

电弱规范理论

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讲义大纲

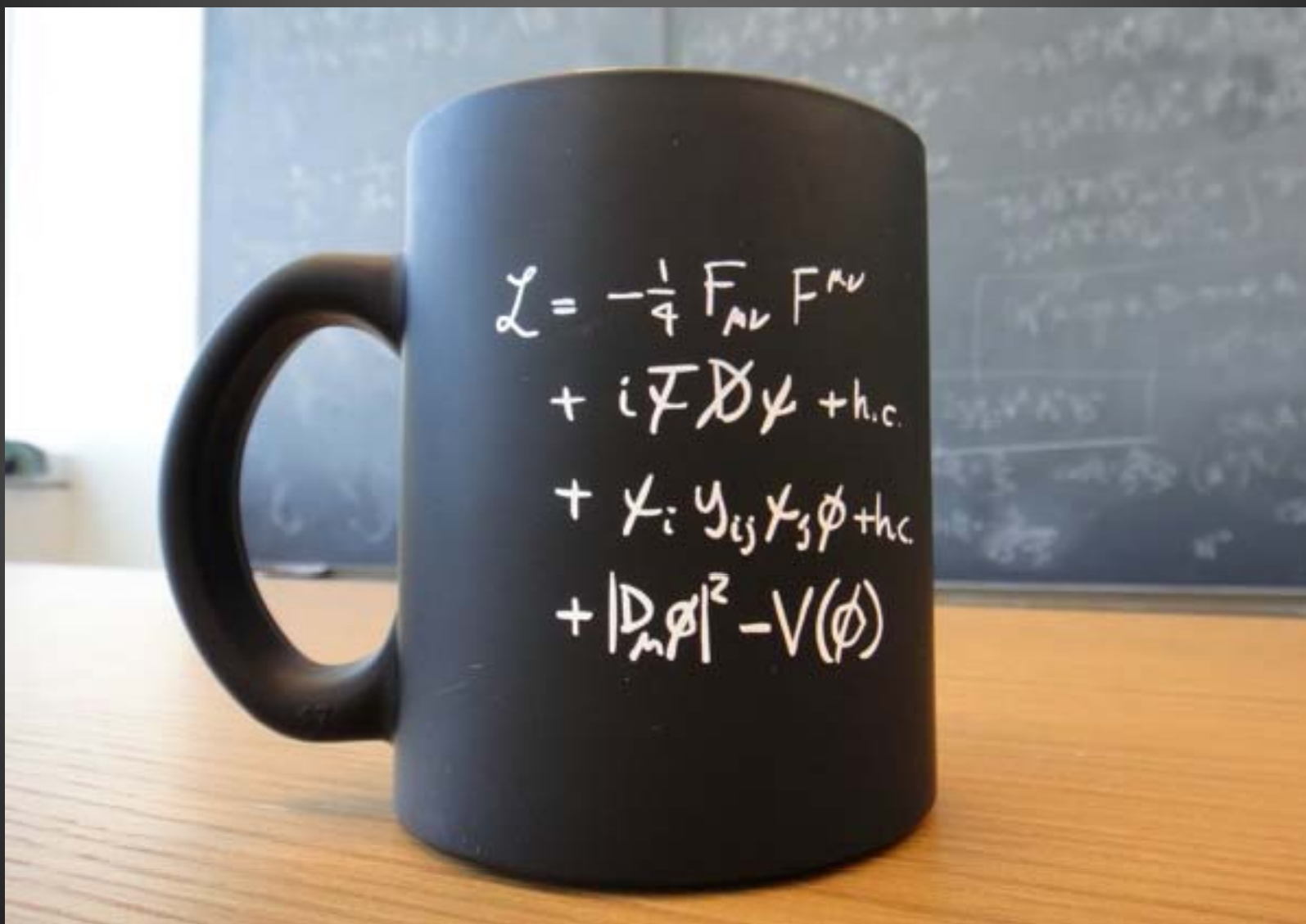
- 标准模型的基本粒子及其量子数
- 标准模型的反常相消
- 规范变换
- 整体对称性的自发破缺与Goldstone定理
- 局域对称性的自发破缺与Higgs机制
- 标准模型电弱部分的拉氏量
- 电弱精确测量参量 S 、 T 、 U

标准模型的基本粒子及其量子数

标准模型的基本粒子

	up	charm	top	gluon	Higgs boson
mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	$2/3$	$2/3$	$2/3$	0	0
spin →	$1/2$	$1/2$	$1/2$	1	0
QUARKS	down	strange	bottom	photon	
	mass →	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0
	charge →	$-1/3$	$-1/3$	$-1/3$	0
spin →	$1/2$	$1/2$	$1/2$	1	
LEPTONS	electron	muon	tau	Z boson	
	mass →	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$
	charge →	-1	-1	-1	0
spin →	$1/2$	$1/2$	$1/2$	1	
GAUGE BOSONS	electron neutrino	muon neutrino	tau neutrino	W boson	
	mass →	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$
	charge →	0	0	0	± 1
spin →	$1/2$	$1/2$	$1/2$	1	

标准模型的拉格朗日量



标准模型的规范对称性: $SU(3) \times SU(2) \times U(1)$

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1

标准模型的反常相消

三角规范反常图：1个轴矢流 + 2个矢量流

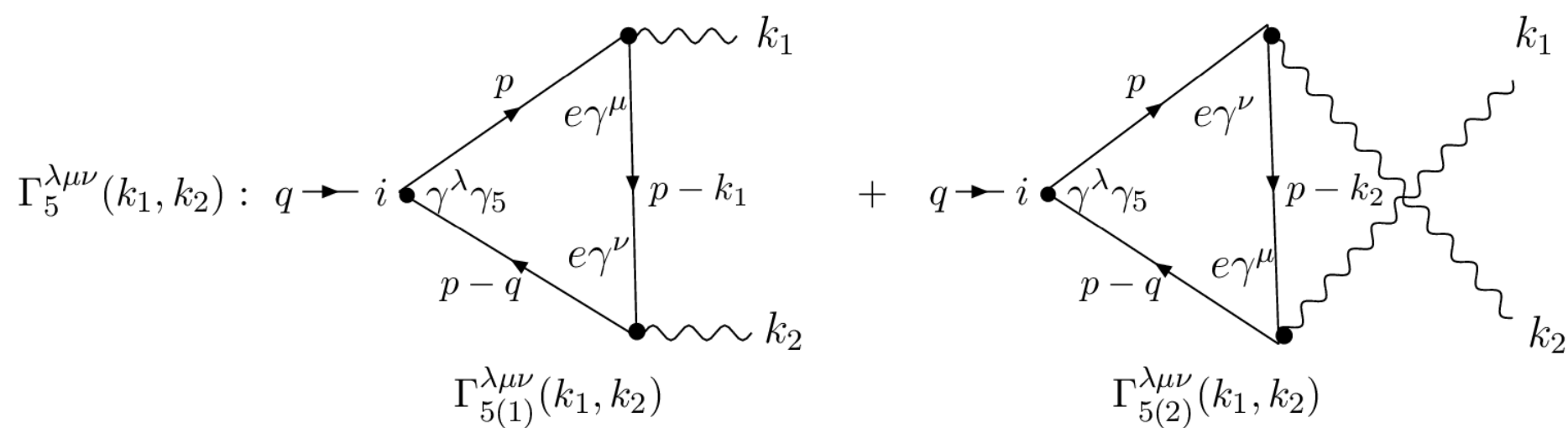
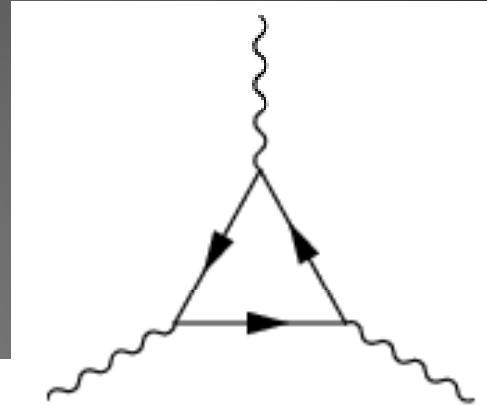


FIG. 2: Lowest order Feynman diagrams contributing to $\Gamma_5^{\lambda\mu\nu}(k_1, k_2)$.

对于手征规范理论，如果规范反常不能相消，则理论是不自洽的。

规范反常正比于 $\text{tr} [\gamma_5 T^a \{T^b, T^c\}]$



- 容易验证: $\text{tr}[T^a \{T^b, T^c\}]$ is totally symmetric in (a, b, c) .
- 实际操作可以只计算 $\text{tr}(T^a T^b T^c)$, 不妨将左手费米子的贡献取正, 将右手费米子的贡献取负.
- 对于 vector-like 的规范理论, 左右手费米子在规范群中的表示相同, 因此反常自动相消.

为了消除引力反常，还需要考虑有一个规范玻色子、两个引力子的反常图。

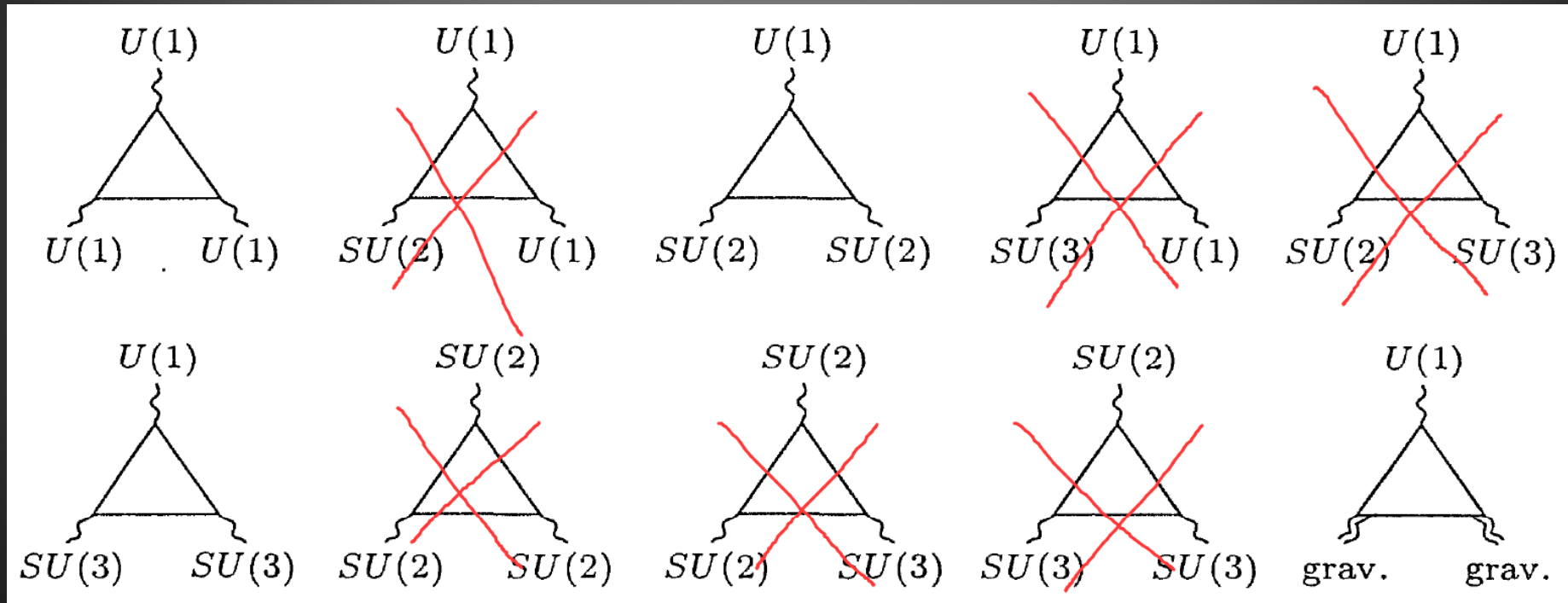
由于 $SU(3)_c$ 是矢量规范理论，左右手费米子对333、3gg反常图的贡献相消，不必考虑333、3gg图，其中g代表引力子。

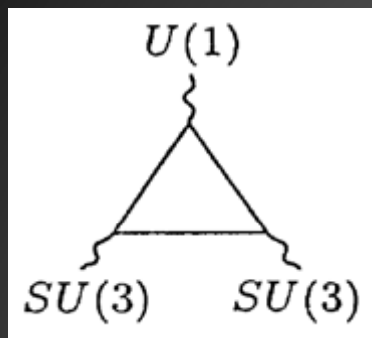
既含有 $SU(3)_c$ 又含有 $SU(2)_L \times U(1)_Y$ 的规范玻色子的反常图有可能非零，需要加以考虑。

因为 $SU(n)$ 群的生成元是无迹的，含有单个2或单个3的图都不必考虑。

因为 $SU(2)$ 群是anomaly free的群，222反常图也不必考虑。

我们下面来验证：
 在标准模型中，每一代的手征费米子的群表示设置恰好是
anomaly free。





331:

$$\text{tr}\left(\frac{\lambda^a}{2} \frac{\lambda^b}{2} Y\right) = \frac{1}{2} \delta^{ab} \sum_f Y_f$$

对于每一代,

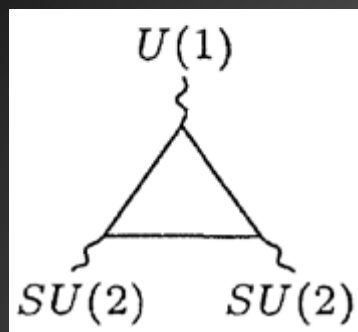
$$\sum_f Y_f = 2 \times \frac{1}{6} - \frac{2}{3} - \left(-\frac{1}{3}\right) = 0$$



$SU(2)_L$ 2重态

→ 331 无反常

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1



221:

$$\text{tr}\left(\frac{\delta^a}{2} \frac{\delta^b}{2} \gamma\right) = \frac{1}{2} \delta^{ab} \sum_{f_L} Y_{f_L}$$

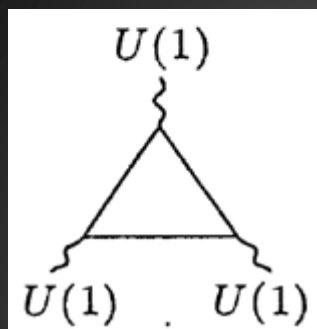
对于每一代,

$$\sum_{f_L} Y_{f_L} = 3 \times \frac{1}{6} + \left(-\frac{1}{2}\right) = 0$$

↓
色

→ 221 无反常

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
<u>$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$</u>	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
<u>$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$</u>	1	2	$-\frac{1}{2}$
e_R	1	1	-1



|||:

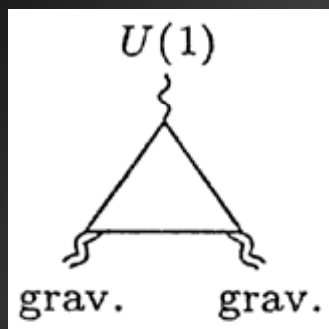
$$\text{tr}(Y^3) = \sum Y^3$$

对于每一代,

$$\begin{aligned} \sum Y^3 &= 3 \times 2 \times \left(\frac{1}{6}\right)^3 - 3 \times \left(\frac{2}{3}\right)^3 - 3 \times \left(-\frac{1}{3}\right)^3 \\ &\quad + 2 \times \left(-\frac{1}{2}\right)^3 - (-1)^3 = 0 \end{aligned}$$

→ ||| 无反常

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1



1gg:

$$\text{tr}(Y) = \sum Y$$

对于每一代,

$$\begin{aligned} \sum Y &= 3 \times 2 \times \frac{1}{6} - 3 \times \frac{2}{3} - 3 \times \left(-\frac{1}{3}\right) \\ &\quad + 2 \times \left(-\frac{1}{2}\right) - (-1) = 0 \end{aligned}$$

→ 1gg 无反常

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1

规范变换

• 规范变换

▲ 阿贝尔情形

若某个场 $\psi(x)$ 具有 $U(1)$ 荷 $(-e)$, 即
它的 $U(1)$ 局域变换是

$$\psi(x) \rightarrow e^{i \underbrace{(-e)}_{\text{荷}} \theta(x)} \psi(x) = U(x) \psi(x)$$

其中 $\theta(x)$ 是依赖于时空坐标的群参数。

ψ 的协变导数可这样引入:

$$D_\mu \psi = (\partial_\mu + ie A_\mu) \psi$$

其中 A_μ 是 $U(1)$ 规范场。

引入协变导数的目的是为了使

$D_\mu \psi$ 与 ψ

的 $U(1)$ 变换相同, 即 $D_\mu \psi \longrightarrow U(x) D_\mu \psi$

即 D_μ 的变换是

$$D_\mu \rightarrow U(x) D_\mu U^\dagger(x)$$

显式地, 有

$$\partial_\mu + ieA_\mu \rightarrow U(x)(\partial_\mu + ieA_\mu)U^\dagger(x)$$

$$\Rightarrow A_\mu \rightarrow A_\mu - \frac{i}{e} U(x) \underbrace{[\partial_\mu U^\dagger(x)]}_{"U^\dagger ie \partial_\mu \theta"}$$

$$\Rightarrow A_\mu \rightarrow A_\mu + \partial_\mu \theta$$

由 $D_\mu \rightarrow U(x) D_\mu U^\dagger(x)$ 知

$$[D_\mu, D_\nu] \rightarrow U(x) [D_\mu, D_\nu] U^\dagger(x)$$

$= [D_\mu, D_\nu]$ 是规范不变量。

而 $[D_\mu, D_\nu] = [\partial_\mu + ieA_\mu, \partial_\nu + ieA_\nu]$

$$= ie (\partial_\mu A_\nu - \partial_\nu A_\mu) = ie F_{\mu\nu}$$

$\Rightarrow F_{\mu\nu}$ 是规范不变量。

▲ 非阿贝尔情形

设某个多重态处于某个非阿贝尔规范群的基础表示, 则它的协变导数是

$$D_\mu = \partial_\mu - ig T^a A_\mu^a(x)$$

其中 g 是规范群的耦合常数, $A_\mu^a(x)$ 是规范场, T^a 是规范群的基础表示的生成元。

这些生成元满足如下的李代数:

$$[T^a, T^b] = i f^{abc} T^c$$

其中 f^{abc} 是规范群的结构常数, 它是全反对称的。

若该多重态的规范变换是 $U(x) = e^{iT^a \alpha^a(x)}$

则协变导数的规范变换是

$$D_\mu \rightarrow U(x) D_\mu U^\dagger(x)$$

即 $\partial_\mu - ig T^a A_\mu^a \rightarrow U(x) (\partial_\mu - ig T^a A_\mu^a) U^\dagger(x)$

$$\Leftrightarrow T^a A_\mu^a \rightarrow U(x) (T^a A_\mu^a + \frac{i}{g} \partial_\mu) U^\dagger(x)$$

特别地, 对于无穷小的规范变换:

$$U(x) \simeq 1 + iT^a \alpha^a(x)$$

$$\Rightarrow U^\dagger(x) \simeq 1 - iT^a \alpha^a(x)$$

$$\Rightarrow A_\mu^a \rightarrow A_\mu^a + \frac{1}{g} \left[\partial_\mu \alpha^a - ig \overset{ifabc}{\parallel} (T_G^b)_{ac} A_\mu^b \alpha^c \right]$$

$(D_\mu \alpha)^a$

$$\begin{aligned}
 [D_\mu, D_\nu] &= [\partial_\mu - ig T^a A_\mu^a, \partial_\nu - ig T^b A_\nu^b] \\
 &= -ig T^a (\underbrace{\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c}_{\equiv F_{\mu\nu}^a})
 \end{aligned}$$

的规范变换是

$$[D_\mu, D_\nu] \rightarrow U(x) [D_\mu, D_\nu] U^{-1}(x)$$

故可构造规范不变量: $\text{tr}([D_\mu, D_\nu][D^\mu, D^\nu])$

若约定 $\text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$, 则

$$\mathcal{L} \supset \frac{1}{2g^2} \text{tr}([D_\mu, D_\nu][D^\mu, D^\nu]) = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

整体对称性的自发破缺与Goldstone定理

• 对称性自发破缺

a real scalar field $\phi(x)$

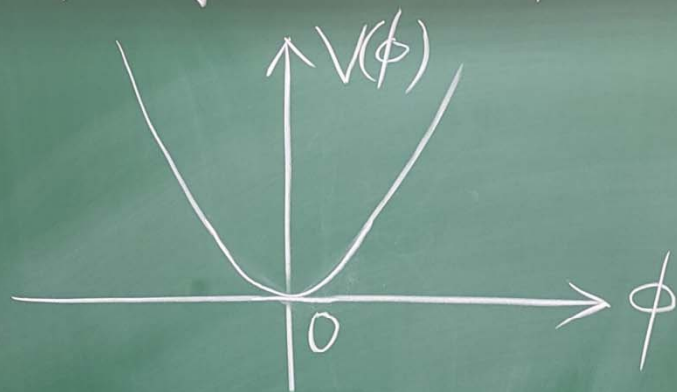
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi),$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$V(\phi) \text{ 有下界} \Rightarrow \lambda > 0$$

