2022年理论物理前沿讲习班：暗物质与新物理暑期学校，2022/07/06-07/07

Introduction to B Physics

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Outline

- ☐ Flavor physics: what/why/how
- ☐ Quark mixing and CKM matrix
- ☐ CP violation: history, status, why B decays
- ☐ B physics: basics, examples, and status, …
  - ➢ Low-energy effective Hamiltonian
  - ➢ Calculation of Wilson coefficients
  - ➢ Two-loop calculation of hadronic matrix elements
  - ➢ Various B anomalies and possible explanations
- ☐ Summary

Flavor physics:
what/why/how

What is flavor

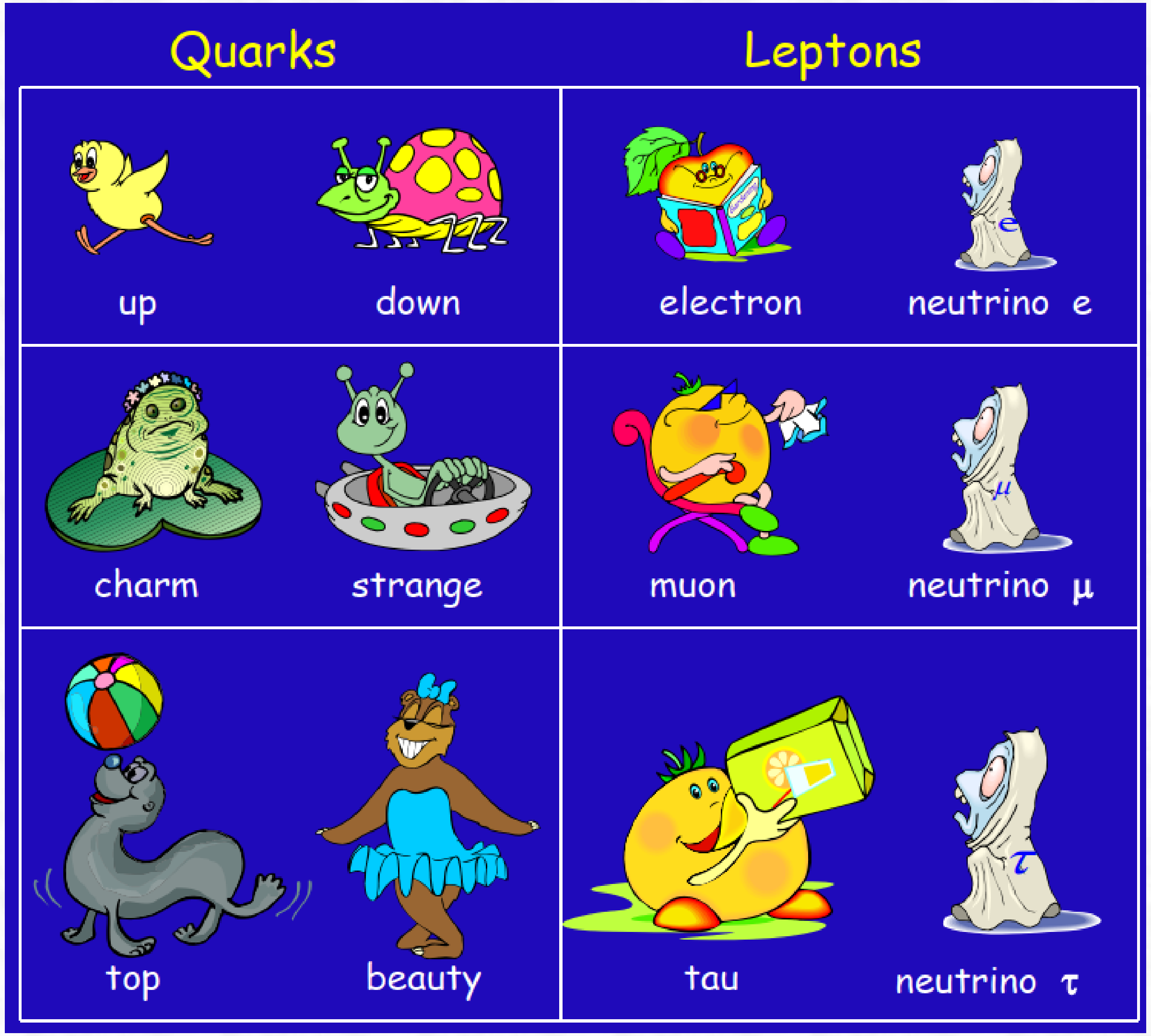
- The term “Flavor” was coined by Harald Fritzsch and Murray Gell-Mann at a Baskin-Robbins ice-cream store in Pasadena in 1971.

“Just as ice cream has both color and flavor so do quarks.”

- In particle physics, flavour or flavor refers to the species of an elementary particle. The Standard Model counts six flavors of quarks and six flavors of leptons. They are conventionally parameterized with flavor quantum numbers that are assigned to all subatomic particles.

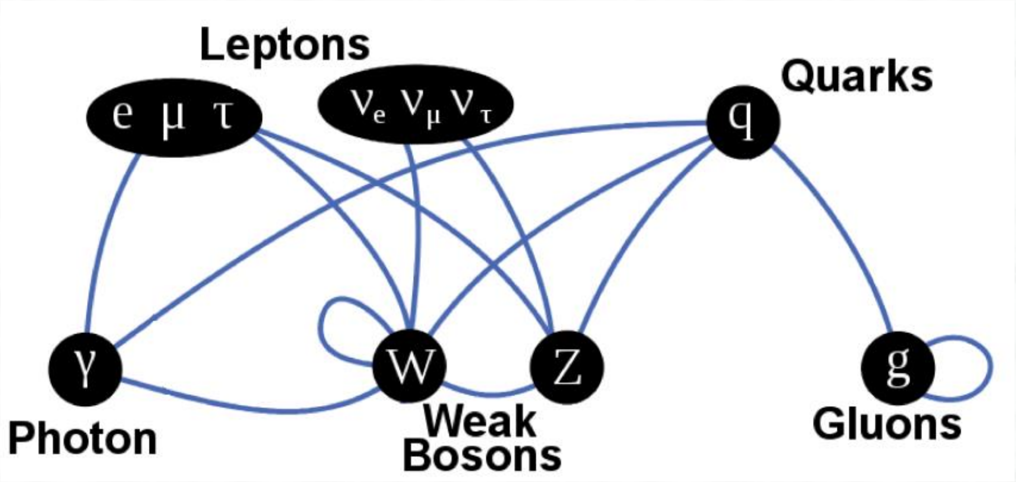
https://en.wikipedia.org/wiki/Flavour_(particle_physics)

What is flavor



Quark flavor **Lepton flavor**

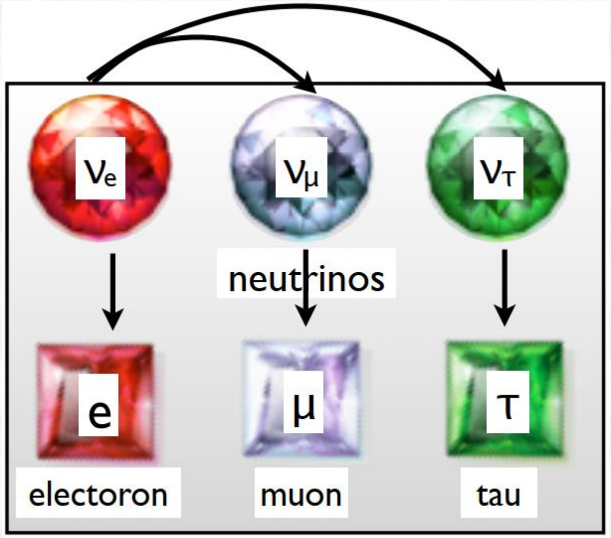
Heavy flavor **Light flavor**

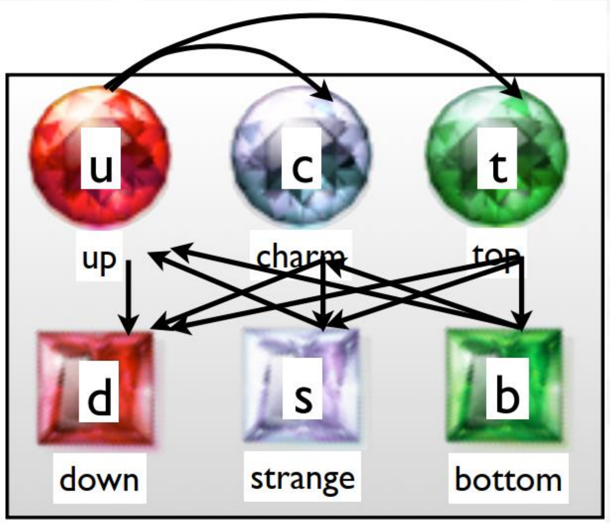


Gauge interactions make a quark a quark, a lepton a lepton; strengths are identical for the 3 generations.

What is flavor physics

- □ Interactions that distinguish among the generations:
  - – Neither strong nor electromagnetic interactions
  - – Within the SM: only weak and Yukawa interactions
- □ In the weak-interaction basis:
  - – Weak interactions are always flavor-universal
  - – Sources of all SM flavor physics: Yukawa interactions among gauge interaction eigenstates
- □ Flavor parameters:
  - – Parameters with flavor indices (m_i, V_{ij})





Very wide topic: • Neutrinos • Charged leptons • Kaon (strange) physics • Charm physics • B physics • (Some) top-quark physics

Flavor parameters in the SM quark sector

□ **Question:** How many independent parameters does the SM quark flavor sector contain?

Sector	Lagrangian
$\mathcal{L}_{\rm kin}^{\rm gauge}$	$-\frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu}-\frac{1}{4}W^{a\mu\nu}W^{a}_{\mu\nu}-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}$
$\mathcal{L}_{\rm kin}^{\rm fermion}$	$\overline{q^0_{L\alpha}}i\not{D}q^0_{L\alpha}+\overline{u^0_{R\alpha}}i\not{D}u^0_{R\alpha}+\overline{d^0_{R\alpha}}i\not{D}d^0_{R\alpha}+\overline{\ell^0_{L\alpha}}i\not{D}\ell^0_{L\alpha}+\overline{e^0_{R\alpha}}i\not{D}e^0_{R\alpha}$
$\mathcal{L}_{\rm Higgs}$	$(D_\mu\phi)^\dagger(D^\mu\phi)-V(\phi)$
$\boxed{\mathcal{L}_{\rm Yukawa}}$	$-Y^d_{\alpha\beta}\overline{q^0_{L\alpha}}\phi d^0_{R\beta}-Y^u_{\alpha\beta}\overline{q^0_{L\alpha}}\tilde{\phi}u^0_{R\beta}-Y^\ell_{\alpha\beta}\overline{\ell^0_{L\alpha}}\phi e^0_{R\beta}+\text{h.c.}$

WBTs: $\begin{cases} q^0_L=W^q_L q'_L, \quad u^0_R=W^u_R u'_L, \quad d^0_R=W^d_R d'_R \\ \ell^0_L=W^\ell_L \ell'_L, \quad e^0_R=W^e_R e'_R \end{cases}$

- **The two complex Yukawa matrices Y^u and Y^d introduce 36 free parameters**
- **The chiral quark fields q_L, u_R, d_R can be chosen in arbitrary flavor basis (i.e., WBTs)**

  ⟶ **9+9+9=27 symmetry generators of $U(3)_{q_L}\otimes U(3)_{u_R}\otimes U(3)_{d_R}$**

- **One overall $U(1)$ symmetry (baryon number conservation) accidentally remains after EWSB**
- **The # of independent physical parameters in the SM quark sector:**

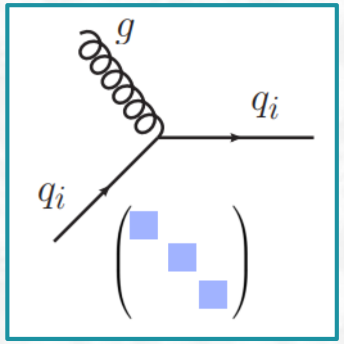
  ⟶ $\boxed{(\#\text{of entries})-(\#\text{of broken symmetry generators})=36-(27-1)=10}$

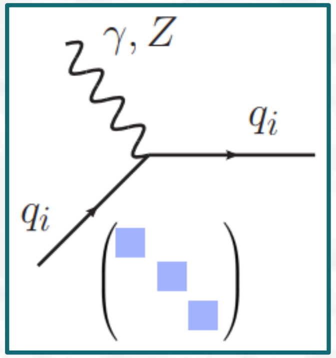
6 quark masses

3 CKM angles

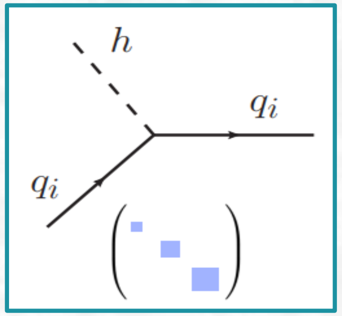
1 CP-violating phase

Flavor dictionary

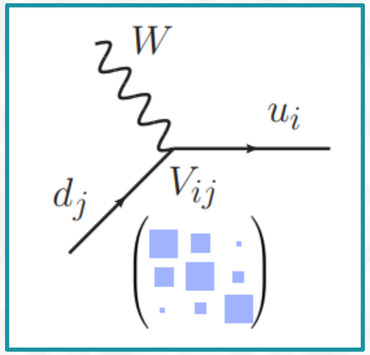


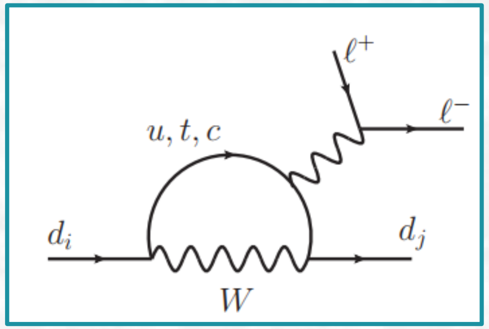


- **Flavor universal (flavor blind):**
  - **Couplings/parameters $\propto 1_{ij}$ in flavor space**
  - **Example: strong and electromagnetic interactions**



- **Flavor diagonal:**
  - **Coupling/parameters diagonal, but not necessarily universal**
  - **Example: Yukawa interactions in mass-eigenstate basis**

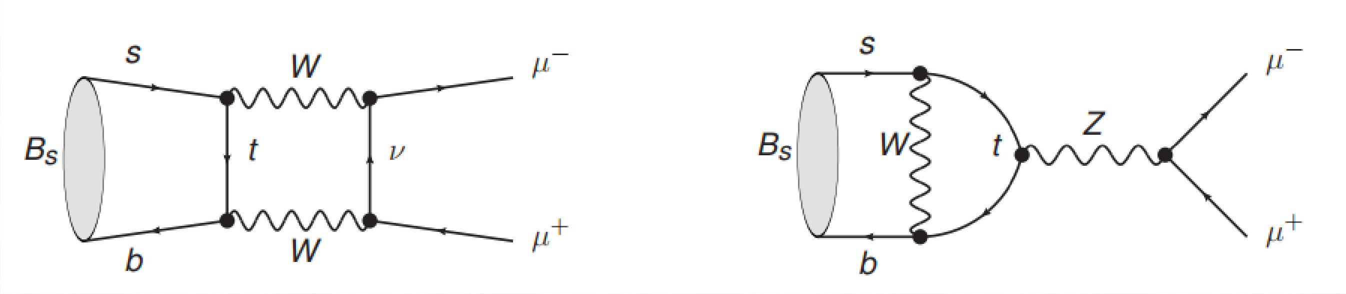




- **Flavor changing (FC):**
  - **Initial flavor number $\neq$ final flavor number**
  - **Flavor number = # particles – # antiparticles**
- **"FCCC" processes: both U and D involved**
- **"FCNC" processes: either U or D but not both involved**

Why flavor physics

- **A tool for indirect discovery: flavor physics is sensitive to NP at $\Lambda_{NP} \gg \Lambda_{exp}$**



decay constant

Effect of lifetime difference of Bs and Bs-bar

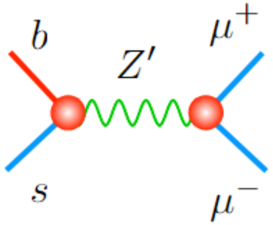
$$BR(B_s \to \mu^+\mu^-)_{SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}$$

helicity suppression

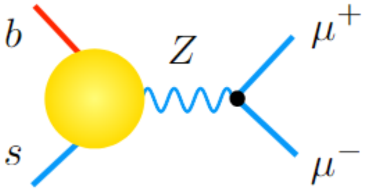
loop suppression

CKM suppression

- a weakly-coupled Z′ boson with generic flavor-changing tree-level quark couplings



$$\mu_{B_s \to \mu^+\mu^-} \simeq 1 \pm \frac{4\pi}{g^2 |V_{tb}^* V_{ts}|^2} \frac{v^2}{\Lambda^2},$$



$$\mu_{B_s \to \mu^+\mu^-} \simeq 1 \pm \frac{v^2}{\Lambda^2},$$

$$\boxed{\mu_{B_s \to \mu^+\mu^-} = \frac{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)}{\mathrm{Br}\left(B_s \to \mu^+\mu^-\right)_{\mathrm{SM}}} = 0.78 \pm 0.18}$$

$$\Lambda \gtrsim \frac{v}{\sqrt{0.2}} \times \begin{cases} \dfrac{\sqrt{4\pi}}{g\,|V_{tb}^* V_{ts}|} \\ 1 \end{cases} \simeq \begin{cases} 50\,\mathrm{TeV}\,, & \text{anarchic tree} \\ 0.6\,\mathrm{TeV}\,, & \text{MFV loop} \end{cases}$$

[U. Haisch, arXiv:1510.03341]

- one-loop modifications of the Z-penguin diagram assuming MFV (e.g., due to triple-gauge coupling)

Why flavor physics

□ Flavor physics has a good track record:

GIM mechanism in $K^0 \to \mu\mu$	CP violation, $K_L^0 \to \pi\pi$	$B^0 \leftrightarrow \overline{B}^0$ mixing

GIM mechanism in $K^0 \to \mu\mu$

Weak Interactions with Lepton-Hadron Symmetry*

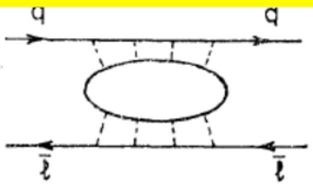
S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

splitting, beginning at order $G(G\Lambda^2)$, as well as contributions to such unobserved decay modes as $K_2 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., involving neutral lepton

We wish to propose a simple model in which the divergences are properly ordered. Our model is founded in a quark model, but one involving four, not three, fundamental fermions; the weak interactions are medi-

new quantum number $\mathcal{C}$ for charm.



Glashow, Iliopoulos, Maiani,
Phys.Rev. D2 (1970) 1285

CP violation, $K_L^0 \to \pi\pi$

27 July 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have

three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$

Christenson, Cronin, Fitch, Turlay,
Phys.Rev.Lett. 13 (1964) 138-140

$B^0 \leftrightarrow \overline{B}^0$ mixing

DESY 87-029
April 1987

OBSERVATION OF B^0-$\overline{B}^0$ MIXING

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pairs, lepton pairs and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that B^0-$\overline{B}^0$ mixing has been observed and is substantial.

Parameters	Comments
$r > 0.09$ 90%CL	This experiment
$x > 0.44$	This experiment
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	B meson ($\approx$ pion) decay constant
$m_b < 5\text{GeV}/c^2$	b-quark mass
$\tau_b < 1.4 \cdot 10^{-12}$s	B meson lifetime
$\lvert V_{td}\rvert < 0.018$	Kobayashi-Maskawa matrix element
$\eta_{QCD} < 0.86$	QCD correction factor [17]
$m_t > 50\text{GeV}/c^2$	t quark mass

ARGUS Coll.
Phys.Lett.B192:245,1987

Rare decay implies charm quark

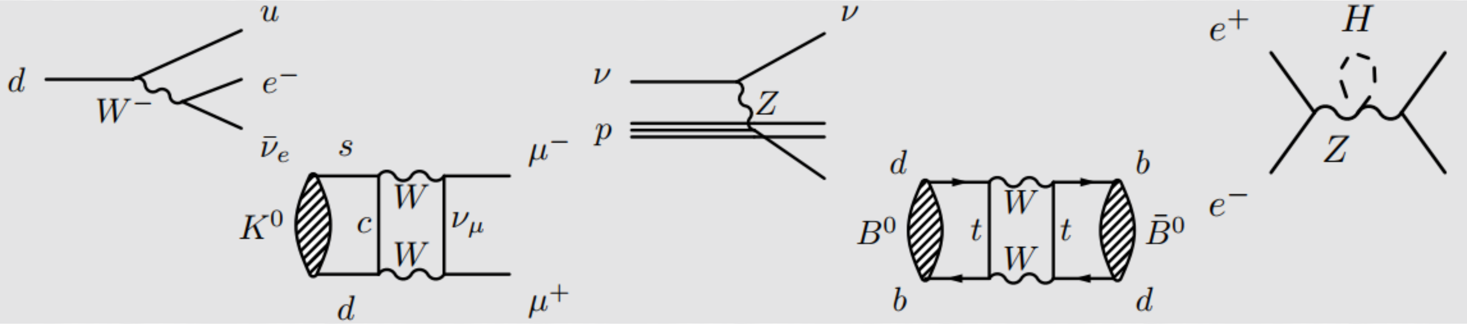
CP violation implies 3rd family

Mixing implies a heavy top quark

Why flavor physics

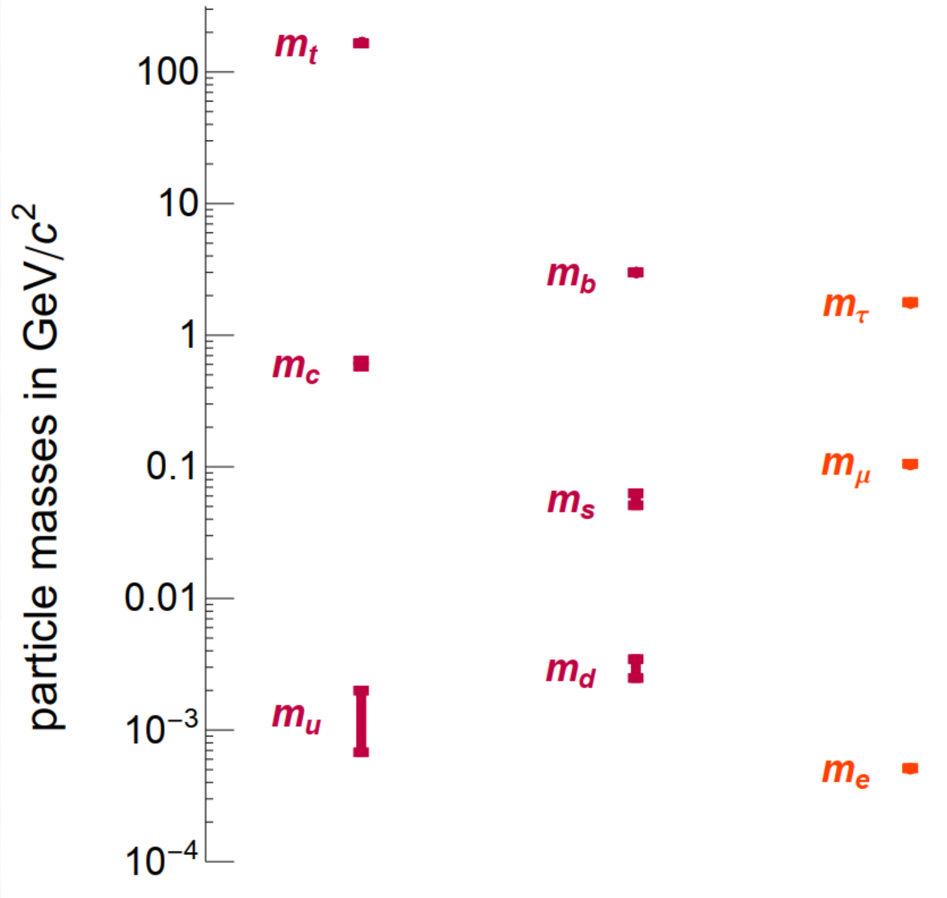
□ Historical records of indirect discoveries through flavor physics:

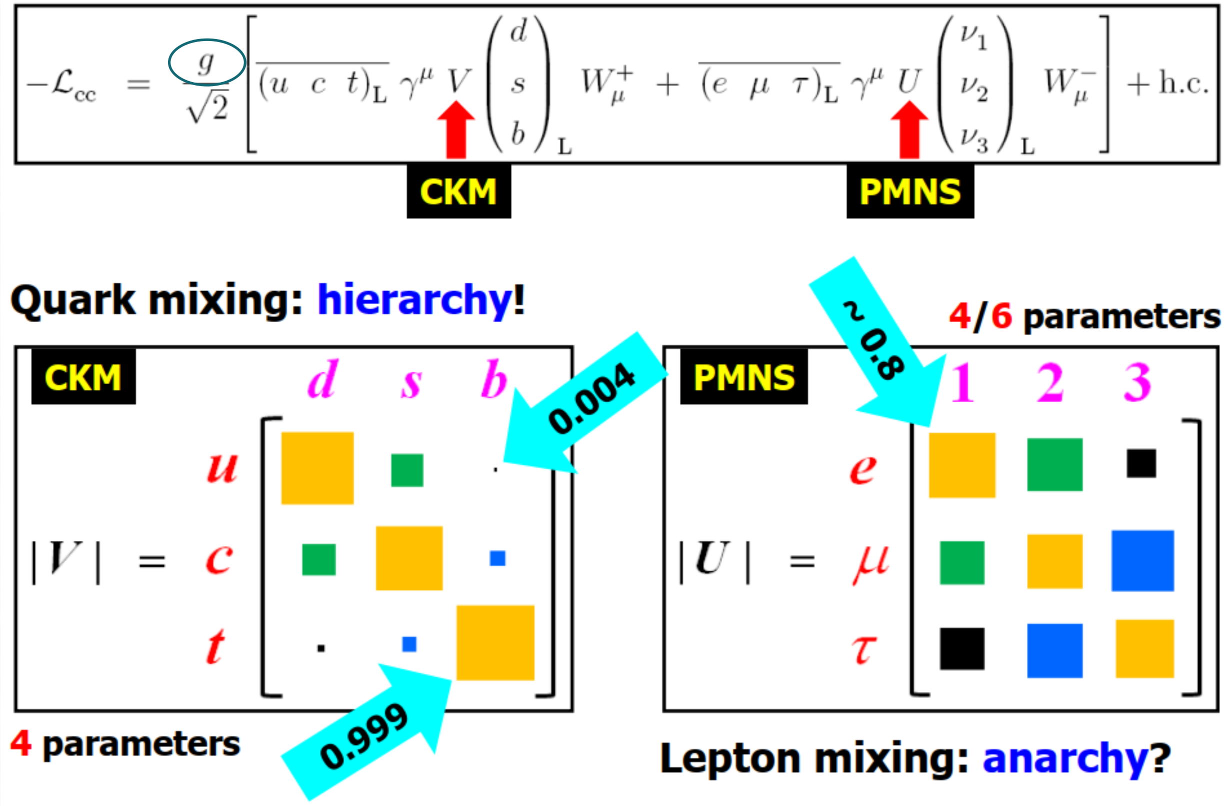
Particle	Indirect			Direct		
ν	β decay	Fermi	1932	Reactor ν-CC	Cowan, Reines	1956
W	β decay	Fermi	1932	$W \to e\nu$	UA1, UA2	1983
c	$K^0 \to \mu\mu$	GIM	1970	J/ψ	Richter, Ting	1974
b	CPV $K^0 \to \pi\pi$	CKM, 3rd gen	1964/72	Υ	Ledermann	1977
Z	ν-NC	Gargamelle	1973	$Z \to e^+e^-$	UA1	1983
t	B mixing	ARGUS	1987	$t \to Wb$	D0, CDF	1995
H	e^+e^-	EW fit, LEP	2000	$H \to 4\mu/\gamma\gamma$	CMS, ATLAS	2012



Why flavor physics

- **Helpful for understanding the SM flavor puzzle: why flavor parameters so hierarchical?**

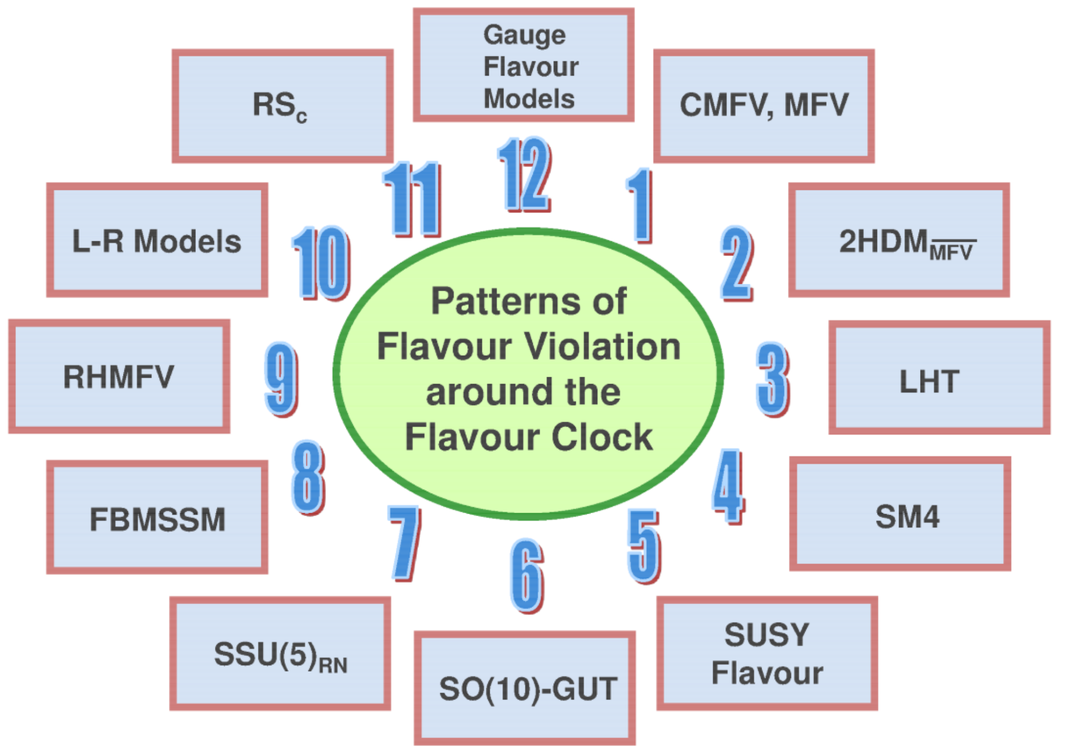


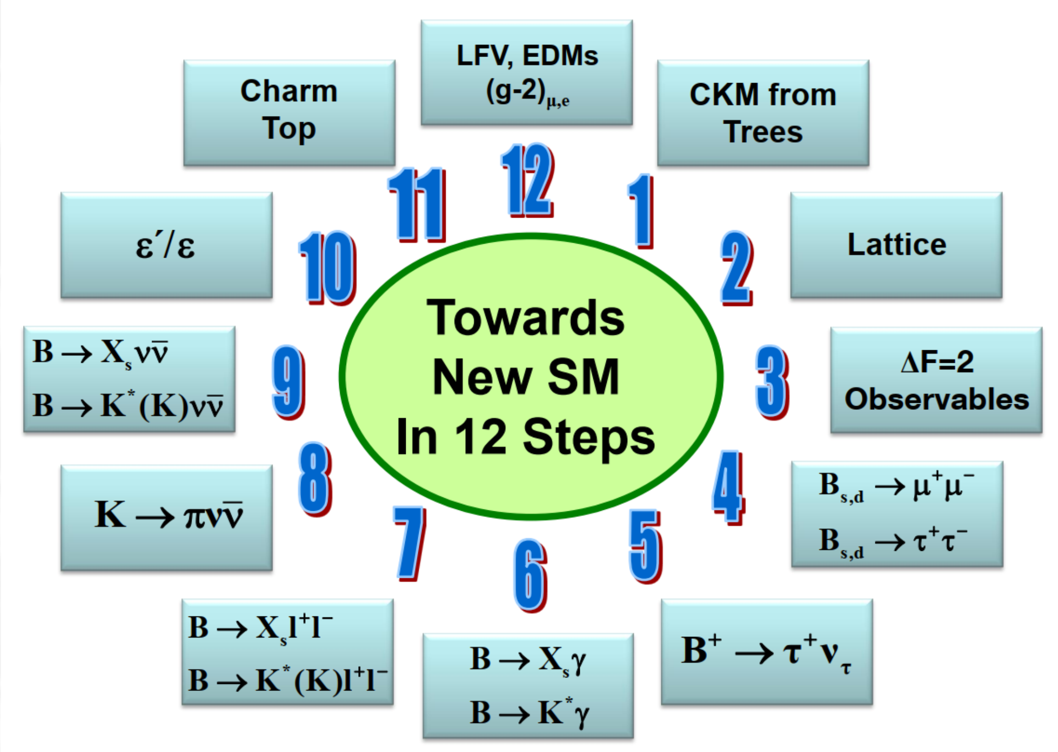


- **Spectrum spans five orders of magnitude!**
- **Different structures between quark and lepton mixing matrices!**

Why flavor physics

- □ **NP flavor puzzle: shortcomings of the SM imply that NP at or below TeV scale must exist;**





[A. Buras and J. Girrbach,1306.3755]

- □ **If NP were there, new flavor- and CP-violating sources arise, then why FCNCs so small?**

  what and why such a specific flavor structure? ⟹ flavor physics always plays a key role!

