Axion, a brief introduction

杨峤立 Qiaoli Yang
Jinan Univ. (Guangzhou)

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The standard model

Space-time: GR

Matter: Quarks and leptons

Forces: Yang-Mills fields

The Yang-Mills

The Yang-Mills part $\mathcal{L}_Y$ which describes elementary spin-1 bosons mediating the fundamental interactions is written as:

$$\mathcal{L}_Y = -\frac{1}{4g_3^2}\sum G^A_{\mu\nu}G^{\mu\nu A} - \frac{1}{4g_2^2}\sum F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4g_1^2}B_{\mu\nu}B^{\mu\nu}, \qquad (1)$$

where g_i are dimension-less constants that determine the respective interaction strength, $A = 1...8$, and $a = 1...3$. $G^A_{\mu\nu}$, $F^a_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength tensors corresponding to the SU(3), the SU(2), and the U(1) group respectively.

The Weyl-Dirac:

The Weyl-Dirac part $\mathcal{L}_W$ describes the spin-1/2 fundamental fermions (the quarks and the leptons) and their couplings to the gauge particles. Since the left-handed fermions are SU(2) doublets and the right-handed fermions are SU(2) singlets, the standard model is chiral. The quarks are SU(3) triplets and the leptons are SU(3) singlets so the leptons do not participate the strong interactions. One can write the Weyl-Dirac Lagrangian as:

$$\mathcal{L}_W = L_i^\dagger \sigma^\mu D_\mu L_i + \bar{e}_i^\dagger \sigma^\mu D_\mu \bar{e}_i + Q_i^\dagger \sigma^\mu D_\mu Q_i + \bar{u}_i^\dagger \sigma^\mu D_\mu \bar{u}_i + \bar{d}\,_i^\dagger \sigma^\mu D_\mu \bar{d}_i \quad , \tag{2}$$

where D_μ are the covariant derivatives of the respective fermion fields. For example, $D_\mu \bar{e}_i = (\partial_\mu + iB_\mu)\bar{e}_i$, et al.

The Higgs part $\mathcal{L}_H$ is composed by the Higgs doublet: $H = \binom{H_1}{H_2}$. The Lagrangian is:

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 \quad .$$

Finally the Yukawa part $\mathcal{L}_{Yu}$ can be written as:

$$\mathcal{L}_{Yu} = iL_i^T \sigma_2 \bar{e}_j H^* Y_{ij}^e + iQ_i^T \sigma_2 \bar{d}_j H^* Y_{ij}^d + iQ_i^T \sigma_2 \bar{u}_j \tau_2 H Y_{ij}^u + c.c. \tag{4}$$

where Y_{ij}^k are 3×3 matrices of the respective Yukawa couplings. The matrix of the leptons can be always written as a real diagonal matrix in an expense of mixing the lepton fields. For the quark part, there are two matrices appearing in the Lagrangian thus one finds:

$$\mathcal{L}_{Yu\ Quark} = iQ_i^T \sigma_2 y_{ii}^d \bar{d}_i H^* + iQ_i^T \sigma_2 U_{ji} y_{jj}^u \bar{u}_j \tau_2 H + c.c. \tag{5}$$

where $y_{ii}^{d,u}$ are real numbers, and U_{ji} is the Cabibbo-Kobayashi-Maskawa matrix which has three mixing angles plus one phase.

The three gauge couplings, four parameters of the CKM matrix, the nine masses of the fermions, the two term in the Higgs part=18

We have one additional term: the QCD vacuum angle

The $U(1)_A$ Problem

- The light u-d quark Lagrangian in the chiral limit:

$$\mathcal{L}_{QCD} \simeq -\frac{1}{4} G^{A}_{\mu\nu} G^{\mu\nu A} + \sum_{i=1}^{2} \bar{q}_i \gamma_\mu D^\mu q_i,$$

- It has following symmetries:

$$SU_{L,R}(2): \; q \to U_{L,R} q_{L,R}; \quad U(1): \; q_{L,R} \to e^{i\alpha} q_{L,R};$$

$$U_A(1): \; q_L \to e^{i\theta} q_L; \quad q_R \to e^{-i\theta} q_R$$

- The symmetry is broken due to quark pair condensation

$$< q_L^{\dagger i} q_{Rj} >= \delta^i_j \mu^3.$$

- The vector symmetry is unbroken so

$$SU_L(2) \times SU_R(2) \to SU_{L+R}(2).$$

- Three generators are broken so there are three pseudo Nambu-Goldstone bosons.

$$\pi^+, \pi^-, \pi^0.$$

- What happened to the $U(1)_A$? It should also be broken and give the fourth pseudo Nambu-Goldstone which is not observed.
- Solution: $U(1)_A$ is explicitly broken due to QCD instanton effect.

