

微扰QCD和对撞机物理

朱华星 (浙江大学)

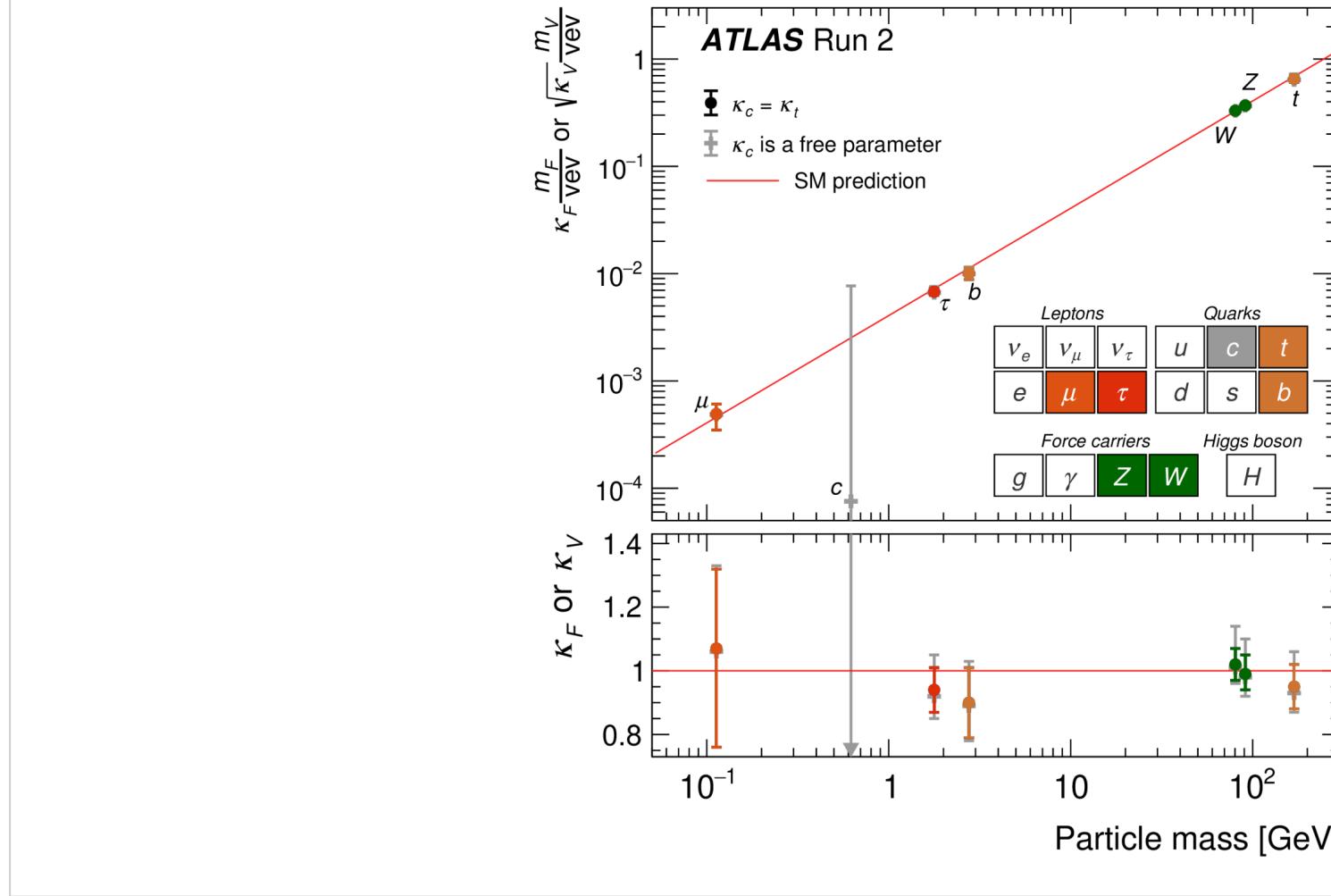
暗物质与新物理暑期学校

南京师范大学，2022年7月

- 2012年7月4日ATLAS和CMS宣布发现希格斯玻色子



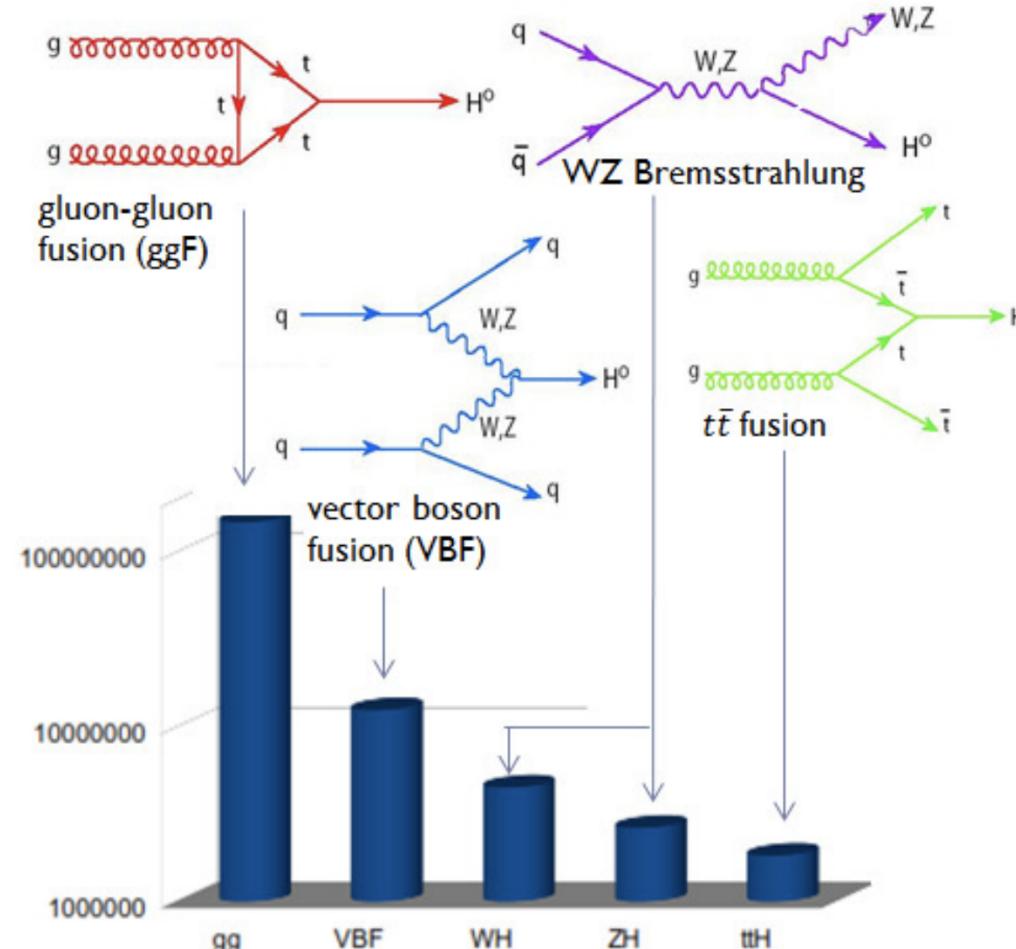
- 2022年7月4日CERN庆祝希格斯玻色子发现10周年



- 希格斯粒子的发现以及对它性质的精细测量是系统工程，有赖实验学家和理论学家的长期通力合作。
- 今天我想向大家介绍的是微扰QCD在希格斯发现和精细测量中的作用

希格斯的产生模式

- 希格斯粒子与标准模型其它粒子的耦合正比于粒子质量
- 与希格斯耦合最强的是顶夸克, Z 玻色子, W 玻色子
- 但LHC是质子-质子对撞, 为了产生希格斯玻色子, 我们需要从质子中先得到顶夸克, Z 玻色子, W 玻色子



