$B \to D^*$ form factors beyond leading power

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Motivations

- Understanding strong interaction dynamics in heavy hadron system
  - Factorization property
  - Higher order (α_s) impact
  - Renormalization and asymptotic properties of the (higher-twist) B-meson DAs
  - Interplay of different QCD techniques
- Understanding the general properties of power expansion in EFTs
- Extracting $|V_{cb}|$ exclusively (inclusive vs. exclusive)
- Testing $R(D^*)$, ΔA_{FB} $\cdots$

State of art of $B \to D^{(*)}$ FF

- Heavy-quark expansion [Isgur/Wise 89' 90']
  - Power corrections at $1/m_Q$ and $1/m_Q^2$ [Luke 90', Neubert 92', Ball 92', Mannel 94' ; Falk/Neubert 92' 93']
  - QCD corrections at $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ [Gounaris 83' 84', Shifman 88'; Neubert 92', Czarnecki 96', 97']
  - QCDSR for subleading I-W func [Ligeti/Nir/Neubert 92' 92' 93']
  - QED correction at $\mathcal{O}(\alpha)$ [Sirlin 82']
- LCSR with B-meson LCDAs [Faller *et al.* 09']
  - LCSR at $\mathcal{O}(\alpha_s)$ [Wang *et al.* 16']
  - LCSR with higher-twist contributions [Gubernari *et al.* 19']
  - LCSR NLL + 3 NLP [Gao *et al.*21']
- Parametrization [BGL 97'; CLN 98']
- Lattice QCD [FNAL/MILC04' 08' 13' 14' 15' 21'; HPQCD15' 17' 17' 20' 21'; UKQCD 19'; JLQCD ?]
- Experiment [Belle 16' 17' 19' 20'; BarBar 08' 09' 10']

Leading power factorization formulae

- Factorization formulae for semileptonic decay at leading power

$$F_i^{B\to D^*}(n\cdot p) = C_i^{(\mathrm{A0})}(n\cdot p)\,\xi_a(n\cdot p) + \int d\tau\, C_i^{(\mathrm{B1})}(\tau, n\cdot p)\,\Xi_a(\tau, n\cdot p)\,,(a = \parallel, \perp)$$

- Hard matching coefficients $C^{(A0)}$
  - One-loop perturbative calculations [Bauer/Fleming/Pirjol/Stewart 01'; Beneke/Kiyo/Yang 04']
  - Two-loop perturbative calculations [Bonciani/Ferroglia 08'; Asatrian/Greub/Pecjak 08'; Beneke/Huber/Li 09'; Bell 09'; Bell/Beneke/Huber/Li 11']
- Hard matching coefficients $C^{(B1)}$
  - One-loop perturbative calculations [Becher/Hill 04'; Hill/Becher/Lee/Neubert 04'; Beneke/Yang 06']
- Hard-collinear matching coefficients $J^{(A0)}_{\parallel(\perp),-(+)}, J^{(B1)}_{\parallel(\perp),+}$
  - This work (power counting: $m_c \sim \mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$)
- B-meson LCDAs [KKQT 01'; Braun,Ji,Manashov 17'; Beneke/Braun/Ji/Wei 18']
- NLL resummation for C, J, ϕ_B

Subleading Power Corrections

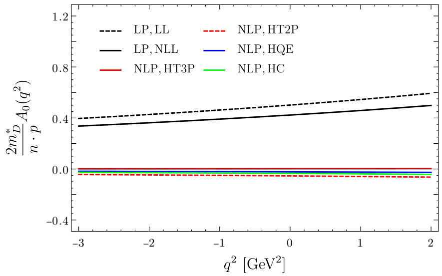
- Higher-twist B-meson LCDAs including both 2-& 3-particle (up to twist-6)
- HQET representation of QCD b-quark field

$$\bar{q}\,\gamma_\mu\, b \to \bar{q}\,\gamma_\mu\, h_v + \frac{1}{2m_b}\,\bar{q}\,\gamma_\mu\, i\vec{\not{D}} h_v + \mathcal{O}(\frac{1}{m_b^2}) + \cdots$$