- A classical field configuration of QCD vacuum: $A_\mu = i/gU\partial_\mu U^\dagger$
  with the winding number n

$$n = -\frac{1}{24\pi^2}\int d^3x\epsilon^{ijk}\, Tr[(U\partial_i U^\dagger)(U\partial_j U^\dagger)(U\partial_k U^\dagger)].$$

cannot be smoothly deformed into others with a different winding number without passing energy barriers

- However these field configurations with different winding numbers can tunnel to each other due to instantons.

$$< n2|H|n1 > \sim e^{-S}$$

- So the physical vacuum has to include field configuration with all possible winding numbers thus has the form:

$$|\omega >= \sum_n e^{in\theta}|n >.$$

The θ vacuum has observable effects. Considering the vacuum to vacuum transition amplitude:

$$\begin{aligned} < \theta_1|e^{-Ht}|\theta > &= \sum_{n_1}\sum_{n} e^{-i(n_1\theta_1 - n\theta)} < n_1|e^{-Ht}|n > \\ &= \sum_{n_1} e^{-in_1(\theta_1-\theta)} \sum_{n_1-n} \int [DA_\mu]_{n_1-n} exp[-\int d^4xL - i(n_1-n)\theta] \\ &= \delta(\theta_1-\theta)\int [DA_\mu]_q exp[-\int d^4x(L + \frac{\theta}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu a})], \end{aligned} \tag{7}$$

where $\int[DA_\mu]_{n_1-n}$ is the functional integration with respect to gauge configurations $n_1 - n = q$. An effective interaction term is presented: $\theta/32\pi^2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ due to the θ vacuum. Since $F\tilde{F}$ is CP odd, QCD will be not CP invariant when $\theta \neq 0$.

The following transformation:

$$\theta F\tilde{F} \to (\theta - \sum \alpha_i)F\tilde{F} \tag{8}$$

$$\bar{q}_i m_i q_i \to \bar{q}_i e^{(i\alpha_i\gamma_5/2)} m_i e^{(i\alpha_i\gamma_5/2)} q_i \tag{9}$$

can annihilate the θ term if $\sum \alpha_i = \theta$. However, the expense is the additional mass terms:

$$\sum (m_i cos\alpha_i \bar{q}_i q_i + m_i sin\alpha_i \bar{q}_i \gamma_5 q_i). \tag{10}$$

Thus the parameter determines the CP violation of the QCD is $\bar{\theta} = \theta + argdetM$. The $\bar{\theta}$ term gives rise to a contribution to the neutron electric dipole moment.

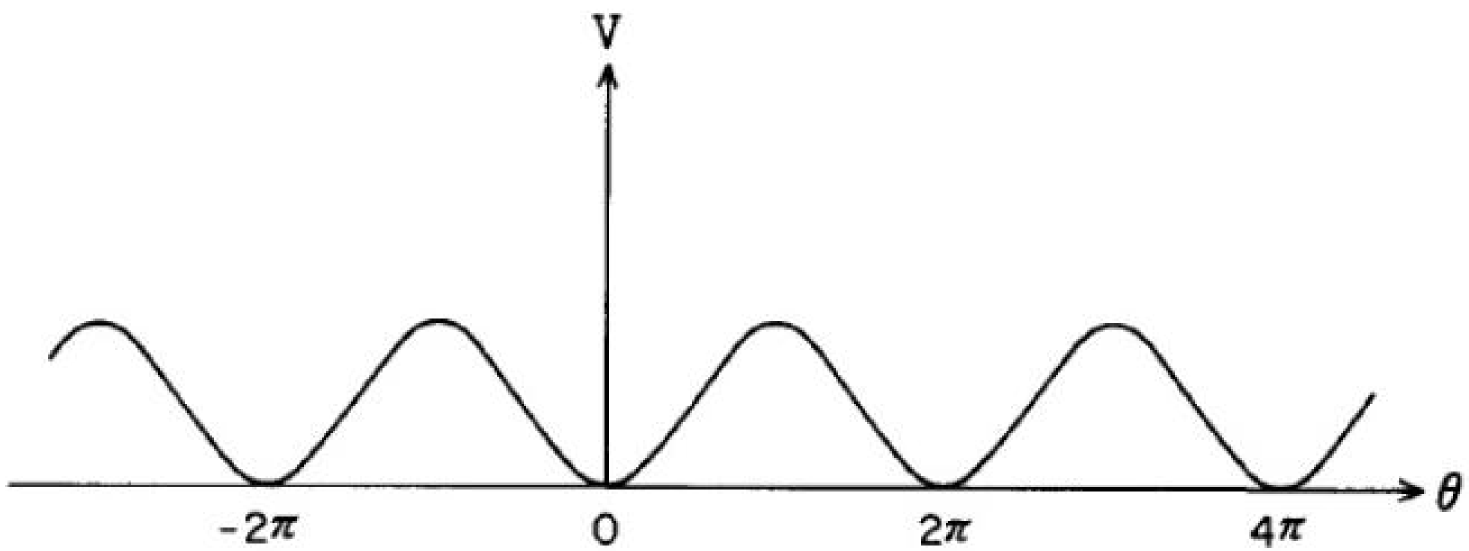
- This term violates CP in-variance if $\theta \neq 0$
- The measurement of electric dipole moment of neutron gives a upper limit:

$$|\theta| \leq 10^{-9}$$

A proof that the energy is minimized when $\bar{\theta} = 0$ is given by C. Vafa and E. Witten [4]. Considering the path integral of the QCD action in the Euclidean space, the QCD Lagrangian is: $\mathcal{L} = -1/4g^2 Tr(G_{\mu\nu}G_{\mu\nu}) + \sum \bar{q}_(D_\mu \gamma_\mu + m_i) q_i + i\bar{\theta}/32\pi^2 Tr(G_{\mu\nu}\tilde{G}_{\mu\nu})$. After integration of the fermions one finds:

$$e^{-VE} = \int DA_\mu det(D_\mu \gamma_\mu + M) e^{\int d^4x [1/4g^2 Tr G_{\mu\nu} G_{\mu\nu} - i\bar{\theta}/32\pi^2 G_{\mu\nu}\tilde{G}_{\mu\nu}]} \quad . \tag{11}$$

The energy due to the vacuum angle



To solve the strong CP problem, one introduces the $U(1)_{PQ}$ symmetry which is spontaneously broken

$$\begin{aligned} L &= -1/4g^2 Tr(G_{\mu\nu}G_{\mu\nu}) + \sum \bar{q}_(D_\mu \gamma_\mu + m_i) q_i \\ &+ \theta/32\pi^2 Tr G_{\mu\nu}\tilde{G}_{\mu\nu} + 1/2\partial_\mu a \partial^\mu a + a/(f_a 32\pi^2) Tr G_{\mu\nu}\tilde{G}_{\mu\nu}, \end{aligned}$$

$\theta + a/f_a \rightarrow 0$ relaxes to zero during QCD phase transition.

A example: the KSVZ axion

- One introduces an new complex scalar and a new heavy quark Q.

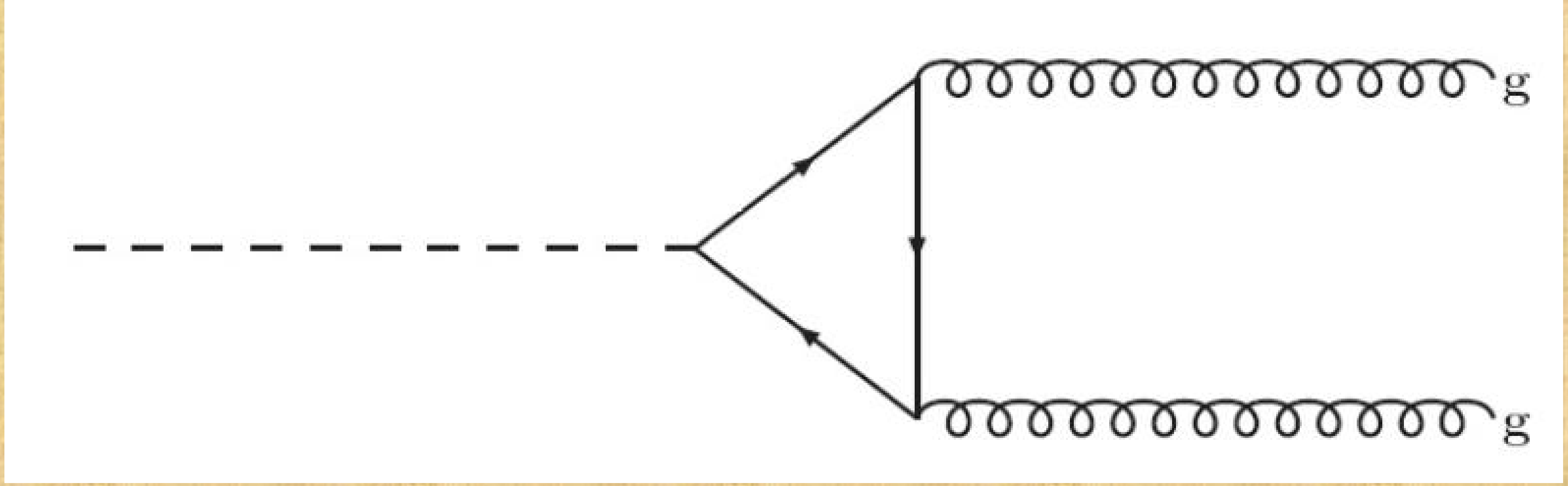
$$L_{Yu} = -fQ_L^\dagger \sigma Q_R - f^* Q_R^\dagger \sigma^* Q_R$$

