, Eur. Phys. J. C77, 587(2017);77, 863(E) (2017)

R. N. Faustov, V. O. Galkin, and X. W. Kang, Phys. Rev. D101, 013004 (2020)





Phys. Rev. D 101, 072005 (2020)

➤ $N_{sig} = 194 \pm 20$

➤ $\mathcal{B}(D^+ \rightarrow \omega \mu^+ \nu_\mu) = (17.7 \pm 1.8 \pm 1.1) \times 10^{-4}$

与多个理论计算结果一致

(LFQM, CCQM, and LCSR methods).

➤ $\mathcal{R}_{\mu/e} = 1.05 \pm 0.14$ **vs** SM: 0.93-0.99 -> No LFUV



H. Y. Cheng and X. W. Kang, Eur. Phys. J. C77, 587(2017);77, 863(E) (2017)

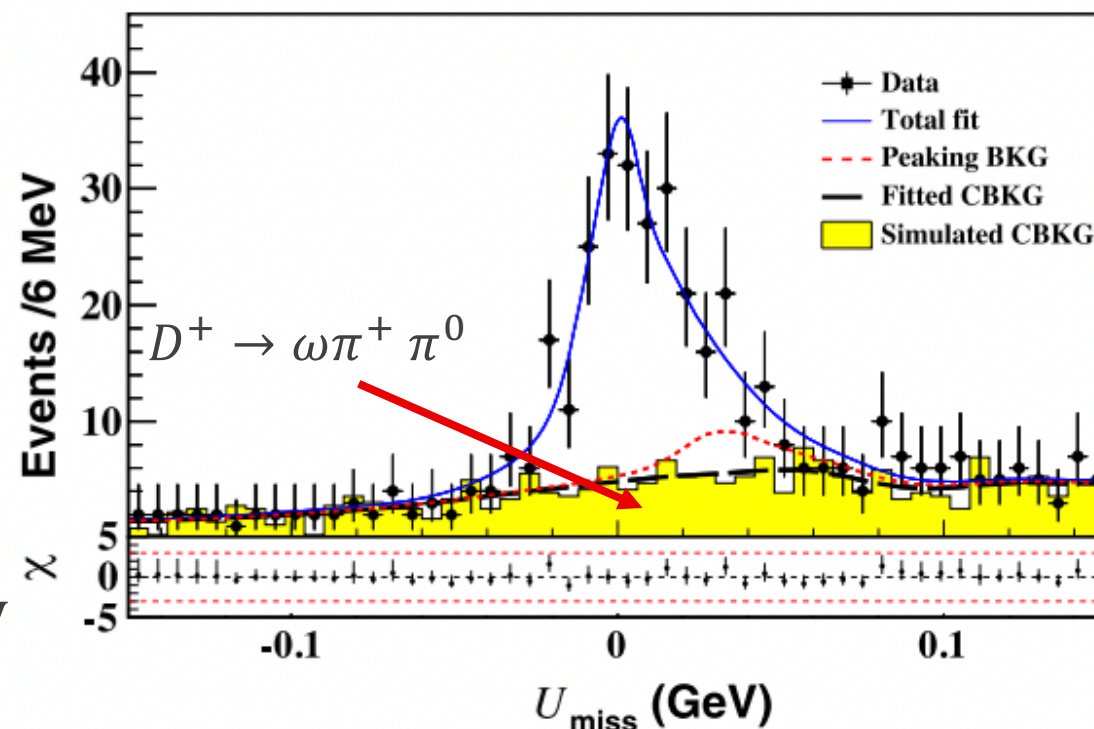
T. Sekihara and E. Oset, Phys. Rev. D92, 054038 (2015)

N. R. Soni, M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, and C. T. Tran, Phys. Rev. D98, 114031 (2018)

M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C. T. Tran, Front. Phys.14, 64401 (2019)

H.B. Fu, W. Cheng, L. Zheng, D.D. Hu, T. Zhong, Phys. Rev. Research 2, 043129 (2020)

