强子物理中的逆散射问题：Compositeness和CDD零点

吴佳俊（中国科学院大学）

合作者：李严，郭奉坤，庞锦毅

Phys.Rev.D 105 (2022) 11, 116024

Phys.Rev.D 105 (2022) 7, L071502

第四届重味物理与量子色动力学研讨会　湖南大学　2022.7.29

University of Chinese Academy of Sciences

目录

介绍

- 散射问题：已知势能，求解散射T矩阵拟合数据。

QCD $\not\longrightarrow$ 势能　　**模型假设**

- 逆散射问题：已知散射T矩阵，求解势能。

相当于从数据出发确定势能。

介绍

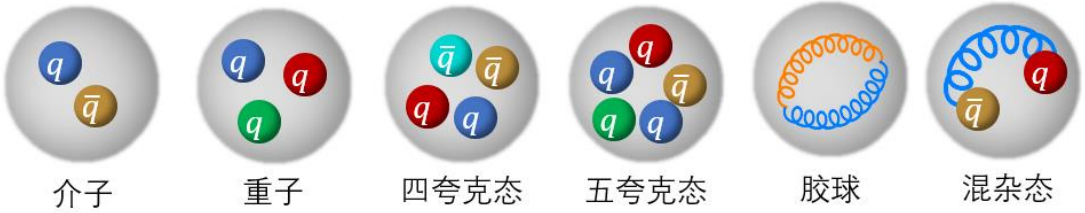
- 逆散射问题：已知散射T矩阵，求解势能。
- 逆散射问题是一个比较古老的问题，大家可以参考如下文献。

  K. Chadan, P. C. Sabatier, and R. G. Newton, Inverse Problems in Quantum Scattering Theory (Springer Berlin Heidelberg, Berlin, Heidelberg, 1989).

- 逆散射问题的关键问题是通常情况下无法唯一确定势能。往往有很多不同解同时存在。

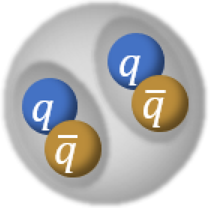
介绍

- 强子的结构是强子物理的核心问题。
- 大体上，强子的形成有两种源头。
- 一种是在夸克和胶子层次上形成了一个束缚态。



需要QCD理论来描述它们

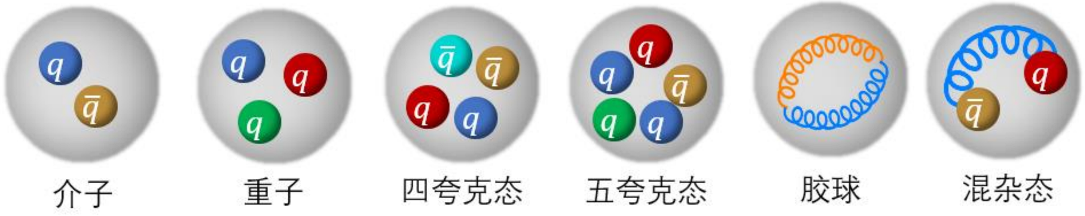
- 另一种是在强子层次上形成一个束缚态。
  无色集团通过吸引相互作用形成的束缚态。



强子分子态

介绍

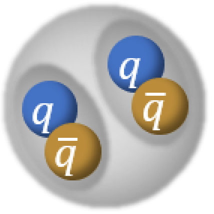
- 强子的结构是强子物理的核心问题。
- 大体上，强子的形成有两种源头。
- 一种是在夸克和胶子层次上形成了一个束缚态。



需要QCD理论来描述它们

裸态

- 另一种是在强子层次上形成一个束缚态。
  无色集团通过吸引相互作用形成的束缚态。



多强子耦合道

含有裸态的逆散射问题

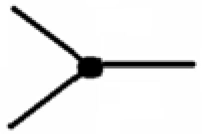
$$H = H_0 + V = H_0 + \hat{g} + \hat{v}$$

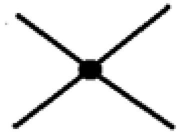
$$H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p|$$

$$\hat{g} = \int \frac{p^2 dp}{(2\pi)^3} \lambda_b \left[|p\rangle f(p) \langle b| + |b\rangle f(p) \langle p| \right]$$

$$\hat{v} = \int \frac{p^2 dp}{(2\pi)^3} \frac{k^2 dk}{(2\pi)^3} \lambda_c \left[|p\rangle \tilde{f}(p,k) \langle k| \right]$$