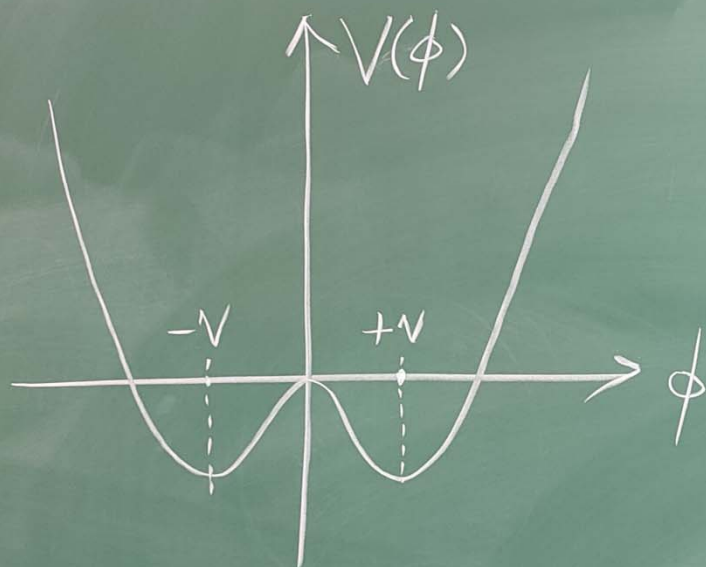
$\mathcal{L}$ 具有一个 Z_2 对称性: $\phi \rightarrow -\phi$

▲ 若 $\mu^2 > 0$, 则 V 的极小值在 $\phi = 0$ 处取得。



▲ 若 $\mu^2 < 0$, 则 $V(\phi)$ 的极值点应满足

$$0 = \frac{\partial V}{\partial \phi} = \mu^2 \phi + \lambda \phi^3 = \phi (\mu^2 + \lambda \phi^2)$$



如图可知:

$V(\phi)$ 的极小值点是

$$\phi^2 = \frac{(-\mu^2)}{\lambda} \equiv v^2$$

$$\Rightarrow \phi = \pm v, \quad v \equiv \sqrt{\frac{(-\mu^2)}{\lambda}}$$

将 $\phi(x)$ 在其中一个极小值点 v 邻域作展开:

$$\phi(x) = v + \delta(x)$$

练习1：请检验这个式子。

则拉氏量可写为

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \underbrace{(-\mu^2)}_{\equiv \frac{1}{2}m_\phi^2}\phi^2 - \sqrt{(-\mu^2)\lambda}\phi^3 - \frac{\lambda}{4}\phi^4 + \text{Const.}$$

由于这里存在三次项 ϕ^3 ，原先的 Z_2 对称性已被自发破缺。

离散对称性的自发破缺不会产生零质量的Goldstone玻色子。

练习1解答:

1/3

$$\text{由 } V(\phi) = \frac{\lambda}{4} \phi^4 - \frac{(-\mu^2)}{2} \phi^2 = \frac{\lambda}{4} \left[\phi^4 - 2 \frac{(-\mu^2)}{\lambda} \phi^2 \right]$$

$$= \frac{\lambda}{4} \left[\phi^2 - \underbrace{\left(\frac{-\mu^2}{\lambda} \right)}_{\equiv v^2} \right]^2 + \underbrace{\text{Const.}}_{\text{可忽略}}$$

$$= \frac{\lambda}{4} (\phi^2 - v^2)^2$$

$$\text{又由 } \phi(x) = v + \delta(x) \Rightarrow \begin{cases} \phi(x) - v = \delta(x) \\ \phi(x) + v = 2v + \delta(x) \end{cases}$$
$$\Rightarrow \phi^2 - v^2 = (\phi - v)(\phi + v) = \delta(2v + \delta)$$

$$\Rightarrow V(\phi) = \frac{\lambda}{4} (\phi^2 - v^2)^2 = \frac{\lambda}{4} [\phi(2v + \phi)]^2$$
$$= \frac{\lambda}{4} \phi^2 (\phi^2 + 4v\phi + 4v^2)$$

而 $\phi(x) = v + \delta(x) \Rightarrow \partial_\mu \phi = \partial_\mu \delta$

$$\Rightarrow \frac{1}{2} (\partial \phi)^2 = \frac{1}{2} (\partial \delta)^2$$

因此, $\mathcal{L} = \frac{1}{2} (\partial \phi)^2 - V(\phi)$

$$= \frac{1}{2} (\partial \delta)^2 - \frac{\lambda}{4} \delta^2 (\delta^2 + 4v\delta + 4v^2)$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \underbrace{(\lambda v^2)}_{\substack{= \\ \lambda \cdot \frac{(-\mu^2)}{\lambda} \\ = \\ (-\mu^2)}} \phi^2 - \underbrace{(\lambda v)}_{\substack{= \\ \lambda \sqrt{\frac{(-\mu^2)}{\lambda}} \\ = \\ \sqrt{(-\mu^2)\lambda}}} \phi^3 - \frac{\lambda}{4}\phi^4$$

$$= \frac{1}{2}(\partial\phi)^2 - \underbrace{(-\mu^2)}_{\substack{= \\ \frac{1}{2}m_\phi^2}} \phi^2 - \sqrt{(-\mu^2)\lambda} \phi^3 - \frac{\lambda}{4}\phi^4$$

$$\Rightarrow m_\phi^2 = 2(-\mu^2)$$



• 连续对称性的自发破缺

4个实标量场 $\phi_i(x)$ ($i=1, \dots, 4$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - \frac{1}{2}m^2(\phi_i \phi_i) - \frac{1}{4}\lambda(\phi_i \phi_i)^2$$

其中 $\lambda > 0$.

$\mathcal{L}$ 具有一个 $SO(4)$ 对称性: $\phi_i(x) \rightarrow R_{ij} \phi_j(x)$,
其中 $R \in SO(4)$

同样, 若 $\mu^2 < 0$, 则

$$\begin{aligned}
 V &= \frac{\lambda}{4}(\phi_i \phi_i)^2 - \frac{(-\mu^2)}{2}(\phi_i \phi_i) \\
 &= \frac{\lambda}{4} \left[(\phi_i \phi_i)^2 - 2 \frac{(-\mu^2)}{\lambda} (\phi_i \phi_i) \right] \\
 &= \frac{\lambda}{4} \left[\phi_i \phi_i - \underbrace{\left(\frac{(-\mu^2)}{\lambda} \right)}_{\equiv v^2} \right]^2 + \text{Const.}
 \end{aligned}$$

的极小值点是

$$\phi_i^2 = \frac{(-\mu^2)}{\lambda} \equiv v^2,$$

其中 $v \equiv \sqrt{\frac{(-\mu^2)}{\lambda}}$

不妨在其中一个极小值点 $\phi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix}$ 邻域作展开.

$$\phi(x) \equiv \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \end{pmatrix} = \begin{pmatrix} \pi_1(x) \\ \pi_2(x) \\ \pi_3(x) \\ v + \delta(x) \end{pmatrix}$$

则拉氏量变为

$$\mathcal{L} = \frac{1}{2}(\partial\delta)^2 - \frac{1}{2}(-2\overset{m_\delta^2}{\mu^2})\delta^2 - \lambda v \delta^3 - \frac{\lambda}{4}\delta^4 \\ + \frac{1}{2}(\partial\pi_i)(\partial\pi_i) - \frac{\lambda}{4}(\pi_i\pi_i)^2 - \lambda v (\pi_i\pi_i)\delta - \frac{\lambda}{2}(\pi_i\pi_i)\delta^2$$

$\Rightarrow \pi_i (i=1,2,3)$ 没有质量项!

练习2: 请检验这个式子。

练习2解答:

1/2

$$\text{由 } \phi(x) = \begin{pmatrix} \pi_1(x) \\ \pi_2(x) \\ \pi_3(x) \\ v + \delta(x) \end{pmatrix} \Rightarrow \begin{cases} \partial_\mu \phi = \begin{pmatrix} \partial_\mu \pi_1 \\ \partial_\mu \pi_2 \\ \partial_\mu \pi_3 \\ \partial_\mu \delta \end{pmatrix} \\ \phi^\top \phi = (v + \delta)^2 + \vec{\pi}^2 \end{cases}$$

$$\Rightarrow \mathcal{L} \supset \frac{1}{2} (\partial_\mu \phi^\top) (\partial^\mu \phi) = \frac{1}{2} (\partial \delta)^2 + \frac{1}{2} (\partial \vec{\pi}) \cdot (\partial \vec{\pi})$$

$$\begin{aligned} \text{而 } V &= \frac{\lambda}{4} (\phi^\top \phi)^2 - \frac{(-\mu^2)}{2} (\phi^\top \phi) = \frac{\lambda}{4} \left[(\phi^\top \phi)^2 - 2 \frac{(-\mu^2)}{\lambda} \phi^\top \phi \right] \\ &= \frac{\lambda}{4} \left[\underbrace{(\phi^\top \phi)}_{(v+\delta)^2 + \vec{\pi}^2} - \underbrace{\left(\frac{-\mu^2}{\lambda} \right)}_{v^2} \right]^2 + \text{Const.} \\ &\quad \downarrow \text{可忽略.} \end{aligned}$$

$$\Rightarrow V = \frac{\lambda}{4} (\delta^2 + 2\nu\delta + \vec{\pi}^2)^2$$

$$\begin{aligned} & \delta^4 + 4\nu^2\delta^2 + (\vec{\pi}^2)^2 + 4\nu\delta^3 + 4\nu\delta\vec{\pi}^2 \\ & + 2\delta^2\vec{\pi}^2 \end{aligned}$$

$$= \frac{\lambda}{4} \delta^4 + \lambda\nu\delta^3 + \lambda\nu^2\delta^2$$

$$+ \frac{\lambda}{4} (\vec{\pi}^2)^2 + \frac{\lambda}{2} \vec{\pi}^2\delta^2 + \lambda\nu\vec{\pi}^2\delta$$

$$\begin{aligned} \Rightarrow \mathcal{L} & \supset \frac{1}{2}(\partial\phi^T)(\partial\phi) - V \\ & = \frac{1}{2}(\partial\delta)^2 - \lambda\nu^2\delta^2 - \lambda\nu\delta^3 - \frac{\lambda}{4}\delta^4 + \frac{1}{2}(\partial\vec{\pi})^2 - \frac{\lambda}{4}(\vec{\pi}^2)^2 - \lambda\nu\vec{\pi}^2\delta \\ & \quad - \frac{\lambda}{2}\vec{\pi}^2\delta^2 \quad \checkmark \end{aligned}$$

注意：在 3 个 π_i 场中仍然有一个 $SO(3)$ 对称性。

	$SO(4)$	$SO(3)$	破缺的生成元数目
生成元数目:	$\frac{4 \times 3}{2} = 6$	$\frac{3 \times 2}{2} = 3$	$6 - 3 = 3$

零质量的 π_i 数目

归纳可得：

→ Goldstone 定理

连续对称性的自发破缺会产生零质量的标量玻色子（称为 Goldstone 玻色子），其数目等于破缺生成元的数目。

局域对称性的自发破缺与Higgs机制

• Higgs 机制

考虑一个复标量场 $\phi(x)$ 与一个阿贝尔规范场 $A_\mu(x)$ 耦合的理论:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi^\dagger)(D^\mu\phi) - V(\phi)$$

其中 $V(\phi) = m^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$

$$D_\mu\phi = (\partial_\mu + ieA_\mu)\phi \Rightarrow D_\mu\phi^\dagger = \phi^\dagger(\overleftarrow{\partial}_\mu - ieA_\mu)$$

▲ 当 $\mu^2 > 0$ 时, $\mathcal{L}$ 是一个质量为 μ , 电荷为 $(-e)$, 且具有 ϕ^4 自相互作用的带电标量粒子的 QED 拉氏量。

▲ 当 $\mu^2 < 0$ 时, ϕ 会获得 VEV:

$$\langle \phi \rangle = \sqrt{\frac{(-\mu^2)}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

在 VEV 邻域对 $\phi(x)$ 作展开:

$$\phi(x) = \frac{v + \phi_1(x) + i\phi_2(x)}{\sqrt{2}}$$

拉氏量可表达为 (为简化起见, 我们略去一些相互作用项):

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overbrace{(ev)^2}^{M_A^2}A_\mu A^\mu \\ & + \frac{1}{2}(\partial\phi_1)^2 - \underbrace{(\lambda v^2)\phi_1^2}_{\rightarrow \frac{1}{2}m_1^2} \\ & + \frac{1}{2}(\partial\phi_2)^2 + evA_\mu(\partial^\mu\phi_2) \end{aligned}$$

练习3: 请检验这个式子。

练习3解答:

1/6

$$V = \lambda(\phi^+\phi)^2 - (-\mu^2)\phi^+\phi = \lambda\left[(\phi^+\phi)^2 - \frac{(-\mu^2)}{\lambda}\phi^+\phi\right]$$

$$= \lambda\left[\phi^+\phi - \underbrace{\frac{(-\mu^2)}{2\lambda}}_{\equiv \frac{v^2}{2}}\right]^2 + \underbrace{\text{Const.}}_{\text{可省略.}}$$

$$= \lambda\left(\phi^+\phi - \frac{v^2}{2}\right)^2$$

$$\phi = \frac{v + \phi_1 + i\phi_2}{\sqrt{2}} \Rightarrow \phi^+ = \frac{v + \phi_1 - i\phi_2}{\sqrt{2}}$$

$$\Rightarrow \phi^+\phi = \frac{1}{2}\left[(v + \phi_1)^2 + \phi_2^2\right] = \frac{1}{2}(v^2 + 2v\phi_1 + \phi_1^2 + \phi_2^2)$$

$$\Rightarrow \phi^\dagger \phi - \frac{v^2}{2} = \frac{1}{2}(2v\phi_1 + \phi_1^2 + \phi_2^2)$$

$$\Rightarrow V = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

$$= \frac{\lambda}{4} (2v\phi_1 + \phi_1^2 + \phi_2^2)^2$$

$$\equiv (4v^2\phi_1^2 + \phi_1^4 + \phi_2^4 + 4v\phi_1^3 + 4v\phi_1\phi_2^2 + 2\phi_1^2\phi_2^2)$$

$$\supset \lambda v^2 \phi_1^2 \quad (\text{这里暂时只考虑质量项})$$

$$\Rightarrow \mathcal{L} \supset -V \supset -\underbrace{(\lambda v^2)}_{\equiv \frac{1}{2}m_1^2} \phi_1^2$$

再来看 $\mathcal{L} \supset (D_\mu \phi^\dagger)(D^\mu \phi)$

$$= \phi^\dagger (\overleftarrow{\partial}_\mu - ie A_\mu) (\partial^\mu + ie A^\mu) \phi$$

$$= \underbrace{(\partial_\mu \phi^\dagger)(\partial^\mu \phi)}_{(a)} + \underbrace{e A_\mu (-i) \phi^\dagger (\overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu) \phi}_{(b)}$$

$$+ \underbrace{e^2 A_\mu A^\mu \phi^\dagger \phi}_{(c)}$$

由 $\phi(x) = \frac{V + \phi_1(x) + i\phi_2(x)}{\sqrt{2}}$

$$\Rightarrow \begin{cases} \partial_\mu \phi = \frac{\partial_\mu \phi_1 + i \partial_\mu \phi_2}{\sqrt{2}} \\ \partial_\mu \phi^\dagger = \frac{\partial_\mu \phi_1 - i \partial_\mu \phi_2}{\sqrt{2}} \end{cases}$$

$$\Rightarrow (a) = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) = \frac{1}{2}(\partial \phi_1)^2 + \frac{1}{2}(\partial \phi_2)^2$$

由前面求得 $\phi^\dagger \phi = \frac{1}{2}(v^2 + 2v\phi_1 + \phi_1^2 + \phi_2^2)$

$$\Rightarrow \mathcal{C} = e^2 A_\mu A^\mu \frac{1}{2}(v^2 + 2v\phi_1 + \phi_1^2 + \phi_2^2)$$

$$\supset \frac{1}{2} e^2 v^2 A_\mu A^\mu$$

$$\text{由 } \phi(x) = \frac{v + \phi_1(x) + i\phi_2(x)}{\sqrt{2}}$$

$$\Rightarrow (-i)\phi^\dagger \partial_\mu \phi = (-i) \frac{(v + \phi_1 - i\phi_2)}{\sqrt{2}} \frac{[(\partial_\mu \phi_1) + i(\partial_\mu \phi_2)]}{\sqrt{2}}$$

$$= \frac{(-i)}{2} \left[(v + \phi_1)(\partial_\mu \phi_1) + \phi_2(\partial_\mu \phi_2) + i(v + \phi_1)(\partial_\mu \phi_2) - i\phi_2(\partial_\mu \phi_1) \right]$$

$$= \frac{1}{2}(v + \phi_1)(\partial_\mu \phi_2) - \frac{1}{2}\phi_2(\partial_\mu \phi_1) + (\text{Imaginary part})$$

而 $i\phi\partial_\mu\phi^\dagger$ 是 $(-i)\phi^\dagger\partial_\mu\phi$ 的复共轭,

$$\begin{aligned} \text{因此, } & (-i)\phi^\dagger\partial_\mu\phi + i\phi\partial_\mu\phi^\dagger \\ &= 2\text{Re}[(-i)\phi^\dagger\partial_\mu\phi] \\ &= (v+\phi_1)(\partial_\mu\phi_2) - \phi_2(\partial_\mu\phi_1) \end{aligned}$$

$$\begin{aligned} \Rightarrow \textcircled{b} &= eA_\mu(-i)\phi^\dagger(\vec{\partial}^\mu - \overleftarrow{\partial}^\mu)\phi \\ &= eA_\mu[(v+\phi_1)(\partial^\mu\phi_2) - \phi_2(\partial^\mu\phi_1)] \\ &\supset evA_\mu(\partial^\mu\phi_2) \quad (\text{这里暂时只考虑二次项}) \end{aligned}$$

综上,有

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A_\mu A^\mu \\ & + \frac{1}{2}(\partial\phi_1)^2 - \lambda v^2\phi_1^2 \\ & + \frac{1}{2}(\partial\phi_2)^2 + evA_\mu(\partial^\mu\phi_2) \end{aligned}$$



讨论:

① 规范场 A_μ 获得了质量:

$$\frac{1}{2} M_A^2 A_\mu A^\mu \quad \text{其中} \quad M_A = ev = e \sqrt{\frac{(-\mu^2)}{\lambda}}$$

② 在 VEV 方向的标量粒子激发 ϕ_1 具有质量:

$$m_1^2 = 2\lambda v^2 = 2(-\mu^2)$$

③ 表面上来看, 有一个零质量的标量 ϕ_2 (称为 would-be Goldstone 玻色子)。但存在一些问题:

③.1 在 A_μ 与 ϕ 之间有一个混合项:

$$\mathcal{L} \supset eV A_\mu (\partial^\mu \phi_2)$$

③.2 自由度前后不一致:

破缺前有	{	2 (复标量 ϕ)	破缺后有	{	1 (ϕ_1)
4个自由度		2 (零质量的矢量 A_μ)			5个自由度
					3 (有质量的矢量 A_μ)

可见, 在破缺后, 有一个场是非物理的, 应该被去除。

我们通过作一个**规范变换**, 可以消掉 ϕ_2 这个自由度 (见下页)。

$$\mathcal{L} \supset \frac{1}{2}(ev)^2 A_\mu A^\mu + \frac{1}{2}(\partial_\mu \phi_2)^2 + ev A_\mu (\partial^\mu \phi_2)$$

$$= \frac{1}{2}(ev A_\mu + \partial_\mu \phi_2)^2$$

$$= \frac{1}{2}(ev)^2 \left(A_\mu + \frac{1}{ev} \partial_\mu \phi_2 \right)^2$$

$$= \frac{1}{2}(ev)^2 \underbrace{\left(A_\mu + \frac{1}{ev} \partial_\mu \phi_2 \right)}_{\equiv \tilde{A}_\mu}$$

可见, 只须作规范变换: $A_\mu \rightarrow \tilde{A}_\mu = A_\mu + \frac{1}{ev} \partial_\mu \phi_2$
就可将 ϕ_2 这个自由度去除。

我们还可用极坐标来参数化 $\phi(x)$:

$$\phi(x) = \frac{[v + \eta(x)]}{\sqrt{2}} e^{i \frac{\zeta(x)}{v}}$$

通过一个规范变换, 我们可以完全消去相位 $\zeta(x)$:

$$\phi(x) \rightarrow U(x) \phi(x) = \frac{v + \eta(x)}{\sqrt{2}}$$

$$\text{其中 } U(x) = e^{-i \frac{\zeta(x)}{v}}$$

相应地, 规范场的变换是: $A_\mu \rightarrow \tilde{A}_\mu = A_\mu + \frac{1}{ev} (\partial_\mu \zeta)$

$$D_\mu \rightarrow U D_\mu U^{-1}$$

$$\text{即 } \partial_\mu + ieA_\mu \rightarrow U(x)(\partial_\mu + ieA_\mu)U^{-1}(x)$$

$$\Leftrightarrow A_\mu \rightarrow U(x)(A_\mu - \frac{i}{e}\partial_\mu)U^{-1}(x)$$

$$= A_\mu - \frac{i}{e} U(\partial_\mu U^{-1})$$

$$= \underbrace{A_\mu + \frac{1}{ev}(\partial_\mu \zeta)}_{\text{III } \tilde{A}_\mu} \quad \begin{matrix} \text{II } e^{i\frac{\zeta(x)}{v}} \\ \partial_\mu e^{i\frac{\zeta(x)}{v}} \\ \text{II } e^{i\frac{\zeta(x)}{v}} \frac{i}{v}(\partial_\mu \zeta) \end{matrix}$$