R. N. Faustov, V. O. Galkin, and X. W. Kang, Phys. Rev. D101, 013004 (2020)





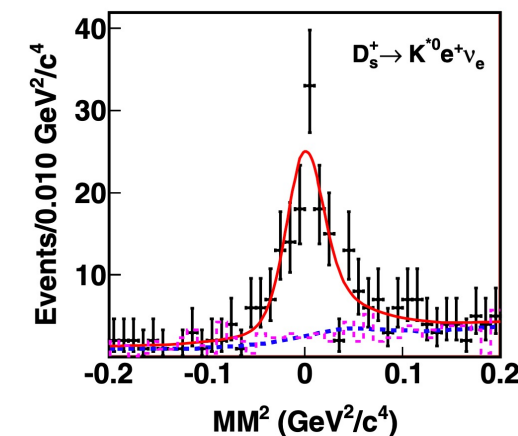
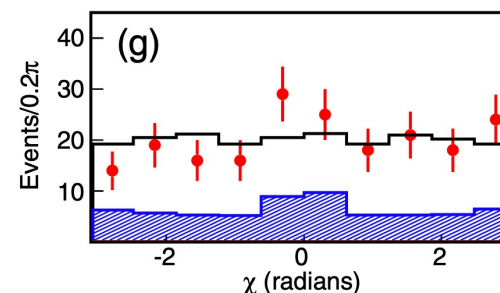
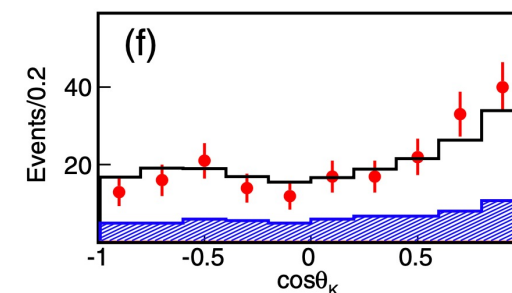
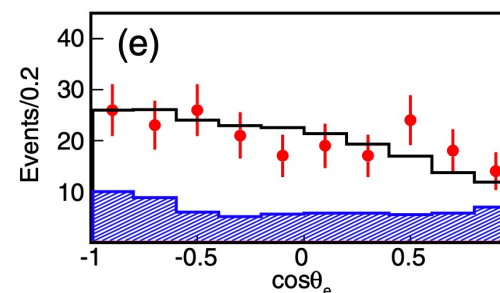
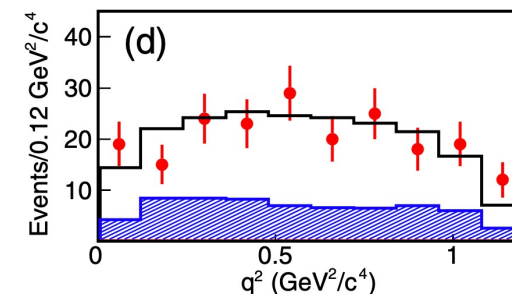
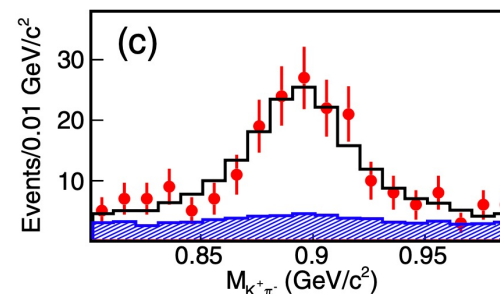
c -> d 类型半轻衰变: $D_s^+ \rightarrow K^{*0} e^+ \nu_e$

Phys. Rev. Lett. 122, 061801 (2019)

- 采用数据@4.18 GeV
- $N_{sig}(D_s^+ \rightarrow K^{*0} e^+ \nu_e) = 155.0 \pm 17.2$
- $\mathcal{B}(D_s^+ \rightarrow K^{*0} e^+ \nu_e) = (2.37 \pm 0.26 \pm 0.20) \times 10^{-3}$
- 首次测量形状因子：
 $r_V = 1.67 \pm 0.34 \pm 0.16, r_2 = 0.77 \pm 0.28 \pm 0.07$
- 符合LQCD和 U-spin ($d \leftrightarrow s$) 对称性的预期:

Use **BESIII** and **CLEO** measurement

	Values
$f_+^{D_s^+ \rightarrow K^0}(0)/f_+^{D^+ \rightarrow \pi^0}(0)$	$1.16 \pm 0.14 \pm 0.02$
$r_V^{D_s^+ \rightarrow K^{*0}}/r_V^{D^+ \rightarrow \rho^0}$	$1.13 \pm 0.26 \pm 0.11$
$r_2^{D_s^+ \rightarrow K^{*0}}/r_2^{D^+ \rightarrow \rho^0}$	$0.93 \pm 0.36 \pm 0.10$



利用阈值处粲介子对产生和半轻衰变的独特优势， BESIII实验对 $D_{(s)}$ 介子半轻衰变到矢量介子末态 (ρ, ω, K^*) 开展了一系列高精度测量的研究。

- 通过双标记方法测量绝对分支比
- 通过振幅分析测量强子化形状因子等物理参数
- 检验轻子味道普适性
- 更多衰变过程（尤其是 μ 子道）的研究正在BESIII上开展

总结与展望

➤ 预期未来1~2年收集数据:

20 fb⁻¹ @ 3.773 GeV

➤ 目前(截止2022.6.20):

2.93 fb⁻¹ + 5.0 fb⁻¹

BESIII go after Best!

Thank you!

	BESIII	BESIII	Belle	Belle II
Luminosity	2.9 fb ⁻¹ @3.773 GeV	20 fb ⁻¹ @3.773 GeV	0.28 ab ⁻¹	50 ab ⁻¹
$D^0 \rightarrow K^- e^+ \nu_e$	0.4% _{stat.} 0.5% _{syst.}	0.2% _{stat.} 0.4% _{syst.}	1.0% _{stat.} 3.2% _{syst.} *	0.1% _{stat.} 1.6% _{syst.} *
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	0.5% _{stat.} 0.4% _{syst.}	0.2% _{stat.} 0.4% _{syst.}		
$D^0 \rightarrow \pi^- e^+ \nu_e$	1.3% _{stat.} 0.7% _{syst.}	0.5% _{stat.} 0.4% _{stat.}	3.2% _{stat.} 4.8% _{syst.} *	0.2% _{stat.} 2.4% _{syst.} *
$D^0 \rightarrow \pi^- \mu^+ \nu_\mu$	NA	0.8% _{stat.} 0.8% _{syst.}		
$D^0 \rightarrow K^{*-} e^+ \nu_e$				
r_V	5.0% _{stat.} 2.0% _{syst.}	2.0% _{stat.} 2.0% _{syst.}	—	—
r_A	10.% _{stat.} 2.0% _{syst.}	4.0% _{stat.} 2.0% _{syst.}	—	—
$D^0 \rightarrow a_0^-(980) e^+ \nu_e$	NA	10.% _{stat.} 5.0% _{syst.}	—	—
$D^0 \rightarrow K_1^-(1270) e^+ \nu_e$	NA	10.% _{stat.} 5.0% _{syst.}	—	—
$D^+ \rightarrow \bar{K}^0 e^+ \nu_e$	0.6% _{stat.} 1.7% _{syst.}	0.2% _{stat.} 1.0% _{syst.}	—	—
$D^+ \rightarrow \bar{K}_1^0 e^+ \nu_e$	0.9% _{stat.} 1.6% _{syst.}	0.4% _{stat.} 1.0% _{syst.}	—	—
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	NA	0.3% _{stat.} 1.0% _{syst.}	—	—
$D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$				
$A_1(0)$	1.7% _{stat.} 2.0% _{syst.}	0.7% _{stat.} 1.0% _{syst.}	—	—
r_V	4.0% _{stat.} 0.5% _{syst.}	1.6% _{stat.} 0.5% _{syst.}	—	—
r_A	5.0% _{stat.} 1.0% _{syst.}	2.0% _{stat.} 1.0% _{syst.}	—	—
$D^+ \rightarrow \pi^0 e^+ \nu_e$	1.9% _{stat.} 0.5% _{syst.}	0.7% _{stat.} 0.5% _{syst.}	—	—
$D^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	NA	1.0% _{stat.} 1.0% _{syst.}	—	—
$D^+ \rightarrow \eta e^+ \nu_e$	4.5% _{stat.} 2.0% _{syst.}	2.0% _{stat.} 2.0% _{syst.}	—	—
$D^+ \rightarrow \eta' e^+ \nu_e$	NA	10.% _{stat.} 5.0% _{syst.}	—	—
$D^+ \rightarrow \omega e^+ \nu_e$				
r_V	7.2% _{stat.} 4.8% _{syst.}	3.0% _{stat.} 2.0% _{syst.}	—	—
r_A	14% _{stat.} 5.0% _{syst.}	3.0% _{stat.} 2.0% _{syst.}	—	—
$D^+ \rightarrow a_0^0(980) e^+ \nu_e$	NA	10.% _{stat.} 5.0% _{syst.}	—	—
$D^+ \rightarrow \bar{K}_1^0(1270) e^+ \nu_e$	NA	10.% _{stat.} 5.0% _{syst.}	—	—
$D^{0(+)} \rightarrow \rho^{-(0)} e^+ \nu_e$				
r_V	5.0% _{stat.} 4.0% _{syst.}	2.0% _{stat.} 2.0% _{syst.}	—	—
r_A	8.0% _{stat.} 4.0% _{syst.}	3.0% _{stat.} 2.0% _{syst.}	—	—