逆散射问题比较复杂，我们现阶段只考虑了只含有**一个分波**，且含有**单个裸态**和**单个耦合道**的过程





缺点是模型的假设，分离势能。优点没有对形状因子的具体形式进行假定。

这里我们介绍两种情况的通解形式

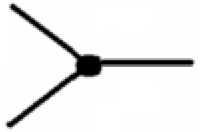
1. $\lambda_c = 0$，即忽略耦合道的相互作用。
2. $\lambda_c \neq 0$，但是 $\tilde{f}(p,k) = f(p)f(k)$，即形状因子相同。

含有裸态的逆散射问题

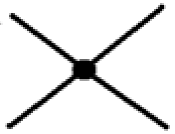
$$H = H_0 + V = H_0 + \hat{g} + \hat{v}$$

$$H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p|$$

$$\hat{g} = \int \frac{p^2 dp}{(2\pi)^3} \lambda_b \left[|p\rangle f(p) \langle b| + |b\rangle f(p) \langle p| \right] = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$



$$\hat{v} = \int \frac{p^2 dp}{(2\pi)^3} \frac{k^2 dk}{(2\pi)^3} \lambda_c \left[|p\rangle \tilde{f}(p,k) \langle k| \right] = \lambda_c |f\rangle\langle f|$$



逆散射问题比较复杂，我们现阶段只考虑了只含有**一个分波**，且含有**单个裸态**和**单个耦合道**的过程

缺点是模型的假设，**分离势能**。优点没有对形状因子的具体形式进行假定。

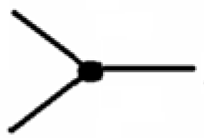
$$|f\rangle \equiv \int \frac{p^2 dp}{(2\pi)^3} f(p) |p\rangle$$

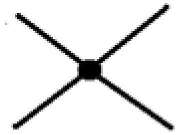
这里我们介绍两种情况的通解形式

1. $\lambda_c = 0$，即忽略耦合道的相互作用。
2. $\lambda_c \neq 0$，但是 $\tilde{f}(p,k) = f(p) f(k)$，即形状因子相同。

含有裸态的逆散射问题

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





逆散射问题一般解法，根据弗雷德霍尔姆行列式（Fredholm determinant），

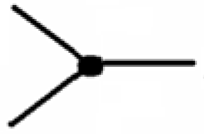
$$D(W) \equiv \det\left(1 - \frac{1}{W - H_0} V\right) = 1 - \left(\frac{\lambda_b^2}{W - m} + \lambda_c\right) F(W) \qquad F(W) = \left\langle f \left| \frac{1}{W - H_0} \right| f \right\rangle$$

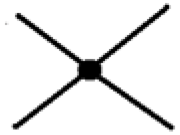
为了避免一些W奇点（裸质量点）和零点（束缚态质量点），

$$C(W) \equiv \frac{\prod_{i=1}^{N_B} \frac{W - E_{B_i}}{W - E_{th}}}{\prod_{i=1}^{N_b} \frac{W - m_{b_i}}{W - E_{th}}} \qquad \delta(E) = -\arg \frac{D(E + i\epsilon)}{C(E + i\epsilon)}$$

含有裸态的逆散射问题

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





逆散射问题一般解法，根据弗雷德霍尔姆行列式（Fredholm determinant），

$$D(W) \equiv \det\left(1 - \frac{1}{W - H_0} V\right) = 1 - \left(\frac{\lambda_b^2}{W - m} + \lambda_c\right) F(W) \qquad F(W) = \left\langle f \left| \frac{1}{W - H_0} \right| f \right\rangle$$

为了避免一些W奇点（裸质量点）和零点（束缚态质量点），

$$C(W) \equiv \frac{\prod_{i=1}^{N_B} \frac{W - E_{B_i}}{W - E_{th}}}{\prod_{i=1}^{N_b} \frac{W - m_{b_i}}{W - E_{th}}} \qquad \delta(E) = -\arg \frac{D(E + i\epsilon)}{C(E + i\epsilon)}$$

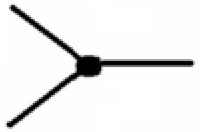
列文森定理（Levinson theorem）

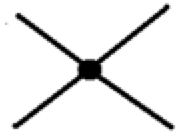
$$\delta(\infty) - \delta(E_{th}) = -\pi(N_B - N_b)$$

