Three-loop QCD corrections to quarkonium electroweak decays

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- Matching the Short Distance Coefficients
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- Phenomenology
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Motivation

Motivation

- Quarkonium leptonic decays are fundamental processes in high energy physics experiments.
- Theoretically, It plays an important role of probing the decay constant, which is a basic nonperturbative parameter.
- Previous works: Vector quakonium leptonic decay
  - Tree: Van Royen, Weisskopf, Nuovo Cim, 1967.
  - One loop: Barbieri, R. Gatto, et al., PLB1975; Celmaster, PRD1979.
  - Two loops: Beneke, Signer, Smirnov, PRL1998; Czarnecki, Melniko, PRL1998; Kniehl, Onishchenko, et al., PLB2006; Egner, Fael, et al., PRD2021.
  - Three loops: Marquard, Piclum, et al., NPB2006, PLB2009, PRD2014; Beneke, Kiyo, et al., PRL2014 without singlet and charm mass effect.

  B_c meson:
  - One loop: Braaten, Fleming, PRD1995.
  - Two loops: Onishchenko, Veretin, EPJC2007; Chen, Qiao, PLB2015

In our work, we calculate the complete three-loop correction for the Υ decay constant with massive charm and singlet/indirect contributions, and new three-loop correction to the B_c decay constant.

Matching the Short Distance Coefficients (SDCs)

Fractorization of the decay constants

NRQCD Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\text{NRQCD}} =& \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L} \\
\mathcal{L}_{\text{light}} =& \frac{1}{2}\mathrm{tr}\, G_{\mu\nu}^2 + \sum \bar{q}\mathrm{i}\not{D} q, \\
\mathcal{L}_{\text{heavy}} =& \psi^\dagger \left(\mathrm{i}D_t + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger \left(\mathrm{i}D_t - \frac{\mathbf{D}^2}{2M}\right)\chi \\
\delta\mathcal{L}_{\text{bilinear}} =& \frac{c_1}{8M^3}\psi^\dagger \left(\mathbf{D}^2\right)^2 \psi + \frac{c_2}{8M^2}\psi^\dagger \left(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D}\right)\psi \\
&+ \frac{c_3}{8M^2}\psi^\dagger \left(\mathrm{i}\mathbf{D}\times g\mathbf{E} - \mathrm{i}g\mathbf{E}\times\mathbf{D}\right)\psi + \frac{c_4}{2M}\psi^\dagger \left(g\mathbf{B}\cdot\boldsymbol{\sigma}\right)\psi \\
&+ \text{charge conjugation terms}
\end{aligned}
$$

Fractorization of the decay constants

The leptonic decay constants f_V for vector quarkonia $V = J/\psi, \Upsilon$ and pseudoscalar B_c are given by

$$\langle 0|\mathcal{J}^{\mu}_{\rm EM}|V(\epsilon)\rangle = M_V f_V \epsilon^{\mu}_V, \quad \mathcal{J}^{\mu}_{\rm EM} = \sum_f e_f \bar{\Psi}_f \gamma^{\mu} \Psi_f$$

$$\langle 0|\bar{b}\gamma^{\mu}\gamma_5 c|B_c^+(P)\rangle = iP^{\mu} f_{B_c}$$

According to the NRQCD factorization,

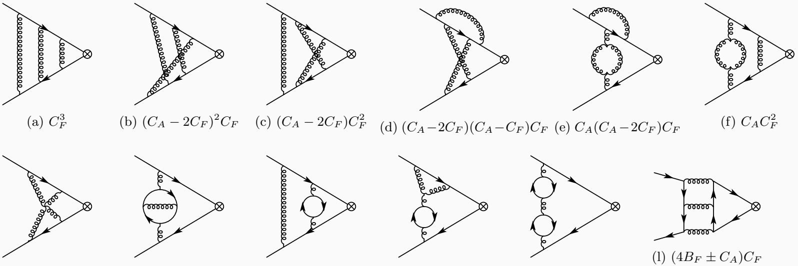
$$\langle 0|\mathcal{J}^{i}_{\rm EM}|V(\epsilon)\rangle = \sqrt{2M_V} e_Q \left(\mathcal{C}_{\rm dir} + \sum_{f\neq Q} \mathcal{C}_{\rm ind,f} \frac{e_f}{e_Q}\right) \langle 0|\chi^{\dagger}\sigma^{i}\psi|V(\epsilon)\rangle_{\rm NR} + \mathcal{O}(v^2)$$

$$\langle 0|\bar{b}\gamma^{\mu}\gamma_5 c|B_c^+\rangle = \sqrt{2M_{B_c}}\mathcal{C}_0 \langle 0|\chi_b^{\dagger}\psi_c|B_c^+\rangle_{\rm NR} + \mathcal{O}(v^2)$$

