Institute of Theoretical Physics
Chinese Academy of Sciences

NNU · 南京师范大学
NANJING NORMAL UNIVERSITY

有效场论
Effective Field Theories

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暗物质与新物理暑期学校

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Outline

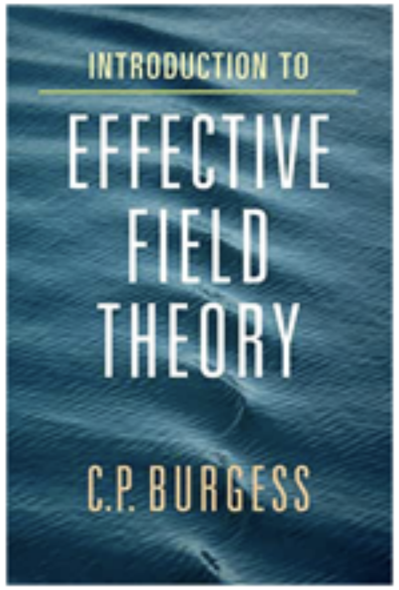
- Conceptual Overview on Effective Field Theory

- Practical Calculation on Matching and Running

- Quest for New Physics in the Standard Model EFT

- Chiral Lagrangian for QCD and Electroweak Theory

References



Introduction to Effective Field Theory: Thinking Effectively about Hierarchies of Scale
by C. P. Burgess | Jan 21, 2021
Hardcover
80^{82}

Introduction to Effective Field Theories #3

Aneesh V. Manohar (UC, San Diego) (Apr 16, 2018)

Published in: *Les Houches Lect.Notes* 108 (2020) • Contribution to: Les Houches summer school • e-Print: 1804.05863 [hep-ph]

As Scales Become Separated: Lectures on Effective Field Theory

Timothy Cohen (Oregon U.) (Mar 8, 2019)

Published in: *PoS* TASI2018 (2019) 011 • Contribution to: TASI 2018, 011 • e-Print: 1903.03622

Effective Field Theory and Precision Electroweak Measurements

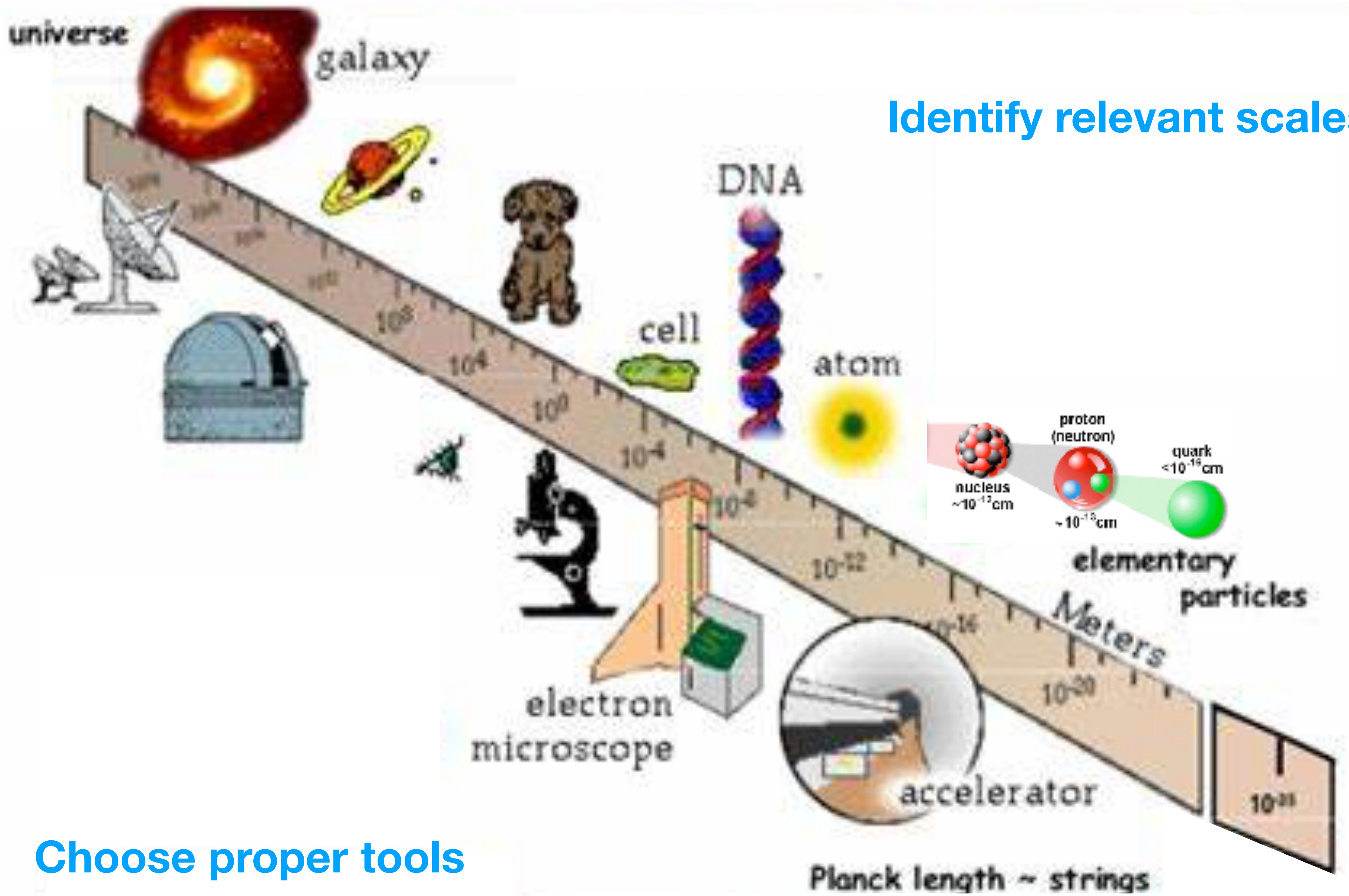
Witold Skiba (Yale U.) (Jun, 2010)

Published in: • Contribution to: TASI 2009, 5-70 • e-Print: 1006.2142 [hep-ph]

Basic Concepts in EFT

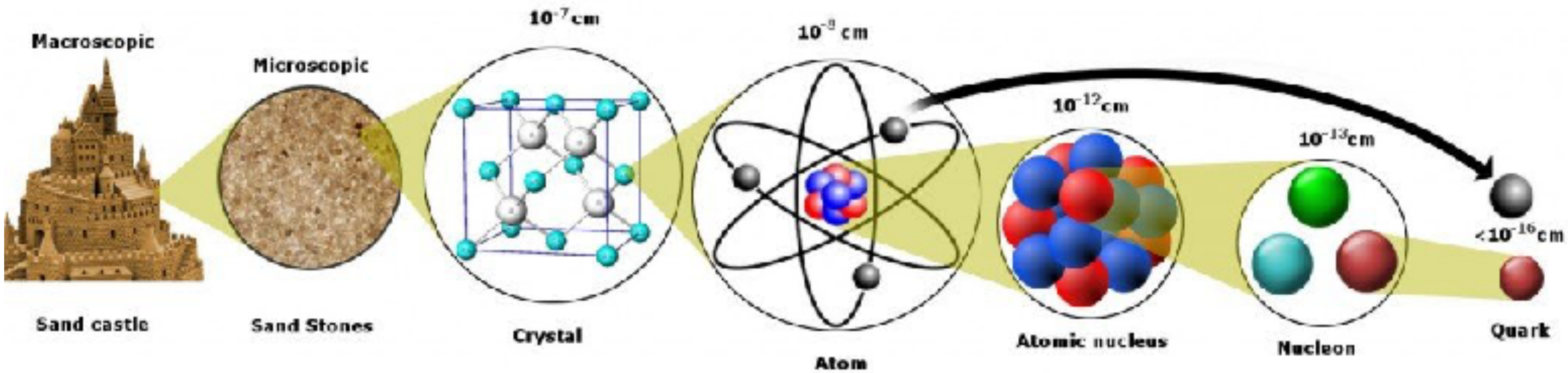
Scales of Nature

Identify relevant scales

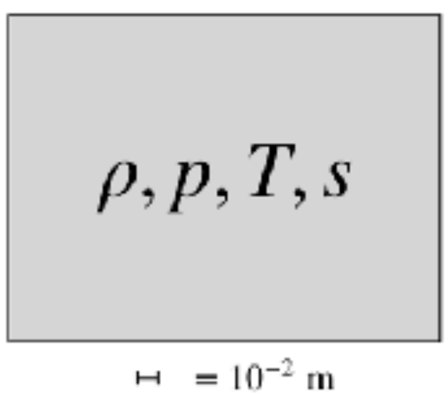


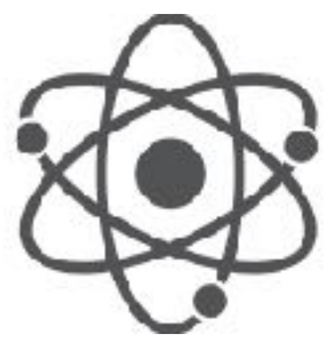
Choose proper tools

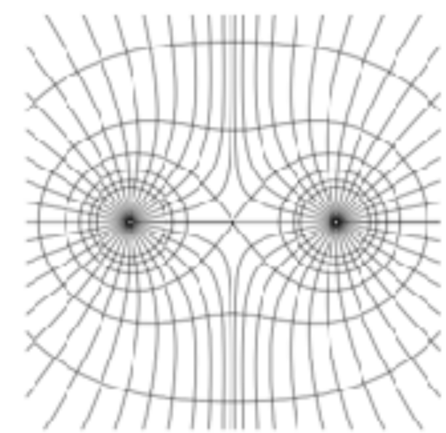
Decoupling Among Scales

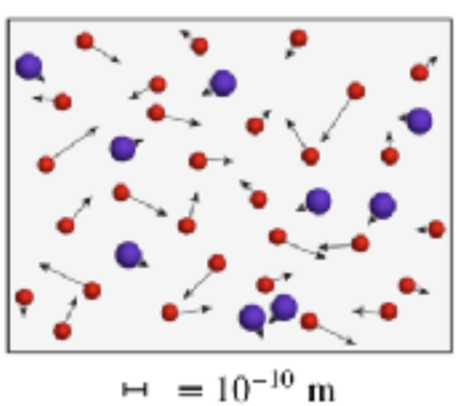


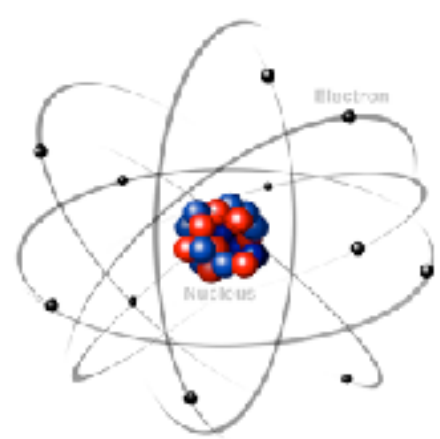
Not all scales relevant at the energy of interest

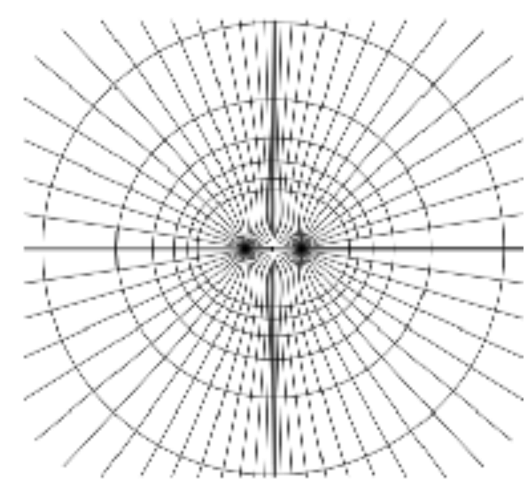












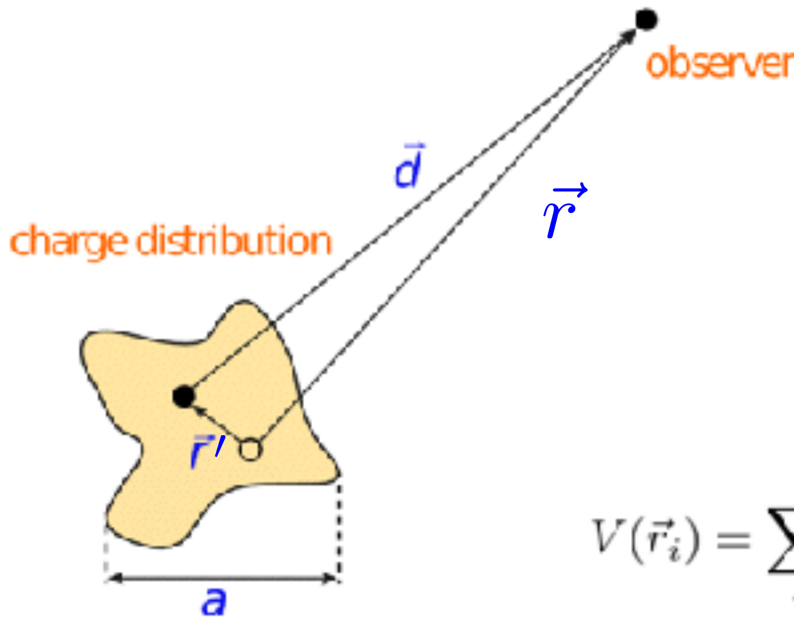
Hyperfine splitting

Multipole expansion

Scale Separation

When involving multiple, disparate scales

two scales r and a, with $r \gg a$



Tayler expansion on $\delta = r'/r$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\frac{1}{[1+(\frac{r'}{r})^2 - \frac{2\vec{r}\cdot\vec{r}'}{r^2}]^{1/2}} \qquad \frac{2\vec{r}\cdot\vec{r}'}{r^2} = 2(r'/r)cos\theta$$

$$= \frac{1}{r}[1 + \frac{r'}{r}cos\theta + (\frac{r'}{r})^2(\frac{3cos^2\theta}{2} - \frac{1}{2}) + (\frac{r'}{r})^3(\frac{5cos^3\theta}{2} - \frac{3cos\theta}{2}) + O((\frac{r}{r'})^4$$

$$V(\vec{r}_i) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|} = \sum_i \frac{kq_i}{r} + \sum_i \frac{kq_i r_i cos\theta_i}{r^2} + \sum_i \frac{kq_i r_i^2}{2r^3}[3cos^2\theta_i - 1] + O(\frac{1}{r^4})$$