F. Vidal and J. LeTourneux, PRC 45, 418 (1992)

含有裸态的逆散射问题

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





逆散射问题一般解法，根据弗雷德霍尔姆行列式（Fredholm determinant），

$$D(W) \equiv \det\left(1 - \frac{1}{W - H_0} V\right) = 1 - \left(\frac{\lambda_b^2}{W - m} + \lambda_c\right) F(W) \qquad F(W) = \left\langle f \left| \frac{1}{W - H_0} \right| f \right\rangle$$

为了避免一些W奇点（裸质量点）和零点（束缚态质量点），

$$C(W) \equiv \frac{\prod_{i=1}^{N_B} \frac{W - E_{B_i}}{W - E_{th}}}{\prod_{i=1}^{N_b} \frac{W - m_{b_i}}{W - E_{th}}} \qquad \delta(E) = -\arg \frac{D(E + i\epsilon)}{C(E + i\epsilon)}$$

列文森定理（Levinson theorem）

$$\delta(\infty) - \delta(E_{th}) = -\pi (N_B - N_b)$$

F. Vidal and J. LeTourneux, PRC 45, 418 (1992)

标准的色散分析

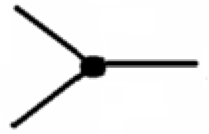
$$\ln \frac{D(W)}{C(W)} = -\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - W} \Longrightarrow D(W) = C(W) e^{-\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - W}}$$

$$-\mathrm{Tr}\, V = \lim_{|W| \to \infty} W \ln D(W) = \frac{1}{\pi} \int dE\, \delta(E) - \sum_{i=1}^{N_B} (E_{B_i} - E_{th}) + \sum_{i=1}^{N_b} (m_{b_i} - E_{th})$$

含有裸态的逆散射问题

$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$



$$D(W) = 1 - \left(\frac{\lambda_b^2}{W - m} + \cancel{\lambda_c} \right) \left\langle f \middle| \frac{1}{W - H_0} \middle| f \right\rangle \qquad C_1(W) \equiv (W - m)\, C(W) = \begin{cases} W - E_{th}, & N_B = 0 \\ W - E_B, & N_B = 1 \end{cases}$$

$$\operatorname{Im} D(h_p + i\epsilon) = \frac{\lambda_b^2}{h_p - m} \pi \langle f | \delta(h_p - H_0) | f \rangle = \frac{\pi p^2}{(2\pi)^3 h_p'} \frac{\lambda_b^2 f^2(p)}{h_p - m} \qquad D(W) = C(W) e^{-\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - W}}$$

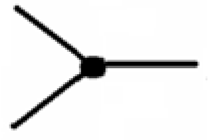
$$\boldsymbol{\lambda_b^2 f^2(p) = \frac{(2\pi)^3 h_p'}{\pi p^2} e^{-\frac{1}{\pi} \mathrm{P} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - h_p}} |\sin \delta(h_p)| C_1(h_p)}$$

$$-\operatorname{Tr} V = \frac{1}{\pi} \int dE \delta(E) - \sum_{i=1}^{N_B} \left(E_{B_i} - E_{th} \right) + \sum_{i=1}^{N_b} \left(m_{b_i} - E_{th} \right) = 0 \qquad \boldsymbol{m = \begin{cases} E_{th} - \frac{1}{\pi} \int dE\, \delta(E), & N_B = 0 \\ E_B - \frac{1}{\pi} \int dE\, \delta(E), & N_B = 1 \end{cases}}$$

含有裸态的逆散射问题

$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$

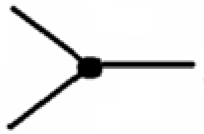


$$\boldsymbol{\lambda_b^2 f^2(p) = \frac{(2\pi)^3 h_p'}{\pi p^2} e^{-\frac{1}{\pi} \mathrm{P} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - h_p}} |\sin \delta(h_p)| C_1(h_p)}$$

$$|B\rangle = \sqrt{Z} \left(|b\rangle + \frac{\lambda_b}{E_B - H_0} |f\rangle \right)$$

$$Z = |\langle b|B\rangle|^2 = \frac{1}{1 + \langle f| \frac{\lambda_b^2}{(E_B - H_0)^2} |f\rangle} = \exp\left(\frac{1}{\pi} \int dE \frac{\delta(E)}{E - E_B} \right)$$