$$V = -\mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2$$

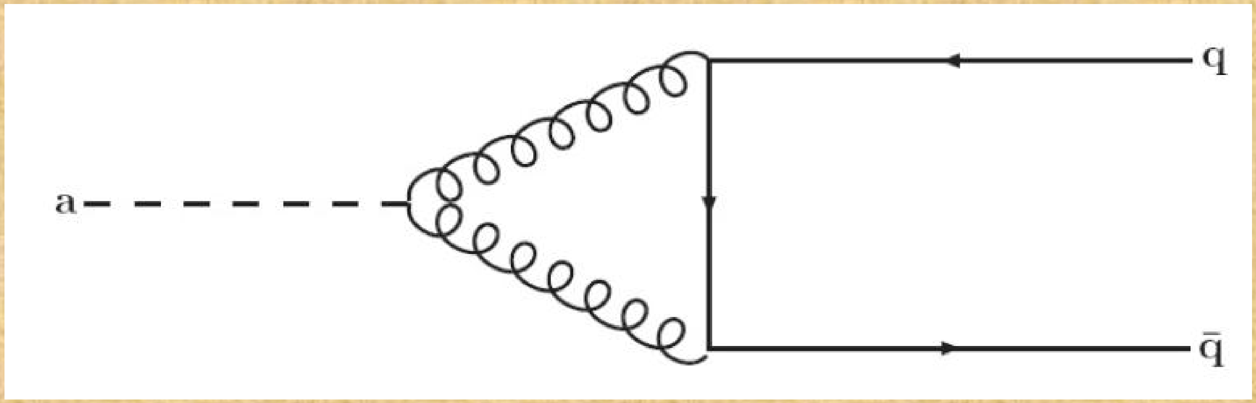
=> $\sigma = (v + \rho) exp(i\frac{a}{v}).$

$$\begin{aligned} U(1)_{PQ}: \quad a &\rightarrow a + f_a \alpha \\ \sigma &\rightarrow exp(iq\alpha)\sigma \\ Q_L &\rightarrow exp(iQ\alpha/2) Q_L \\ Q_R &\rightarrow exp(-iQ\alpha/2) Q_R \end{aligned}$$

- The ABJ anomaly gives the required effective axion-gluon-gluon coupling.



- After QCD phase transition, the “PQ” Nambu-Goldstone boson acquires mass due to instanton effects, hence becoming a quasi-Nambu-Goldstone boson, the “axion”.



Axion like particles

interested background fields in string theory:

- the metric $g_{\mu\nu}(X)$
- the two-form gauge antisymmetric field $B_{\mu\nu}$
- dilaton $\Phi(X)$

...

Axion like particles

- alps arises due to compactification of the antisymmetric tensor fields

$$B = \frac{1}{2\pi} \sum b^i(x)\omega_i(y) + \dots ,$$

- the x are non-compact coordinate, y are compact coordinates.

Axion like particles

- the zero mode acquires a potential due to non-perturbative effects on the compactification cycle.
- The effective Lagrangian in four dimension:

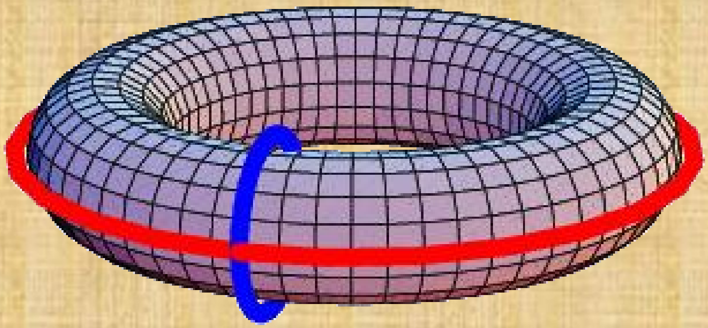
$$\mathcal{L} = \frac{f_{ALPs}^2}{2}(\partial a)^2 - \Lambda_{ALPs}^4 U(a)$$

The Axiverse

Svrcek and Witten, arXiv:hep-th/0605206

- String theory has extra dimensions which can be compactified.

- Axions are KK zero-modes of gauge fields compactified on closed cycles.



- Potentials from non-perturbative physics (D-branes, instantons etc.) give rise to axion masses.

