Glueballs on the lattice

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Outline

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III. Glueballs in J/ψ radiative decays

  - Scalar glueball
  - Tensor glueball
  - Pseudoscalar glueball

IV. Preliminary results from lattice QCD at the physical point

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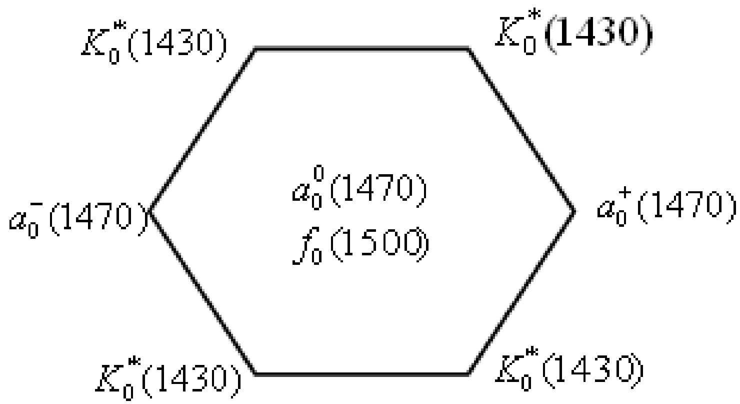
I. Introduction

1. Experimental candidates for glueballs

Scalar mesons



$q\bar{q}$??

+ $f_0(1370)$
$f_0(1710)$

Pseudoscalar glueball candidate —— $X(2370)$???

$\eta(1295)/\eta(1405)/\eta(1475)$???

If only two states, there is unnecessarily a glueball candidate here (J.-J. Wu et al., PRL 109 (2012) 081803)

Tensor glueball candidate ——$f_2(2340)$???

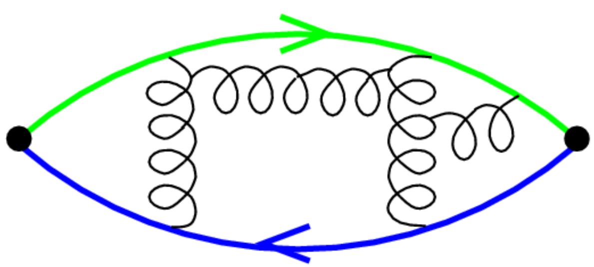
2. Formalism of Lattice QCD

$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$

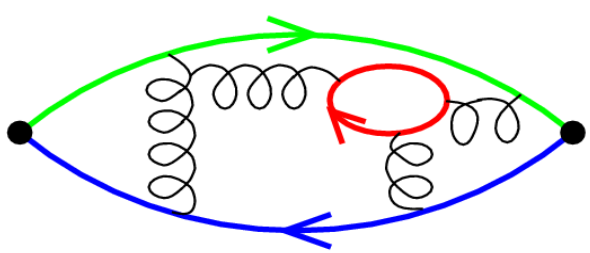
$$S = S_{gauge} + S_{quarks} = \int d^4x \, \left(\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det}\, M_i)$$

$$Z = \int \mathcal{D}A_\mu \; \det M \; e^{\int d^4x \, \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)} .$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S} .$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

QCD in quenched approximation (QQCD) vs. Full-QCD
(for glueball relevant studies——slow progress!)

QQCD:

- glueballs are well-defined objects
- large statistics can be easily achieved

- not a unitary (physical) theory
- the systematical uncertainties due to the neglect of sea quarks are not under control

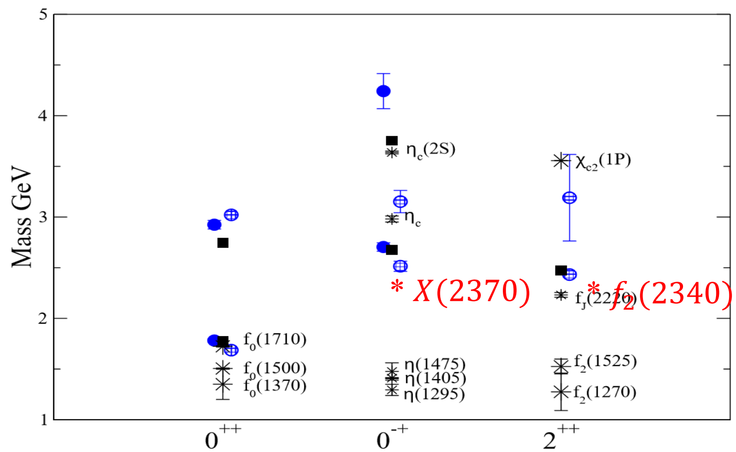
Full-QCD——theoretically much more complicated for glueballs

- most of hadrons are observed as resonances
- how to define a glueball state, even a qqbar meson?
- the mixing between glueballs and conventional mesons
- glueball decays
- should be done in the framework of hadron-hadron scattering.
- Far beyond the capability of present lattice QCD calculation

II. Glueball spectrum from lattice QCD

	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 2+1$ [22]	360	1795(60)	2620(50)	—
quenched [13]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [14]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

$N_f = 2$: W. Sun et al (CLQCD), Chin. Phys. C 42, 093103 (2018)
[14] C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999
[13] Y. Chen et al, Phys. Rev. D 73, 014516, 2006
[22] E. Gregory et al., JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)



Filled Squares: QQCD
Open circles: full QCD, coarse lattice
Closed circles: full QCD, fine lattice

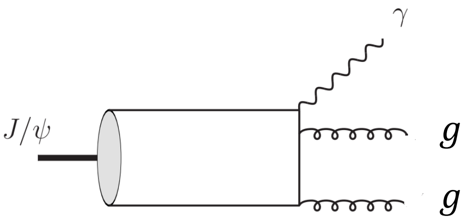
C.M. Richards et al., [UKQCD Collab.], Phys. Rev. D82, 034501 (2010).

No meson or two-meson operators have been involved yet!

III. Glueball in J/ψ radiative decays

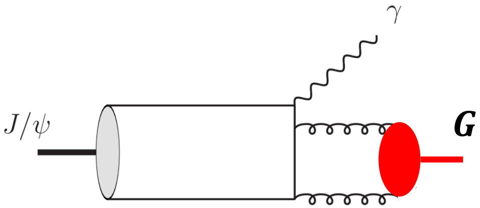
- J/ψ radiative decays —— best hunting ground for glueballs

Gluon abundant in J/ψ decays

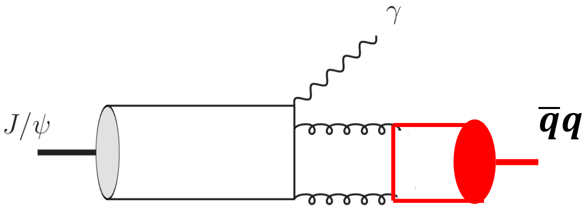


Gluon is flavor singlet —— isospin filter

- J/ψ radiative decay products—— $\bar{q}q$ meson vs. glb



$O(1)$



Suppressed by $O(\alpha_s^2)$

- Serve as criteria for the experimental identification of glueball.