温伯格Compositeness公式的拓展



$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$

Compositeness $X = 1 - e^{\frac{1}{\pi}\int dE \frac{\delta(E)}{E - E_B}}$

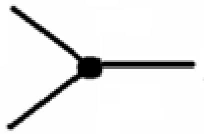
Low Equation $T_{p,k} = \cancel{V_{p,k}} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q}$

$$a = -\frac{2X_W}{1 + X_W} R + O(m_\pi^{-1}) \qquad X_W = \sqrt{\frac{1}{1 + 2r/a}}$$

$$r = -\frac{1 - X_W}{X_W} R + O(m_\pi^{-1}) \qquad R = 1/\sqrt{2\mu |E_B|}$$

Weinberg, Phys. Rev. 137, B672 (1965).

温伯格Compositeness公式的拓展



$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$

Compositeness $X = 1 - e^{\frac{1}{\pi}\int dE \frac{\delta(E)}{E-E_B}}$

Low Equation $T_{p,k} = \cancel{V_{p,k}} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q}$

$$a = -\frac{2X_W}{1+X_W} R + O(m_\pi^{-1}) \qquad X_W = \sqrt{\frac{1}{1+2r/a}}$$

$$r = -\frac{1-X_W}{X_W} R + O(m_\pi^{-1}) \qquad R = 1/\sqrt{2\mu|E_B|}$$

Weinberg, Phys. Rev. 137, B672 (1965).

ERE 近似下，$p\cot\delta(E) \approx \frac{1}{a} + \frac{r}{2}p^2 + O(p^4)$

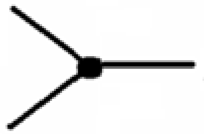
当$r < 0, a \in [-R, 0]$时 $X = X_W$

当$r > 0, a \notin [-R, 0]$时 $X = 1$，$X_W > 1$

$\delta(E_{th}) = 0^-, (r < 0) \cot\delta(\infty) \sim -\infty \Rightarrow \delta(\infty) = 0^-$

$\delta(E_{th}) = 0^-, (r > 0) \cot\delta(\infty) \sim +\infty \Rightarrow \delta(\infty) = -\pi$

温伯格Compositeness公式的拓展



$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$

Compositeness $X = 1 - e^{\frac{1}{\pi}\int dE \frac{\delta(E)}{E - E_B}}$

Low Equation $T_{p,k} = \cancel{V_{p,k}} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q}$

$$a = -\frac{2X_W}{1 + X_W} R + O(m_\pi^{-1}) \quad X_W = \sqrt{\frac{1}{1 + 2r/a}}$$

$$r = -\frac{1 - X_W}{X_W} R + O(m_\pi^{-1}) \quad R = 1/\sqrt{2\mu |E_B|}$$

Weinberg, Phys. Rev. 137, B672 (1965).

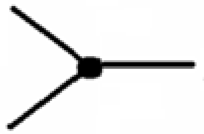
ERE 近似下，$p \cot\delta(E) \approx \frac{1}{a} + \frac{r}{2} p^2 + O(p^4)$

当$r < 0, a \in [-R, 0]$时 $X = X_W$

当$r > 0, a \notin [-R, 0]$时 $X = 1$，$X_W > 1$

氘核的散射长度-5.419(7)fm,有效力程1.766(8)fm,束缚能-2.224575(9)MeV，用温伯格公式得到X_W=168%

温伯格Compositeness公式的拓展



$\lambda_c = 0$

$$H = H_0 + V \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad V = \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right]$$

Compositeness $\quad X = 1 - e^{\frac{1}{\pi}\int dE \frac{\delta(E)}{E-E_B}}$

Low Equation $\quad T_{p,k} = \cancel{V_{p,k}} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q}$

$$a = -\frac{2X_W}{1+X_W} R + O(m_\pi^{-1}) \qquad X_W = \sqrt{\frac{1}{1+2r/a}}$$

$$r = -\frac{1-X_W}{X_W} R + O(m_\pi^{-1}) \qquad R = 1/\sqrt{2\mu |E_B|}$$

Weinberg, Phys. Rev. 137, B672 (1965).