So there are many different axions

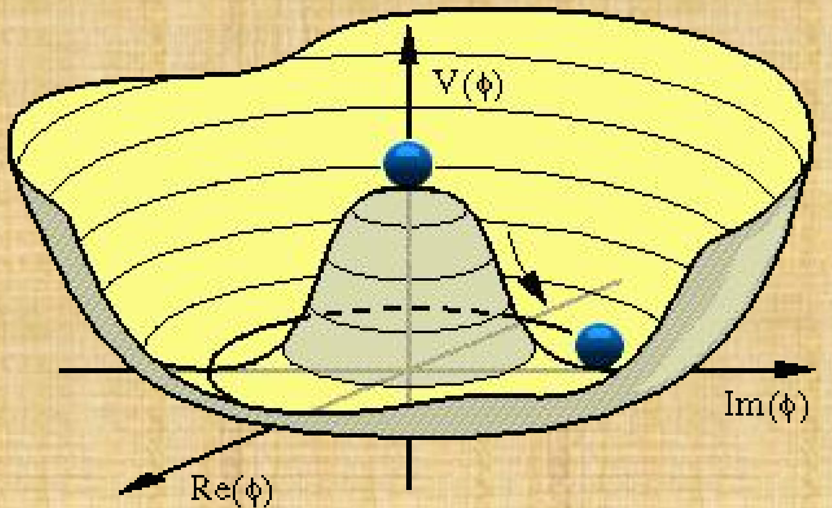
One of the axions in the axiverse coupled to the QCD sector

- This axion solves the strong CP problem:

$$L \supset \phi \tilde{F} F$$

- SSB after f then instantons tilt the hat.

- So the QCD axion is light.



Other axion species have less constraints

ALPs

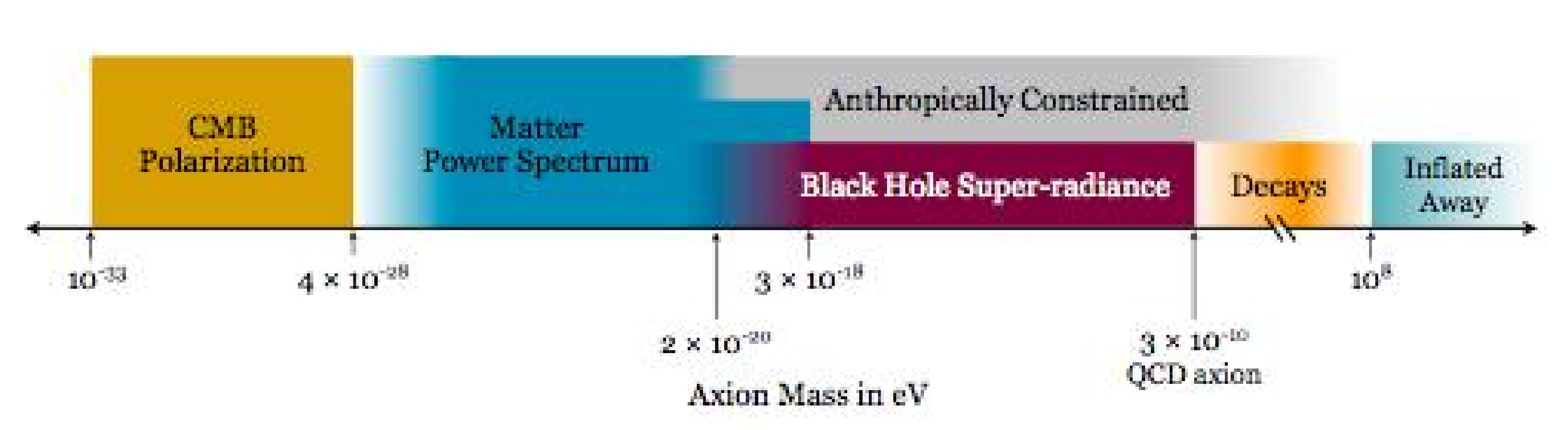
So if the string theory is true, alps is inevitably.

Question is: what is the mass?

The other known BSM phenomena:

- Massive neutrinos
- Inflation
- Dark energy
- ...

The Axiverse may explain a lot of



Arvanitaki et al PRD 81, 123530 (2010)

a side-note

- Particle physics always try to find fundamental particles which constitute matter and mediate forces
- Axion can be both matter (massive, stable) and force mediator (boson)

Cold Dark Matter (CDM)

- CDM is widely believed to be an important part of the universe based on a large number of observations:

a. Dynamics of galaxy clusters.
b. Rotation curves of galaxies.
c. Abundance of light elements.
d. Gravitational lensing.
e. Anisotropies of the CMBR.