The SDCs can be obtained by matching the on-shell quark-antiquark vertex function

$$\boxed{\sqrt{Z_{2,1}Z_{2,2}}\Gamma = \sqrt{2M}\mathcal{C}_0(\mu_\Lambda)\sqrt{\tilde{Z}_{2,1}\tilde{Z}_{2,2}}\tilde{Z}^{-1}(\mu_\Lambda)\tilde{\Gamma} + \mathcal{O}\left(v^2\right)}$$

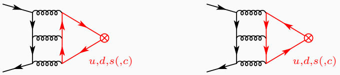
Feynman diagrams



(a) C_F^3 (b) $(C_A-2C_F)^2C_F$ (c) $(C_A-2C_F)C_F^2$ (d) $(C_A-2C_F)(C_A-C_F)C_F$ (e) $C_A(C_A-2C_F)C_F$ (f) $C_AC_F^2$

(g) $C_A^2C_F$ (h) $(C_A-2C_F)C_F$ (i) C_AC_F (j) C_F^2 (k) C_F (l) $(4B_F\pm C_A)C_F$

Figure 1: Representative diagrams of the direct channel. The color factor B_F is defined as $\sum_{bc} d^{abc}d^{ebc} = 4B_F\delta^{ae}$ and $B_F = (N_c^2-4)/(4N_c)$ for $SU(N_c)$ group.



(a) $(4B_F+C_A)\,C_F$ (b) $(4B_F-C_A)\,C_F$

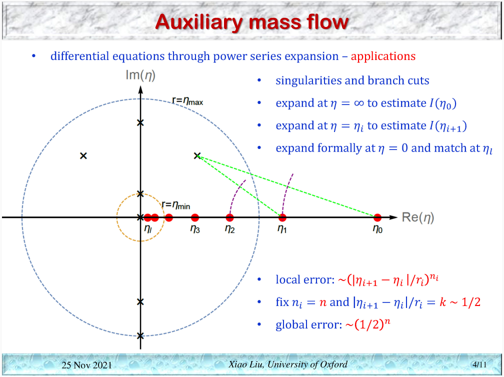
Figure 2: Representative diagrams of the indirect channel.

Tool chain

The more than 300 Feynman diagrams are generated by the packages `QGraf`/`FeynArts` for crosscheck. `FormLink`/`FeynCalc` are then utilized to deal with Dirac and color matrices. After applying partial fraction by `Apart` and IBP reduction by `FIRE`, we get roughly 300 master integrals.

The master integrals are evaluated by the auxiliary mass flow method implemented as the package `AMFlow` [arXiv: 1711.09572, 2201.11669].

See Yan-Qing Ma's talk



SDCs

$$\mathcal{C}_0\left(\frac{\mu_R}{m_Q},\frac{\mu_\Lambda}{m_Q},x\right)=$$

$$1+\frac{\alpha_s(\mu_R)}{\pi}\mathcal{C}^{(1)}(x)+\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^2\left[\mathcal{C}^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_Q^2}+\gamma^{(2)}\ln\frac{\mu_\Lambda^2}{m_Q^2}+\mathcal{C}^{(2)}(x)\right]$$

$$+\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^3\left\{\frac{\mathcal{C}^{(1)}}{16}\beta_0^2\ln^2\frac{\mu_R^2}{m_Q^2}+\left[\frac{\mathcal{C}^{(1)}}{16}\beta_1+\mathcal{C}^{(2)}(x)\frac{\beta_0}{2}\right]\ln\frac{\mu_R^2}{m_Q^2}\right.$$

$$+\gamma^{(2)}\frac{\beta_0}{2}\ln\frac{\mu_\Lambda^2}{m_Q^2}\ln\frac{\mu_R^2}{m_Q^2}+\frac{1}{4}\left[2\frac{\mathrm{d}\gamma^{(3)}(\mu_\Lambda)}{\mathrm{d}\ln\mu_\Lambda^2}-\beta_0\gamma^{(2)}\right]\ln^2\frac{\mu_\Lambda^2}{m_Q^2}$$

$$\left.+\left[\mathcal{C}^{(1)}\gamma^{(2)}+\gamma^{(3)}(m_Q)\right]\ln\frac{\mu_\Lambda^2}{m_Q^2}+\mathcal{C}^{(3)}(x)\right\}+\mathcal{O}\left(\alpha_s^4\right).$$