此时, 拉氏量变为 (省略了相互作用项):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(ev)^2\tilde{A}_\mu\tilde{A}^\mu + \frac{1}{2}(\partial\eta)^2 - \lambda v^2\eta^2$$

其中 $F_{\mu\nu} \equiv \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu$

我们看到: 所有的 λ 项都不出现在拉氏量中。

这种在拉氏量中只含有物理粒子的规范称为么正规范。

- 因此, 光子(2个自由度)吸收了 would-be Goldstone 玻色子(1个自由度), 变成了有质量的矢量玻色子(3个自由度): 它的纵向极化就是 Goldstone 玻色子。
- $U(1)$ 规范对称性被自发破缺, 规范玻色子获得了质量。这就是 Higgs 机制。

标准模型电弱部分的拉氏量

$$SU(2)_L \times U(1)_Y$$

夸克: $q_L^j = \begin{pmatrix} u^j \\ d^j \end{pmatrix}_L \sim (2, \frac{1}{6})$

$$u_R^j \sim (1, \frac{2}{3})$$

$$d_R^j \sim (1, -\frac{1}{3})$$

轻子: $l_L^j = \begin{pmatrix} \nu^j \\ e^j \end{pmatrix}_L \sim (2, -\frac{1}{2})$

$$e_R^j \sim (1, -1)$$

有可能存在(尚未观测到) $N_R^j \sim (1, 0)$

Higgs二重态: $H \sim (2, \frac{1}{2})$

$$\Rightarrow H^* \sim (2^*, -\frac{1}{2})$$

$$\Rightarrow \tilde{H} \equiv i\sigma_2 H^* \sim (2, -\frac{1}{2})$$

当 Higgs 获得 VEV:

有

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix} \text{ 时,}$$

→ 给出下型费米子(d, e)
的质量,

$$\langle \tilde{H} \rangle = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}$$

→ 给出上型费米子(u)
的质量,

标准模型的拉氏量 (暂时忽略 QCD 部分) 可写为

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}$$

其中

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\begin{aligned} \mathcal{L}_{\text{fermion}} = & \bar{q}_L^j i \not{D} q_L^j + \bar{u}_R^j i \not{D} u_R^j + \bar{d}_R^j i \not{D} d_R^j \\ & + \bar{l}_L^j i \not{D} l_L^j + \bar{e}_R^j i \not{D} e_R^j \end{aligned}$$

$$\mathcal{L}_{\text{Yukawa}} = - \underbrace{y_d^{ij}}_{\substack{SU(2)_L \times U(1)_Y: \\ (\bar{2}, -\frac{1}{6})}} \underbrace{\bar{q}_L^i}_{(1, -\frac{1}{3})} \underbrace{d_R^j}_{(2, \frac{1}{2})} \underline{H} - \underbrace{y_u^{ij}}_{\substack{SU(2)_L \times U(1)_Y: \\ (\bar{2}, -\frac{1}{6})}} \underbrace{\bar{q}_L^i}_{(1, -\frac{1}{3})} \underbrace{u_R^j}_{(1, \frac{2}{3})} \tilde{\underline{H}} - \underbrace{y_e^{ij}}_{\substack{SU(2)_L \times U(1)_Y: \\ (\bar{2}, \frac{1}{2})}} \underbrace{\bar{l}_L^i}_{(1, -1)} \underbrace{e_R^j}_{(2, \frac{1}{2})} \underline{H} + h.c.$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H^\dagger)(D^\mu H) + \underbrace{\mu^2 H^\dagger H - \lambda (H^\dagger H)^2}_{-V}$$

其中 $\lambda > 0$,
 $\mu^2 > 0$.

即 Higgs potential 是

$$\begin{aligned} V &= \lambda (H^\dagger H)^2 - \mu^2 H^\dagger H \\ &= \lambda \left(H^\dagger H - \frac{\mu^2}{2\lambda} \right)^2 - \frac{\mu^4}{4\lambda} \end{aligned}$$

V 的极小值点是

$$\langle H^\dagger H \rangle = \frac{\mu^2}{2\lambda}$$

$\Rightarrow$ 不妨取

$$\boxed{\langle H \rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}}$$

在么正规范下, $H(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \left(1 + \frac{h(x)}{v}\right)$

$$\Rightarrow H^\dagger(x) = \left(1 + \frac{h(x)}{v}\right) \left(0, \frac{v}{\sqrt{2}}\right)$$

$$\Rightarrow H^\dagger H = \frac{v^2}{2} \left(1 + \frac{h}{v}\right)^2 = \frac{v^2}{2} \left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right)$$

$$\Rightarrow H^\dagger H - \frac{v^2}{2} = v h + \frac{h^2}{2}$$

$$\Rightarrow V(H) = \frac{\lambda \left(H^\dagger H - \frac{v^2}{2}\right)^2}{\lambda \left(v h + \frac{h^2}{2}\right)^2} + \text{Const.}$$

$$\lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

$$\Rightarrow \mathcal{L}_{\text{Higgs}} = (D_\mu H^\dagger)(D^\mu H) - V(H)$$

$$\text{(舍去常数项)} \quad \frac{1}{2}(\partial h)^2 - \underbrace{\lambda v^2}_{\frac{1}{2}m_h^2} h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4$$

$$= \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h^2 h^2 - \sqrt{\frac{\lambda}{2}} m_h h^3 - \frac{\lambda}{4} h^4$$

其中 $\frac{1}{2}m_h^2 = \lambda v^2 = \mu^2$

$$\Rightarrow m_h = \sqrt{2\lambda} v = \sqrt{2} \mu$$

在标准模型的 Higgs 部分的势能 $V(H)$ 实际上具有一个 $SO(4) \sim SU(2) \times SU(2)$ 的整体对称性。

可以这样看出：将 Higgs 二重态 H 参数化为

$$H = \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix} \Rightarrow H^\dagger H = h_1^2 + h_2^2 + h_3^2 + h_4^2$$

而 $V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$ 只依赖于 $H^\dagger H$,

故 $V(H)$ 具有 $SO(4)$ 对称性。

当其中一个 h_i (不妨选取为 h_3) 获得非零的 VEV 时,

这个对称性被自发破缺为 $SO(3) \sim SU(2)$, 这个未破缺的 $SU(2)$ 对称性被称为 custodial 对称性。

为了更容易看出这个 $SO(4) \sim SU(2) \times SU(2)$ 对称性，
我们构造一个 2×2 矩阵 $\Phi \equiv (\tilde{H}, H)$

其中 $H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim (2, \frac{1}{2})$ 是标准模型的 Higgs 二重态，

$$\tilde{H} \equiv i\sigma_2 H = \begin{pmatrix} h^{0*} \\ -h^- \end{pmatrix} \sim (2, -\frac{1}{2})$$

Φ 的 $SU(2)_L \times U(1)_Y$ 变换是

$$\Phi \rightarrow e^{i\frac{\sigma^a}{2}\theta_L^a} \Phi e^{-i\frac{\sigma_3}{2}\theta_Y}$$

Proof: 由于 H 和 $\tilde{H}$ 都处于 $SU(2)_L$ 的基础表示 2,
故 $\Phi = (\tilde{H}, H)$ 的 $SU(2)_L$ 变换是

$$\begin{aligned}\Phi &\rightarrow (e^{i\frac{\delta^a}{2}\theta_L^a} \tilde{H}, e^{i\frac{\delta^a}{2}\theta_L^a} H) \\ &= e^{i\frac{\delta^a}{2}\theta_L^a} (\tilde{H}, H) = e^{i\frac{\delta^a}{2}\theta_L^a} \Phi \quad \checkmark\end{aligned}$$

再来看超荷 Y : $\Phi = (\tilde{H}, H)$

超荷: $-\frac{1}{2} \quad \frac{1}{2}$

故 Φ 的 $U(1)_Y$ 变换是

$$\begin{aligned}\Phi &\rightarrow (e^{i(-\frac{1}{2})\theta_Y} \tilde{H}, e^{i\frac{1}{2}\theta_Y} H) = (\tilde{H}, H) \begin{pmatrix} e^{-i\frac{\delta^3}{2}\theta_Y} & \\ & e^{i\frac{1}{2}\theta_Y} \end{pmatrix} \\ &= \Phi e^{-i\frac{\delta^3}{2}\theta_Y} \quad \checkmark\end{aligned}$$

由 Φ 的 $SU(2)_L \times U(1)_Y$ 变换: $\Phi \rightarrow e^{i\frac{\sigma^a}{2}\theta_L^a} \Phi e^{-i\frac{\sigma_3}{2}\theta_Y}$,

可知 Φ 的协变导数是

$$D_\mu \Phi = \partial_\mu \Phi - i g \frac{\sigma^a}{2} W_\mu^a \Phi + \Phi i g' \frac{\sigma_3}{2} B_\mu$$

作为对比: H 的 $SU(2)_L \times U(1)_Y$ 变换是

$$H \rightarrow e^{i\frac{\sigma^a}{2}\theta_L^a} e^{i\frac{1}{2}\theta_Y} H$$

$$\Rightarrow D_\mu H = (\partial_\mu - i g \frac{\sigma^a}{2} W_\mu^a - i g' \frac{1}{2} B_\mu) H$$

通过直截了当的计算, 可推导得到:

$$\text{tr}[(D_\mu \Phi^\dagger)(D^\mu \Phi)] = 2(D_\mu H^\dagger)(D^\mu H)$$

此外, 容易证明: $\tilde{H}^+ \tilde{H} = H^+ H$.

因此,
$$\text{tr}(\Phi^\dagger \Phi) = H^+ H + \tilde{H}^+ \tilde{H} = 2 H^+ H$$

$$\begin{aligned} \Rightarrow V(H) &= \lambda (H^+ H)^2 - \mu^2 H^+ H \\ &= \frac{\lambda}{4} [\text{tr}(\Phi^\dagger \Phi)]^2 - \frac{\mu^2}{2} \text{tr}(\Phi^\dagger \Phi) \end{aligned}$$

它具有 $SU(2)_L \times SU(2)_R$ 的整体对称性。

当 H 获得 VEV 时, $\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \langle \tilde{H} \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \langle \Phi \rangle = (\langle \tilde{H} \rangle, \langle H \rangle) = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

它导致对称性被自发破缺为 $SU(2)_{L+R}$.

custodial 对称性

$\mathcal{L}_{\text{Higgs}}$ 也可用 Φ 改写为

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H^\dagger)(D^\mu H) + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2} \text{tr}[(D_\mu \Phi^\dagger)(D^\mu \Phi)] + \frac{\mu^2}{2} \text{tr}(\Phi^\dagger \Phi) - \frac{\lambda}{4} [\text{tr}(\Phi^\dagger \Phi)]^2$$

注意：这一项明显破缺了 $SU(2)_L \times SU(2)_R$ 对称性，
因为当 $SU(2)_R$ 的子群 $U(1)_Y$ 被 gauged 时，会破坏 $SU(2)_R$ 对称性。

$\langle H \rangle \neq 0$ 使电弱规范玻色子获得质量:

$$\text{由 } H \sim (2, \frac{1}{2}) \Rightarrow H^+ \sim (\bar{2}, -\frac{1}{2})$$

$$\Rightarrow D_\mu H = (\partial_\mu - ig T^a W_\mu^a - ig' Y B_\mu) H$$

$$D_\mu H^+ = H^+ (\overleftarrow{\partial}_\mu + ig T^a W_\mu^a + ig' Y B_\mu)$$

$$\text{其中 } T^a = \frac{\sigma^a}{2} \quad (a=1, 2, 3)$$

$$Y = \frac{1}{2}$$

$$\begin{aligned} \Rightarrow \mathcal{L}_{\text{Higgs}} &\supset (D_\mu H^+) (D^\mu H) = H^+ (\overleftarrow{\partial}_\mu + ig T \cdot W_\mu + ig' Y B_\mu) (\partial^\mu - ig T \cdot W^\mu - ig' Y B^\mu) H \\ &\supset H^+ (g T^a W_\mu^a + g' Y B_\mu)^2 H \end{aligned}$$

$$\Rightarrow \mathcal{L}_{\text{Higgs}} \supset \langle H^+ \rangle (g T^a W_\mu^a + g' Y B_\mu)^2 \langle H \rangle$$

将 $T^a = \frac{\sigma^a}{2}$, $Y = \frac{1}{2}$ 代入 $g T^a W_\mu^a + g' Y B_\mu$, 得

$$g T^a W^a + g' Y B = g \left[\frac{1}{2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} W_1 + \frac{1}{2} \begin{pmatrix} & -i \\ i & \end{pmatrix} W_2 + \frac{1}{2} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} W_3 \right]$$

$$+ g' \frac{1}{2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} B$$

$$= \frac{1}{2} \begin{pmatrix} g W_3 + g' B & g(W_1 - i W_2) \\ g(W_1 + i W_2) & -g W_3 + g' B \end{pmatrix}$$

$$\Rightarrow (g T^a W^a + g' Y B)^2 = \frac{1}{4} \begin{pmatrix} * & * \\ * & g^2(W_1^2 + W_2^2) + (g W_3 - g' B)^2 \end{pmatrix}$$

$$\text{而 } \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow \langle H^+ \rangle = \frac{1}{\sqrt{2}} (0 \quad v)$$

$$\Rightarrow \mathcal{L}_{\text{Higgs}} \supset \langle H^+ \rangle (g T^a W^a + g' Y B)^2 \langle H \rangle$$

$$= \frac{1}{\sqrt{2}} (0 \quad v) \frac{1}{4} \begin{pmatrix} * & * \\ * & g^2(W_1^2 + W_2^2) + (gW_3 - g'B)^2 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v^2}{8} \left[g^2(W_1^2 + W_2^2) + \underline{(gW_3 - g'B)^2} \right]$$

$$g^2 W_3^2 + g'^2 B^2 - 2gg'W_3B$$

$$= \frac{1}{2} (W_1 \quad W_2 \quad W_3 \quad B) \underbrace{\left(\frac{v^2}{4} \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix} \right)}_{M_{\text{gauge}}^2} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

也即是说, $\mathcal{L}_{\text{Higgs}} \supset \underbrace{\frac{1}{2} \frac{(g_V)^2}{4} (W_1^2 + W_2^2)}_{\textcircled{1}} + \underbrace{\frac{1}{2} \frac{V^2}{4} (gW_3 - g'B)^2}_{\textcircled{2}}$

定义 $W^\pm \equiv \frac{1}{\sqrt{2}} (W_1 \mp iW_2) \Rightarrow W^+ W^- = \frac{1}{2} (W_1^2 + W_2^2)$

$\Rightarrow \textcircled{1} = \underbrace{\left(\frac{(g_V)^2}{4} \right)}_{\equiv M_W^2} W^+ W^-$

另一方面, $\textcircled{2} = \frac{1}{2} \frac{V^2}{4} (gW_3 - g'B)^2$

$= \frac{1}{2} \frac{(g^2 + g'^2) V^2}{4} \left(\underbrace{\left(\frac{g}{\sqrt{g^2 + g'^2}} \right)}_{\equiv C_W} W_3 - \underbrace{\left(\frac{g'}{\sqrt{g^2 + g'^2}} \right)}_{\equiv S_W} B \right)^2$

$= \frac{1}{2} \underbrace{\left(\frac{(g^2 + g'^2) V^2}{4} \right)}_{\equiv M_Z^2} \underbrace{\left(C_W W_3 - S_W B \right)}_{\equiv Z}^2 = \frac{1}{2} M_Z^2 Z^2$

这里将 Weinberg 角 θ_w 定义为

$$\cos \theta_w \equiv c_w \equiv \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_w \equiv s_w \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$$

Z_μ 被定义为

$$Z_\mu \equiv c_w W_{3\mu} - s_w B_\mu$$

与 Z_μ 正交的方向是

$$A_\mu \equiv c_w B_\mu + s_w W_{3\mu}$$

也即是说

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

• W_1, W_2 既是规范本征态, 也是质量本征态, 但不是电荷本征态。

• 电荷算符 $Q = T_3 + Y$

对于规范场 $W^a \sim (3, 0)$, 其电荷 $Q = T_3$

$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}$ 处于 $SU(2)_L$ 的矢量表示, $T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

↓ 尚未对角化

T_3 的本征值及对应的本征矢量是

$$\lambda_+ = +1, \quad N_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

$$\lambda_- = -1, \quad N_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$\lambda_0 = 0, \quad N_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{令 } S \equiv (V_{+1}, V_{-1}, V_0) = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow S^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{ 此时有 } S^{-1} T_3 S = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

现在, 作表象变换:

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \rightarrow S^{-1} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{W_1 - iW_2}{\sqrt{2}} \\ \frac{W_1 + iW_2}{\sqrt{2}} \\ W_3 \end{pmatrix} \equiv \begin{pmatrix} W^+ \text{ (电荷为 } +1 \text{)} \\ W^- \text{ (电荷为 } -1 \text{)} \\ W_3 \end{pmatrix}$$

$$\left. \begin{aligned} \text{由 } M_W^2 &= \frac{g^2 v^2}{4} \\ M_Z^2 &= \frac{(g^2 + g'^2) v^2}{4} \end{aligned} \right\} \Rightarrow \frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2} = c_W^2$$

$$\Rightarrow \rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1$$

我们来考察 h 与 $W^\pm, Z$ 的相互作用.

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &\supset (D_\mu H^\dagger)(D^\mu H) \\ &= H^\dagger (\overleftarrow{\partial}_\mu + ig T^a W_\mu^a + ig' Y B_\mu) (\partial^\mu - ig T^b W^{b\mu} - ig' Y B^\mu) H \\ &= (\partial_\mu H^\dagger)(\partial^\mu H) + \underbrace{H^\dagger (g T^a W_\mu^a + g' Y B_\mu)^2 H}_{\text{a}} \\ &\quad + \underbrace{(g T^a W_\mu^a + g' Y B_\mu)_{ij} H_i^\dagger i (\overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu) H_j}_{\text{b}} \end{aligned}$$

$$\text{由 } H(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow \partial_\mu H = \begin{pmatrix} 0 \\ \frac{\partial_\mu h}{\sqrt{2}} \end{pmatrix}$$

$$H^\dagger(x) = \left(0, \frac{v+h(x)}{\sqrt{2}} \right) \Rightarrow \partial_\mu H^\dagger = \left(0, \frac{\partial_\mu h}{\sqrt{2}} \right)$$

$$\Rightarrow H_i^\dagger i(\vec{\partial}^\mu - \vec{\partial}^\mu) H_j = \delta_{i2} \delta_{j2} i \left[\underbrace{H_2^\dagger (\partial^\mu H_2) - (\partial^\mu H_2^\dagger) H_2}_{=0} \right]$$

$$\Rightarrow \textcircled{b} = 0 \quad = 0 \quad \frac{(v+h)}{\sqrt{2}} \frac{(\partial^\mu h)}{\sqrt{2}} - \frac{(\partial^\mu h)}{\sqrt{2}} \frac{(v+h)}{\sqrt{2}} = 0$$

另一方面, $\textcircled{a} = H^\dagger \left(g \frac{\sigma^a}{2} W^a + g' \frac{1}{2} B \right)^2 H$

$$\begin{aligned} (M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 Z^2) &= \left(0, \frac{v+h}{\sqrt{2}} \right) \left(g \frac{\sigma^a}{2} W^a + g' \frac{1}{2} B \right)^2 \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \\ &= \left(\left(0, \frac{v}{\sqrt{2}} \right) \left(g \frac{\sigma^a}{2} W^a + g' \frac{1}{2} B \right)^2 \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right) \cdot \left(1 + \frac{h}{v} \right)^2 \end{aligned}$$

$$\Rightarrow \textcircled{a} = (M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 Z^2) \underbrace{\left(1 + \frac{h}{v}\right)^2}_{\parallel \left(1 + \frac{2h}{v} + \frac{h^2}{v^2}\right)}$$

以上就是在标准模型中, h 与 W, Z 的相互作用。
现在来看 $\mathcal{L}_{\text{Yukawa}}$:

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &\supset -y_d^{ij} \bar{q}_L^i d_R^j \underbrace{\langle H \rangle}_{\parallel \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} - y_u^{ij} \bar{q}_L^i u_R^j \underbrace{\langle \tilde{H} \rangle}_{\parallel \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} - y_e^{ij} \bar{l}_L^i e_R^j \underbrace{\langle H \rangle}_{\parallel \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} + h.c. \\ &= -\underbrace{\left(y_d^{ij} \frac{v}{\sqrt{2}}\right)}_{\parallel M_{ij}^{(d)}} \bar{d}_L^i d_R^j - \underbrace{\left(y_u^{ij} \frac{v}{\sqrt{2}}\right)}_{\parallel M_{ij}^{(u)}} \bar{u}_L^i u_R^j - \underbrace{\left(y_e^{ij} \frac{v}{\sqrt{2}}\right)}_{\parallel M_{ij}^{(e)}} \bar{e}_L^i e_R^j + h.c. \end{aligned}$$

这些费米子质量矩阵可以用双么正变换对角化:

$$M_{\text{diag}}^{(u)} = V_L^{(u)} M^{(u)} (V_R^{(u)})^\dagger,$$

$$M_{\text{diag}}^{(e)} = V_L^{(e)} M^{(e)} (V_R^{(e)})^\dagger,$$

$$M_{\text{diag}}^{(d)} = V_L^{(d)} M^{(d)} (V_R^{(d)})^\dagger,$$

费米子质量本征态是

$$f_L^{\text{mass}} = V_L^{(f)} f_L$$

$$f_R^{\text{mass}} = V_R^{(f)} f_R$$

例如:

$$\mathcal{L} = - \underbrace{(\bar{u}_L)}_{\text{}} M^{(u)} \underbrace{(u_R)}_{\text{}} + \text{h.c.}$$

$$\bar{u}_L^{\text{mass}} V_L^{(u)} \quad (V_R^{(u)})^+ u_R^{\text{mass}}$$

$$= - \bar{u}_L^{\text{mass}} \underbrace{V_L^{(u)} M^{(u)} (V_R^{(u)})^+}_{\text{}} u_R^{\text{mass}} + \text{h.c.}$$

$$M_{\text{diag}}^{(u)} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\begin{aligned}
 \Rightarrow \mathcal{L} \supset & \underbrace{-m_u (\bar{u}_L^{\text{mass}} u_R^{\text{mass}} + \bar{u}_R^{\text{mass}} u_L^{\text{mass}})}_{\bar{u}^{\text{mass}} u^{\text{mass}}} \\
 & -m_c (\bar{c}_L^{\text{mass}} c_R^{\text{mass}} + \bar{c}_R^{\text{mass}} c_L^{\text{mass}}) \\
 & \underbrace{-m_t (\bar{t}_L^{\text{mass}} t_R^{\text{mass}} + \bar{t}_R^{\text{mass}} t_L^{\text{mass}})}_{\bar{t}^{\text{mass}} t^{\text{mass}}}
 \end{aligned}$$

同理,有

$$\begin{aligned}
 \mathcal{L} \supset & -m_d \bar{d}d - m_s \bar{s}s - m_b \bar{b}b \\
 & -m_e \bar{e}e - m_\mu \bar{\mu}\mu - m_\tau \bar{\tau}\tau
 \end{aligned}$$

← (这里都是质量本征态, 我们省略了上标 mass.)