ERE 近似下，$p \cot\delta(E) \approx \frac{1}{a} + \frac{r}{2}p^2 + O(p^4)$

当$r < 0, a \in [-R, 0]$时 $X = X_W$

当$r > 0, a \notin [-R, 0]$时 $X = 1$，$X_W > 1$

氘核的散射长度-5.419(7)fm,有效力程1.766(8)fm,束缚能-2.224575(9)MeV，用温伯格公式得到X_W=168%

原因：

$$\tilde{g}(p) \equiv \langle p|V|B\rangle = \sqrt{Z}\lambda_b f(p) = \frac{8\pi^2}{\mu^2 R} \begin{cases} \sqrt{\frac{1}{1+2r/a}} + O(p^4) & \left(\sqrt{\frac{1}{1+\frac{2r}{a}}} \in [0.1]\right) \\ \frac{a^2}{R^2} \frac{1}{1+(a+R)^2 p^2} + O(p^4) & \left(\sqrt{\frac{1}{1+\frac{2r}{a}}} > 1\right) \end{cases}$$

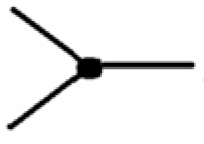
耦合道和束缚态的耦合形式(Dressed Coupling)

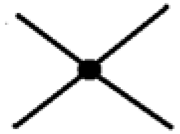
温伯格在计算中就只取了$\tilde{g}(p)$的领头阶，即常数。这和ERE取到次零头阶其实是不自洽的。

$\lambda_c \neq 0$

含有裸态的逆散射问题

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





$$D(W) = 1 - \left(\frac{\lambda_b^2}{W-m} + \lambda_c\right) F(W) \equiv 1 - \frac{E_C - W}{E_C - m}\frac{\lambda_b^2}{W-m} F(W)$$

$$E_C \equiv m - \frac{\lambda_b^2}{\lambda_c}$$

$$\mathrm{Im}\, D(h_p + i\epsilon) = \frac{E_C - h_p}{E_C - m}\frac{\pi p^2}{(2\pi)^3 h_p'}\frac{\lambda_b^2 f^2(p)}{h_p - m}$$

$$C_1(W) \equiv (W-m)\, C(W) = \begin{cases} W - E_{th}, & N_B = 0 \\ W - E_B, & N_B = 1 \\ (W - E_{B_1})\dfrac{W - E_{B_2}}{W - E_{th}}, & N_B = 2 \end{cases}$$

$$D(W) = C(W) e^{-\frac{1}{\pi}\int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E-W}}$$

$$D(E_C + i\epsilon) = 1 = C(E_C) e^{-\frac{1}{\pi}\int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - E_C - i\epsilon}}$$

$$\boldsymbol{\lambda_b^2 f^2(p) = \frac{(2\pi)^3 h_p'}{\pi p^2}\frac{E_C - m}{E_C - h_p} e^{-\frac{1}{\pi}\mathrm{P}\int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - h_p}} |\sin\delta(h_p)| C_1(h_p)}$$

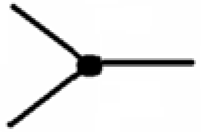
$$\boldsymbol{m = E_C - C_1(E_C) e^{-\frac{1}{\pi}\int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - E_C - i\epsilon}}}$$

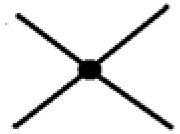
含有裸态的逆散射问题

$\lambda_c \neq 0$

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p|$$

$$\hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





$$D(W) = 1 - \left(\frac{\lambda_b^2}{W-m} + \lambda_c \right) F(W) \equiv 1 - \frac{E_C - W}{E_C - m} \frac{\lambda_b^2}{W-m} F(W)$$

$$\boxed{E_C \equiv m - \frac{\lambda_b^2}{\lambda_c}}$$

$$\mathrm{Im}\, D(h_p + i\epsilon) = \frac{E_C - h_p}{E_C - m} \frac{\pi p^2}{(2\pi)^3 h_p'} \frac{\lambda_b^2 f^2(p)}{h_p - m}$$

$$C_1(W) \equiv (W-m)\, C(W) = \begin{cases} W - E_{th}, & N_B = 0 \\ W - E_B, & N_B = 1 \\ \left(W - E_{B_1}\right) \dfrac{W - E_{B_1}}{W - E_{th}}, & N_B = 2 \end{cases}$$

$$D(W) = C(W) e^{-\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E-W}}$$

$$D(E_C + i\epsilon) = 1 = C(E_C) e^{-\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - E_C - i\epsilon}}$$