For unknown charge distribution, parametrize

charge dipole quadrupole

$$= \frac{kQ}{r} + \frac{k\vec{p}\cdot\hat{r}}{r^2} + \frac{k\hat{r}\cdot\tilde{Q}_2\cdot\hat{r}}{r^3} + O(1/r^4)$$

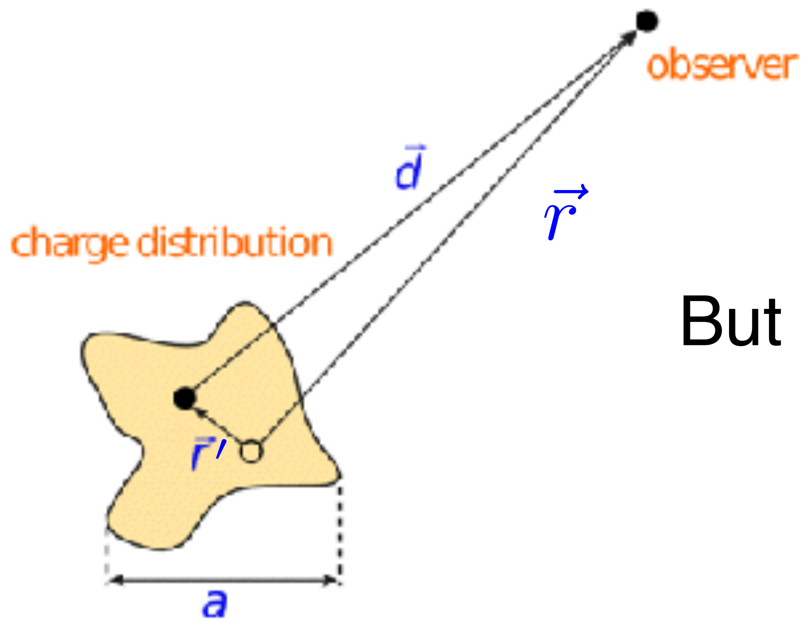
Determine these unknown coefficients up to certain moments

for finite experimental resolution

Then predict electric potential up to certain accuracy

Decoupling

Short distance scale a is not important since moments determined by exp



$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} b_{lm}\frac{1}{r^l}Y_{lm}(\Omega)$$

But if we know a, then we can predict b (matching)

$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} c_{lm}\left(\frac{a}{r}\right)^l Y_{lm}(\Omega)$$

$$b_{lm} \equiv c_{lm}a^l$$

Matching

$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} b_{lm}\frac{1}{r^l}Y_{lm}(\Omega)$$

Momentum space via Fourier transformation

Short distance a	$\longleftrightarrow$	UV scale $\Lambda \sim 1/a$
Long distance r	$\longleftrightarrow$	IR scale $p \sim 1/r$
Tayler expansion $\frac{a}{r}$	$\longleftrightarrow$	power counting $\frac{p}{\Lambda}$

Effective Field Theory (EFT)

In QFT, particles as excitations of the underlying fields

the relevant scales are particle masses, scattering momenta

Take scattering amplitude (two mass scales m, M) at CM energy E

$E^2 \sim m^2 \ll M^2$

- Describe by an **expansion** in $(m/M)^n$, $(E/M)^n$ (power counting)
- Effects of heavy physics with mass M, 'decouple' at low momenta, p
- When applied at the right scale, EFTs can predict with arbitrary precision

Range of validity

Why using EFT?

A. If 'full theory' is **known**: greatly simplify calculations — Top-down

B. If 'full theory' is **unknown**: universally parametrise UV effects — Bottom-up

How to Build an EFT?

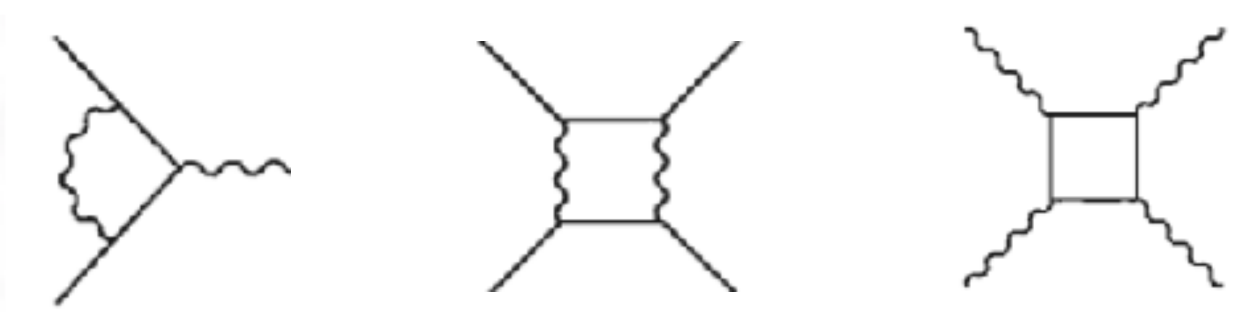
Start with QED theory in QFT course

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu$$

Add higher dimensional terms

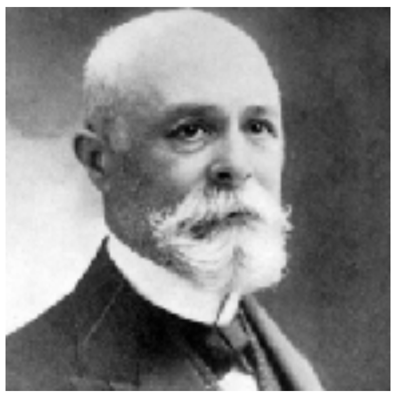
$$+\frac{c_5}{\Lambda}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} + \frac{c_6}{\Lambda^2}(\bar{\psi}\psi)^2 + \frac{c_8}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \cdots$$

Generate such terms from renormalizible QED?

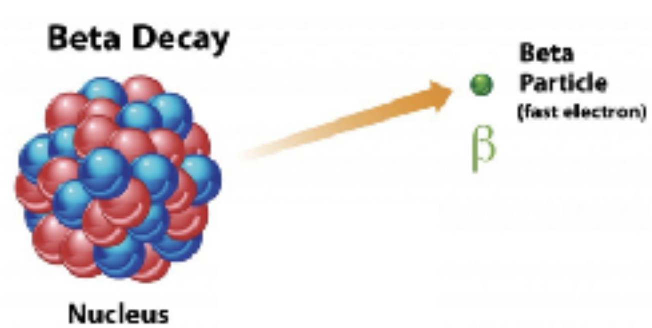


Relevant vs irrelevant

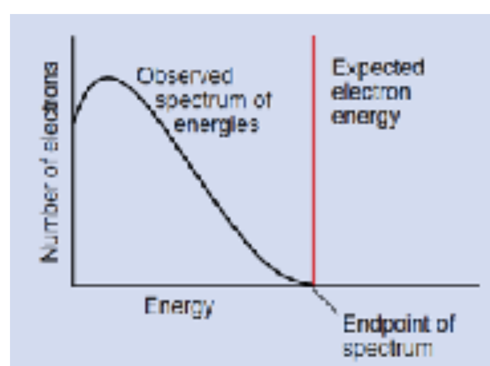
The First EFTs!



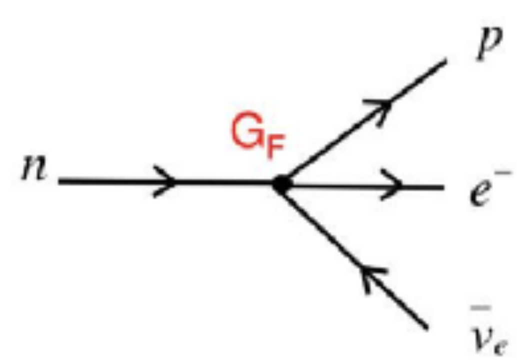
Becquerel
1896



Pauli
1933

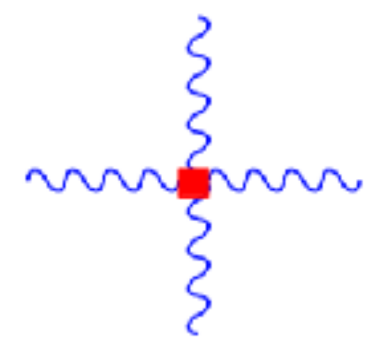


Fermi
1934



$$G_F \left[\overline{\psi_n}\gamma^\mu\psi_p\right]\left[\overline{\psi_e}\gamma^\mu\psi_\nu\right]$$

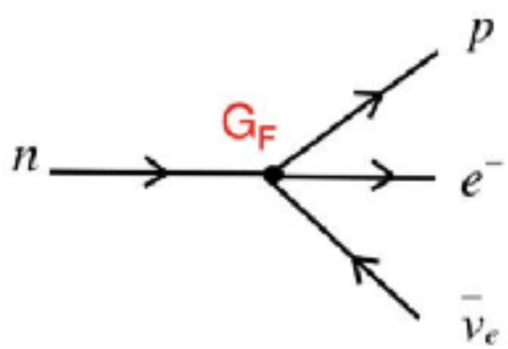
Euler-Heisenberg
1936



$$(F_{\mu\nu}F^{\mu\nu})^2$$

Four-Fermion Theory

According to the Fermi's Golden rule, predict the electron energy spectra



$$W_{fi} = 2\pi G_F^2 |M_{if}|^2 \, p_e^2 (E_f - E_e)^2 dp_e$$

$$M_{if} = \langle p| \, J_\mu^{wk} \, |n\rangle \, \langle e\nu| \, J_\mu^{wk} \, |0\rangle$$

$$E_e\sqrt{E_e^2 - m_e^2 c^4} E_\nu^2 \delta(E_e + E_\nu - E_0) dE_e dE_\nu$$

$$\boxed{G_F \left[\overline{\psi_n}\gamma^\mu\psi_p\right] \left[\overline{\psi_e}\gamma^\mu\psi_\nu\right]}$$

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$$

First higher dimensional operator (1934 rejected by nature)!

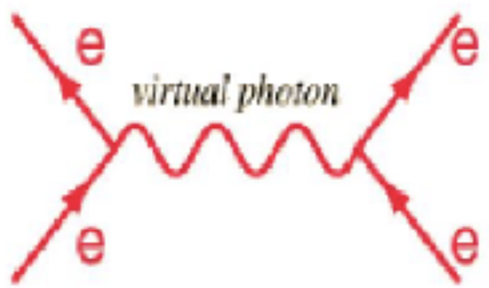
Why does G_F has dimensions of GeV^{-2}?

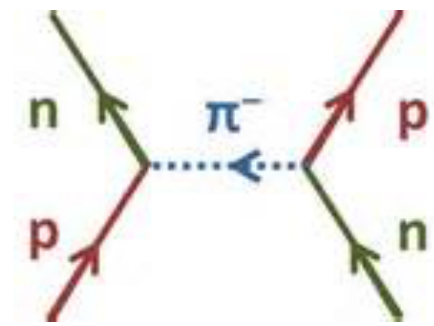
Suppose the four-fermion theory were right

$\sigma(e\nu \to e\nu) \propto G_F^2 s$ $\quad$ $\sqrt{s} \sim 500$ GeV, unitary it violated

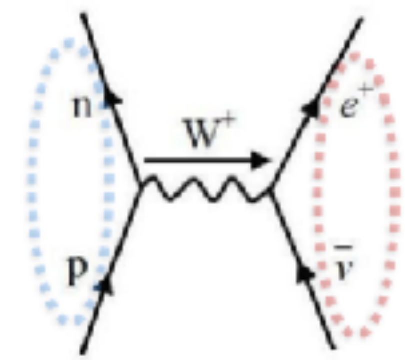
Intermediate Vector Boson

Current-current interaction through exchange of mediator boson





Strong isospin



Strong isospin (gauge SU2)

Weak isospin?

$$-e[\bar{\psi}\gamma_\mu\psi]\frac{g^{\mu\nu}}{q^2}[\bar{\psi}\gamma_\mu\psi] \longrightarrow -g[\bar{\psi}\gamma_\mu\psi]\frac{g^{\mu\nu}}{q^2 - m_W^2}[\bar{\psi}\gamma_\mu\psi]$$

$$\downarrow$$

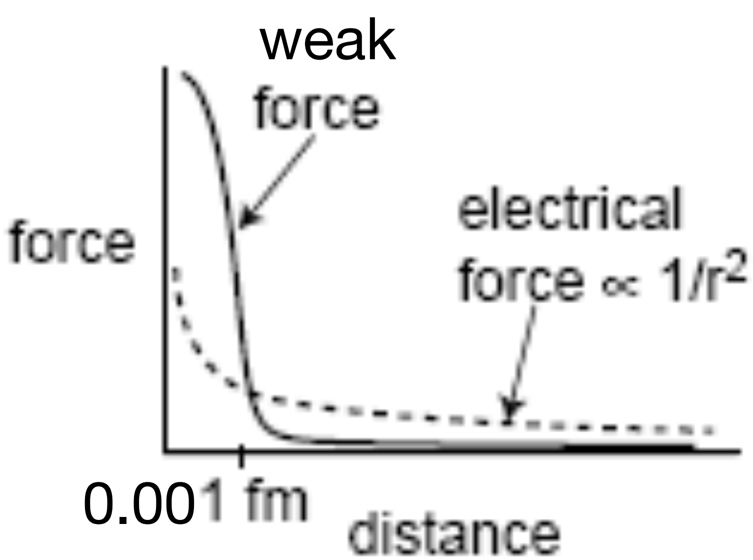
$$\frac{g}{m_W^2}[\bar{\psi}\gamma_\mu\psi][\bar{\psi}\gamma_\mu\psi]$$