$$x=m_M/m_H,$$

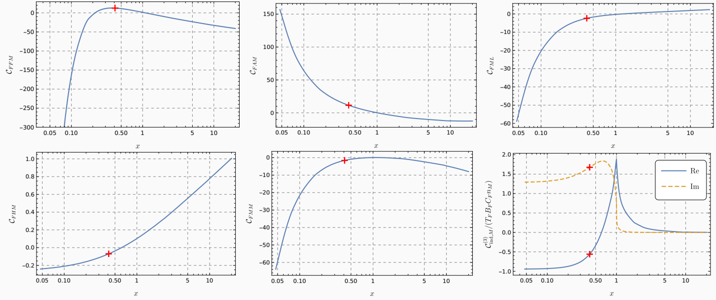
$$\begin{aligned}\mathcal{C}_{\text{dir}}^{(3)}=&C_F\left[C_F^2\mathcal{C}_{FFF}+C_FC_A\mathcal{C}_{FFA}+C_A^2\mathcal{C}_{FAA}\right.\\&+T_Fn_L\left(C_F\mathcal{C}_{FFL}+C_A\mathcal{C}_{FAL}+T_Fn_H\mathcal{C}_{FHL}+T_Fn_M\mathcal{C}_{FML}+T_Fn_L\mathcal{C}_{FLL}\right)\\&+T_Fn_H\left(C_F\mathcal{C}_{FFH}+C_A\mathcal{C}_{FAH}+T_Fn_H\mathcal{C}_{FHH}+T_Fn_M\mathcal{C}_{FHM}+B_F\mathcal{C}_{BFH}\right)\\&\left.+T_Fn_M\left(C_F\mathcal{C}_{FFM}+C_A\mathcal{C}_{FAM}+T_Fn_M\mathcal{C}_{FMM}\right)\right].\end{aligned}$$

SDCs of J/ψ and Υ decay constants

With 4 active flavors in α_s and β_i, terms that are independent of x are given by

$$
\begin{aligned}
\mathcal{C}_{FFF} =& 36.4948624588059253763347618987279203166418 1,\\
\mathcal{C}_{FFA} =& -188.077841659880713905799940232784764503891 05,\\
\mathcal{C}_{FAA} =& -97.734973269918386342345245004574098439887181,\\
\mathcal{C}_{FFL} =& 46.691692905515132467558267641260536017779126774,\\
\mathcal{C}_{FAL} =& 39.623718554524419077342047422053477518698120476 7,\\
\mathcal{C}_{FHL} =& -0.27025043915650217173213869139777864792399772 1,\\
\mathcal{C}_{FLL} =& -2.4683364544823741163705418765248618965896838 6,\\
\mathcal{C}_{FFH} =& -0.843562291159500145305509373641959358579825 2,\\
\mathcal{C}_{FAH} =& -0.10247416149293174085748359719938021201631 06,\\
\mathcal{C}_{FHH} =& 0.0512396075119837249349311858899964136984463561 7,\\
\mathcal{C}_{BFH} =& 2.11557826798090649843682222191394437004433 56\\
& +\mathrm{i}\,0.4942127107006720402412181080201603811552204 87.
\end{aligned}
$$

Terms that are dependent of x. Red crosses are the physical values with three-loop pole masses of quarks $m_Q \equiv m_H \equiv m_b = 4.98$ GeV, $m_M \equiv m_c = 2.04$ GeV:



SDCs of B_c decay constant

$m_H \equiv m_b = 4.98$ GeV, $m_M \equiv m_c = 2.04$ GeV, $m_Q = \sqrt{m_b m_c}$, with 3 active flavors in α_s and β_i.

$$
\begin{aligned}
\mathcal{C}_{FFF} &= -17.64812525464175353 9131, & \mathcal{C}_{FFA} &= -192.15179822434790874 7121,\\
\mathcal{C}_{FAA} &= -106.55700074027885859242, & \mathcal{C}_{FFL} &= 53.59088238032099883 98528,\\
\mathcal{C}_{FAL} &= 40.04194395562570772 8391, & \mathcal{C}_{FCL} &= -0.59955659588604920607755,\\
\mathcal{C}_{FBL} &= -0.05567360504047408860700, & \mathcal{C}_{FLL} &= -1.32484367522413099859707,\\
\mathcal{C}_{FBC} &= 0.15047037340977620584792, & \mathcal{C}_{FFC} &= 4.46892700776466970 1991,\\
\mathcal{C}_{FAC} &= -0.90391224294954408 74057, & \mathcal{C}_{FCC} &= 0.18738217573423910690057,\\
\mathcal{C}_{FFB} &= 1.97991279879730446 94123, & \mathcal{C}_{FAB} &= -0.72105476302894669 43049,\\
\mathcal{C}_{FBB} &= 0.03474911743391490676344.
\end{aligned}
$$