还可以求出 h 与费米子的耦合:

$$\text{在么正规范, } H(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow \tilde{H}(x) = \begin{pmatrix} \frac{v+h(x)}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &\supset - y_d^{ij} \bar{q}_L^i d_R^j \underbrace{\left(\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \right)}_{\parallel} - y_u^{ij} \bar{q}_L^i u_R^j \underbrace{\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)}_{\parallel} - y_e^{ij} \bar{l}_L^i e_R^j \underbrace{\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)}_{\parallel} + \text{h.c.} \\ &= - (m_u \bar{u} u + m_c \bar{c} c + m_t \bar{t} t + m_d \bar{d} d + m_s \bar{s} s + m_b \bar{b} b \\ &\quad + m_e \bar{e} e + m_\mu \bar{\mu} \mu + m_\tau \bar{\tau} \tau) \left(1 + \frac{h}{v} \right) \end{aligned}$$

在 $\mathcal{L}_{\text{fermion}}$ 中包含带电流相互作用。

$$\begin{aligned}\mathcal{L}_{\text{fermion}} &\supset \bar{q}_L i \cancel{\partial} q_L + \bar{l}_L i \cancel{\partial} l_L \\&= \bar{q}_L i \gamma^\mu (\partial_\mu - i g T^a W_\mu^a - i g' Y B_\mu) q_L \\&\quad + \bar{l}_L i \gamma^\mu (\partial_\mu - i g T^a W_\mu^a - i g' Y B_\mu) l_L \\&\supset \bar{q}_L \gamma^\mu (g T^a W_\mu^a + g' Y B_\mu) q_L \\&\quad + \bar{l}_L \gamma^\mu (g T^a W_\mu^a + g' Y B_\mu) l_L\end{aligned}$$

容易求得: $gT^a W^a + g' Y B$

$$= g(T^1 W^1 + T^2 W^2) + gT^3 W^3 + g' Y B$$

$$\equiv \frac{1}{\sqrt{2}}(T^+ W^+ + T^- W^-)$$

$$\text{其中 } T^\pm \equiv T^1 \pm iT^2$$

$$W^\pm \equiv \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

$$\equiv eQA + \frac{e}{cs}(T^3 - s^2 Q)Z$$

$$\text{其中 } s \equiv \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$c \equiv \cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$Q \equiv T^3 + Y$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} \equiv \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

因此, $\mathcal{L}_{\text{fermion}} \supset \bar{g}_L \gamma^\mu g (T^1 W'_\mu + T^2 W''_\mu) g_L$
 $+ \bar{l}_L \gamma^\mu g (T^1 W'_\mu + T^2 W''_\mu) l_L$
 $= \bar{g}_L \gamma^\mu \frac{g}{\sqrt{2}} (T^+ W_\mu^+ + T^- W_\mu^-) g_L$
 $+ \bar{l}_L \gamma^\mu \frac{g}{\sqrt{2}} (T^+ W_\mu^+ + T^- W_\mu^-) l_L \equiv \mathcal{L}_{\text{cc}}$

对于 $T^a = \frac{\sigma^a}{2}$, 有

$$T^+ \equiv T^1 + iT^2 = \frac{\sigma^1}{2} + i\frac{\sigma^2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- \equiv T^1 - iT^2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow T^+ W^+ + T^- W^- = \begin{pmatrix} & W^+ \\ W^- & \end{pmatrix}$$

$$\begin{aligned}
\Rightarrow \mathcal{L}_{cc} &= \frac{g}{\sqrt{2}} (\bar{u}_L \quad \bar{d}_L) \gamma^\mu \begin{pmatrix} W^+ \\ W^- \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\
&\quad + \frac{g}{\sqrt{2}} (\bar{\nu}_L \quad \bar{e}_L) \gamma^\mu \begin{pmatrix} W^+ \\ W^- \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \\
&= \frac{g}{\sqrt{2}} \left[(\bar{u}_L \gamma^\mu W_\mu^+ d_L + \bar{d}_L \gamma^\mu W_\mu^- u_L) \right. \\
&\quad \left. + (\bar{\nu}_L \gamma^\mu W_\mu^+ e_L + \bar{e}_L \gamma^\mu W_\mu^- \nu_L) \right] \\
&= \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^\mu W_\mu^+ d_L + \bar{e}_L \gamma^\mu W_\mu^- \nu_L) + h.c.
\end{aligned}$$

即

$$\begin{aligned}
 \mathcal{L}_{cc} &= \frac{g}{\sqrt{2}} \left(\underbrace{(\overline{u}_L)}_{\overline{u}_L^{\text{mass}} V_L^{(u)}} \gamma^\mu W_\mu^+ \underbrace{(d_L)}_{(V_L^{(d)})^\dagger d_L^{\text{mass}}} + \underbrace{(\overline{e}_L)}_{\overline{e}_L^{\text{mass}} V_L^{(e)}} \gamma^\mu W_\mu^- \underbrace{(\nu_L)}_{(V_L^{(\nu)})^\dagger \nu_L^{\text{mass}}} \right) + h.c. \\
 &= \frac{g}{\sqrt{2}} \left[\overline{u}_L^{\text{mass}} \gamma^\mu W_\mu^+ \underbrace{V_L^{(u)} (V_L^{(d)})^\dagger}_{V_{CKM}} d_L^{\text{mass}} \right. \\
 &\quad \left. + \overline{e}_L^{\text{mass}} \gamma^\mu W_\mu^- \underbrace{V_L^{(e)} (V_L^{(\nu)})^\dagger}_{V_{PMNS}} \nu_L^{\text{mass}} \right] + h.c.
 \end{aligned}$$

显式地, 有 (以下省略质量本征态的上标 mass),

$$\mathcal{L}_c = \frac{g}{\sqrt{2}} (\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L) \gamma^\mu W_\mu^+ V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

$$+ \frac{g}{\sqrt{2}} (\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L) \gamma^\mu W_\mu^- V_{PMNS} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + h.c.$$

再来看 $\mathcal{L}_{\text{fermion}}$ 中的中性流相互作用。

$$\mathcal{L}_{\text{fermion}} \supset e A_\mu \sum_f \bar{f} \gamma^\mu Q f + \frac{e}{c s} Z_\mu \sum_f \bar{f} \gamma^\mu \left(T_f^3 \overset{\frac{1-\gamma_5}{2}}{\textcircled{P_L}} - s^2 Q \right) f$$

|||

$\mathcal{L}_{\text{NC}}$

显式地, $\mathcal{L}_{\text{NC}}$ 中的电磁流相互作用是

$$\mathcal{L}_{\text{NC}} \supset e A_\mu \sum_f \bar{f} \gamma^\mu Q f$$

$$= e A_\mu \left(\frac{2}{3} \bar{u}^j \gamma^\mu u^j - \frac{1}{3} \bar{d}^j \gamma^\mu d^j - \bar{e}^j \gamma^\mu e^j \right)$$

$\mathcal{L}_{NC}$ 中的 Z_μ 流相互作用是

$$\mathcal{L}_{NC} \supset \frac{e}{cs} Z_\mu \sum_f \bar{f} \gamma^\mu (T_f^3 P_L - s^2 Q) f$$

$$\begin{aligned} = & \frac{e}{cs} Z_\mu \left[\bar{u}_L^j \gamma^\mu \left(\frac{1}{2} - \frac{2}{3}s^2 \right) u_L^j + \bar{u}_R^j \gamma^\mu \left(-\frac{2}{3}s^2 \right) u_R^j \right. \\ & + \bar{d}_L^j \gamma^\mu \left(-\frac{1}{2} + \frac{1}{3}s^2 \right) d_L^j + \bar{d}_R^j \gamma^\mu \left(\frac{1}{3}s^2 \right) d_R^j \\ & + \bar{\nu}_L^j \gamma^\mu \frac{1}{2} \nu_L^j \\ & \left. + \bar{e}_L^j \gamma^\mu \left(-\frac{1}{2} + s^2 \right) e_L^j + \bar{e}_R^j \gamma^\mu s^2 e_R^j \right] \end{aligned}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$= -\frac{1}{4} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c) (\partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g \epsilon^{ade} W^{\mu d} W^{e\nu})$$

$$- \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

其中的二次项是

$$(\text{quadratic term}) = \left(-\frac{1}{4} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) (\partial^\mu W^{a\nu} - \partial^\nu W^{a\mu}) \right)$$

$$- \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

$$\begin{aligned} & \text{“} \frac{1}{2} W_\mu^a \delta^{ab} (\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) W_\nu^b \\ & \text{“} \frac{1}{2} B_\mu (\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) B_\nu \end{aligned}$$

$$\text{由 } W^\pm \equiv \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \Rightarrow (W^1)^2 + (W^2)^2 = W^+ W^-$$

$$\begin{pmatrix} W^3 \\ B \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \Rightarrow (W^3)^2 + B^2 = Z^2 + A^2$$

因此, 二次项

$$\begin{aligned} (\text{quadratic term}) &= -\frac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\ &= \frac{1}{2}W_\mu^a \delta^{ab}(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)W_\nu^b + \frac{1}{2}B_\mu(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)B_\nu \end{aligned}$$

(还可用质量本征态写为)

$$\begin{aligned} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ &\quad -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 \\ &= W_\mu^+(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)W_\nu^- + \frac{1}{2}Z_\mu(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)Z_\nu + \frac{1}{2}A_\mu(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)A_\nu \end{aligned}$$

三次项是

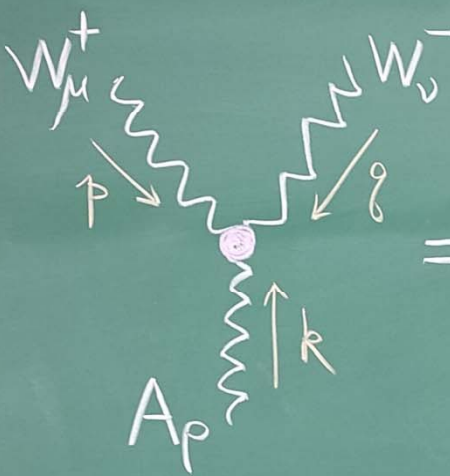
$$\begin{aligned}(\text{cubic term}) &= -\frac{1}{4} 2 (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) g \epsilon^{abc} W^{b\mu} W^{c\nu} \\ &= -\frac{1}{2} g \epsilon^{abc} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) W^{b\mu} W^{c\nu}\end{aligned}$$

(还可质量本征态写为)

$$\begin{aligned}&= ie \left[(\partial_\mu A_\nu - \partial_\nu A_\mu) W^{+\mu} W^{-\nu} + (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) W^{-\mu} A^\nu \right. \\ &\quad \left. + (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) A^\mu W^{+\nu} \right] \\ &\quad + igc \left[(\partial_\mu Z_\nu - \partial_\nu Z_\mu) W^{+\mu} W^{-\nu} + (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) W^{-\mu} Z^\nu \right. \\ &\quad \left. + (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) Z^\mu W^{+\nu} \right]\end{aligned}$$

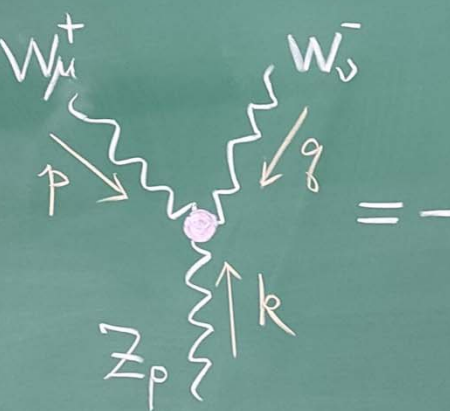
下面来看划一线和划~线部分给出的Feynman规则。

划—线部分 $\Rightarrow$



$$= -ie \left[\eta^{\mu\nu} (p-q)^\rho + \eta^{\nu\rho} (q-k)^\mu + \eta^{\rho\mu} (k-p)^\nu \right]$$

划~线部分 $\Rightarrow$



$$= -igc \left[\eta^{\mu\nu} (p-q)^\rho + \eta^{\nu\rho} (q-k)^\mu + \eta^{\rho\mu} (k-p)^\nu \right]$$

四次项是

$$(\text{quartic term}) = -\frac{1}{4}g^2 \epsilon^{abc} \epsilon^{ade} W_\mu^b W_\nu^c W^{d\mu} W^{e\nu}$$

(还可质量本征态写为)

$$\begin{aligned} &= g^2 c^2 [(W^+ \cdot Z)(W^- \cdot Z) - (W^+ \cdot W^-)(Z \cdot Z)] \\ &\quad + g^2 s^2 [(W^+ \cdot A)(W^- \cdot A) - (W^+ \cdot W^-)(A \cdot A)] \\ &\quad + g^2 cs [(W^+ \cdot Z)(W^- \cdot A) + (W^+ \cdot A)(W^- \cdot Z) - 2(W^+ \cdot W^-)(Z \cdot A)] \\ &\quad + \frac{g^2}{2} [(W^+)^2 (W^-)^2 - (W^+ \cdot W^-)^2] \end{aligned}$$

其中 $W^+ \cdot Z \equiv W_\mu^+ Z^\mu$.

第1行

$$\Rightarrow \begin{array}{c} W_\mu^+ \\ \diagdown \\ \text{---} \bullet \text{---} \\ \diagup \\ W_\nu^- \\ \diagdown \\ Z_\rho \\ \diagup \\ Z_\delta \end{array} = -ig^2 c^2 \underbrace{S^{\mu\nu,\rho\delta}}_{\text{III}}$$

第2行

$$\Rightarrow \begin{array}{c} W_\mu^+ \\ \diagdown \\ \text{---} \bullet \text{---} \\ \diagup \\ W_\nu^- \\ \diagdown \\ A_\rho \\ \diagup \\ A_\delta \end{array} = -i \underbrace{g^2 s^2}_{e^2} S^{\mu\nu,\rho\delta}$$

$$(2\eta^{\mu\nu}\eta^{\rho\delta} - \eta^{\mu\rho}\eta^{\nu\delta} - \eta^{\mu\delta}\eta^{\nu\rho})$$

第3行

$$\Rightarrow \begin{array}{c} W_\mu^+ \\ \diagdown \\ \text{---} \bullet \text{---} \\ \diagup \\ W_\nu^- \\ \diagdown \\ A_\rho \\ \diagup \\ Z_\delta \end{array} = -ig^2 cs S^{\mu\nu,\rho\delta}$$

第4行

$$\Rightarrow \begin{array}{c} W_\mu^+ \\ \diagdown \\ \text{---} \bullet \text{---} \\ \diagup \\ W_\nu^+ \\ \diagdown \\ W_\rho^- \\ \diagup \\ W_\delta^- \end{array} = ig^2 S^{\mu\nu,\rho\delta}$$

电弱精确测量参量 S 、 T 、 U

Reference:

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Estimation of oblique electroweak corrections

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We review the general analysis of the contributions of electroweak vacuum-polarization diagrams to precision experiments. We first review the representation of these contributions by three parameters S , T , and U and discuss the assumptions involved in this reduction. We then discuss the contributions to these parameters from various models of new physics. We show that S can be computed by a dispersion relation, and we use this technique to estimate S in technicolor models of the Higgs sector. We discuss the reliability and the gauge invariance of this estimate. Finally, we present the limits on S and T imposed by current experimental results.

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<https://journals.aps.org/prd/abstract/10.1103/PhysRevD.46.381>

设与电弱规范玻色子 W_μ^a, B_μ 耦合的流分别是 J_a^μ, J_Y^μ , 即

$$\mathcal{L} \supset g W_\mu^a J_a^\mu + g' B_\mu J_Y^\mu$$

上式还可表达为

$$\mathcal{L} \supset \frac{e}{\sqrt{2}s} (W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu) + \frac{e}{cs} Z_\mu (J_3^\mu - s^2 J_Q^\mu) + e A_\mu J_Q^\mu$$

其中

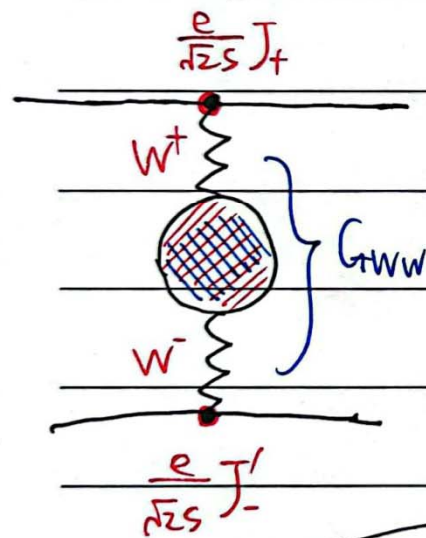
$$J_\pm^\mu \equiv J_1^\mu \pm i J_2^\mu, \quad W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

$$J_Q^\mu \equiv J_3^\mu + J_Y^\mu, \quad gs = g'c \equiv e, \quad s \equiv \sin \theta_W, \quad c \equiv \cos \theta_W$$

$$\begin{pmatrix} W^3 \\ B \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \Leftrightarrow \begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

将规范玻色子传播子的 $g^{\mu\nu}$ 系数分别记为 G_{AA}, G_{ZZ}, G_{WW} ,
 A, Z, W 的 将 A-Z 混合的 $g^{\mu\nu}$ 系数记为 $G_{AZ} = G_{ZA}$.