Why flavor physics

- **Helps us to understand the origin of CPV**

- **CPV closely related to quark flavor physics**
  - **In the SM, CPV due to the KM mechanism**

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2+(-1/8-A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2+A\lambda^4(1/2-\rho - i\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Expansion in order λ^3

Expansion in order λ^4

- **All CPVs in meson systems well explained**

$\not{C}\not{P}$ category	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
	Hadronic system										
decay	✓	✗	✗	✓	✗	✗	✗	✓✓	✓✓	✓	✗
mixing	✓✓	–	–	✗	–	–	–	✗	–	✗	–
decay/mixing interf.	✓	–	–	✗	–	–	–	✓✓	–	✓	–

✓ Observed
✓✓ Several observations
✗ Not observed (yet)
– Not expected

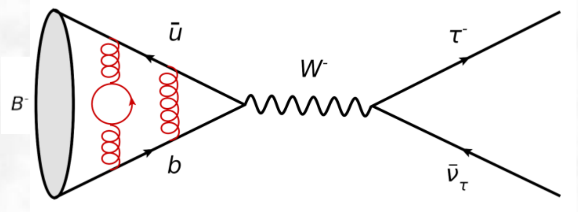
- **CPV in the SM not enough to explain BAU: new sources of CPV must exist!**

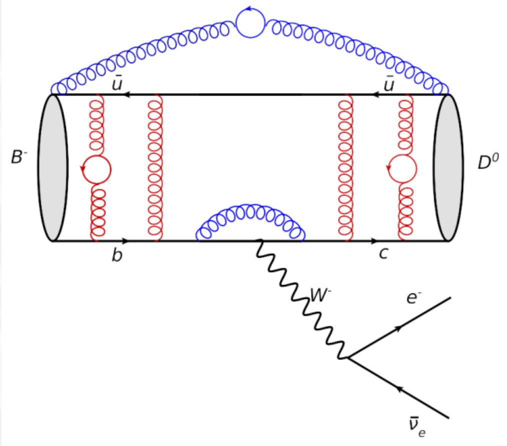
→ **Necessary to search for new CPVs in various different systems and processes!**

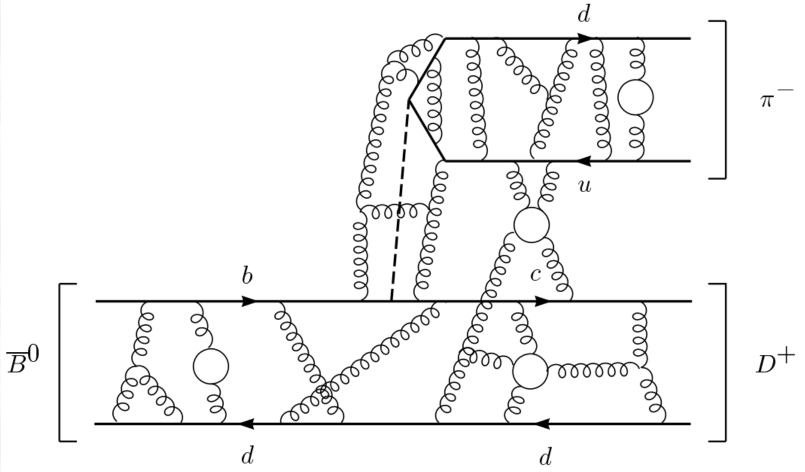
Why flavor physics

- **Helps us to understand various aspects of QCD**

- **For hadron decays, QCD effects always matters:** in real world, quarks are confined inside hadrons; the simplicity of weak interactions overshadowed by the complexity of strong interactions!







multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

HQET, SCET, NRQCD, ...

QCDF, pQCD, LCSR, LQCD, ...

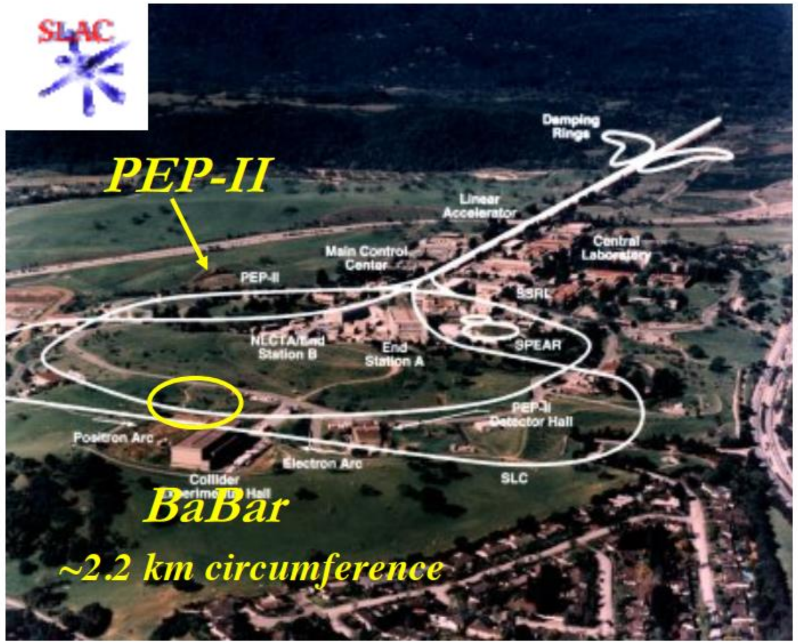
SU(3), Isospin, U-spin, ...

How to do flavor physics

- **B-factories (e^+e^-): Belle and BaBar**
- **Hadron colliders ($p\bar{p}$): CDF and D0 @ Tevatron**

Observation of B_s mixing

3.5 GeV e^+ 8 GeV e^-



3.1 GeV e^+ 9 GeV e^-

Nobel Prize 2008 for

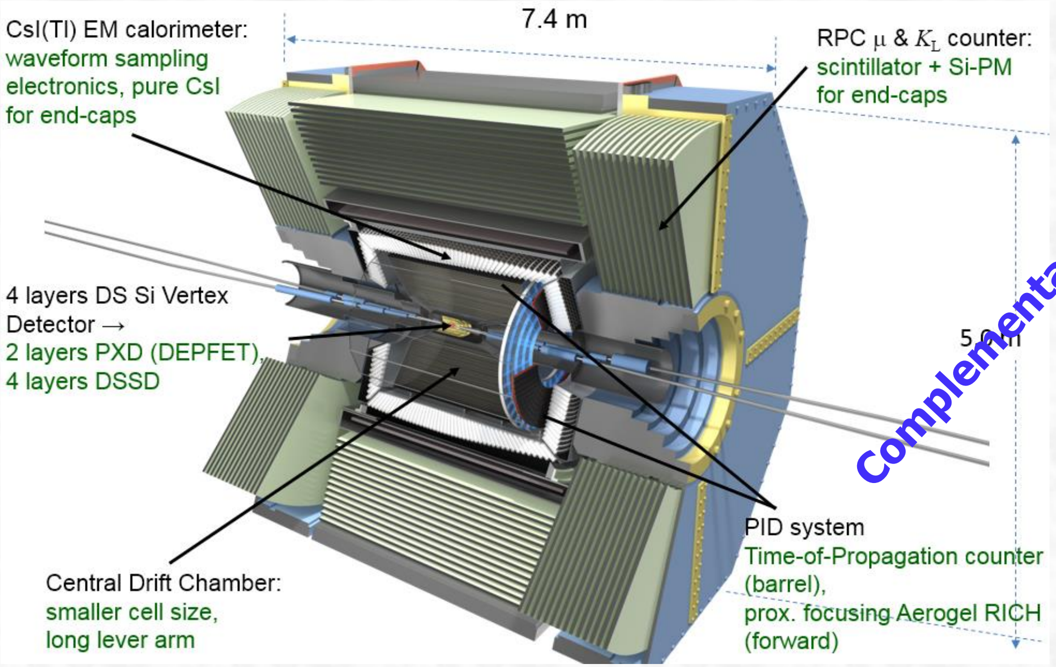
Makoto Kobayashi

Toshihide Maskawa

BaBar & Belle confirmed the KM mechanism of CPV in the SM!

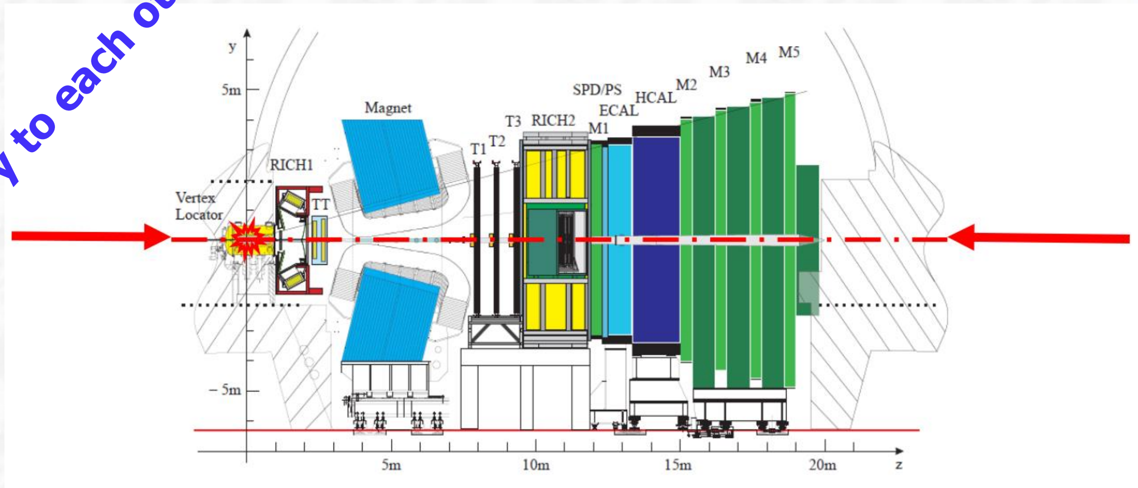
How to do flavor physics

□ **Super B-factories (e^+e^-): Belle II @ KEK**



□ **Hadron colliders (pp): LHCb @ LHC**

Complementary to each other!



Belle II @ KEK and LHCb @ LHC: dedicated detectors for quark flavor physics, and can perform quite a wide range of measurements! [The Belle II Physics Book, 1808.10567; LHCb Collaboration, 1808.08865]

□ **Other dedicated experiments:** BESIII, KOTO, Mu2e, MEG II, Muon g-2, …

B-factories vs Hadron colliders

	B Factories *Belle (1999-2010)* *BaBar (1999-2008)*	**Hadron colliders** *Tevatron (<2 TeV, 1983–2011)* *LHC (<14 TeV, 2008–)*
Collision environment	Asymmetric $e^+e^- \to \Upsilon(4S)$ Clean! Pure $B\bar{B}$ event ✔	pp or $p\bar{p}$ (also ions…) Messy! Proton remnants give background particles
Flavour tagging (initial B^0 or $\bar{B}^0$)	Excellent ✔ (30% 'tagging power')	Challenging (~5%)
Production σ(B)	1 nb	~100–500 μb ✔
B hadron boost	Small ($\beta\gamma \approx 0.5$)	Large ($\beta\gamma \approx 100$) ✔
B hadrons created	B^+B^- (50%), $B^0\bar{B}^0$ (50%)	$B^\pm$ (40%), B^0 (40%), B_s^0 (10%) ✔ b baryons (10%)

Complementary to each other!

Quark mixing and CKM matrix

SM Lagrangian

$$\mathcal{G}_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

gauge sector	Higgs sector	flavor sector
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + \text{h.c.}$	$+ \lvert D_\mu \phi \rvert^2 - V(\phi)$	$+ \bar{\psi}_i y_{ij} \psi_j \phi + \text{h.c.}$
describes the gauge interactions of the quarks and leptons	breaks electro-weak symmetry and gives mass to the $W^\pm$ and Z bosons	leads to masses and mixings of the quarks and leptons
parametrized by 3 gauge couplings g_1, g_2, g_3	2 free parameters Higgs mass Higgs vev	22 free parameters to describe the masses and mixings of the quarks and leptons

The flavor sector is the most puzzling part of the SM!

SM Lagrangian

$$\mathcal{G}_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

- **The following Lagrangian describes almost everything we have ever observed**

$$\mathscr{L}_{\text{SM}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \theta_{\text{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$$
$$+\bar{L}_L i\not{D}L_L + \bar{Q}_L i\not{D}Q_L + \bar{e}_R i\not{D}e_R + \bar{d}_R i\not{D}d_R + \bar{u}_R i\not{D}u_R$$
$$+|D_\mu H|^2 + m_H^2|H^\dagger H| - \lambda|H^\dagger H|^2$$
$$-[\bar{L}_L H Y_E e_R + \bar{Q}_L H Y_D d_R + \bar{Q}_L H^c Y_U u_R + \text{h.c.}]$$

hypercharge = $-\frac{1}{2}$ $\frac{1}{2}$ -1 $\frac{1}{6}$ $\frac{1}{2}$ $-\frac{1}{3}$ $\frac{1}{6}$ $-\frac{1}{2}$ $\frac{2}{3}$

$$H^c = i\sigma_2 H^*$$

- **After EWSB via Higgs mechanism, $H \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, fermions & $W^\pm, Z^0$ obtain their masses**

- **θ_{QCD} violates CP, but constrained to be very small (<10^{-10}), the so-called strong CP issue!**

Higgs mechanism

- **SM is a local gauge field theory**

$$\mathcal{G}_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

Fields in representations under the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$

Higgs: $\Phi(1,2,1/2)$ hypercharge $Y = Q - T_3$

quarks: $Q_L(3,2,1/6)_i$, $D_R(3,1,-1/3)_i$, $U_R(3,1,2/3)_i$

leptons: $L_L(1,2,-1/2)_i$, $E_R(1,1,-1)_i$ L: doublet, R:singlet under $SU(2)_L$

- **Mass terms for quarks and leptons break the $SU(2)_L$ gauge invariance**

$$\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

SU(2) singlet SU(2) doublet

- **The $SU(2)_L$ gauge invariance requires the EW gauge bosons to be exactly massless**

$$\frac{M^2}{2}A_\mu A^\mu, \qquad \text{or} \qquad \frac{M^2}{2}A^a_\mu A^{\mu a},$$

- **Both problems are solved by spontaneously breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$**

$$\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \Rightarrow (D_\mu\phi)^\dagger(D^\mu\phi) \sim m_W^2 W_\mu^+ W^{\mu-} + \frac{1}{2}m_Z^2 Z_\mu^0 Z^{\mu 0} + \cdots, \quad \text{with:} \begin{cases} m_W^2 = \dfrac{g^2v^2}{4}, & m_Z^2 = \dfrac{g^2v^2}{4c_W^2}, \\ m_A = 0 \ \text{and} & m_G = 0. \end{cases}$$

Fermion masses

Weak Eigenstates: $\begin{pmatrix} \nu'_j \\ l'_j \end{pmatrix}, \begin{pmatrix} u'_j \\ d'_j \end{pmatrix}$

- Scalar – fermion Yukawa couplings allowed by gauge symmetry

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \quad \Downarrow \quad \text{SSB}$$

$$\mathcal{L}_Y = -\left(1+\frac{h}{\mathrm{v}}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

- Fermion masses are arbitrary and non-diagonal 3 X 3 complex matrices

$$\left[\mathbf{M}'_d, \mathbf{M}'_u, \mathbf{M}'_l \right]_{jk} = \left[c^{(d)}_{jk}, c^{(u)}_{jk}, c^{(l)}_{jk} \right] \frac{\mathrm{v}}{\sqrt{2}}$$

Fermion masses

Mass Eigenstates: $\begin{pmatrix} \nu_j \\ l_j \end{pmatrix}, \begin{pmatrix} u_j \\ d_j \end{pmatrix}$

- **Diagonalization of fermion mass matrices**

$$\mathcal{L}_Y = -\left(1+\frac{h}{\mathrm{v}}\right)\left\{\bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.}\right\}$$

$$\Downarrow$$

$$d_L \equiv \mathbf{S}_d \cdot d'_L, \quad u_L \equiv \mathbf{S}_u \cdot u'_L, \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$d_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot d'_R, \quad u_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot u'_R, \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates $\neq$ Weak Eigenstates

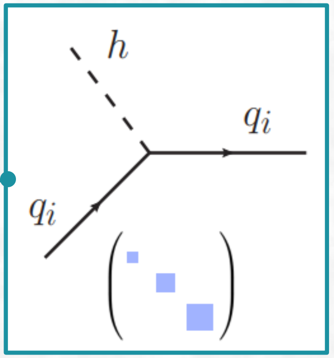
$$\Downarrow$$

$$\mathcal{L}_Y = -\left(1+\frac{h}{\mathrm{v}}\right)\left\{\bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l\right\}$$

$$\mathcal{M}_u = \mathrm{diag}(m_u, m_c, m_t) \;;\; \mathcal{M}_d = \mathrm{diag}(m_d, m_s, m_b) \;;\; \mathcal{M}_l = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

- **Higgs – fermion couplings: diagonal, but not universal!**

$$\mathcal{L}_{\text{int}} = m_d \bar{d}d + m_u \bar{u}u + \frac{m_d}{\mathrm{v}}\bar{d}dh + \frac{m_u}{\mathrm{v}}\bar{u}uh$$



Quark flavor mixings

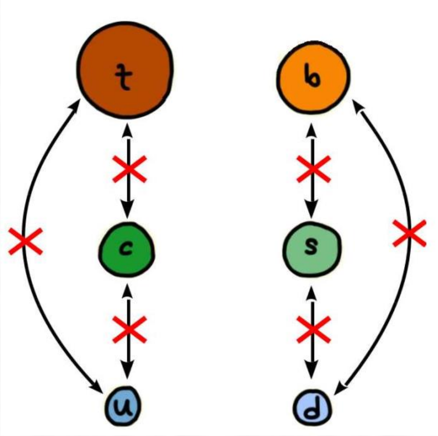
- **Replace the weak eigenstates by the mass eigenstates**

$$\bar{L}'_L i\not{D} L'_L + \bar{Q}'_L \not{D} Q'_L + \bar{e}_R i\not{D} e_R + \bar{d}'_R i\not{D} d'_R + \bar{u}'_R i\not{D} u'_R$$

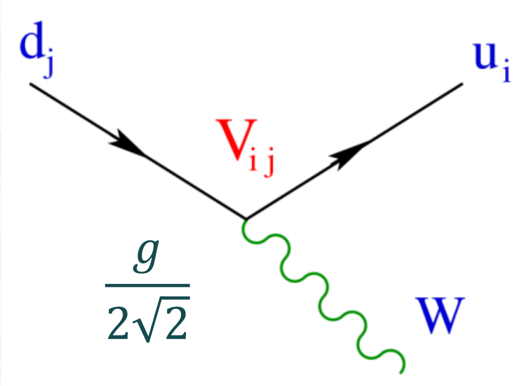
$$d_L \equiv \mathbf{S}_d \cdot d'_L, \quad u_L \equiv \mathbf{S}_u \cdot u'_L, \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$

$$d_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot d'_R, \quad u_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot u'_R, \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

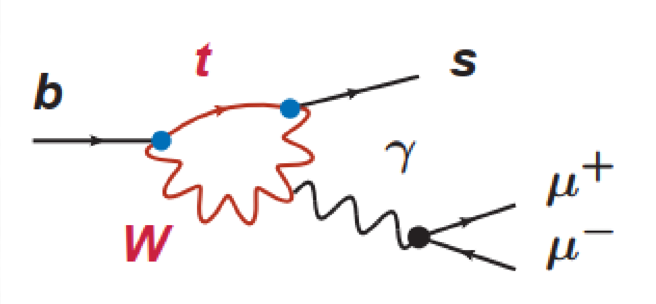
$$\bar{f}'_L f'_L = \bar{f}_L f_L, \qquad \bar{f}'_R f'_R = \bar{f}_R f_R, \qquad \bar{u}'_L d'_L = \bar{u}_L \cdot V_{CKM} \cdot d_L, \qquad V_{CKM} = S_u \cdot S_d^\dagger$$



- No tree-level FCNCs in the SM!



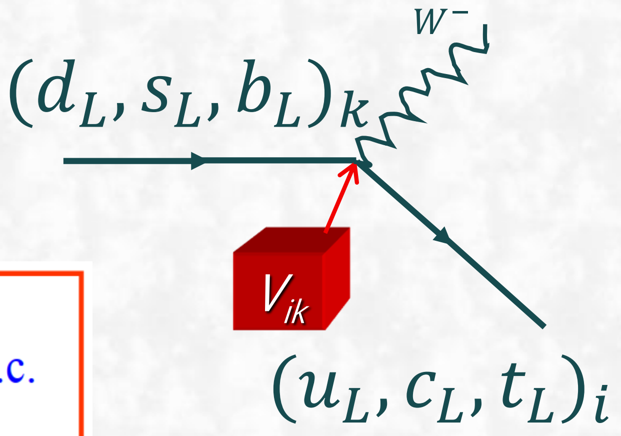
- FCCCs determined also by V_{ij}!



- FCNCs arise only at the loop level!

$$\sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^*$$

CKM matrix



- **FCCCs determined, in addition to g, also by CKM matrix**

$$\boxed{\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}}$$

- **Counting independent parameters contained in V_{CKM}**

- complex NxN matrix: $2N^2$ parameters
- must be unitary:
  - eg. t must decay to either b, s or d, so $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$
  - in general: $V^{*T} V = I \rightarrow N^2$ constraints
- freedom to change phase of quark fields $|q_j\rangle \rightarrow e^{i\phi_j} |q_j\rangle$
  - 2N-1 phases are irrelevant:

$$\langle q_i | V_{ij} | q_j \rangle \rightarrow \langle q_i | e^{-i\phi_i} V_{ij} e^{i\phi_j} | q_j \rangle$$

$$V_{ij} \rightarrow e^{i(\phi_j - \phi_i)} V_{ij}$$

- number of 'physical' parameters = N^2-2N+1
- how many can be rotation angles? N(N-1)/2
- For N=2 : 1 parameter, with 1 rotation angle (Cabbibo!)
- For N=3: 4 parameters = 3 rotations + 1 *irreducible* complex phase!

> **Physical parameters in V_{CKM}:**
>
> $\frac{1}{2}N(N-1)$ **rotation angle**
>
> $\frac{1}{2}(N-1)(N-2)$ **phases**

CKM matrix

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

- For N=3 and six-quark case: **3 rotational angles and 1 phase**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates   Mass eigenstates

- PDG standard parametrization:

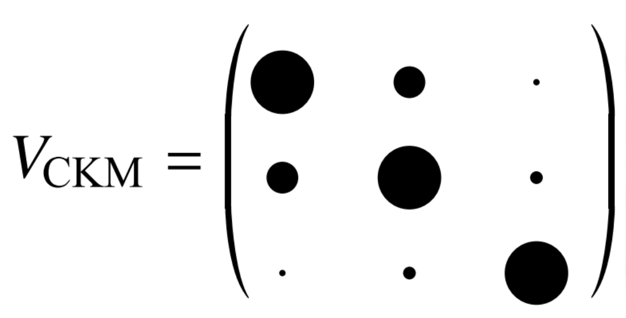
$$\theta_{12} \cong 13^o, \theta_{13} \cong 0.21^o, \theta_{23} \cong 2.4^o, \delta_{13} \cong 71^o$$

$$\mathbf{V} = \begin{bmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{bmatrix}$$

only source of CP violation

$$\lambda \equiv s_{12}\,,\ A \equiv s_{23}/\lambda^2\,,\ \rho + i\eta \equiv s_{13}e^{i\delta}/A\lambda^3$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$



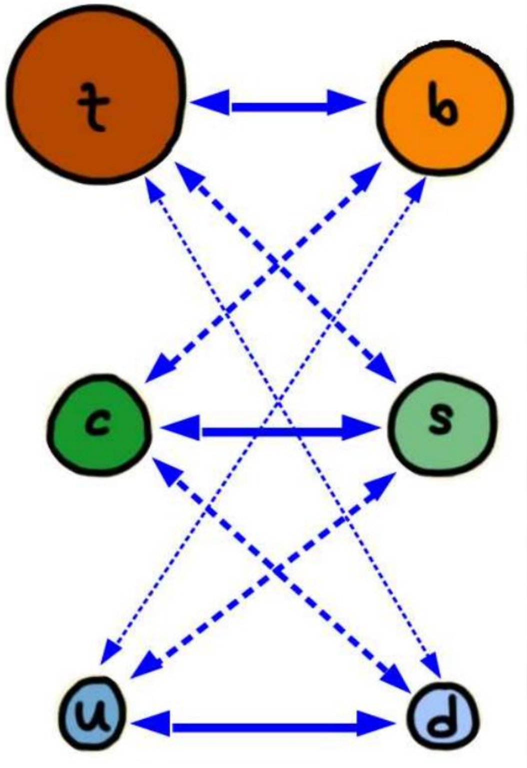
- Wolfenstein parametrization:

$$\lambda = 0.22650 \pm 0.00048\,, \qquad A = 0.790^{+0.017}_{-0.012}\,,$$
$$\bar{\rho} = 0.141^{+0.016}_{-0.017}\,, \qquad \bar{\eta} = 0.357 \pm 0.011\,.$$

CKM matrix

- **Hierarchical structures**

  among different flavors:



$$\mathbf{V} \approx \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \end{array} + \mathcal{O}\left(\lambda^4\right)$$

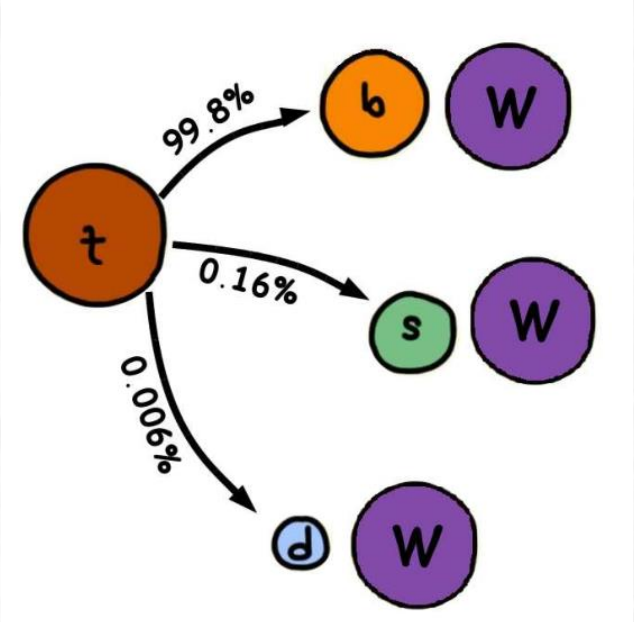
$$\lambda \approx \sin\theta_C \approx 0.223 \quad ; \quad A \approx 0.84 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.4$$

$$A(b \to c) \sim V_{cb} \sim 4 \times 10^{-2}$$

$$(\text{e.g. } B \to D\mu\nu)$$

$$A(b \to u) \sim V_{ub} \sim 4 \times 10^{-3}$$

$$(\text{e.g. } B \to \pi\mu\nu)$$



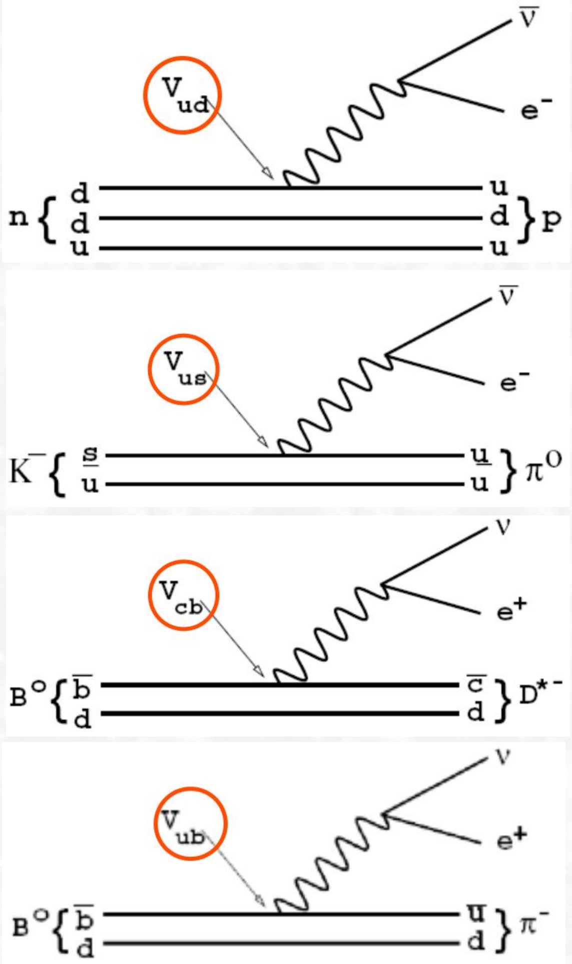
Determination of V_{CKM}

□ **Direct determination of moduli of the CKM matrix elements**

- □ $|V_{ud}|=0.97420(21)$: $0^+\to0^+$ nuclear β decay
- □ $|V_{us}|=0.2243(5)$: $K^0\to\pi^- l\nu$ + lattice QCD
- □ $|V_{cd}|=0.218(4)$: $\nu_\mu N\to\mu^- X$ vs $\nu_\mu N\to\mu^-\mu^+\nu_\mu X$. $D\to\pi l\nu$ and $D\to l\nu$ + lattice QCD
- □ $|V_{cs}|=0.997(17)$: $D_s\to\mu\nu$, $D\to K l\nu$, $W\to cs$ + lattice QCD
- □ $|V_{cb}|_{\rm incl}=(42.46\pm0.88)\ 10^{-3}$: $B\to X_c l\nu$
  $|V_{cb}|_{\rm excl}=(39.08\pm0.91)\ 10^{-3}$: $B\to D^{(*)} l\nu$ and $B\to\tau\nu$ + lattice QCD
- □ $|V_{ub}|_{\rm incl}=(4.52\pm0.20)\ 10^{-3}$: $B\to X_u l\nu$
  $|V_{ub}|_{\rm excl}=(3.73\pm0.14)\ 10^{-3}$: $B\to\pi l\nu$ + lattice QCD
- □ $|V_{tb}|=1.019(25)$: single top production

□ $|V_{td}|$ and $|V_{ts}|$ from neutral B-meson mixings

$$|V_{td}| = (8.0 \pm 0.3) \times 10^{-3}\,, \qquad |V_{ts}| = (38.8 \pm 1.1) \times 10^{-3}$$



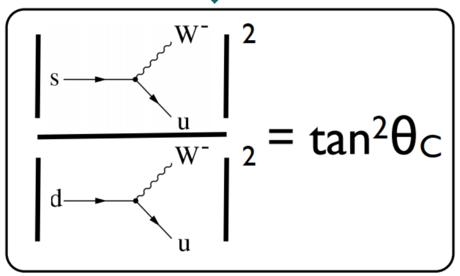
PDG2022: very precise!