Explain GF $\quad G_F = \frac{g}{m_W^2}$

Preserve unitarity

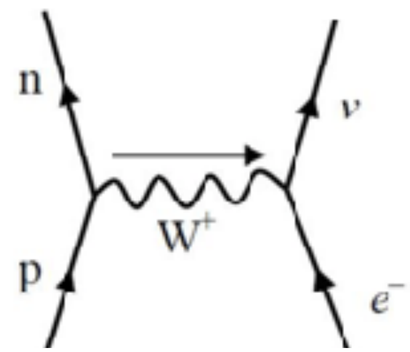
EFT vs UV Theory

$$\lambda \sim \frac{1}{p} < \Delta r_W \sim \frac{1}{m_W}$$

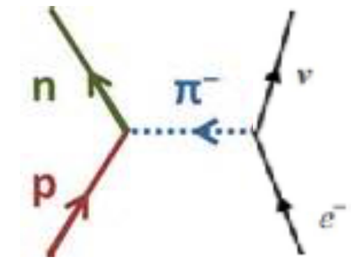


$$V(r) = -\frac{g^2}{4\pi}\frac{1}{r}e^{-m_\phi r}$$

$$\lambda \sim \frac{1}{p} > \Delta r_W \sim \frac{1}{m_W}$$

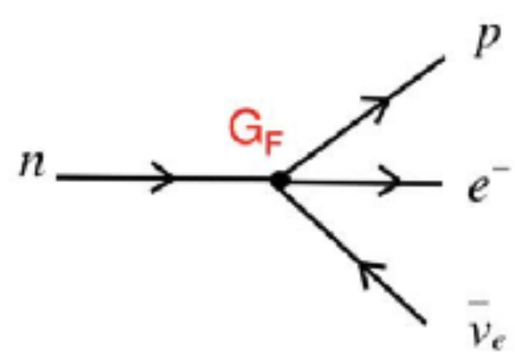


Top-down



Long-range interaction at UV

Other UV?



Bottom-up

EFT contact interaction at low energy

Euler-Heisenberg EFT

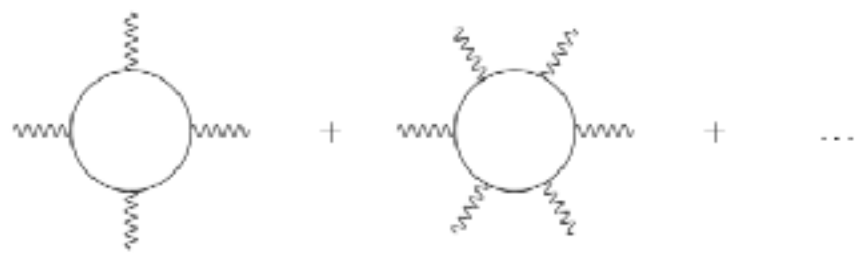
Light-by-light scattering at very low energy scale

$(\mathbf{E_\gamma \ll m_e})$

- Gauge, Lorentz, Charge Conjugation & Parity
- Energy expansion (E_γ / m_e)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6 / m_e^8)$$

Give the UV theory (QED)



$$a = -\frac{1}{36} \alpha^2 \quad , \quad b = \frac{7}{90} \alpha^2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \propto \frac{\alpha^4 E^6}{m_e^8}$$

photon does interact with itself

Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \ll \Delta E \sim \alpha^2 m_e \ll a_0^{-1} \sim \alpha m_e \ll M_A$$

- Neutral atom + gauge invariance $\longrightarrow$ $F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description: $\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{1}{2M} \vec{\nabla}^2 \right) \psi + \mathcal{L}_{\rm int}$

$$\mathcal{L}_{\rm int} = a_0^3 \, \psi^\dagger \psi \left(c_1 \vec{E}^2 + c_2 \vec{B}^2 \right) + \ldots \quad , \quad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \sim c_i \, a_0^3 \, E_\gamma^2 \longrightarrow \sigma \propto a_0^6 \, \mathbf{E}_\gamma^4$$

Blue light is scattered more strongly than red one

Dimensional Analysis

Fermi interaction is a higher dimensional operator

4D QFT functional integral: $Z = \int \mathcal{D}\phi\, e^{iS[\phi]}, \quad S = \int d^4x\, \mathcal{L}[\phi(x)]$

Natural units, h=c=1: [Length] = Mass^{-1}

From kinetic terms

$$[\mathcal{L}] = 4 : \quad [\phi] = 1,\ [\psi] = \frac{3}{2},\ [D_\mu] = 1,\ [A_\mu] = 1,\ [g] = 0$$

Renormalisable interactions have couplings [c] ≥ 0

$$\mathcal{L}_{\text{int.}} = c\,\mathcal{O}, \quad [\mathcal{O}] \leq 4$$

- Renormalisable: need a **finite number** of counter-terms (CT) to absorb divergences in loop computations to **all orders** in perturbation theory

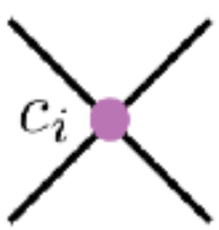
$[\mathcal{O}] < 4,\ [c] > 0$	$[\mathcal{O}] = 4,\ [c] = 0$	$[\mathcal{O}] > 4,\ [c] < 0$
'Relevant'	'Marginal'	'Irrelevant'
$I\,,\ \phi^2\,,\ \phi^3\,,\ \bar{\psi}\psi$	$\phi^4\,,\ \phi\bar{\psi}\psi\,,\ V_\mu\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\psi\bar{\psi}\psi\,,\ \partial_\mu\phi\bar{\psi}\gamma^\mu\psi\,,\ \phi^2\bar{\psi}\psi\,,\ \cdots$

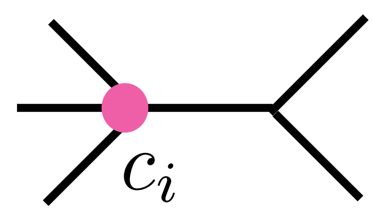
Power Counting

EFT Lagrangian expansion based on canonical dimension

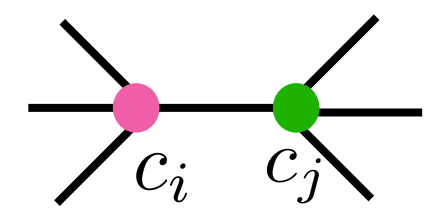
$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}^{d_i}$$

Normalized scattering amplitude follows the EFT power counting formula





$$\mathcal{A} \sim c_i \left(\frac{p}{\Lambda}\right)^{d_i-4}$$



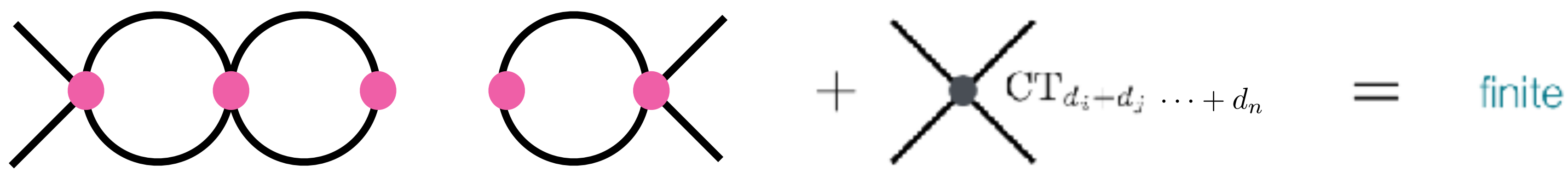
$$\mathcal{A} \sim c_i c_j \left(\frac{p}{\Lambda}\right)^{n_i+n_j} \qquad n_i = d_i - 4$$

How about beyond tree-level?

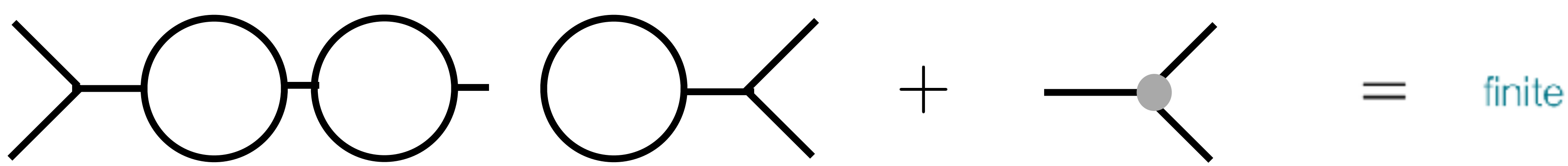
$$\longrightarrow \quad \mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n_i+n_j}$$

Counter terms

Counterterms from higher dim operators



Infinite counter terms, formally **non-renormalizable**!

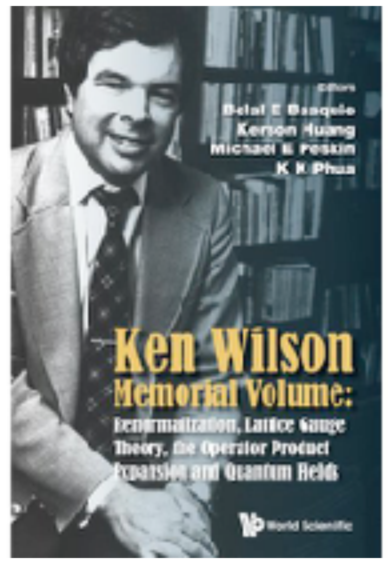


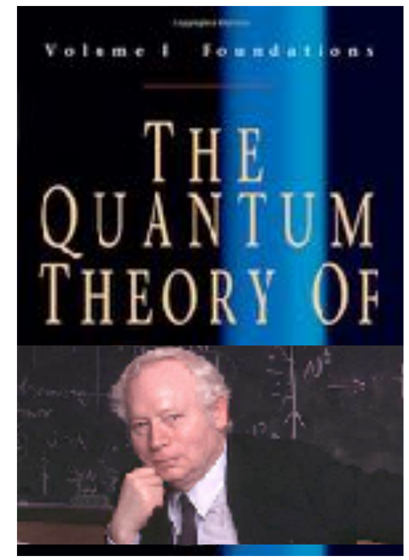
$$\left(\frac{p}{\Lambda}\right)^{d_i-4\ \leq 0} \qquad\qquad d_i \ \leq 4$$

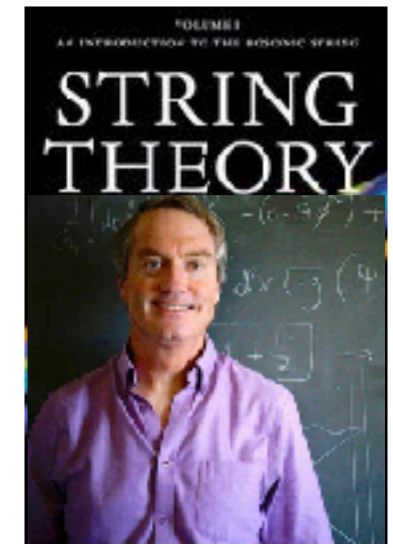
$$\mathrm{div} = 4 - B - \frac{3}{2}F - V + \sum_i n_i(\Delta_i - 4)$$

If only dim-3,4 operators, then renormalizable!

Developments on EFT







1949~1970
QFT should be Renormalizable

1970
Wilsonian EFT

1979
Folk theorem

1984
EFT should be Renormalizable

"What bothered me was that the proofs that renormalization works seemed extremely combinatoric and technical, but the results in the end came down to dimensional analysis. What I realized was that things would become nearly trivial if, instead of describing the path integral order by order in perturbation theory, as nearly always done, we described it scale-by-scale in energy. As soon as I thought those words, I knew I could prove them...It took just three weeks for me to work out the proof and write it up."