不难看出, 由 $W^\pm$ 传递的带电流相互作用的振幅是



$$M_{cc} = \frac{e}{\sqrt{2}s} J_+ G_{WW} \frac{e}{\sqrt{2}s} J'_-$$

$$= \frac{e^2}{2s^2} J_+ J'_- G_{WW}$$

由 A, Z 传递的中性流相互作用的振幅是

$$\frac{eJ_Q}{A} \frac{eJ'_Q}{A} G_{AA} + \frac{\frac{e}{c}(J_3 - s^2 J_Q)}{Z} \frac{\frac{e}{c}(J'_3 - s^2 J'_Q)}{Z} G_{ZZ} + \frac{eJ_Q}{A} \frac{\frac{e}{c}(J'_3 - s^2 J'_Q)}{Z} G_{AZ} + \frac{\frac{e}{c}(J_3 - s^2 J_Q)}{Z} \frac{eJ'_Q}{A} G_{ZA}$$

$$M_{nc} = e^2 J_Q J'_Q G_{AA} + \frac{e^2}{c^2 s^2} (J_3 - s^2 J_Q) (J'_3 - s^2 J'_Q) G_{ZZ} + \frac{e^2}{cs} [J_Q (J'_3 - s^2 J'_Q) + (J_3 - s^2 J_Q) J'_Q] G_{AZ}$$

在树图水平, A与Z之间没有混合: $G_{AZ} = G_{ZA} = 0$

13

其它三个传播子是

$$D_{BB} = \frac{1}{q^2 - M_{B(0)}^2}$$

其中 $B = A, Z, W$, 在传播子中的裸质量是

$$M_{A(0)}^2 = 0, \quad M_{Z(0)}^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4}$$

$$M_{W(0)}^2 = \frac{e^2}{s^2} \frac{v^2}{4}$$

此时显然有 $\rho \equiv \frac{M_{W(0)}^2}{M_{Z(0)}^2 c^2} = 1$

真空极化会改变规范玻色子传播子 G_{AA} , G_{AZ} , G_{ZZ} 和 G_{WW} , 从而会改变前面的带电流振幅 M_{CC} 和中性流振幅 M_{NC} .

- 因为真空极化不会直接改变 M_{cc} , M_{nc} 的形式, 故它们被称为间接 (oblique) 修正。
- 真空极化振幅被定义为

$$\begin{aligned} \text{Diagram: } \mu \text{ wavy line} \rightarrow \text{circle with cross-hatch} \rightarrow \nu \text{ wavy line} &= i\pi_{XY}^{\mu\nu}(q^2) = \int d^4x e^{-iqx} \langle J_X^\mu(x) J_Y^\nu(0) \rangle \\ &= i g^{\mu\nu} \pi_{XY}(q^2) + (g^\mu g^\nu \text{ terms}) \end{aligned}$$

其中 $(XY) = (11), (22), (33), (3Q), (QQ)$.

- 我们定义 $\pi'_{XY}(q^2)$ 为

$$\pi_{XY}(q^2) = \pi_{XY}(0) + q^2 \pi'_{XY}(q^2)$$

显然有, $\pi'_{XY}(0) = \left. \frac{d\pi_{XY}}{dq^2} \right|_{q^2=0}$. 但在其它点 $q^2 \neq 0$, $\pi'_{XY}(q^2)$ 并不等于 $\frac{d\pi_{XY}(q^2)}{dq^2}$.

由未破缺的 $U(1)_{em} (= SO(2))$ 对称性知

5

$$\pi_{11}(q^2) = \pi_{22}(q^2)$$

由 QED 的 Ward 恒等式 $\partial_\mu J_Q^\mu(x) = 0$, 可得

$$\pi_{3Q}(0) = \pi_{QQ}(0) = 0$$

Proof: $\langle J_X^\mu(x) J_Y^\nu(0) \rangle = \int \frac{d^4 q}{(2\pi)^4} e^{i q x} i \pi_{XY}^{\mu\nu}(q^2)$

$$\partial_\mu J_Q^\mu(x) = 0 \Rightarrow \partial_\mu \langle J_Q^\mu(x) J_*^\nu(0) \rangle = 0$$

$$\Rightarrow 0 = \partial_\mu \pi_{Q*}^{\mu\nu}(q^2) = \partial_\mu [q^{\mu\nu} \pi_{Q*}(q^2) + (q^\mu q^\nu \text{ terms})]$$

$$= q^\nu \pi_{Q*}(q^2) + (q^2 q^\nu \text{ terms})$$

现在, 取 $q^2 = 0$, 立得

$$\parallel$$
$$[\pi_{Q*}(0) + q^2 \pi'_{Q*}(q^2)]$$

$$\pi_{Q*}(0) = 0$$

✓

光子、W和Z的单粒子不可约的(1PI)自能, 以及1PI的光子-Z混合
都可用前面定义的 π_{XY} 的组合表达:

$$\pi_{AA} = e^2 \pi_{QQ}, \quad \pi_{WW} = \frac{e^2}{s^2} \pi_{11}$$

$$\pi_{ZZ} = \frac{e^2}{s^2 c^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})$$

$$\pi_{ZA} = \frac{e^2}{sc} (\pi_{3Q} - s^2 \pi_{QQ})$$

Proof: 由 $\mathcal{L} \supset e A_\mu J_Q^\mu \equiv A_\mu J_A^\mu \Rightarrow J_A^\mu = e J_Q^\mu$

$$\Rightarrow \langle J_A J_A \rangle = e^2 \langle J_Q J_Q \rangle$$

$$\Rightarrow \pi_{AA} = e^2 \pi_{QQ} \quad \checkmark$$

由 $\mathcal{L} \supset \frac{e}{\sqrt{2}s} (W^+ J^+ + W^- J^-) \Rightarrow J_{W^+} = \frac{e}{\sqrt{2}s} J^+, J_{W^-} = \frac{e}{\sqrt{2}s} J^-$

$$\Rightarrow \langle J_{W^+} J_{W^-} \rangle = \frac{e^2}{2s^2} \langle J^+ J^- \rangle = \frac{e^2}{2s^2} \langle (J_1 + iJ_2)(J_1 - iJ_2) \rangle$$

$$\Rightarrow \langle J_{W^+} J_{W^-} \rangle = \frac{e^2}{2s^2} \left(\underbrace{\langle J_1 J_1 \rangle}_{\text{"} \langle J_1 J_1 \rangle \text{"}} + \underbrace{\langle J_2 J_2 \rangle}_{\text{"} 0 \text{"}} + i(\langle J_2 J_1 \rangle - \langle J_1 J_2 \rangle) \right)$$

$$= \frac{e^2}{s^2} \langle J_1 J_1 \rangle$$

$$\Rightarrow \pi_{WW} = \frac{e^2}{s^2} \pi_{11} \quad \checkmark$$

$$\text{由 } \mathcal{L} \supset \frac{e}{cs} \sum_{\mu} (J_3^{\mu} - s^2 J_Q^{\mu}) \Rightarrow J_Z = \frac{e}{cs} (J_3 - s^2 J_Q)$$

$$\Rightarrow \langle J_Z J_Z \rangle = \frac{e^2}{c^2 s^2} \underbrace{\langle (J_3 - s^2 J_Q)(J_3 - s^2 J_Q) \rangle}_{\text{"} (\langle J_3 J_3 \rangle - 2s^2 \langle J_3 J_Q \rangle + s^4 \langle J_Q J_Q \rangle) \text{"}}$$

$$\Rightarrow \pi_{ZZ} = \frac{e^2}{c^2 s^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ}) \quad \checkmark$$

$$\text{另一方面, } \langle J_Z J_A \rangle = \frac{e^2}{cs} \langle (J_3 - s^2 J_Q) J_Q \rangle = \frac{e^2}{cs} (\langle J_3 J_Q \rangle - s^2 \langle J_Q J_Q \rangle)$$

$$\Rightarrow \pi_{ZA} = \frac{e^2}{cs} (\pi_{3Q} - s^2 \pi_{QQ}) \quad \checkmark$$

我们可以用 Dyson 方程将真空极化振幅的所有阶都考虑进来。

先来看 $W^\pm$ 的完全传播子 G_{WW} , 由 Dyson 方程:

$$G_{WW} = D_{WW} + \underbrace{D_{WW} \Pi_{WW} D_{WW} + D_{WW} \Pi_{WW} D_{WW} \Pi_{WW} D_{WW} + \dots}_{\text{二}}$$

$$D_{WW} \Pi_{WW} (\underbrace{D_{WW} + D_{WW} \Pi_{WW} D_{WW} + \dots}_{\text{二}})$$

|| G_{WW}

即 $G_{WW} = D_{WW} + D_{WW} \Pi_{WW} G_{WW}$

$$\Rightarrow (1 - D_{WW} \Pi_{WW}) G_{WW} = D_{WW}$$

$$\Rightarrow G_{WW} = \frac{D_{WW}}{1 - D_{WW} \Pi_{WW}} = \frac{\frac{1}{q^2 - M_{W(0)}^2}}{1 - \frac{\Pi_{WW}}{q^2 - M_{W(0)}^2}} = \frac{1}{q^2 - M_{W(0)}^2 - \Pi_{WW}}$$

$$\Rightarrow M_{cc} = \frac{e^2}{2s^2} J_+ J_-' G_{WW} = \frac{e^2}{2s^2} \frac{J_+ J_-'}{q^2 - M_{W(0)}^2 - \Pi_{WW}}$$

再来求解 A, Z 的完全传播子 $G_{AA}, G_{ZZ}, G_{AZ} = G_{ZA}$.

由于 A, Z 在圈图水平存在混合: $\pi_{AZ} = \pi_{ZA} \neq 0$, 一个方便的做法是用矩阵来求解:

设 (A, Z) 完全传播子矩阵为 $\hat{G} \equiv \begin{pmatrix} G_{AA} & G_{AZ} \\ G_{ZA} & G_{ZZ} \end{pmatrix}$,

则它的逆可用真空极化表达为

$$\hat{G}^{-1} = \begin{pmatrix} \textcircled{D_{AA}^{-1}} - \pi_{AA} & -\pi_{AZ} \\ -\pi_{ZA} & \textcircled{D_{ZZ}^{-1}} - \pi_{ZZ} \end{pmatrix}$$

2x2 矩阵 $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 的逆是

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} g^2 - \pi_{AA} & -\pi_{AZ} \\ -\pi_{ZA} & g^2 - M_{Z(0)}^2 - \pi_{ZZ} \end{pmatrix}$$

$$\Rightarrow \hat{G} = \frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \begin{pmatrix} g^2 - M_{Z(0)}^2 - \pi_{ZZ} & \pi_{AZ} \\ \pi_{ZA} & g^2 - \pi_{AA} \end{pmatrix}$$

对比可得

$$\begin{cases} G_{TAA} = \frac{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \\ G_{TZZ} = \frac{g^2 - \pi_{AA}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \\ G_{TAZ} = G_{TZA} = \frac{\pi_{AZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \end{cases}$$

$$\Rightarrow M_{NC} = (eJ_Q, \frac{e}{cs}(J_3 - s^2 J_Q)) \begin{pmatrix} G_{TAA} & G_{TAZ} \\ G_{TAZ} & G_{TZZ} \end{pmatrix} \begin{pmatrix} eJ'_Q \\ \frac{e}{cs}(J'_3 - s^2 J'_Q) \end{pmatrix}$$

$$= \frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} (eJ_Q, \frac{e}{cs}(J_3 - s^2 J_Q)) \begin{pmatrix} g^2 - M_{Z(0)}^2 - \pi_{ZZ} & \pi_{AZ} \\ \pi_{AZ} & g^2 - \pi_{AA} \end{pmatrix} \begin{pmatrix} eJ'_Q \\ \frac{e}{cs}(J'_3 - s^2 J'_Q) \end{pmatrix}$$

$$= \frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \left[e^2 J_Q J'_Q (g^2 - M_{Z(0)}^2 - \pi_{ZZ}) + \frac{e^2}{c^2 s^2} (J_3 - s^2 J_Q)(J'_3 - s^2 J'_Q)(g^2 - \pi_{AA}) \right. \\ \left. + \frac{e}{cs} [J_Q(J'_3 - s^2 J'_Q) + (J_3 - s^2 J_Q)J'_Q] \pi_{AZ} \right]$$

11

$$= \frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \left[e^2 J_Q J'_Q \left[\underbrace{(g^2 - M_{Z(0)}^2 - \pi_{ZZ})}_{(1)} - \underbrace{\frac{\pi_{AZ}^2}{g^2 - \pi_{AA}}}_{(2)} + \underbrace{\frac{\pi_{AZ}^2}{g^2 - \pi_{AA}}}_{(2)} \right] \right. \\ \left. + \underbrace{\frac{e^2}{c^2 s^2} (J_3 - s^2 J_Q)(J'_3 - s^2 J'_Q)(g^2 - \pi_{AA})}_{(3)} \right. \\ \left. + \underbrace{\frac{e}{cs} [J_Q(J'_3 - s^2 J'_Q) + (J_3 - s^2 J_Q)J'_Q]}_{(4)} \pi_{AZ} \right]$$

其中 (1) =
$$\frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} e^2 J_Q J'_Q \left[\underbrace{(g^2 - M_{Z(0)}^2 - \pi_{ZZ})}_{(1)} - \frac{\pi_{AZ}^2}{g^2 - \pi_{AA}} \right]$$

$$= e^2 \frac{J_Q J'_Q}{g^2 - \pi_{AA}} \frac{(g^2 - M_{Z(0)}^2 - \pi_{ZZ})(g^2 - \pi_{AA}) - \pi_{AZ}^2}{g^2 - \pi_{AA}}$$

$$\textcircled{2} + \textcircled{3} + \textcircled{4} = \frac{1}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \left[\underbrace{e^2 J_Q J'_Q \frac{\pi_{AZ}^2}{g^2 - \pi_{AA}} + \frac{e^2}{c^2 s^2} (J_3 - s^2 J_Q)(J'_3 - s^2 J'_Q)(g^2 - \pi_{AA})}_{\text{wavy line}} \right. \\ \left. + \frac{e}{cs} [J_Q(J'_3 - s^2 J'_Q) + (J_3 - s^2 J_Q)J'_Q] \pi_{AZ} \right]$$

上面的划波浪线部分等于

$$\frac{e^2}{c^2 s^2} (g^2 - \pi_{AA}) \left[(J_3 - s^2 J_Q)(J'_3 - s^2 J'_Q) + c^2 s^2 J_Q J'_Q \frac{\pi_{AZ}^2}{(g^2 - \pi_{AA})^2} + cs \frac{\pi_{AZ}}{g^2 - \pi_{AA}} [J_Q(J'_3 - s^2 J'_Q) + (J_3 - s^2 J_Q)J'_Q] \right]$$

$$= \frac{e^2}{c^2 s^2} (g^2 - \pi_{AA}) \left[(J_3 - s^2 J_Q) + cs J_Q \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right] \left[(J'_3 - s^2 J'_Q) + cs J'_Q \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right]$$

12

$$\parallel$$

$$\left[J_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J_Q \right]$$

$$= \frac{e^2}{c^2 s^2} (g^2 - \pi_{AA}) \left[J_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J_Q \right] \left[J'_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J'_Q \right]$$

$$\Rightarrow \textcircled{2} + \textcircled{3} + \textcircled{4} = \frac{\frac{e^2}{c^2 s^2} (g^2 - \pi_{AA})}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{Az}^2} \left[J_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J_Q \right] \left[J'_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J'_Q \right]$$

$$\parallel \frac{e^2}{c^2 s^2} \frac{1}{g^2 - M_{Z(0)}^2 - \pi_{ZZ} - \frac{\pi_{Az}^2}{g^2 - \pi_{AA}}}$$

$$\Rightarrow M_{Nc} = \textcircled{1} + (\textcircled{2} + \textcircled{3} + \textcircled{4}) = e^2 \frac{J_Q J'_Q}{g^2 - \pi_{AA}}$$

$$+ \frac{e^2}{c^2 s^2} \frac{\left[J_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J_Q \right] \left[J'_3 - \left(s^2 - cs \frac{\pi_{Az}}{g^2 - \pi_{AA}} \right) J'_Q \right]}{g^2 - M_{Z(0)}^2 - \pi_{ZZ} - \frac{\pi_{Az}^2}{g^2 - \pi_{AA}}}$$

11

插播：用Dyson方程求解完全传播子

• 用 Dyson 方程求解完全传播子

1

已知: 光子的树图传播子 $D_{AA} = \frac{1}{q^2}$, Z^0 的树图传播子 $D_{ZZ} = \frac{1}{q^2 - M_{Z(0)}^2}$

我们可以根据以下的 Dyson 方程组:

其中 $M_{Z(0)}^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4}$

$$\begin{cases} G_{AA} = D_{AA} + D_{AA} \Pi_{AA} G_{AA} + D_{AA} \Pi_{AZ} G_{ZA} & \text{--- ①} \\ G_{ZZ} = D_{ZZ} + D_{ZZ} \Pi_{ZZ} G_{ZZ} + D_{ZZ} \Pi_{ZA} G_{AZ} & \text{--- ②} \\ G_{ZA} = D_{ZZ} \Pi_{ZZ} G_{ZA} + D_{ZZ} \Pi_{ZA} G_{AA} & \text{--- ③} \end{cases}$$

求解出完全传播子 G_{AA} , G_{ZZ} , G_{ZA} .

由①得: $G_{AA}(1 - D_{AA}\pi_{AA}) = D_{AA} + D_{AA}\pi_{AZ}G_{ZA}$ ----- ①'

由③得: $G_{ZA}(1 - D_{ZZ}\pi_{ZZ}) = D_{ZZ}\pi_{ZA}G_{AA}$

$$\Rightarrow G_{ZA} = \frac{D_{ZZ}\pi_{ZA}}{1 - D_{ZZ}\pi_{ZZ}} G_{AA} \text{ ----- ③'}$$

将③'代入①', 得

$$G_{AA}(1 - D_{AA}\pi_{AA}) = D_{AA} + D_{AA}\pi_{AZ} \frac{D_{ZZ}\pi_{ZA}}{1 - D_{ZZ}\pi_{ZZ}} G_{AA}$$

$$\Rightarrow G_{AA} \left(1 - D_{AA}\pi_{AA} - \frac{D_{AA}\pi_{AZ}D_{ZZ}\pi_{ZA}}{1 - D_{ZZ}\pi_{ZZ}} \right) = D_{AA}$$

$$\Rightarrow G_{AA} = \frac{D_{AA} = \frac{1}{g^2}}{1 - \underbrace{D_{AA}}_{\frac{1}{g^2}} \pi_{AA} - \frac{D_{AA} \pi_{AZ}^2 D_{ZZ}}{1 - \underbrace{D_{ZZ}}_{\frac{1}{g^2 - M_{Z(0)}^2}} \pi_{ZZ}}} = \frac{\frac{1}{g^2}}{1 - \frac{\pi_{AA}}{g^2} - \frac{\frac{1}{g^2} \pi_{AZ}^2 \frac{1}{g^2 - M_{Z(0)}^2}}{1 - \frac{\pi_{ZZ}}{g^2 - M_{Z(0)}^2}}}$$

$$= \frac{1}{g^2 - \pi_{AA} - \frac{\pi_{AZ}^2 \frac{1}{g^2 - M_{Z(0)}^2}}{1 - \frac{\pi_{ZZ}}{g^2 - M_{Z(0)}^2}}} = \frac{1}{g^2 - \pi_{AA} - \frac{\pi_{AZ}^2}{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}}$$

$$= \frac{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2}$$

✓

3

现在, 将上页求得的 G_{AA} 的表达式代入第2页的 (3):

$$G_{AZ} = G_{ZA} = \frac{D_{ZZ} \pi_{ZA}}{1 - D_{ZZ} \pi_{ZZ}} G_{AA}$$

$$= \frac{\frac{1}{g^2 - M_{Z(0)}^2} \pi_{ZA}}{1 - \frac{\pi_{ZZ}}{g^2 - M_{Z(0)}^2}} \cdot \frac{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2}$$

$$\frac{\pi_{ZA}}{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}$$

$$= \frac{\pi_{ZA}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2}$$



最后, 由第1页的 ② 式得

$$G_{zz}(1 - D_{zz}\pi_{zz}) = D_{zz} + D_{zz}\pi_{za}G_{az}$$

$$\Rightarrow G_{zz} = \frac{D_{zz} + D_{zz}\pi_{za}G_{az}}{1 - D_{zz}\pi_{zz}}$$

$$= \frac{\frac{1}{g^2 - M_{z(0)}^2} + \frac{1}{g^2 - M_{z(0)}^2}\pi_{za} \frac{\pi_{za}}{(g^2 - \pi_{aa})(g^2 - M_{z(0)}^2 - \pi_{zz}) - \pi_{az}^2}}{1 - \frac{\pi_{zz}}{g^2 - M_{z(0)}^2}}$$

$$= \frac{1 + \frac{\pi_{za}^2}{(g^2 - \pi_{aa})(g^2 - M_{z(0)}^2 - \pi_{zz}) - \pi_{az}^2}}{g^2 - M_{z(0)}^2 - \pi_{zz}} = \frac{(g^2 - \pi_{aa})(g^2 - M_{z(0)}^2 - \pi_{zz})}{(g^2 - M_{z(0)}^2 - \pi_{zz})[(g^2 - \pi_{aa})(g^2 - M_{z(0)}^2 - \pi_{zz}) - \pi_{az}^2]}$$

$$= \frac{g^2 - \pi_{aa}}{(g^2 - \pi_{aa})(g^2 - M_{z(0)}^2 - \pi_{zz}) - \pi_{az}^2} \quad \checkmark$$

小结: 完全传播子的解是

$$\left\{ \begin{aligned} G_{AA} &= \frac{g^2 - M_{Z(0)}^2 - \pi_{ZZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \\ G_{ZZ} &= \frac{g^2 - \pi_{AA}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \\ G_{AZ} = G_{ZA} &= \frac{\pi_{ZA} = \pi_{AZ}}{(g^2 - \pi_{AA})(g^2 - M_{Z(0)}^2 - \pi_{ZZ}) - \pi_{AZ}^2} \end{aligned} \right.$$

注: 第1页的 ③ 式也可由以下方程替代:

$$G_{AZ} = D_{AA} \pi_{AA} G_{AZ} + D_{AA} \pi_{AZ} G_{ZZ} \quad \text{--- ③}$$

将它与第1页的 ② 式联立, 可首先求解出 G_{ZZ} , 结果相同!