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435\pm0.00016 & 0.22500\pm0.00067 & 0.00369\pm0.00011 \\ 0.22486\pm0.00067 & 0.97349\pm0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

GIM mechanism & Cabibbo theory

weak-isospin doublet

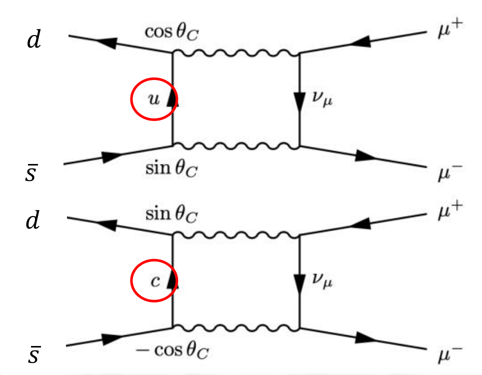
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$

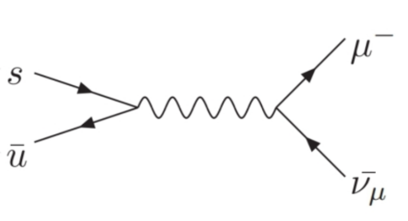


- ◻ **How to explain why $\Gamma_{\Delta S=1}(\Lambda \to pe^-\bar{\nu}) \cong 1/20\ \Gamma_{\Delta S=0}(n \to pe^-\bar{\nu})$?**

- ➢ **Cabibbo theory: weak interaction couples an up quark to an orthogonal combination of down & strange quarks**

- ◻ **How to explain why $\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)} \simeq 10^{-8}$?**

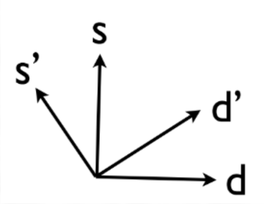




- ◻ **In 1970, Glashow, Iliopoulos & Maiani introduce a fourth quark (charm)**

weak-isospin doublet

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

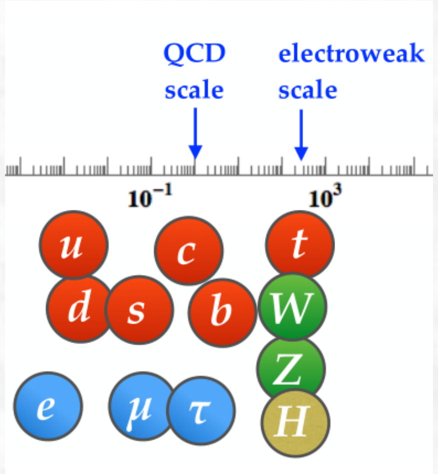


- ➢ **This adds an additional decay amplitude being almost identical to the original one, but with an opposite sign ⇒ (almost) fully destructive interference, and thus explains data!**

Generalized GIM mechanism

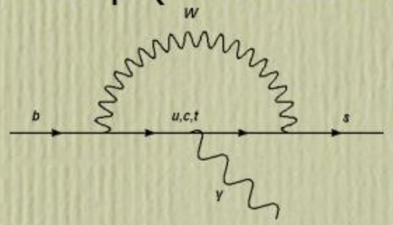
$$\sum_i V_{ij}^* V_{ik} = \sum_i V_{ji}^* V_{ki} = \delta_{jk}$$

- **Modern GIM: unitarity of the CKM matrix**



- **The GIM mechanism less effective in the down-type FCNCs than in the up-type FCNCs!**

- 1-loop (down sector):



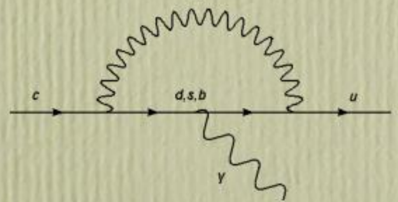
$b \to s\gamma$

$$= V_{ub}^* V_{us} f\left(\frac{m_u^2}{m_W^2}\right) + \overset{\sim 10^{-4}}{V_{cb}^* V_{cs}} f\left(\frac{m_c^2}{m_W^2}\right) + \overset{\sim 4}{V_{tb}^* V_{ts}} f\left(\frac{m_t^2}{m_W^2}\right)$$

$$= V_{tb}^* V_{ts}\left(f\left(\frac{m_t^2}{m_W^2}\right) - f\left(\frac{m_u^2}{m_W^2}\right)\right) + V_{cb}^* V_{cs}\left(f\left(\frac{m_c^2}{m_W^2}\right) - f\left(\frac{m_u^2}{m_W^2}\right)\right)$$

$$\simeq V_{tb}^* V_{ts}\left(f\left(\frac{m_t^2}{m_W^2}\right) - f(0)\right) \propto V_{tb}^* V_{ts}\frac{m_t^2}{m_W^2}$$

- 1-loop (up sector):



$c \to u\gamma$

$$= V_{ud}^* V_{cd} f\left(\frac{m_d^2}{m_W^2}\right) + \overset{\sim \times 10^{-6}}{V_{us}^* V_{cs}} f\left(\frac{m_s^2}{m_W^2}\right) + \overset{\sim 4\times 10^{-3}}{V_{ub}^* V_{cb}} f\left(\frac{m_b^2}{m_W^2}\right)$$

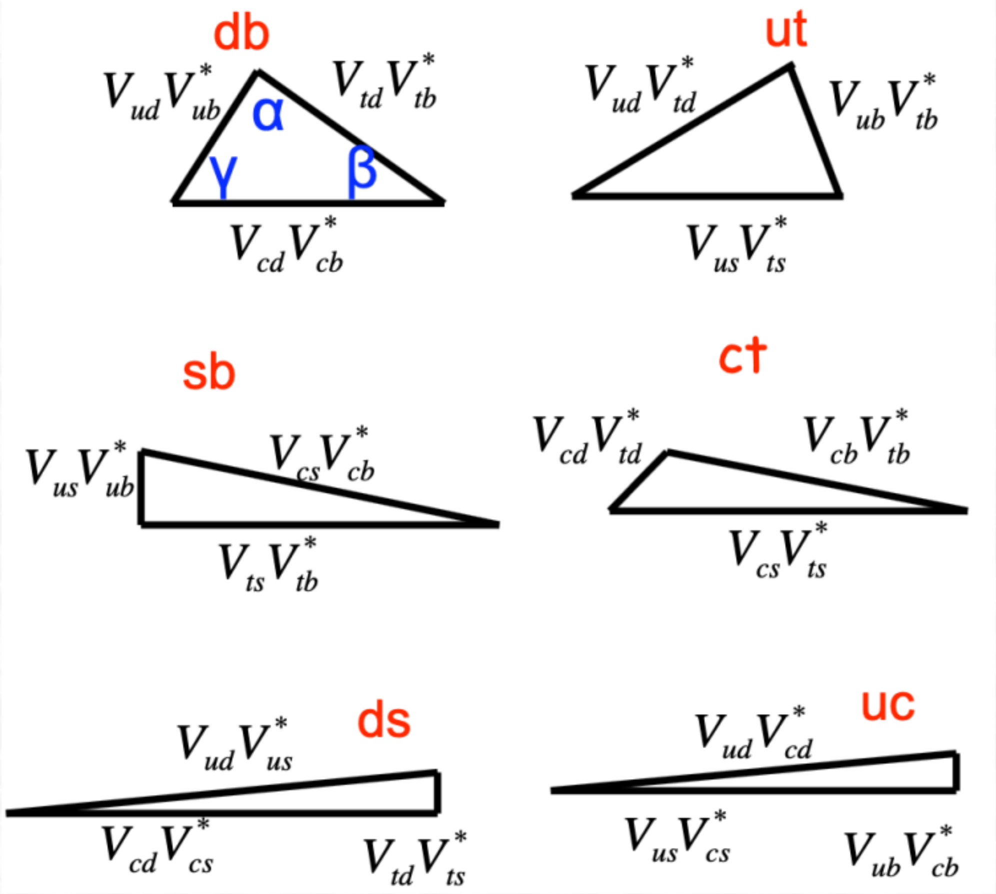
$$= V_{ub}^* V_{cb}\left(f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right)\right) + V_{us}^* V_{cs}\left(f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right)\right)$$

$$\simeq V_{ub}^* V_{cb}\left(f\left(\frac{m_b^2}{m_W^2}\right) - f(0)\right) \propto V_{ub}^* V_{cb}\frac{m_b^2}{m_W^2}$$

$$\sim \text{(combination of CKM entries)}\ \frac{m_b^2}{m_W^2}$$

CKM unitary triangle

- Unitarity of V_{CKM} implies $\sum_i V_{ij}V_{ik}^* = \delta_{jk}$ and $\sum_i V_{ji}V_{ki}^* = \delta_{jk}$
- Each of these 6 unitary constraints can be cast as a triangle in the complex plane

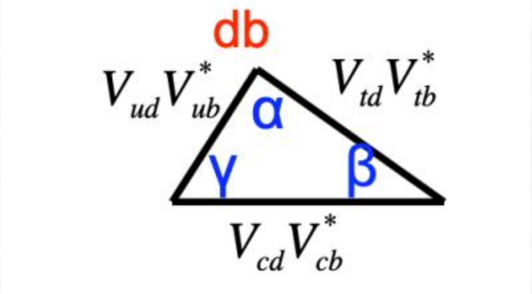


- All triangles have **the same area = ½ J**

$$J = 2a = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta \simeq \lambda^6 A^2 \eta \simeq 10^{-5}$$

- The Jarlskog invariant J is a measure of CPV within the SM
- Only the first two not squashed
- The first one most relevant in B physics

CKM unitary triangle



- The **db** UT triangle: involving **B_d-meson** decays

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

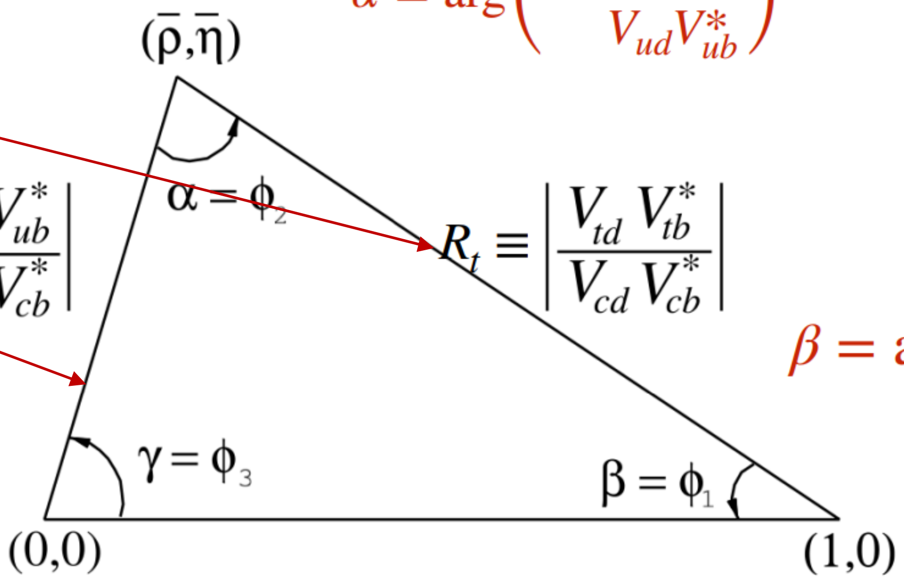
$$\mathcal{O}(\lambda^3) \qquad \mathcal{O}(\lambda^3) \qquad \mathcal{O}(\lambda^3)$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$\bar{\rho} \equiv \rho\left(1-\lambda^2/2\right), \bar{\eta} \equiv \eta\left(1-\lambda^2/2\right)$$

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$R_u \equiv \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| \qquad R_t \equiv \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right|$$



$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

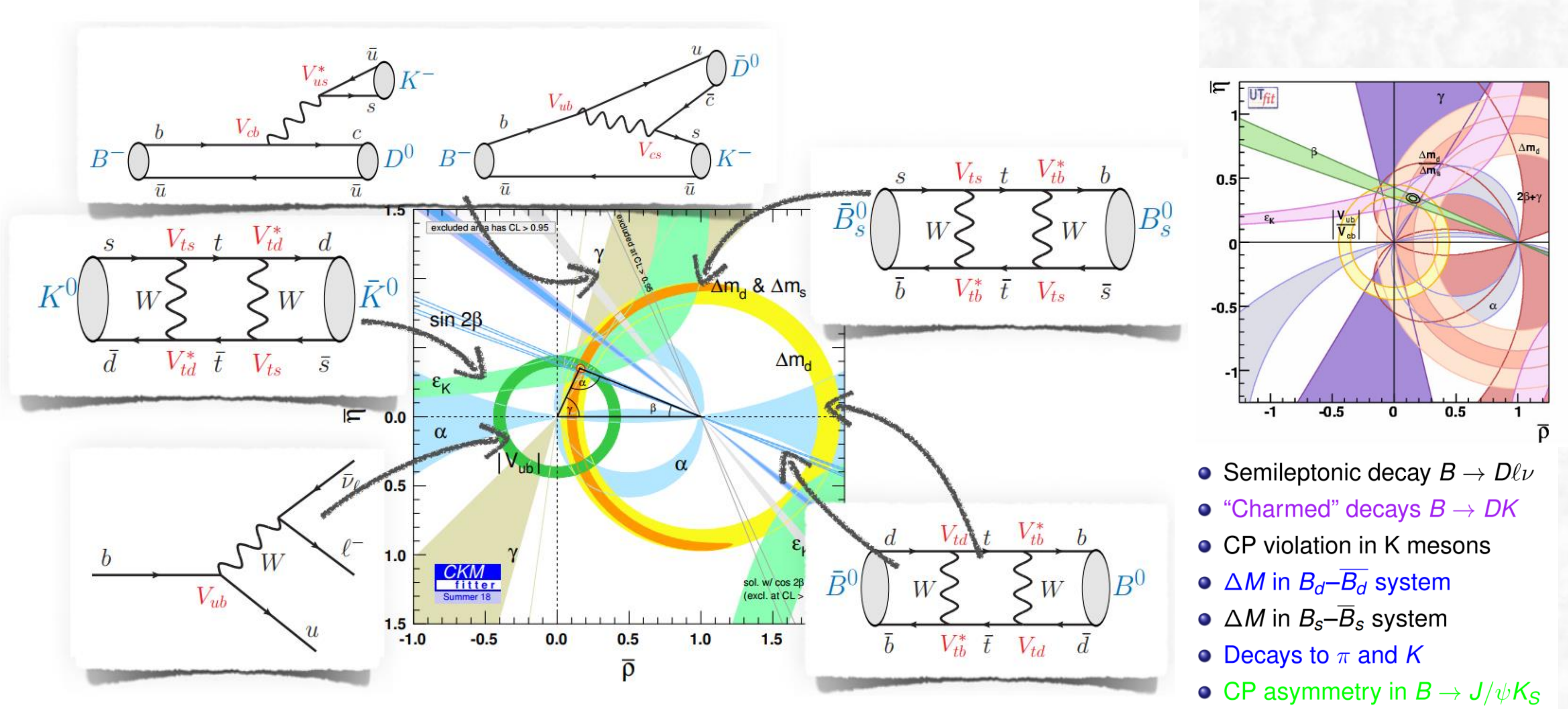
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- **Huge improvement on the knowledge of the CKM elements in the last decades!**

CKM unitary triangle

$\{$ http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

- **Determination of the CKM UT**



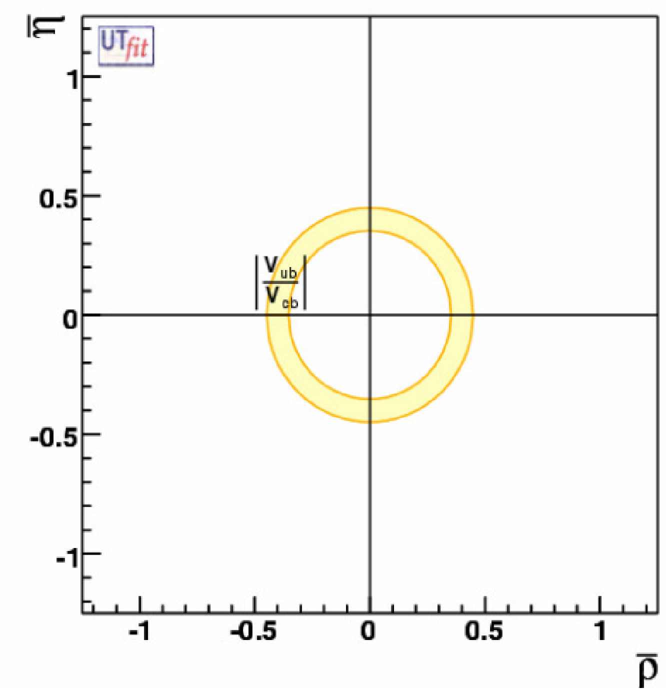
- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons
- ΔM in B_d–$\overline{B}_d$ system
- ΔM in B_s–$\overline{B}_s$ system
- Decays to π and K
- CP asymmetry in $B \to J/\psi K_S$

- **All measurements agree with the SM picture of CKM matrix within errors!**

CKM unitary triangle

{ http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

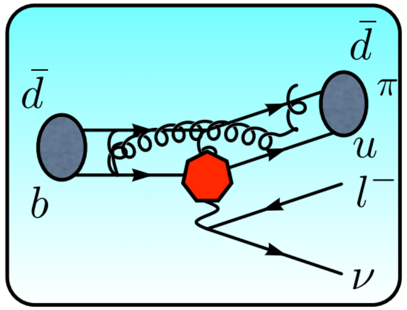
- **Demonstrate how to constrain the CKM UT**



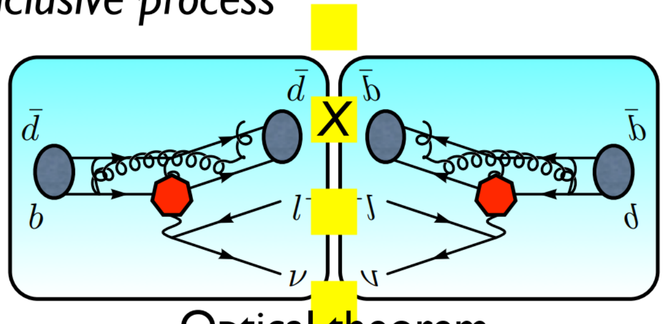
$$V_{ub} = A\lambda^3(\bar{\rho} - i\bar{\eta})$$

- Semileptonic decay $B \to D\ell\nu$ and $B \to \pi l\nu$ or inclusive decays

Exclusive process



Inclusive process



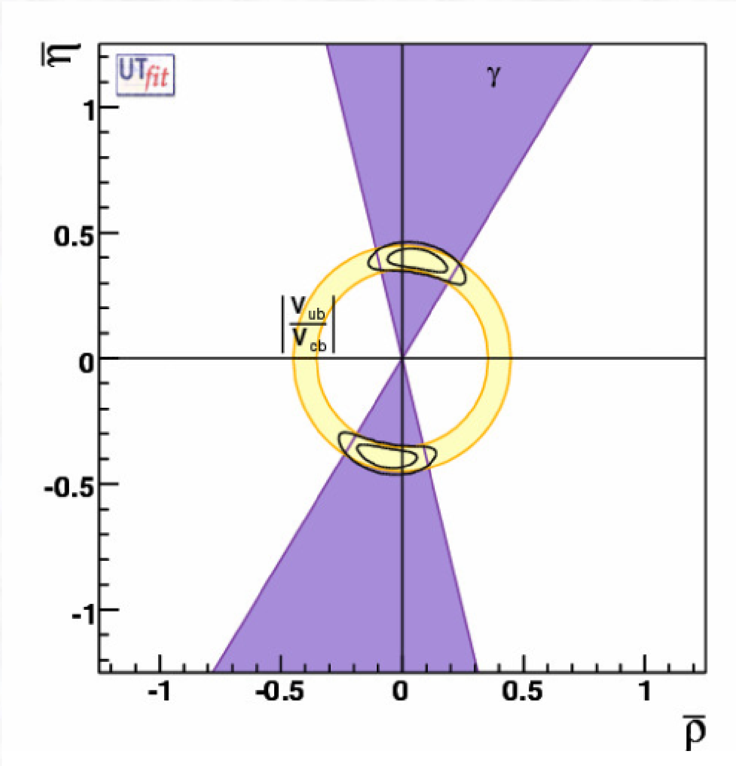
Optical theorem

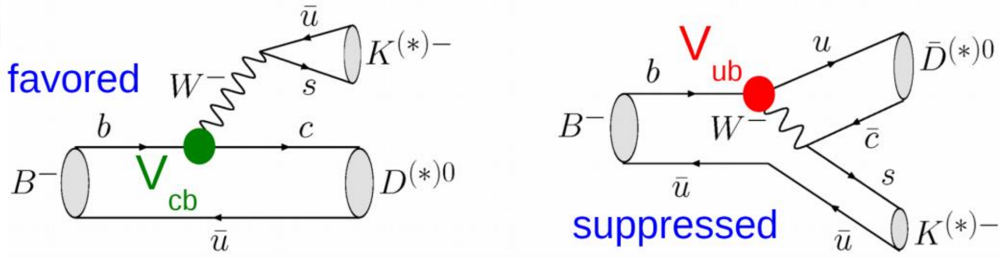
$$\mathcal{A}(B \to \pi l\nu) \propto |V_{ub}| F^{B\to\pi}(q^2) \qquad \sum |\mathcal{A}(B \to X_u l\nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \ldots)$$

CKM unitary triangle

http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

- Demonstrate how to constrain the CKM UT





- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- Also $B_s \to D_s^{\pm} K^{\mp}$ decays

By considering six decay rates $B^{\pm} \to D^0_{CP}K^{\pm}$, $B^+ \to D^0K^+$, $\bar{D}^0K^+$ and $B^- \to D^0K^-$, $\bar{D}^0K^-$ where $D^0_{CP} = (D^0 + \bar{D}^0)/\sqrt{2}$ is a CP eigenstate, and noting that

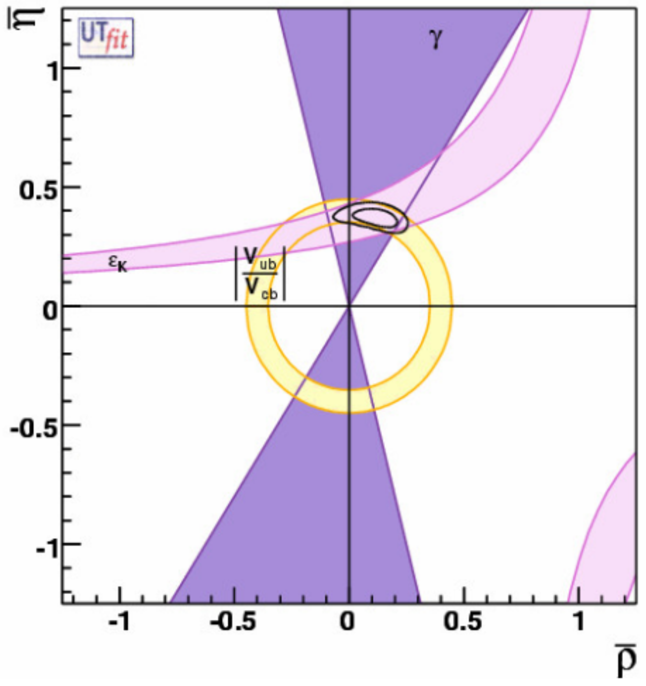
$$A(B^+ \to \bar{D}^0K^+) = A(B^- \to D^0K^-),$$

$$A(B^+ \to D^0K^+) = A(B^- \to \bar{D}^0K^-)e^{2i\gamma},$$

CKM unitary triangle

{ http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

- Demonstrate how to constrain the UT



- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons

$$\bar{\eta}\left[(1-\bar{\varrho})A^2\eta_2 S_0(x_t) + P_c(\varepsilon)\right] A^2\hat{B}_K\kappa_\varepsilon = 0.187,$$

$$\varepsilon = C_\varepsilon\kappa_\varepsilon\hat{B}_K \mathrm{Im}\lambda_t\left\{\mathrm{Re}\lambda_c\left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right] - \mathrm{Re}\lambda_t\eta_2 S_0(x_t)\right\} e^{i\phi_\varepsilon}$$

CKM unitary triangle

http://ckmfitter.in2p3.fr/; frequentist
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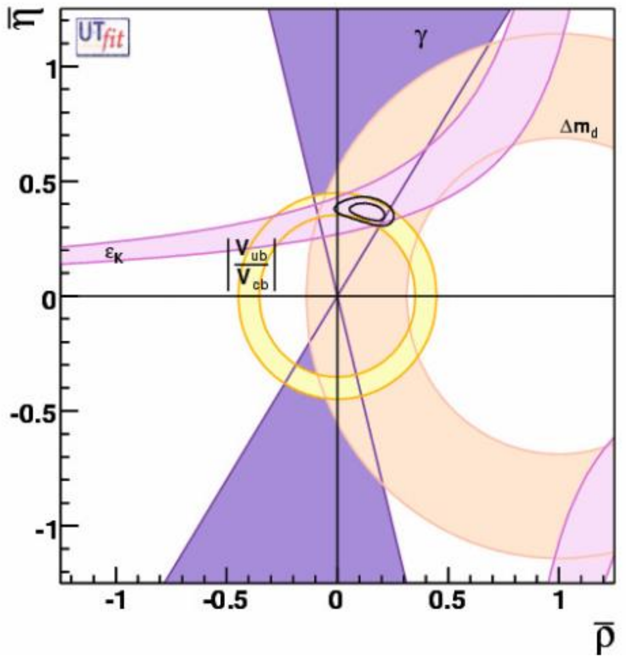
- Demonstrate how to constrain the UT



- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons
- ΔM in B_d–$\overline{B_d}$ system

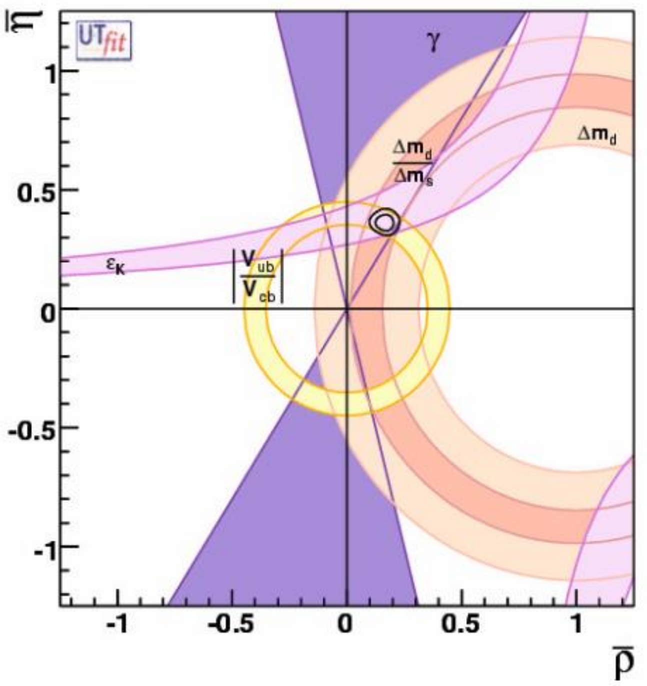
$$\Delta M_d = 0.5055/\mathrm{ps} \cdot \left[\frac{\sqrt{\hat{B}_{B_d}} F_{B_d}}{227.7\,\mathrm{MeV}}\right]^2 \left[\frac{S(x_t)}{2.322}\right] \left[\frac{|V_{td}|}{8.00 \cdot 10^{-3}}\right]^2 \left[\frac{\eta_B}{0.5521}\right].$$

$$V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta})$$

CKM unitary triangle

http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

- Demonstrate how to constrain the UT



- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons
- ΔM in B_d–$\overline{B}_d$ system
- ΔM in B_s–$\overline{B}_s$ system

$$\Delta M_s = 17.757/\text{ps} \cdot \left[\frac{\sqrt{\hat{B}_{B_s}}F_{B_s}}{274.6\,\text{MeV}}\right]^2 \left[\frac{S(x_t)}{2.322}\right]\left[\frac{|V_{ts}|}{0.0390}\right]^2\left[\frac{\eta_B}{0.5521}\right],$$

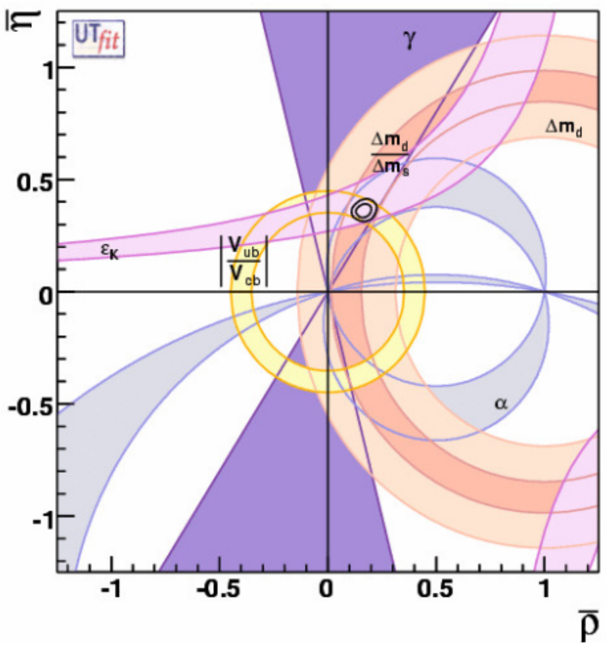
$$V_{td} = A\lambda^3(1-\bar\rho-i\bar\eta)$$

$$V_{ts} = -A\lambda^2 + \frac{1}{2}A(1-2\rho)\lambda^4 - i\eta A\lambda^4$$

CKM unitary triangle

- http://ckmfitter.in2p3.fr/; frequentist
- http://utfit.org/UTfit; Bayesian

□ **Demonstrate how to constrain the UT**



- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons
- ΔM in B_d–$\overline{B_d}$ system
- ΔM in B_s–$\overline{B}_s$ system
- Decays to π and K

The time-dependent CP asymmetry is:

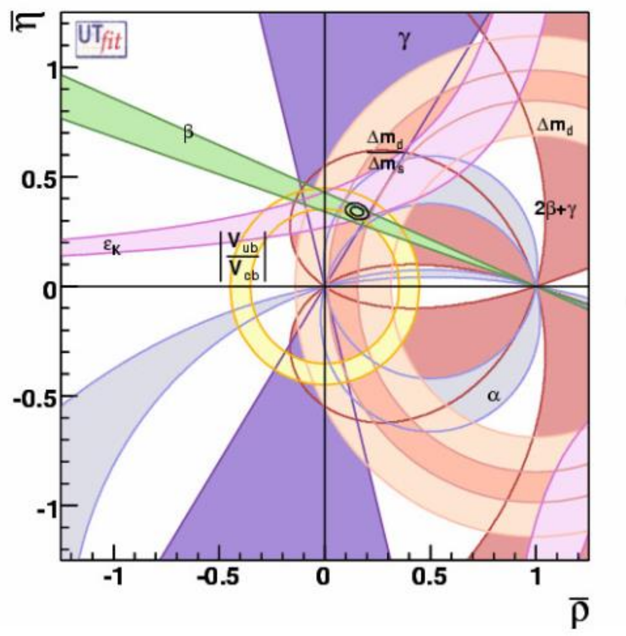
$$\begin{aligned} \mathcal{A}_{\pi\pi} &= -C_{\pi\pi}\cos(\Delta m_{B_d} t) + S_{\pi\pi}\sin(\Delta m_{B_d} t) \\ S_{\pi\pi} &= \sin(2\alpha) + O(r_{\pi\pi}) \\ C_{\pi\pi} &= O(r_{\pi\pi}) \end{aligned}$$

$$\lambda_{\pi\pi} = \frac{q}{p}\frac{A_{\pi\pi}}{\bar A_{\pi\pi}} = \underbrace{\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}}_{e^{-2i\beta}} \underbrace{\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}}_{e^{-2i\gamma}} \frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*} = e^{2i\alpha}\frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*}$$

CKM unitary triangle

http://ckmfitter.in2p3.fr/; frequentist
http://utfit.org/UTfit; Bayesian

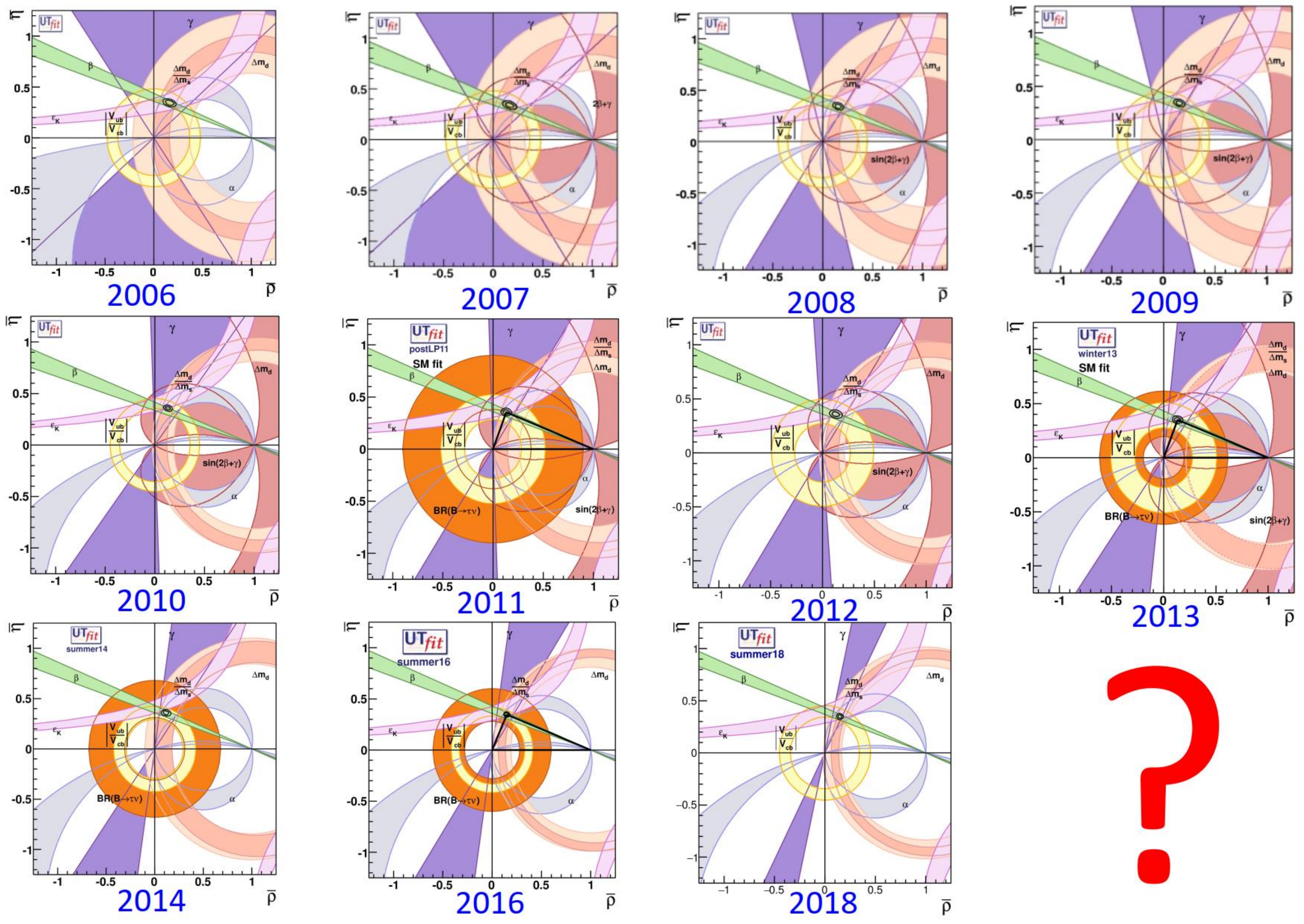
- Demonstrate how to constrain the UT

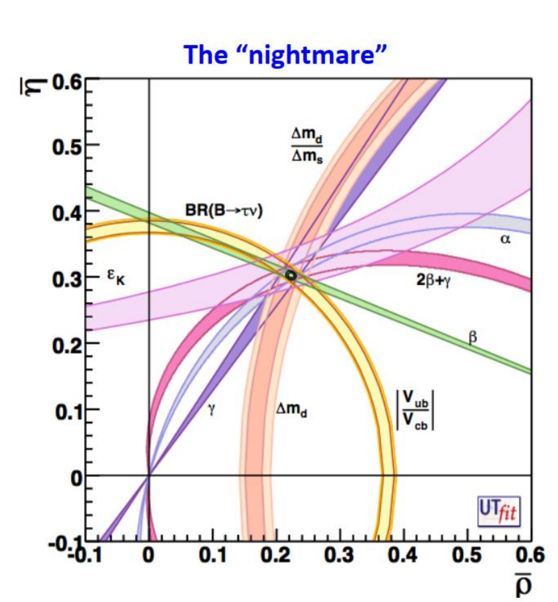


- Semileptonic decay $B \to D\ell\nu$
- "Charmed" decays $B \to DK$
- CP violation in K mesons
- ΔM in B_d–$\overline{B_d}$ system
- ΔM in B_s–$\overline{B}_s$ system
- Decays to π and K
- CP asymmetry in $B \to J/\psi K_S$

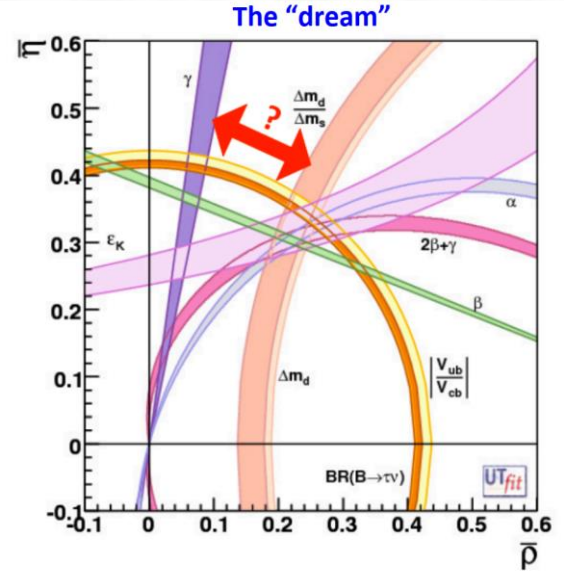
$$\boxed{A_{CP,\psi K_S}(t) = S_{\psi K_S}\sin(\Delta M_d t), \qquad S_{\psi K_S} = \sin\left(\phi_M^{(d)}\right) = \sin(2\beta),}$$

Evolution of the CKM UT fit





https://arxiv.org/abs/0710.3799



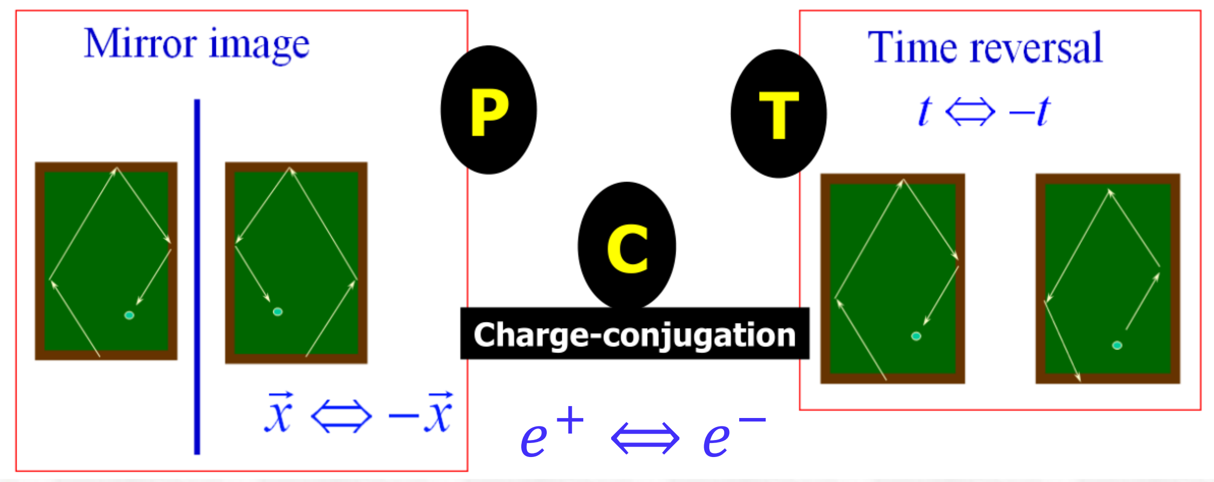
CP violation:
history, status, why B decays

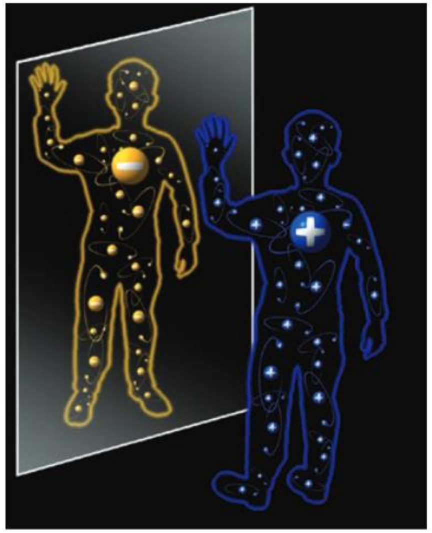
Why symmetries

- **Symmetries: crucial for understanding the laws of Nature**
  - **SU(3) flavor symmetry $\Longrightarrow$ the quark model**
  - **Continuous space-time (translational/rotational) symmetries $\Longrightarrow$ energy-momentum conservation laws**
  - **Gauge symmetries $\Longrightarrow$ electroweak and strong interactions**
- **Symmetries may keep exact or be broken, and both are important!**
  - **SU(3) flavor symmetry: broken $\Longrightarrow$ fertile hadron spectrum**
  - **U(1) electromagnetic gauge symmetry: exact $\Longrightarrow$ massless photon**
  - **SU(2) weak gauge symmetry: broken $\Longrightarrow$ massive $W^{\pm}, Z^{0}$, fermion masses**
  - **SU(3) color gauge symmetry: exact $\Longrightarrow$ massless gluons**

Discrete symmetries: P, C, T

- **Definition**





- **P violation: a new and revolutionary idea,**

  ➡ **V-A theory of weak interactions**

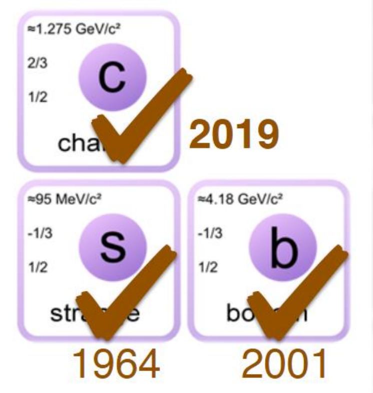
1956 **1957**

- **CP violation: first observed in K decays in 1964**

$$|\eta_{+-}| = |A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-)| = (2.236 \pm 0.007) \times 10^{-3}.$$

J.W. Cronin, Val L. Fitch

CPV observed in K, B & D systems



1. All three types of CP violation have been observed in $K \to \pi\pi$ decays:

$$\mathcal{R}e(\epsilon') = \frac{1}{6}\left(\left|\frac{\overline{A}_{\pi^0\pi^0}}{A_{\pi^0\pi^0}}\right| - \left|\frac{\overline{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}}\right|\right) = (2.5 \pm 0.4) \times 10^{-6}\,, \quad \text{(I)}$$
Direct CPV

$$\mathcal{R}e(\epsilon) = \frac{1}{2}\left(1 - \left|\frac{q}{p}\right|\right) = (1.66 \pm 0.02) \times 10^{-3}\,, \quad \text{(II)}$$
Indirect CPV

$$\mathcal{I}m(\epsilon) = -\frac{1}{2}\mathcal{I}m(\lambda_{(\pi\pi)_{I=0}}) = (1.57 \pm 0.02) \times 10^{-3}\,. \quad \text{(III)}$$
CPV due to interplay of decay & mixing

2. For D mesons, CP violation in decay has been established in the difference of asymmetries for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays.