这个课程里我们聚焦在胶子融合顶夸克产生道

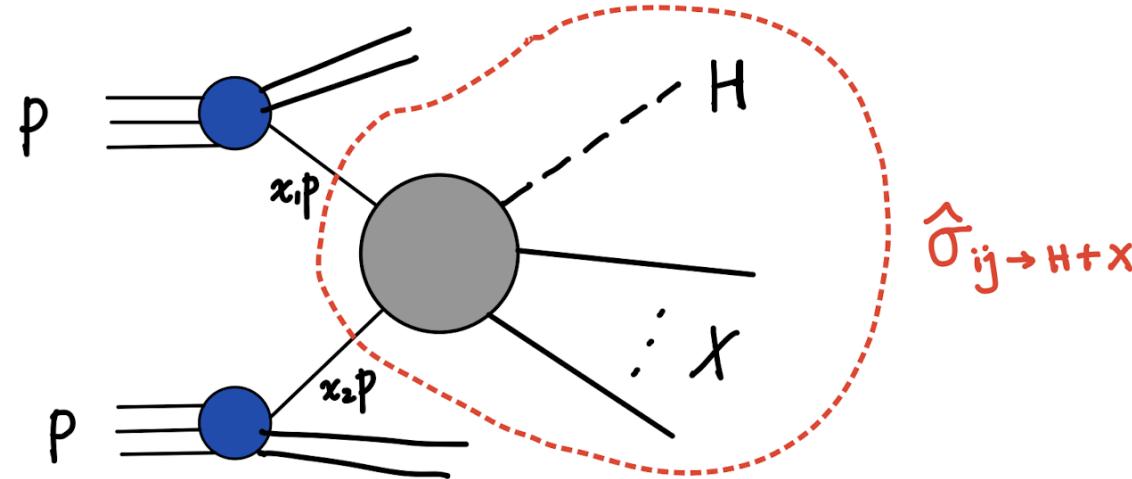
这个课程的目的

- 计算 $pp \rightarrow H$ 在 LHC 13 TeV 的产生截面 $\sigma_{pp \rightarrow H}$ ($\sqrt{S} = 13 \text{TeV}$)
- 尽可能提高理论精度
- 这个过程中我们会涉猎微扰 QCD 的一些基本知识和计算方法
- 并且将这些计算在计算机上实现



QCD共线因子化定理

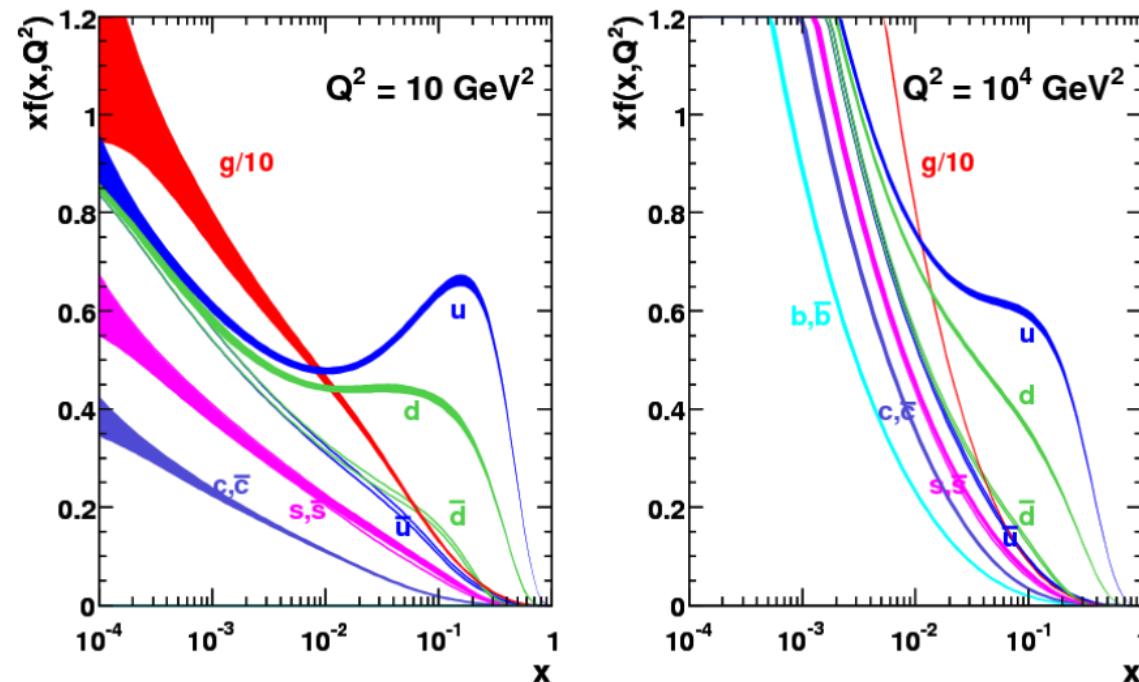
$$\sigma(H + X) = \sum_{i,j} \int dx_1 f_i(x_1, \mu_F) \int dx_2 f_j(x_2, \mu_F) \times \hat{\sigma}_{ij \rightarrow H+X}$$