插播结束

$$\text{由 } \pi_{AA} = e^2 \underbrace{\pi_{QQ}}_{\substack{|| \\ g^2 \pi'_{QQ}(q^2)}} = e^2 g^2 \pi'_{QQ}(q^2) \Rightarrow M_{Nc} \supset e^2 \frac{J_Q J'_Q}{q^2 - \pi_{AA}} = e^2 \frac{J_Q J'_Q}{q^2 - e^2 g^2 \pi'_{QQ}(q^2)} \quad |13$$

$$= \underbrace{\frac{e^2}{1 - e^2 \pi'_{QQ}(q^2)}}_{\substack{||| \\ e_*^2(q^2)}} \frac{J_Q J'_Q}{g^2}$$

我们可定义跑动的耦合常数: $e_*^2(q^2) \equiv \frac{e^2}{1 - e^2 \pi'_{QQ}(q^2)} \simeq e^2 [1 + e^2 \pi'_{QQ}(q^2)]$

又由 M_{Nc} 中的组合:

$$s^2 - s_c \frac{\pi_{AZ}(q^2)}{q^2 - \pi_{AA}(q^2)} \rightarrow \frac{e^2}{s_c} (\pi_{3Q} - s^2 \pi_{QQ})$$

$$= s^2 - s_c \frac{\frac{e^2}{s_c} (g^2 \pi'_{3Q} - g^2 \pi'_{QQ})}{q^2 - e^2 g^2 \pi'_{QQ}} e^2 \pi_{QQ}$$

$$= \left(s^2 - \frac{e^2 (\pi'_{3Q} - \pi'_{QQ})}{1 - e^2 \pi'_{QQ}} \right) \equiv S_*^2(q^2)$$

可定义

$$S_*^2(q^2) \equiv s^2 - \frac{e^2 (\pi'_{3Q} - \pi'_{QQ})}{1 - e^2 \pi'_{QQ}} \simeq s^2 - e^2 [\pi'_{3Q}(q^2) - \pi'_{QQ}(q^2)]$$

这里只须保留到
 π 的一阶, 因为
它们是小量。

此时, $M_{Nc} = e_*^2(g^2) \frac{J_Q J'_Q}{g^2} + \frac{e^2}{c^2 s^2} \frac{[J_3 - s_*^2(g^2) J_Q][J'_3 - s_*^2(g^2) J'_Q]}{g^2 - M_{Z(0)}^2 - \Pi_{ZZ} - \frac{\Pi_{AZ}^2}{g^2 - \Pi_{AA}}}$

注意到 $g^2 - \underbrace{M_{Z(0)}^2}_{\frac{e^2}{c^2 s^2} \frac{v^2}{4}} - \underbrace{\Pi_{ZZ}}_{\frac{e^2}{c^2 s^2} (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ})} - \frac{\Pi_{AZ}^2}{g^2 - \Pi_{AA}} = \frac{\frac{e^4}{c^2 s^2} (g^2 \Pi'_{3Q} - s^2 g^2 \Pi'_{QQ})^2}{g^2 - e^2 g^2 \Pi'_{QQ}} = g^2 \frac{e^4}{c^2 s^2} \frac{(\Pi'_{3Q} - s^2 \Pi'_{QQ})^2}{(1 - e^2 \Pi'_{QQ})}$

$\approx g^2 - \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ} \right)$

$\mathcal{O}(\Pi^2) \rightarrow \text{可忽略!}$

$$\Rightarrow M_{Nc} = e_*^2(g^2) \frac{J_Q J'_Q}{g^2} + \frac{e^2}{c^2 s^2} \frac{[J_3 - s_*^2(g^2) J_Q][J'_3 - s_*^2(g^2) J'_Q]}{g^2 - \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ} \right)}$$

同理, 由于 $g^2 - \underbrace{M_{W(0)}^2}_{\frac{e^2}{s^2} \frac{v^2}{4}} - \underbrace{\Pi_{WW}}_{\frac{e^2}{s^2} \Pi_{11}} = g^2 - \frac{e^2}{s^2} \left(\frac{v^2}{4} + \Pi_{11} \right)$, 第8页的 M_{cc} 可写为

$$M_{cc} = \frac{e^2}{2s^2} \frac{J_+ J'_+}{g^2 - \frac{e^2}{s^2} \left(\frac{v^2}{4} + \Pi_{11} \right)}$$

规范玻色子W和Z的质量是它们各自传播子的极点, 即

15

$$\begin{cases} \left[q^2 - \frac{e^2}{s^2} \left(\frac{v^2}{4} + \pi_{11} \right) \right]_{q^2=M_W^2} = 0 \\ \left[q^2 - \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ} \right) \right]_{q^2=M_Z^2} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} M_W^2 = \frac{e^2}{s^2} \left(\frac{v^2}{4} + \pi_{11} \right) \Big|_{q^2=M_W^2} \\ M_Z^2 = \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ} \right) \Big|_{q^2=M_Z^2} \end{cases}$$

对 $\frac{e^2}{s^2} \left[\frac{v^2}{4} + \pi_{11}(q^2) \right]$ 在 $q^2=M_W^2$ 邻域作 Taylor 展开:

$$\begin{aligned} \frac{e^2}{s^2} \left[\frac{v^2}{4} + \pi_{11}(q^2) \right] &= \frac{e^2}{s^2} \left(\frac{v^2}{4} + \pi_{11} \right) \Big|_{q^2=M_W^2} + \frac{e^2}{s^2} \frac{d\pi_{11}}{dq^2} \Big|_{q^2=M_W^2} \cdot (q^2 - M_W^2) + O[(q^2 - M_W^2)^2] \\ &= \overset{M_W^2}{M_W^2} + \frac{e^2}{s^2} \frac{d\pi_{11}}{dq^2} \Big|_{q^2=M_W^2} \cdot (q^2 - M_W^2) + O[(q^2 - M_W^2)^2] \end{aligned}$$

$$\Rightarrow q^2 - \frac{e^2}{s^2} \left[\frac{v^2}{4} + \Pi_{11}(q^2) \right] = \underbrace{q^2 - M_W^2}_{II} - \frac{e^2}{s^2} \left. \frac{d\Pi_{11}}{dq^2} \right|_{q^2=M_W^2} \cdot (q^2 - M_W^2) + O[(q^2 - M_W^2)^2] \quad [16]$$

$$\underbrace{(q^2 - M_W^2) \left(1 - \frac{e^2}{s^2} \left. \frac{d\Pi_{11}}{dq^2} \right|_{q^2=M_W^2} \right)}_{III}$$

Z_W^{-1}

$$= Z_W^{-1} (q^2 - M_W^2) + O[(q^2 - M_W^2)^2]$$

$$\Rightarrow \frac{I}{q^2 - \frac{e^2}{s^2} \left[\frac{v^2}{4} + \Pi_{11}(q^2) \right]} = \frac{I}{Z_W^{-1} (q^2 - M_W^2) + O[(q^2 - M_W^2)^2]} = \frac{Z_W}{(q^2 - M_W^2) + O[(q^2 - M_W^2)^2]}$$

$$\equiv \frac{Z_W}{q^2 - \underbrace{M_{W*}^2(q^2)}_{\text{称为跑动的W质量平方}}}$$

显然有

显然有

$$M_{W*}^2(M_W^2) = M_W^2,$$

$$\left. \frac{dM_{W*}^2}{dq^2} \right|_{q^2=M_W^2} = 0$$

同理, 对 $\frac{e^2}{c^2 s^2} \left[\frac{v^2}{4} + \pi_{33}(q^2) - 2s^2 \pi_{3Q}(q^2) + s^4 \pi_{QQ}(q^2) \right]$ 在 $q^2 = M_Z^2$ 邻域作 Taylor 17

展开:

$$\frac{e^2}{c^2 s^2} \left[\frac{v^2}{4} + \pi_{33}(q^2) - 2s^2 \pi_{3Q}(q^2) + s^4 \pi_{QQ}(q^2) \right]$$

$$= \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ} \right) \Big|_{q^2=M_Z^2} + \frac{e^2}{c^2 s^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right) \Big|_{q^2=M_Z^2} \cdot (q^2 - M_Z^2)$$

||
 M_Z^2

$$+ O[(q^2 - M_Z^2)^2]$$

$$= M_Z^2 + \frac{e^2}{c^2 s^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right) \Big|_{q^2=M_Z^2} \cdot (q^2 - M_Z^2) + O[(q^2 - M_Z^2)^2]$$

$$\Rightarrow q^2 - \frac{e^2}{c^2 s^2} \left(\frac{v^2}{4} + \pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ} \right) = (q^2 - M_Z^2) \left[1 - \frac{e^2}{c^2 s^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right) \Big|_{q^2=M_Z^2} \right]$$

|||
 Z_Z^{-1}

$$+ O[(q^2 - M_Z^2)^2]$$

$$= Z_Z^{-1} (q^2 - M_Z^2) + O[(q^2 - M_Z^2)^2]$$

$$\Rightarrow \frac{1}{q^2 - \frac{e^2}{c^2 s^2} \left(\frac{\gamma^2}{4} + \pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ} \right)} = \frac{1}{Z_Z^{-1} (q^2 - M_Z^2) + O[(q^2 - M_Z^2)^2]}$$

$$\frac{Z_Z}{q^2 - M_{Z*}^2(q^2)} = \frac{Z_Z}{(q^2 - M_Z^2) + O[(q^2 - M_Z^2)^2]}$$

→ 称为跑动的Z质量平方

显然有

$$M_{Z*}^2(M_Z^2) = M_Z^2,$$

$$\left. \frac{dM_{Z*}^2}{dq^2} \right|_{q^2=M_Z^2} = 0$$

根据这些定义, 可将第14页的带电流振幅 M_{cc} 和中性流振幅 M_{Nc} 改写为

$$M_{cc} = \frac{e^2}{2s^2} J_+ \frac{Z_W}{q^2 - M_{W*}^2(q^2)} J_-'$$

$$M_{Nc} = e_*^2(q^2) \frac{J_Q J_Q'}{q^2} + \frac{e^2}{c^2 s^2} [J_3 - s_*^2(q^2) J_Q] \frac{Z_Z}{q^2 - M_{Z*}^2(q^2)} [J_3' - s_*^2(q^2) J_Q']$$

由以上表达式, 我们可定义跑动的波函数重整化常数 $Z_{W*}(q^2)$ 和 $Z_{Z*}(q^2)$ 为

$$\frac{e^2}{s^2} Z_W \equiv \frac{e_*^2(q^2)}{s_*^2(q^2)} Z_{W*}(q^2), \quad \frac{e^2}{s^2 c^2} Z_Z \equiv \frac{e_*^2(q^2)}{s_*^2(q^2) c_*^2(q^2)} Z_{Z*}(q^2)$$

其中 $c_*^2(q^2) \equiv 1 - s_*^2(q^2)$. 此时, M_{cc} 和 M_{Nc} 可表达为

$$M_{cc} = \frac{e_*^2(q^2)}{2s_*^2(q^2)} J_+ \frac{Z_{W*}(q^2)}{q^2 - M_{W*}^2(q^2)} J_-'$$

$$M_{Nc} = e_*^2(q^2) \frac{J_Q J_Q'}{q^2} + \frac{e_*^2(q^2)}{s_*^2(q^2) c_*^2(q^2)} [J_3 - s_*^2(q^2) J_Q] \frac{Z_{Z*}(q^2)}{q^2 - M_{Z*}^2(q^2)} [J_3' - s_*^2(q^2) J_Q']$$

上式与树图水平的振幅(见第2页, 注意 G_{AZ} 在树图水平为零)具有相同的形式, 不同之处是所有的耦合常数和规范玻色子参数都用带*的参数替代, 这些带*的参数是 q^2 的函数. 这种仅仅通过带*的参数来影响弱相互作用的可观测量修正就是间接的 (oblique) 修正.

上面的 $Z_{W*}(q^2)$ 和 $Z_{Z*}(q^2)$ 也可用真空极化 $\Pi's$ 来表达.

$$Z_{W*}(q^2) \simeq 1 + \frac{e^2}{s^2} \left. \frac{d\pi_{11}}{dq^2} \right|_{q^2=M_W^2} - \frac{e^2}{s^2} \pi'_{3Q}(q^2)$$

$$Z_{Z*}(q^2) \simeq 1 + \frac{e^2}{s^2 c^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right)_{q^2=M_Z^2} - \frac{e^2 s^2}{c^2} \pi'_{QQ}(q^2) - \frac{e^2(c^2-s^2)}{s^2 c^2} \pi'_{3Q}(q^2)$$

Proof: $\frac{e^2}{s^2} Z_W = \frac{e_*^2}{s_*^2} Z_{W*}$, $Z_W = \frac{1}{1 - \frac{e^2}{s^2} \left. \frac{d\pi_{11}}{dq^2} \right|_{q^2=M_W^2}}$, $e_*^2 \simeq e^2 (1 + e^2 \pi'_{QQ})$,

$$s_*^2 \simeq s^2 - e^2 (\pi'_{3Q} - s^2 \pi'_{QQ})$$

$$= s^2 \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right]$$

$$\Rightarrow Z_{W*} = Z_W \frac{e^2}{s^2} \cdot \frac{s_*^2}{e_*^2} \simeq Z_W \frac{e^2}{s^2} \cdot \frac{s^2 \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right]}{e^2 (1 + e^2 \pi'_{QQ})}$$

$$\simeq \underbrace{Z_W}_{12} \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) - e^2 \pi'_{QQ} \right]$$

$$\left(1 + \frac{e^2}{s^2} \left. \frac{d\pi_{11}}{dq^2} \right|_{q^2=M_W^2} \right)$$

$$\simeq 1 + \frac{e^2}{s^2} \left. \frac{d\pi_{11}}{dq^2} \right|_{q^2=M_W^2} - \frac{e^2}{s^2} \pi'_{3Q} \quad \checkmark$$

$$\frac{e^2}{s^2 c^2} Z_Z = \frac{e_*^2}{s_*^2 c_*^2} Z_{Z_*}, \quad Z_Z = \frac{1}{1 - \frac{e^2}{s^2 c^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right)_{q^2=M_Z^2}},$$

$$e_*^2 \simeq e^2 (1 + e^2 \pi'_{QQ})$$

$$s_*^2 \simeq s^2 - e^2 (\pi'_{3Q} - s^2 \pi'_{QQ}) = s^2 \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right]$$

$$\Rightarrow c_*^2 \equiv 1 - s_*^2 = \underbrace{1 - s^2}_{\parallel c^2} + e^2 (\pi'_{3Q} - s^2 \pi'_{QQ}) = c^2 \left[1 + \frac{e^2}{c^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right]$$

$$\Rightarrow Z_{Z_*} = Z_Z \frac{e^2}{s^2 c^2} \frac{s_*^2 c_*^2}{e_*^2} \simeq Z_Z \frac{e^2}{s^2 c^2} \frac{s^2 \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right] \left[1 + \frac{e^2}{c^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) \right]}{e^2 (1 + e^2 \pi'_{QQ})}$$

$$\simeq \underbrace{Z_Z}_{\parallel} \left[1 - \frac{e^2}{s^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) + \frac{e^2}{c^2} (\pi'_{3Q} - s^2 \pi'_{QQ}) - e^2 \pi'_{QQ} \right]$$

$$\underbrace{1 + \frac{e^2}{s^2 c^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right)_{q^2=M_Z^2}}_{\parallel} \quad \parallel \quad 1 - \left(\frac{e^2}{s^2} - \frac{e^2}{c^2} \right) \pi'_{3Q} - \frac{e^2 s^2}{c^2} \pi'_{QQ}$$

$$\parallel \quad \frac{e^2 (c^2 - s^2)}{s^2 c^2}$$

$$\simeq 1 + \frac{e^2}{s^2 c^2} \left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right)_{q^2=M_Z^2} - \frac{e^2 (c^2 - s^2)}{s^2 c^2} \pi'_{3Q} - \frac{e^2 s^2}{c^2} \pi'_{QQ} \quad \checkmark$$

对于一些电弱可观测量, 若我们知道它们在树图水平用裸参数表达的公式, 则必须将裸参数替换为相应的带*的参数(它们依赖于动量), 就可以包含来自真空极化的修正。

例: 被间接修正的左右不对称性 A_{LR} 是

$$A_{LR}(q^2) = \frac{2[1 - 4s_*^2(q^2)]}{1 + [1 - 4s_*^2(q^2)]^2}$$

Z 规范玻色子的衰变宽度是

$$\Gamma_Z = \sum_{f^*} \frac{\alpha_* M_Z}{6 s_*^2 c_*^2} \sum_f N_f (T_{3f} - s_*^2 Q_f) \Big|_{q^2 = M_Z^2} \rightarrow \alpha_s(M_Z^2) = 0.12 \pm 0.01$$

其中 $\alpha_*(q^2) = \frac{e_*^2(q^2)}{4\pi}$

费米子味道 f 的有效色数目 $N_f = \begin{cases} 1 & \text{(对于轻子)} \\ 3(1 + \frac{\alpha_s}{\pi} + \dots) = 3.12 \pm 0.01 & \text{(对于夸克)} \end{cases}$
(at $q^2 = M_Z^2$)

虽然第19页的 M_{cc} 和 M_{Nc} 表达式包含了所有的信息,但在应用于低能实验时有些笨拙。我们不妨从第14页的 M_{cc} 和 M_{Nc} 表达式出发。

在 $q^2 \rightarrow 0$ 极限下, M_{cc} 变成

$$M_{cc} = \frac{e^2}{2s^2} \frac{J_+ J_-}{q^2 - \frac{e^2}{s^2} \left(\frac{\nu^2}{4} + \pi_{11} \right)} \xrightarrow{(q^2 \rightarrow 0)} \frac{e^2}{2s^2} \frac{J_+ J_-}{\left(-\frac{e^2}{s} \right) \left[\frac{\nu^2}{4} + \pi_{11}(0) \right]} = -\frac{1}{2} \frac{J_+ J_-}{\frac{\nu^2}{4} + \pi_{11}(0)}$$

而 M_{Nc} 中交换部分的振幅在 $q^2 \rightarrow 0$ 极限下化简为

$$\begin{aligned} M_{Nc} &\supset \frac{e^2}{c^2 s^2} \frac{[J_3 - s_x^2(q^2) J_R][J'_3 - s_x^2(q^2) J'_R]}{q^2 - \frac{e^2}{c^2 s^2} \left(\frac{\nu^2}{4} + \pi_{33} - 2s^2 \pi_{3R} + s^4 \pi_{RR} \right)} \\ &\xrightarrow{(q^2 \rightarrow 0)} \frac{e^2}{c^2 s^2} \frac{[J_3 - s_x^2(0) J_R][J'_3 - s_x^2(0) J'_R]}{\left(-\frac{e^2}{c^2 s^2} \right) \left[\frac{\nu^2}{4} + \pi_{33}(0) - 2s^2 \pi_{3R}(0) + s^4 \pi_{RR}(0) \right]} \\ &= -\frac{[J_3 - s_x^2(0) J_R][J'_3 - s_x^2(0) J'_R]}{\frac{\nu^2}{4} + \pi_{33}(0)} \end{aligned}$$

上页的 M_{cc} 和 M_{Nc} 中的 Z 交换部分:

$$M_{cc} \xrightarrow{q^2 \rightarrow 0} -\frac{1}{2} \frac{J_+ J'_-}{\frac{N^2}{4} + \pi_{11}(0)}$$

$$(M_{Nc})_{Z \text{ 交换部分}} \xrightarrow{q^2 \rightarrow 0} -\frac{[J_3 - s_*^2(0) J_Q][J'_3 - s_*^2(0) J'_Q]}{\frac{N^2}{4} + \pi_{33}(0)}$$

也可看成由如下的低能有效拉氏量得到:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2[\frac{N^2}{4} + \pi_{11}(0)]} J_{+\mu} J_-^\mu - \frac{1}{2[\frac{N^2}{4} + \pi_{33}(0)]} [J_{3\mu} - s_*^2(0) J_{Q\mu}][J_3^\mu - s_*^2(0) J_Q^\mu]$$

$$= -\underbrace{\frac{1}{2[\frac{N^2}{4} + \pi_{11}(0)]}}_{\text{III}} \left[J_{+\mu} J_-^\mu + \underbrace{\frac{\frac{N^2}{4} + \pi_{11}(0)}{\frac{N^2}{4} + \pi_{33}(0)}}_{\text{III}} (J_3^\mu - s_*^2 J_Q^\mu)^2 \right]$$