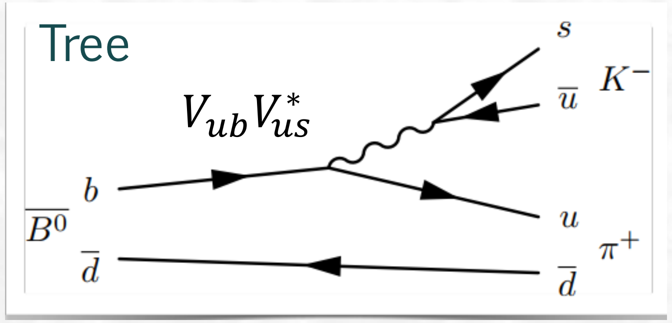
$$\Delta a_{CP} = \frac{|\overline{A}_{K^+K^-}/A_{K^+K^-}|^2 - 1}{|\overline{A}_{K^+K^-}/A_{K^+K^-}|^2 + 1} - \frac{|\overline{A}_{\pi^+\pi^-}/A_{\pi^+\pi^-}|^2 - 1}{|\overline{A}_{\pi^+\pi^-}/A_{\pi^+\pi^-}|^2 + 1} = (-0.164 \pm 0.028) \times 10^{-3}\,, \quad \text{(I)}$$
Direct CPV

3. In the B meson system, CP violation in decay has been observed in, for example, $B^0 \to K^+\pi^-$ transitions, while CP violation in interference of decays with and without mixing has been observed in, for example, the $B^0 \to J/\psi K_S$ channel:

$$\mathcal{A}_{K^+\pi^-} = \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.084 \pm 0.004\,, \quad \text{(I)}$$
Direct CPV

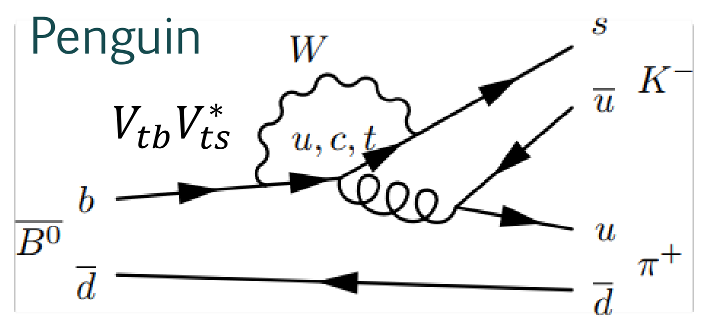
$$S_{\psi K} = \mathcal{I}m(\lambda_{\psi K}) = +0.699 \pm 0.017\,. \quad \text{(III)}$$
Indirect CPV

Three types of CP violation



- **Type I: direct CP violation, requiring that $A(i \to f) \neq \bar{A}(i \to f)$**

$$\mathcal{A}_{\rm CP}^{\rm decay} = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})} = \frac{1 - |\bar{A}/A|^2}{1 + |\bar{A}/A|^2}$$



- **Direct CP asymmetry: needs (at least!) two interfering amplitudes with different strong and weak phases**

$$A = \sum_{i=1,2} A_i e^{i(\delta_i + \phi_i)}, \quad \bar{A} = \sum_{i=1,2} A_i e^{i(\delta_i - \phi_i)}$$

$$\mathcal{A}_{\rm CP}^{\rm decay} = \frac{-2A_1A_2\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)}$$

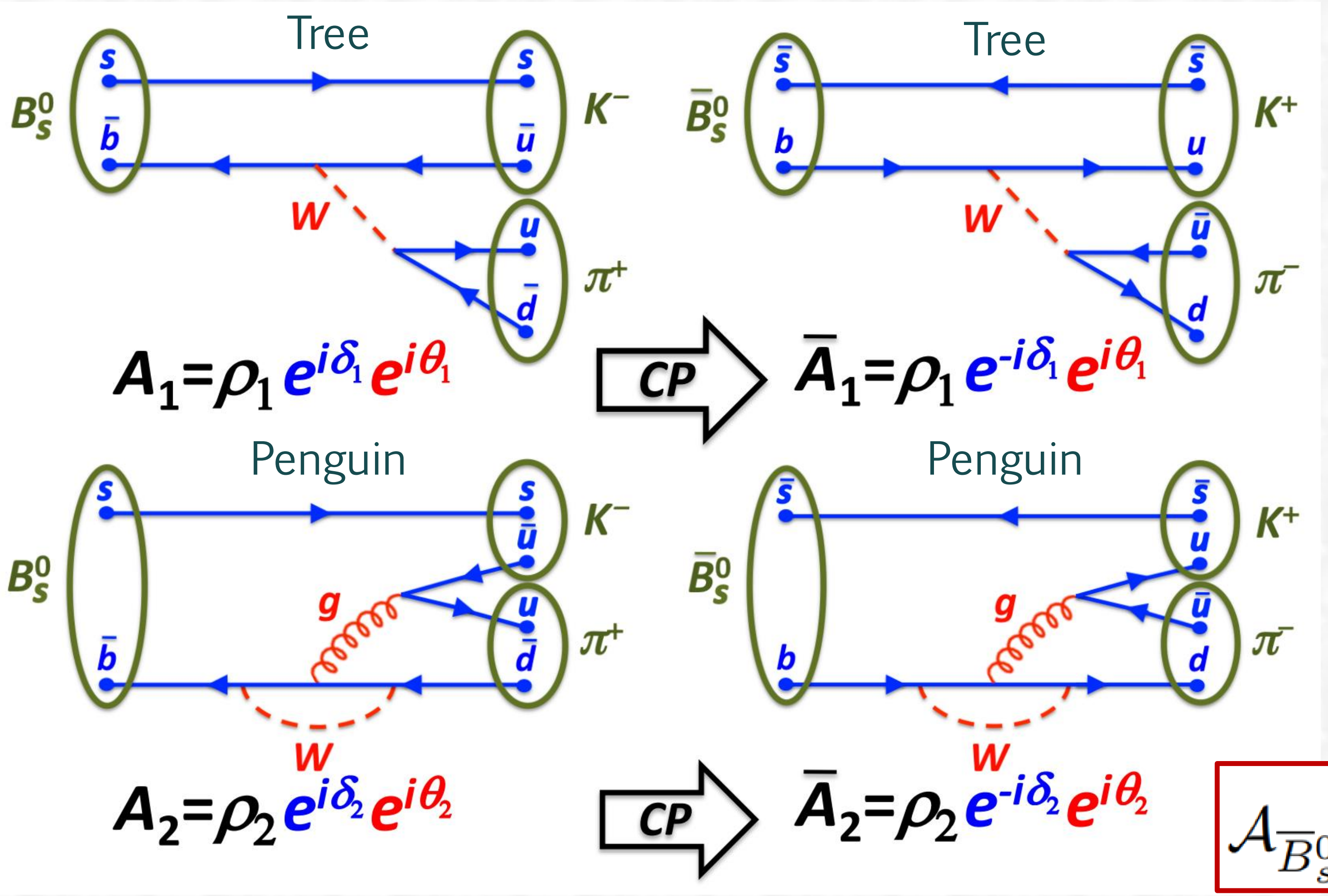
- Strong phases $\delta_{1,2}$: difficult to calculate reliably
- Weak phases $\phi_{1,2}$: arising from CKM in SM or NP

Three types of CP violation

$$\mathcal{A}_{CP}^{\text{decay}} = \frac{\Gamma(B\to f)-\Gamma(\bar{B}\to \bar{f})}{\Gamma(B\to f)+\Gamma(\bar{B}\to \bar{f})} = \frac{1-|\bar{A}/A|^2}{1+|\bar{A}/A|^2}$$

$$= \frac{-2A_1A_2\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{A_1^2+A_2^2+2A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)}$$

- **Example of direct CP asymmetry: $B_s^0 \to K^-\pi^+$**



- Tree amplitude:

$$V_{ub}V_{ud}^* = A\lambda^3(\bar{\rho}-i\bar{\eta})$$

- Penguin amplitude:

$$V_{tb}V_{td}^* = A\lambda^3(1-\bar{\rho}+i\bar{\eta})$$

$$|\bar{A}_1+\bar{A}_2|^2 - |A_1+A_2|^2$$

$$= 4\rho_1\rho_2\sin(\delta_1-\delta_2)\sin(\theta_1-\theta_2)$$

$$\boxed{\mathcal{A}_{\overline{B}_s^0\to K^+\pi^-} = +0.213 \pm 0.017}$$

Three types of CP violation

$$|B_H\rangle = p|B\rangle + q|\overline{B}\rangle$$
$$|B_L\rangle = p|B\rangle - q|\overline{B}\rangle$$

- **Type II: indirect CP violation, and arises only from neutral-meson mixings**
  - needs to be neutral and has distinct anti-particle
  - needs to have a non-zero lifetime

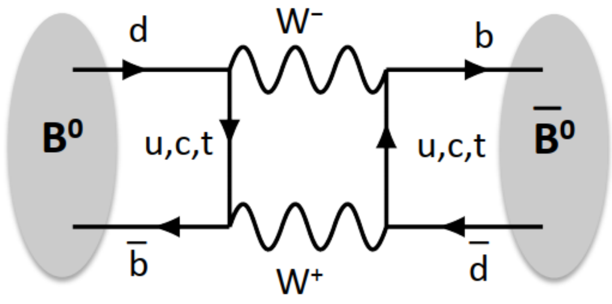
	u	c	t
$\bar{u}$	$\times$	D^0	$\diamond$
$\bar{c}$	$\overline{D^0}$	$\times$	$\diamond$
$\bar{t}$	$\diamond$	$\diamond$	$\times$

	$\bar{d}$	$\bar{s}$	$\bar{b}$
d	$\times$	K^0	B^0
s	$\overline{K^0}$	$\times$	B_s
b	$\overline{B^0}$	$\overline{B_s}$	$\times$

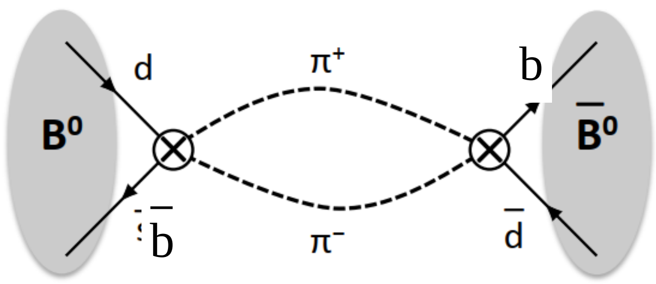
- **For B_q^0 meson, flavor eigenstates different from CP eigenstates/mass eigenstates, and can mix with each other via box diagrams due to weak interactions**

$$B_d^0 = (\bar{b}d), \qquad \bar{B}_d^0 = (b\bar{d}), \qquad B_s^0 = (\bar{b}s), \qquad \bar{B}_s^0 = (b\bar{s})$$

Box diagram



"short-distance"
(=virtual particle exchange)



"long-distance"
(=real particle exchange)

Three types of CP violation

- Due to $B^0-\bar{B}^0$ mixing, a given state $|\psi(t)\rangle$ at time t is given by $|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle$, and the time evolution is determined by the Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \hat{H}\begin{pmatrix} a \\ b \end{pmatrix} \equiv (\hat{M} - \frac{i}{2}\hat{\Gamma})\begin{pmatrix} a \\ b \end{pmatrix}$$

$$H = \begin{pmatrix} m - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* + \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}$$

CPT invariance assumed!

- Two mass eigenstates and their time evolution

$$|B_H\rangle = p|B\rangle + q|\bar{B}\rangle$$
$$|B_L\rangle = p|B\rangle - q|\bar{B}\rangle$$

$$\begin{aligned} |B_H(t)\rangle &= |B_H\rangle\, e^{-i\left(M+\frac{1}{2}\Delta m-\frac{i}{2}(\Gamma-\Delta\Gamma)\right)t} \\ |B_L(t)\rangle &= |B_L\rangle\, e^{-i\left(M-\frac{1}{2}\Delta m+\frac{i}{2}(\Gamma+\Delta\Gamma)\right)t} \end{aligned}$$

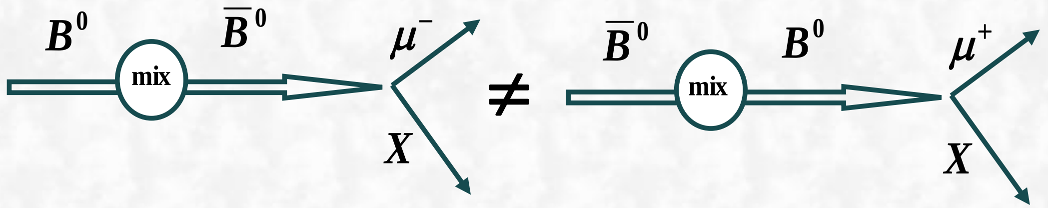
$$\boxed{\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}}$$

- Time-dependent decay width and then

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt[\overline{M}^0_{\rm phys}(t)\to \ell^+X] - d\Gamma/dt[M^0_{\rm phys}(t)\to \ell^-X]}{d\Gamma/dt[\overline{M}^0_{\rm phys}(t)\to \ell^+X] + d\Gamma/dt[M^0_{\rm phys}(t)\to \ell^-X]} = \frac{1-|q/p|^4}{1+|q/p|^4}\,.$$

requiring that $|q/p| \neq 1$

➢ Type II: indirect CP violation, also known as flavor-specific (semi-leptonic) CPV!



Difference among K, B and D mixings

□ Mixing observables

- $\Delta m \equiv (m_H - m_L)$, $\quad x = \Delta m/\Gamma$
- $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$, $\quad y = \Delta\Gamma/2\Gamma$
- $\Delta\Gamma \equiv \Gamma_L - \Gamma_H$

□ Δm determined by short-distance box diagram

CKM involved

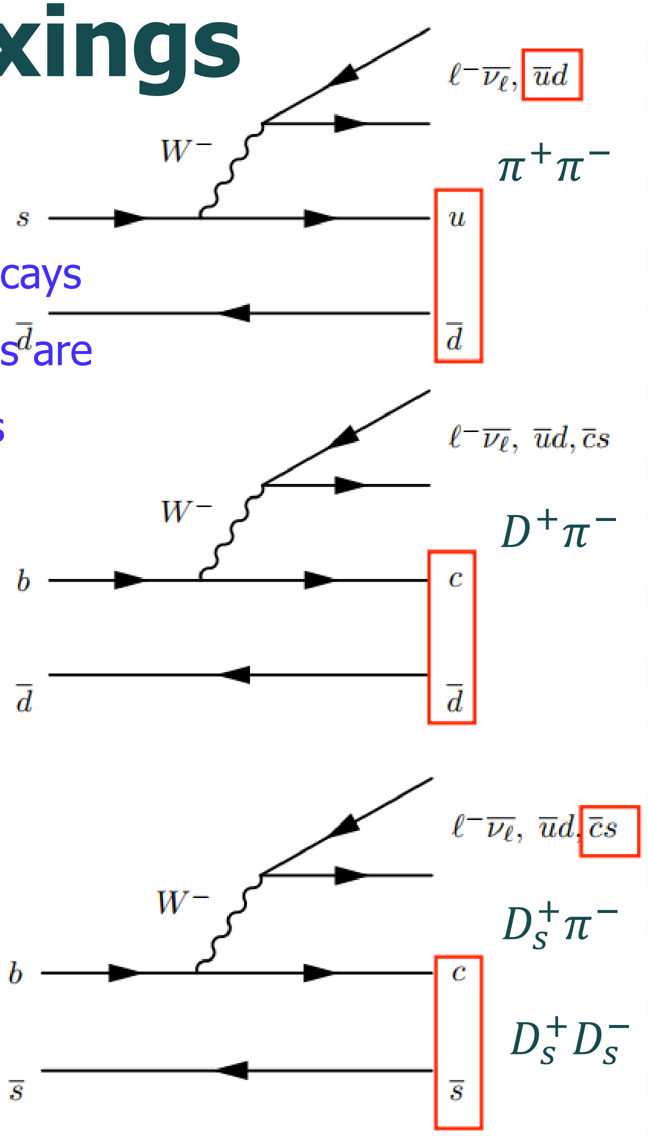
CKM		$\Delta m(\mathrm{ps}^{-1})$ $(x = \Delta m/\Gamma)$	$\Delta\Gamma(\mathrm{ps}^{-1})$ $(y = \Delta\Gamma/(2\Gamma))$
$\lvert V_{cb}V_{ub}^*\rvert^2 \simeq \lambda^{10}$	D^0	small $(0.95 \pm 0.44)\%$	small $(0.75 \pm 0.12)\%$
$\lvert V_{tb}V_{td}^*\rvert^2 \simeq \lambda^6$	B^0	medium 0.5065 ± 0.0019	small 0.000 ± 0.007
$\lvert V_{tb}V_{ts}^*\rvert^2 \simeq \lambda^4$	B_s^0	large 17.765 ± 0.006	medium 0.084 ± 0.005

□ $\Delta\Gamma$ dictated by fraction of decays into common final states

- For K^0, all decays without leptons are CP eigenstates

- For B^0, the dominant decays are not CP eigenstates

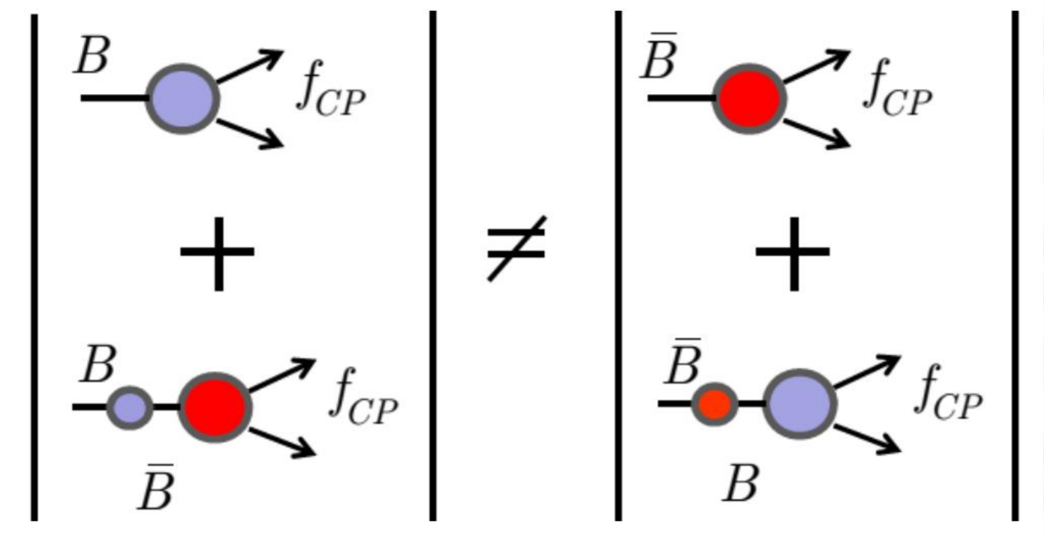
- For B_s, to a somewhat lesser extent, the dominant decays are not CP eigenstates



Dominant decay amplitudes

Three types of CP violation

- **Type III: CP violation arising from the interplay of decay & mixing**



$$\mathcal{A}_f^2 + \mathcal{S}_f^2 + \mathcal{H}_f^2 = 1$$

$$\mathcal{A}_f = \frac{|\lambda_f|^2 - 1}{1 + |\lambda_f|^2}, \quad \mathcal{S}_f = \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}, \quad \mathcal{H}_f = \frac{2\mathrm{Re}\lambda_f}{1 + |\lambda_f|^2},$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = \eta_f\, e^{2i\phi}\frac{A(\bar{B}_q \to f)}{A(B_q \to f)}$$

$$CP|f_{CP}>= \eta_f|f_{CP}>$$

$$\mathcal{A}(t)|_{B^0} = \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} = \mathcal{A}_f \cos(\Delta M_d t) + \mathcal{S}_f \sin(\Delta M_d t),$$

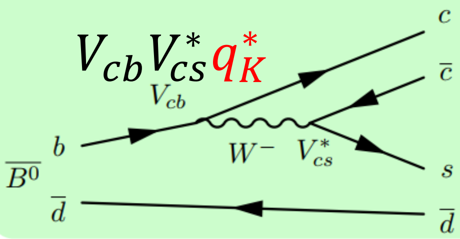
$$\mathcal{A}(t)|_{B_s^0} = \frac{\Gamma(\bar{B}_s^0(t) \to f) - \Gamma(B_s^0(t) \to f)}{\Gamma(\bar{B}_s^0(t) \to f) + \Gamma(B_s^0(t) \to f)} = \frac{\mathcal{A}_f \cos(\Delta M_s\, t) + \mathcal{S}_f \sin(\Delta M_s\, t)}{\cosh\left(\frac{\Delta\Gamma_s}{2} t\right) + \mathcal{H}_f \sinh\left(\frac{\Delta\Gamma_s}{2} t\right)},$$

Three types of CP violation

□ Example of type-III CP violation: $B^0 \to J/\psi K_S$

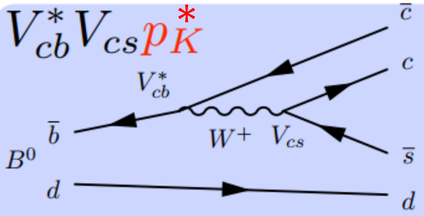
$$\lambda_{J/\psi K_S} \equiv \frac{q}{p}\frac{\overline{A}_{J/\psi K_S}}{A_{J/\psi K_S}} = -\frac{q}{p}\frac{\overline{A}_{J/\psi \overline{K^0}, \overline{K^0}\to K_S}}{\boxed{A_{J/\psi K^0, K^0\to K_S}}}$$

$V_{cb}V_{cs}^* q_K^*$



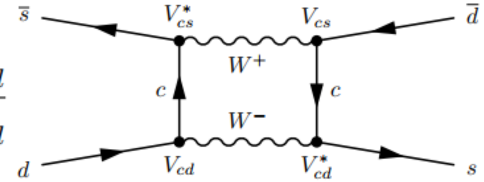
$\times \left\langle K_s | \overline{K^0} \right\rangle$

$V_{cb}^* V_{cs} p_K^*$



$\times \left\langle K_s | K^0 \right\rangle$

$$\frac{q_K}{p_K} \approx \frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*}$$

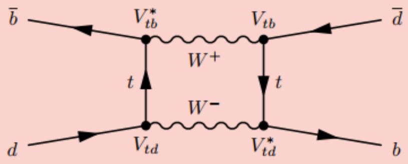


$$|K_s\rangle = p_K \left|K^0\right\rangle + q_K \left|\overline{K^0}\right\rangle$$

$$\overline{A}_f = \eta_{CP} \overline{A}_{\overline{f}}$$

$$\begin{aligned} CP\,|J/\psi K_S\rangle &= (-1^1)\,[CP\,|J/\psi\rangle]\,[CP\,|K_S\rangle] \\ &= (-1^1)\,[(-1^1)(-)\,|J/\psi\rangle]\,[(+)\,|K_S\rangle] \\ &= -\,|J/\psi K_S\rangle \end{aligned}$$

$$\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$



$$\begin{aligned} \lambda_{J/\psi K_S} &\equiv \frac{q}{p}\frac{\overline{A}_{J/\psi K_S}}{A_{J/\psi K_S}} \\ &= -\frac{q}{p}\frac{\overline{A}_{J/\psi \overline{K^0}, \overline{K^0}\to K_S}}{A_{J/\psi K^0, K^0\to K_S}} \\ &= -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \\ &= -e^{-2i\beta} \end{aligned}$$

$$\boxed{\mathcal{A}_{CP} = \frac{\Gamma(\overline{B^0} \to J/\psi K_S) - \Gamma(B^0 \to J/\psi K_S)}{\Gamma(\overline{B^0} \to J/\psi K_S) + \Gamma(B^0 \to J/\psi K_S)} = \sin(2\beta)\sin(\Delta m t)}$$

CP violation within the SM

- **CPV observables in meson decays can be explained by the KM mechanism**

$$\begin{array}{cc} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \end{array}$$

- **For bottom, large CP asymmetries expected within the SM**

$$\mathcal{A}_{\overline{B}^0_s\to K^+\pi^-} = +0.213\pm 0.017$$

- **For charm, CP asymmetries typically of the order of 10^{-4}-10^{-3} within the SM**

$$\Delta a_{CP} = (-0.164\pm 0.028)\times 10^{-3}$$

- **For strange, also small CP asymmetries expected within the SM**

$$|\epsilon| = (2.228\pm 0.011)\times 10^{-3} \qquad \mathcal{R}e(\epsilon'/\epsilon) = (1.65\pm 0.26)\times 10^{-3}$$

B physics:

basics, examples, and status, …

Bottom quark

- **Predicted by KM in 1973, and discovered by Lederman in 1977 [**S. Herb et al., PRL.39,

$\boxed{b}$ $\quad I(J^P) = 0(\tfrac{1}{2}^+)$

$m_b = 4.18^{+0.03}_{-0.02}$ GeV $\quad$ Charge $= -\tfrac{1}{3}\ e$ $\quad$ Bottom $= -1$

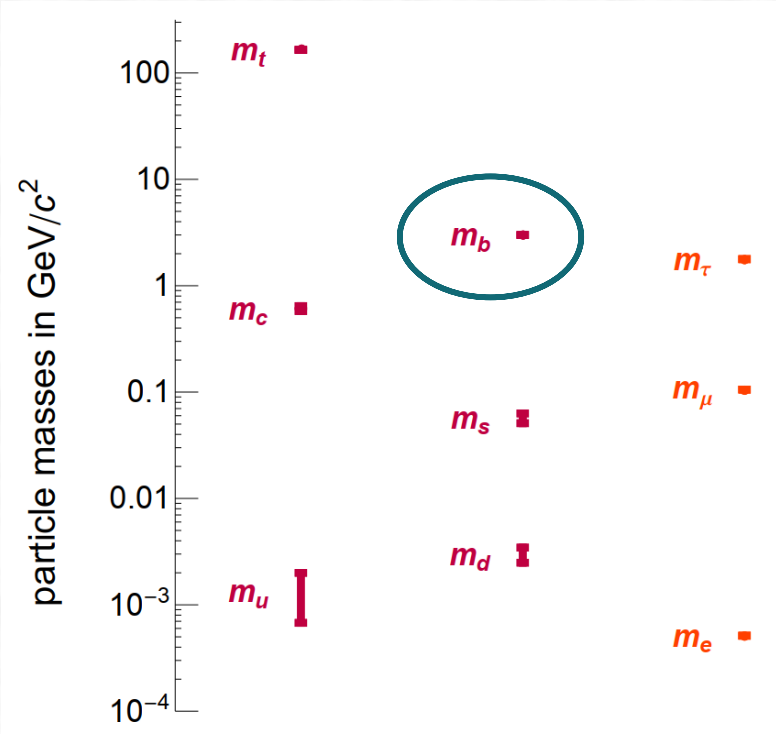
- **Massive enough to form various bound states**

B-**mesons**

	$B_d = (\bar{b}d)$	$B^+ = (\bar{b}u)$	$B_s = (\bar{b}s)$	$B_c^+ = (\bar{b}c)$
Mass (GeV)	5.27961(16)	5.27929(15)	5.36679(23)	6.2751(10)
Lifetime (ps)	1.520(4)	1.638(4)	1.505(5)	0.507(9)
$\tau(X)/\tau(B_d)$	1	1.076 ± 0.004	0.990 ± 0.004	$0.334 \pm 0.006*$

b-**baryons**

	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Mass (GeV)	5.61951(23)	5.7918(5)	5.7944(12)	6.0480(19)
Lifetime (ps)	1.470(10)	1.479(31)	1.571(40)	$1.64\ \binom{+18}{-17}$
$\tau(X)/\tau(B_d)$	0.967 ± 0.007	$0.973 \pm 0.020*$	$1.034 \pm 0.026*$	$1.08\ \binom{+12}{-11}\ *$



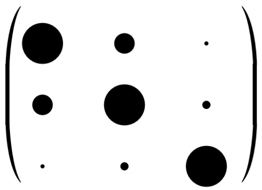
* Status as of February 2017, by A. Lenz

B-hadron weak decays

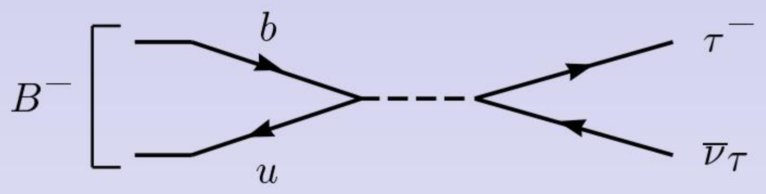
- **Weak decays:** at the quark level, all by flavor-changing charged-currents mediated by *W*-boson in the SM.

$$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}} J^{\mu}_{CC}\, W^{\dagger}_{\mu} + \text{h.c.}$$

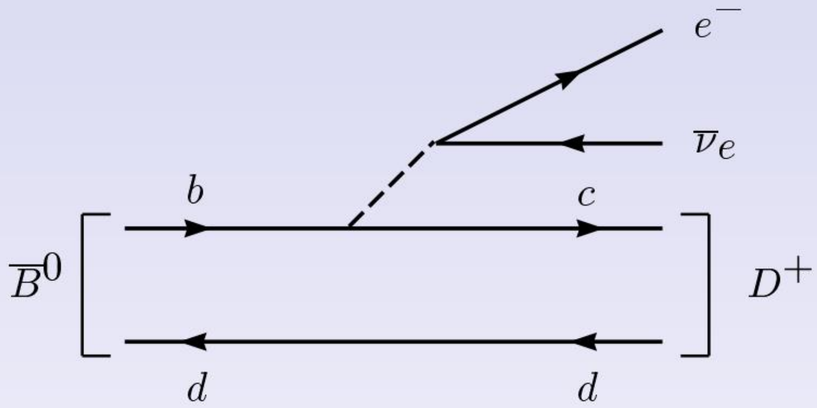
$$J^{\mu}_{CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\mu \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + (\bar{u}_L, \bar{c}_L, \bar{t}_L)\,\gamma^\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$V_{CKM} =$ 