部分子分布函数

$f_i(x, Q)$:在质子中找到动量分数在 $(x, x + dx)$ 间的部分子*i*的概率

MSTW 2008 NLO PDFs (68% C.L.)

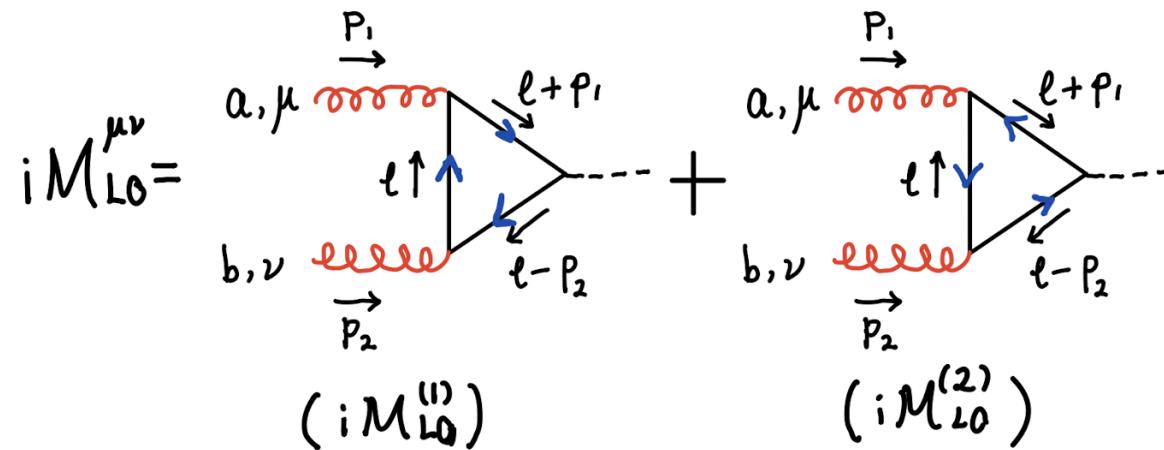


散射截面的微扰展开

$$\begin{aligned}\hat{\sigma}_{ij \rightarrow H+X} = & \hat{\sigma}^{(0)}(ij \rightarrow H) \quad (\text{LO, 领头阶}) \\ & + \hat{\sigma}^{(1)}(ij \rightarrow H + \text{up to 1 parton}) \quad (\text{NLO, 次领头阶}) \\ & + \hat{\sigma}^{(2)}(ij \rightarrow H + \text{up to 2 partons}) \quad (\text{NNLO, 次次领头阶}) \\ & + \dots\end{aligned}$$

领头阶计算

领头阶费曼图 (单圈图)



- $p_1^2 = p_2^2 = 0$
- $p_H = p_1 + p_2$
- $p_H^2 = (p_1 + p_2)^2 = 2p_1 \cdot p_2 = M_H^2$
- 无量纲比值: $a = 4M_Q^2/M_H^2$, M_Q 夸克质量

$$iM_{\text{LO}}^{\mu\nu} = \frac{P_1}{a, \mu} \text{---} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} + \frac{P_1}{a, \mu} \text{---} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array}$$

$\ell \uparrow$

b, ν $\overrightarrow{P_2}$

$\ell + P_1$

$\ell - P_2$

$(iM_{\text{LO}}^{(1)})$

$\ell \uparrow$

b, ν $\overrightarrow{P_2}$

$\ell + P_1$

$\ell - P_2$

$(iM_{\text{LO}}^{(2)})$

- 洛伦兹对称性 + 量纲分析 + Ward恒等式

$$\begin{aligned} M_{\text{LO}}^{\mu\nu} &= M_{\text{LO}}^{(1),\mu\nu} + M_{\text{LO}}^{(2),\mu\nu} \\ M_{\text{LO}}^{(2),\mu\nu} &= M_{\text{LO}}^{(1),\mu\nu}(p_1 \leftrightarrow p_2, \mu \leftrightarrow \nu, a \leftrightarrow b) \\ iM_{\text{LO}}^{\mu\nu} &= i \frac{1}{v} (p_2^\mu p_1^\nu - g^{\mu\nu} p_1 \cdot p_2) F(4M_Q^2/M_H^2) \end{aligned}$$

- $F(a)$ 是一个无量纲形状因子。最后散射振幅还需乘上胶子的极化矢量

$$iM = iM_{\text{LO}}^{\mu\nu} \epsilon_\mu(p_1) \epsilon_\nu(p_2)$$

费曼规则

A Feynman diagram showing a quark loop with a magnetic field H . The loop consists of two solid lines and two dashed lines. The top solid line is labeled j , the bottom solid line is labeled i , the left dashed line is labeled Q , and the right dashed line is labeled H . The loop is oriented clockwise.

$$-i \frac{M_Q}{v}$$

A Feynman diagram showing a quark loop with a gluon exchange between vertices i and j . The loop consists of two solid lines and two dashed lines. The top solid line is labeled j , the bottom solid line is labeled i , the left dashed line is labeled μ, a , and the right dashed line is labeled γ . The loop is oriented clockwise.

$$-i g_s \gamma^\mu T^a_{ji}$$

A Feynman diagram showing a quark loop with a propagator from vertex i to vertex j . The loop consists of two solid lines and two dashed lines. The top solid line is labeled j , the bottom solid line is labeled i , the left dashed line is labeled P , and the right dashed line is labeled Q .