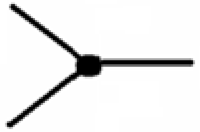
$$\lambda_b^2 f^2(p) = \frac{(2\pi)^3 h_p'}{\pi p^2} \frac{E_C - m}{E_C - h_p} e^{-\frac{1}{\pi} \mathrm{P} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - h_p}} |\sin \delta(h_p)| C_1(h_p)$$

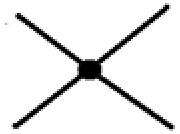
$$m = E_C - C_1(E_C) e^{-\frac{1}{\pi} \int_{E_{th}}^{\infty} dE \frac{\delta(E)}{E - E_C - i\epsilon}}$$

CDD零点

$\lambda_c \neq 0$

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





$$D(W) = 1 - \left(\frac{\lambda_b^2}{W - m} + \lambda_c \right) F(W) \equiv 1 - \frac{E_C - W}{E_C - m} \frac{\lambda_b^2}{W - m} F(W)$$

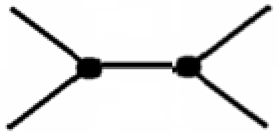
$$\boxed{E_C \equiv m - \frac{\lambda_b^2}{\lambda_c}}$$

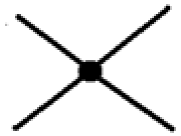
the Castillejo-Dalitz-Dyson (CDD) zero

$$\delta(E_C) = -\arg \frac{D(E_C + i\epsilon)}{C(E_C + i\epsilon)} = -\arg \frac{1}{C(E_C + i\epsilon)} = \begin{cases} 0 & E_C > m \\ -\pi & E_C < m \end{cases}$$

$$T(E_C) = 0$$

$$0 = \frac{\lambda_b^2}{E_C - m} + \lambda_c$$



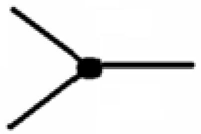


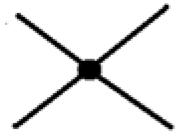
$\lambda_c \neq 0$

CDD零点

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p|$$

$$\hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





$$D(W) = 1 - \left(\frac{\lambda_b^2}{W - m} + \lambda_c \right) F(W) \equiv 1 - \frac{E_C - W}{E_C - m} \frac{\lambda_b^2}{W - m} F(W)$$

$$\boxed{E_C \equiv m - \frac{\lambda_b^2}{\lambda_c}}$$

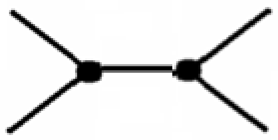
the Castillejo-Dalitz-Dyson (CDD) zero

$$\delta(E_C) = -\arg \frac{D(E_C + i\epsilon)}{C(E_C + i\epsilon)} = -\arg \frac{1}{C(E_C + i\epsilon)} = \begin{cases} 0 & E_C > m \\ -\pi & E_C < m \end{cases}$$

$$T(E_C) = 0$$

$$0 = \frac{\lambda_b^2}{E_C - m} + \lambda_c$$

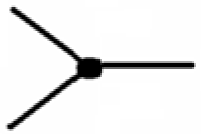
图像上，对于两强子态的散射，有两种不同的相互作用机制，在CDD零点，好像是相互抵消了。

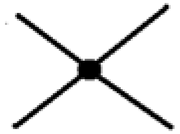


CDD零点

$\lambda_c \neq 0$

$$H = H_0 + \hat{g} + \hat{v} \qquad H_0 = |b\rangle m_b \langle b| + \int \frac{p^2 dp}{(2\pi)^3} |p\rangle h(p) \langle p| \qquad \hat{g} = \lambda_b \left[|f\rangle\langle b| + |b\rangle\langle f| \right] \qquad \hat{v} = \lambda_c |f\rangle\langle f|$$





$$H = H_{0b} + H_{0c} + \hat{g} + \hat{v}$$

$$\langle B|H_{0b} + H_{0c} + \hat{g} + \hat{v}|B\rangle \equiv E_{0b} + E_{0c} + E_{bc} + E_{cc}$$