Analytic Anomalous Dimensions

Renormalization constant of NRQCD operators and anomalous dimensions

Thanks to the extremely high precision of the results generated by `AMFlow`, one can reconstruct the analytic expressions for the non-renormalized poles of the SDCs to infer the renormalization constant of the NRQCD operator and corresponding anomalous dimension.

$$\tilde{Z} \equiv 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \tilde{Z}^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \tilde{Z}^{(3)} + \mathcal{O}\left(\alpha_s^4\right),$$

$$\gamma \equiv \frac{\mathrm{d}\ln \tilde{Z}}{\mathrm{d}\ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right).$$

We implement the Thiele's interpolation formula to reconstruct rational functions and use PSLQ algorithm to speculate transcendental functions.

Renormalization constant of the NRQCD current for Υ

$$
\begin{aligned}
\tilde{Z}_v =& 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{C_F \pi^2}{\epsilon}\left(\frac{1}{12}C_F + \frac{1}{8}C_A\right) + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 C_F \pi^2 \\
& \times \left\{ C_F^2 \left[\frac{5}{144\epsilon^2} + \left(\frac{43}{144} - \frac{1}{2}\ln 2 + \frac{5}{48}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right]\right. \\
& + C_F C_A \left[\frac{1}{864\epsilon^2} + \left(\frac{113}{324} + \frac{1}{4}\ln 2 + \frac{5}{32}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + C_A^2 \left[-\frac{1}{16\epsilon^2} + \left(\frac{2}{27} + \frac{1}{4}\ln 2 + \frac{1}{24}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + T_F n_L \left[C_F\left(\frac{1}{54\epsilon^2} - \frac{25}{324\epsilon}\right) + C_A\left(\frac{1}{36\epsilon^2} - \frac{37}{432\epsilon}\right)\right] \\
& \left. + T_F n_H \frac{C_F}{60\epsilon} + T_F n_M \frac{1}{\epsilon}\left[\frac{C_F m_Q^2}{60 m_M^2} - \left(\frac{C_F}{18} + \frac{C_A}{12}\right)\ln\frac{\mu_\Lambda^2}{m_M^2}\right]\right\} \\
& + \mathcal{O}\left(\alpha_s^4\right).
\end{aligned}
$$

Renormalization constant of the NRQCD current for B_c

$$\tilde{Z}_p = 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{\pi^2 C_F}{\epsilon} \left(\frac{3+z}{8(1+z)} C_F + \frac{1}{8} C_A\right) + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 Z_p^{(3)}$$

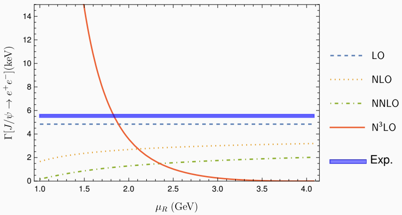
where $z = \frac{1}{2}\left(x + \frac{1}{x}\right)$. $Z_p^{(3)}$ to appear in arXiv:2207.xxxxx.

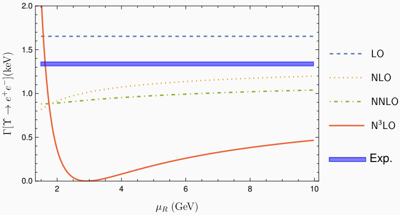
Phenomenology

Leptonic decay width of J/ψ and Υ

$\Gamma(\text{keV})$ / V	LO	NLO	NNLO	N^3LO			PDG
				Direct ($m_M = 0$)	Direct ($m_M \neq 0$)	Total	
Υ	1.6529	$1.1095^{+0.0888}_{-0.2922}$	$0.9750^{+0.0642}_{-0.0942}$	$0.1948^{+1.5900}_{-0.1948}$	$0.1763^{+1.9577}_{-0.1763}$	$0.1764^{+1.9560}_{-0.1764}$	1.340 ± 0.018
J/ψ	4.8392	$2.6999^{+0.4925}_{-1.0391}$	$1.3138^{+0.7094}_{-1.1444}$	$3.2219^{+123.4838}_{-3.2219}$			5.53 ± 0.10

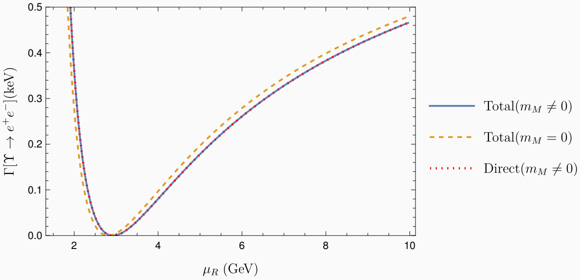
Table 1: Decay width for J/ψ and Υ. The central values of predictions are obtained by setting $\mu_R = m_Q$, while the errors are estimated by varying μ_R from μ_Λ to $2m_Q$.