$$\delta_{ij} \frac{i(P + M_Q)}{P^2 - M_Q^2 + i\epsilon}$$

$$iM_{LO}^{\mu\nu} =$$

The equation shows the LO contribution to the quark loop amplitude $iM_{LO}^{\mu\nu}$ as a sum of two diagrams. Both diagrams involve a quark loop with a gluon exchange between vertices a, μ and b, ν . The top diagram has a red wavy line entering vertex a, μ with momentum P_1 and a red wavy line exiting vertex b, ν with momentum P_2 . The loop is oriented clockwise. The bottom diagram is similar but with a different internal loop orientation. Blue arrows indicate momenta ℓ and ℓ' flowing through the loop.

$$(iM_{LO}^{(1)}) \quad (iM_{LO}^{(2)})$$

$$\begin{aligned}
iM_{\text{LO}}^{(1),\mu\nu} &= \int \frac{d^4l}{(2\pi)^4} (-) \text{Tr} \left[(-ig_s \gamma^\mu T_{ji}^a) \frac{i(l_\rho \gamma^\rho + M_Q)}{l^2 - M_Q^2 + i\varepsilon} (-ig_s \gamma^\nu T_{ij}^a) \frac{i((l-p_2)_\lambda \gamma^\lambda + M_Q)}{(l-p_2)^2 - M_Q^2 + i\varepsilon} \left(-i \frac{M_Q}{v}\right) \frac{i((l+p_1)_\sigma \gamma^\sigma + M_Q)}{(l+p_1)^2 - M_Q^2 + i\varepsilon} \right] \\
&= -g_s^2 \frac{M_Q}{v} \frac{\delta^{ab}}{2} \int \frac{d^4l}{(2\pi)^4} \frac{N^{\mu\nu}(l, p_1, p_2)}{(l^2 - M_Q^2)((l-p_2)^2 - M_Q^2)((l+p_1)^2 - M_Q^2)}
\end{aligned}$$

$$N^{\mu\nu}(l, p_1, p_2) = \text{Tr} [\gamma^\mu (l + M_Q) \gamma^\nu (l - p_2 + M_Q) (l + p_1 + M_Q)]$$

Let's evaluate the numerator Dirac trace first.

You can do it by hand but nowadays it's much convenient to do it using computer algebra tools, such as Mathematica and many high energy physics package.

In this lecture we will use the FeynCalc package.

For installation instruction of FeynCalc see <https://feyncalc.github.io/> (<https://feyncalc.github.io/>)

Once installed, we can load the package using

In [1]: `<<FeynCalc``

```
FeynCalc 9.3.1 (stable version). For help, use the documentation center,\

>     check out the wiki or visit the forum.
To save your and our time, please check our FAQ

>     for answers to some common FeynCalc questions.
See also the supplied examples. If you use FeynCalc in your research, please\

>     cite
\[Bullet] V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256\

>     (2020) 107478, arXiv:2001.04407.
\[Bullet] V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207\

>     (2016) 432–444, arXiv:1601.01167.
\[Bullet] R. Mertig, M. Bohm, and A. Denner, Comput. Phys. Commun. 64\

>     (1991) 345–359.
```

In FeynCalc a D-dimensional Dirac Gamma matrix γ^μ is represented as

```
In [2]: GAD[a]//HoldForm//TeXForm
```

```
Out[2]:  $\gamma^a$ 
```

Feynman's slash notation $\not{p} \equiv \gamma^\mu p_\mu$ is represented as

```
In [3]: GSD[p]//HoldForm//TeXForm
```

```
Out[3]:  $\gamma \cdot p$ 
```

And a Dirac trace can be calculated as

```
In [4]: Tr[GAD[\mu].GAD[v]]//HoldForm//TeXForm
```

```
Out[4]:  $\text{Tr} [\gamma^\mu \cdot \gamma^\nu]$ 
```

```
In [5]: Tr[GAD[\mu].GAD[v]]
```

```
Out[5]:  $4 g^{\mu \nu}$ 
```

Now we are ready to evaluate the numerator trace

$$N^{\mu\nu}(l, p_1, p_2) = \text{Tr} [\gamma^\mu (l + M_Q) \gamma^\nu (l - \not{p}_2 + M_Q) (l + \not{p}_1 + M_Q)]$$

```
In [6]: Tr[GAD[\mu].(GSD[1]+MQ).GAD[v].(GSD[1]-GSD[p2]+MQ).(GSD[1]+GSD[p1]+MQ)]//HoldForm//TeXForm
```

```
Out[6]:  $\text{Tr} [\gamma^\mu \cdot (\gamma \cdot l + MQ) \cdot \gamma^\nu \cdot (\gamma \cdot l - \gamma \cdot p_2 + MQ) \cdot (\gamma \cdot l + \gamma \cdot p_1 + MQ)]$ 
```

```
In [7]: SPD[p1,p2]=MH^2/2;  
num = Tr[GAD[\mu].(GSD[1]+MQ).GAD[v].(GSD[1]-GSD[p2]+MQ).(GSD[1]+GSD[p1]+MQ)]//Simplify
```

```
Out[8]:  $2 MQ \left( -g^{\mu \nu} \left( 2 l^2 + MH^2 - 2 MQ^2 \right) + 4 l^\nu p_1^\mu + l^\mu (8 l^\nu - 4 p_2^\nu) + 2 p_1^\nu p_2^\mu - 2 p_1^\mu p_2^\nu \right)$ 
```

Next we combine the denominator using Feynman's trick

$$\frac{1}{ABC} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[Ax + By + C(1-x-y)]^3}$$