- Hard-collinar charm quark propagator

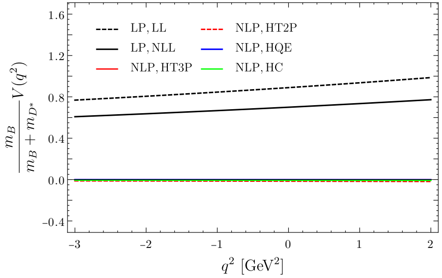
$$\frac{\not{p} - \not{k} + m_c}{(p-k)^2 - m_c^2} \to \underbrace{\frac{1}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}}_{\text{LP}} + \underbrace{\frac{1}{n\cdot p\bar{n}\cdot\hat{p}}\left[\bar{n}\cdot p\frac{\not{n}}{2} - \not{k} + \frac{n\cdot k\bar{n}\cdot p}{\bar{n}\cdot\hat{p}}\frac{\not{\bar{n}}}{2}\right]}_{\text{NLP}} + \underbrace{\frac{m_c}{n\cdot p}\frac{1}{\bar{n}\cdot\hat{p}}\left[1 + \frac{n\cdot k\bar{n}\cdot p}{n\cdot p\bar{n}\cdot\hat{p}}\right]}_{m_c\ \text{NLP}}$$

Predictions for form factors



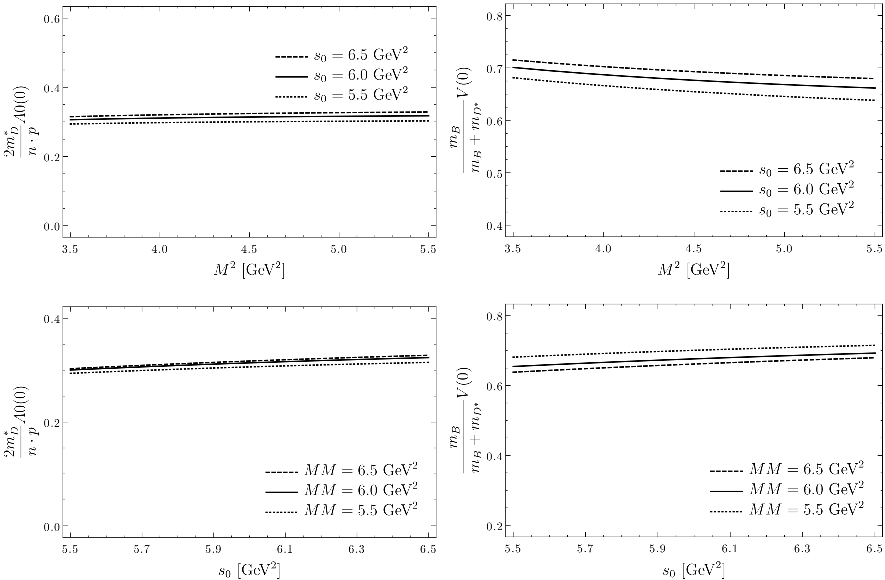
At $q^2 = 0$

- NLL $\sim$ -15.6%
- HT3P $\sim$ 0.4%
- HT2P $\sim$ -10.8%
- HQE $\sim$ -4.5%
- HC $\sim$ -6.9%



- NLL $\sim$ -21.3%
- HT3P $\sim$ -0.0%
- HT2P $\sim$ -0.17%
- HQE $\sim$ -0.0%
- HC $\sim$ -0.7%