- **Radiative decay width:**

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i+1} \times \sum_{r_i, r_j, r_\gamma} \left| M_{r_i, r_j, r_\gamma} \right|^2,$$

- **Transition amplitudes:** $M_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma) \langle f(\vec{p}_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(\vec{p}_i, r_i) \rangle$

- **Multipole decomposition:**

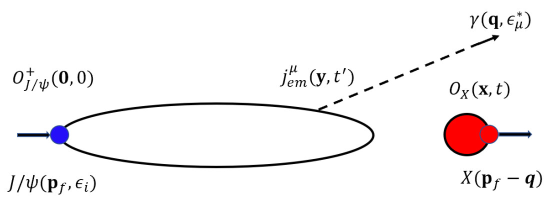
$$\langle f(\vec{p}_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(\vec{p}_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_i, p_f) F_k(Q^2).$$

- **Decay width expressed in terms of the form factors**

$$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0).$$

- **So the major task is to calculate the matrix elements, which can be derived from the three-point functions on the lattice**

$$\Gamma^{(3)\mu i}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T} \sum_{\tau=0}^{T-1} \sum_{\vec{y}} e^{+i\vec{q}\cdot\vec{y}} \left\langle O_G(\vec{p}_f, t_f + \tau) j^\mu(\vec{y}, t+\tau) O_{J/\psi}^{i,+}(\tau) \right\rangle$$



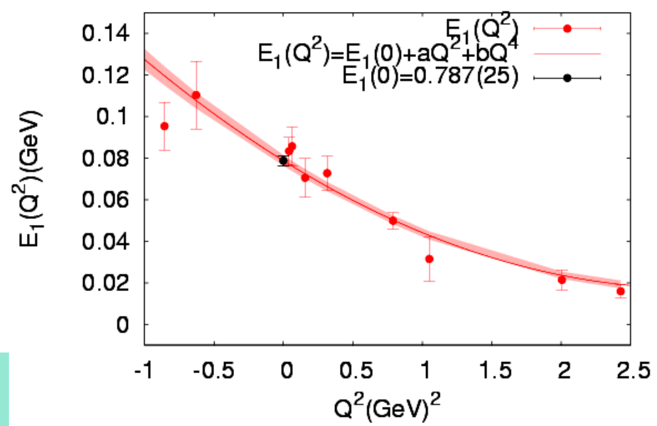
1. J/psi radiatively decaying to the scalar glueball

(L.Gui, et al. (CLQCD Collaboration), Phys. Rev. Lett. 110, 021601 (2013))

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2$$

Interpolated on-shell form factor E1(0) and its continuum limit

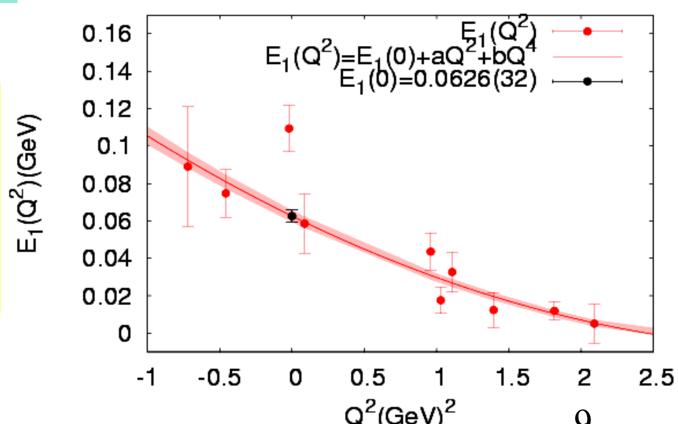
β	M_G(GeV)	$Z_V^{(s)}(a)$	$E_1(0,a)$ (GeV)	Γ(keV)
2.4	1.360(9)	1.39(2)	0.0787(25)	-
2.8	1.537(7)	1.11(1)	0.0626(32)	-
∞	1.710(90) [3]	-	0.0536(57)	0.35(8)



The predicted width and the branch ratio

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2 = 0.35(8)keV$$

$$\Gamma / \Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$



Experimental results of $f_0(1500)/f_0(1710)$???

LQCD prediction of the partial ew

$$\Gamma(J/\psi \to \gamma G_{0^+}) = 0.35(8)keV, \quad \Gamma/\Gamma_{tot} = 3.8(9) \times 10^{-3}$$

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys., 083C01 (2020)

$$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K} \quad \left(9.5^{+1.0}_{-0.5}\right) \times 10^{-4}$$
$$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi \quad (3.8 \pm 0.5) \times 10^{-4}$$
$$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega \quad (3.1 \pm 1.0) \times 10^{-4}$$
$$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta \quad \left(2.4^{+1.2}_{-0.7}\right) \times 10^{-4}$$

$$\Rightarrow Br(J/\psi \to \gamma f_0(1710)) > 1.9 \times 10^{-3}$$

Using $Br(f_0(1710) \to K\underline{K}) = 0.36 \quad \Rightarrow \quad Br(J/\psi \to \gamma f_0(1710)) = 2.4 \times 10^{-3}$

$Br(f_0(1710) \to \pi\pi) = 0.15 \quad \Rightarrow \quad Br(J/\psi \to \gamma f_0(1710)) = 2.7 \times 10^{-3}$

In contrast,

$$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \quad (1.01 \pm 0.34) \times 10^{-4}$$
$$J/\psi \to \gamma f_0(1500) \to \gamma K_S^0 K_S^0 \quad \left(1.59 \pm 0.16^{+0.18}_{-0.56}\right) \times 10^{-5}$$

$$Br(f_0(1500) \to \pi\pi) = (34.5 \pm 2.2)\%$$
$$Br(f_0(1500) \to K\bar{K}) = (8.5 \pm 1.0)\%$$

$$\Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9 \times 10^{-4}$$

Recent BESIII results from PWA

$J/\psi \to \gamma X \to \gamma\pi\pi$

BES, PLB642(2006)441

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi) = (4.01 \pm 1.0) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi) = (1.01 \pm 0.34) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1370) \to \gamma\pi\pi) = ----$$

$J/\psi \to \gamma X \to \gamma\eta\eta$

BESIII, PRD87(2013)092009

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta) = \left(2.35^{+1.27}_{-0.77}\right) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta) = \left(1.65^{+0.57}_{-1.50}\right) \times 10^{-5}$$

$$Br(J/\psi \to \gamma f_0(1370) \to \gamma\eta\eta) = ----$$

$J/\psi \to \gamma X \to \gamma K_s K_s$

BESIII, arXiv:1808.06946 (hep-ex)

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma K_s K_s) = \left(2.00^{+0.03+0.31}_{-0.02-0.10}\right) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1500) \to \gamma K_s K_s) = \left(1.59^{+0.16+0.18}_{-0.16-0.59}\right) \times 10^{-5}$$

$$Br(J/\psi \to \gamma f_0(1370) \to \gamma K_s K_s) = \left(1.07^{+0.08+0.36}_{-0.07-0.34}\right) \times 10^{-5}$$

Obviously, in each process,
$f_0(1710)$ are produced 10 times more than $f_0(1500)$.

Flavor-blindness of glueball decays

Theor.

$$\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$$

BES

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi) = (4.01 \pm 1.0) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma K_s K_s) = \left(2.00^{+0.03+0.31}_{-0.02-0.10}\right) \times 10^{-4}$$

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}) = Br(J/\psi \to \gamma f_0(1710) \to \gamma K_s K_s) \times 4$$

$$Br(J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta) = \left(2.35^{+1.27}_{-0.77}\right) \times 10^{-4}$$

$$\Gamma\left(G \to \pi\pi : K\bar{K} : \eta\eta\right) \approx 2:4:1.2$$

$$P.S.(G \to \pi\pi : K\bar{K} : \eta\eta) \approx 0.5:0.41:0.38$$

$$\frac{1}{P.S.}\Gamma\left(G \to \pi\pi : K\bar{K} : \eta\eta\right) \approx 1.3:3.16:1$$

Chiral suppression in glueball decays

Chanowitz
(PRL 95(2005)172001):

$$\Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K}) \approx O\left(\frac{m_n^2}{m_s^2}\right)$$

K.T. Chao, X.G. He, J.P. Ma
(PRL 98(2007)149103):

$$\Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K}) \approx \frac{f_\pi^4}{f_K^4} \approx 0.48$$

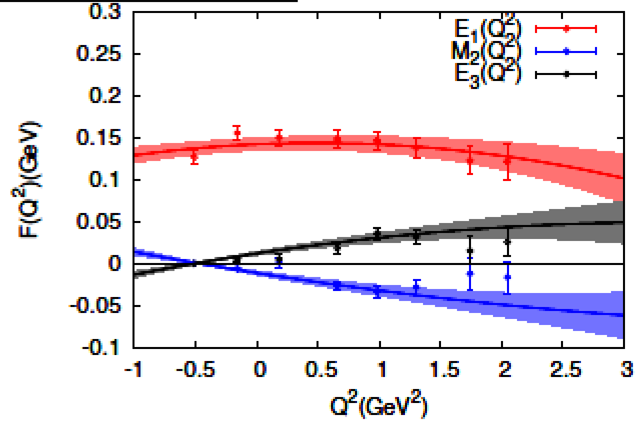
2. J/psi radiatively decaying to the tensor glueball