Applying the trick to our propagator

$$\begin{aligned}
 & \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}(l, p_1, p_2)}{(l^2 - M_Q^2)((l - p_2)^2 - M_Q^2)((l + p_1)^2 - M_Q^2)} \\
 &= 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}(l, p_1, p_2)}{[x((l - p_2)^2 - M_Q^2) + y((l + p_1)^2 - M_Q^2) + (1 - x - y)(l^2 - M_Q^2)]^3} \\
 &= 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}(l, p_1, p_2)}{[l^2 - M_Q^2 + 2l \cdot (yp_1 - xp_2)]^3} \\
 &= 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}(l, p_1, p_2)}{[(l + (yp_1 - xp_2))^2 - M_Q^2 + xyM_H^2]^3}
 \end{aligned}$$

```
$$
2 \int_0^1 dx \int_0^{1-x} dy \frac{N^{\mu\nu}(l - (yp_1 - xp_2), p_1, p_2)}{[l^2 - M_Q^2 + xyM_H^2]^3}
$$
```

The denominator suggests that we should shift the loop momentum as

$$l \rightarrow l - (yp_1 - xp_2)$$

We then have

$$2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}(l - (yp_1 - xp_2), p_1, p_2)}{[l^2 - M_Q^2 + xyM_H^2]^3}$$

The new numerator now becomes

In [9]: num

Out[9]: $2MQ(-g^{\mu\nu}(2l^2 + MH^2 - 2MQ^2) + 4l^\nu p_1^\mu + l^\mu(8l^\nu - 4p_2^\nu) + 2p_1^\nu p_2^\mu - 2p_1^\mu p_2^\nu)$

In [10]: num1=num/.FCI[{{SPD[1,1]->SPD[1,1]-2*y*SPD[1,p1]+2*x*SPD[1,p2]-x*y*MH^2,FVD[1,a_]:>FVD[1,a]-y*FVD[p1,a]+x*FVD[p2,a]}]/.Expand

Out[10]: $2MQ(-2l^2 g^{\mu\nu} + 4y g^{\mu\nu}(l \cdot p_1) - 4x g^{\mu\nu}(l \cdot p_2) - MH^2 g^{\mu\nu} + 2MH^2 xy g^{\mu\nu} + 2MQ^2 g^{\mu\nu} + 4l^\nu(p_1^\mu - 2y p_1^\mu + 2x p_2^\mu) + l^\mu(8l^\nu - 8y p_1^\nu - 4p_2^\nu + 8x p_2^\nu) + 2p_1^\nu p_2^\mu - 2p_1^\mu p_2^\nu + 4x p_1^\mu p_2^\nu - 8xy p_1^\nu p_2^\mu - 8xy p_1^\mu p_2^\nu + 4y p_1^\mu p_2^\nu + 8y^2 p_1^\mu p_1^\nu - 4y p_1^\mu p_1^\nu + 8x^2 p_2^\mu p_2^\nu - 4xp_2^\mu p_2^\nu)$

In [11]: num1/MQ//Expand//TeXForm

Out[11]: $-4lg^{\mu\nu} + 8yg^{\mu\nu}(l \cdot p_1) - 8xg^{\mu\nu}(l \cdot p_2) - 2MH^2 g^{\mu\nu} + 4MH^2 xy g^{\mu\nu} + 4MQ^2 g^{\mu\nu} + 16l^\mu l^\nu + 8l^\nu p_1^\mu - 16yl^\nu p_1^\mu - 16yl^\mu p_1^\nu - 8l^\mu p_2^\nu + 16xl^\nu p_2^\mu + 16xl^\mu p_2^\nu + 4p_1^\nu p_2^\mu - 4p_1^\mu p_2^\nu + 8xp_1^\mu p_2^\nu - 16xyp_1^\nu p_2^\mu - 16xyp_1^\mu p_2^\nu + 8yp_1^\mu p_2^\nu + 16y^2 p_1^\mu p_1^\nu - 8yp_1^\mu p_1^\nu + 16x^2 p_2^\mu p_2^\nu - 8xp_2^\mu p_2^\nu$

We need to simplify this expression a bit.

- First we note that linear terms in loop momentum can be dropped.
- Second we can use $l^\mu l^\nu \rightarrow \frac{1}{D} l^2 g^{\mu\nu}$
- Third we note that amplitudes is proportional to $N^{\mu\nu} \epsilon_\mu(p_1) \epsilon_\nu(p_2)$
- For physical transverse gluon, we have $p \cdot \epsilon(p) = 0$.
- Therefore we can also drop terms with p_1^μ and with p_2^ν .

In [12]: `num1//TeXForm`

Out[12]:
$$2MQ(-2lg^{\mu\nu} + 4yg^{\mu\nu}(l \cdot p1) - 4xg^{\mu\nu}(l \cdot p2) - MH^2g^{\mu\nu} + 2MH^2xyg^{\mu\nu} + 2MQ^2g^{\mu\nu} + 4l^\nu(p1^\mu - 2yp1^\mu + 2xp2^\mu) + l^\mu(8l^\nu - 8yp1^\nu - 4p2^\nu + 8xp2^\nu) + 2p1^\nu p2^\mu - 2p1^\mu p2^\nu + 4xp1^\mu p2^\nu - 8xyp1^\nu p2^\mu - 8xyp1^\mu p2^\nu + 4yp1^\mu p2^\nu + 8y^2p1^\mu p1^\nu - 4yp1^\mu p1^\nu + 8x^2p2^\mu p2^\nu - 4xp2^\mu p2^\nu)$$

In [13]: `num2=Expand[FCI[num1]]/.FCI[FVD[p1,μ]->0]/.FCI[FVD[p2,ν]->0]`

Out[13]:
$$-4l^2MQg^{\mu\nu} + 8MQyg^{\mu\nu}(l \cdot p1) - 8MQxg^{\mu\nu}(l \cdot p2) - 2MH^2MQg^{\mu\nu} + 4MH^2MQxyg^{\mu\nu} + 4MQ^3g^{\mu\nu} + 16MQl^\mu l^\nu - 16MQyl^\mu p1^\nu + 16MQxl^\nu p2^\mu + 4MQp1^\nu p2^\mu - 16MQxyp1^\nu p2^\mu$$

In [14]: `num3=num2/.FCI[SPD[1,p2]->0]/.FCI[SPD[1,p1]->0]/.FCI[FVD[1,μ]*FVD[1,ν]->MTD[μ,ν]*SPD[1,1]/D]`