Dependence on the M^2 and s_0



BGL parametrization

- Parametrization

$$
\begin{aligned}
g(z) &= \frac{1}{P_{1^-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n z^n , \\
f(z) &= \frac{1}{P_{1^+}(z)\phi_f(z)} \sum_{n=0}^{\infty} b_n z^n , \\
\mathcal{F}_1(z) &= \frac{1}{P_{1^+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} c_n z^n , \\
\mathcal{F}_2(z) &= \frac{1}{P_{0^-}(z)\phi_{\mathcal{F}_2}(z)} \sum_{n=0}^{\infty} d_n z^n
\end{aligned}
$$

$$
P(z) = \prod_{P=1}^{n} \frac{z - z_P}{1 - z z_p}, \qquad z_P = \frac{\sqrt{t_+ - m_P^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - m_P^2} + \sqrt{t_+ - t_0}}
$$

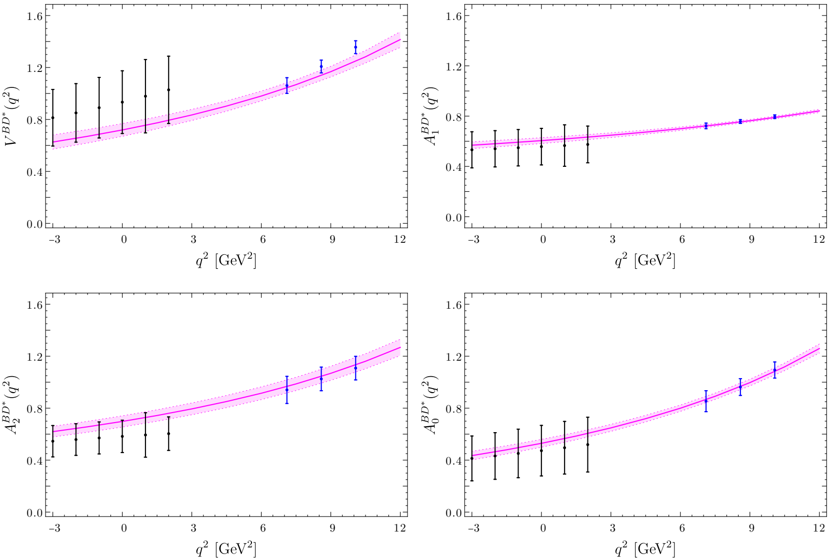
- Constrains

$$
\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0) \qquad \mathcal{F}_1(z = z_{max}) = \frac{1+r}{(1-r) m_B^2 (1+\omega)} \mathcal{F}_2(z = z_{max})
$$

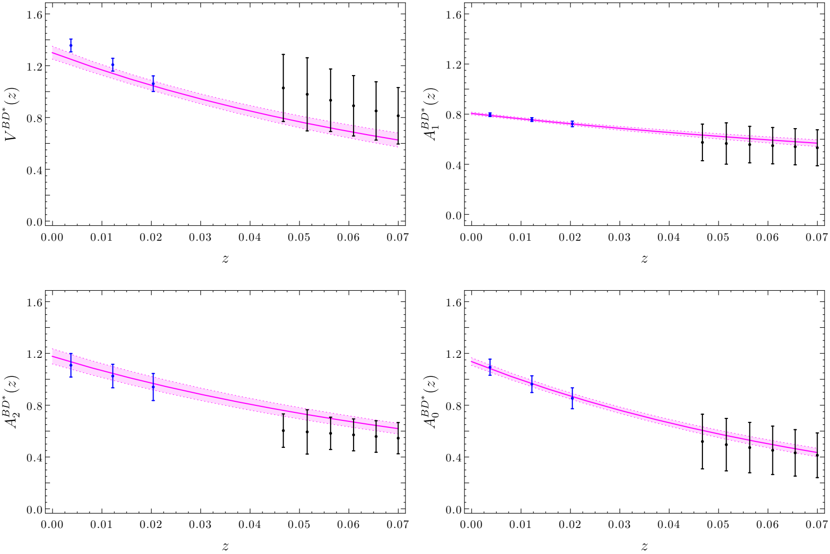
- (Weak) Unitary bound

$$
\sum_0^{\infty} a_n^2 < 1 \qquad \sum_0^{\infty} b_n^2 + c_n^2 < 1 \qquad \sum_0^{\infty} d_n^2 < 1
$$