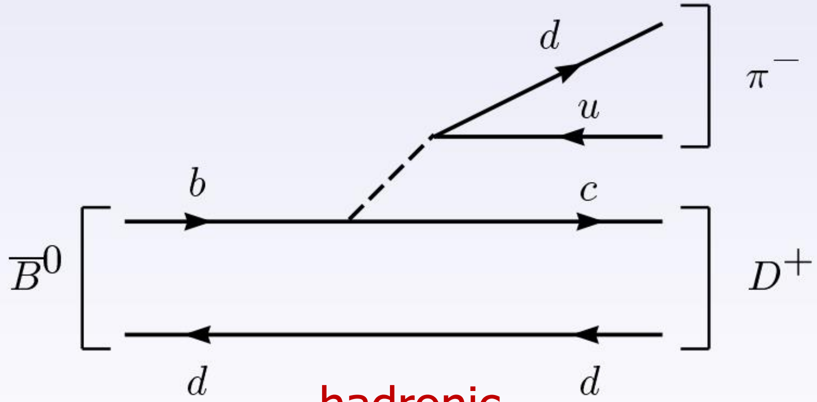
$$b \to \begin{cases} c \\ u \end{cases} + W^{*-} \to \begin{cases} c \\ u \end{cases} + \begin{cases} \bar{u} + d \\ \bar{c} + s \\ \bar{u} + s \\ \bar{c} + d \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{cases}$$



purely leptonic



semi-leptonic



hadronic

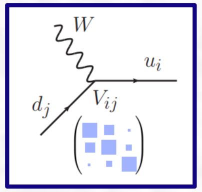
- **Classification of b-flavored weak decays:** three classes;

  purely leptonic, semi-leptonic, hadronic

Tree vs rare FCNC decays

□ Hierarchical structures among CKM elements:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

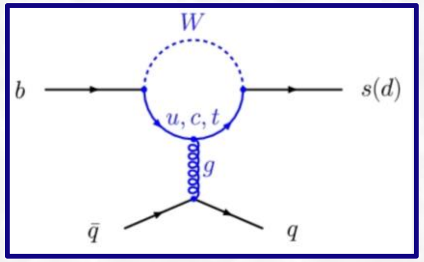


$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

➢ **FCNC decays:** arise firstly at one-loop level in SM;

$$A(b\to s) \sim \frac{1}{16\pi^2} V_{ts}^* V_{tb} \sim 2.5 \times 10^{-4}$$
(e.g. $B \to K^* \mu^+ \mu^-$)

$$A(b\to d) \sim \frac{1}{16\pi^2} V_{td}^* V_{tb} \sim 5 \times 10^{-5}$$
(e.g. $B \to \pi \mu^+ \mu^-$)



Rare decays

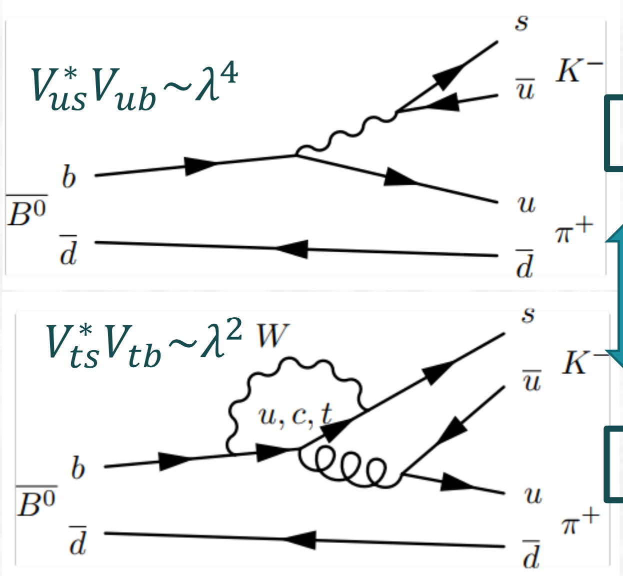
➢ **Tree-level charged-current decays:**

$$A(b\to c) \sim V_{cb} \sim 4 \times 10^{-2}$$
(e.g. $B \to D \mu \nu$)

$$A(b\to u) \sim V_{ub} \sim 4 \times 10^{-3}$$
(e.g. $B \to \pi \mu \nu$)

Tree decays

□ Interference between tree & penguin:



Tree amplitude

$$A_{CP}(\bar{B}^0 \to K^- \pi^+) = -0.084 \pm 0.004$$

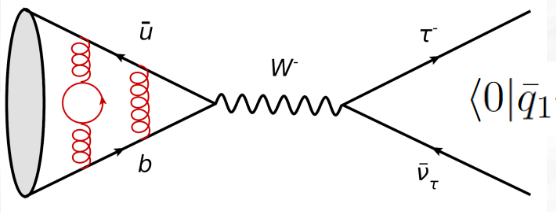
Penguin amplitude

Interplay between weak & strong forces

- **QCD effect matters:** in real world, quarks are always confined inside hadrons by QCD;

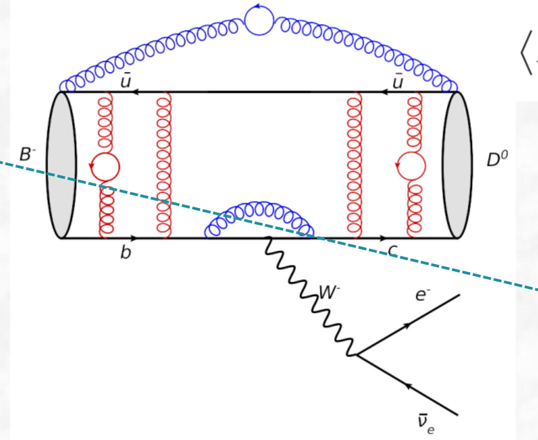
  the simplicity of weak interactions overshadowed by the complexity of strong interactions!

- **Purely leptonic decays:** decay constant



$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = ip_\mu f_P$$

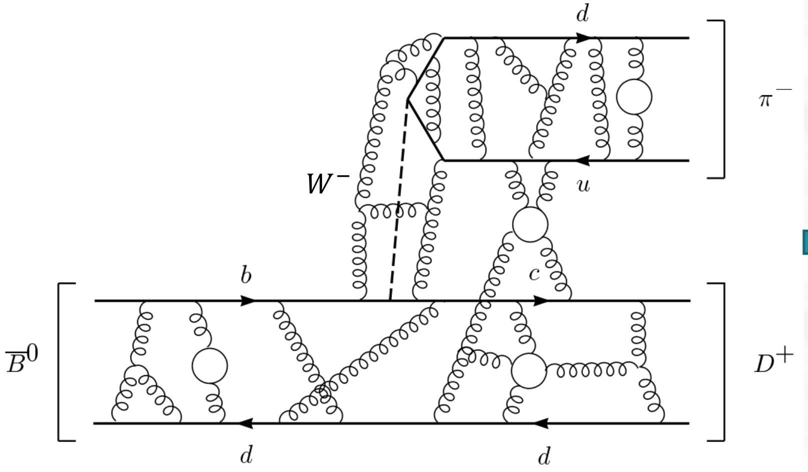
- **Semi-leptonic decays:** transition form factors



$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu + [f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

LQCD or LCSR etc.

- **Hadronic decays:** hadronic matrix elements



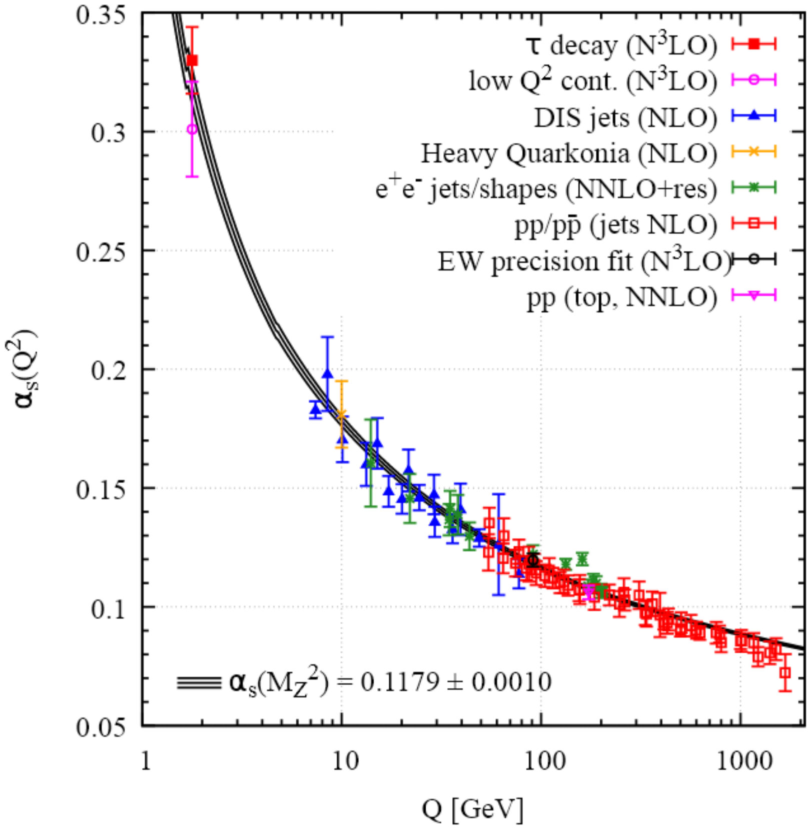
multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

very difficult to deal with such an object!

Strong interactions and QCD

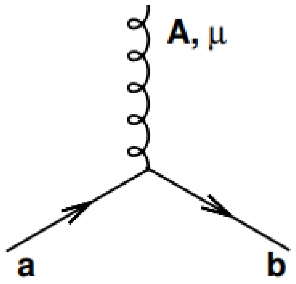
David Gross , David Politzer, Frank Wilczek



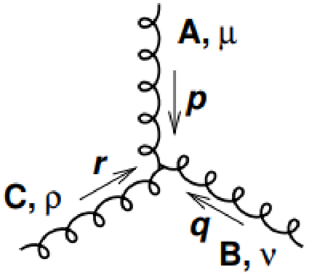
□ QCD Lagrangian for quark and gluon interactions

$$
\begin{aligned}
\mathcal{L}_{\rm QCD} \;=\; & -\frac{1}{4}\left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right)\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right) + \sum_f \bar{q}_f^\alpha\,\left(i\gamma^\mu\partial_\mu - m_f\right)\,q_f^\alpha \\
& + g_s\, G_a^\mu \sum_f \bar{q}_f^\alpha\,\gamma_\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^\beta \\
& - \frac{g_s}{2} f^{abc}\left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G_\mu^b\, G_\nu^c - \frac{g_s^2}{4} f^{abc} f_{ade}\, G_b^\mu\, G_c^\nu\, G_\mu^d\, G_\nu^e
\end{aligned}
$$

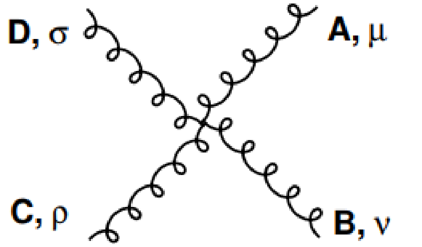
□ Basic interaction vertices: non-abelian



$-ig_s t^A_{ba}\gamma^\mu$



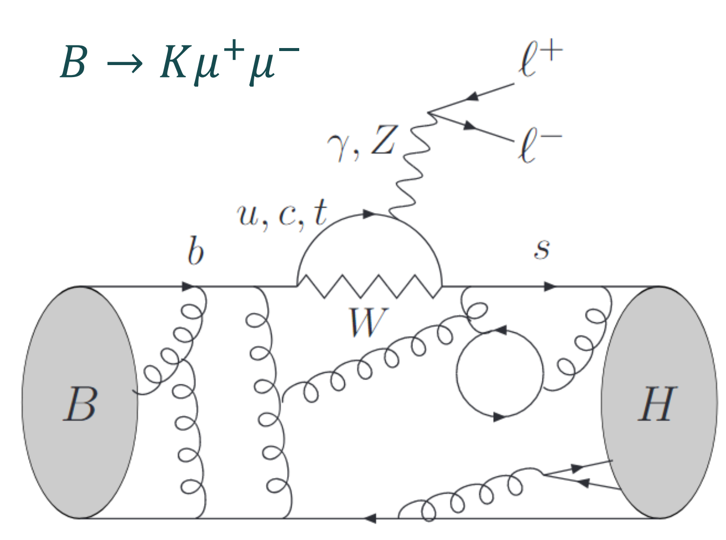
$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$



$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$

Weak effective Hamiltonian for B decays

- **For a weak decay processes: several different typical energy/length scales involved**



New physics	:	$\delta x \sim 1/\Lambda_{\rm NP}$
Electroweak interactions	:	$\delta x \sim 1/M_W$
Short-distance QCD(QED) corrections	:	$\delta x \sim 1/M_W \to 1/m_{b(c)}$
Hadronic effects	:	$\delta x > 1/m_{b(c)}$

- **OPE and factorization idea:** separate these different scales using the *RG-improved perturbation theory*

$$\int d^4x\, e^{iq\cdot x}\, T(\phi(x)\,\phi(0)) = \sum_i c_i(q^2)\,\mathcal{O}_i(0)$$

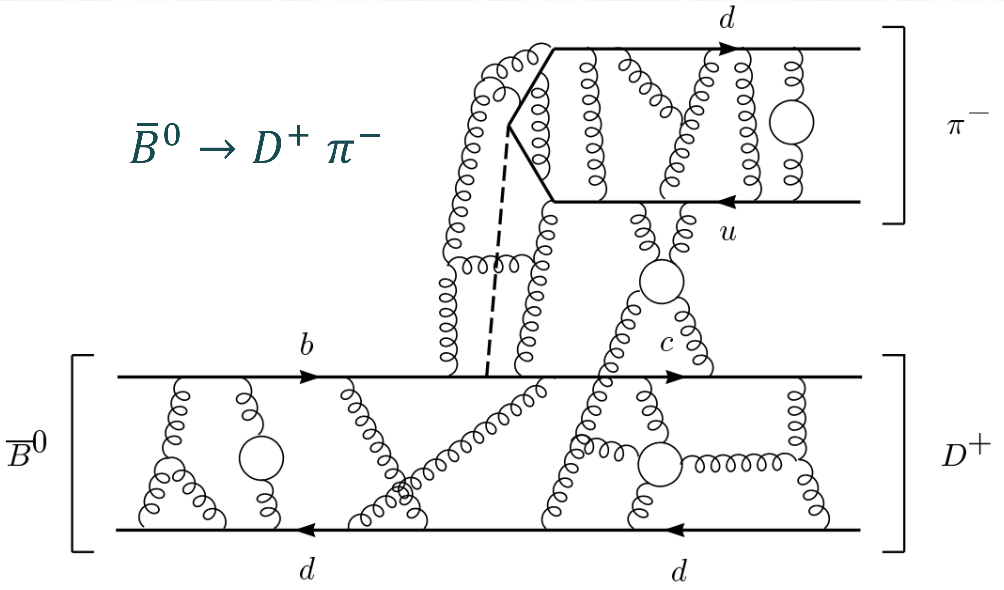
"Wilson Coefficients" $c_i(q^2)$

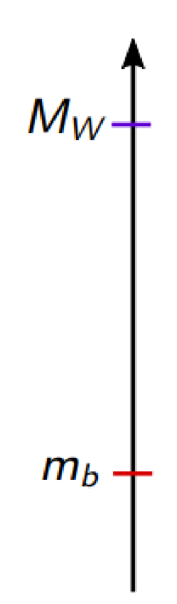
"Effective" Operators $\mathcal{O}_i(0)$

- **Low-energy effective Hamiltonian/Lagrangian:**
  - high-energy (short-distance) information contained in the Wilson coefficients (functions)
  - matrix elements of effective operators reproduce the low-energy dynamics

Weak effective Hamiltonian for B decays

- **RG-improved perturbation theory:** goes beyond the usual perturbation theory





- **Problem:** Strong interactions with multiple and vastly different scales can lead to uncontrolled perturbative series, thus spoiling the perturbative convergence due to large logs

$$P(M_W, m_b) = 1 + \alpha_s \left(\# \ln \frac{M_W}{m_b} + * \right) + \alpha_s^2 \left(\# \ln^2 \frac{M_W}{m_b} + * \right) + \ldots$$

Weak effective Hamiltonian for B decays

Solution: The perturbative series needs to be reorganized, and thus all $(\alpha_s \ln \frac{m_W}{m_b})^n$ re-summed

- step1: through matching to achieve a separation of scales, sometimes also called "factorization"

$$\left[1+\alpha_s\left(\#\ln\frac{M_W}{\mu}+*\right)+\dots\right]\cdot\left[1+\alpha_s\left(\#\ln\frac{\mu}{m_b}+*\right)+\dots\right]$$

$$P(M_W, m_b) = C(M_W,\mu)D(m_b,\mu)$$

at the cost of introducing a "factorization scale" μ.

- step2a: while $P(MW\ ,m_b)$ is formally μ-independent, the factors C and D by themselves are not, and obey

$$\text{RGEs:}\ \left\{\begin{array}{lcl}\mu\frac{d}{d\mu}C(M_W,\mu) & = & \gamma(\mu)\,C(M_W,\mu)\\ \mu\frac{d}{d\mu}D(M_W,\mu) & = & -\gamma(\mu)\,D(M_W,\mu)\end{array}\right\}\Rightarrow \mu\frac{d}{d\mu}(CD)=0$$

["C and D run with μ."]

- step2b: solve the RGEs and then evolve

$$\boxed{\mu_{\text{high}}\sim m_W}$$

$$\begin{array}{rcl} C(M_W,\mu) & = & C(M_W,\mu_{\text{high}})\,U(\mu_{\text{high}},\mu)\\ D(m_b,\mu) & = & D(m_b,\mu_{\text{low}})\,U(\mu,\mu_{\text{low}})\end{array}$$

$$\boxed{\mu\ \text{arbitrary}} \qquad \boxed{\mu_{\text{low}}\sim m_b}$$

Final result for $P(M_W, m_b)$:

$$\boxed{P(M_W,m_b)=\underbrace{C(M_W,\mu_{\text{high}})U(\mu_{\text{high}},\mu_{\text{low}})}_{C_{\text{RGimproved}}(M_W,\mu_{\text{low}})}D(m_b,\mu_{\text{low}})}$$

$U(\mu_{\text{high}},\mu_{\text{low}})$ generally an exponential, and thus re-sums large logs $(\alpha_s \ln \frac{\mu_{\text{high}}}{\mu_{\text{low}}})^n$

Weak effective Hamiltonian for B decays

☐ **Philosophy of Effective Field Theory:**

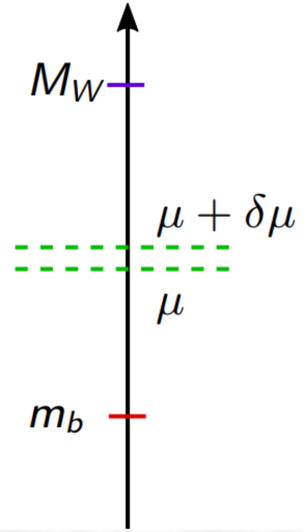
$$\boxed{P(M_W, m_b) = \underbrace{C(M_W, \mu_{\rm high})\, U(\mu_{\rm high}, \mu_{\rm low})}_{C_{\rm RGimproved}(M_W, \mu_{\rm low})}\, D(m_b, \mu_{\rm low})}$$

☐ **The two ingredients in the factorized physical observable *$P = C \cdot D$* are connected to**

$$\langle \text{Full theory} \rangle = C(M_W, \mu)\, \langle \text{EFT}, \mu \rangle$$

- key point 1: The EFT reproduces the IR physics of the full theory to any desired precision

- key point 2: The couplings [Wilson coefficients $C_i(\mu)$] capture the UV physics above $\mu_{\rm low}$ of the full theory



$$\sum_i C_i(\mu + \delta\mu)\, \langle O_i(\mu + \delta\mu) \rangle$$

$$\updownarrow$$

$$\sum_i C_i(\mu)\, \langle O_i(\mu) \rangle$$

Weak effective Hamiltonian for B decays

□ **Example of RG-improved perturbative theory**

QCD coupling

$$\frac{d\alpha_s}{d\ln\mu} = -2\beta_0\frac{\alpha_s^2}{4\pi} - 2\beta_1\frac{\alpha_s^3}{(4\pi)^2}$$

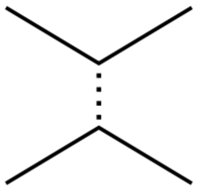
$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{v(\mu)}\left[1 - \frac{\beta_1}{\beta_0}\frac{\alpha_s(M_Z)}{4\pi}\frac{\ln v(\mu)}{v(\mu)}\right]$$

$$v(\mu) = 1 - \beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right),$$

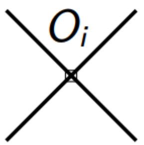
$$\alpha_s(\mu) = \alpha_s(M_Z)\left[1 + \sum_{n=1}^{\infty}\left(\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\frac{M_Z}{\mu}\right)^n\right]$$

The solution of the RGE sums automatically large logarithms $\ln\frac{M_Z}{\mu}$ ***for*** $\mu \ll M_Z$

□ **Example of OPE**



$$= \sum_i C_i(M_W)$$



$$\frac{-1}{k^2 - M_W^2} = \frac{1}{M_W^2}\left(1 + \frac{k^2}{M_W^2} + \frac{k^4}{M_W^4} + \ldots\right)$$

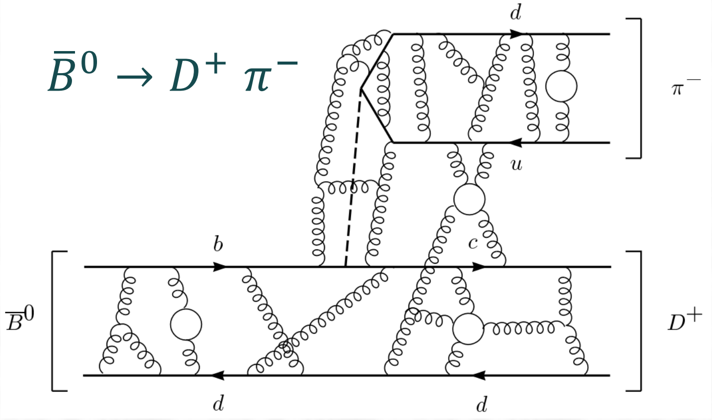
$$\mathcal{L}_{\rm eff} \ni \frac{g^2}{M_W^2}(\bar\psi\psi)(\bar\psi\psi) + \frac{g^2}{M_W^4}(\bar\psi\psi)(i\partial)^2(\bar\psi\psi) + \ldots$$

More precisely, "integrating out" the d.o.f. of massive particles and modes

By OPE, non-local interactions can be expanded in local operators

Effective Hamiltonian for $b \to c\bar{u}d$ decay

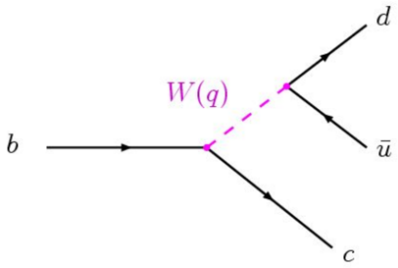
- In **full theory**: described in terms of current-current interaction weighted by CKM elements

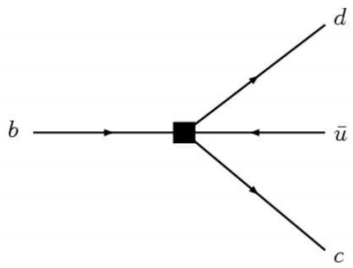


$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar{c}\,\gamma_\alpha(1-\gamma_5)\,b]\,, \qquad \bar{J}_\beta^{(d\to u)} = V_{ud}^*\,[\bar{d}\,\gamma_\beta(1-\gamma_5)\,u]$$

- **At Born level:** $|q| \le m_b \ll m_W$

Full theory (SM) $\rightarrow$ Fermi model





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)}\,\frac{-g^{\alpha\beta}+\frac{q^\alpha q^\beta}{M_W^2}}{q^2-M_W^2}\,\bar{J}_\beta^{(d\to u)} \xrightarrow{|q|\ll M_W} \frac{G_F}{\sqrt{2}}\,J_\alpha^{(b\to c)}\,g^{\alpha\beta}\,\bar{J}_\beta^{(d\to u)}$$

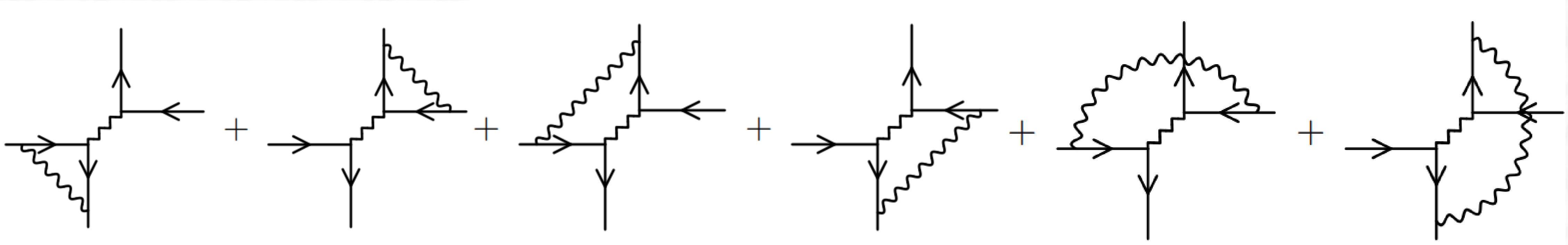
- In **effective theory**: described in terms of the local four-fermion operators with only light fields and weighted by G_F

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

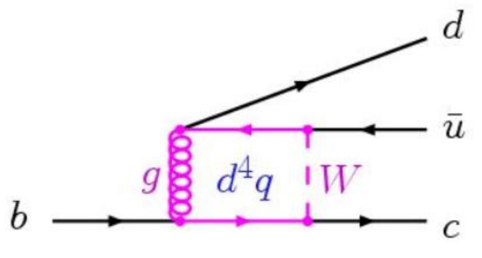
$$\boxed{G_F \simeq 1.16639 \cdot 10^{-5}\,\text{GeV}^{-2}}$$

Effective Hamiltonian for $b \to c\bar{u}d$ decay

- ☐ **One-loop QCD corrections to $b \to c\bar{u}d$ decays in full theory**



- ☐ **key point: compared to the Born term, the W-boson momentum $|q|$ is an internal loop momentum that is integrated between 0 and ∞**



➡
- **we cannot simply expand in $|q|/m_W$**
- **need a method to separate the cases $|q| \geq m_W$ and $|q| \ll m_W$**

- ☐ **Factorization/OPE/RG-improved perturbative theory**

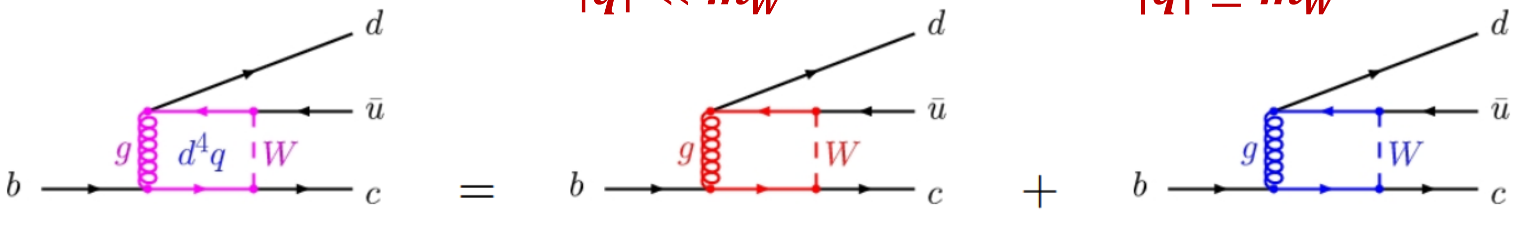
Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **Procedure and basic idea:**

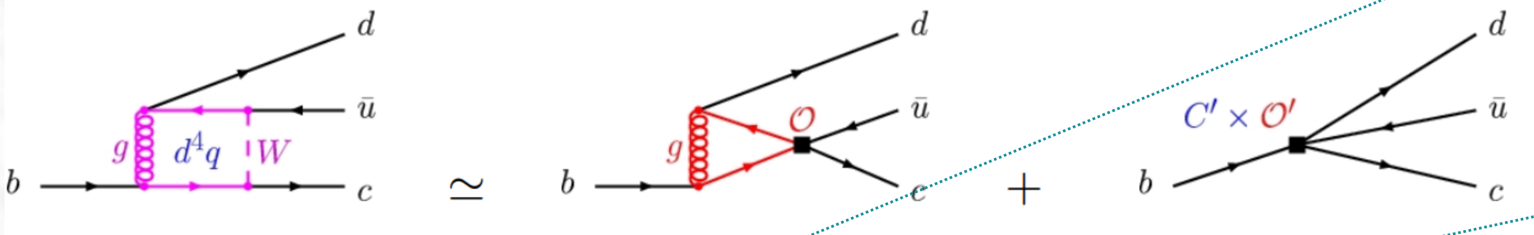
$$\boxed{\int_{-p^2}^{M_W^2}\frac{dk^2}{k^2} = \int_{-p^2}^{\mu^2}\frac{dk^2}{k^2} + \int_{\mu^2}^{M_W^2}\frac{dk^2}{k^2}}$$

full theory = IR region $(M_W \to \infty)$ + UV region $(m_{b,c} \to 0)$

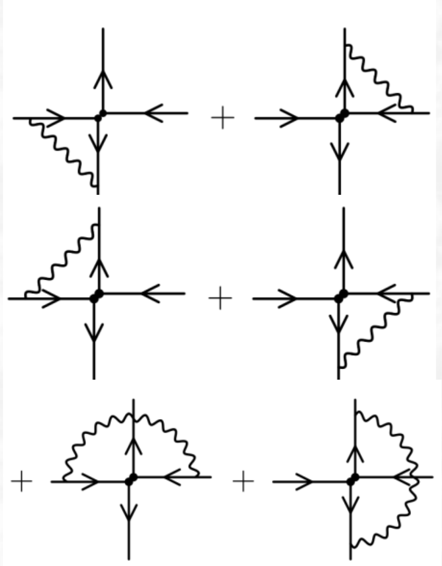
$|q| \ll m_W$ $\qquad$ $|q| \geq m_W$



$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq I_{IR}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + I_{UV}(\alpha_s;\ \frac{\mu}{m_W})$$



$$I(\alpha_s;\ \frac{m_b}{M_W},\ \frac{m_c}{m_b})/G_F \simeq \langle\mathcal{O}\rangle^{\text{loop}}(\alpha_s;\ \frac{m_b}{\mu},\ \frac{m_c}{m_b}) + C'(\alpha_s;\ \frac{\mu}{m_W}) \times \langle\mathcal{O}'\rangle^{\text{tree}}$$



1-loop matrix element of operator $\mathcal{O}$ in Eff. Th.
- independent of M_W

1-loop coefficient for new operator $\mathcal{O}'$ in ET
- independent of $m_{b,c}$

Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **Effective Hamiltonian for $b \to c\bar{u}d$ decay**

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i + \text{h.c.}$$

- **Effective operators:** there are two dim-6 local current-current operators

$$\mathcal{O}_1 = (\bar{d}_L^i \gamma_\alpha u_L^j)(\bar{c}_L^j \gamma^\alpha b_L^i)$$
$$\mathcal{O}_2 = (\bar{d}_L^i \gamma_\alpha u_L^i)(\bar{c}_L^j \gamma^\alpha b_L^j)$$

- **Wilson coefficients $C_i(\mu)$:** contain all information on the short-distance physics above the scale μ

$$C_i(\mu) = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} + \frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{\mu^2}{M_W^2} + \frac{11}{6}\right)\begin{Bmatrix} 3 \\ -1 \end{Bmatrix} + \mathcal{O}(\alpha_s^2)$$

- short-distance QCD corrections preserve chirality, so both $(V-A)\otimes(V-A)$
- quark-gluon vertices induce a second color structure

Effective Hamiltonian for $b \to c\bar{u}d$ decay

- **Wilson coefficients $C_i(\mu)$:**

$$C_i(\mu) = \boxed{\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}} + \frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{\mu^2}{M_W^2} + \frac{11}{6}\right)\begin{Bmatrix} 3 \\ -1 \end{Bmatrix} + \mathcal{O}(\alpha_s^2)$$

Question : How do we choose the renormalization scale μ ?