Complicated loop calculation can be organized by renorm. EFT

Wilsonian EFT

Consider a QFT with a fundamental high energy scale M

Interested in observables at energy E << M

Choose a cutoff $\Lambda < M$ and divide all quantum fields

$\phi_{\rm H}: \quad \omega > \Lambda$

$\phi_{\rm L}: \quad \omega < \Lambda.$

$$\phi = \phi_{\rm H} + \phi_{\rm L}$$

$$Z[J_L] = \int \mathcal{D}\phi_L\, \mathcal{D}\phi_H\, e^{iS(\phi_L,\phi_H)+i\int d^Dx\, J_L(x)\,\phi_L(x)}$$

Integrate-out

$$\int \mathcal{D}\phi_{\rm L}\, \mathcal{D}\phi_{\rm H}\, e^{iS(\phi_{\rm L},\phi_{\rm H})} = \int \mathcal{D}\phi_{\rm L}\, e^{iS_\Lambda(\phi_{\rm L})}$$

$$Z[J_L] \equiv \int \mathcal{D}\phi_L\, e^{iS_\Lambda(\phi_L)+i\int d^Dx\, J_L(x)\,\phi_L(x)}$$

$$e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H\, e^{iS(\phi_L,\phi_H)}$$

Effective action

$$S_\Lambda(\phi_L) = \int d^Dx\, \mathcal{L}^{\rm eff}_\Lambda(x)$$

Local EFT

$$\boxed{\mathcal{L}^{\rm eff}_\Lambda(x) = \sum_i g_i\, Q_i(\phi_L(x))}$$

coupling constants (Wilson coefficients)

local operators built out of fields ϕ_L and their derivatives

Integrate-out procedure

The generating functional in the UV theory of light fields φ and heavy fields H

$$Z_{\rm UV}[J_\phi, J_H] = \int [D\phi][DH]\exp\left[i\int d^4x\left(\mathscr{L}_{\rm UV}(\phi,H) + J_\phi\phi + J_H H\right)\right]$$

The generating functional in the EFT of light fields φ

$$Z_{\rm EFT}[J_\phi] = \int [D\phi]\exp\left[i\int d^4x\left(\mathscr{L}_{\rm EFT}(\phi) + J_\phi\phi\right)\right]$$

Matching consists in imposing the condition

$$Z_{\rm EFT}[J_\phi] = Z_{\rm UV}[J_\phi, 0]$$

At leading order (tree-level), the field configurations contributing to the path integral are the ones that extremize the action:

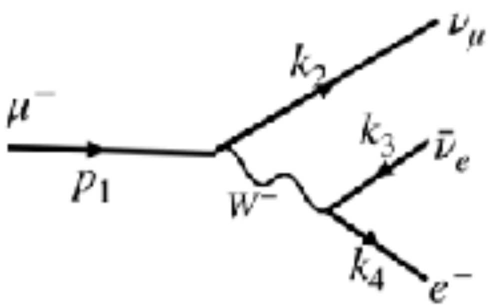
$$Z_{\rm UV}[J_\phi, 0] = \int [D\phi]\exp\left[i\int d^4x\left(\mathscr{L}_{\rm UV}(\phi, H_{\rm cl}(\phi)) + J_\phi\phi\right)\right] \qquad 0 = \frac{\delta S}{\delta H}\Big|_{H=H_{\rm cl}(\phi)}$$

that is, $H_{\rm cl}(\varphi)$ solves the classical equations of motion in the UV Lagrangian

Hence $\quad \mathscr{L}_{\rm EFT}(\phi) = \mathscr{L}_{\rm UV}(\phi, H_{\rm cl}(\phi))$

Wilsonian EFT

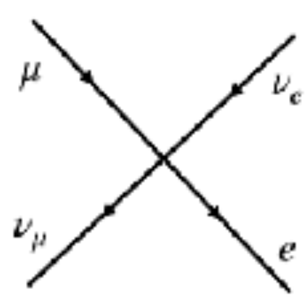
Diagrammatic approach



$$\mathcal{M} = \frac{g_L^2}{2}\bar u(k_2)\gamma_\rho P_L u(p_1)\frac{1}{q^2 - m_W^2}\bar u(k_4)\gamma_\rho P_L v(k_3)$$

$$q^2 \lesssim m_\mu^2 \ll m_W^2$$

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots\right)$$



$$-i\frac{g_L^2}{2m_W^2}[\gamma_\rho P_L][\gamma_\rho P_L]$$

$$\mathcal{M} = -\frac{g_L^2}{2m_W^2}\bar u(k_2)\gamma_\rho P_L u(p_1)\bar u(k_4)\gamma_\rho P_L v(k_3) + \mathcal{O}(q^2/m_W^4)$$

Path integral approach

$$\mathcal{L}_{\rm UV} \supset -W_\rho^+(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right]W_\rho^+ + \mathrm{h.c.}$$

$$-(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right] = 0$$

$$W_\rho^- = \frac{g_L}{\sqrt{2}}(\Box - m_W^2)^{-1}\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right]$$

(Non-local) Effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \frac{g_L^2}{2}\left[\bar e_L\gamma_\rho\nu_e + \bar\mu_L\gamma_\rho\nu_\mu\right](\Box - m_W^2)^{-1}\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right]$$

$$\frac{1}{\Box - m_W^2} = -\frac{1}{m_W^2} - \frac{\Box}{m_W^4} - \frac{\Box^2}{m_W^6} - \ldots$$

Leading (local) Effective Lagrangian:

$$\mathcal{L}_{\rm eff} = -\frac{g_L^2}{2m_W^2}\left[\bar e_L\gamma_\rho\nu_e + \bar\mu_L\gamma_\rho\nu_\mu\right]\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right] + \mathcal{O}\left(\frac{1}{m_W^4}\right)$$

$$-\frac{g_L^2}{2m_W^4}\left[\bar e_L\gamma_\rho\nu_e + \bar\mu_L\gamma_\rho\nu_\mu\right]\Box\left[\bar\nu_e\gamma_\rho e_L + \bar\nu_\mu\gamma_\rho\mu_L\right] + \ldots$$

Continuum EFT

Wilsonian EFT: cutoff as parametrization of underlying dynamics

Mass-dependent scheme

Cutoff regularization
momentum subtraction

Mass-independent scheme

Dimensional regularization
MS-bar scheme

Four-fermion theory as example $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{a}{\Lambda^2}(\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4}(\bar{\psi}\Box\psi)(\bar{\psi}\psi) + \cdots$

$$\delta m \sim 2i\,\frac{a}{\Lambda^2}\,m\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m^2}$$

- **Cut-off regularization:** $\delta m \sim m\,\frac{a}{\Lambda^2}\Lambda^2 \sim m\,a$ **Not suppressed!**
- **Dimensional regularization:** Mass independent

$$\delta m \sim 2a\,m\,\frac{m^2}{16\pi^2\Lambda^2}\left\{\Delta_\infty(\mu) + \log\left(\frac{m^2}{\mu^2}\right) - 1 + \mathcal{O}(D-4)\right\}$$

$$\Delta_\infty(\mu) = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

Well-defined power counting

Naive Dimensional Analysis

Organize/estimate the size of the infinite effective operators using NDA

Valid at loop level

$$\mathcal{L}_{\Lambda}^{\text{eff}}(x) = \sum g_i \, Q_i(\phi_L(x))$$

$$C_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0 \\ \ll 1\,; & \text{if } \gamma_i > 0 \\ \gg 1\,; & \text{if } \gamma_i < 0 \end{cases}$$

$$\text{div} = 4 - B - \frac{3}{2}F - V + \sum_i n_i(\Delta_i - 4)$$

$$\delta_i = [Q_i] = D + \gamma_i$$

Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

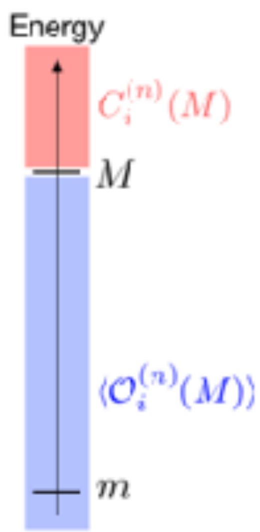
only operators with $\gamma_i \leq 0$ are important for $E \ll M$

Only a **finite number** of relevant and marginal operators exist!

Depending on the precision goal, one can truncate the infinite sum over terms by only retaining operators whose γ_i value is smaller than a certain value

Relevant or Irrelevant?

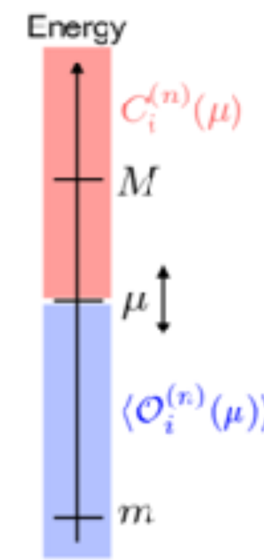
$$\mathcal{L}_{\rm EFT} = \sum_{n=0}^{\infty} \sum_i \frac{C_i^{(n)}(M)}{M^n} \mathcal{O}_i^{(n)}(M)$$



factorization

$$\mathcal{L}_{\rm EFT} = \sum_{n=0}^{\infty} \sum_i \frac{C_i^{(n)}(\mu)}{M^n} \mathcal{O}_i^{(n)}(\mu)$$

$$\mathcal{O}_{i,0}^{(n)} = \sum_j Z_{ij}^{(n)}(\mu)\, \mathcal{O}_j^{(n)}(\mu)$$



RG running effect

$$\frac{dC_i(\mu)}{d\ln\mu} \mathcal{O}_i(\mu) + C_i(\mu) \frac{d\mathcal{O}_i(\mu)}{d\ln\mu} = \left[\frac{dC_i(\mu)}{d\ln\mu} \delta_{ij} - C_i(\mu)\, \gamma_{ij}(\mu) \right] \mathcal{O}_j(\mu) = 0$$

$$\frac{d\vec{C}(\alpha_s)}{d\alpha_s} = \frac{\gamma^T(\alpha_s)}{\beta(\alpha_s)} \vec{C}(\alpha_s)$$

$$C(\alpha_s(\mu)) \approx \left(\frac{\alpha_s(\mu)}{\alpha_s(Q)} \right)^{-\gamma_0/2\beta_0} C(\alpha_s(M))$$

Anomalous scaling

Relevant operator $d < 4$ $\quad C(\mu) \gg C(M), \quad \mu \ll M$

Irrelevant operator $d > 4$ $\quad C(\mu) \ll C(M), \quad \mu \ll M$

Marginal $\quad \beta(\alpha_s(\mu)) = \mu \frac{d\alpha_s(\mu)}{d\mu}$

Renormalize EFT

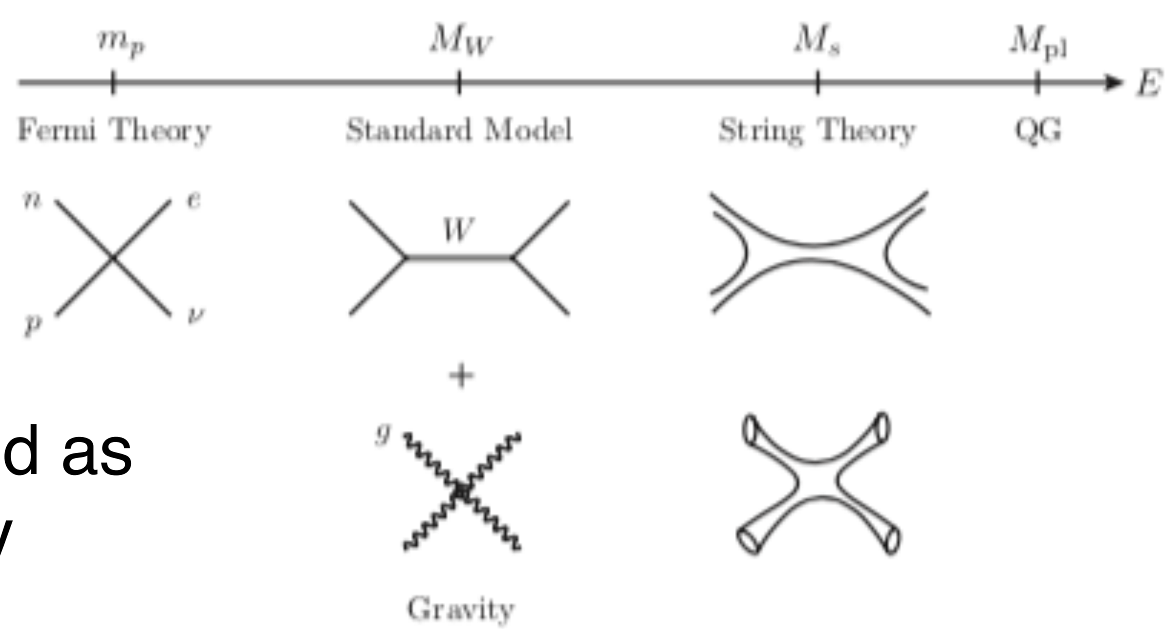
The EFT describes the low energy physics, to a given precision, in terms of a finite set of parameters

$$(\mathsf{E}/\mathsf{M})^{\mathsf{d_i}-4} \gtrsim \epsilon \quad \Longleftrightarrow \quad \mathsf{d_i} \lesssim 4 + \frac{\log(1/\epsilon)}{\log(\mathsf{M}/\mathsf{E})}$$

Renormalize non-renormalizable theory:

1. Write the most general operator up to certain truncated order
2. Determine Wilson coefficients using enough input data (absorb all div.)
3. Make predictions on other observables using the truncated theory