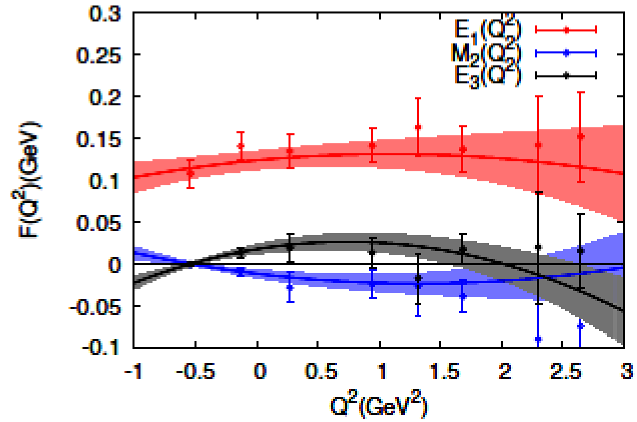
(Y.B. Yang ,et al .(CLQCD Collaboration), Phys. Rev. Lett. 111, 091601 (2013))

$$\Gamma(J/\psi \to \gamma G_{2^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left[|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2\right]$$

- **The form factors we obtained from the lattice QCD**

β	M_T (GeV)	E_1 (GeV)	M_2 (GeV)	E_3 (GeV)
2.4	2.360(20)	0.142(07)	−0.012(2)	0.012(2)
2.8	2.367(25)	0.125(10)	−0.011(4)	0.019(6)
∞	2.372(28)	0.114(12)	−0.011(5)	0.023(8)





- **We also carry out a similar lattice study on the tensor glueball production rate in J/psi radiative decay.**

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

LQCD prediction

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$
$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2) \times 10^{-2}$$

$J/\psi \to \gamma X \to \gamma\eta\eta$ BESIII, PRD87(2013)092009

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = \left(5.60^{+0.62+2.37}_{-0.65-2.07}\right) \times 10^{-5}$$

$J/\psi \to \gamma X \to \gamma\eta'\eta'$ BESIII, arXiv:2201.09710 (hep-ex)

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta'\eta') = \left(8.67 \pm 0.70^{+0.61}_{-1.67}\right) \times 10^{-6}$$

$$\frac{\Gamma(f_2 \to \eta\eta)}{\Gamma(f_2 \to \eta'\eta')} \sim \frac{g^2_{f_2\eta\eta}}{g^2_{f_2\eta'\eta'}}\left(\frac{k_\eta}{k_{\eta'}}\right)^5 \sim 8.5\,\frac{g^2_{\eta\eta}}{g^2_{\eta'\eta'}}$$

$J/\psi \to \gamma X \to \gamma K_s K_s$ BESIII, PRD98(2018)072003

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma K_s K_s) = \left(5.54^{+0.34+3.82}_{-0.40-1.49}\right) \times 10^{-5}$$

$J/\psi \to \gamma X \to \gamma\varphi\varphi$ BESIII, PRD93(2016)112011

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\phi\phi) = \left(1.91 \pm 0.14^{+0.72}_{-0.73}\right) \times 10^{-4}$$

It is desirable to do a systematic analysis of decay modes $J/\psi \to \gamma VV$

Flavor-blindness of glueball decays

Theor.

$$\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$$

BES

$$\Gamma\big(G_2 \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta'\big) = ? : 4 : 1 : ? : ?$$

PP final states in the tensor glueball decays should be in D-wave, considering the centrifugal barrier effects,

$$\Gamma(G \to M\bar{M}) = \eta\alpha \frac{k^{2L+1}}{{m_G}^{2L}} = \eta\alpha m_G \left(\frac{k}{m_G}\right)^{2L+1}$$

$$\frac{k}{m_G} = \frac{1}{2}\sqrt{1-\left(\frac{2m_M}{m_G}\right)^2} \sim 0.5-0.3 \Rightarrow \left(\frac{k}{m_G}\right)^4 \sim O(0.1)$$

$$Br(G_{2^+} \to PP) \sim O(10\%)$$

$$\frac{\Gamma(G_2 \to \eta\eta)}{\Gamma(G_2 \to PP)} \sim O(10\%)$$

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) \approx 5.6 \times 10^{-5}$$

$$Br(J/\psi \to \gamma f_2(2340) \sim 10^{-2}$$

Compatible with LQCD prediction

3. Pseudoscalar glueball relevant

- The production rate of the pseudoscalar glueball in J/ψ radiative decays from LQCD (L.-C. Gui et al. , Phys. Rev. D 100, 054511 (2019))

$$\Gamma(J/\psi \to \gamma G_{ps}) = 0.022(7)\ \text{keV}$$

$$Br(J/\psi \to \gamma X(2370)) = 2.3(8) \times 10^{-4}$$

Not that large!

- If the kenetic factor is subtracted, we have the effective couplings

$$\Gamma(J/\psi \to \gamma X) = \frac{1}{3}\alpha g_X^2 \frac{|\mathbf{q}|^3}{m_{J/\psi}^2},$$

$$g_X = \left[\frac{24\Gamma_{J/\psi}}{\alpha}\frac{Br(J/\psi \to \gamma X)m_{J/\psi}^5}{(m_{J/\psi}^2 - m_X^2)^3}\right]^{\frac{1}{2}}$$

TABLE VI: The g_X of flavor-singlet pseudoscalar mesons.

Pseudoscalar (X)	g_X
η	0.0108(2)
η'	0.0259(8)
$\eta(1405/1475)$	0.0313(41)
$\eta(1760)$	0.0255(25)
$X(1835)$	0.0123(12)
$\eta(2225)$	0.0167(17)
$G(0^{-+})$	0.0144(27)

- The effective couplings are comparable for glueball and mesons.
- The $U_A(1)$ anomaly may play an important role here.

1) Pseudoscalar glueball candidate (0^{-+})

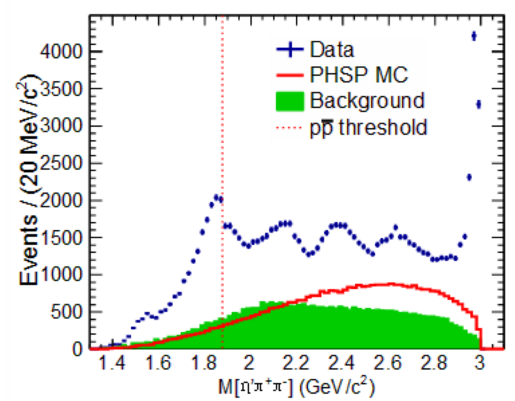
- **BESIII new results for** $J/\psi \rightarrow \gamma\varphi\varphi$

TABLE I. Mass, width, $\mathcal{B}(J/\psi \rightarrow \gamma X \rightarrow \gamma\phi\phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

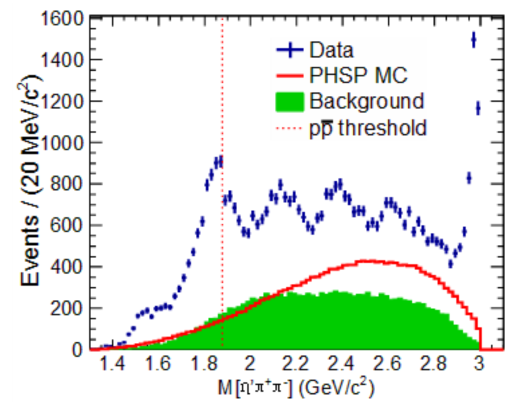
Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