"Matching"

Answer : For $\mu \sim M_W$ the logarithmic term is small, and $C_i(M_W)$ can be calculated in **Fixed-Order Perturbation Theory,** since

$$\frac{\alpha_s(M_W)}{\pi} \ll 1\,.$$

In this context, $\mu \sim M_W$ is called the **Matching Scale.**

Numerical values for $C_{1,2}$ in the SM [Buchalla/Buras/Lautenbacher 96]

operator:	$\mathcal{O}_1 = (\bar{d}^i_L\gamma_\mu u^j_L)(\bar{c}^j_L\gamma^\mu b^i_L)$	$\mathcal{O}_2 = (\bar{d}^i_L\gamma_\mu T u^i_L)(\bar{c}^j_L\gamma^\mu b^j_L)$
$C_i(m_b)$:	-0.514 (LL) -0.303 (NLL)	1.026 (LL) 1.008 (NLL)

- **Reality:** to compare with experiment of hadronic models, $\langle D^+\pi^-|O_i|\bar{B}^0\rangle$ needed at scales $\mu \sim m_b$

- **Remember:** Only the combination $C_i(\mu)\,\langle O_i\rangle(\mu)$ are scale independent

  ⇒ ***need Wilson coefficients at low scale !***

- **RGE of $C_i(\mu)$:** can be calculated using the RG-improved PT

$$C_\pm(\mu) \simeq C_\pm(M_W)\cdot\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\gamma_\pm^{(1)}/2\beta_0}$$

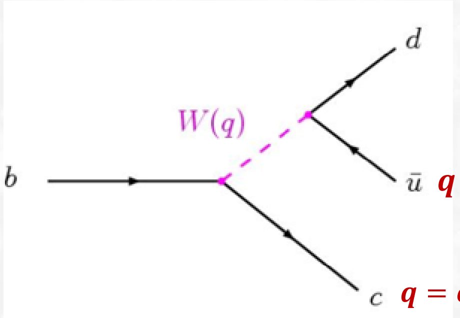
NNLL

$$\vec{C}(\mu) = \hat{K}(\mu)\hat{U}^{(0)}(\mu,\mu_0)\left(\vec{A}^{(0)} + \frac{\alpha_s(\mu_0)}{4\pi}\left[\vec{A}^{(1)} - \hat{R}^{(1)}\vec{A}^{(0)}\right] + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2\left[\vec{A}^{(2)} - \hat{R}^{(1)}\vec{A}^{(1)} - \left(\hat{R}^{(2)} - \left(\hat{R}^{(1)}\right)^2\right)\vec{A}^{(0)}\right]\right)$$

[Gorbahn and Haisch '04]

Effective Hamiltonian for $b \to s(d)\bar{q}q$ decays

- **Current-current operators:** *two possible flavor structures*



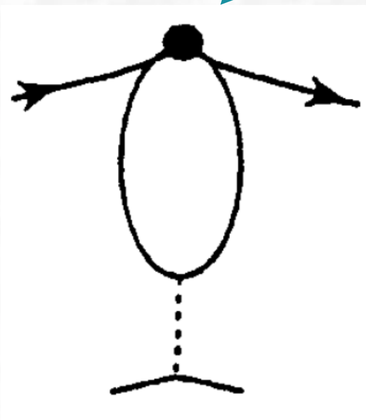
$q = \bar{c}, \bar{u}$

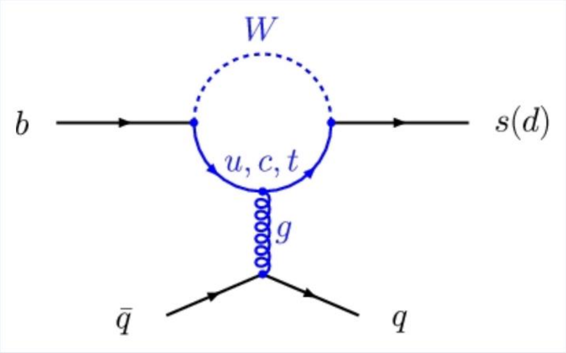
$q = c, u$

$$V_{ub} V^*_{us(d)}\ (\bar{u}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu u_L) \equiv \lambda_u\, \mathcal{O}_2^{(u)}$$

$$V_{cb} V^*_{cs(d)}\ (\bar{c}_L \gamma_\mu b_L)(\bar{s}(d)_L \gamma^\mu c_L) \equiv \lambda_c\, \mathcal{O}_2^{(c)}$$

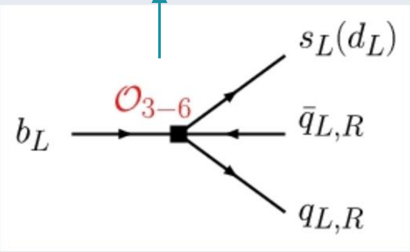
- **New feature: penguin diagrams** as QCD cannot distinguish between $\bar{q}q = \bar{u}u, \bar{d}d, \bar{c}c, \bar{s}s, \bar{b}b$; invented in 1975 by Shifman/Vainshtein/Zakharov and baptised by John Ellis in 1977 [V. Shifman, hep-ph/9510397]





$\rightarrow$

QCD penguin operators



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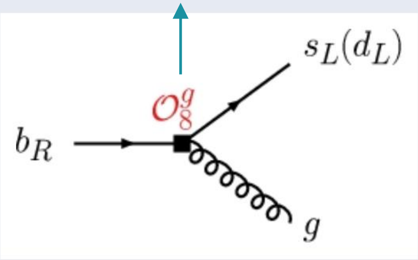
Chromo-magnetic operator



Their Wilson coefficients numerically suppressed by α_s and loop factor!

Effective Hamiltonian for $b \to s(d)\bar{q}q$ decays

□ Final result for the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(V_{ub} V_{us}^* \, \mathcal{O}_i^{(u)} + V_{cb} V_{cs}^* \, \mathcal{O}_i^{(c)} \right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=3}^{6} \boxed{C_i(\mu, x_t)} \mathcal{O}_i + \boxed{C_8(\mu, x_t)} \mathcal{O}_8^g \right) ,$$

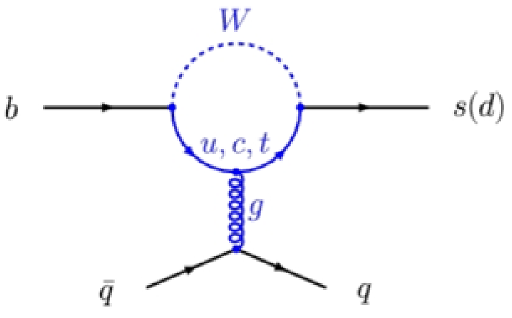
□ Full operators

$$O_{1c} = (\bar{c}_{Li} \gamma^\alpha b_L^i)(\bar{s}_{Lj} \gamma_\alpha c_L^j)$$
$$O_{2c} = (\bar{c}_{Li} \gamma^\alpha b_L^j)(\bar{s}_{Lj} \gamma_\alpha c_L^i)$$

$$O_{1u} = (\bar{u}_{Li} \gamma^\alpha b_L^i)(\bar{s}_{Lj} \gamma_\alpha u_L^j)$$
$$O_{2u} = (\bar{u}_{Li} \gamma^\alpha b_L^j)(\bar{s}_{Lj} \gamma_\alpha u_L^i)$$

$$\mathcal{O}_{3,5} = (\bar{s}\gamma_\mu(1-\gamma_5)b) \sum_{q=u,d,s,c,b} (\bar{q}\gamma^\mu(1\pm\gamma_5)q)$$
$$\mathcal{O}_{4,6} = (\bar{s}^i\gamma_\mu(1-\gamma_5)b^j) \sum_{q=u,d,s,c,b} (\bar{q}^j\gamma^\mu(1\pm\gamma_5)q^i)$$

$$\mathcal{O}_8^g = \frac{g_s m_b}{8\pi^2} (\bar{s}\sigma^{\mu\nu} T^A (1+\gamma_5) b) \, G_{\mu\nu}^A$$



Assume $m_{u,c}$=0 and the GIM mechanism used

Effective Hamiltonian for $b \to s(d)\bar{q}q$ decays

- **Wilson coefficients at low scale**

  - Matching calculation at initial scale

$$\vec{C}(\mu_W) = \vec{A}^{(0)} + \frac{\alpha_s(\mu_W)}{4\pi}\left(\vec{A}^{(1)} - \hat{r}^T\vec{A}^{(0)}\right).$$

  - ADM calculation and get RGE

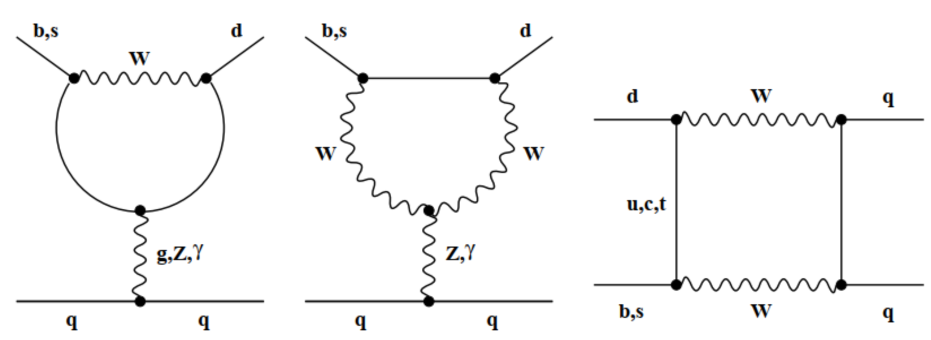
$$\frac{d\vec{C}(\mu)}{d\ln\mu} = \hat{\gamma}^T(g)\vec{C}(\mu),$$

  - Running from the high to the low scale

$$\vec{C}(\mu) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}\hat{J}\right)\hat{U}^{(0)}(\mu,\mu_W)\left(\vec{A}^{(0)} + \frac{\alpha_s(\mu_W)}{4\pi}\left[\vec{A}^{(1)} - (\hat{r}^T + \hat{J})\vec{A}^{(0)}\right]\right).$$

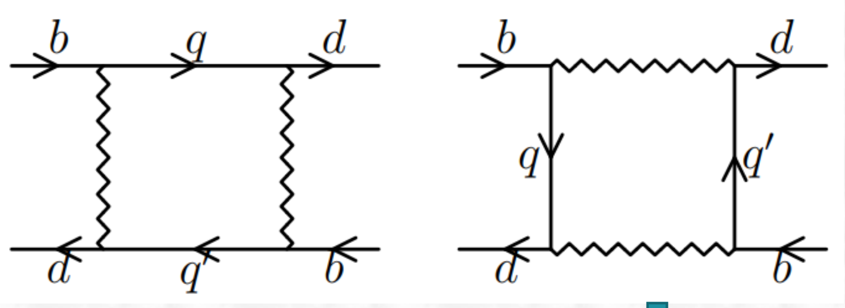
LO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu = m_b/2$	1.185	−0.387	0.018	−0.038	0.010	−0.053
$\mu = m_b$	1.117	−0.268	0.012	−0.027	0.008	−0.034
$\mu = 2m_b$	1.074	−0.181	0.008	−0.019	0.006	−0.022
	C_7/α	C_8/α	C_9/α	C_{10}/α	$C_{7\gamma}^{\text{eff}}$	C_{8g}^{eff}
$\mu = m_b/2$	−0.012	0.045	−1.358	0.418	−0.364	−0.169
$\mu = m_b$	−0.001	0.029	−1.276	0.288	−0.318	−0.151
$\mu = 2m_b$	0.018	0.019	−1.212	0.193	−0.281	−0.136

EW penguin operators　electro-magnetic operator



Effective Hamiltonian for $B_q^0 - \bar{B}_q^0$ mixings

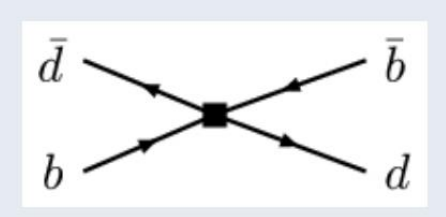
- $B_q^0 - \bar{B}_q^0$ mixings occur through **box diagrams**



$$\boxed{\mathcal{H}_{\mathrm{eff}}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{td}\right)^2 C_Q(\mu_b) Q(\Delta B = 2) + h.c.,}$$

$$\boxed{Q(\Delta B = 2) = (\bar{b}_\alpha d_\alpha)_{V-A} (\bar{b}_\beta d_\beta)_{V-A} \,.}$$

$$\boxed{C_Q(\mu_b) = \left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}\right]^{6/23} S_0(x_t)}$$



- **Physical amplitude: depend on the hadronic matrix elements**

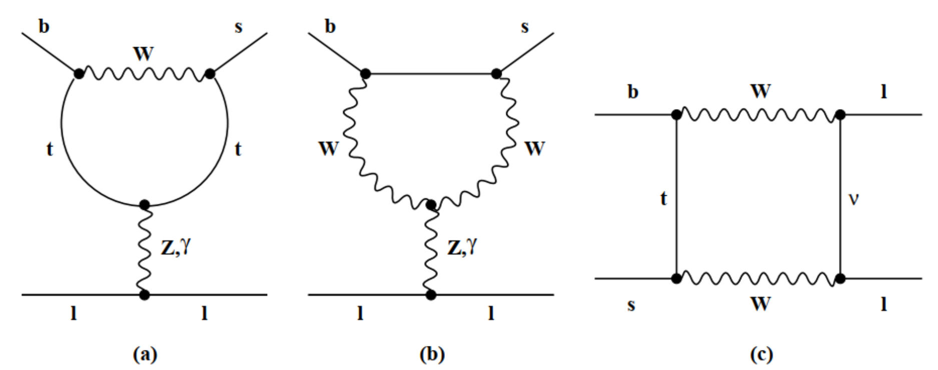
$$\langle \bar{B}_d^0 | \mathcal{H}_{\mathrm{eff}}^{\Delta B=2} | B_d^0 \rangle = \frac{G_F^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{td}\right)^2 C_Q(\mu_b) \langle \bar{B}_d^0 | Q(\Delta B = 2)(\mu_b) | B_d^0 \rangle,$$

$$\langle \bar{B}_d^0 | Q(\Delta B = 2)(\mu_b) | B_d^0 \rangle \equiv \frac{4}{3} \boxed{B_{B_d}(\mu_b)} F_{B_d}^2 m_{B_d}$$

The Bag parameter characterizes the deviation from VIA, and can be calculated by LQCD or HQE

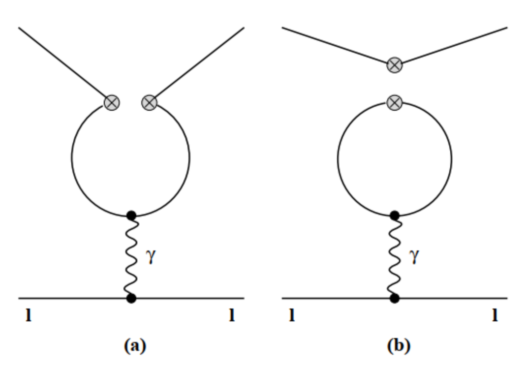
Effective Hamiltonian for $b \to s(d)ll$ decays

- Representative diagrams in **full theory**



EW penguin diagram　　Box diagram

- Representative diagrams in **effective theory**



- **The effective Hamiltonian for $B \to X_s\gamma$, $B \to X_s l^+ l^-$, $B \to K^*\gamma$, $B \to K^* l^+ l^-$ and $B_s \to l^+ l^-$**

$$
\begin{aligned}
H_{\rm eff} = & \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(V_{ub} V_{us}^* \mathcal{O}_i^{(u)} + V_{cb} V_{cs}^* \mathcal{O}_i^{(c)} \right) \\
& - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=3}^{10} C_i(\mu, x_t)\, \mathcal{O}_i + C_8(\mu, x_t)\, \mathcal{O}_8^g \right) \\
& - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=9}^{10} C_i(\mu, x_t)\, \mathcal{O}_i^{\ell\ell} + C_7(\mu, x_t)\, \mathcal{O}_7^{\gamma} \right)
\end{aligned}
$$

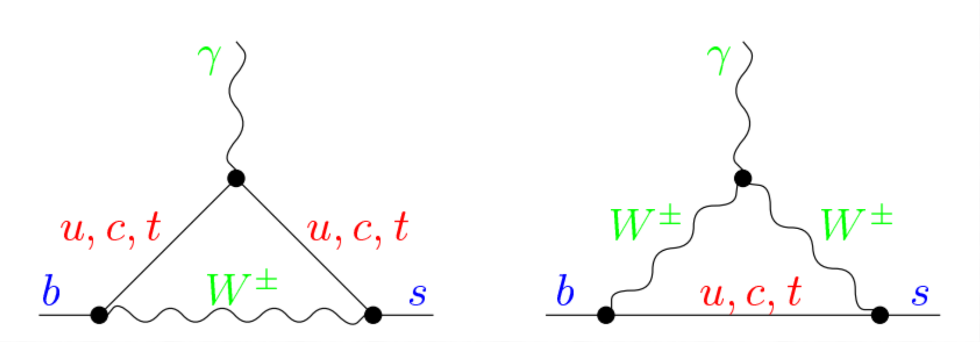
$$
O_i = \begin{cases}
(\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\
(\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\
\frac{em_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\
\frac{gm_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\
\frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4
\end{cases}
$$

How to calculate matrix elements of O_i?

Effective Hamiltonian for $b \to s(d)$ decays

- **Question:** why the associated CKM factor of the penguin operators are $V_{tb}V_{ts}^*$ or $V_{tb}V_{td}^*$?

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{2} C_i(\mu) \left(V_{ub}V_{us}^* \mathcal{O}_i^{(u)} + V_{cb}V_{cs}^* \mathcal{O}_i^{(c)} \right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(\sum_{i=3}^{10} C_i(\mu, x_t)\, \mathcal{O}_i + C_8(\mu, x_t)\, \mathcal{O}_8^g \right)$$
$$- \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left(\sum_{i=9}^{10} C_i(\mu, x_t)\, \mathcal{O}_i^{\ell\ell} + C_7(\mu, x_t)\, \mathcal{O}_7^{\gamma} \right)$$



- These loop diagrams yield a function of the internal quark mass which is multiplied by the corresponding combination of CKM elements

$$\sum_{q=u,c,t} V_{qb}V_{qs(d)}^*\, f(m_q) = \lambda_u\, f(m_u) + \lambda_c\, f(m_c) + \lambda_t\, f(m_t)$$

- Setting the lighter quark masses $0 = m_{u,c} \ll m_t$ and using the CKM unitarity $\sum_{q=u,c,t} \lambda_q = 0$

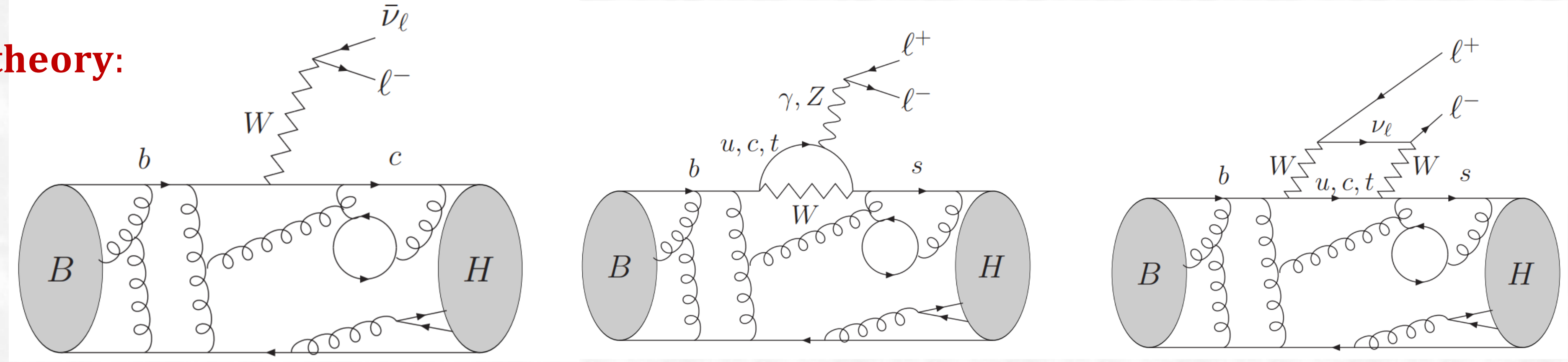
$$(\lambda_u + \lambda_c)\, f(0) + \lambda_t\, f(m_t) = \lambda_t\, (f(m_t) - f(0))$$

$$\boxed{\lambda_t = -(\lambda_u + \lambda_c) = V_{tb}V_{ts(d)}^*}$$

Effective Hamiltonian for $b \to s(d)$ decays

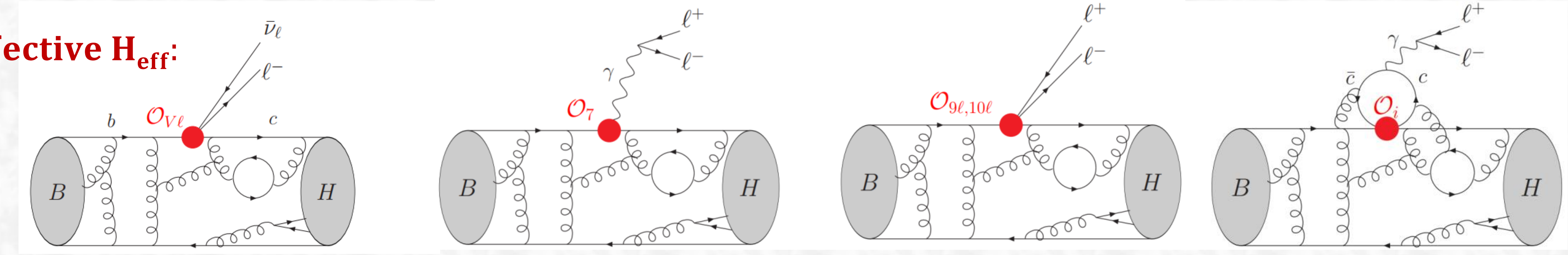
- **Language used by experimentalists vs by theorists**

Full theory:



vs

Effective H$_{\text{eff}}$:



Summary of WET for b-quark decays

"Full theory" $\leftrightarrow$ all modes propagate Parameters: $M_{W,Z}, M_H, m_t, m_q, g, g', \alpha_s \ldots$	$\uparrow \mu > M_W$	
$C_i(M_W) = C_i\big\vert_{\text{tree}} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$	matching: $\mu \sim M_W$	Step 1
"Eff. theory" $\leftrightarrow$ Low-energy modes propagate. High-energy modes are "integrated out". Parameters: $m_b, m_c, G_F, \alpha_s, C_i(\mu) \ldots$	$\downarrow \mu < M_W$	
$\frac{\partial}{\partial \ln \mu} C_i(\mu) = \gamma_{ji}(\mu)\, C_j(\mu)$	anomalous dimensions	Step 2
Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$. All dependence on $\ln \frac{M_W}{m_b}$ absorbed into $C_i(m_b)$	resummation of logs	Step 3

B-hadron lifetime

- **Exp. data from HFLAV 2020:**
  - All lifetimes are of the same order of magnitude (ps) and differ at most by a factor of 25
  - For hadrons containing one b-quark (and no c-quark), all lifetimes are equal within about 10%

b-hadron species	average lifetime	lifetime ratio
B^0	1.519 ± 0.004 ps	
B^+	1.638 ± 0.004 ps	$B^+/B^0 = 1.076 \pm 0.004$
$B_s{}^0$	1.515 ± 0.004 ps	$B_s{}^0/B^0 = 0.998 \pm 0.004$
$B_{s\mathrm{L}}$	1.423 ± 0.005 ps	
$B_{s\mathrm{H}}$	1.620 ± 0.007 ps	
$B_c{}^+$	0.510 ± 0.009 ps	
Λ_b	1.471 ± 0.009 ps	$\Lambda_b/B^0 = 0.969 \pm 0.006$
$\Xi_b{}^-$	1.572 ± 0.040 ps	
$\Xi_b{}^0$	1.480 ± 0.030 ps	$\Xi_b{}^0/\Xi_b{}^- = 0.929 \pm 0.028$
$\Omega_b{}^-$	1.64 +0.18 −0.17 ps	

- **Theoretical framework:** Heavy Quark Expansion (HQE) **and** quark-hadron duality [A. Lenz 1405.3601]

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X \int_{\mathrm{PS}} (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X|\mathcal{H}_{eff}|B\rangle|^2$$

$$= \frac{1}{2m_B} \langle B|\mathcal{T}|B\rangle$$

transition operator

non-local double insertion of H_{eff}

$$\mathcal{T} = \mathrm{Im}\, i \int d^4x T\left[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right]$$

$$= \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b}\bar{b}b + \frac{c_{5,b}}{m_b^2}\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b + 2\frac{c_{6,b}}{m_b^3}(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma + \ldots\right]$$

B-hadron lifetime

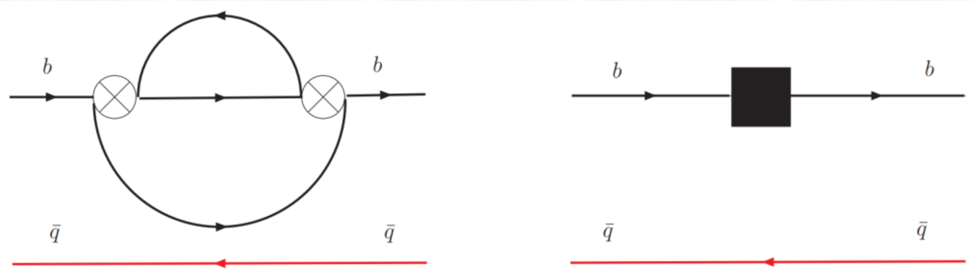
- **Total decay rate of an inclusive $B \to X$ decay**

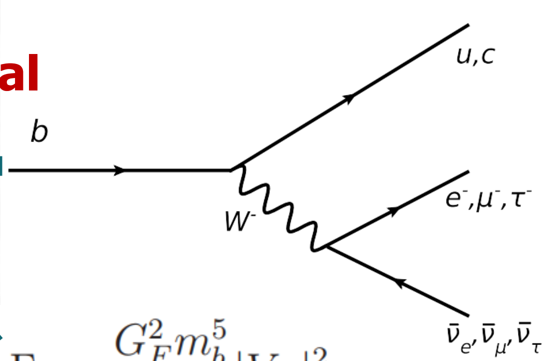
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right]$$

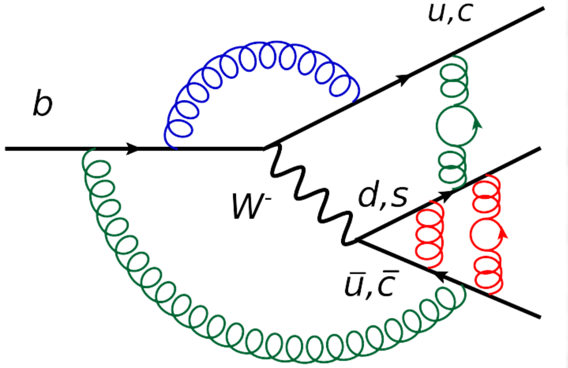
$$= \frac{G_F^2 m_b^5}{192\pi^3}V_{cb}^2\left\{c_{3,b}\left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right]\right.$$

$$\left. + 2c_{5,b}\left[\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right\}$$

- **Leading term (=free quark decay): universal**







$$\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 c_{3,b}$$

$$\boxed{\tau_b = (1.65 \pm 0.24)\ \mathrm{ps}}$$

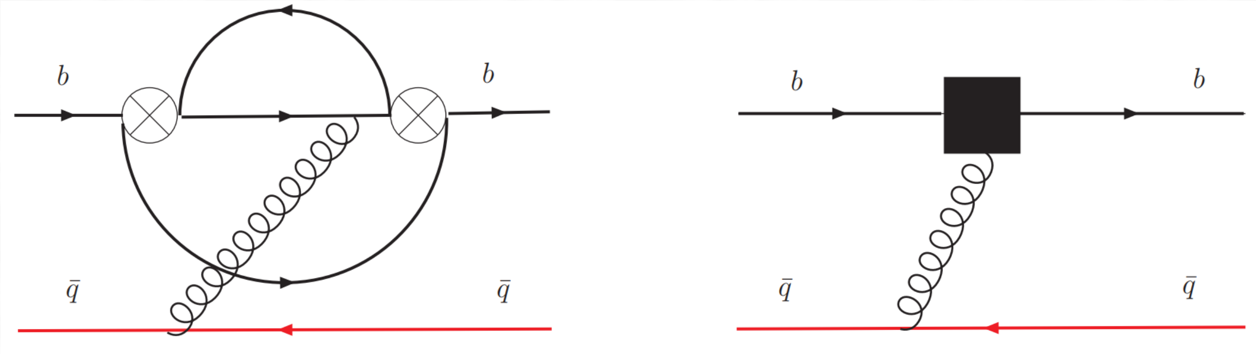
Spectator quark not involved, thus same lifetimes for all b-hadrons

B-hadron lifetime

- **Total decay rate of an inclusive $B \to X$ decay**

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right]$$
$$= \frac{G_F^2 m_b^5}{192\pi^3}V_{cb}^2\left\{c_{3,b}\left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right]\right.$$
$$\left. + 2c_{5,b}\left[\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right\}$$

- **Second term: a gluon is emitted from one of the internal quarks of the 2-loop diagram, and thus obtains the so-called dim-5 chromo-magnetic operator**



$$c_{5,b} = \begin{cases} \approx -9 & (m_c = 0 = \alpha_s) \\ -3.8 \pm 0.3 & (\bar{m}_c(\bar{m}_b)\ ,\alpha_s(m_b)) \end{cases}$$

Spectator quark now relevant and the decay rate reduced

B-hadron lifetime

- **Total decay rate of an inclusive $B \to X$ decay**

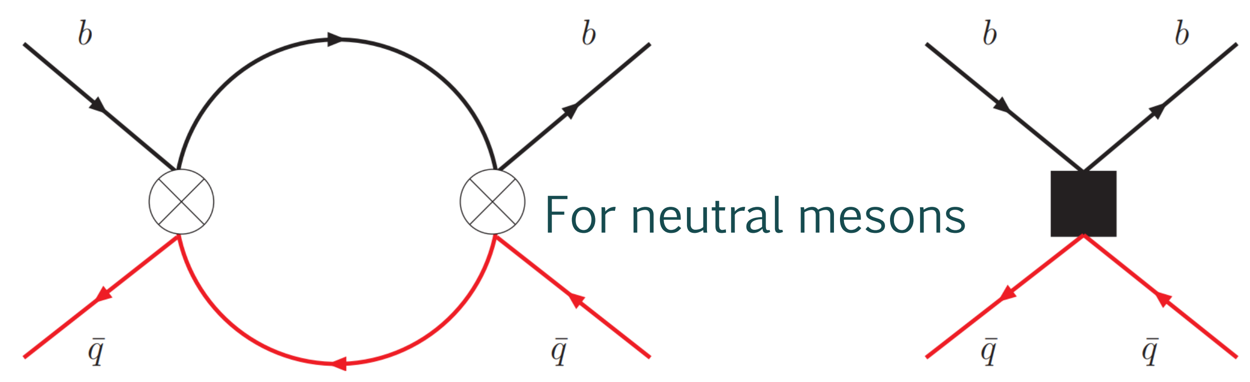
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right]$$

$$= \frac{G_F^2 m_b^5}{192\pi^3}V_{cb}^2\left\{c_{3,b}\left[1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right]\right.$$

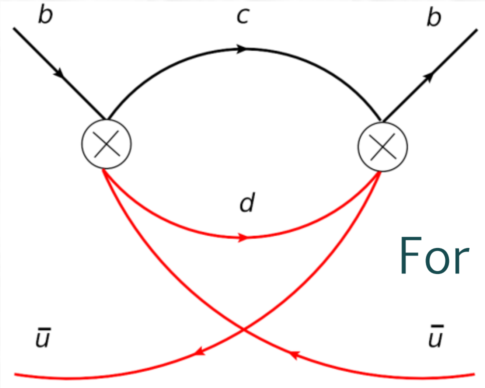
$$\left. + 2c_{5,b}\left[\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\right] + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots\right\}$$

Lifetime differences almost entirely due to these diagrams

- **Third term: contract only two quark lines in the product of H_{eff}; spectator- and b-quark not contracted; weak annihilation (for $B_{d,s}$) or Pauli interference (for B_u) diagrams**



For neutral mesons

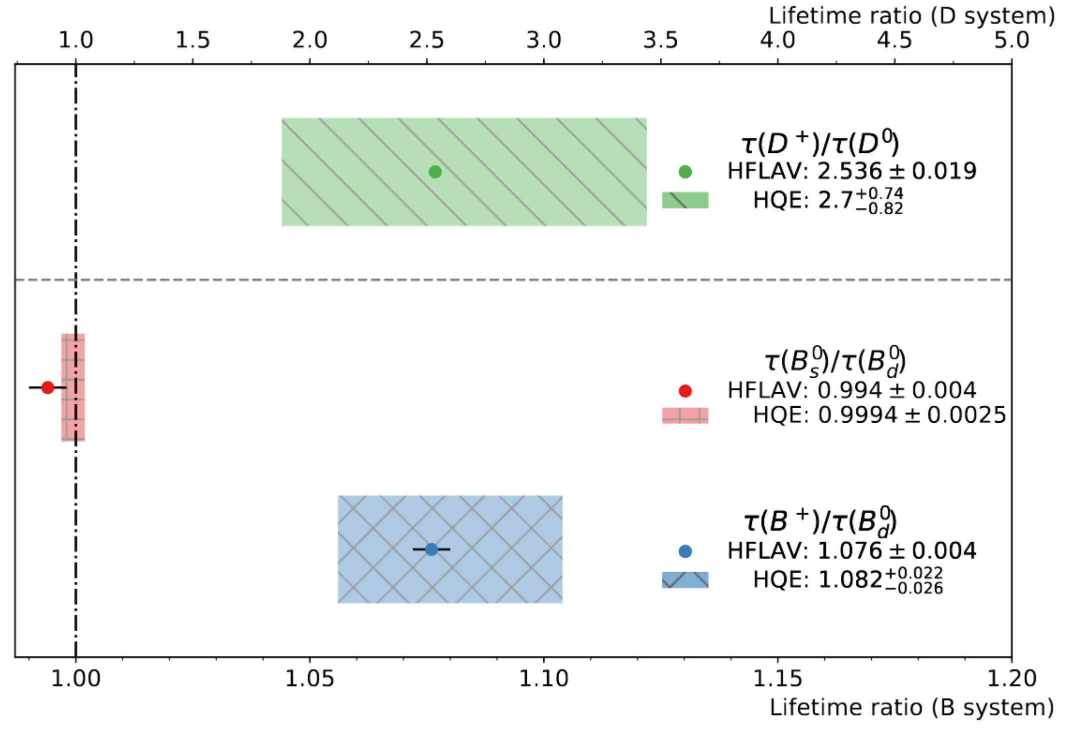


For charged mesons

B-hadron lifetime

- **Predicted lifetime ratios and compared to the exp. data** [Kirk, Lenz, Rauh 1711.02100]

$$\frac{\tau(H_1)}{\tau(H_2)} = \frac{\Gamma_2}{\Gamma_1} = 1 + \frac{\Gamma_2 - \Gamma_1}{\Gamma_1} = 1 + \frac{\mu_\pi^2(H_1) - \mu_\pi^2(H_2)}{2m_b^2} + \frac{c_G}{c_3}\frac{\mu_G^2(H_2) - \mu_G^2(H_1)}{2m_b^2} + \frac{c_6(H_2)}{c_3}\frac{\langle H_2|Q|H_2\rangle}{m_b^3 M_B} - \frac{c_6(H_1)}{c_3}\frac{\langle H_1|Q|H_1\rangle}{m_b^3 M_B} + \mathcal{O}\left(\frac{\Lambda^4}{m_b^4}\right)$$



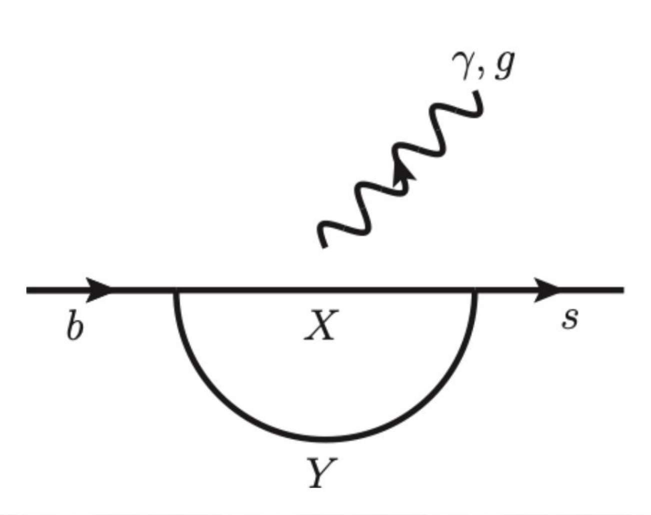
$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|^{\rm th} = 1.0006 \pm 0.0025\,, \quad \left.\frac{\tau(B^+)}{\tau(B_d)}\right|^{\rm th} = 1.076 \pm 0.004\,, \quad \left.\frac{\tau(\Lambda_b)}{\tau(B_d)}\right|^{\rm th} = 0.935 \pm 0.054\,,$$

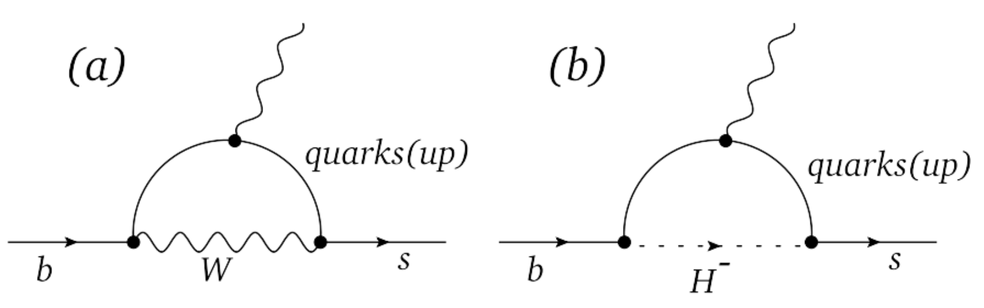
$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|^{\rm exp} = 0.994 \pm 0.004\,, \quad \left.\frac{\tau(B^+)}{\tau(B_d)}\right|^{\rm exp} = 1.076 \pm 0.004\,, \quad \left.\frac{\tau(\Lambda_b)}{\tau(B_d)}\right|^{\rm exp} = 0.969 \pm 0.006\,,$$

- ***The HQE technique works quite well for bottom-hadron lifetimes, and even for charmed-hadron lifetimes***

Inclusive radiative B decays

- **Why $B \to X_s\gamma$ decay:**
  - **forbidden at tree-level, and occurs firstly at one-loop level;**
  - **sensitive to new particles in the loop;**
  - **one of the most suitable processes for indirect searches of NP in the quark flavor sector!**





$M_{H^\pm} > 580$ GeV at 95%C.L.