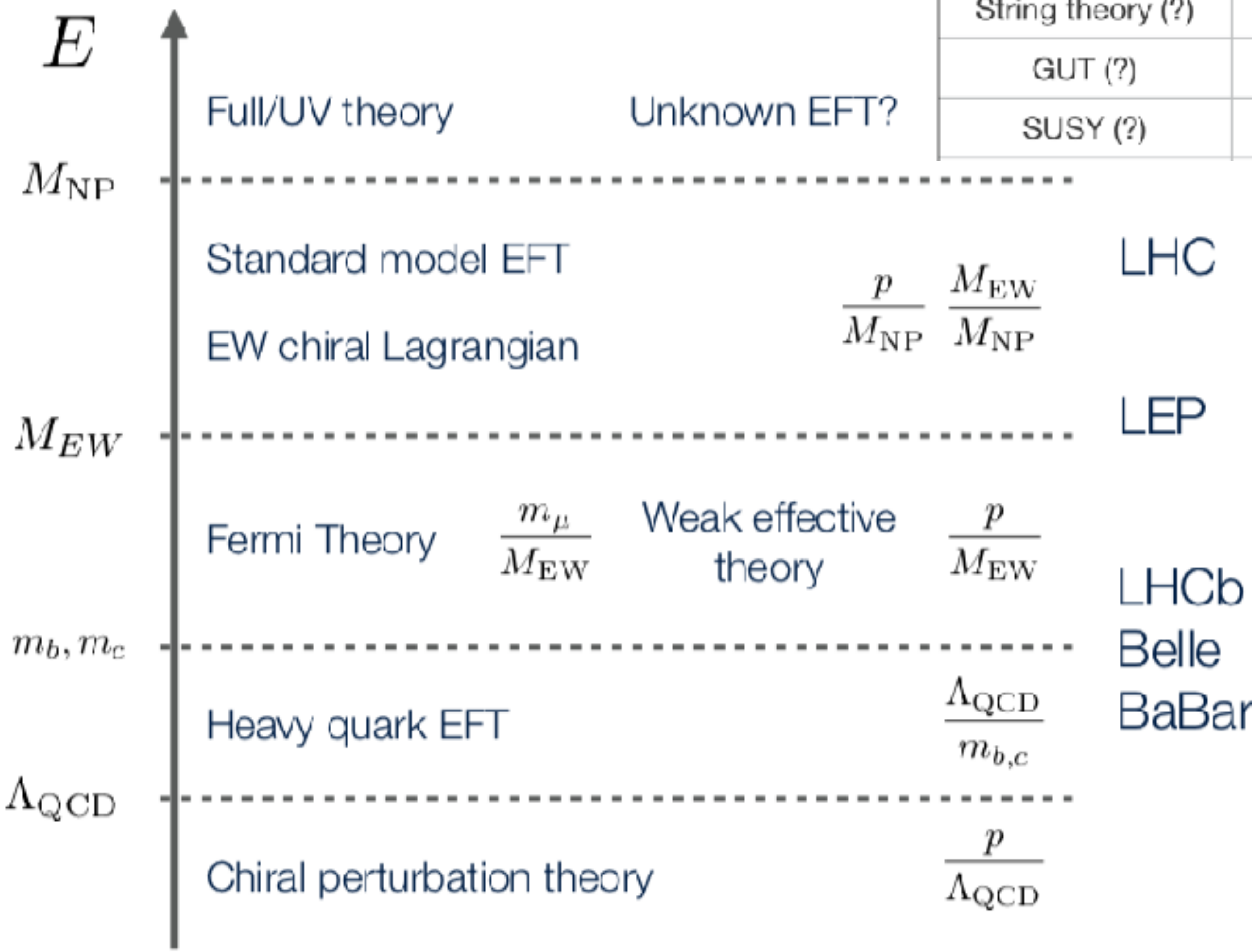
To a given precision, EFT is renormalizable and predictive!



Fermi/Gravity as good as underlying theory

EFT Ladder

High-energy theory	Fundamental scale	Low-energy theory
11-d M theory (?)	?	String theory (?)
String theory (?)	$M_S \sim 10^{18}$ GeV	QFT/Gravity
GUT (?)	$M_{GUT} \sim 10^{16}$ GeV	SUSY (?)
SUSY (?)	$M_{SUSY} \sim 10^{4+x}$ GeV	SMEFT



EFT = Modern QFT

"Theorem of modesty":

- no QFT ever is complete on all length and energy scales
- all QFTs are low-energy effective theories valid in some energy range, up to some cutoff Λ

Give up renormalizability as a construction criterion for "decent" QFTs:

Forget the folklore about "cancellations of infinities"

- at low energy, any effective theory will automatically reduce to a "renormalizable" QFT, meaning that "non-renormalizable" interactions give rise to small contributions $\sim(E/M)^n$
- low-energy physics depends on the **short-distance dynamics** of the fundamental theory only through a small number of **relevant and marginal couplings,** and possibly through some irrelevant couplings if our measurements are sufficiently precise
- this finite number of couplings can be renormalized (i.e., infinities can be removed consistently) using a finite number of experimental data

Matching and Running

Real Scalar EFT

EFT with a single real scalar with Z2 symmetry

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$\hat{O}_6 \equiv (\Box\phi)^2, \quad \tilde{O}_6 \equiv \phi\Box\phi^3, \quad \tilde{O}'_6 \equiv \phi^2\Box\phi^2, \quad \tilde{O}''_6 \equiv \phi^2\partial_\mu\phi\partial_\mu\phi, \quad \ldots$$

Use Leibniz rule + integration by parts:

$$\phi^2\partial_\mu\phi\partial_\mu\phi = -2\phi\partial_\mu\phi\partial_\mu\phi\phi - \phi^3\Box\phi \quad \Rightarrow \quad \tilde{O}''_6 = -\frac{1}{3}\phi^3\Box\phi = -\frac{1}{3}\tilde{O}_6$$

$$\phi^2\Box\phi^2 = 2\phi^2\partial_\mu(\phi\partial_\mu\phi) = 2\phi^3\Box\phi + 2\phi^2(\partial_\mu\phi)^2 \quad \Rightarrow \quad \tilde{O}'_6 = 2\tilde{O}_6 + 2\tilde{O}''_6 = \frac{4}{3}\tilde{O}_6$$

Use equations of motion: $\Box\phi = -m^2\phi - \frac{C_4}{6}\phi^3 + \mathcal{O}(\Lambda^{-2})$

$$\tilde{O}_6 \equiv \phi^3\Box\phi = -m^2\phi^4 - \frac{C_4}{6}\phi^6 = -m^2O_4 - \frac{C_4}{6}O_6$$

$$\hat{O}_6 \equiv (\Box\phi)^2 = m^4\phi^2 + \frac{m^2C_4}{3}\phi^4 + \frac{C_4^2}{36}\phi^6 = m^4O_2 + \frac{m^2C_4}{3}O_4 + \frac{C_4^2}{36}O_6$$

Equivalent Lagrangian

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3\Box\phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

$$\tilde{C}_6 = -\frac{C_6}{5C_4}$$

$$\tilde{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{C_6}{5C_4}$$

Operator Bases

Consider 2 to 2 scattering in the two kinds of Lagrangian

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3\Box\phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{M}^{\rm unbox}_{EFT} = -C_4 + \mathcal{O}(\Lambda^{-4})$$

$$\begin{aligned}\mathscr{M}^{\rm box}_{EFT} &= -\tilde{C}_4 + \frac{\tilde{C}_6}{4\Lambda^2}(p_1^2+p_2^2+p_3^2+p_4^2) + \mathcal{O}(\Lambda^{-4}) \\ &= -\tilde{C}_4 + \tilde{C}_6\frac{m^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})\end{aligned}$$

$$\tilde{C}_6 = -\frac{C_6}{5C_4}$$

$$\tilde{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{C_6}{5C_4}$$

$$\mathscr{M}^{\rm unbox}_{EFT} = \mathscr{M}^{\rm box}_{EFT} + \mathcal{O}(\Lambda^{-4})$$

Origin: field redefinition!

Field Redefinition

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i \exp\left(i \int d^4x \left[\mathcal{L}_0 + \eta \mathcal{L}_1 + \sum_i j_i \varphi_i + \mathcal{O}(\eta^2)\right]\right)$$

$$\phi^\dagger = (\phi')^\dagger + \eta T[\varphi]$$ $T[\varphi]$ is any local function of any of the fields φ

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta\phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}_0' + \delta\mathcal{L}_0' + \eta\mathcal{L}_1' + \eta\delta\mathcal{L}_1' + \sum_i j_i\varphi_i + j_{\phi^\dagger}\eta T + \mathcal{O}(\eta^2)\right]\right)$$

$$\mathcal{L}_i' \equiv \mathcal{L}_i\left((\phi')^\dagger, \partial_\mu(\phi')^\dagger\right) \qquad \delta\phi^\dagger \equiv \phi^\dagger - (\phi')^\dagger = \eta T[\varphi]$$

$$\delta\mathcal{L}_i' \equiv \frac{\delta\mathcal{L}_i'}{\delta(\phi')^\dagger}\delta\phi^\dagger - \frac{\delta\mathcal{L}_i'}{\delta\partial_\mu(\phi')^\dagger}\delta\partial_\mu\phi^\dagger = \left(\frac{\delta\mathcal{L}_i'}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}_i'}{\delta\partial_\mu(\phi')^\dagger}\right)\delta\phi^\dagger = \left(\frac{\delta\mathcal{L}_i'}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}_i'}{\delta\partial_\mu(\phi')^\dagger}\right)\eta T[\varphi]$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta\phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}_0' + \left(\frac{\delta\mathcal{L}_0'}{\delta(\phi')^\dagger} - \partial_\mu\frac{\delta\mathcal{L}_0'}{\delta\partial_\mu(\phi')^\dagger}\right)\eta T[\varphi] + \eta\mathcal{L}_1' + \sum_i j_i\varphi_i + j_{\phi^\dagger}\eta T + \mathcal{O}(\eta^2)\right]\right)$$

the source term and the Jacobian can be neglected

Gaussian theorem on action

Equation of Motion (EOM)

Integration by part (IBP) $\partial_\mu \mathcal{O}^\mu$

Two equivalent operators related by EOM

Total derivatives are removed

Exercise

How many independent operators of the form $\partial^{2n}\phi^4$?

$$\partial^{2n}\phi^2,\ \partial^{2n}\phi^3,\ \partial^{2n}\phi^4,\ \cdots$$

$2n$	0	2	4	6	8	10	12	14	16
# independent $\partial^{2n}\phi^4$ operators	1	0	1	1	1	1	2	1	2

1

$s+t+u=0$

$s^2+t^2+u^2$

$s^3+t^3+u^3 \sim stu$

$(s^2+t^2+u^2)^2$

$stu\,(s^2+t^2+u^2)$

$(s^2+t^2+u^2)^3$ & $(stu)^2$

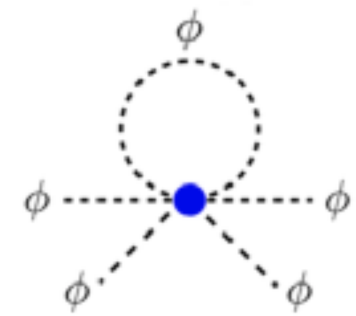
$stu\,(s^2+t^2+u^2)^2$

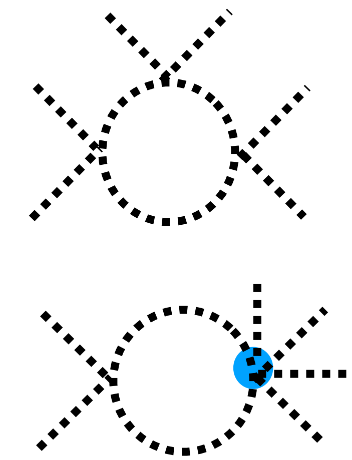
$(s^2+t^2+u^2)^4$ & $(stu)^2(s^2+t^2+u^2)$

Magic things happen?!

One loop and RGE

Renormalize the scalar EFT using the MSbar scheme





$$\delta M_2^{\rm EFT} = -\frac{C_4}{2}\int \frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2}$$

$$= C_4\frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

$$M_4^{\rm EFT} = -C_4(\mu) + \frac{C_4^2}{32\pi^2}\left[f(s,m)+f(t,m)+f(u,m)\right]$$

$$+\frac{3C_4^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right)+2\right) + \frac{C_6 m^2}{32\pi^2\Lambda^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right)+1\right)$$

$$f(s,m) \equiv \sqrt{1-\frac{4m^2}{s}}\log\left(\frac{2m^2 - s + \sqrt{s(s-m^2)}}{2m^2}\right)$$

$$\frac{dm^2}{d\log\mu} = C_4\frac{m^2}{16\pi^2}$$

$$\frac{dC_4}{d\log\mu} = \frac{3C_4^2}{16\pi^2} + \frac{C_6 m^2}{16\pi^2\Lambda^2}$$

$$m^2(\mu) = m^2(\Lambda)\left(\frac{\mu}{\Lambda}\right)^{\frac{C_4}{16\pi^2}}$$

$$C_4(\mu) \approx C_4(m) + \frac{3C_4^2}{16\pi^2}\log\left(\frac{\mu}{m}\right) + \frac{C_6 m^2}{16\pi^2\Lambda^2}\log\left(\frac{\mu}{m}\right)$$

the low energy observable after running

$$C_4^r(\tilde{\mu}_L) = \frac{C_4^r(\tilde{\mu}_H)}{1 + C_4^r(\tilde{\mu}_H)\,\frac{3}{32\pi^2}\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_L^2}}$$

$$i\,\mathcal{A}^{\rm EFT} = -im_{\rm E}^2(\tilde{\mu}_L) + \frac{iC_4(\tilde{\mu}_L)}{32\pi^2}\,m_{\rm E}^2(\tilde{\mu}_L)\left[\log\frac{\tilde{\mu}_L^2}{m_{\rm E}^2(\tilde{\mu}_L)} + 1\right]$$

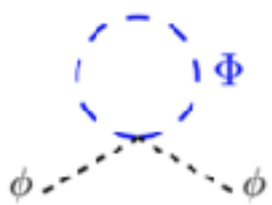
$$i\,\mathcal{A}^{\rm EFT} = -iC_4(\tilde{\mu}_L)\left[1 - \frac{3}{32\pi^2}C_4(\tilde{\mu}_L)\log\frac{\tilde{\mu}_L^2}{m^2}\right] + \frac{iC_6}{32\pi^2}\frac{m^2}{M^2}\left[\log\frac{\tilde{\mu}_L^2}{m^2}+1\right]$$

$$= -iC_4^r(\tilde{\mu}_H)\left\{1 - C_4^r(\tilde{\mu}_H)\left(\log\frac{\tilde{\mu}_H^2}{m^2} + \frac{2}{3}\right) + \mathcal{O}\left((C_4^r)^2\log^2\frac{\tilde{\mu}_H^2}{\tilde{\mu}_L^2}\right)\right\}$$