$D^0 \rightarrow K^{*-} e^+ \nu_e$ (Backup)

$$d^5\Gamma = \frac{G_F^2 \|V_{cs}\|^2}{(4\pi)^6 m_D^3} X \beta \mathcal{I}(m^2, q^2, \theta_K, \theta_e, \chi) \times dm^2 dq^2 d\cos(\theta_K) d\cos(\theta_e) d\chi$$

Where $X = p_{K\pi} m_D$ and $\beta = 2p^*/m$, $p_{K\pi}$ is the momentum of the $K^-\pi^0$ system in the rest D^0 system and p^* is the momentum of K^- in the $K^-\pi^0$ rest frame

$$\begin{aligned} \mathcal{I} = & \mathcal{I}_1 + \mathcal{I}_2 \cos 2\theta_e + \mathcal{I}_3 \sin^2 \theta_e \cos 2\chi + \mathcal{I}_4 \sin 2\theta_e \cos \chi + \mathcal{I}_5 \sin \theta_e \cos \chi \\ & + \mathcal{I}_6 \cos \theta_e + \mathcal{I}_7 \sin \theta_e \sin \chi + \mathcal{I}_8 \sin 2\theta_e \sin \chi + \mathcal{I}_9 \sin^2 \theta_e \sin 2\chi \end{aligned}$$

$$\mathcal{I}_1 = \frac{1}{4} \{ |\mathcal{F}_1|^2 + \frac{3}{2} \sin^2 \theta_k (|\mathcal{F}_2|^2 + |\mathcal{F}_3|^2) \}$$

$$\mathcal{I}_2 = -\frac{1}{4} \{ |\mathcal{F}_1|^2 - \frac{1}{2} \sin^2 \theta_k (|\mathcal{F}_2|^2 + |\mathcal{F}_3|^2) \}$$

$$\mathcal{I}_3 = -\frac{1}{4} \{ |\mathcal{F}_2|^2 - |\mathcal{F}_3|^2 \} \sin^2 \theta_K$$

$$\mathcal{I}_4 = \frac{1}{2} \text{Re}(\mathcal{F}_1^* \mathcal{F}_2) \sin \theta_K$$