Joint fit with FNAL/MILC



Joint fit with FNAL/MILC

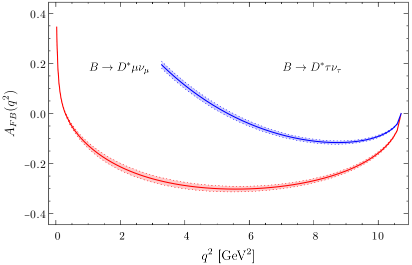


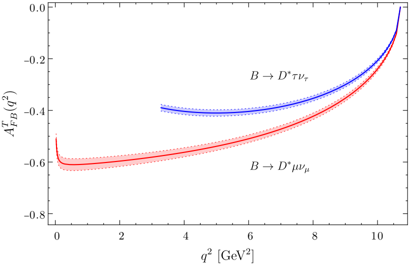
Results from joint fit

q^2 [GeV^2]	-3.0	-2.0	-1.0	0.0	1.0	2.0
A_0	0.413(172)	0.432(179)	0.452(187)	0.473(194)	0.495(203)	0.520(211)
A_1	0.532(144)	0.540(144)	0.549(145)	0.557(145)	0.566(164)	0.575(146)
A_2	0.546(121)	0.558(122)	0.571(123)	0.583(125)	0.594(171)	0.604(129)
V	0.813(218)	0.851(225)	0.891(233)	0.933(241)	0.979(282)	1.028(259)

		correlation matrix					
	value	a_0	a_1	b_0	b_1	c_1	d_1
a_0	0.0305(12)	1	-0.4673	0.0904	-0.0409	0.0395	-0.1570
a_1	$-0.0632(366)$		1	-0.0476	0.2351	0.1468	0.1986
b_0	0.0124(2)			1	-0.2547	-0.2545	-0.3396
b_1	0.0014(87)				1	0.5990	0.6753
c_1	0.0061(18)					1	0.8699
d_1	$-0.4411(678)$						1

Forward-backward asymmetry

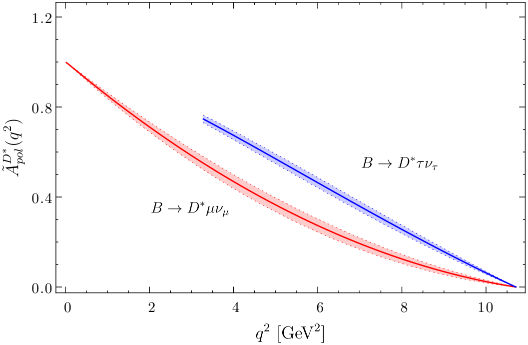




$$A_{FB}(q^2) = \frac{3q^2(H_+^2 - H_-^2) - 6m_\ell^2 H_0 H_t}{2m_\ell^2(H_0^2 + 3H_t^2 + H_+^2 H_-^2) + 4q^2(H_0^2 + +H_+^2 H_-^2)}$$

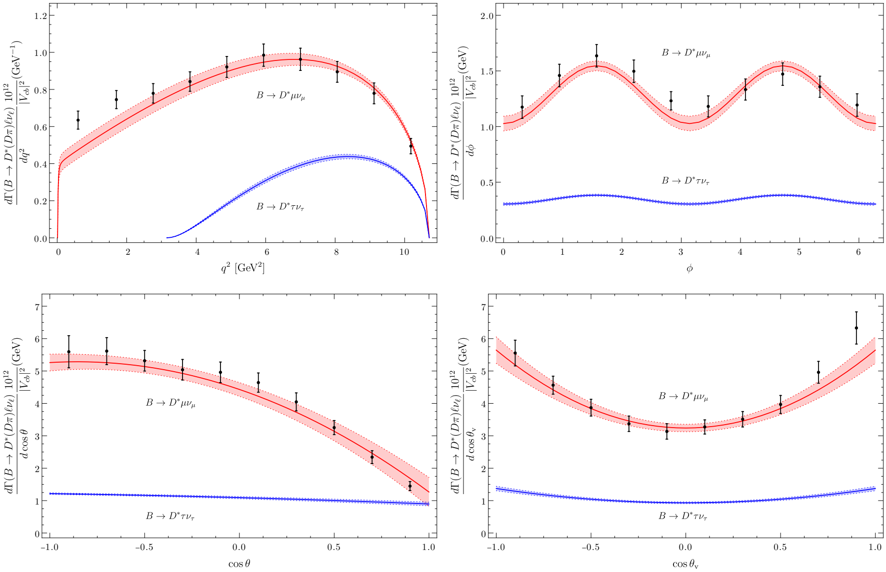
$$A^T_{FB}(q^2) = \frac{3q^2(H_+^2 - H_-^2)}{(m_\ell^2 + 2q^2)(H_+^2 H_-^2)}$$