Out[14]:
$$\frac{16l^2MQg^{\mu\nu}}{D} - 4l^2MQg^{\mu\nu} - 2MH^2MQg^{\mu\nu} + 4MH^2MQxyg^{\mu\nu} + 4MQ^3g^{\mu\nu} - 16MQyl^\mu p1^\nu + 16MQxl^\nu p2^\mu + 4MQp1^\nu p2^\mu - 16MQxyp1^\nu p2^\mu$$

In [15]: `num4=num3/.FCI[FVD[1,_]->0]//Simplify`

Out[15]:
$$\frac{2MQg^{\mu\nu}(-2(D-4)l^2 + D MH^2(2xy-1) + 2D MQ^2)}{D} + 4MQp1^\nu p2^\mu(1-4xy)$$

Our amplitude is now simplified to

$$M^{(1),\mu\nu} \propto M_Q \int \frac{d^4l}{(2\pi)^4} \frac{-4\frac{D-4}{D}l^2g^{\mu\nu} + (2M_H^2(2xy-1) + 4M_Q^2)g^{\mu\nu} + 4(1-4xy)p_1^\nu p_2^\mu}{(l^2 - \Delta)^3}$$

$$\Delta = M_Q^2 - xyM_H^2 - i\varepsilon$$

Dimensional regularization: $D = 4 - 2\epsilon$

$$\int \frac{d^4l}{(2\pi)^4} \rightarrow \int \frac{d^Dl}{(2\pi)^D}$$

Now we can use the following formula from Peskin:

$$I_1 = \int \frac{d^Dl}{(2\pi)^D} \frac{1}{(l^2 - \Delta)^3} = \frac{-i}{(4\pi)^{D/2}} \frac{\Gamma(1+\epsilon)}{\Gamma(3)} \left(\frac{1}{\Delta}\right)^{3-D/2}$$

$$I_2 = \int \frac{d^D l}{(2\pi)^D} \frac{l^2}{(l^2 - \Delta)^3} = \frac{i}{(4\pi)^{D/2}} \frac{D}{2} \frac{\Gamma(\epsilon)}{\Gamma(3)} \left(\frac{1}{\Delta} \right)^{2-D/2}$$

and our integral becomes

$$M^{(1),\mu\nu} \propto -4M_Q \frac{D-4}{D} g^{\mu\nu} I_2 + M_Q \left[(2M_H^2(2xy-1) + 4M_Q^2)g^{\mu\nu} + 4(1-4xy)p_1^\nu p_2^\mu \right] I_1$$

In [16]: `num4//TeXForm`

$$\text{Out[16]: } \frac{2MQg^{\mu\nu}(-2(D-4)l + DMH^2(2xy-1) + 2DMQ^2)}{D} + 4MQp1^\nu p2^\mu(1-4xy)$$

In [17]: `I1=-I/(4*Pi)^(D/2)*Gamma[1+eps]/Gamma[3]*(1/del)^(3-D/2);
I2=I/(4*Pi)^(D/2)*D/2*Gamma[eps]/Gamma[3]*(1/del)^(2-D/2);
num5=Coefficient[num4,FCI[SPD[1,1]]]*I2 + I1*(num4/.FCI[SPD[1,1]->0])`

$$\text{Out[19]: } i 2^{-D-2} D \pi^{-D/2} \left(\frac{1}{\text{del}} \right)^{2-\frac{D}{2}} \Gamma(\text{eps}) \left(\frac{16 MQ g^{\mu\nu}}{D} - 4 MQ g^{\mu\nu} \right) - i 2^{-D-1} \pi^{-D/2} \left(\frac{1}{\text{del}} \right)^{3-\frac{D}{2}} \Gamma(\text{eps}+1) \left(\frac{2 MQ g^{\mu\nu} (D MH^2 (2 xy - 1) + 2 D MQ^2)}{D} + 4 MQ p1^\nu p2^\mu (1 - 4 xy) \right)$$

In [20]: `num6=Series[num5/.D->4-2*eps,{eps,0,0}]//Normal//Simplify`

$$\text{Out[20]: } \frac{i MQ (g^{\mu\nu} (2 \text{del} - 2 MH^2 x y + MH^2 - 2 MQ^2) + 2 (4 x y - 1) \bar{p1}^\nu \bar{p2}^\mu)}{16 \pi^2 \text{del}}$$

In [22]: `num7=num6/.del->MQ^2-x*y*MH^2//Simplify//TeXForm`

$$\text{Out[22]: } -\frac{i MQ (4 x y - 1) \left(M H^2 \bar{g}^{\mu\nu} - 2 \bar{p1}^\nu \bar{p2}^\mu \right)}{16 \pi^2 (M Q^2 - M H^2 x y)}$$

After all these tedious calculation, the result turns out to be rather simple.

$$\begin{aligned} i M^{(1),\mu\nu} &= -g_s^2 \frac{M_Q}{v} \frac{\delta^{ab}}{2} 2(i) \frac{M_Q (M_H^2 g^{\mu\nu} - 2 p_1^\nu p_2^\mu)}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{M_Q^2 - M_H^2 xy} \\ M^{\mu\nu} &= 2 M^{(1),\mu\nu} = -\frac{\alpha_s}{2\pi} \frac{\delta^{ab}}{v} (M_H^2 g^{\mu\nu} - 2 p_1^\nu p_2^\mu) F\left(\frac{4M_Q^2}{M_H^2}\right) \\ F(a) &= \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{1 - \frac{4}{a} xy - i\varepsilon} \end{aligned}$$

We shall evaluate this integral for $a > 1$ (relevant for top quark), and leave the case $a < 1$ (relevant for b quark) as an exercise.