CDM

- Accounts 23% of total energy density of universe while baryonic matter accounts 4%.
- Properties:

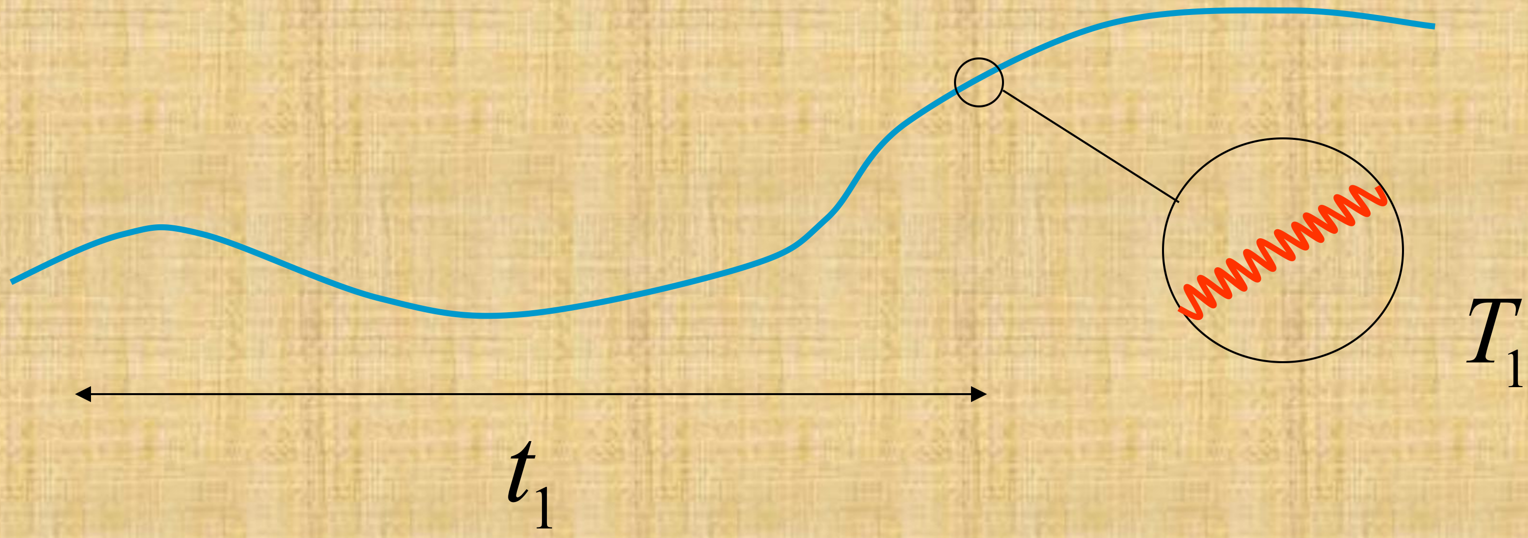
a. Pressureless

Primordial velocity is very small , at most ~ $10^{-8}c$ today .

b.Collisionless

Cold dark matter is weakly interacting (so dark), except for gravity.

There are two cosmic axion populations: hot and cold.



When the axion mass turns on, at QCD time,

$$t_1 \simeq 2 \cdot 10^{-7} \text{ sec}$$

$$T_1 \simeq 1 \text{ GeV}$$

Axion field in early universe

The axion field evolves in an expanding flat FRW universe as

$$\partial_t^2 a + 3H\partial_t a - \frac{1}{R^2}\nabla^2 a + \partial_a V(a) = 0 \ , \qquad (10)$$

where R is the scale factor, $H = \dot{R}/R$ is the Hubble parameter, and $V(a)$ is the potential of the axion field.

The potential term is

$$V(a) \approx f_a^2 m_a^2(T)\left[1 - \cos(\frac{a}{f_a})\right] \quad . \qquad (11)$$

If $T > \Lambda_Q \sim 200\text{MeV}$, the mass is temperature dependent

$$m_a(T) \propto m_0 b \left(\frac{\Lambda_Q}{T}\right)^4 \quad , \qquad (12)$$

where $b \sim \mathcal{O}(10^{-2})$ and when $T \lesssim \Lambda_Q$, the axion mass m_0 is almost a constant.

The axion field fluctuations

$$\langle a_0^2 \rangle = (\theta_0^2 + \sigma_\theta^2) f_a^2 \quad , \tag{14}$$

Notice that the axion energy density was transferred from the QCD sector so the total energy density was conserved

$$\delta\rho_{total} = m_a \delta n_a + m_i \delta n_i + 4\rho_{rad} \frac{\delta T}{T} = 0. \tag{15}$$

The fluctuation of number density n_i to entropy density is

$$\delta S_i = \frac{\delta(n_i/s)}{n_i/s} = \delta n_i/n_i - 3\delta T/T. \tag{16}$$

The axion field fluctuations

As the $U(1)$ PQ symmetry breaking happened before inflation, the standard deviation is at the order of

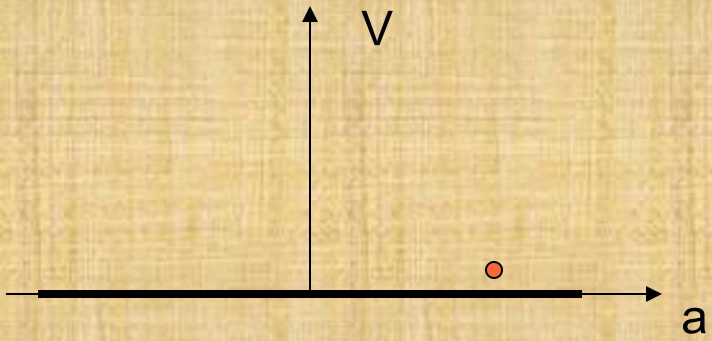
$$\sigma_\theta \sim \frac{H_I}{2\pi f_a} \quad , \tag{19}$$

due to the Gibbons-Hawking temperature at inflation.