D^* polarization asymmetry

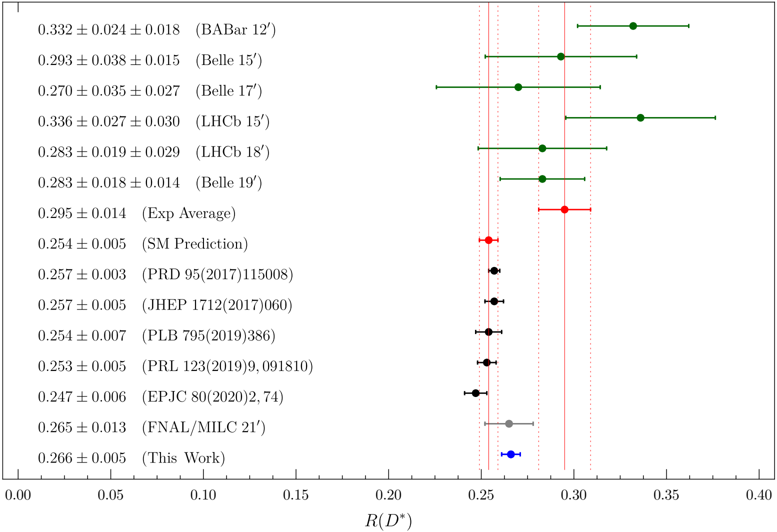


$$A^{D^*}_{\rm pol}(q^2) = 1 - \frac{(m_\ell^2 + 2q^2)(H_+^2 + H_-^2)}{6m_\ell^2 H_t^2 + 2(m_\ell^2 + 2q^2)H_0^2}$$

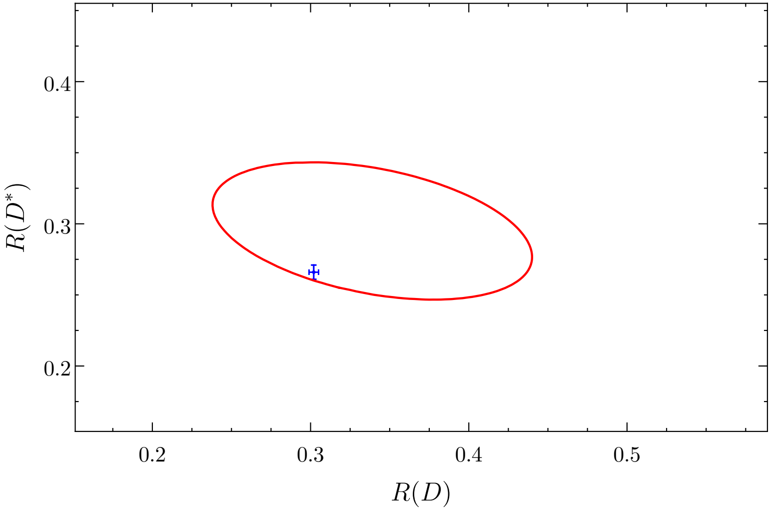
Differential decay rates and EXP from BELLE 17'