A detail account of this integral can be found at [\(http://scipp.ucsc.edu/~haber/webpage/oneloopfun4.pdf\)](http://scipp.ucsc.edu/~haber/webpage/oneloopfun4.pdf)

For $a > 1$, there is no imaginary part. We can drop the $i\varepsilon$ term. The dy integral is easily calculated with MMA:

In [23]: `Integrate[(1-4 x y)/(1-4/a*x*y),{y,0,1-x},Assumptions->{1>x>0,a>1}]//Collect[#,Log[_]]&//TeXForm`

Out[23]:
$$-a(x-1) + \frac{(1-a)a \log(a)}{4x} + \frac{(a-1)a \log(a+4(x-1)x)}{4x}$$

We are left with the integral

$$\begin{aligned} F(a) &= \int_0^1 dx \left((1-x)a + \frac{(a-1)a}{4} \frac{\log\left(1 - \frac{4x(1-x)}{a}\right)}{x} \right) \\ &= \frac{a}{2} + \frac{(a-1)a}{4} g(a), \quad g(a) = \int_0^1 dx \frac{\log\left(1 - \frac{4x(1-x)}{a}\right)}{x} \end{aligned}$$

$$g(a) = \int_0^1 dx \frac{\log\left(1 - \frac{4x(1-x)}{a}\right)}{x}$$

Let $a = 1/\sin^2 \theta$, $0 < \theta < \pi/2$, then

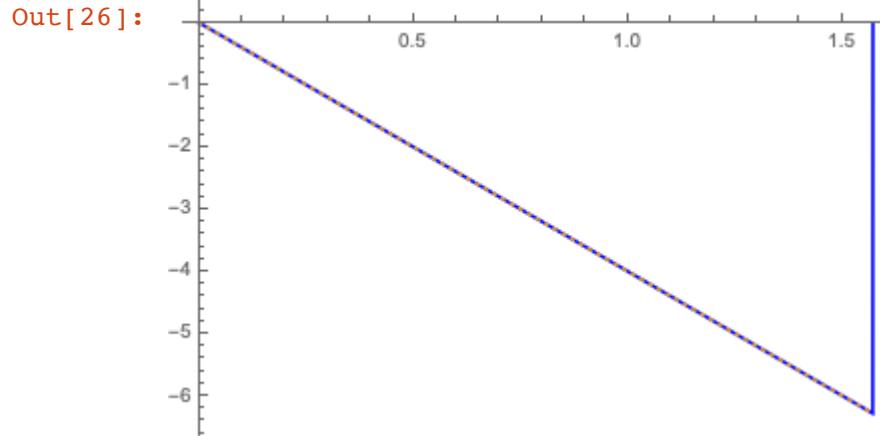
$$g(a) = \int_0^1 dx \frac{\log(1 - 4x(1-x)\sin^2 \theta)}{x}$$

Take derivative of θ and exchange derivative and integration, we get

$$\frac{dg(a)}{d\theta} = -4 \sin(2\theta) \int_0^1 \frac{(1-x)dx}{1 - 4x(1-x)\sin^2 \theta}$$

$$\frac{dg(a)}{d\theta} = -4 \sin(2\theta) \int_0^1 \frac{(1-x)dx}{1 - 4x(1-x)\sin^2 \theta}$$

```
In [26]: Plot[{NIntegrate[-4*Sin[2*th]*(1 - x)/(1 - 4*x*(1 - x)*Sin[th]^2), {x, 0, 1}], -4*th}, {th, 0, Pi/2}, PlotStyle -> {Blue, {Dotted}}}]
```



$$\frac{dg(\theta)}{d\theta} = -4\theta$$

$$g(\theta) = -2\theta^2, \quad \theta = \arcsin 1/\sqrt{a}$$

$$F(a) = \frac{a}{2} - \frac{(1-a)a}{4} g(a) = \frac{a}{2} + \frac{(1-a)a}{2} \arcsin^2 \frac{1}{\sqrt{a}}$$

We have finished the calculation of one-loop (leading order) amplitude:

$$M = -\frac{\alpha_s}{2\pi} \frac{\delta^{ab}}{v} (M_H^2 g^{\mu\nu} - 2p_1^\nu p_2^\mu) F\left(\frac{4M_Q^2}{M_H^2}\right) \epsilon_\mu(p_1) \epsilon_\nu(p_2)$$

For $a > 1$,

$$F(a) = \frac{a}{2} + \frac{(1-a)a}{2} \arcsin^2 \frac{1}{\sqrt{a}}$$

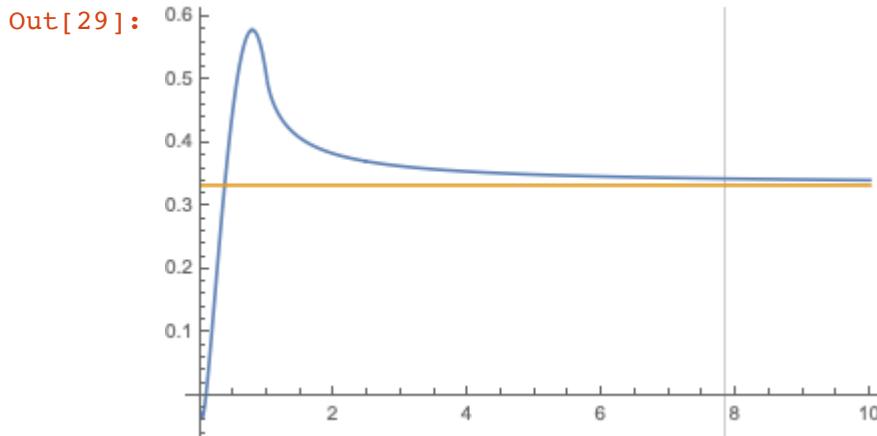
For $a < 1$,

$$F(a) = \frac{a}{2} - \frac{(1-a)a}{8} \left[\log \frac{1+\sqrt{1-a}}{1-\sqrt{1-a}} - i\pi \right]^2$$

```
In [27]: F[a_]:=Piecewise[{{a/2 + (1 - a)*a/2*ArcSin[1/Sqrt[a]]^2,
  a > 1}, {a/
  2 + (1 - a)*
  a/-8*(Log[(1 + Sqrt[1 - a])/(1 - Sqrt[1 - a])] - I*Pi)^2,
  0 < a < 1}}];
Limit[F[a],a->Infinity]//TeXForm
```

Out[28]: $\frac{1}{3}$

```
In [29]: Plot[{Re[F[a]],1/3},{a,10^-2,10},GridLines->{{4*175^2/125^2,0},{4*175^2/125^2,10}},PlotRange->All]
```



Question: why the amplitude does not vanish for infinite top quark mass?