UV Origin of EFT Operators

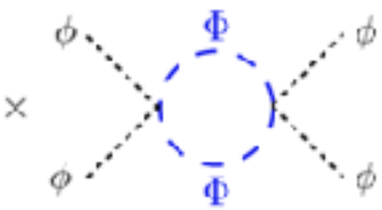
Suppose we know the UV renormalizable theory

$$\mathcal{L}^{\text{FULL}} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m_{\text{F}}^2 \phi^2 + \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} M^2 \Phi^2 - \frac{1}{4}\kappa \phi^2 \Phi^2 - \frac{1}{4!}\eta \phi^4$$



$$= \frac{i\kappa}{32\pi^2} M^2 \left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}_M^2}{M^2} + 1 + \mathcal{O}(\epsilon)\right]$$

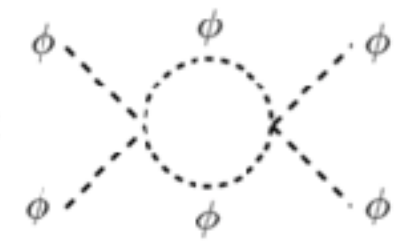
3 ×



$$= \frac{3i}{32\pi^2} \mu^{2\epsilon} \kappa^2 \left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{M^2}\right) + \mathcal{O}(\lambda^2)$$

$$= \frac{i\eta}{32\pi^2} m_{\text{F}}^2 \left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}_M^2}{m_{\text{F}}^2} + 1 + \mathcal{O}(\epsilon)\right]$$

3 ×



$$= \frac{3i}{32\pi^2} \mu^{2\epsilon} \eta^2 \left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{m^2} + \frac{2}{3}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}\log\tilde{\mu}^2} m_{\text{F}}^2 = \gamma_{m_{\text{F}}^2} m_{\text{F}}^2$$

$$\gamma_{m_{\text{F}}^2} = \frac{\eta}{32\pi^2} + \frac{\kappa}{32\pi^2}\frac{M^2}{m_{\text{F}}^2}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\log\tilde{\mu}^2} = \frac{3}{32\pi^2}\eta^2 + \frac{3}{32\pi^2}\kappa^2$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}\log\tilde{\mu}^2} = \frac{1}{8\pi^2}\kappa^2 + \frac{1}{32\pi^2}\kappa\eta$$

Running down to the scale M

$$m_{\text{F}}^2(\tilde{\mu}_M)_{\text{Expanded}} = m_{\text{F}}^2(\tilde{\mu}_H) - \frac{\eta}{32\pi^2} m_{\text{F}}^2 \log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} - \frac{\kappa}{32\pi^2} M^2 \log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2}$$

$$i\mathcal{A}^{\text{FULL}} = -i\eta(\tilde{\mu}) + i\frac{3}{32\pi^2}(\eta(\tilde{\mu}))^2\left(\log\frac{\tilde{\mu}^2}{m^2} + \frac{2}{3}\right) + i\frac{3}{32\pi^2}(\kappa(\tilde{\mu}))^2 \log\frac{\tilde{\mu}^2}{M^2}$$

$$\eta(\tilde{\mu}_L)_{\text{Expanded}} = \eta(\tilde{\mu}_H) - \frac{3}{32\pi^2}(\eta^2 + \kappa^2)\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_L^2}$$

Large Log Resummation

Scattering amplitude carries large log term

$$i\mathcal{A}^{\text{FULL}} = -i\eta(\tilde{\mu}) + i\frac{3}{32\pi^2}\left(\eta(\tilde{\mu})\right)^2\left(\log\frac{\tilde{\mu}^2}{m^2}+\frac{2}{3}\right) + i\frac{3}{32\pi^2}\left(\kappa(\tilde{\mu})\right)^2\log\frac{\tilde{\mu}^2}{M^2} \qquad [\text{LL+NLO}]$$

EFT does not have large log term

$$i\mathcal{A}^{\text{EFT}} = -iC_4(\tilde{\mu}_L) + i\frac{3}{32\pi^2}\left(C_4(\tilde{\mu}_L)\right)^2\left(\log\frac{\tilde{\mu}_L^2}{m^2}+\frac{2}{3}\right) + \frac{iC_6}{32\pi^2}\frac{m^2}{M^2}\left[\log\frac{\tilde{\mu}_L^2}{m^2}+1\right]$$

Perform matching and running

$$= -i\eta(\tilde{\mu}_H) + \frac{3i}{32\pi^2}\left[\eta^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} + C_4^2\left(\log\frac{\tilde{\mu}_M^2}{m^2}+\frac{2}{3}\right)\right] + \frac{3i}{32\pi^2}\kappa^2\log\frac{\tilde{\mu}_H^2}{M^2} \qquad [\text{LL+NLO}]$$

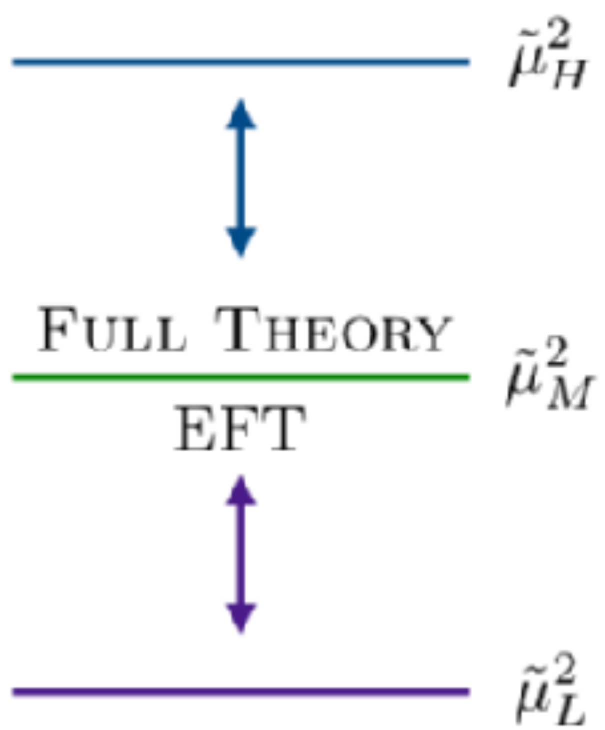
Recover the UV result!

1. UV log M is absorbed into the Wilson coefficients

2. UV non-log m/M are suppressed (two loop in above example)

Integrate-out

Typically integrate out procedure in the path integral approach



$$Z[J_\Phi, J_\varphi] = \int [D\Phi]\,[D\varphi]\; e^{i\int d^dx(\mathscr{L}[\Phi,\varphi]+J_\Phi\Phi+J_\varphi\varphi)}$$

Equation of motion (EOM): $\frac{\delta\mathscr{L}_{\rm UV}}{\delta\Phi}=0\,.$

Set $\Phi = \Phi_c[\phi]$ (EOM solution)

$$Z[J_\varphi] = \int [D\varphi]\; e^{i\int d^dx(\mathscr{L}_{\rm eft}[\varphi,\Phi_c]+J_\varphi\varphi)}$$

Based on the decoupling theorem

Decoupling: Appelquist–Carazzone

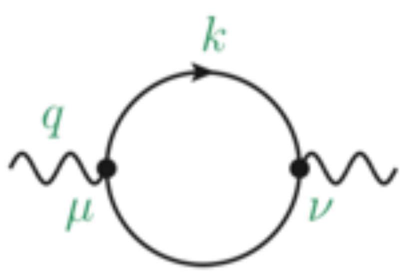
The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

Decoupling or Not?

How about diagrammatic approach? marginal operator as example

$$\frac{d\alpha_s(\mu)}{d\ln\mu} = -2\beta_0\,\frac{\alpha_s^2(\mu)}{4\pi} \qquad \alpha_s(\mu) = \frac{\alpha_s(Q)}{1+\alpha_s(Q)\,\frac{\beta_0}{4\pi}\,\ln\frac{\mu^2}{Q^2}}$$

Different scheme choices:



$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty(\mu)+6\int_0^1 dx\; x(1-x)\;\log\left(\frac{m_f^2-q^2x(1-x)}{\mu^2}\right)\right\}$$

MSbar scheme

$$\Delta\Pi_\epsilon(\mu^2) \;\equiv\; -Q_f^2\,\frac{\alpha_0}{3\pi}\,\Delta_\infty(\mu)$$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\; x(1-x)\;\log\left[\frac{m_f^2-q^2x(1-x)}{\mu^2}\right]$$

$$\beta_1 = \frac{2}{3}\,Q_f^2$$

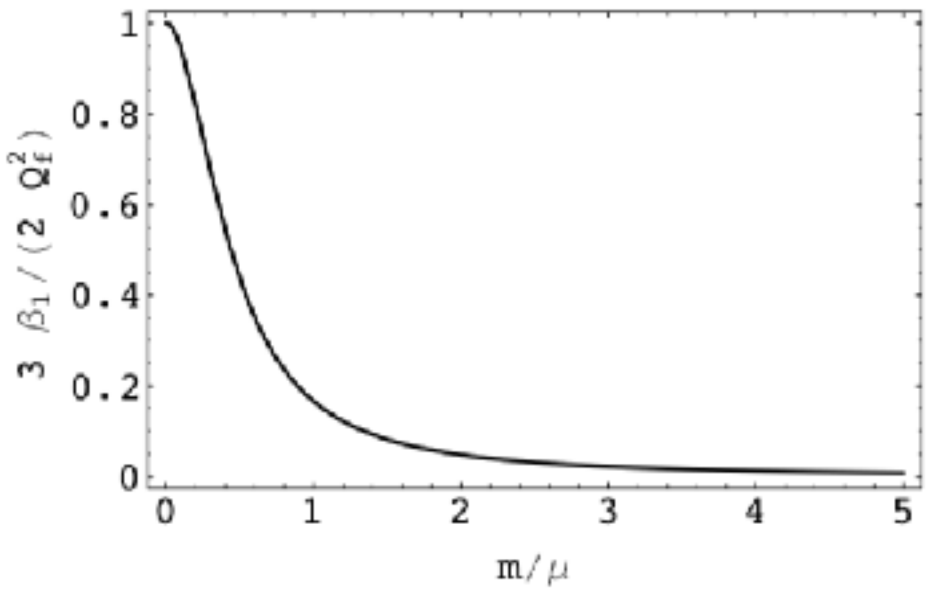
Mass-dependent scheme

$$\Delta\Pi_\epsilon(\mu^2) \;\equiv\; \Pi(-\mu^2)$$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\; x(1-x)\;\log\left[\frac{m_f^2-q^2x(1-x)}{m_f^2+\mu^2x(1-x)}\right]$$

$$\beta_1 = 4\,Q_f^2\int_0^1 dx\;\frac{\mu^2x^2(1-x)^2}{m_f^2+\mu^2x(1-x)}$$

Integrate-out by Hand

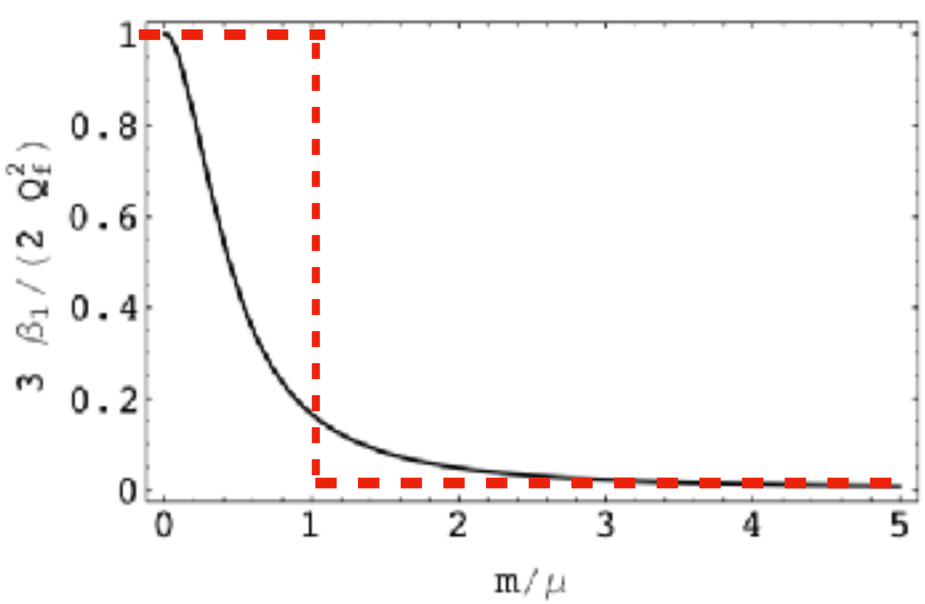


two different EFT regime

- $m_f^2 \ll \mu^2, q^2$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$
- $m_f^2 \gg \mu^2, q^2$: $\beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2}$, $\Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2+\mu^2}{m_f^2}$

Decoupling = heavy scale suppressed

Choose the momentum independent scheme, but force decoupling by



μ

EFT with heavy fermion $\qquad \beta_1 = \frac{2}{3} Q_f^2 + \beta_1^{\text{light}}$

m_f - - - - - - - - - - - - - - - - -

EFT without heavy fermion $\qquad \beta_1 = \beta_1^{\text{light}}$

In MSbar scheme, remove the particle by hand (integrate-out)

Matching

Calculate one-loop diagrams and RGEs at full theory and EFT separately

Then match two theories at the matching scale M

Large μ

$\mathcal{L}(\phi_i) + \mathcal{L}(\phi_i, \Phi)$ $\qquad$ ϕ_i, Φ

Renormalization Group

$\mu = M$ **Matching** $= \left[\begin{array}{c}\text{1-loop}\\ \text{full theory}\end{array}\right]_{\overline{MS}} - \left[\begin{array}{c}\text{1-loop}\\ \text{EFT}\end{array}\right]_{\overline{MS}}$