- **Low-energy weak effective Hamiltonian:**

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}^* V_{qs} \left(C_1 Q_1^q + \sum_{i=2}^{6} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g}\right) + \text{h.c.}$$

- Most important operators are $Q_{7\gamma}$, Q_{8g} and Q_1^q.
- $Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} F^{\mu\nu}(1+\gamma_5)b$
- $Q_1^q = (\bar{q}b)_{V-A}(\bar{s}q)_{V-A} \quad Q_{8g} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu} G^{\mu\nu}(1+\gamma_5)b$

current-current	photonic dipole	gluonic dipole	penguin
$Q_{1,2}$	Q_7	Q_8	$Q_{3,4,5,6}$
$C_{1,2}(m_b) \sim 1$	$C_7(m_b) \sim -0.3$	$C_8(m_b) \sim -0.15$	$C_{3,4,5,6}(m_b) \sim 0.07$

$|C_7| : |C_{1,2}| : |C_8| \simeq 1 : 3 : 1/2$

Inclusive radiative B decays

- **Decay rate can be effectively evaluated by exploiting the HQE:**

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \begin{pmatrix} \text{Non-perturbative} \\ \sim (\pm 5)\% \\ \text{arXiv : 1003.5012} \end{pmatrix} \quad b \in \bar{B} = B^-(b\bar{u}) \text{ or } \bar{B}^0(b\bar{d})$$

Provided that E_0 is large ($\sim m_b/2$) but not close to endpoint ($m_b - 2E_0 \gg \Lambda_{QCD}$).
$E_0 \sim m_b/3 \simeq 1.6$ GeV is now conventional.

- **Due to $m_b \gg \Lambda_{QCD}$, the inclusive decay rate well approximated by the partonic decay rate**

$$\Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} = N \sum_{i,j=1}^{8} C_i^{\text{eff}}(\mu_b) C_j^{\text{eff}}(\mu_b) \boldsymbol{\widetilde{G}_{ij}}(E_0, \mu_b)$$

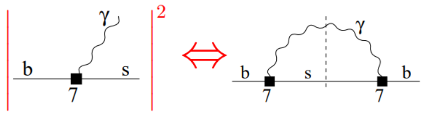
$$\widetilde{G}_{ij}(E_0,\mu_b) = \left[\widetilde{G}_{ij}^{(0)}(E_0) + \frac{\alpha_s}{4\pi}\widetilde{G}_{ij}^{(1)}(E_0,\mu_b) + \left(\frac{\alpha_s}{4\pi}\right)^2 \widetilde{G}_{ij}^{(2)}(E_0,\mu_b) + \mathcal{O}(\alpha_s^3)\right] + \ldots$$

describe interferences between amplitudes generated by Q_i and Q_j, and known up to the NNLO

Inclusive radiative B decays

□ Perturbative evaluation of G_{ij}:

LO: $G_{77} = 1$

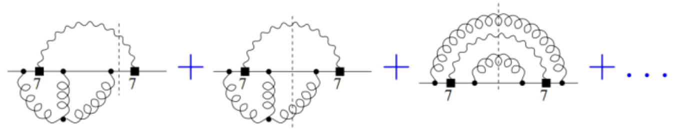


NLO: 1996: Quasi-complete G_{ij}

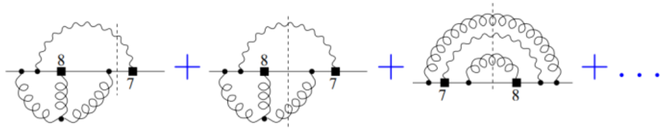
2002: Complete[(*)] G_{ij}

NNLO: We are still on the way to the quasi-complete case:

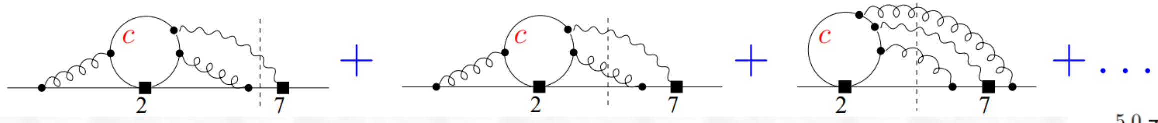
G_{77} is fully known:



G_{78} is fully known:



G_{27}:
(and analogous G_{17})



□ State-of-the-art SM prediction vs exp. data:

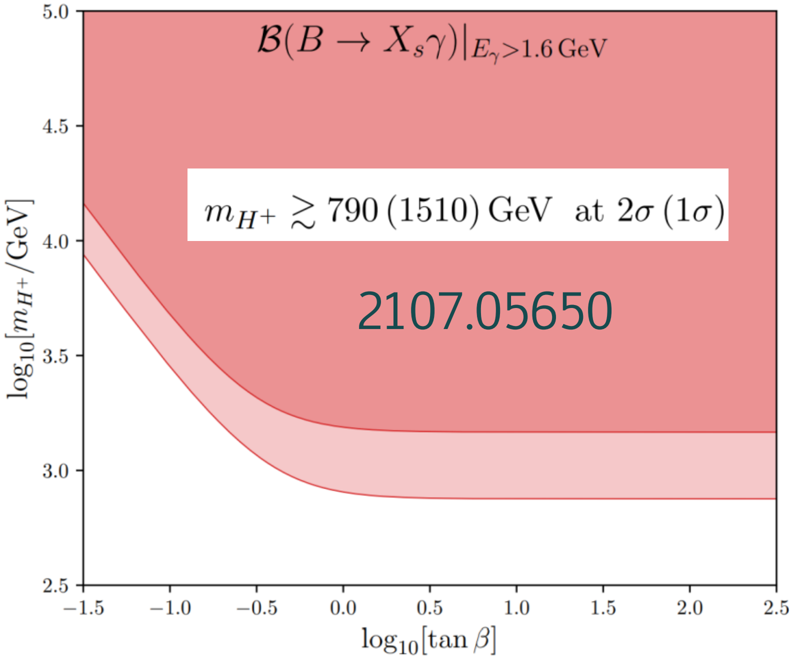
M. Misiak, H. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kamiński, G. Ossola, M. Poradziński, AR, T. Schutzmeier, M. Steinhauser and J. Virto
Phys. Rev. Lett. **114** (2015) 22, 221801 [arXiv:1503.01789].

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{SM}_{E_\gamma > 1.6\mathrm{GeV}} = (3.36 \pm 0.23) \cdot 10^{-4}$$

HFLAV, Eur. Phys. J. C77 (2017) 895, [arXiv:1612.07233].

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{Exp}_{E_\gamma > 1.6\mathrm{GeV}} = (3.32 \pm 0.15) \cdot 10^{-4}$$

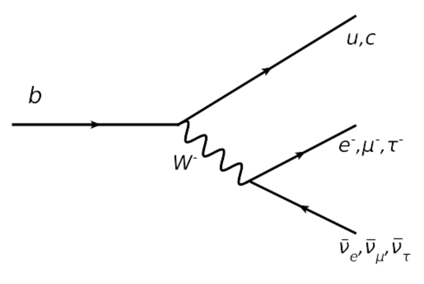
□ Good agreement provides strong constraints on NP:



Different types of b-quark weak decays

- **Dominant tree-level processes:**

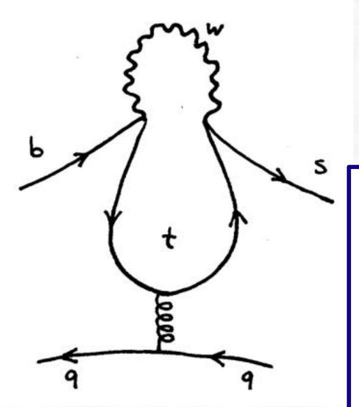
$$b \to \begin{cases} c \\ u \end{cases} + W^{*-} \to \begin{cases} c \\ u \end{cases} + \begin{cases} \bar{u} + d \\ \bar{c} + d \\ \bar{u} + s \\ \bar{c} + s \\ e^- + \bar{\nu}_e \\ \mu^- + \bar{\nu}_\mu \\ \tau^- + \bar{\nu}_\tau \end{cases}$$



$$A(b \to c) \sim V_{cb} \sim 4 \times 10^{-2}$$
(e.g. $B \to D\mu\nu$)

$$A(b \to u) \sim V_{ub} \sim 4 \times 10^{-3}$$
(e.g. $B \to \pi\mu\nu$)

- **Rare loop-level processes:**



$$b \to s\bar{s}s,\ d\bar{s}s,\ d\bar{d}d,\ d\bar{s}s,\ s\gamma,\ d\gamma,\ sl^+l^-,\ dl^+l^-$$

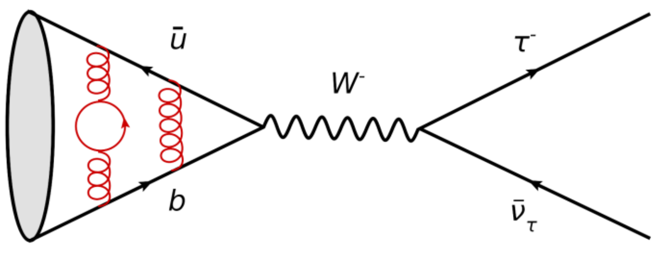
$$A(b \to s) \sim \frac{1}{16\pi^2} V_{ts}^* V_{tb} \sim 2.5 \times 10^{-4}$$
(e.g. $B \to K^*\mu^+\mu^-$)

$$A(b \to d) \sim \frac{1}{16\pi^2} V_{td}^* V_{tb} \sim 5 \times 10^{-5}$$
(e.g. $B \to \pi\mu^+\mu^-$)

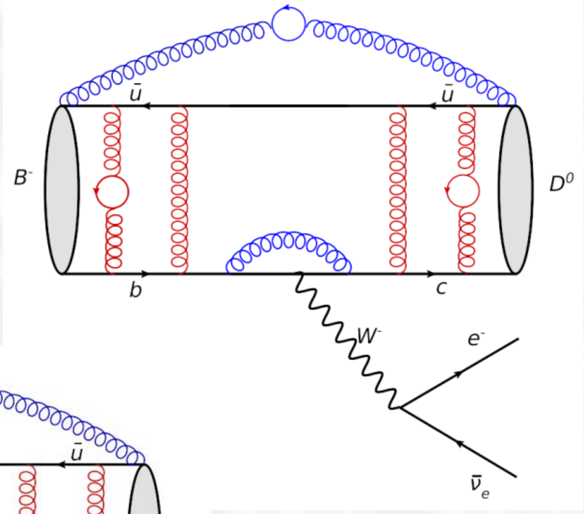
- **Decays that proceed at both tree and loop levels:**

$$b \to \bar{c}cs,\ \bar{c}cd,\ \bar{u}us,\ \bar{u}ud$$

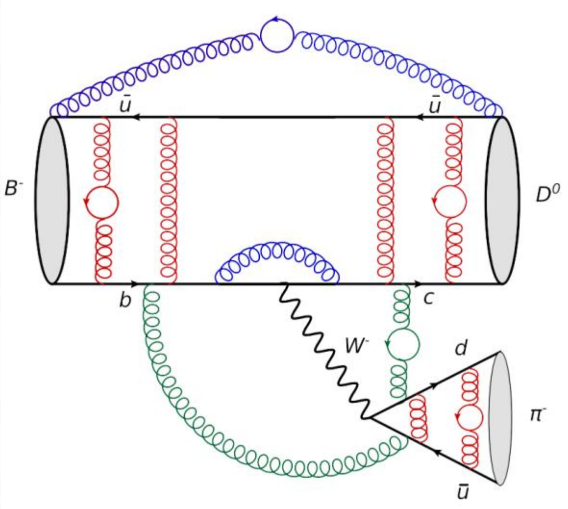
Tree-level charged-current decays



- **Purely leptonic decay modes: only for charged B mesons; e.g. $B^+ \to \tau^+\nu_\tau$, $B^+ \to \mu^+\nu_\mu$**



- **Semi-leptonic decay modes: both for charged and neutral B mesons;**
  - **Exclusive modes: e.g. $B \to D\tau\nu_\tau$, $B \to \pi\mu\nu_\mu$**
  - **Inclusive modes: e.g. $B \to X_c\tau\nu_\tau$, $B \to X_u\mu\nu_\mu$**



- **Hadronic decay modes: both for charged and neutral B mesons;**
  - **hundreds of possible final states:**

    e.g. $B \to D\pi, DK, \ldots$

Loop-level rare FCNC decays

- **Purely leptonic decay modes: only for neutral B mesons; e.g. $B_s \to \mu^+\mu^-$, $B_d \to \tau^+\tau^-$**

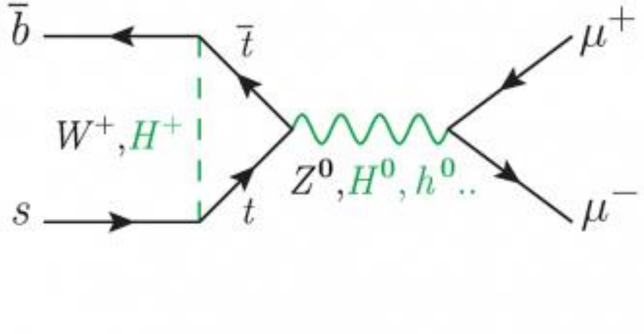
- **Radiative decay modes: both for charged and neutral B mesons;**
  - **- Exclusive modes: e.g. $B \to K^*\gamma$, $B \to \rho\gamma$**
  - **- Inclusive modes: e.g. $B \to X_s\gamma$**

- **Semi-leptonic decay modes: both for charged and neutral B mesons;**
  - **- Exclusive modes: e.g. $B \to K\mu^+\mu^-$, $B_s \to \phi e^+e^-$, $B \to K^*\nu\bar{\nu}$**
  - **- Inclusive modes: e.g. $B \to X_s\mu^+\mu^-$**

- **Hadronic decay modes: both for charged and neutral B mesons;**
  - **- hundreds of possible final states: e.g. $B \to \phi K_S$, $B_s \to \phi\phi$, …**

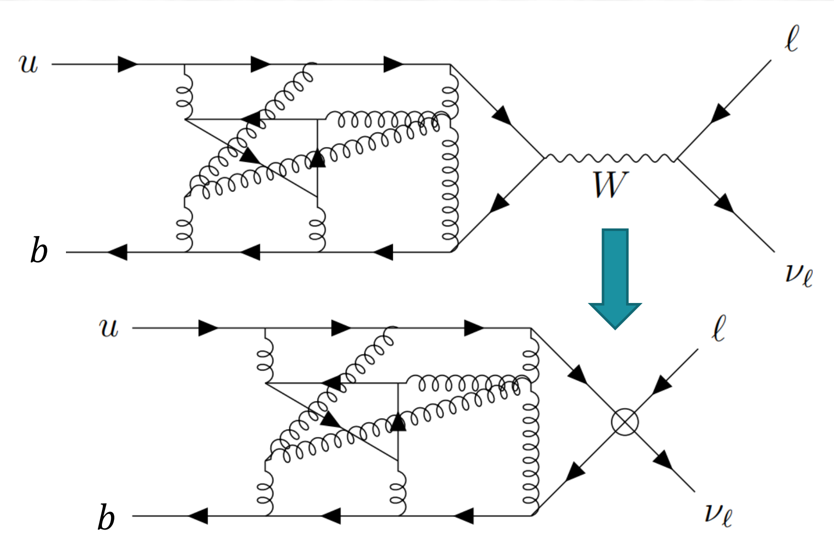
> ➢ **Sensitive to New Physics, and thus optimal for New Physics searches,**

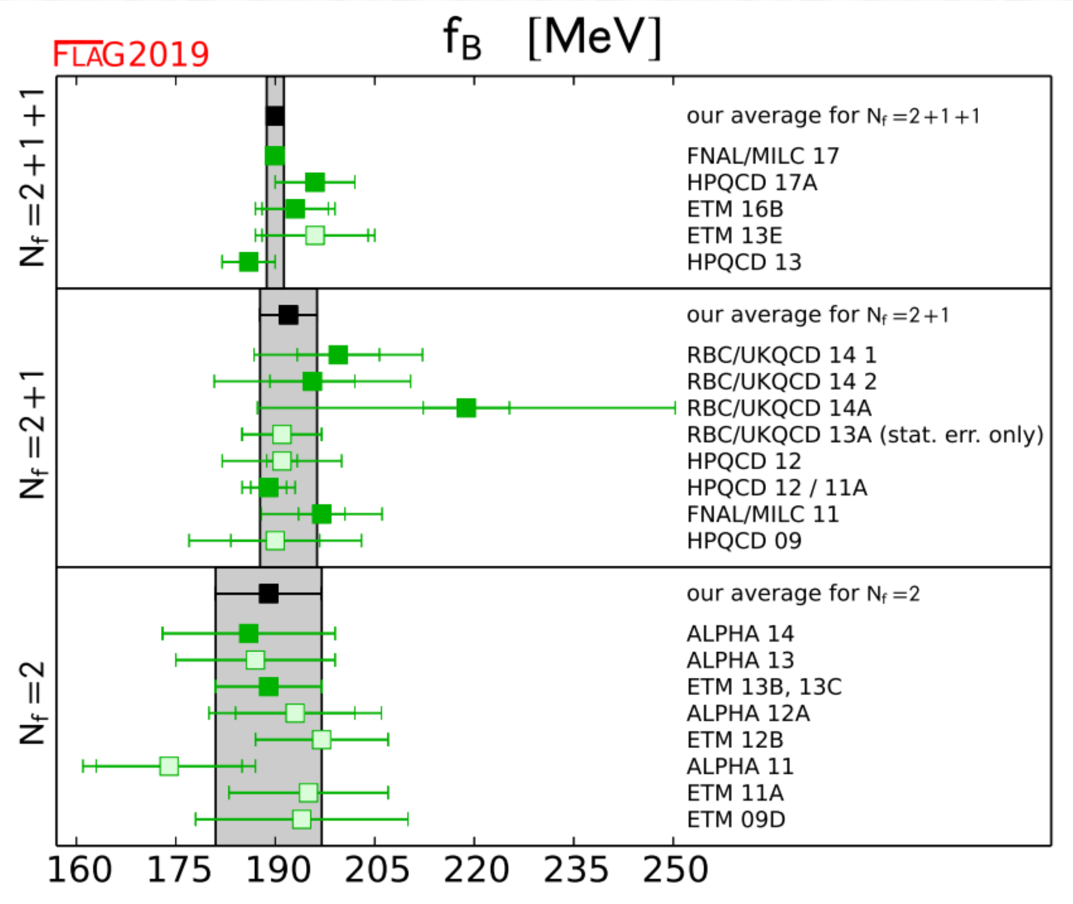
$B_s \to \mu^+\mu^-$



Examples of B decays

- **Purely leptonic decay modes:** $B^+ \to \tau^+ \nu_\tau$



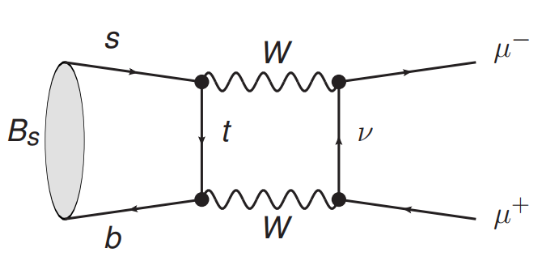


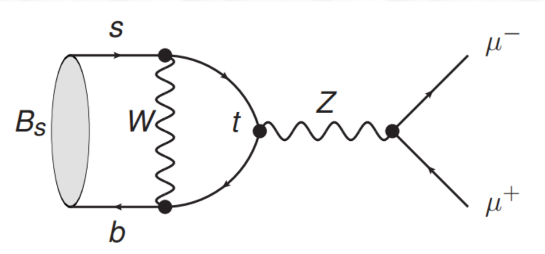
$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = ip_\mu f_P$$

$$\Gamma(B^+ \to \ell^+\nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

The decay constant can be thought of as the "wave-function overlap" of the quark and anti-quark.

Examples of B decays





- **Purely leptonic decay modes: $B_s \to \mu^+\mu^-$**

$$\langle\mu^+\mu^-|\mathcal{H}_{\rm eff}|\bar{B}_s\rangle = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{e^2}{16\pi^2}C_{10}\langle\mu^+\mu^-|(\bar{\mu}\gamma^\alpha\gamma_5\mu)|0\rangle\langle 0|(\bar{s}\gamma_\alpha P_L b)|\bar{B}_s\rangle$$

$$\langle 0|(\bar{s}\gamma^\alpha b)|\bar{B}_s\rangle = 0$$

$$\langle 0|(\bar{s}\gamma^\alpha\gamma_5 b)|\bar{B}_s\rangle = i f_{B_s} p^\alpha_{B_s}$$

decay constant

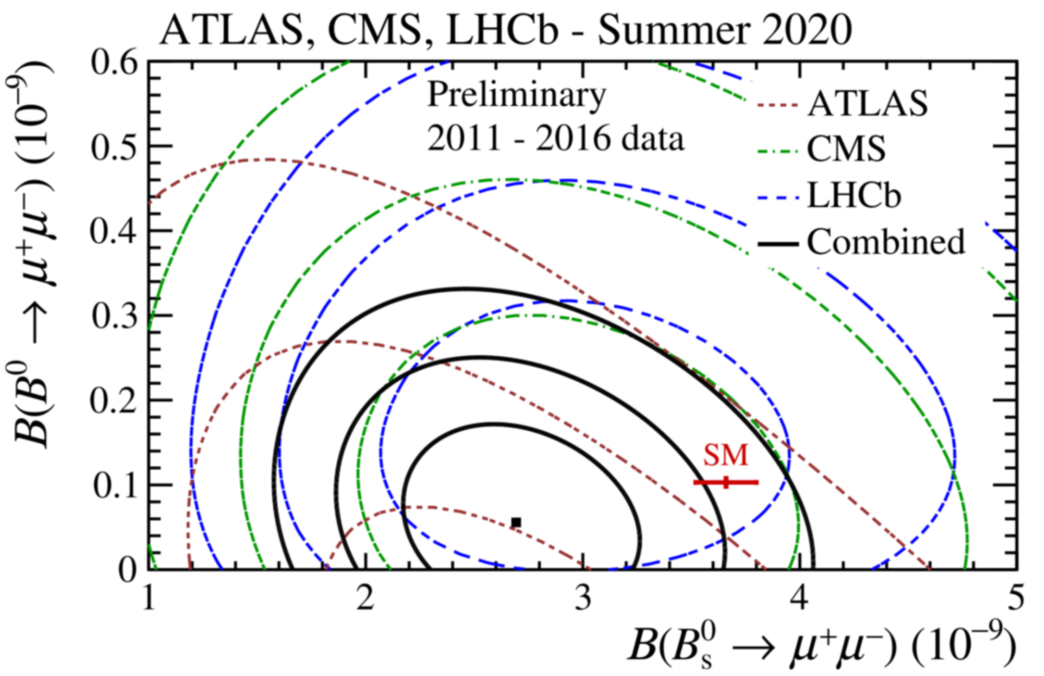
Effect of lifetime difference of Bs and Bs-bar

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = \tau_{B_s}\frac{G_F^2}{\pi}m_{B_s}f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2}|V_{ts}^* V_{tb}|^2\frac{Y(x_t)^2}{s_W^4}\frac{1}{1-y_s}$$

helicity suppression

loop suppression

CKM suppression



a truly rare decay

Examples of B decays

□ **Exclusive semi-leptonic decay modes:** $B \to D^{(*)}\mu\nu_\mu$

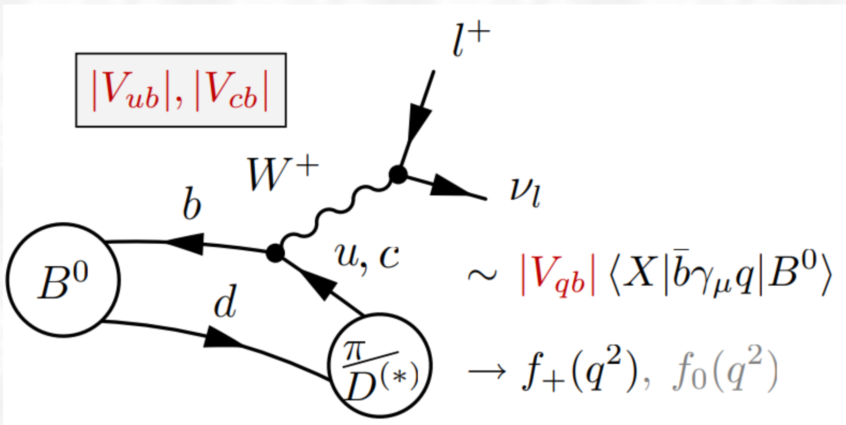
$$\langle D^{(*)}\ell\nu|\mathcal{H}_{\text{eff}}|B\rangle = \frac{4G_F}{\sqrt{2}} V_{cb} C \langle \ell\bar{\nu}|(\bar{\ell}\gamma^\mu P_L \nu)|0\rangle \langle D^{(*)}|(\bar{c}\gamma_\mu P_L b)|B\rangle$$

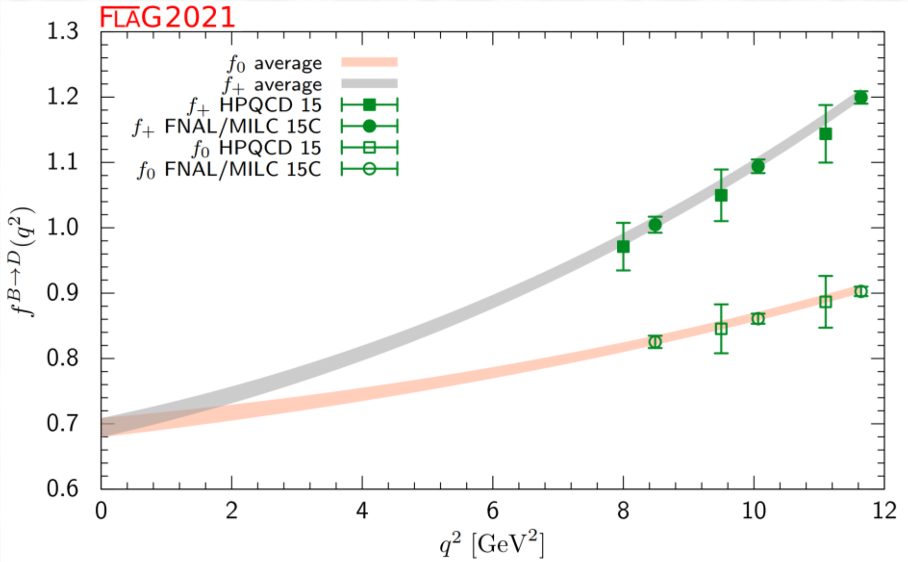
$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu + [f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

$$\langle D^*|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv -ig(q^2)\,\epsilon^{\mu\nu\rho\sigma}\,\varepsilon^*_\nu\,(p_B+p_{D^*})_\rho\, q_\sigma\,,$$

$$\langle D^*|\,\bar{c}\gamma^\mu\gamma^5 b\,|\bar{B}\rangle \equiv \varepsilon^{*\mu} f(q^2) + a_+(q^2)\,\varepsilon^*\cdot p_B\,(p_B+p_{D^*})^\mu + a_-(q^2)\,\varepsilon^*\cdot p_B\, q^\mu$$

$$\frac{d\Gamma(B\to D\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3}\,\frac{(q^2-m_\ell^2)^2\sqrt{E_D^2-m_D^2}}{q^4 m_B^2}\times \left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_B^2(E_D^2-m_D^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_B^2-m_D^2)^2|f_0(q^2)|^2\right]$$



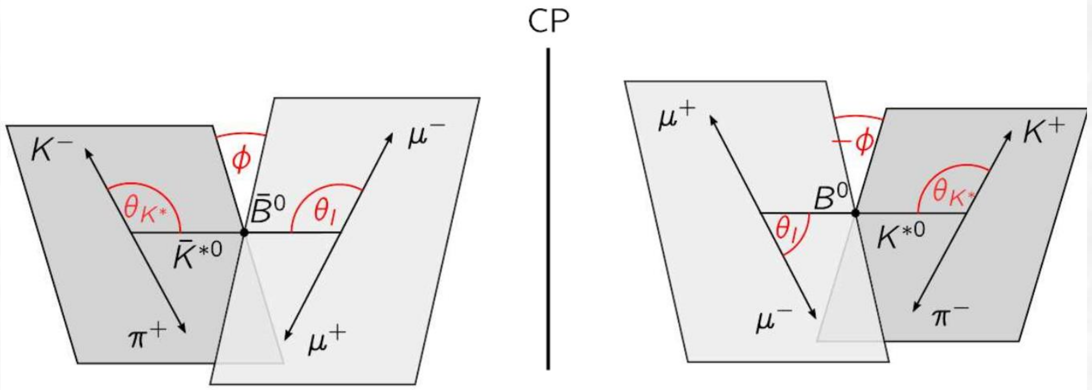


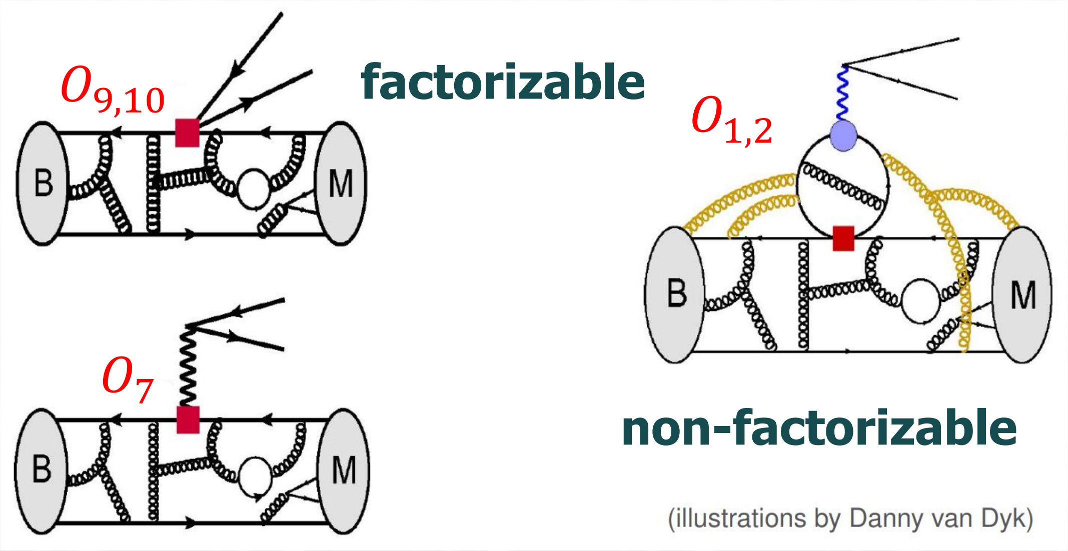
$$R_{D^{(*)}} = \frac{BR(B\to D^{(*)}\tau\nu)}{BR(B\to D^{(*)}\ell\nu)}$$

Hot topic since 2012 BaBar results

Examples of B decays

- **Semi-leptonic decay modes:** $B \to K^*\mu^+\mu^-$





$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

$$\boxed{\frac{d^4\bar{\Gamma}}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi)}$$

$$\begin{aligned}
\bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi) &= \\
&= \bar{I}_1^s \sin^2\theta_{K^*} + \bar{I}_1^c \cos^2\theta_{K^*} + (\bar{I}_2^s \sin^2\theta_{K^*} + \bar{I}_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\
&+ \bar{I}_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + \bar{I}_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi \\
&- \bar{I}_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\
&- (\bar{I}_6^s \sin^2\theta_{K^*} + \bar{I}_6^c \cos^2\theta_{K^*}) \cos\theta_\ell + \bar{I}_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\
&- \bar{I}_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi - \bar{I}_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi
\end{aligned}$$