Lepton flavor universality $R(D^*)$



$R(D) - R(D^*)$ & HFLAV averaged (Babar 12',15', Belle 19') 3σ



Extracting $|V_{cb}|$

- BaBar 07'

$$|V_{cb}| = \left(44.2^{+1.1}_{-1.0}\Big|_{\rm th}\ {}^{+1.2}_{-1.2}\Big|_{\rm exp}\right) \times 10^{-3}$$

- Belle 20'

$$|V_{cb}| = \left(40.1^{+1.0}_{-0.9}\Big|_{\rm th}\ {}^{+0.3}_{-0.3}\Big|_{\rm exp}\right) \times 10^{-3}$$

Summary and outlook

- Leading power
  - Factorization formulae
  - NLO calculations for hard-collinear matching Coe
- Subleading power
  - Two-& Three-particle higher-twist
  - HQET Rep. of b-quark field
  - Hard-collinear charm-quark propagator
- Joint fit LCSR at small q^2 and Lattice at large q^2
- Observales
  - $d\Gamma/d[q^2, \cos\theta_D, \cos\theta_\ell, \phi]$, A_{FB}, A_{FB}^T, $A_{pol}^{D^*}$, $R(D^*)$
  - Extracting $|V_{cb}|$
- Same operation for $B_s \to D_s^* \ell\nu$ with HPQCD 21'

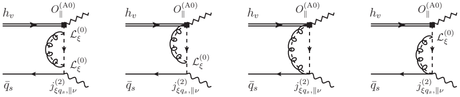
Outlook:

- Theoretical correlation between diff FF, q^2, processes
- Joint fit with Belle17' (40-Bin)

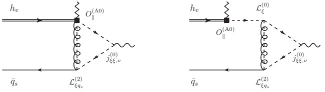
Thank You

Back-up

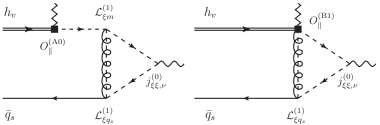
Diagrammatic representations

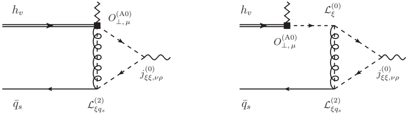


(a) (b) (c) (d)



(a) (b)





(a) (b)