BESIII, PRD93(2016)112011

$J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$



$\eta' \rightarrow \gamma\pi^+\pi^-$



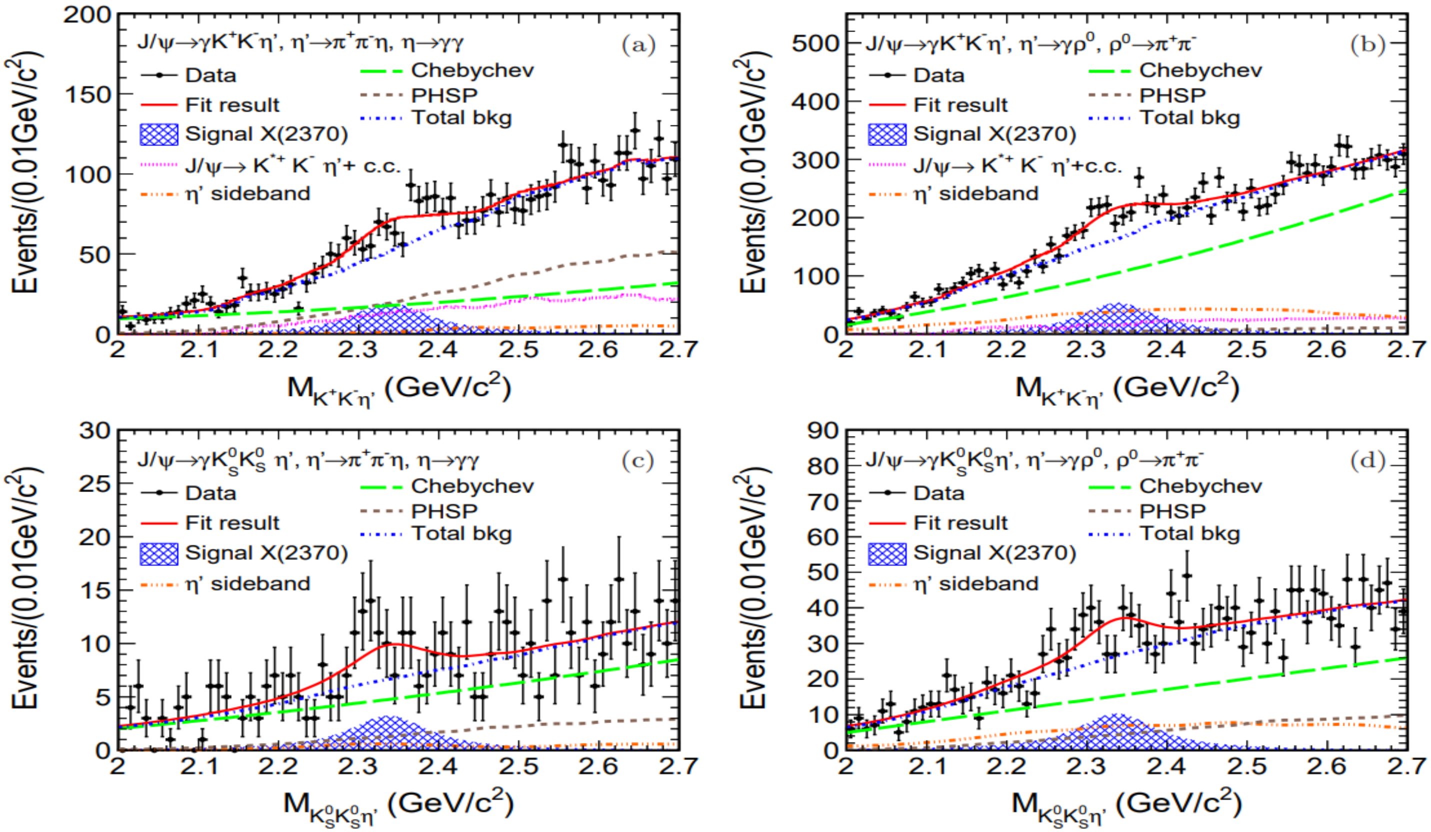
$\eta' \rightarrow \eta(\rightarrow \gamma\gamma)\pi^+\pi^-$

BESIII, PRL117(2016)042002
arXiv:1603.09653

Lattice QCD result: $Br(J/\psi \to \gamma X(2370))$=2.3(8)× 10^{-4}

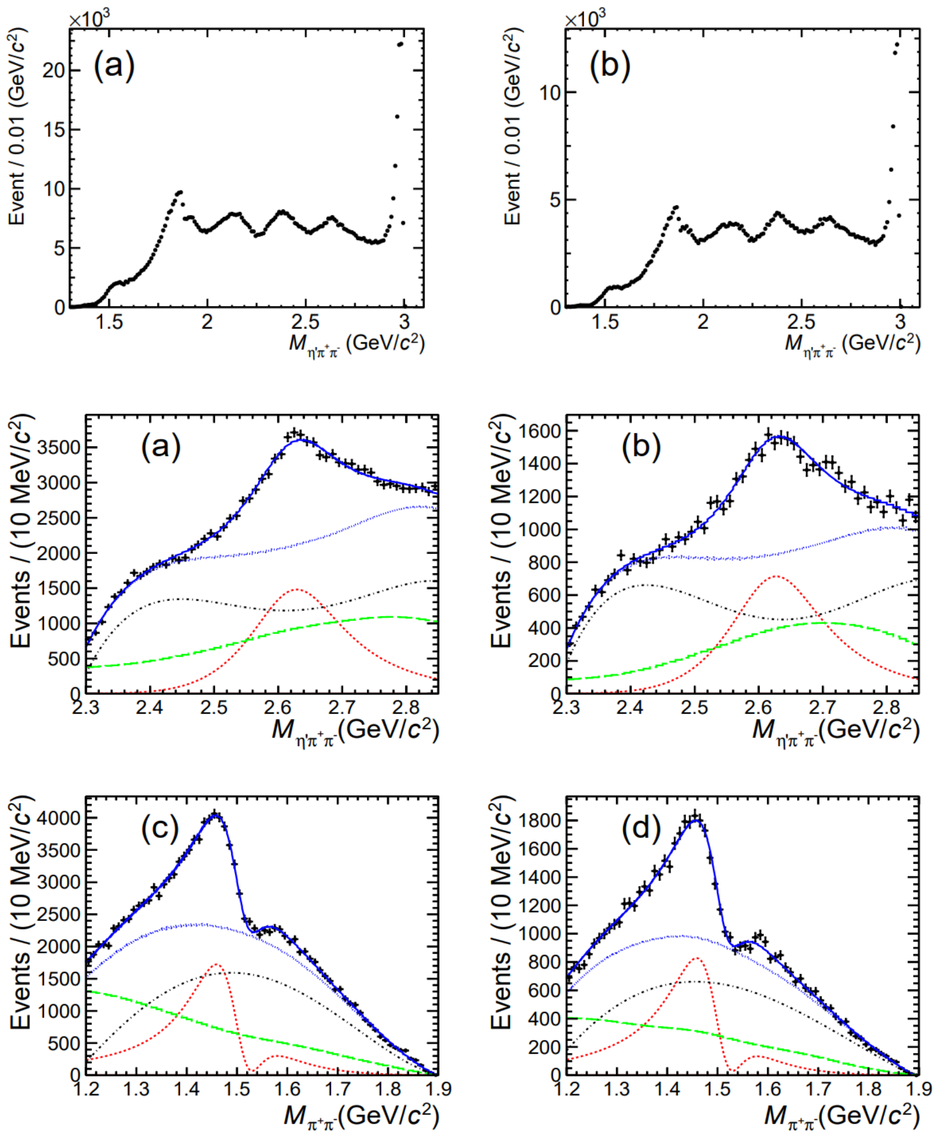
$J/\psi \to \gamma X(2370) \to \gamma K^+ K^- \eta'$ $(1.79 \pm 0.23 \pm 0.65) \times 10^{-5}$

$J/\psi \to \gamma X(2370) \to \gamma K_S^0 K_S^0 \eta'$ $(1.18 \pm 0.32 \pm 0.39) \times 10^{-5}$



BESIII, Eur. Phys. J. C 80 (2020) 8, 746.

BESIII, arXiv:2201.10769 (hep-ex)



$J/\psi \to \gamma\eta'\pi^+\pi^-$

$\eta' \to \gamma\pi^+\pi^-$

$\eta' \to \eta\pi^+\pi^-$

$X(2600) \to f_0(1500)\eta'$

$X(2600) \to f_2'(1525)\eta'$

$B(\psi \to \gamma X) \cdot B(X \to f_0\eta')$
$\cdot\, B(f_0 \to \pi\pi) \approx 3.4 \times 10^{-5}$

$B(f_0 \to \pi\pi) \approx 34.5\%$

$B(\psi \to \gamma X) \cdot B(X \to f_2'\eta')$
$\cdot\, B(f_2' \to \pi\pi) \approx 2.4 \times 10^{-5}$???

