$$\chi^{2} = \sum_{i=0}^{N} \left(\frac{y_{meas}^{i} - y_{fit}^{i}}{\sigma_{i}} \right)^{2}$$
$$f(x) = F(a_{i}, x) \ i = 0, 1, \dots, M$$
$$\chi^{2} = (\mathbf{y} - \mathbf{G}\mathbf{a})^{T} \mathbf{W}(\mathbf{y} - \mathbf{G}\mathbf{a})$$
$$\mathbf{W} = \mathbf{C}_{y}^{-1}$$
$$\mathbf{C}_{a} = \left(\mathbf{G}^{T} \mathbf{C}_{y}^{-1} \mathbf{G}\right)^{-1}$$
$$\Delta a_{i} = \sqrt{(\mathbf{C}_{a})_{ii}}$$

$$y_n = f(x_n) + u_n + \sum_{m=0}^{n-1} \alpha_m (x_n - x_m)$$

$$\left(\boldsymbol{C}_{\boldsymbol{y}}\right)_{mn} = \sigma_n^2 \delta_{mn} + \sum_{j=0}^{Min[m,n]-1} \sigma_{\alpha_j}^2 (x_m - x_j) (x_n - x_j)$$

$$\sigma_{\alpha_j} = \frac{0.0136 GeV/c}{\beta P_t} \sqrt{\frac{d_j}{X_0}} \left(1 + 0.038 \ln \frac{d_j}{X_0}\right)$$

$$\boldsymbol{G_{mn}} = \frac{\partial F(a_i, x_n)}{\partial a_m}$$

- $a = (a_0, a_1, ..., a_M)$ $x = (x_0, x_1, ..., x_N)$ $y = (y_0, y_1, ..., y_N)$ detector layers : 0, 1, ..., N
 track fitting parameters : 0, 1, ..., M
- **y** : measurement quantity
- **a** : helix parameter
- **G** : parameter function relation matrix
- C_y : covariance matrix for y
- C_a : covariance matrix for a
- σ_n : spatial resolution
- σ_{α_i} : error by multiple scattering

Reference : Nuclear Inst. and Methods in Physics Research, A 910 (2018) 127–132

$$\lambda = \frac{\pi}{2} - \theta$$

$$\alpha = \frac{1(m)}{0.299792458 \cdot B}$$

$$\beta = \frac{P_t}{\sqrt{P_t^2 + m^2}}$$

$$R = \alpha \cdot P_t$$

$$\varphi = \cos^{-1} \left[\frac{(d_0 + R)^2 + R^2 - r^2}{2 \cdot R(d_0 + R)} \right]$$

$$\varphi_{e^-} = -\varphi_{e^+}$$

$$\omega = \frac{\varphi}{2} = \sin^{-1} \frac{\frac{r}{2}}{R}$$

$$s_{xy} = R \cdot \varphi$$

$$s = \sqrt{(R \cdot \varphi)^2 + z^2}$$

helix parameters
$$(d_0, z_0, \theta, \phi, P_t)$$

 $x = d_0 \cos \phi + R[\cos \phi - \cos(\phi + \phi)]$
 $y = d_0 \sin \phi + R[\sin \phi - \sin(\phi + \phi)]$
 $z = z_0 - R \tan \lambda \cdot \phi$
 $xy_{meas} = r \cdot \tan^{-1} \frac{y}{x}$
 $z_{meas} = z$

For RES only :