$\frac{4G_F}{\sqrt{2}}$
 $\rho_*(0)$

线

$$= -\frac{4G_F}{\sqrt{2}} [J_{+\mu} J_-^\mu + \rho_*(0) (J_3^\mu - s_*^2 J_Q^\mu)^2]$$

$$\text{由 } \frac{4G_F}{\sqrt{2}} \equiv \frac{1}{2\left[\frac{v^2}{4} + \pi_{11}(0)\right]} \Rightarrow 4\sqrt{2}G_F \equiv \frac{1}{\frac{v^2}{4} + \pi_{11}(0)} \Rightarrow \frac{1}{4\sqrt{2}G_F} = \frac{v^2}{4} + \pi_{11}(0) \quad |25$$

$$\rho_*(0) \equiv \frac{\frac{v^2}{4} + \pi_{11}(0)}{\frac{v^2}{4} + \pi_{33}(0)} \Rightarrow \frac{1}{\rho_*(0)} = \frac{\frac{v^2}{4} + \pi_{33}(0)}{\frac{v^2}{4} + \pi_{11}(0)} = \frac{\frac{v^2}{4} + \pi_{11} + \pi_{33} - \pi_{11}}{\frac{v^2}{4} + \pi_{11}}$$

$$= 1 + \underbrace{\frac{1}{\frac{v^2}{4} + \pi_{11}}}_{4\sqrt{2}G_F} [\pi_{33}(0) - \pi_{11}(0)] \quad \checkmark$$

· 我们已将对弱相互作用的可观测量的间接修正效应用带*的参数来表达, 但这些公式仍不够实用, 因为即使在最低阶, 这些公式仍然依赖于3个参数: e^2 , s^2 和 v^2 (或等价地, g , g' 和 v). 我们需要用3个实验可观测量来确定这3个参数。

· 若我们考虑一个破坏custodial对称性的理论, 即需要包括第4个参数 ρ , 此时就需要多考虑一个实验可观测量来确定它。

- 三个输入的可观测量可自然地选择为: α , G_F 和 M_Z . 因为它们是电弱相互作用的测得最精确的参数。它们的测量值是

$$\frac{1}{\alpha} = 137.0359895(61),$$

$$G_F = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = 91.174(21) \text{ GeV}$$

- 文献中对弱混合角 (即 Weinberg 角) 有多种定义:

▲ Sirlin 定义 (基于 M_W 和 M_Z)

$$S_W^2 \equiv \sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}$$

▲ 基于 Z^0 不对称性的定义:

$$\bar{S}_W^2 \equiv \sin^2 \bar{\theta}_W \equiv S_*^2(M_Z^2)$$

▲ 基于三个输入可观测量 α , G_F , M_Z 的定义:

$$\sin^2 \theta_0 \equiv \sqrt{\frac{4\pi \alpha_{s,0}(M_Z^2)}{\sqrt{2} G_F M_Z^2}}$$

$$\text{其中 } \frac{1}{\alpha_{s,0}(M_Z^2)} = 128.80 \pm 0.12$$

是跑动的电荷在 M_Z 标度的取值, 而从 $q^2=0$ 到 $q^2=M_Z^2$ 的跑动是从已知的物理计算得到的。

这个定义是基于在标准模型中的 Weinberg 角 θ 与 α , G_F , M_Z 的如下关系:

$$\sin^2 2\theta = \frac{e^2}{\sqrt{2} G_F M_Z^2}$$

对 $\sin^2\theta_w$ 的各种不同定义在树图阶都等价于裸量 s^2 ，但它们的辐射修正很不一样。

由于 s_0^2 只依赖于

$$\begin{cases} \frac{1}{\alpha_{e,0}(M_Z)} = 128.80 \pm 0.12 \\ G_F = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2} \\ M_Z = 91.174(21) \text{ GeV} \end{cases}$$

它的值非常精确：

$$s_0^2 = 0.23146 \pm 0.00034$$

取这个值为参考点，我们可以给出其它定义的 $\sin^2\theta_w$ 值，以及其它任意的电弱可观测测量，它们都可表达为辐射修正。

我们先来推导一个数学恒等式：

$$\delta(s^2) = \frac{2s^2c^2}{c^2 - s^2} \frac{\delta(\sin 2\theta)}{\sin 2\theta}$$

其中 $s \equiv \sin\theta$
 $c \equiv \cos\theta$

Proof: $\delta(\sin^2\theta) = 2 \sin\theta \frac{\delta(\sin\theta)}{\cos\theta} = 2sc \delta\theta$

另一方面, $\delta(\sin 2\theta) = \cos 2\theta \cdot 2\delta\theta \Rightarrow \delta\theta = \frac{\delta(\sin 2\theta)}{2\cos 2\theta}$

$$\Rightarrow \delta(s^2) = \frac{2sc}{2\cos 2\theta} \delta(\sin 2\theta) = \frac{sc}{\cos 2\theta} \cdot 2sc \frac{\delta(\sin 2\theta)}{\sin 2\theta}$$

✓

由于 α , G_F , M_Z 都有间接修正的贡献, s_0^2 也存在间接的修正:

$$s_0^2 = s^2 + \frac{2s^2c^2}{c^2-s^2} \cdot \frac{1}{2} \left[\frac{\delta\alpha_{*,0}}{\alpha_{*,0}} - \frac{\delta G_F}{G_F} - \frac{\delta M_Z^2}{M_Z^2} \right]$$

$$= s^2 + \frac{s^2c^2}{c^2-s^2} \left[e^2 \pi'_{QQ}(0) + \frac{e^2}{s^2c^2 M_Z^2} \pi_{11}(0) - \frac{e^2}{s^2c^2 M_Z^2} (\pi_{33} - 2s^2\pi_{3Q} + s^4\pi_{QQ})(M_Z^2) \right]$$

Proof: 我们来考察 s_0^2 与裸参数 s^2 之间的偏离:

$$\delta(s_0^2) \equiv s_0^2 - s^2 = \frac{2s^2c^2}{c^2-s^2} \delta[\ln(\sin 2\theta_0)]$$

$$\text{由 } \sin 2\theta_0 \equiv \sqrt{\frac{4\pi\alpha_{*,0}(M_Z^2)}{\sqrt{2} G_F M_Z^2}} \Rightarrow \ln(\sin 2\theta_0) = \frac{1}{2} (\ln \alpha_{*,0} - \ln G_F - \ln M_Z^2) + \text{Const.}$$

$$\Rightarrow \delta[\ln(\sin 2\theta_0)] = \frac{1}{2} \delta(\ln \alpha_{*,0} - \ln G_F - \ln M_Z^2)$$

$$= \frac{1}{2} \left(\underbrace{\frac{\delta\alpha_{*,0}}{\alpha_{*,0}}}_{\textcircled{1}} - \underbrace{\frac{\delta G_F}{G_F}}_{\textcircled{2}} - \underbrace{\frac{\delta M_Z^2}{M_Z^2}}_{\textcircled{3}} \right)$$

$$\text{由第13页: } e_*^2(q^2) \simeq e^2 [1 + e^2 \pi'_{QQ}(q^2)] \Rightarrow \alpha_*(q^2) \equiv \frac{e_*^2(q^2)}{4\pi} \simeq \underbrace{\left(\frac{e^2}{4\pi} \right)}_{\alpha} [1 + e^2 \pi'_{QQ}(q^2)]$$

$$\Rightarrow \delta\alpha_x(q^2) \equiv \alpha_x(q^2) - \alpha \simeq \alpha e^2 \pi'_{\text{QQ}}(q^2)$$

$$\Rightarrow \textcircled{1} = \frac{\delta\alpha_x(M_Z^2)}{\alpha_x(M_Z^2)} \simeq \frac{\delta\alpha_x(M_Z^2)}{\alpha} \simeq e^2 \pi'_{\text{QQ}}(M_Z^2)$$

在树图水平, $\left. \begin{array}{l} \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \\ M_W^2 = \frac{g^2 v^2}{4} \end{array} \right\} \Rightarrow \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \Rightarrow G_F = \frac{1}{\sqrt{2}v^2} \text{ (tree level)}$

在低能极限 ($q^2 \rightarrow 0$) 下被间接修正的 G_F 是 $4\sqrt{2}G_F \equiv \frac{1}{\frac{v^2}{4} + \pi_{11}(0)} \Rightarrow G_F = \frac{1}{\sqrt{2}v^2 + 4\sqrt{2}\pi_{11}(0)}$

$$\Rightarrow \delta G_F = G_F - G_F^{(\text{tree})} = \frac{1}{\sqrt{2}v^2 + 4\sqrt{2}\pi_{11}(0)} - \frac{1}{\sqrt{2}v^2} = \frac{-4\sqrt{2}\pi_{11}(0)}{[\sqrt{2}v^2 + 4\sqrt{2}\pi_{11}(0)]\sqrt{2}v^2}$$

$$\simeq \frac{-4\sqrt{2}\pi_{11}(0)}{(\sqrt{2}v^2)^2}$$

线 $\Rightarrow \frac{\delta G_F}{G_F} \simeq \frac{\delta G_F}{G_F^{(\text{tree})}} \simeq \frac{\frac{-4\sqrt{2}\pi_{11}(0)}{(\sqrt{2}v^2)^2}}{\frac{1}{\sqrt{2}v^2}} = -\frac{4\sqrt{2}\pi_{11}(0)}{\sqrt{2}v^2} = -\frac{\pi_{11}(0)}{\frac{v^2}{4}}$

在树图水平, $M_Z^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4} \Rightarrow -\frac{1}{\frac{v^2}{4}} = -\frac{e^2}{s^2 c^2 M_Z^2}$

$$\Rightarrow \frac{\delta G_F}{G_F} = -\frac{\frac{\pi_{11}(0)}{\frac{v^2}{4}}}{\frac{v^2}{4}} \simeq -\frac{e^2}{s^2 c^2 M_Z^2} \pi_{11}(0)$$

$$\Rightarrow \textcircled{2} = -\frac{\delta G_F}{G_F} \simeq \frac{e^2}{s^2 c^2 M_Z^2} \pi_{11}(0)$$

由第15页: $M_Z^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4} + \frac{e^2}{s^2 c^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2)$

$$\Rightarrow \delta M_Z^2 = M_Z^2 - \frac{e^2}{s^2 c^2} \frac{v^2}{4} = \frac{e^2}{s^2 c^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2)$$

$$\Rightarrow \textcircled{3} = -\frac{\delta M_Z^2}{M_Z^2} = -\frac{e^2}{s^2 c^2 M_Z^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2)$$

綜上, 有 $s_0^2 - s^2 = \frac{2s^2 c^2}{c^2 - s^2} \delta[\ln(\sin 2\theta_0)] = \frac{s^2 c^2}{c^2 - s^2} (\textcircled{1} + \textcircled{2} + \textcircled{3})$

$$\parallel \frac{1}{2} (\textcircled{1} + \textcircled{2} + \textcircled{3})$$

$$= \frac{s^2 c^2}{c^2 - s^2} \left[e^2 \pi'_{QQ}(M_Z^2) + \frac{e^2}{s^2 c^2 M_Z^2} \pi_{11}(0) - \frac{e^2}{s^2 c^2 M_Z^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2) \right] \checkmark$$

我们还可求出 s_0^2 与 s_W^2 的差别:

$$\begin{aligned} s_0^2 - s_W^2 &= \frac{M_W^2}{M_Z^2} - G^2 \\ &= \frac{e^2 c^2}{s^2(c^2 - s^2)M_Z^2} \left[\frac{c^2 - s^2}{c^2} \Pi_{11}(M_W^2) + \frac{s^2}{c^2} \Pi_{11}(0) - \Pi_{33}(M_Z^2) + 2s^2 \Pi_{3Q}(M_Z^2) \right] \\ &\quad + \frac{e^2 s^2 c^2}{c^2 - s^2} [\Pi'_{QQ}(0) - \Pi'_{QQ}(M_Z^2)] \end{aligned}$$

Proof: 由第28页结果: $s_0^2 = s^2 + \frac{s^2 c^2}{c^2 - s^2} \left[e^2 \Pi'_{QQ}(0) + \frac{e^2}{s^2 c^2 M_Z^2} \Pi_{11}(0) - \frac{e^2}{s^2 c^2 M_Z^2} (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ})(M_Z^2) \right]$

$$\Rightarrow G^2 \equiv 1 - s_0^2 = c^2 - \frac{s^2 c^2}{c^2 - s^2} \left[e^2 \Pi'_{QQ}(0) + \frac{e^2}{s^2 c^2 M_Z^2} \Pi_{11}(0) - \frac{e^2}{s^2 c^2 M_Z^2} (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ})(M_Z^2) \right] \quad (1)$$

另一方面, 由于 M_W 是 $W^\pm$ 传播子的极点: $M_W^2 = \frac{e^2}{s^2} \left[\frac{v^2}{4} + \Pi_{11}(M_W^2) \right]$

$$\Rightarrow \frac{M_W^2}{M_Z^2} = \frac{e^2}{s^2 M_Z^2} \left[\frac{v^2}{4} + \Pi_{11}(M_W^2) \right] \quad (2)$$

又由于 M_Z 是 Z^0 传播子的极点: $M_Z^2 = \frac{e^2}{s^2 c^2} \left[\frac{v^2}{4} + (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ})(M_Z^2) \right]$

$$\Rightarrow \frac{v^2}{4} = \frac{s^2 c^2 M_Z^2}{e^2} - (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ})(M_Z^2) \quad (3)$$

将上页的③式代入②式, 可得:

$$\frac{M_W^2}{M_Z^2} = c^2 - \frac{e^2}{s^2 M_Z^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2) + \frac{e^2}{s^2 M_Z^2} \pi_{11}(M_W^2) \quad (4)$$

将④式与上页的①式相减, 得

$$\begin{aligned} \frac{M_W^2}{M_Z^2} - c^2 &= \frac{e^2 c^2}{s^2 (c^2 - s^2) M_Z^2} \left[\frac{c^2 - s^2}{c^2} \pi_{11}(M_W^2) + \frac{s^2}{c^2} \pi_{11}(0) - (\pi_{33} - 2s^2 \pi_{3Q})(M_Z^2) \right] \\ &\quad + \frac{e^2 s^2 c^2}{c^2 - s^2} [\pi'_{QQ}(0) - \pi'_{QQ}(M_Z^2)] \end{aligned}$$

同样, 可求出 $s_*^2(q^2)$ 与 s_0^2 的差别:

$$\begin{aligned} s_*^2(q^2) - s_0^2 &= \frac{e^2}{c^2 - s^2} \left[\frac{\pi_{33}(M_Z^2) - 2s^2 \pi_{3Q}(M_Z^2) - \pi_{11}(0)}{M_Z^2} - (c^2 - s^2) \pi'_{3Q}(q^2) \right] \\ &\quad + \frac{e^2 s^2}{c^2 - s^2} [s^2 \pi'_{QQ}(M_Z^2) - c^2 \pi'_{QQ}(0) + (c^2 - s^2) \pi'_{QQ}(q^2)] \end{aligned}$$

Proof:

由 $s_*^2(q^2) \simeq s^2 - e^2 [\pi'_{3Q}(q^2) - s^2 \pi'_{QQ}(q^2)]$

$$s_0^2 = s^2 + \frac{s^2 c^2}{c^2 - s^2} \left[e^2 \pi'_{QQ}(0) + \frac{e^2}{s^2 c^2 M_Z^2} \pi_{11}(0) - \frac{e^2}{s^2 c^2 M_Z^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})(M_Z^2) \right]$$

两式相减,

即得证. ✓

剩下的带*的参数可表达为它们与1的偏离:

$$\rho_*(0) - 1 = \frac{e^2}{s^2 c^2 M_Z^2} [\pi_{11}(0) - \pi_{33}(0)]$$

$$Z_{Z*}(q^2) - 1 = \frac{e^2}{s^2 c^2} \left[\left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right) \Big|_{q^2=M_Z^2} - (c^2 - s^2) \pi'_{3Q}(q^2) - s^4 \pi'_{QQ}(q^2) \right]$$

$$Z_{W*}(q^2) - 1 = \frac{e^2}{s^2} \left[\frac{d\pi_{11}}{dq^2} \Big|_{q^2=M_W^2} - \pi'_{3Q}(q^2) \right]$$

Proof: 由第25页: $\frac{1}{\rho_*(0)} = 1 + \frac{1}{\frac{\nu^2}{4} + \pi_{11}(0)} [\pi_{33}(0) - \pi_{11}(0)]$

$$\Rightarrow \rho_*(0) \simeq 1 - \frac{1}{\frac{\nu^2}{4} + \pi_{11}(0)} [\pi_{33}(0) - \pi_{11}(0)] = 1 + \frac{1}{\frac{\nu^2}{4} + \pi_{11}(0)} [\pi_{11}(0) - \pi_{33}(0)]$$

而准确到树图水平, 有 $\frac{\nu^2}{4} + \pi_{11}(0) \simeq \frac{\nu^2}{4} + \pi_{11}(M_W^2) = \frac{M_W^2 s^2}{e^2} \simeq \frac{M_Z^2 c^2 s^2}{e^2}$

$$\Rightarrow \rho_*(0) - 1 \simeq \frac{e^2}{s^2 c^2 M_Z^2} [\pi_{11}(0) - \pi_{33}(0)] \quad \checkmark$$

其余两式根据第20页关于 $Z_{Z*}(q^2)$ 和 $Z_{W*}(q^2)$ 的表达式立得. $\checkmark$

若在真空极化图的内线重粒子的质量 M_T 远大于 M_Z , 则真空极化振幅将会在关于 g^2 的级数展开中快速收敛, 因此我们可以将真空极化振幅 $\Pi's$ 只保留到 g^2 阶, 而忽略掉 g^4 以及更高阶, 即可以取如下近似:

$$\Pi_{QQ}(g^2) \approx g^2 \Pi'_{QQ}(0)$$

$$\Pi_{3Q}(g^2) \approx g^2 \Pi'_{3Q}(0)$$

$$\Pi_{33}(g^2) \approx \Pi_{33}(0) + g^2 \Pi'_{33}(0)$$

$$\Pi_{11}(g^2) \approx \Pi_{11}(0) + g^2 \Pi'_{11}(0)$$

这个近似的相对误差应该是 $O\left(\frac{M_Z^2}{M_T^2}\right)$ 量级。

上面有6个Taylor级数系数: $\Pi'_{QQ}(0)$, $\Pi'_{3Q}(0)$, $\Pi_{33}(0)$, $\Pi_{11}(0)$, $\Pi'_{33}(0)$, $\Pi'_{11}(0)$ 。

由于我们有3个输入的可观测量: α , G_F , M_Z , 这6个Taylor级数系数只有 $6-3=3$ 个自由度。我们可将这3个自由度选取为如下的等外有限的组合:

$$\alpha S \equiv 4e^2 [\pi'_{33}(0) - \pi'_{3Q}(0)]$$

$$\alpha T \equiv \frac{e^2}{s^2 c^2 M_Z^2} [\pi_{11}(0) - \pi_{33}(0)]$$

$$\alpha U \equiv 4e^2 [\pi'_{11}(0) - \pi'_{33}(0)]$$

此时, 带*的参数以及 Weinberg 角定义的差别都可表达为 S, T, U 的线性函数:

$$\rho_*(0) - 1 \approx \alpha T$$

$$Z_{Z*}(q^2) - 1 \approx \frac{\alpha}{4s^2 c^2} S$$

$$Z_{W*}(q^2) - 1 \approx \frac{\alpha}{4s^2} (S + U)$$

$$s_0^2 - s_w^2 = \frac{M_W^2}{M_Z^2} - c_0^2 \approx \frac{\alpha c^2}{c^2 - s^2} \left(-\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4s^2} U \right)$$

$$s_*^2(q^2) - s_0^2 \approx \frac{\alpha}{c^2 - s^2} \left(\frac{1}{4} S - s^2 c^2 T \right)$$

Proof: $\rho_*(0) - 1 = \frac{e^2}{s^2 c^2 M_Z^2} [\pi_{11}(0) - \pi_{33}(0)] \equiv \alpha T \quad \checkmark$

36

III
 $\propto T$

$$Z_{Z^*}(q^2) - 1 = \frac{e^2}{s^2 c^2} \left[\left(\frac{d\pi_{33}}{dq^2} - 2s^2 \frac{d\pi_{3Q}}{dq^2} + s^4 \frac{d\pi_{QQ}}{dq^2} \right)_{q^2=M_Z^2} - (c^2 - s^2) \pi'_{3Q}(q^2) - s^4 \pi'_{QQ}(q^2) \right]$$

$\underbrace{\pi'_{3Q}(q^2)}_{\pi'_{3Q}(0)} \quad \underbrace{\pi'_{QQ}(q^2)}_{\pi'_{QQ}(0)}$

由 $\pi_{33}(q^2) \approx \pi_{33}(0) + q^2 \pi'_{33}(0) \Rightarrow \frac{d\pi_{33}}{dq^2} \approx \pi'_{33}(0)$

$\pi_{3Q}(q^2) \approx q^2 \pi'_{3Q}(0) \Rightarrow \frac{d\pi_{3Q}}{dq^2} \approx \pi'_{3Q}(0)$