$B(f'_2 \to \pi\pi) \approx 0.8\%$

2) Glueball component of η_c and its implication

(R.Q. Zhang et al. , arXiv: 2107.12749 (hep-lat))

- η_c total width is quite large: $\Gamma_{\eta_c} = 32.0(7)$ MeV
- Lattice QCD predict the mass of the pseudoscalar glueball to be around 2.4-2.6 GeV
- This motivates the possibility of sizable gluonic component in η_c
- If $\eta(1405)$ and $\eta(1475)$ are the same state, there is no need for a pseudoscalar glueball round 1.3-1.5 GeV

  J.-J. Wu et al., Phys. Rev. Lett. 108, 081803 (2012)

- There may be mixing between $\bar{c}c(\,^1S_0\,)$ and the PS glueball

  Y.-D. Tsai, H.-n. Li, and Q. Zhao, Phys. Rev. D 85, 034002 (2012)

  W. Qin, Q. Zhao, and X.-H. Zhong, Phys. Rev. D 97, 096002 (2018)

Gauge configurations: $N_f = 2$ degenerate charm quarks permit the mixing between $c\bar{c}$ and glueball

ensemble	$L^3 \times T$	β	a_s(fm)	ξ	$m_{J/\psi}$(MeV)	$N_{\rm cfg}$
I	$16^3 \times 128$	2.8	0.1026	5	2743	~ 6000
II	$16^3 \times 128$	2.8	0.1026	5	3068	~ 6000

- $c\bar{c}$ operator: $O_{\bar{c}c} = \bar{c}\gamma_5 c$ Glueball operator: O_G
- Mixing model

$$\hat{H} = \begin{pmatrix} m_G & x \\ x & m_{c\bar{c}} \end{pmatrix} \qquad \begin{pmatrix} |X\rangle \\ |\eta_c\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |G\rangle \\ |c\bar{c}\rangle \end{pmatrix}$$

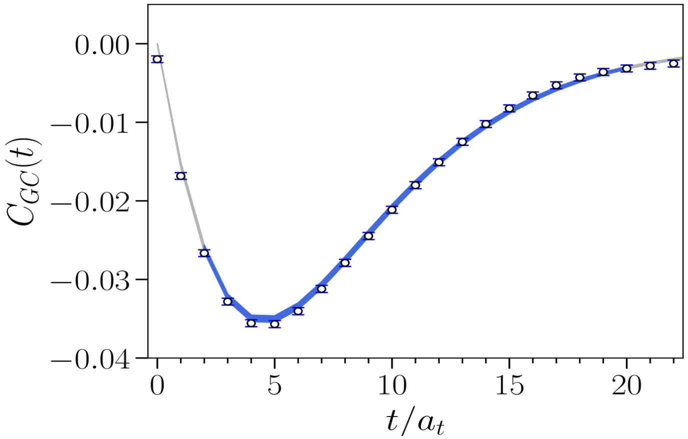
- Under some assumptions, one has

$$C_{GC}(t) \approx -Z \sin\theta \left(e^{-m_X t} - e^{-m_{\eta_c}}\right) + \cdots$$

$$\sin\theta \approx \frac{x}{m_{c\bar{c}} - m_G}$$

- **Very precise data**
- **Well described by the function form**

$$C_{GC}(t) \approx -Z\sin\theta\,(e^{-m_X t} - e^{-m_{\eta_c}}) + \cdots$$



ensemble	Γ	m_{η_1}(MeV)	m_{g_1}(MeV)	θ_1	x_1(MeV)
I	γ_5	2691(2)	2317(51)	7.7(1.1)°	48(7)
	$\gamma_5\gamma_4$	2685(1)	2317(43)	6.8(8)°	43(6)
	avg.	2686(1)	2317(46)	7.1(9)°	46(7)
II	γ_5	2987(9)	2308(63)	4.9(6)°	59(8)
	$\gamma_5\gamma_4$	3013(3)	2385(40)	4.2(3)°	46(6)
	avg.	3010(4)	2363(47)	4.3(4)°	49(7)

Effects of the mixing on the total width of η_c

- Assuming $X(2370)$ is predominantly a glueball, $\Gamma_X \approx 100$ MeV

$J/\psi \to \gamma X(2370) \to \gamma\pi^+\pi^-\eta'$:

$$M_{X(2370)} = 2341.6 \pm 6.5(\text{stat.}) \pm 5.7(\text{syst.})\ \text{MeV}$$
$$\Gamma_{X(2370)} = 117 \pm 10(\text{stat.}) \pm 8(\text{syst.})\ \text{MeV}$$

$J/\psi \to \gamma X(2370) \to \gamma K\bar{K}\eta'$:

$$M_{X(2370)} = 2376.3 \pm 8.7(\text{stat.})^{+3.2}_{-4.3}(\text{syst.})\ \text{MeV}$$
$$\Gamma_{X(2370)} = 83 \pm 17(\text{stat.})^{+44}_{-6}(\text{syst.})\ \text{MeV},$$

- The mixing angle and the mass shift of PS charmonium

$$\sin\theta = \frac{x}{m_{\eta_c} - m_X} \approx 0.080(10)$$

$$\delta m_{c\bar{c}} = m_{\eta_c} - m_{c\bar{c}} \approx \frac{x^2}{m_{\eta_c} - m_X} \approx +3.9(9)\ \text{MeV}$$

- **The decays of η_c and $X(2370)$ into light hadrons:**
  can be viewed as decaying into two gluons first and then hadronizing

$$\Gamma_{\eta_c} \approx \Gamma(\eta_c \to gg) = \frac{1}{32\pi}\frac{1}{m_{\eta_c}}|\mathcal{M}(c\bar{c} \to gg)|^2$$

$$\Gamma_X \approx \Gamma(X \to gg) = \frac{1}{32\pi}\frac{1}{m_X}|\mathcal{M}(X \to gg)|^2$$

- **This also applies to PS charmonium, such that ($\Gamma_X \approx 100$ MeV)**

$$\frac{|\mathcal{M}(X \to gg)|}{|\mathcal{M}(c\bar{c} \to gg)|} \approx \left(\frac{m_X\Gamma_X}{m_{\eta_c}\Gamma_{c\bar{c}}}\right)^{1/2}$$

Expt. known

$$\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}} \approx \left|\cos\theta + \sin\theta\frac{|\mathcal{M}(X \to gg)|}{|\mathcal{M}(c\bar{c} \to gg)|}\right|^2 \approx 1 + 2\sin\theta\left(\frac{m_X\Gamma_X}{m_{\eta_c}\Gamma_{\eta_c}}\right)^{1/2}\left(\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}}\right)^{1/2}$$

- **Thus, even though we cannot calculate the width directly, we have**

$$\Gamma_{\eta_c}/\Gamma_{c\bar{c}} \approx 1.27(4), \qquad \delta\Gamma_{\eta_c} \approx +7.2(8)\ \text{MeV}$$

IV. Preliminary results at the physical point

F. Chen, X. Jiang, **Y. Chen**, K.-F. Liu, W. Sun and Y.-B. Yang , arXiv: 2111.11929 (hep-lat)

- $N_f = 2 + 1$ **dynamical confiugrations generated by RBC/UKQCD Collaboration.**

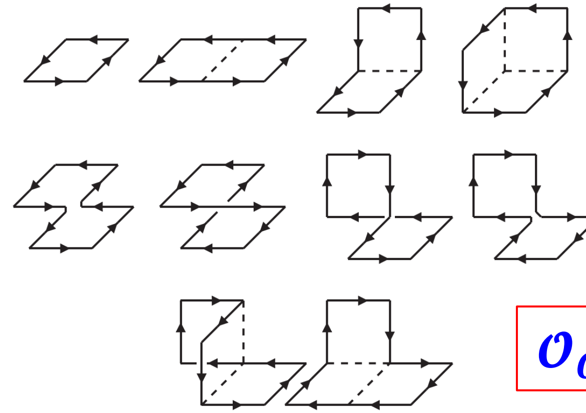
- **Accessed through the agreement between χQCD Collaboration (PI: Prof. K.-F. Liu of Univ. Kentucky)**

TABLE I. Parameters of 48I and 64I ensemble.