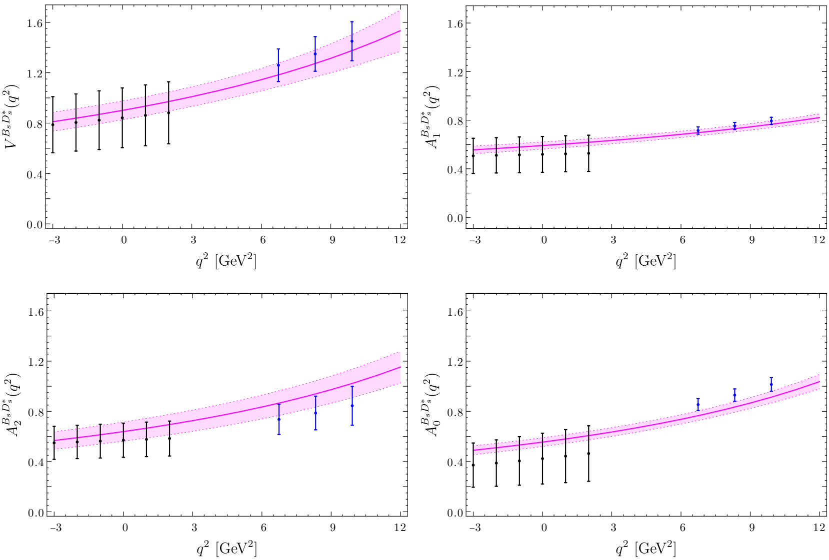
Angular distribution

$$\begin{aligned}
\frac{d^4\Gamma(\bar{B}\to D^*(\to D\pi)\ell^-\bar{\nu}_\ell)}{dq^2\,d\cos\theta\,d\phi\,d\cos\theta_V} &= \mathcal{N}_\pi|\vec{p}_{D^*}|\left(1-\frac{m_\ell^2}{q^2}\right)^2\Big\{I_{1s}^\pi\,\sin^2\theta_V+I_{1c}^\pi\,\cos^2\theta_V \\
&+ \left(I_{2s}^\pi\,\sin^2\theta_V+I_{2c}^\pi\,\cos^2\theta_V\right)\cos 2\theta \\
&+ I_3^\pi\,\sin^2\theta_V\,\sin^2\theta\cos 2\phi+I_4^\pi\,\sin 2\theta_V\,\sin 2\theta\cos\phi \\
&+ I_5^\pi\,\sin 2\theta_V\,\sin\theta\cos\phi+\left(I_{6s}^\pi\,\sin^2\theta_V+I_{6c}^\pi\,\cos^2\theta_V\right)\cos\theta \\
&+ I_7^\pi\sin 2\theta_V\,\sin\theta\sin\phi\Big\}\ ,
\end{aligned}$$

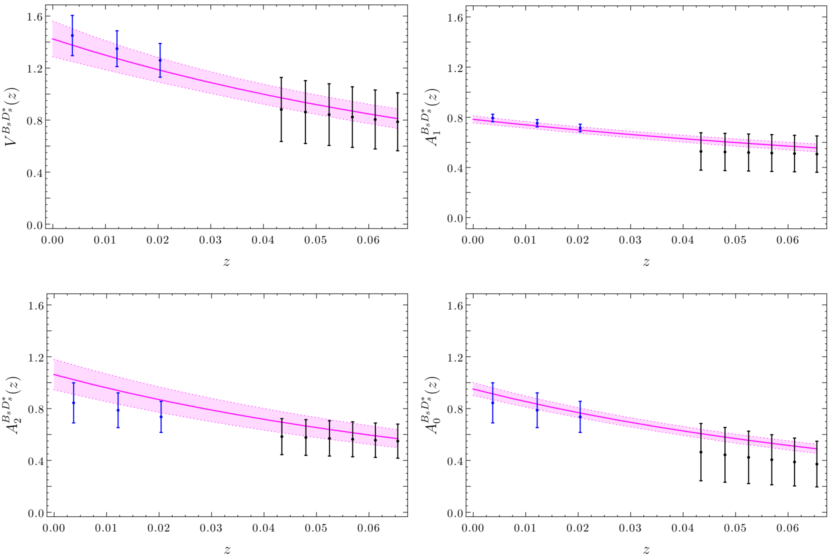
with $\mathcal{N}_\pi=\dfrac{3G_F^2|V_{cb}|^2\mathcal{B}(D^*\to D\pi)}{128(2\pi)^4m_B^2}$, and coefficients of the angular terms are related to the helicity amplitudes

$$\begin{aligned}
I_{1s}^\pi &= \frac{1}{2}(H_+^2+H_-^2)(m_\ell^2+3q^2)\ , & I_{1c}^\pi &= 2(2m_\ell^2H_t^2+H_0^2(m_\ell^2+q^2))\ , \\
I_{2s}^\pi &= \frac{1}{2}(H_+^2+H_-^2)(q^2-m_\ell^2)\ , & I_{2c}^\pi &= 2H_0^2(m_\ell^2-q^2)\ , \\
I_3^\pi &= 2H_+H_-(m_\ell^2-q^2)\ , & I_4^\pi &= H_0(H_++H_-)(m_\ell^2-q^2)\ , \\
I_5^\pi &= -2(H_++H_-)H_tm_\ell^2-2H_0(H_+-H_-)q^2\ , & & \\
I_{6s}^\pi &= 2(H_+^2-H_-^2)q^2\ , & I_{6c}^\pi &= -8H_0H_tm_\ell^2\ , \\
I_7^\pi &= 0 & &
\end{aligned}$$

Joint fit with HPQCD ($B_s \to D_s^* \ell\nu$)



Joint fit with HPQCD ($B_s \to D_s^* \ell \nu$)



Normalized Differential decay rates and exp from LHCb 20'