$$\boxed{E_C \equiv m - \frac{\lambda_b^2}{\lambda_c}}$$

the Castillejo-Dalitz-Dyson (CDD) zero

$$R = \frac{E_{cc}}{E_{bc}} = \frac{1}{2} \frac{m - E_B}{E_C - m}$$

$R > 1$ 说明耦合道之间的相互作用对形成束缚态很重要

$R < 1$ 说明裸态在形成该束缚态中很重要

CDD zero 怎么找 ？？？

$D_{s0}^*(2317)$ in $DK - (c\bar{s})$

$E_B = 2318$ MeV, $m = 2480$ or 2406 MeV,

要求 $R \geq 1$ 即 $E_C \leq m + \frac{m - E_B}{2}$

$E_C \leq 2561\,\mathrm{MeV} \quad 2450\,\mathrm{MeV}$

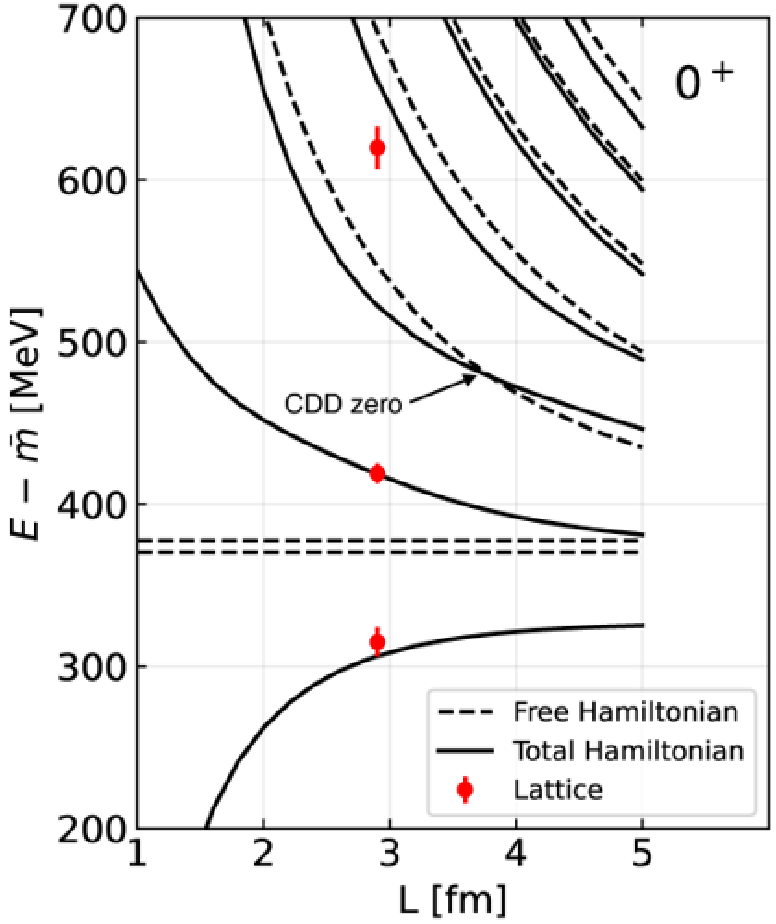
如果不存在CDD零点，那么裸态和耦合道的相互作用强于耦合道耦合道相互作用

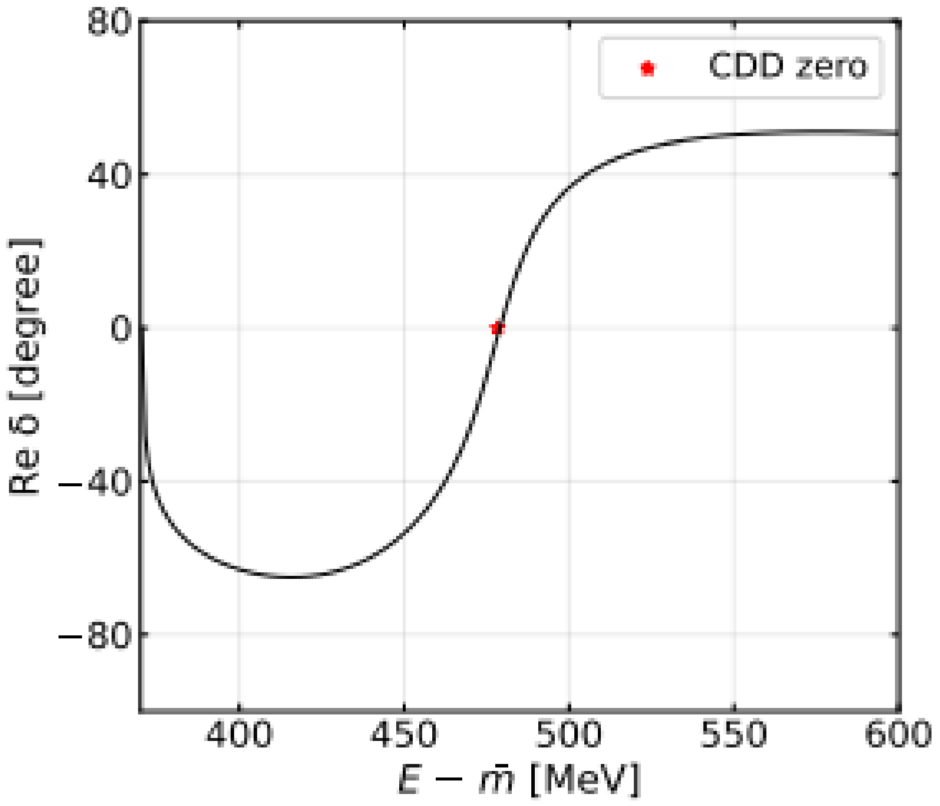
CDD零点

Z. Yang, G.-J. Wang, J.-j. Wu, S.-l. Zhu, M. Oka
arXiv:2207. 07320

$D_{s0}^*(2317)$
伙伴态
$B_{s0}^*(5730)$

$(s\bar{b}) - B\bar{K}$





小结

- 简单介绍了在两种含有裸态的模型假定下，其逆散射的解。
- 第一种模型下忽略了耦合道之间的相互作用，这对于对近阈束缚态得到其复合度的公式，该公式是对温伯格早年公式的拓展。
- 第二种两种机制下的CDD零点对理解两体散射的相互作用会有重要的意义，报告中简单分析了CDD零点对$D_{s0}^*(2317)$形成的意义。

谢谢

Backup

温伯格在Low Equation中忽略V_{pk}

$$T_{p,k} = \cancel{V_{p,k}} + \frac{g(p)\,g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{T_{p,q}T^*_{k,q}}{h_k + i\varepsilon - h_q}$$

$$T_{p,k} = \frac{g(p)g^*(k)}{h_k - E_B - \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|g(p)|^2}{h_k - h_q + i\epsilon}\left(\frac{h_k - E_B}{h_q - E_B}\right)^2}$$