Next we use the amplitude to calculate cross section.

$$|M|^2 = 4 \frac{\alpha_s^2}{\pi^2 v^2} M_H^4 \left| F \left(\frac{4M_Q^2}{M_H^2} \right) \right|^2$$

Let's consider the partonic cross section first.

$$\begin{aligned} \hat{\sigma}_{gg \rightarrow H} &= \frac{1}{2 \times 2^2 8^2 \hat{s}} \int \frac{d^3 P_H}{(2\pi)^3 2E_H} (2\pi)^4 \delta^4(p_1 + p_2 - P_H) |M|^2 \\ &= \frac{1}{512 \hat{s}} |M|^2 2\pi \delta(\hat{s} - M_H^2) \\ &= \frac{\alpha_s^2}{64\pi v^2} \left| F \left(\frac{4M_Q^2}{M_H^2} \right) \right|^2 \delta \left(1 - \frac{\tau}{x_1 x_2} \right), \quad \tau = \frac{M_H^2}{S} \end{aligned}$$

Hadronic cross section is then given by

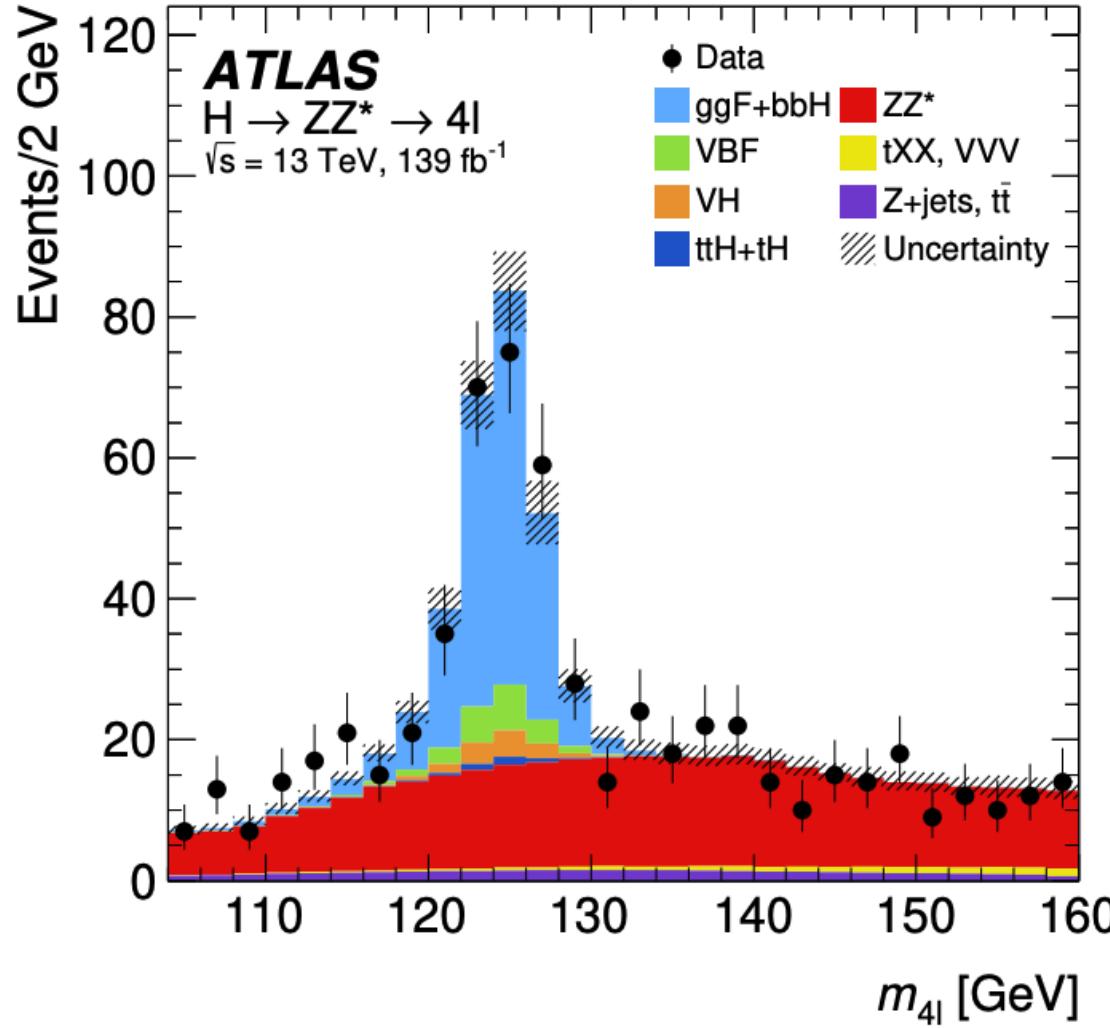
$$\begin{aligned}\sigma_{pp \rightarrow H} &= \int dx_1 f_g(x_1, \mu_F) \\ &\quad \int dx_2 f_g(x_2, \mu_F) \frac{\alpha_s^2}{64\pi v^2} \left| F\left(\frac{4M_Q^2}{M_H^2}\right) \right|^2 \delta\left(1 - \frac{\tau}{x_1 x_2}\right) \\ &= \tau \frac{\alpha_s^2(\mu_R)}{64\pi v^2} \left| F\left(\frac{4M_Q^2}{M_H^2}\right) \right|^2 \int_\tau^1 dx_1 \frac{f_g(x_1, \mu_F)}{x_1} f_g\left(\frac{\tau}{x_1}, \mu_F\right)\end{aligned}$$





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$$\sigma_H \cdot \mathcal{B}_{H \rightarrow ZZ^*} = 1.34 \pm 0.11(\text{stat.}) \pm 0.04(\text{exp.}) \text{ pb}$$

$$\mathcal{B}_{H \rightarrow ZZ^*} = 2.64 \times 10^{-2}$$

$$\sigma_H \approx 50 \text{ pb}$$



```
In [ ]: https://github.com/JohnChen14/higgs_production
```

```
In [ ]:
```