$\mathcal{L}(\phi_i) + \delta\mathcal{L}(\phi_i)$ $\qquad$ ϕ_i

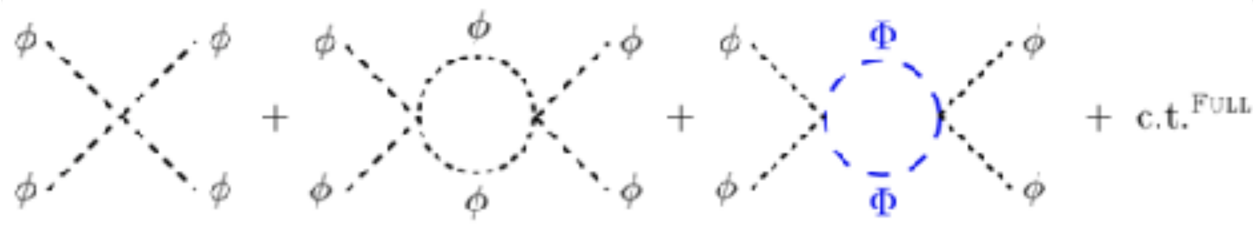
Renormalization Group

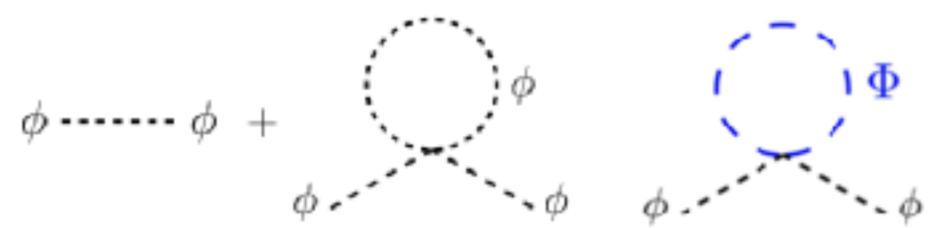
Small μ

Matching and Running

Two theories should be matched together at scale M

$\tilde{\mu}_H^2$





$\tilde{\mu}_M^2$

$$C_4(\tilde{\mu}_M) = \eta(\tilde{\mu}_M) - \frac{3}{32\pi^2}\left(\kappa(\tilde{\mu}_M)\right)^2 \log\frac{\tilde{\mu}_M^2}{M^2}$$

$$m_{\rm E}^2(\tilde{\mu}_M) = m_{\rm F}^2(\tilde{\mu}_M) - \frac{\kappa}{32\pi^2} M^2 \left[\log\frac{\tilde{\mu}_M^2}{M^2} + 1\right]$$

$\tilde{\mu}_L^2$

$$C_4(\tilde{\mu}_M)_{\rm Expanded} = \eta(\tilde{\mu}_H) - \frac{3}{32\pi^2}(\eta^2+\kappa^2)\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} - \frac{3}{32\pi^2}\left(\kappa(\tilde{\mu}_M)\right)^2 \log\frac{\tilde{\mu}_M^2}{M^2},$$
$$= \eta(\tilde{\mu}_H) - \frac{3}{32\pi^2}\left(\eta^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} + \kappa^2\log\frac{\tilde{\mu}_H^2}{M^2}\right) \qquad [\mathrm{LL} + \mathrm{NLO}_M]$$

$$m_{\rm E}^2(\tilde{\mu}_M) = m_{\rm F}^2(\tilde{\mu}_H) - \frac{\eta}{32\pi^2} m_{\rm F}^2 \log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} - \frac{\kappa}{32\pi^2} M^2 \log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} - \frac{\kappa}{32\pi^2} M^2\left[\log\frac{\tilde{\mu}_M^2}{M^2} + 1\right]$$

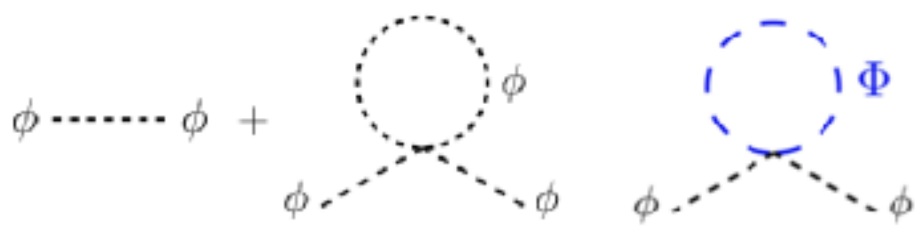
$$C_4(\tilde{\mu}_L)_{\rm Expanded} = \eta(\tilde{\mu}_H) - \frac{3}{32\pi^2}\left(\eta^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} + C_4^2\log\frac{\tilde{\mu}_M^2}{\tilde{\mu}_L^2} + \kappa^2\log\frac{\tilde{\mu}_H^2}{M^2}\right)$$

$$m_{\rm E}^2(\tilde{\mu}_L)_{\rm Expanded} = m_{\rm F}^2(\tilde{\mu}_H) - \frac{\kappa}{32\pi^2}M^2 - \frac{C_4}{32\pi^2}m_{\rm E}^2 - \frac{\eta}{32\pi^2} m_{\rm F}^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} - \frac{C_4}{32\pi^2} m_{\rm E}^2 \log\frac{\tilde{\mu}_M^2}{m_{\rm E}^2} - \frac{\kappa}{32\pi^2}M^2\log\frac{\tilde{\mu}_H^2}{M^2} \qquad [\mathrm{LL} + \mathrm{NLO}]$$

Hierarchy Problem

How the scalar mass connects to the UV physics?

$\tilde{\mu}_H^2$



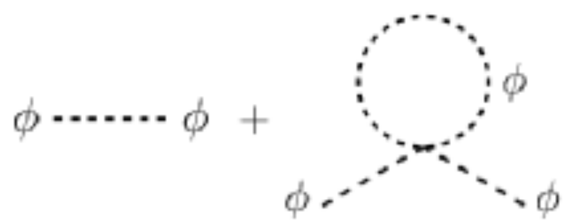
$$-\frac{iy^2}{4\pi^2}\left\{\frac{1}{\varepsilon}(p^2+2m^2)+\frac{p^2}{2}\log\left(\frac{\mu^2}{m^2}\right)+m^2\left[1+\log\left(\frac{\mu^2}{m^2}\right)\right]\right.$$

$\tilde{\mu}_M^2$

$$m_{\mathrm{E}}^2(\tilde{\mu}_M)=m_{\mathrm{F}}^2(\tilde{\mu}_M)-\frac{\kappa}{32\pi^2}M^2\left[\log\frac{\tilde{\mu}_M^2}{M^2}+1\right]$$

$$\mathcal{M}^2(\mu)=M^2+\frac{y^2}{4\pi^2}m^2\left[1+\log\left(\frac{\mu^2}{m^2}\right)\right]$$

$\tilde{\mu}_L^2$



Running down to low scale

$$m_{\mathrm{E}}^2(\tilde{\mu}_M)_{\mathrm{Expanded}}=m_{\mathrm{E}}^2(\tilde{\mu}_L)-\frac{C_4}{32\pi^2}m_{\mathrm{E}}^2\log\frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2}$$

Hierarchy problem

$$m_{\mathrm{F}}^2(M)-\frac{\kappa}{32\pi^2}M^2=m_{\mathrm{E}}^2(m)-\frac{C_4}{32\pi^2}m_{\mathrm{E}}^2\left(1+\log\frac{M^2}{m_{\mathrm{E}}^2}\right)\quad=m_F^2(M)+\frac{y^2}{4\pi^2}M^2$$

SUSY!

$$y^2=\frac{\kappa}{8}$$

Large Log Resummation

Step 1: Determine the Wilson coefficients at a high scale $\mu \approx M$, where they are free of large logarithms

$$\mathcal{L}_{\rm eff} = \sum_{n=0}^{N_{\max}} \sum_i \frac{C_i^{(n)}(\mu)}{M^n} Q_i^{(n)}(\mu)$$

Step 2: Evolve the Wilson coefficients to a low scale $\mu \approx E$, which is characteristic for the observable at hand

$$\boxed{\vec{C}(\alpha_s(\mu)) = \mathrm{T}_{\alpha_s} \exp\left[\int_{\alpha_s(M)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma^T(\alpha_s)}{\beta(\alpha_s)}\right] \vec{C}(\alpha_s(M))}$$

$$\langle O_i \rangle_R = \sum_j \mathbf{Z}_{ij}(\epsilon,\mu) \langle O_j(\mu) \rangle_R \quad ; \quad \gamma_O = \mathbf{Z}^{-1} \mu \frac{d}{d\mu} \mathbf{Z}$$

$$\left(\mu \frac{d}{d\mu} + \gamma_O\right) \langle \vec{O} \rangle_R = 0 \quad ; \quad \left(\mu \frac{d}{d\mu} - \gamma_O^T\right) \langle \vec{c} \rangle_R = 0$$

Diagonalization: $\left(\mathbf{U}^{-1} \gamma_O^T \mathbf{U}\right)_{ij} = \tilde{\gamma}_{O_i} \delta_{ij} \quad ; \quad \tilde{c}_i = \mathbf{U}_{ij}^{-1} c_j$

$$\Rightarrow \boxed{c_i(\mu) = \sum_{j,k} \mathbf{U}_{ij} \exp\left\{\int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \frac{\tilde{\gamma}_{O_j}(\alpha)}{\beta(\alpha)}\right\} \mathbf{U}_{jk}^{-1} c_k(\mu_0)}$$

Step 3: Evaluate the matrix elements of the EFT operators at the scale $\mu \approx E$, where they are free of large logarithms

Large Log Resummation

Scattering amplitude carries large log term

UV and EFT UV div. different!

$$i\mathcal{A}^{\text{FULL}} = -i\eta(\tilde{\mu}) + i\frac{3}{32\pi^2}\left(\eta(\tilde{\mu})\right)^2\left(\log\frac{\tilde{\mu}^2}{m^2}+\frac{2}{3}\right) + i\frac{3}{32\pi^2}\left(\kappa(\tilde{\mu})\right)^2\log\frac{\tilde{\mu}^2}{M^2} \qquad [\text{LL+NLO}]$$

Matching condition $C_4(\tilde{\mu}_M) = \eta(\tilde{\mu}_M) - \frac{3}{32\pi^2}\left(\kappa(\tilde{\mu}_M)\right)^2\log\frac{\tilde{\mu}_M^2}{M^2}$

EFT does not have large log term

$$\begin{aligned} i\mathcal{A}^{\text{EFT}} &= -iC_4(\tilde{\mu}_L) + i\frac{3}{32\pi^2}\left(C_4(\tilde{\mu}_L)\right)^2\left(\log\frac{\tilde{\mu}_L^2}{m^2}+\frac{2}{3}\right) + \frac{iC_6}{32\pi^2}\frac{m^2}{M^2}\left[\log\frac{\tilde{\mu}_L^2}{m^2}+1\right] \\ &= -i\eta(\tilde{\mu}_H) + \frac{3i}{32\pi^2}\left[\eta^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} + C_4^2\left(\log\frac{\tilde{\mu}_M^2}{m^2}+\frac{2}{3}\right)\right] + \frac{3i}{32\pi^2}\kappa^2\log\frac{\tilde{\mu}_H^2}{M^2} \qquad [\text{LL+NLO}] \end{aligned}$$

UV and EFT IR log the same!

Decoupling: Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

Matching Procedure

$$\begin{aligned} I_M &= [I_F + I_{F,\mathrm{c.t.}}] - [I_{\mathrm{EFT}} + I_{\mathrm{EFT,c.t.}}] \\ &= \frac{ig^2}{16\pi^2}\left[\left(\log\frac{\bar{\mu}^2}{M^2}+1\right)+\frac{m^2}{M^2}\left(\log\frac{\bar{\mu}^2}{M^2}+1\right)+\ldots\right] \end{aligned}$$

UV log absorbed

$$\underbrace{\log\frac{m^2}{M^2}}_{\text{UV}} = \underbrace{-\log\frac{M^2}{\bar{\mu}^2}}_{\text{matching}} + \underbrace{\log\frac{m^2}{\bar{\mu}^2}}_{\text{EFT}}$$

$$\underbrace{I_M(m)}_{\text{analytic}} = \underbrace{I_F(m)}_{\text{non-analytic}} - \underbrace{I_{\mathrm{EFT}}(m)}_{\text{non-analytic}}$$

Avoid large logs

Non-analytic m dropped out

$$I_M(\bar{\mu}\sim M) \qquad C_t(\mu_t)$$

$\bar{\mu}$ changes with RGE $\qquad \mu\frac{d}{d\mu}C_t = \gamma_t(\alpha_s)\,C_t$

$$I_{\mathrm{EFT}}(\bar{\mu}\sim m) \qquad C_t(\mu_l) = C_t(\mu_t)\,U(\mu_t,\mu_l)$$

$k \gg m$,

$$I_F^{(\exp)} = g^2\mu^{2\epsilon}\int\frac{\mathrm{d}^{\mathrm{d}}k}{(2\pi)^{\mathrm{d}}}\frac{1}{k^2-M^2}\left[\frac{1}{k^2}+\frac{m^2}{k^4}+\ldots\right]$$