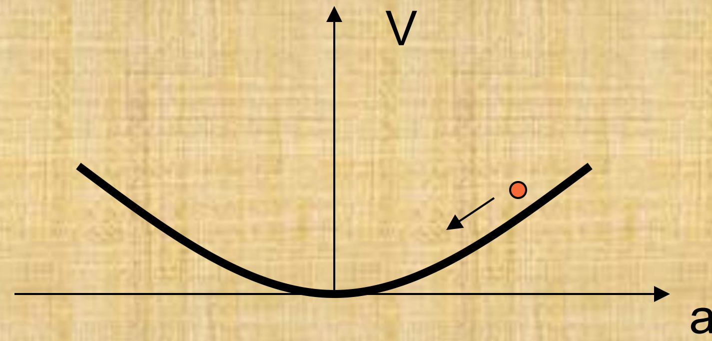
Denoting the energy density of the particle specie i to the photon number density as $\xi_i(T) = \rho_i(T)/n_\gamma(T)$, the isocurvature perturbation is

$$\left\langle \left(\frac{\delta T}{T}\right)^2_{\rm iso} \right\rangle \sim \left(\frac{\xi_a}{3\xi_{\rm matter}}\right)^2 \langle \delta S_a^2 \rangle \lesssim \mathcal{O}(10^{-11}). \tag{20}$$

Axion production by vacuum realignment



$T \geq 1\,\mathrm{GeV}$



$T \leq 1\,\mathrm{GeV}$

$$n_a(t_1) \simeq \frac{1}{2} m_a(t_1)\, a(t_1)^2 \simeq \frac{1}{2t_1} f_a{}^2\, \alpha(t_1)^2$$

initial misalignment angle

$$\rho_a(t_0) \simeq m_a\, n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a{}^{-\frac{7}{6}}$$

Cold axion properties

- number density

$$n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^{\frac{5}{3}} \left(\frac{a(t_1)}{a(t)} \right)^3$$

- velocity dispersion

$$\delta \text{v}(t) \sim \frac{1}{m_a\, t_1} \frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\text{N} \sim n(t) \frac{(2\pi)^3}{\frac{4\pi}{3} (m_a\, \delta \text{v})^3} \sim 10^{61} \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^{\frac{8}{3}}$$

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axion CDM and BEC

www.phys.ufl.edu/~sikivie/Zierler_Interview.pdf

Interviewee: PIERRE SIKIVIE | January 20, 2021
By: DAVID ZIERLER | Videoconference

ZIERLER: Okay. This is David Zierler, Oral Historian for the American Institute of Physics. It is January 20th, 2021. I'm delighted to be here with Professor Pierre Sikivie. Pierre, it's great to see you. Thank you so much for joining me.

SIKIVIE: Thank you. Thank you for having me as your interviewee.

the dark matter must at least in part be axions or an axion-like particle. Maybe not the original axion, but something that behaves like it. This has to do with Bose-Einstein condensation and is a very long argument.

Review of Cold axion properties

1. Small velocity dispersion:

$$\Delta v \sim [a(t_1)/a(t)] \cdot 1/mt_1 \qquad .$$

2. High physical space density:

$$n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left(\frac{f}{10^{12}\,\text{GeV}}\right)^{5/3} \left(\frac{a(t_1)}{a(t)}\right)^3$$

Cold axions form BEC if they thermalize

- BEC results from the impossibility to allocate additional charges to the excited states for given temperature.
- Axion particle number is effectively conserved and is the relevant charge.
- Cold axions’ effective temperature is below the critical temperature which is very high due to axions high number density.

axion BEC in three arenas:

1. in the linear regime within the horizon.

2. in the non-linear regime within the horizon.

3. upon entering the horizon.

Axion BEC in the linear regime within horizon.