$L^3 \times T$	a (fm)	m_π (MeV)	La (fm)	$N_{\rm conf}$
$48^3 \times 96$	0.1141(2)	~ 139	~ 5.5	364
$64^3 \times 128$	0.0836(2)	~ 139	~ 5.3	300

- **Physical m_π, m_K, large volume, but small size of ensembles—— "physical point"**

- **Gauge invariant gluonic operators for glueballs —— build up in terms of Wilson loops**



$\boldsymbol{\mathcal{O}_G}$

- **AA-operators for glueballs**

$$\mathcal{O}_{AA}^{(LM;S)}(\vec{r}) = \frac{1}{N_r}\sum_{|\vec{r}|=r}\sum_{\vec{x}} c_{ij}(S)Y_{LM}(\hat{r})A_i(\vec{x}+\vec{r})A_j(\vec{x})$$

S: the total spin of two gauge field;
(LM): the orbital quantum number between two gauge fields;
N_r: the multiplicity of $\vec{r}$ with $|\vec{r}| = r$

- **AA-operators are not gauge invariant** → **Coulomb gauge!**

- **Bethe-Salpeter wave functions** from the $\mathcal{O}_{AA} - \mathcal{O}_G$ correlation functions

Optimized glueball operators: $\left\langle \mathcal{O}_G^{(n)}(t)\mathcal{O}_G^{(n)}(0)\right\rangle \approx e^{-m_n t} + \cdots$

$\mathcal{O}_{AA} - \mathcal{O}_G$ **correlation functions:**

$$\left\langle \mathcal{O}_{AA}(t)\mathcal{O}_G^{(n)}(0)\right\rangle \propto \langle \Omega|\mathcal{O}_{AA}(r)|n\rangle \left\langle n\left|\mathcal{O}_G^{(n)}\right|\Omega\right\rangle e^{-m_n t} \approx \Phi_n(r)e^{-m_n t} + \cdots$$

0^{++} 0^{-+}

2^{++} 2^{++}

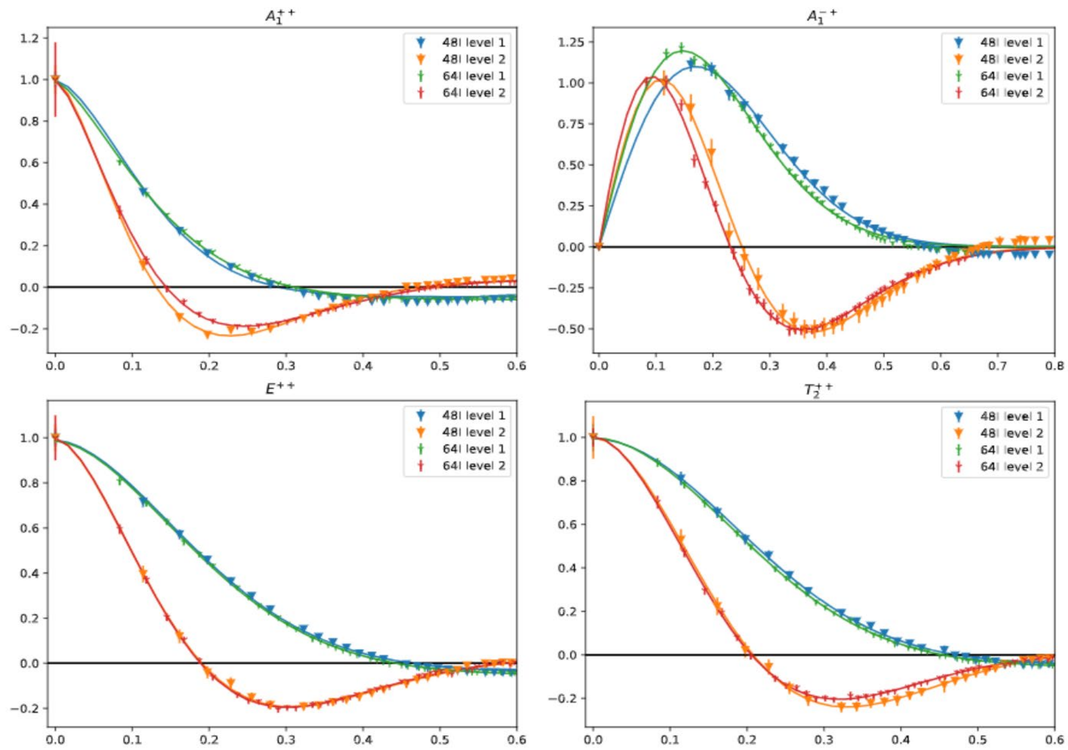
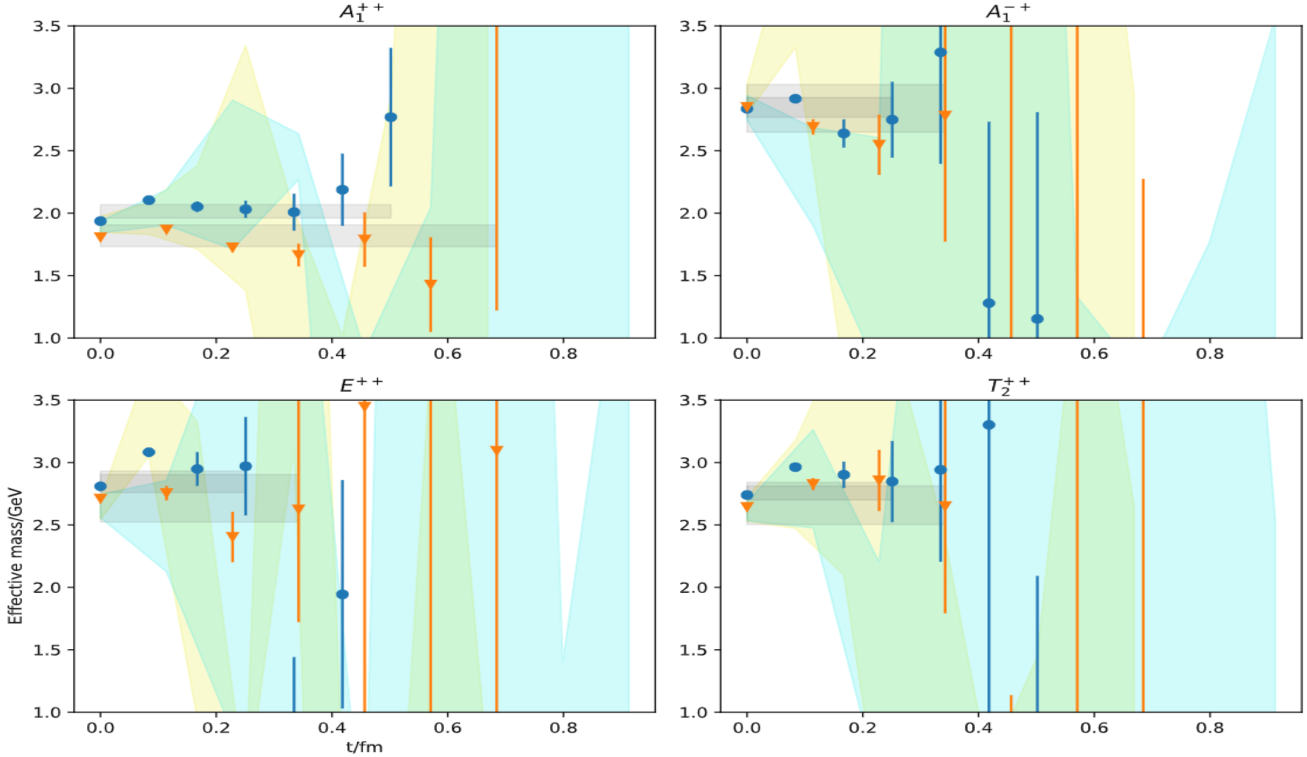


FIG. 2. Normalized BS wave functions of ground and first excited states on 48I and 64I lattice.