Angular distribution of deay width

Examples of B decays

Hadronic matrix elements: from lattice QCD and other non-perturbative methods (e.g., LCSR)

- $B \to K$ & $B \to K^*$ transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \;\; \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

- $\Longrightarrow$ 10 non-perturbative q^2-dependent functions (Form factors)

$$\langle K|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

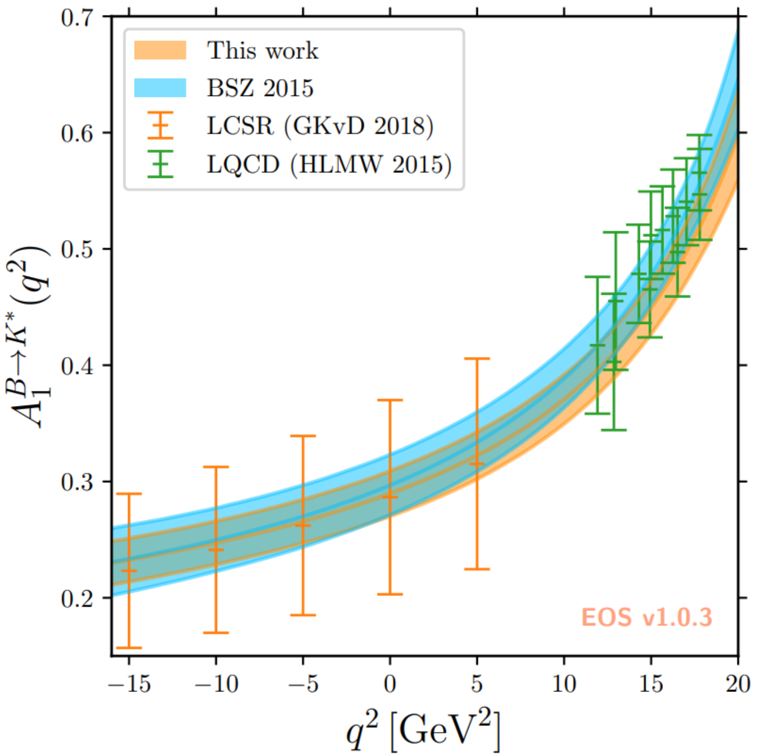
3

$$\langle K|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle K^*|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

7

$$\langle K^*|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$



$$\boxed{\mathcal{F}^{B\to M}_{(T),\lambda}(q^2) = \frac{1}{1-\frac{q^2}{m^2_{J^P}}}\sum_{k=0}^{\infty}\alpha_k^{\mathcal{F}}\left[z(q^2)-z(0)\right]^k}$$

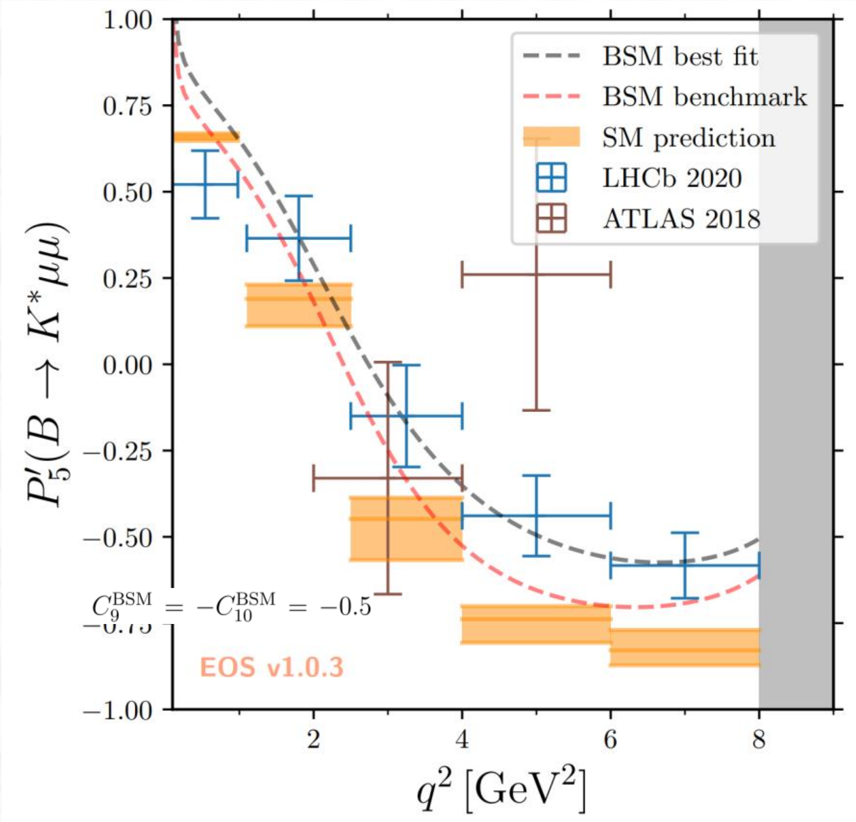
N. Gubernari *et al.*, 2206.03797

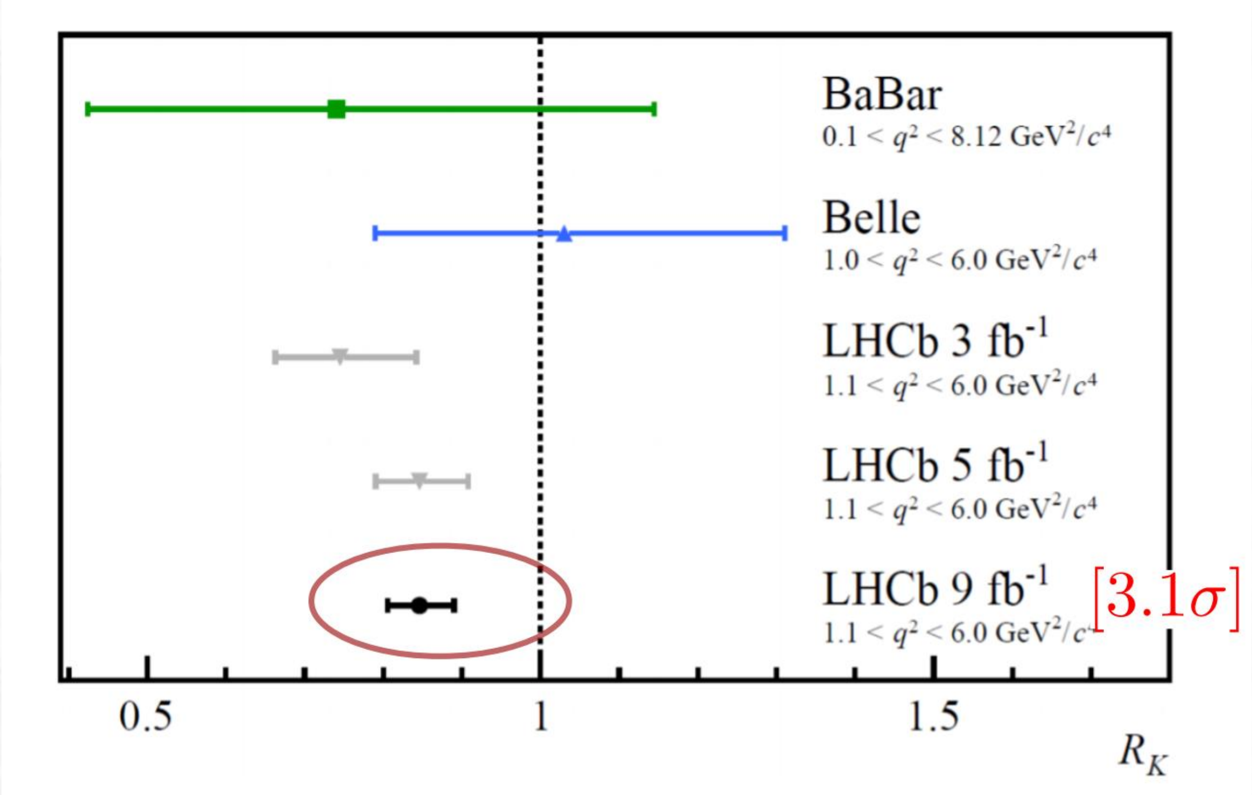
Examples of B decays

B anomalies

- **Optimized variables with reduced FF uncertainties**

$$P_5' = \frac{S_5}{\sqrt{F_L(1-F_L)}} \qquad R_{K^*} = \frac{BR(B\to K^*\mu^+\mu^-)}{BR(B\to K^*e^+e^-)} \qquad R_K = \frac{BR(B\to K\mu^+\mu^-)}{BR(B\to Ke^+e^-)}$$





Examples of B decays

□ Model-independent global fit to B anomalies

EFT for $b \to s\ell\ell$

$$\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i + \sum_{i=7,8,9,10,P,S} \left(C_i(\mu)\mathcal{O}_i + C_i'(\mu)\mathcal{O}_i' \right) \right] + \text{h.c.}$$

- **Semileptonic operators**:

$$\mathcal{O}_9^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_S^{(\prime)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\ell)$$

$$\mathcal{O}_{10}^{(\prime)} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \qquad \mathcal{O}_P^{(\prime)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\gamma_5 \ell)$$

- Dimension-6 **tensor** operators are **not allowed** by $SU(2)_L \times U(1)_Y$

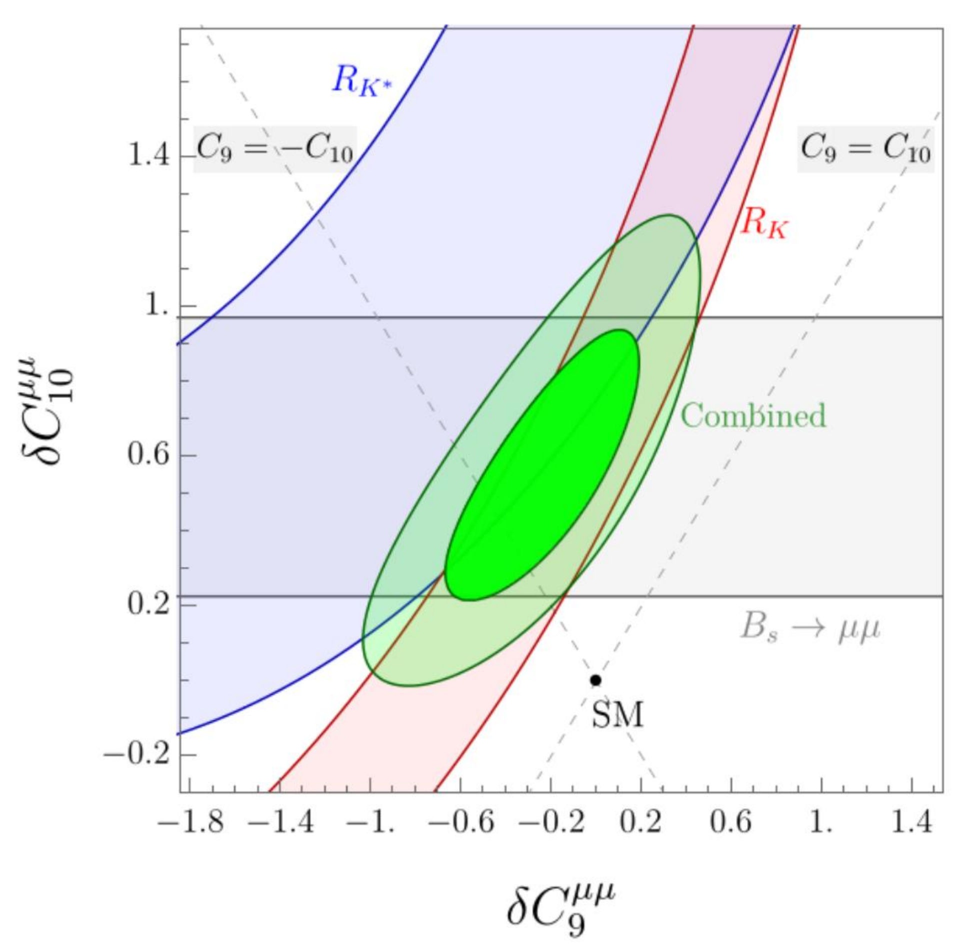
[Buchmuller, Wyler. '85]

- **(Pseudo)scalar** operators are **tightly constrained** by

$$\overline{B}(B_s \to \mu\mu)^{\rm exp} = (2.85 \pm 0.22) \times 10^{-9}$$ [Our average, CMS, ATLAS, LHCb]

$$\overline{B}(B_s \to \mu\mu)^{\rm SM} = (3.66 \pm 0.14) \times 10^{-9}$$ [Beneke et al. '19]

Only vector(axial) coefficients can accommodate the data!



- Purely **left-handed** operator preferred $[4.6\sigma]$:

$$\delta C_9^{\mu\mu} = -\delta C_{10}^{\mu\mu} = -0.41 \pm 0.09$$

A. Angelescu *et al.*, 2103.12504

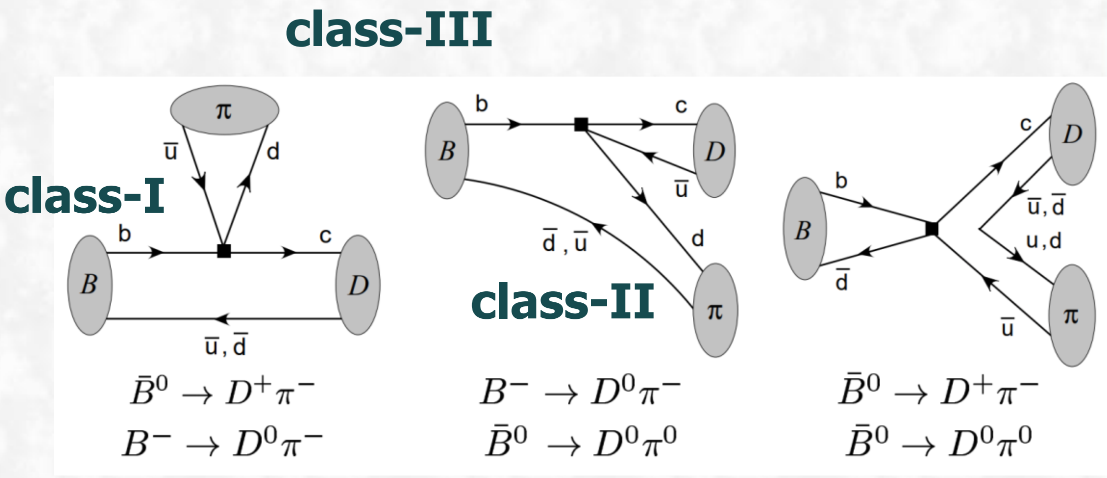
Examples of B decays

□ Hadronic decay modes: $B \to D\pi$

➢ **At quark-level: mediated by $b \to c\bar{u}d(s)$**

all four flavors different from each other, no

penguin operators & no penguin topologies!



$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

➢ **For class-I decays:** QCDF formula much simpler;

[Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+}L^-|\,\mathcal{Q}_i\,|\bar{B}_q^0\rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

i) only color-allowed tree amplitude a_1;

ii) spectator & annihilation power-suppressed;

iii) annihilation absent in $\bar{B}^0_{d(s)} \to D^+_{d(s)}K(\pi)^-$;

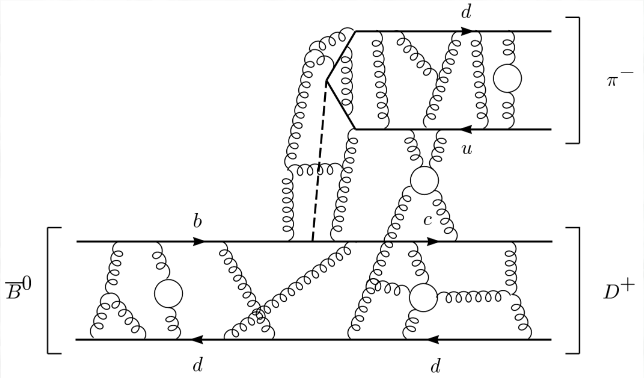
iv) they are theoretically simpler and cleaner!

➢ **Hard kernel T:** both NLO and NNLO results known;

[Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

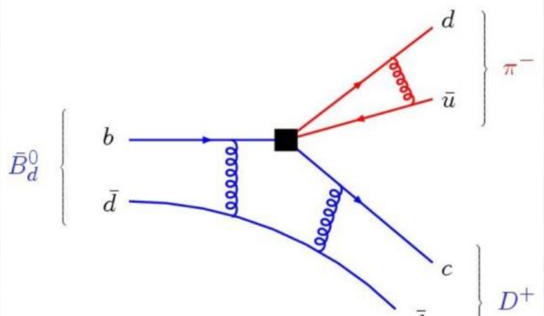
Examples of B decays



◻ Hadronic decay modes: $B \to D\pi$

$$\langle D^+\pi^-|\mathcal{H}_{\rm eff}^{b\to cd\bar{u}}|\bar{B}_d^0\rangle = V_{cb}V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu \sim m_b)\, r_i(\mu \sim m_b)$$

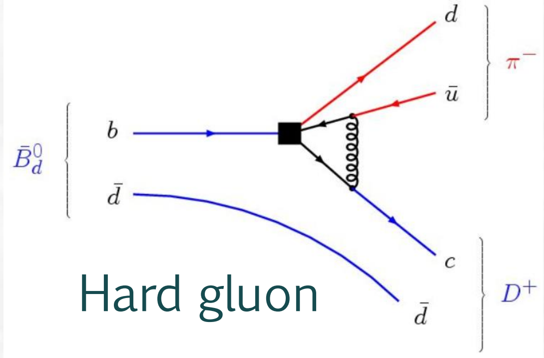
$$r_i(\mu) = \langle D^+\pi^-|\mathcal{O}_i|\bar{B}_d^0\rangle\Big|_{\mu} = \underbrace{\langle D^+|J_i^{(b\to c)}|\bar{B}_d^0\rangle}_{\text{form factor}}\ \underbrace{\langle \pi^-|J_i^{(d\to u)}|0\rangle}_{\text{decay constant}} + \text{corrections}(\mu)$$



Naïve factorization

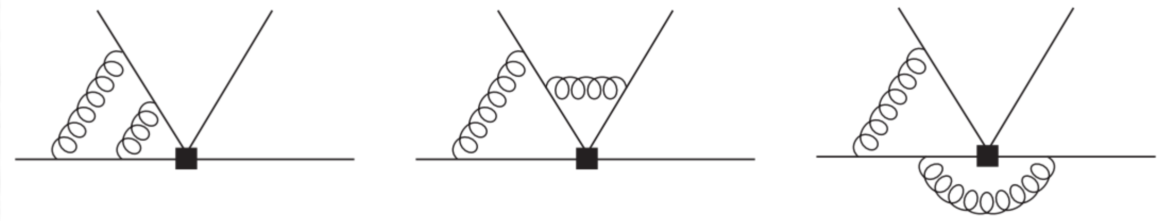
$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)} \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi} \boxed{t_{ij}(u,\mu)} + \dots\right) f_\pi\, \phi_\pi(u,\mu)$$

$$\boxed{T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)}$$



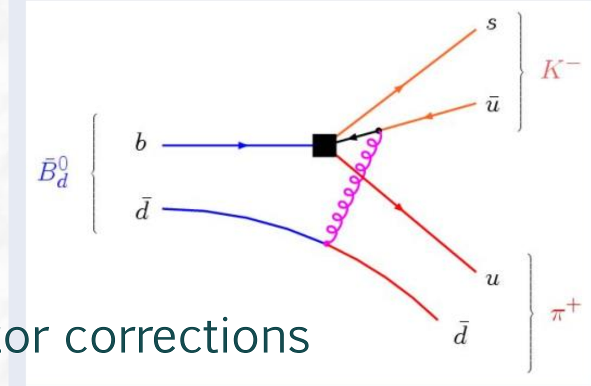
Hard gluon

Sample diagrams of 2-loop vertex corrections



Examples of B decays

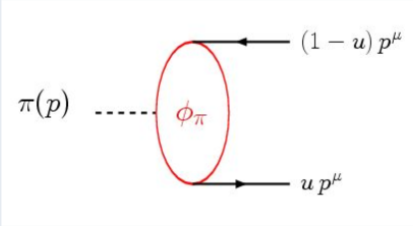
- **Hadronic decay modes: $B \to \pi\pi, \pi K$**

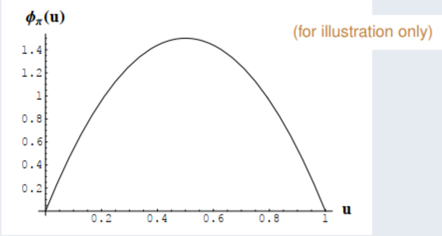


Spectator corrections

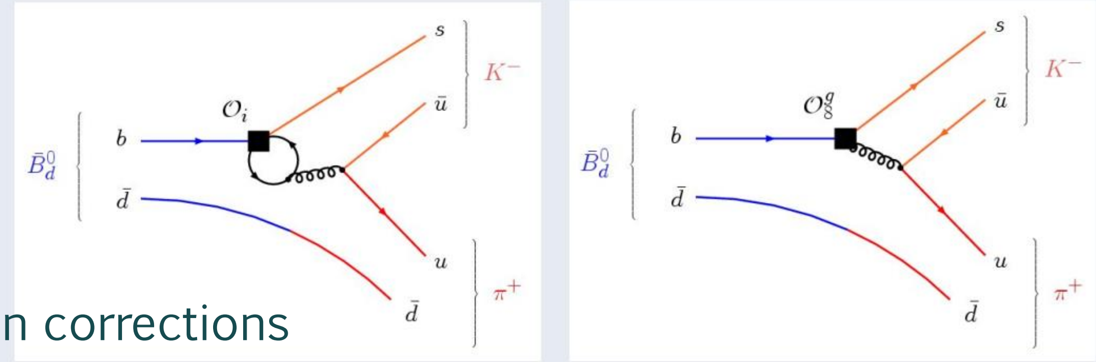
$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi} h_i(u,v,\omega,\mu) + \ldots\right) \times f_K\,\phi_K(u,\mu) \cdot f_\pi\,\phi_\pi(v,\mu) \cdot f_B\,\phi_B(\omega,\mu)$$





Additional diagrams for hard corrections in QCDF (example)

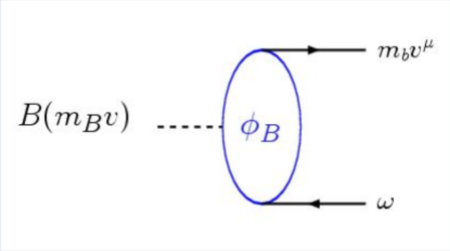


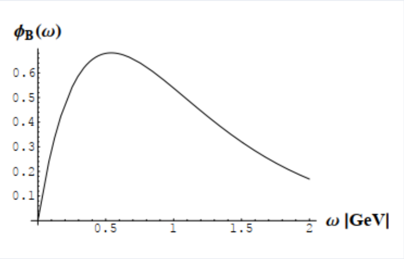
Penguin corrections

$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u,\mu)$

$$r_i(\mu)\Big|_{\text{hard}} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi} t_{ij}(u,\mu) + \ldots\right) f_K\,\phi_K(u,\mu)$$

+ Annihilation corrections

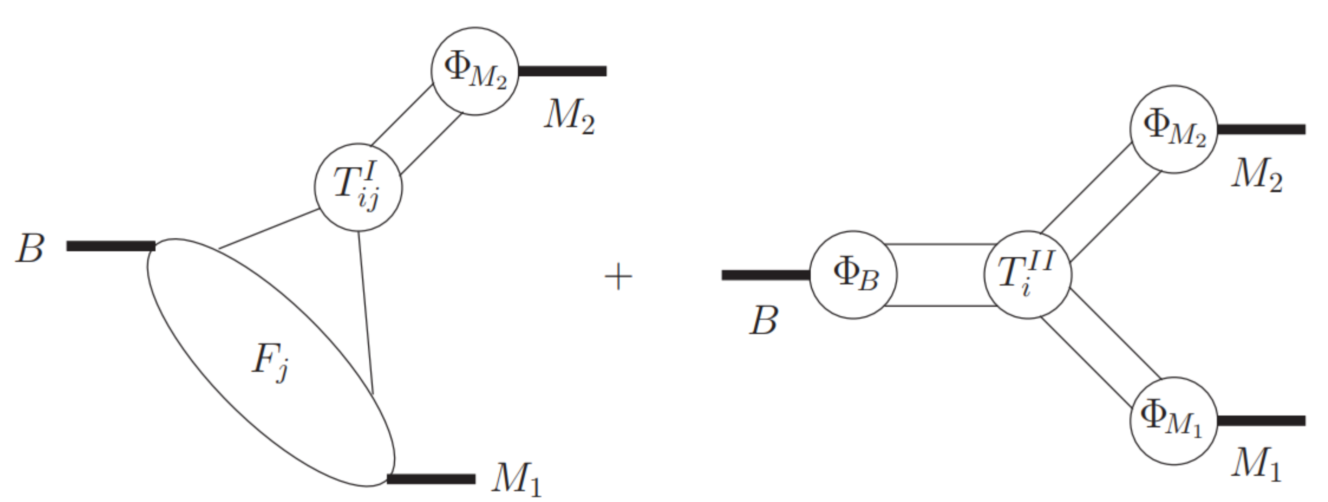




Examples of B decays

QCDF formulae for two-body hadronic B decays [BBNS 99`**]**

$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} = \sum_{\text{terms}} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\text{I}}(\mu_h, \mu_s)}_{1+\alpha_s+\dots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$
$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\text{II}}(\mu_h, \mu_I)}_{1+\dots} \star \underbrace{J^{\text{II}}(\mu_I, \mu_s)}_{\alpha_s+\dots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$



- Higher-order PT can be systematically calculated
- Limited by power corrections

Exclusive B decays

- A meson $|M\rangle$ can be written as a sum over a complete basis of Fock states

light-cone wave function

$$|M(P)\rangle = \sum_{n=1}^{\infty} \int [d\mu_n]\, |n; x_i P^+, \vec{k}_{\perp i} + x_i \vec{P}_\perp, \lambda_i\rangle \Psi_{n/M}\left(x_i, \vec{k}_{\perp i}, \lambda_i\right)$$

phase-space integration

$$\begin{array}{llcll} n=1: & |q\bar{q}: k_i^+, \boldsymbol{k}_{\perp i}, \lambda_i\rangle & = & b^\dagger(q_1)d^\dagger(q_2) & |0\rangle, \\ n=2: & |q\bar{q}g: k_i^+, \boldsymbol{k}_{\perp i}, \lambda_i\rangle & = & b^\dagger(q_1)d^\dagger(q_2)a^\dagger(q_3) & |0\rangle, \\ n=3: & |gg: k_i^+, \boldsymbol{k}_{\perp i}, \lambda_i\rangle & = & a^\dagger(q_1)a^\dagger(q_2) & |0\rangle, \\ \vdots & \vdots & \vdots & \vdots & |0\rangle. \end{array}$$

- Specific to the valence $q\bar{q}$ Fock state, the meson LCDA is defined as an integral over transverse momenta of the meson's Bethe-Salpeter wave function

light-cone distribution amplitude
$$\Phi_\pi\left(x, Q^2\right) = \int_0^{Q^2} \frac{d^2\vec{k}_\perp}{16\pi^3} \Psi_{\bar{q}q/\pi}\left(x, \vec{k}_\perp\right)$$

- Physical meaning: the probability amplitude for finding the pion as a quark-antiquark pair with momentum fractions $x_q = x$ and $x_{\bar{q}} = 1 - x$.

→ *involved in hard exclusive reactions in collinear factorization approach*

Exclusive B decays

- **Leading-twist LCDAs for light mesons and B mesons: only for $q\bar{q}$ Fock state**

$$\langle P(q)|\bar{q}(y)_\alpha q'(x)_\beta|0\rangle\Big|_{(x-y)^2=0} = \frac{if_P}{4}\,(\not{q}\gamma_5)_{\beta\alpha}\int_0^1 du\, e^{i(\bar{u}qx+uqy)}\,\Phi_P(u,\mu),$$

$$\langle V_{||}(q)|\bar{q}(y)_\alpha q'(x)_\beta|0\rangle\Big|_{(x-y)^2=0} = -\frac{if_V}{4}\not{q}_{\beta\alpha}\int_0^1 du\, e^{i(\bar{u}qx+uqy)}\,\Phi_{||}(u,\mu),$$

$$\langle V_{\perp}(q)|\bar{q}(y)_\alpha q'(x)_\beta|0\rangle\Big|_{(x-y)^2=0} = -\frac{if_T(\mu)}{8}\,[\not{\epsilon}^*_\perp,\not{q}]_{\beta\alpha}\int_0^1 du\, e^{i(\bar{u}qx+uqy)}\,\Phi_{\perp}(u,\mu)$$

$$\langle 0|\bar{q}_\alpha(z)[\ldots]b_\beta(0)|\bar{B}_d(p)\rangle\Big|_{z_+=z_\perp=0} = -\frac{if_B}{4}\,[(\not{p}+m_b)\gamma_5]_{\beta\gamma}\int_0^1 d\xi\, e^{-i\xi p_+ z_-}\,[\Phi_{B1}(\xi)+\not{n}_-\Phi_{B2}(\xi)]_{\gamma\alpha}$$

defined in terms of the matrix elements of various non-local composite operators

- **For light mesons, LCDAs usually expanded in Gegenbauer polynomials**

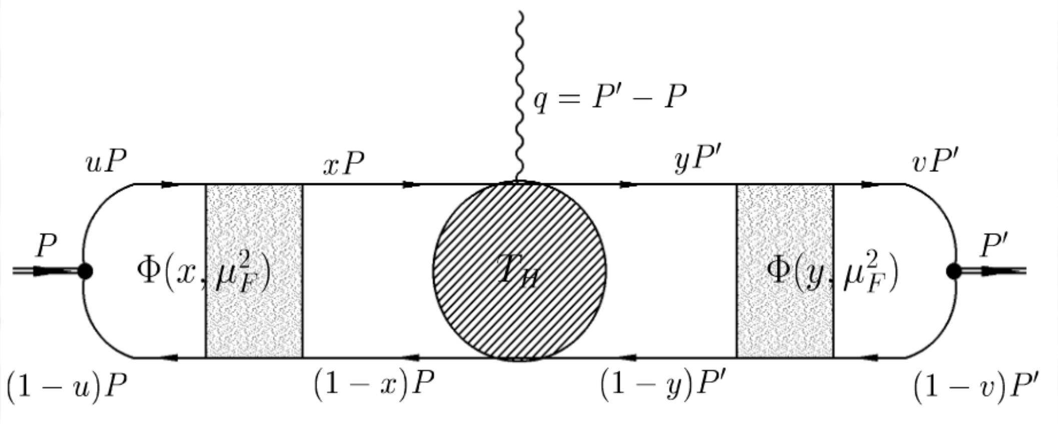
$$\Phi_L(u) = 6u(1-u)\left[1+\sum_{n=1}^{\infty}\alpha_n^L(\mu)\,C_n^{3/2}(2u-1)\right]$$

can be obtained from LQCD or other methods

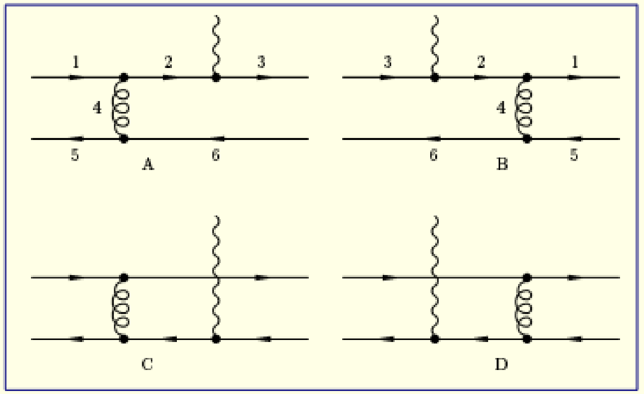
- **These hadronic inputs encode information of the non-pert. strong interaction dynamics**

Exclusive B decays

◻ Example: pion electromagnetic form factor with large momentum transfer

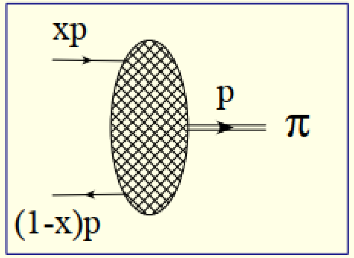


$\gamma^*(q_1\bar{q}_2) \to (q_1\bar{q}_2)$



- $|\pi\rangle \to |q\bar{q}\rangle + |q\bar{q}g\rangle + \cdots$
- collinear approximation:

$$p_q = x\,p, \quad p_{\bar{q}} = (1-x)\,p$$



LO quark-level Feynman diagrams

◻ In the standard hard-scattering picture: collinear factorization theorem

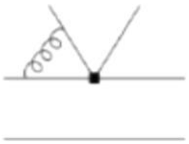
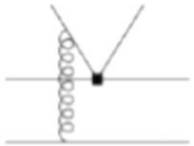
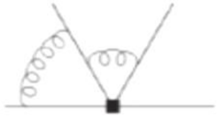
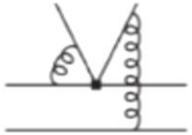
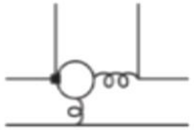
hard scattering amplitude	=	elementary hard-scattering amplitude	⊗	hadron distribution amplitudes

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\ \Phi^*(y, \mu_F^2)\ T_H(x, y, Q^2, \mu_R^2, \mu_F^2)\ \Phi(x, \mu_F^2)$$

Status of the NNLO calculation of T^I & T^{II}

- **For each Q_i insertion, both tree & penguin topologies, and contribute to both T^I & T^{II}.**

$$\langle M_1 M_2|Q_i|B\rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$		$T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$		$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11, Bell Beneke, Huber, Li '15 Bell, Beneke, Huber, Li '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

Further details to be learned, …

- **Specific decay modes: how to calculate theoretically, potential questions in different decay modes,…**

- **Various EFT used in heavy flavor physics, e.g. HQET, LEET, SCET, NRQCD,…**

  - expansion by regions, heavy mass/momentum expansion, modern loop calculations,…

- **New physics probes with flavor physics, model-independent framework vs specific NP models, correlation between high-intensity and high-energy frontiers,…**

Thank You for Your Attention