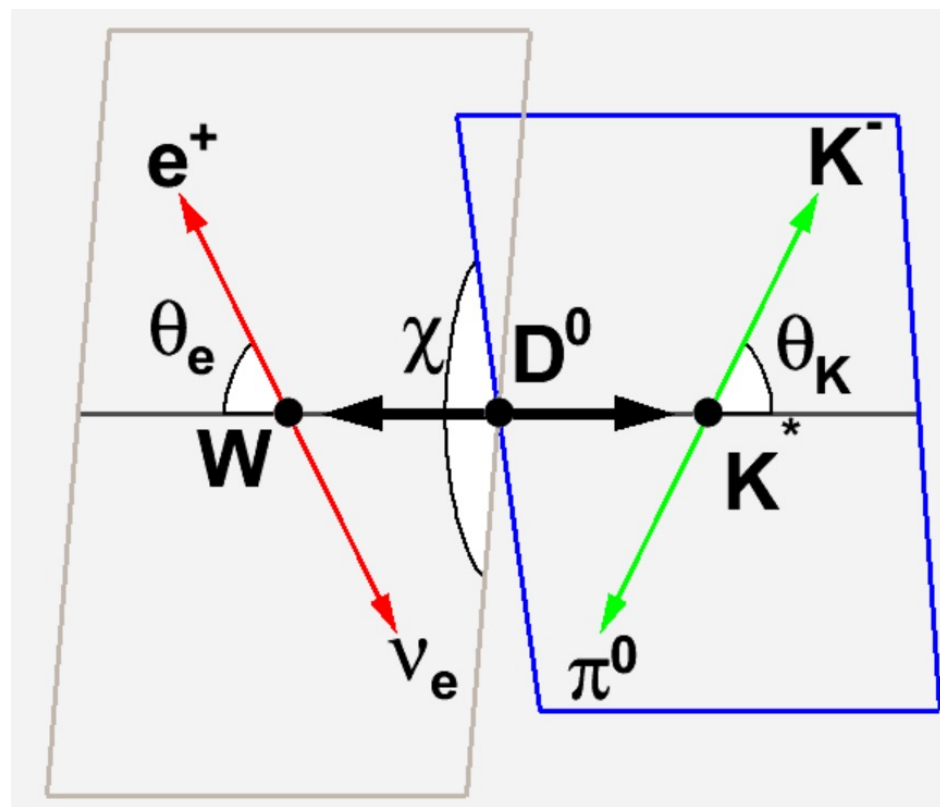
$$\mathcal{I}_5 = \text{Re}(\mathcal{F}_1^* \mathcal{F}_3) \sin \theta_K$$

$$\mathcal{I}_6 = \text{Re}(\mathcal{F}_2^* \mathcal{F}_3) \sin^2 \theta_K$$

$$\mathcal{I}_7 = \text{Im}(\mathcal{F}_1 \mathcal{F}_2^*) \sin \theta_K$$

$$\mathcal{I}_8 = \frac{1}{2} \text{Im}(\mathcal{F}_1 \mathcal{F}_3^*) \sin \theta_K$$

$$\mathcal{I}_9 = -\frac{1}{2} \text{Im}(\mathcal{F}_2 \mathcal{F}_3^*) \sin^2 \theta_K$$





$D^0 \rightarrow K^{*-} e^+ \nu_e$ (Backup)

$$\mathcal{F}_1 = \mathcal{F}_{10} + \mathcal{F}_{11} \cos \theta_K, \mathcal{F}_2 = \frac{1}{\sqrt{2}} \mathcal{F}_{21}, \mathcal{F}_3 = \frac{1}{\sqrt{2}} \mathcal{F}_{31}$$

$$\mathcal{F}_{11} = 2\sqrt{2}\alpha q H_0 \times \mathcal{A}(m), \mathcal{F}_{21} = 2\alpha q (H_+ + H_-) \times \mathcal{A}(m), \mathcal{F}_{31} = 2\alpha q (H_+ - H_-) \times \mathcal{A}(m)$$

$$H_{\pm}(q^2) = (M_D + M_{K\pi}) A_1(q^2) \mp \frac{2M_D p_{K\pi}}{M_D + M_{K\pi}} V(q^2)$$

$$H_0(q^2) = \frac{1}{2M_{K\pi} q} [(M_D^2 - M_{K\pi}^2 - q^2)(M_D + M_{K\pi}) A_1(q^2) - \frac{4M_D^2 p_{K\pi}^2}{M_D + M_{K\pi}} A_2(q^2)]$$

$$\mathcal{F}_{10} = p_{K\pi} m_D \frac{1}{1 - \frac{q^2}{m_A^2}} \mathcal{A}_S(m)$$