- From first order of density perturbation theory within the horizon, we get:

$$\partial_t \delta + \frac{1}{a}\vec{\nabla}\cdot\vec{v} = -3\partial_t\phi + \frac{3H}{4m^2a^2}\nabla^2\delta$$

$$\partial_t\vec{v} + H\vec{v} = -\frac{1}{a}\vec{\nabla}\psi + \frac{1}{4m^2a^3}\vec{\nabla}\,\nabla^2\delta$$

where $\delta(\vec{x},t) \equiv \frac{\delta\rho(\vec{x},t)}{\rho_0(t)}$

This implies a nonzero Jeans length compared with the collisionless DM.

$$\partial_t^2 \delta + 2H\partial_t \delta - \left(4\pi G\rho_0 - \frac{k^4}{4m^2a^4}\right)\delta = 0$$

From equation above, one gets:

$$k_j^{-1} = 1.02 \cdot 10^{14}\,\mathrm{cm}(10^{-5}\,\mathrm{eV}/m)^{1/2}[(10^{-29}\,\mathrm{g}/\mathrm{cm}^3)/\rho]^{1/4}$$

- The equation of motion in non-linear regime within horizon:

$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\vec{\nabla} \times \vec{v} = 0$$

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\vec{F}_g - \vec{\nabla} q$$

$$q = -\nabla^2 (\psi\psi^*)^{1/2} / [2m^2 (\psi\psi^*)^{1/2}]$$

- In the galactic halos BEC has the property
$\vec{\nabla} \times \vec{v} \neq 0$ by appearance of vortices.

- BEC is consistent with small scale structures.

- Most many particle systems are in the particle kinetic regime where:

$$\Gamma >> \delta E$$

- The cold axions are in the opposite regime:

$$\Gamma << \delta E$$

Let us call it “condensed regime”.

- The question is: starting with an arbitrary initial state, how quickly will the average axion state occupation numbers approach a thermal distribution?

- We derive evolution equations for the out of equilibrium system, as an expansion in powers of the coupling strength.

- The first order terms average to zero in the kinetic regime. The second order terms yield the ordinary Boltzmann equation.

- The axions can be written:

$$\varphi(\vec{x},t) = \sum_{\vec{n}} \left[a(t)_{\vec{n}}\ \Phi_{\vec{n}}(x)\ +\ a^{\dagger}_{\vec{n}}(t)\Phi^{\star}_{\vec{n}}\right]$$

- The Hamiltonian is

$$H = \sum_{\vec{n}} \omega_{\vec{n}} a^{\dagger}_{\vec{n}} a_{\vec{n}} + \sum_{\vec{n}_1,\vec{n}_2,\vec{n}_3,\vec{n}_4} \Lambda^{\vec{n}_3,\vec{n}_4}_{\vec{n}_1,\vec{n}_2} a^{\dagger}_{\vec{n}_1} a^{\dagger}_{\vec{n}_2} a_{\vec{n}_3} a_{\vec{n}_4}$$

Solve the Heisenberg equatio $\dot{a}_{\vec{n}} = i[H, a_{\vec{n}}]$
perturbatively,

$$a_{\vec{n}}(t) = e^{-i\omega_{\vec{n}}t}(A_{\vec{n}} + B_{\vec{n}}(t)) + \mathcal{O}(\Lambda^2)$$

where $B_{\vec{n}}(0) = 0,\ A_{\vec{n}} = a_{\vec{n}}(0)$.

One gets:

$$
\begin{aligned}
\dot{N}_{\vec{n}}(t) \;&=\; [-i \sum_{\vec{i},\vec{j},\vec{k}} \Lambda^{\vec{j},\vec{k}}_{\vec{n},\vec{i}} \; A^{\dagger}_{\vec{n}} \; A^{\dagger}_{\vec{i}} \; A_{\vec{j}} \; A_{\vec{k}} \; e^{-i\Omega t} + h.c.] \\
&+ \; [\sum_{\vec{i},\vec{j},\vec{k}} |\Lambda^{\vec{j},\vec{k}}_{\vec{n},\vec{i}}|^2 \{N_{\vec{i}} N_{\vec{j}} (N_{\vec{k}} + 1)(N_{\vec{n}} + 1) - N_{\vec{k}} N_{\vec{n}} (N_{\vec{i}} + 1)(N_{\vec{j}} + 1)\} \frac{sin(\Omega t)}{\Omega} \\
&+ \; off - diagonal \;\; 2rd \;\; order \;\; terms] + \mathcal{O}(\lambda^3) \quad .
\end{aligned}
$$

- The second order terms yield the Boltzmann equation. The first order terms, along with the off diagonal second terms average to zero in the “kinetic regime”.

- In the condensed regime, the first order terms no longer average to zero and dominate. Considering the high occupation numbers of axion states, we can replace the operator A, A^+ with complex numbers whose magnitude is of order $\sqrt{N_{\vec{n}}}$.

We get a c-number equation:

$$< \dot{N}_{\vec{n}} > \simeq \sum_{\vec{i},\vec{j},\vec{k}} \Lambda_{\vec{n},\vec{i}}^{\vec{j},\vec{k}} \sqrt{< N_{\vec{n}} >< N_{\vec{i}} >< N_{\vec{j}} >< N_{\vec{k}} >}$$

which lead to the thermalization rate :

$$\Gamma_g \sim 4\pi G n m^2 l^2,$$

for the gravitational interaction.