$\pi_{QQ}(q^2) \approx q^2 \pi'_{QQ}(0) \Rightarrow \frac{d\pi_{QQ}}{dq^2} \approx \pi'_{QQ}(0)$

$$\Rightarrow Z_{Z^*}(q^2) - 1 \approx \frac{e^2}{s^2 c^2} \left[\pi'_{33} - 2s^2 \pi'_{3Q} + s^4 \pi'_{QQ} - (c^2 - s^2) \pi'_{3Q} - s^4 \pi'_{QQ} \right]$$

$\frac{\alpha}{4} S \leftarrow$

$$= \frac{e^2}{s^2 c^2} (\pi'_{33} - \pi'_{3Q})$$

$$-(c^2 + s^2) \pi'_{3Q} = -\pi'_{3Q}$$

$$= \frac{\alpha}{4s^2 c^2} S \quad \checkmark$$

$$Z_{W*}(q^2) - 1 = \frac{e^2}{s^2} \left[\underbrace{\left. \frac{d\pi_{11}}{dq^2} \right|_{q^2=M_W^2}}_{\pi'_{11}(0)} - \underbrace{\pi'_{3Q}(q^2)}_{\pi'_{3Q}(0)} \right] \approx \frac{e^2}{s^2} \underbrace{(\pi'_{11} - \pi'_{3Q})}_{\text{II}}$$

$$\Rightarrow Z_{W*}(q^2) - 1 \approx \frac{\alpha}{4s^2} (S + U) \quad \checkmark$$

$$\underbrace{\pi'_{11} - \pi'_{33}}_{\text{III}} + \underbrace{\pi'_{33} - \pi'_{3Q}}_{\text{III}} \\ \frac{\alpha}{4e^2} U \quad \frac{\alpha}{4e^2} S$$

$$S_D^2 - S_W^2 = \frac{M_W^2}{M_Z^2} - c^2$$

$$= \frac{e^2 c^2}{s^2 (c^2 - s^2) M_Z^2} \left[\underbrace{\frac{c^2 - s^2}{c^2} \pi_{11}(M_W^2)}_{\pi_{11} + M_W^2 \pi'_{11}} + \frac{s^2}{c^2} \pi_{11}(0) - \underbrace{\pi_{33}(M_Z^2)}_{\pi_{33} + M_Z^2 \pi'_{33}} + 2s^2 \underbrace{\pi_{3Q}(M_Z^2)}_{M_Z^2 \pi'_{3Q}} \right] + \frac{e^2 s^2 c^2}{c^2 - s^2} \underbrace{[\pi'_{0Q}(0) - \pi'_{0Q}(M_Z^2)]}_{0}$$

$$\approx \frac{e^2 c^2}{s^2 (c^2 - s^2) M_Z^2} \left[\underbrace{\frac{c^2 - s^2}{c^2} (\pi_{11} + M_W^2 \pi'_{11})}_{\text{II}} + \frac{s^2}{c^2} \pi_{11} - (\pi_{33} + M_Z^2 \pi'_{33}) + 2s^2 M_Z^2 \pi'_{3Q} \right]$$

$$\underbrace{\left(\frac{c^2 - s^2}{c^2} + \frac{s^2}{c^2} \right) \pi_{11}}_{\text{II}} + (c^2 - s^2) \underbrace{\left(\frac{M_W^2}{c^2} \right) \pi'_{11}}_{M_Z^2} - \pi_{33} - M_Z^2 \pi'_{33} + 2s^2 M_Z^2 \pi'_{3Q}$$

$$= \frac{e^2 c^2}{s^2(c^2 - s^2) M_Z^2} \left[\underbrace{\pi_{11} - \pi_{33}}_{\parallel} + M_Z^2 \underbrace{[(c^2 - s^2) \pi'_{11} - \pi'_{33} + 2s^2 \pi'_{3Q}]}_{\parallel} \right]$$

$$\frac{\alpha s^2 c^2 M_Z^2}{e^2} T \quad \quad \quad (c^2 - s^2) \underbrace{(\pi'_{11} - \pi'_{33})}_{\parallel} - 2s^2 \underbrace{(\pi'_{33} - \pi'_{3Q})}_{\parallel}$$

$$\quad \quad \quad \frac{\alpha}{4e^2} U \quad \quad \quad \frac{\alpha}{4e^2} S$$

$$= \frac{e^2 c^2}{s^2(c^2 - s^2) M_Z^2} \left[\frac{\alpha s^2 c^2 M_Z^2}{e^2} T + M_Z^2 (c^2 - s^2) \frac{\alpha}{4e^2} U - M_Z^2 2s^2 \frac{\alpha}{4e^2} S \right]$$

$$= \frac{\alpha c^2}{c^2 - s^2} \left[-\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4s^2} U \right] \quad \checkmark$$

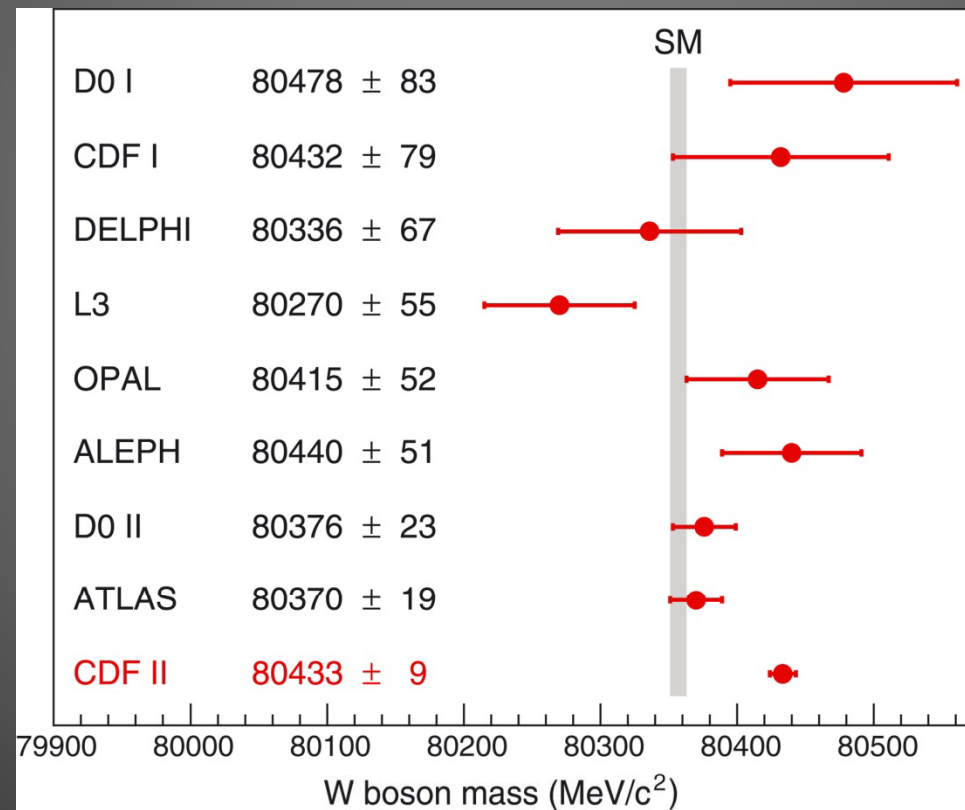
$$S_*^2(q^2) - S_0^2 = \frac{e^2}{c^2 - s^2} \left[\frac{\pi_{33} + M_Z^2 \pi'_{33}}{M_Z^2} - 2s^2 \frac{M_Z^2 \pi'_{3Q}}{M_Z^2} - \pi_{11}(0) - (c^2 - s^2) \pi'_{3Q}(q^2) \right]$$

$$+ \frac{e^2 s^2}{c^2 - s^2} \left[s^2 \pi'_{QQ}(M_Z^2) - c^2 \pi'_{QQ}(0) + (c^2 - s^2) \pi'_{QQ}(q^2) \right]$$

SS
0

$$m_W^2 = m_W^2(\text{ref})$$

$$+ \frac{\alpha c^2}{c^2 - s^2} m_Z^2 \left[-\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4s^2} U \right]$$



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与S参数有关的量纲为6的算符

• 量纲为6的算符 $H^\dagger W_{\mu\nu}^a \delta^a H B^{\mu\nu}$ 对S参数的贡献

1

设 $\mathcal{L} \supset \underbrace{\lambda H^\dagger W_{\mu\nu}^a \delta^a H B^{\mu\nu}}_{\substack{\text{耦合常数} \\ \text{量纲为6}}}$, 其中 $\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

则它对S参数的贡献是

$$\Delta S = \frac{4sc}{\alpha} \lambda v^2$$

其中 $s \equiv \sin\theta_W$, $c \equiv \cos\theta_W$

Proof: $\mathcal{L} \supset \lambda H^\dagger W_{\mu\nu}^a \delta^a H B^{\mu\nu} \supset \lambda \underbrace{\langle H^\dagger \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 1 \end{pmatrix}} \underbrace{W_{\mu\nu}^a \delta^a}_{\begin{pmatrix} * & * \\ * & -W_{\mu\nu}^3 \end{pmatrix}} \underbrace{\langle H \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} B^{\mu\nu}$

$$\Rightarrow \mathcal{L} \supset -\lambda \frac{v^2}{2} W_{\mu\nu}^3 B^{\mu\nu}$$

2

$$\begin{aligned} \text{而 } W_{\mu\nu}^3 B^{\mu\nu} &\supset (\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)(\partial^\mu B^\nu - \partial^\nu B^\mu) \\ &= 2(-\partial_\nu W_\mu^3)(\partial^\mu B^\nu - \partial^\nu B^\mu) \\ &= 2W_\mu^3(\partial_\nu \partial^\mu B^\nu - \partial^2 B^\mu) \\ &= -2W_\mu^3(\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) B_\nu \end{aligned}$$

$$\Rightarrow \mathcal{L} \supset \lambda v^2 W_\mu^3 (\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) B_\nu$$

切换到动量空间: $\partial_\mu \rightarrow -ik_\mu$, 有

$$\mathcal{L} \supset -\lambda v^2 W_\mu^3 (\eta^{\mu\nu} k^2 - k^\mu k^\nu) B_\nu$$

由 $\mathcal{L} \supset -\lambda v^2 W_\mu^3 (\eta^{\mu\nu} k^2 - k^\mu k^\nu) B_\nu$, 可读出 Feynman 规则. [3]

$$W_\mu^3 \text{ --- } \times \text{ --- } B_\nu = -i\lambda v^2 (\eta^{\mu\nu} k^2 - k^\mu k^\nu) \equiv i\pi_{W^3 B}^{\mu\nu}$$

$$\Rightarrow \pi_{W^3 B}^{\mu\nu} = -\lambda v^2 (\eta^{\mu\nu} k^2 - k^\mu k^\nu)$$

|||

$$\pi_{W^3 B}^{\mu\nu} \eta^{\mu\nu} + (k^\mu k^\nu \text{ term}) \Rightarrow \pi_{W^3 B} = -\lambda v^2 k^2$$

$$\text{由 } \pi_{W^3 B}(k^2) \approx \pi_{W^3 B}(0) + k^2 \pi'_{W^3 B}(0)$$

$$\Rightarrow \pi'_{W^3 B} = -\lambda v^2$$

另一方面, 由 $\mathcal{L} \supset g W_\mu^3 J_3^\mu + g' B_\mu J_Y^\mu \equiv W_\mu^3 J_{W^3}^\mu + B_\mu J_B^\mu$ 4

$$\Rightarrow J_{W^3}^\mu = g J_3^\mu, \quad J_B^\mu = g' J_Y^\mu$$

$$\Rightarrow \langle J_{W^3} J_B \rangle = g g' \langle J_3 J_Y \rangle$$

$$\Rightarrow \Pi_{W^3 B} = g g' \Pi_{3Y}$$

$$\left(\begin{array}{l} g = \frac{e}{s}, \\ g' = \frac{e}{c} \end{array} \right)$$

$$\frac{sc}{e^2}$$

因此, 由上页的 $\Pi'_{W^3 B} = -\lambda v^2 \Rightarrow \Pi'_{3Y} = \frac{1}{g g'} (-\lambda v^2)$

由 S 参数定义: $\alpha S \equiv 4e^2 (\Pi'_{33} - \Pi'_{3Q}) = -4e^2 \Pi'_{3Y}$

$$\Rightarrow S = -\frac{4e^2}{\alpha} \Pi'_{3Y}$$

$\Rightarrow$ 该算符的贡献: $\Delta S = -\frac{4e^2}{\alpha} \cdot \frac{sc}{e^2} (-\lambda v^2) = \frac{4sc}{\alpha} \lambda v^2$ ✓

与T参数有关的量纲为6的算符

量纲为6的算符 $|H^\dagger(D_\mu H)|^2 = [(D_\mu H)^\dagger H][H^\dagger(D^\mu H)]$ 对T参数的贡献 11

设 $\mathcal{L} \supset \lambda |H^\dagger(D_\mu H)|^2 = \lambda [(D_\mu H)^\dagger H][H^\dagger(D^\mu H)]$

$\downarrow$ 耦合常数 $\rightarrow$ 量纲为6

则它对T参数的贡献是

$$\Delta T = -\frac{1}{2\alpha} \lambda v^2$$

Proof: $D_\mu H = (\partial_\mu - ig T^a W_\mu^a - ig' Y B_\mu) H$

$$\Rightarrow (D_\mu H)^\dagger = D_\mu H^\dagger = H^\dagger (\partial_\mu + ig T^a W_\mu^a + ig' Y B_\mu)$$

由于 $H \sim (2, \frac{1}{2})$, 可知: $T^a = \frac{\sigma^a}{2}$, $Y = \frac{1}{2}$

当 $T^a = \frac{\sigma^a}{2}$, $Y = \frac{1}{2}$ 时,

2

$$gT^a W^a + g' Y B = \frac{1}{2} \begin{pmatrix} * & * \\ * & -gW^3 + g'B \end{pmatrix}$$

$$\Rightarrow H^\dagger D_\mu H \supset H^\dagger (-i)(gT^a W^a + g' Y B) H$$

$$\begin{aligned} &\supset (-i) \underbrace{\langle H^\dagger \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 1 \end{pmatrix}} \underbrace{(gT^a W^a + g' Y B)}_{\frac{1}{2} \begin{pmatrix} * & * \\ * & -gW^3 + g'B \end{pmatrix}} \underbrace{\langle H \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \\ &= (-i) \frac{v^2}{4} (-gW^3 + g'B) \end{aligned}$$

同理, 有 $(D_\mu H^\dagger) H \supset H^\dagger i(gT^a W^a + g' Y B) H$

$$\supset i \frac{v^2}{4} (-gW^3 + g'B)$$

$$\Rightarrow \mathcal{L} \supset \lambda [(D_\mu H^\dagger) H] [H^\dagger (D^\mu H)]$$

3

$$\supset \lambda \frac{v^4}{16} \frac{(gW^3 - g'B)^2}{\parallel}$$

$$(g^2 + g'^2) \left(\frac{g}{\sqrt{g^2 + g'^2}} W^3 - \frac{g'}{\sqrt{g^2 + g'^2}} B \right)^2$$

$\parallel$

$$(g^2 + g'^2) Z^2$$

$$= \lambda \frac{v^4}{16} (g^2 + g'^2) Z^2$$

由此可读出 Feynman 规则:

$$Z_\mu \text{---} \times \text{---} Z_\nu = i \lambda \frac{v^4}{16} (g^2 + g'^2) \times 2! \eta^{\mu\nu} = i \lambda \frac{v^4}{8} (g^2 + g'^2) \eta^{\mu\nu} \\ \equiv i \Pi_{ZZ}^{\mu\nu}$$

$$\Rightarrow \pi_{zz}^{\mu\nu} = \lambda \frac{v^4}{8} (g^2 + g'^2) \eta^{\mu\nu}$$

$$\pi_{zz} \eta^{\mu\nu} + (k^\mu k^\nu \text{ term})$$

$$\Rightarrow \pi_{zz} = \lambda \frac{v^4}{8} (g^2 + g'^2)$$

$$(\because g = \frac{e}{s}, g' = \frac{e}{c}) \quad \frac{4}{s^2 c^2}$$

$$\frac{e^2}{s^2 c^2}$$

对比 $\pi_{zz} = \frac{e^2}{s^2 c^2} (\pi_{33} - 2s^2 \pi_{3Q} + s^4 \pi_{QQ})$, 可得

$$\pi_{33} = \lambda \frac{v^4}{8}, \quad \pi_{3Q} = \pi_{QQ} = 0$$

由 T 的定义: $\Delta T \equiv \frac{e^2}{s^2 c^2 M_Z^2} (\pi_{11} - \pi_{33}) = \frac{4}{v^2} (\pi_{11} - \pi_{33})$

$M_Z^2 = \frac{(g^2 + g'^2) v^2}{4} = \frac{v^2}{4} \frac{e^2}{s^2 c^2}$

$\Rightarrow \Delta T = -\frac{\lambda v^2}{2\alpha}$ ✓

与U参数有关的量纲为8的算符

· 量纲为8的算符 $(H^\dagger W_{\mu\nu}^a \delta^a H)^2$ 对U参数的贡献

设 $\mathcal{L} \supset \lambda (H^\dagger W_{\mu\nu}^a \delta^a H)^2 = \lambda (H^\dagger W_{\mu\nu}^a \delta^a H)(H^\dagger W^{b\mu\nu} \delta^b H),$

耦合常数

量纲为8 其中 $\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

则它对U参数的贡献是

$$\Delta U = -\frac{4s^2}{\alpha} \lambda v^4$$

其中 $s \equiv \sin\theta_w$

Proof: $H^\dagger W_{\mu\nu}^a \delta^a H \supset \underbrace{\langle H^\dagger \rangle}_{\frac{v}{\sqrt{2}}(0 \ 1)} \underbrace{W_{\mu\nu}^a \delta^a}_{\begin{pmatrix} * & * \\ * & -W_{\mu\nu}^3 \end{pmatrix}} \underbrace{\langle H \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{v^2}{2} (-W_{\mu\nu}^3)$

$$\Rightarrow (H^\dagger W_{\mu\nu}^a \delta^a H)^2 \supset \frac{v^4}{4} W_{\mu\nu}^3 W^{3\mu\nu}$$

因此, $\mathcal{L} \supset \lambda (H^\dagger W_{\mu\nu}^a \delta^a H)^2 \supset \lambda \frac{v^4}{4} \underbrace{W_{\mu\nu}^3 W^{3\mu\nu}}_{\parallel}$

2

$$\parallel -2 W_\mu^3 (\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) W_\nu^3$$

$$\Rightarrow \mathcal{L} \supset -\frac{\lambda v^4}{2} W_\mu^3 (\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) W_\nu^3$$

切换到动量空间: $\partial_\mu \rightarrow -ik_\mu$, 有

$$\mathcal{L} \supset \frac{\lambda v^4}{2} W_\mu^3 (\eta^{\mu\nu} k^2 - k^\mu k^\nu) W_\nu^3$$

由此可读出 Feynman 规则:

$$W_\mu^3 \text{---} \times \text{---} W_\nu^3 = i \frac{\lambda v^4}{2} \times 2! (\eta^{\mu\nu} k^2 - k^\mu k^\nu) \equiv i \Pi_{W^3 W^3}^{\mu\nu}$$

$$\Rightarrow \Pi_{W^3 W^3}^{\mu\nu} = \lambda v^4 (\eta^{\mu\nu} k^2 - k^\mu k^\nu)$$

$$\parallel \Pi_{W^3 W^3}^{\mu\nu} \eta^{\mu\nu} + (k^\mu k^\nu \text{ term})$$

$$\Rightarrow \Pi_{W^3 W^3} = \lambda v^4 k^2$$

$$\parallel \Pi_{W^3 W^3}(0) + k^2 \Pi_{W^3 W^3}(0)$$

$$\Rightarrow \pi'_{W^3 W^3} = \lambda v^4$$

另一方面, 由 $\mathcal{L} \supset g W_\mu^3 J_3^\mu \equiv W_\mu^3 J_{W^3}^\mu$

$$\Rightarrow J_{W^3}^\mu = g J_3^\mu \quad \Rightarrow \langle J_{W^3} J_{W^3} \rangle = g^2 \langle J_3 J_3 \rangle$$

$$\Rightarrow \pi_{W^3 W^3} = g^2 \pi_{33}$$

因此, 由 $\pi'_{W^3 W^3} = \lambda v^4$ 可得 $\pi'_{33} = \frac{1}{g^2} \pi'_{W^3 W^3} = \frac{1}{g^2} \lambda v^4 = \frac{s^2}{e^2} \lambda v^4$

由 U 参数定义: $\alpha U \equiv 4e^2 (\pi'_{11} - \pi'_{33})$

$\Rightarrow$ 该算符对 U 参数的贡献为 $\Delta U = \frac{4e^2}{\alpha} (\underbrace{\pi'_{11}}_0 - \underbrace{\pi'_{33}}_{\frac{s^2}{e^2} \lambda v^4}) = -\frac{4e^2}{\alpha} \cdot \frac{s^2}{e^2} \lambda v^4 = -\frac{4s^2}{\alpha} \lambda v^4$ ✓