Charm mass effect in Υ decay

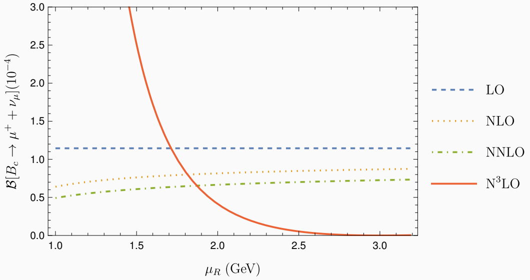
Our complete result is ploted in solid line. The dashed line treats charm quark as massless. The dotted line takes out the contribution from the indirect diagrams. It's shown that indirect channel only has invisible effect on the plot, while charm mass leads to visible small correction.



Leptonic decay width of B_c

$\mathcal{B}(B_c \to \mu^+ + \nu_\mu) \times 10^{-4}$	LO	NLO	NNLO	N^{3}LO
B_c	1.1448	$0.75395^{+0.12070}_{-0.11445}$	$0.60025^{+0.13321}_{-0.10820}$	$3.0872^{+25.070}_{-3.0832}$

Table 2: Branching ratio of the B_c leptonic decay. The uncertainties are given by varying renormalization scale from factorization scale to $m_q = \sqrt{m_b m_c}$. Central value is chosen at $m_r = \frac{m_b m_c}{m_b + m_c}$.



Conclusions

Conclusions

- We confirm the known results of SDCs of Υ, J/ψ and B_c leptonic decays. Furthermore, our results of indirect and charm mass effect to the three-loop QCD correction to Υ decay is new. Our treatment of the phenomenology is different from Beneke, Kiyo, et al., PRL2014. Our three-loop results for B_c is brand new.
- A new term in the Υ NRQCD anomalous dimension corresponding to the intermediate flavor of quark is first obtained. The three-loop anomalous dimension for B_c is completely new.
- Our three-loop results for Υ and B_c share similar patterns that N^3LO corrections heavily depend on the renormalization scale and can grow very large and spoil the convergence of the perturbative expansion. This phenomenon may draw forth a new puzzle which deserves further research.

Thank you for your attention

Backup

Decoupling relation:

$$
\frac{\alpha_s^{(n_L+n_M+n_H)}(\mu_R)}{\pi} = \frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}
$$

$$
+ \left(\frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}\right)^2 T_F n_H \left[\frac{1}{3}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \left(\frac{1}{6}\ln^2\frac{\mu_\Lambda}{m_Q} + \frac{1}{36}\pi^2\right)\epsilon + \mathcal{O}\left(\epsilon^2\right)\right]
$$

$$
+ \left(\frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}\right)^3 T_F n_H \Bigg[\left(\frac{1}{4}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \frac{15}{16}\right) C_F + \left(\frac{5}{12}\ln\frac{\mu_\Lambda^2}{m_Q^2} - \frac{2}{9}\right) C_A
$$

$$
+ \frac{1}{9} T_F n_H \ln^2\frac{\mu_\Lambda}{m_Q} + \mathcal{O}(\epsilon)\Bigg] + \mathcal{O}\left(\alpha_s^4\right)
$$

Backup

Phenomenology of Beneke, Kiyo, et al., PRL2014:

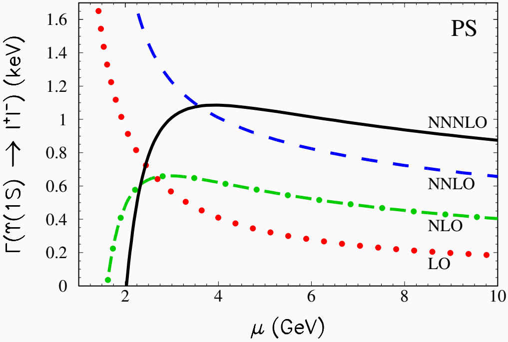


Figure 1: Caption