- **Effective mass plateaus**

  - $m_{eff}(t) = \ln\frac{C(t)}{C(t+1)} \to const.$ if $C(t)$ is exponential



0	A_1^{++}	E^{++}	T_2^{++}	A_1^{-+}
48I	1.82 ± 0.09	2.6 ± 0.2	2.7 ± 0.02	2.8 ± 0.2
64I	1.96 ± 0.08	2.7 ± 0.1	2.7 ± 0.2	2.8 ± 0.2

- **Observations**

a) **At $m_\pi \approx 139$ MeV, we obtain more or less signals for glueballs**

b) **The masses are slightly higher than QQCD predictions but understandable**

c) **BS wave functions (not theoretically rigorous)**

$0^{++}, 2^{++}$: **similar to S-wave (two gluons in S-wave ?)**
0^{-+}: **similar to P-wave (two gluons in P-wave?)**

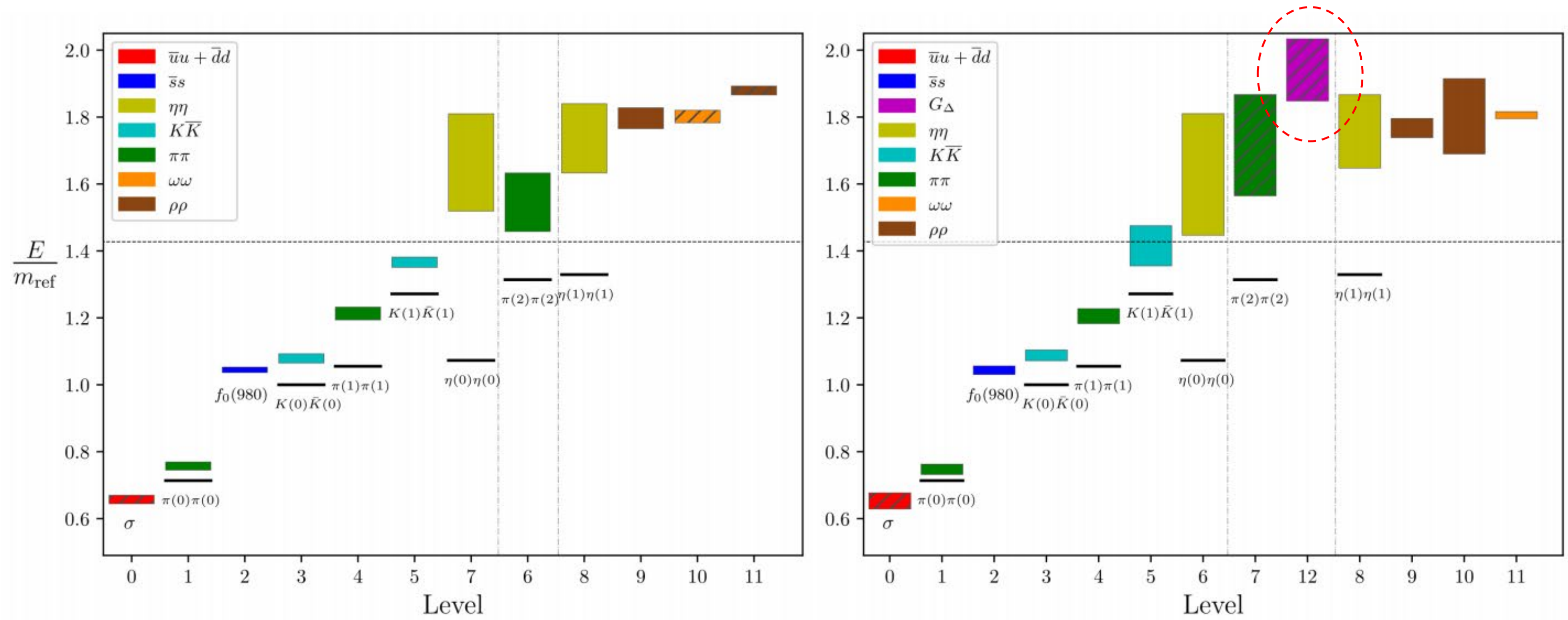
compatible with the QQCD results

P. de Forcrand and K.-F. Liu, Phys. Rev. Lett.69, 245(1992).
J. Liang et al. , Phys. Rev. D91, 054513 (2015)

d) **Statistics is bad, many systematical errors not under control.**

Spectroscopy from lattice QCD: The scalar glueball

R. Brett et al. AIP Conf. Proc. 2249 (2020) 030032 (arXiv: 1909.07306(hep-lat)



- **Hadron Spectrum Collaboration**
- **Quite a lot of operators: $\bar{q}q$, meson-meson, and glueball**
- **Black lines: two-meson thresholds**
- **Colored boxes: lattice energy levels (color corresponds to operator)**
- **Most states close to two-meson thresholds**
- **An additional state (around 1.9 GeV) observed when glueball operators are involved**

V. Summary

- **Glueball spectrum from QQCD and full QCD lattice studies**
- **Scalar, tensor glueballs have large branching fraction in J/psi radiative decays.**
- $\boldsymbol{f_0(1710)}$ **can be the best candidate for the scalar glueball**
- **Charmonium-glueball mixing raise the** $\boldsymbol{\Gamma_{\eta_c}}$ **a lot.**
- **Evidence of glueballs in full-QCD lattice studies at the physical point**
- **BS wave functions shed light on the internal structure of glueball?**
- $\boldsymbol{\Gamma(J/\psi \to \gamma\ (\bar{q}q))}$ **is desired.**
- **Systematics: glueball-meson mixing, glueball decays, etc.**
- **A long way to go!**

Thanks!

The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point for a meson (-like) system with given J^{PC} and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1\Gamma q_2 \quad [\bar{q}_1\Gamma_1 q\,][\bar{q}\Gamma_2 q_2\,] \quad [q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T\,], \ldots\ldots$$

- Two-point functions --- Observables

$$\mathcal{C}_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t)\mathcal{O}_j^+(0) \middle| 0 \right\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle \left\langle n \middle| \mathcal{O}_j^+ \middle| 0 \right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $\mathcal{C}_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue E_n:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + \vec{p}^2} + \Delta E,\ \vec{p} = \frac{2\pi}{L}\vec{n}$
- ……

Comparison of the hadron spectra

Euclidean spacetime lattice		Minkowski continuum spacetime
One particle states	⟷	Stable particles
Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume	⟶	Bound states of hadrons; Resonances
	↑	Continuum scattering states

All the energies are Discretized.

Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\ p\ L}{2\ \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

Resonances

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}, \quad \frac{p^{2l+1}}{\sqrt{s}} \cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

Bound states

$$p\cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2} r_0 |p_B|^2$$

$$T = \frac{}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B), \ p_B = i|p_B|$$

The situation will be much more complicated if the multi-channel coupling is considered and if there are three particle interactions.

Stickiness： two-photon coupling

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

$f_0(1710)$ $\Gamma(i)\Gamma(\gamma\gamma)/\Gamma(\text{total})$

$\Gamma(K\overline{K}) \times \Gamma(\gamma\gamma)/\Gamma_{\text{total}}$ **$\Gamma_1\Gamma_4/\Gamma$**

VALUE (eV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
$\mathbf{12^{+3}_{-2}{}^{+227}_{-\;\;8}}$		UEHARA	13	BELL	$\gamma\gamma \to K^0_S K^0_S$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<480	95	ALBRECHT	90G	ARG	$\gamma\gamma \to K^+ K^-$
<110	95	[1] BEHREND	89C	CELL	$\gamma\gamma \to K^0_S K^0_S$
<280	95	[1] ALTHOFF	85B	TASS	$\gamma\gamma \to K\overline{K}\pi$

[1] Assuming helicity 2.

$\Gamma(\pi\pi) \times \Gamma(\gamma\gamma)/\Gamma_{\text{total}}$ **$\Gamma_3\Gamma_4/\Gamma$**

VALUE (keV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
<0.82	95	[1] BARATE	00E	ALEP	$\gamma\gamma \to \pi^+\pi^-$

[1] Assuming spin 0.

To be confirmed…….