$$\tilde{T}_{p,k} = \frac{\lambda_b f(p)\lambda_b^* f^*(k)}{h_k - m} + \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{\lambda_b f(p)\lambda_b^* f^*(q)}{h_k - m} \frac{1}{h_k - h_q + i\epsilon}\tilde{T}_{q,k}$$

$$\tilde{T}_{p,k} = \frac{\lambda_b f(p)\lambda_b^* f^*(k)}{h_k - m - \Sigma(h_k)} = \frac{\lambda_b f(p)\lambda_b^* f^*(k)}{h_k - E_B + \left(\Sigma(E_B) - \Sigma(h_k)\right)}$$

$$\Sigma(h_k) = \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|\lambda_b f(q)|^2}{h_k - h_q + i\epsilon} \qquad E_B = m + \Sigma(E_B)$$

$$g(p) \equiv \langle p|V|B\rangle = \sqrt{Z}\lambda_b f(p)$$

$$\frac{1}{Z} = 1 + \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|\lambda_b f(q)|^2}{(E_B - h_q)^2}$$

$$T_{p,k} = \frac{\lambda_b f(p)\lambda_b f^*(k)}{h_k - E_B + (h_k - E_B)\int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|\lambda_b f(q)|^2}{(E_B - h_q)^2} - \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|\lambda_b f(p)|^2}{h_k - h_q + i\epsilon}\left(\frac{h_k - E_B}{h_q - E_B}\right)^2}$$

$$T_{p,k} = \frac{\lambda_b f(p)\lambda_b f^*(k)}{\frac{h_k - E_B}{Z} - \int_0^\infty \frac{q^2dq}{(2\pi)^3} \frac{|\lambda_b f(p)|^2}{h_k - h_q + i\epsilon}\left(\frac{h_k - E_B}{h_q - E_B}\right)^2}$$

$$T_{p,k} = \frac{\lambda_b f(p)\lambda_b f^*(k)}{h_k - E_B + \left(\Sigma(E_B) - \Sigma(h_k)\right)} = \tilde{T}_{p,k}$$