Belle, PTEP 2013 (2013) no.12, 123C01

TABLE VIII: Fitted parameters for the $f_0(1710)$ fit and $f_2(1710)$ fit. For the $f_0(1710)$ fit, the first errors are statistical and the second systematic; they are summarized in Table IX. The parameters where the H and L solutions are combined are also shown (explained in Sec. VI B 5).

Parameter	$f_0(1710)$ fit				$f_2(1710)$ fit	
	fit-H	fit-L	H,L combined	PDG	fit-H	fit-L
χ^2/ndf	694.2/585	701.6/585	–	–	796.3/585	831.5/585
Mass(f_J) (MeV/c^2)	1750^{+5+29}_{-6-18}	1749^{+5+31}_{-6-42}	1750^{+6+29}_{-7-18}	1720 ± 6	1750^{+6}_{-7}	1729^{+6}_{-7}
$\Gamma_{\rm tot}(f_J)$ (MeV)	138^{+12+96}_{-11-50}	145^{+11+31}_{-10-54}	139^{+11+96}_{-12-50}	135 ± 6	132^{+12}_{-11}	150 ± 10
$\Gamma_{\gamma\gamma}\mathcal{B}(K\bar{K})_{f_J}$ (eV)	12^{+3+227}_{-2-8}	21^{+6+38}_{-4-26}	12^{+3+227}_{-2-8}	unknown	$2.1^{+0.5}_{-0.3}$	1.6 ± 0.2

In the measurement of the no-tag mode of the process $\gamma\gamma \to K^0_S K^0_S$ [7], the $f'_2(1525)$ resonance with a structure corresponding to the $f_2(1270)$ and the $a_2(1320)$ mesons, and their destructive interference, were observed.

Belle，Phys.Rev. D97 (2018) no.5, 052003

In the present single-tag measurement (Fig. 12), a structure corresponding to the $f'_2(1525)$ state is clearly visible. A structure near the threshold of $K^0_S K^0_S$ is also visible that may be associated with the $f_0(980)$ and the $a_0(980)$ mesons. We do not find any prominent enhancement at the $f_2(1270)$ or the $a_2(1320)$ mass, and this feature is consistent with destructive interference.

Ordinary and extraordinary hadrons

R.L. Jaffe's talk (YKIS-2006, Kyoto), arXiv:hep-ph/0701038.

Ordinary hadrons:

Hadrons that exist in the large N_c limit as confined states.

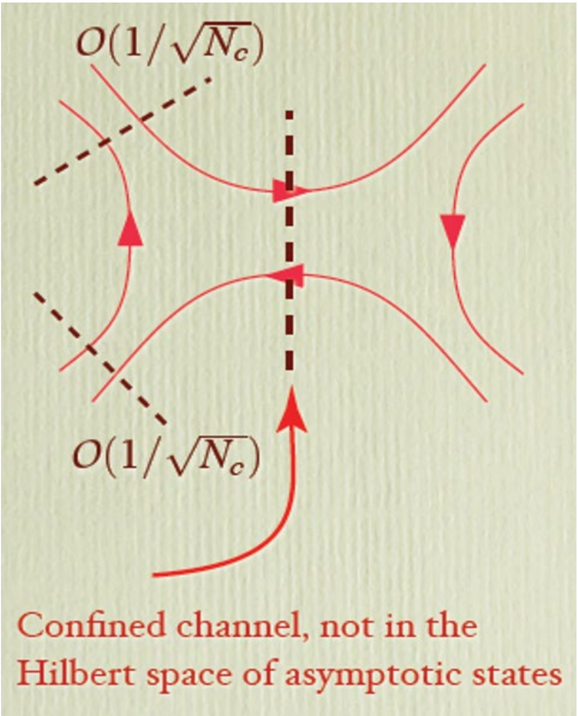
Namely, quark vacumm polarization is switched off.

They appear in the continuum scattering as resonances

Their width shrink to zero when $N_c \to \infty$

Resonance formation takes place by transition from meson-meson continuum (multiquark) to a confined channel that has no asymptotic states.

Extraordinary hadrons: vanish when $N_c \to \infty$

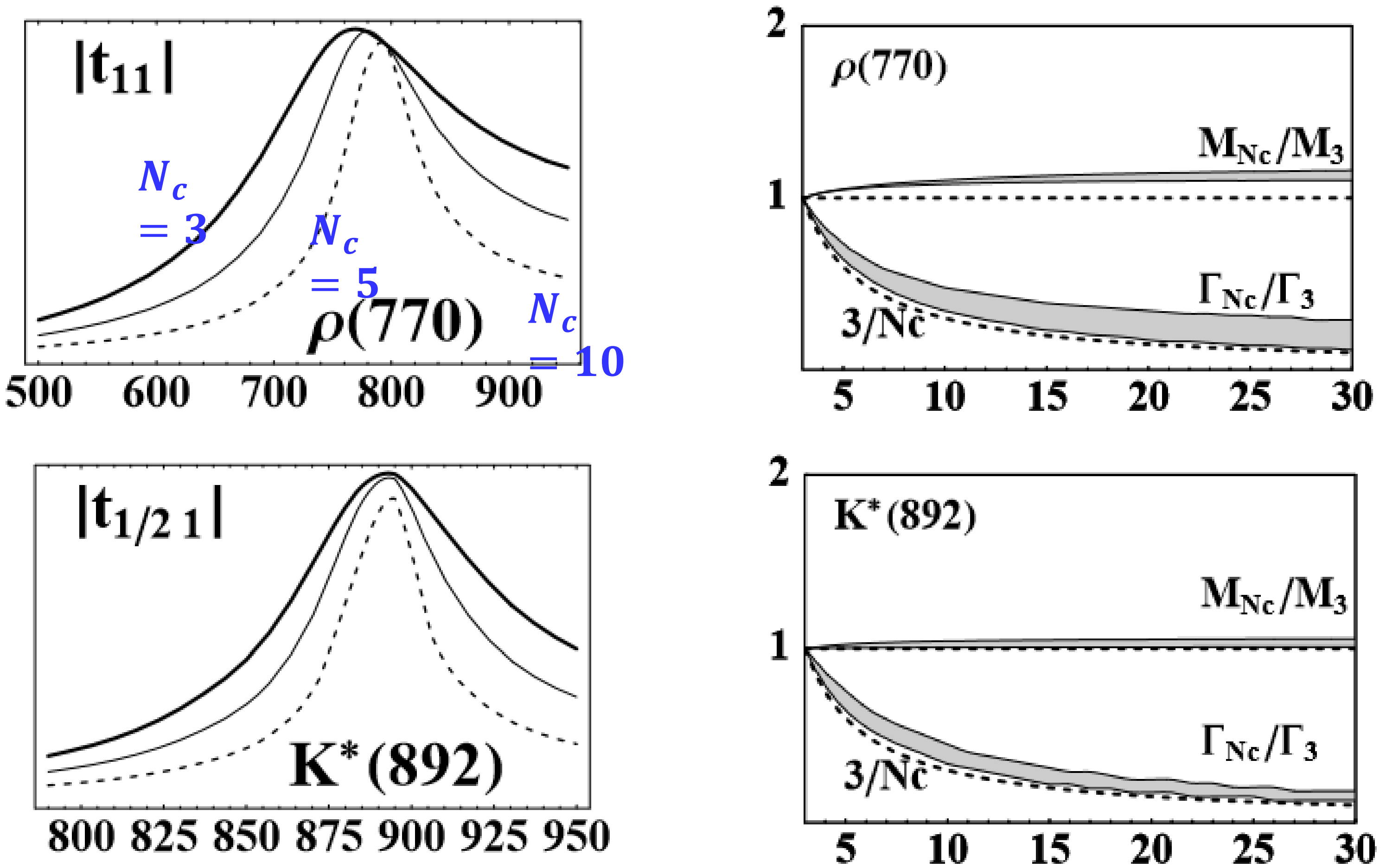


Scattering amplitude: $O\left(\frac{1}{N_c}\right)$

Meson decay width: $O\left(\frac{1}{N_c}\right)$

Meson-meson scattering from ChPT with different N_c

J.R. Pelaez, Phys. Rev. Lett. 92, 102001 (2004)



$\rho(770)$ and $K^*(892)$ hehave as expected. Their masses are roughly independent of N_c, while their widths go to zero when $N_c \to \infty$