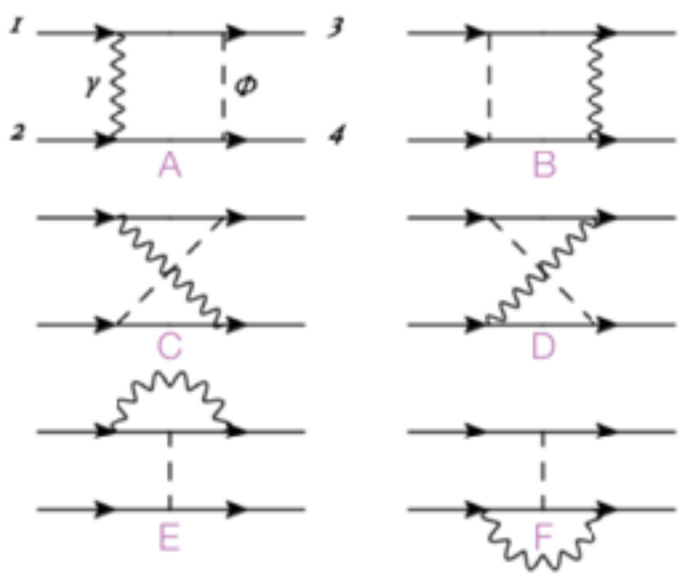
$k \gg m$, $k \ll M$

$$I_{\mathrm{EFT}}^{(\exp)} = g^2\mu^{2\epsilon}\int\frac{\mathrm{d}^{\mathrm{d}}k}{(2\pi)^{\mathrm{d}}}\left[\frac{1}{k^2}+\frac{m^2}{k^4}+\ldots\right]\left[-\frac{1}{M^2}-\frac{k^2}{M^4}-\ldots\right]$$

Separate two scale to two one scale problem

MSbar scheme

Exercise: 4-Fermi Theory

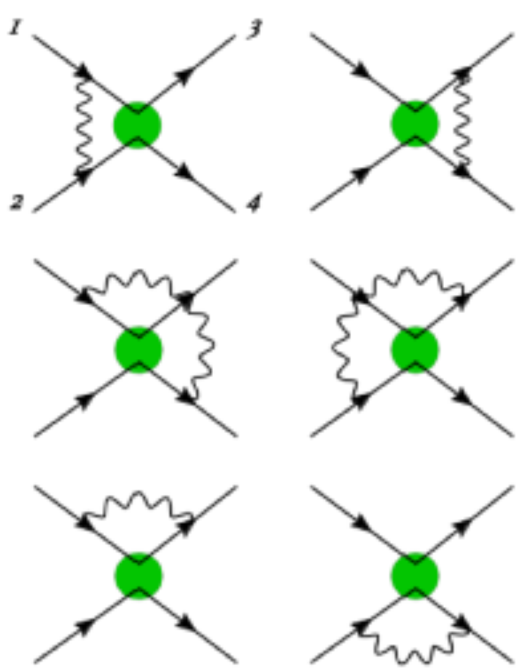


$$\mathcal{U}_S = \bar{u}(p_3)u(p_1)\bar{u}(p_4)u(p_2) - (3 \leftrightarrow 4)$$

$$\mathcal{U}_T = \bar{u}(p_3)\sigma^{\mu\nu}u(p_1)\bar{u}(p_4)\sigma_{\mu\nu}u(p_2) - (3 \leftrightarrow 4)$$

$$\mathcal{U}_S\left(\frac{i\lambda^2}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\boxed{\frac{\sigma^2}{\mu^2}}-1\right)\right]+\mathcal{U}_T\left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(-2\log\boxed{\frac{\sigma^2}{M^2}}-2\right)$$

$$c_S = \lambda^2 + \mathcal{O}(\alpha^2)$$
$$c_T = -\frac{3\alpha Q^2}{8\pi}\lambda^2 + \mathcal{O}(\alpha^2)$$



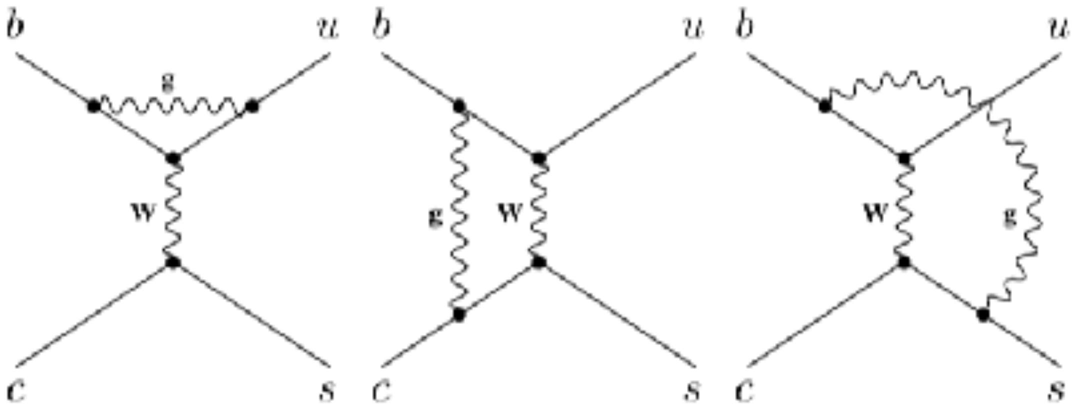
$$\gamma = \frac{2Q^2\alpha}{\pi}\begin{array}{cc} \begin{array}{cc} c_S & c_T \end{array} & \\ \begin{pmatrix} -3/2 & -12 \\ -1/4 & 1/2 \end{pmatrix} & \begin{array}{c} c_S \\ c_T \end{array} \end{array} \qquad \frac{d}{d\ln\mu}c_i = \gamma_{ij}c_j$$

$$\frac{dc_S(\mu)}{d\ln\mu} = -3\frac{Q^2\alpha}{\pi}c_S(\mu) - 24\frac{Q^2\alpha}{\pi}c_T(\mu)$$
$$\frac{dc_T(\mu)}{d\ln\mu} = -\frac{1}{2}\frac{Q^2\alpha}{\pi}c_S(\mu) + \frac{Q^2\alpha}{\pi}c_T(\mu)$$

$$\mathcal{U}_S\underbrace{\left(\frac{ic_S}{M^2}\right)}_{\text{Tree}}\left[1+\underbrace{\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\boxed{\frac{\sigma^2}{\mu^2}}-1\right)}_{\text{Loop}}\right]+\mathcal{U}_T\left[\underbrace{\left(\frac{ic_T}{M^2}\right)}_{\text{Tree}}+\underbrace{\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\epsilon}-2\log\boxed{\frac{\sigma^2}{\mu^2}}+1\right)}_{\text{Loop}}\right]$$

Exercise: b decay

SM:



Calculate one-loop matching and RGE in EFT?

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j\, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j\, \bar{u}_L^j \gamma^\mu b_L^i \right]$$

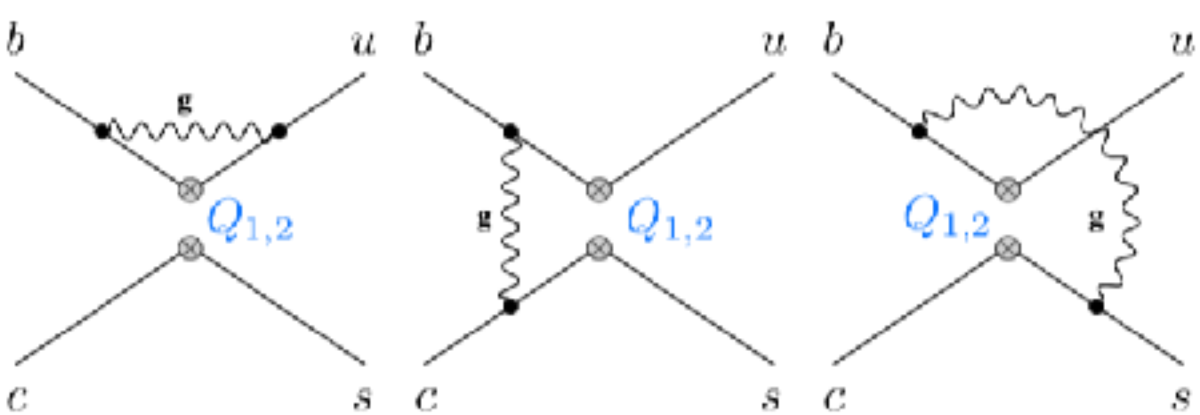
$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

$$C_2(\mu) = -3\,\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

$$\bar{s}_L \gamma_\mu t_a c_L\, \bar{u}_L \gamma^\mu t_a b_L = \frac{1}{2}\bar{s}_L^i \gamma_\mu c_L^j\, \bar{u}_L^j \gamma^\mu b_L^i - \frac{1}{2N_c}\bar{s}_L^j \gamma_\mu c_L^j\, \bar{u}_L^i \gamma^\mu b_L^i$$

$$\boldsymbol{Z}(\mu) = \mathbf{1} + \frac{\alpha_s(\mu)}{4\pi\epsilon}\begin{pmatrix} \frac{3}{N_c} & -3 \\ -3 & \frac{3}{N_c} \end{pmatrix}$$

EFT:



Short Summary

Diagrammatic approach

Feynman diagrams

Easy to miss diagrams

Once matching is done

Running could be easy

Path integral approach

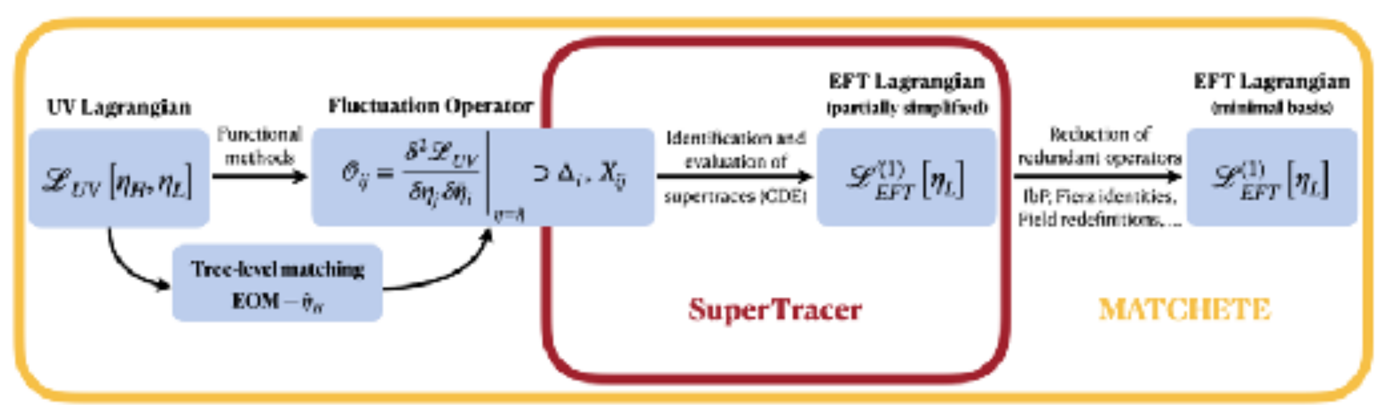
Covariant Derivative Expansion

Systematic but difficult

Matching with loop expansion

Running additional treatment

[Carmona, et.al, 2112.10787]

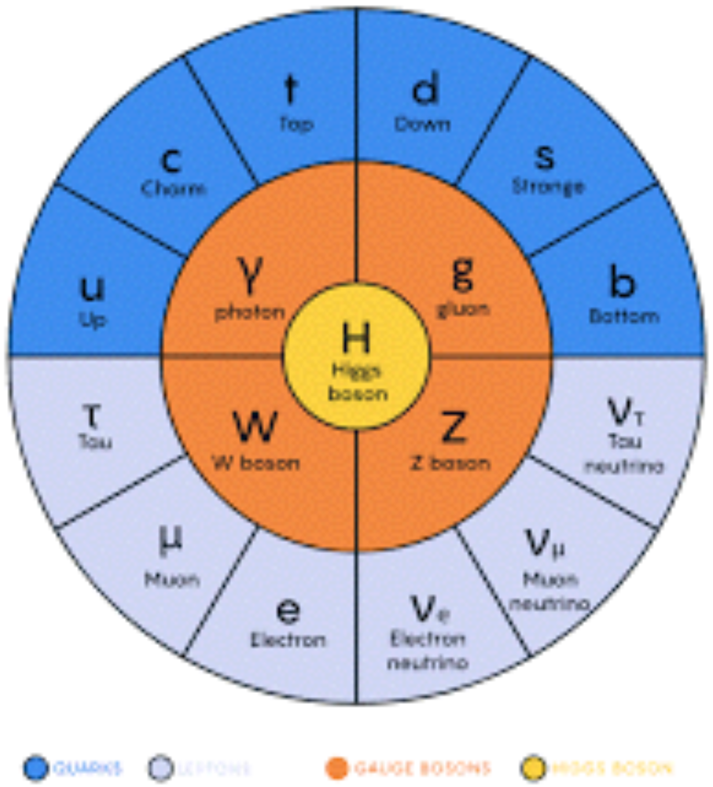


[Cohen, Lu, Zhang, 2012.07851]

[Fuentes-Martin, et.al., 2012.08506]

Standard Model EFT

The Standard Model



$$
\begin{aligned}
\mathcal{L}_{SM} = & \underbrace{\frac{1}{4}W_{\mu\nu}\cdot W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}}_{\text{kinetic energies and self-interactions of the gauge bosons}} \\
& + \underbrace{\bar{L}\gamma^{\mu}\left(i\partial_{\mu} - \frac{1}{2}g\tau\cdot W_{\mu} - \frac{1}{2}g'YB_{\mu}\right)L + \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - \frac{1}{2}g'YB_{\mu}\right)R}_{\text{kinetic energies and electroweak interactions of fermions}} \\
& + \underbrace{\frac{1}{2}\left|\left(i\partial_{\mu} - \frac{1}{2}g\tau\cdot W_{\mu} - \frac{1}{2}g'YB_{\mu}\right)\phi\right|^{2} - V(\phi)}_{W^{\pm},Z,\gamma \text{ and Higgs masses and couplings}} \\
& + \underbrace{g''\left(\bar{q}\gamma^{\mu}T_{a}q\right)G^{a}_{\mu}}_{\text{interactions between quarks and gluons}} + \underbrace{\left(G_{1}\bar{L}\phi R + G_{2}\bar{L}\phi_{c}R + h.c.\right)}_{\text{fermion masses and couplings to Higgs}}
\end{aligned}
$$

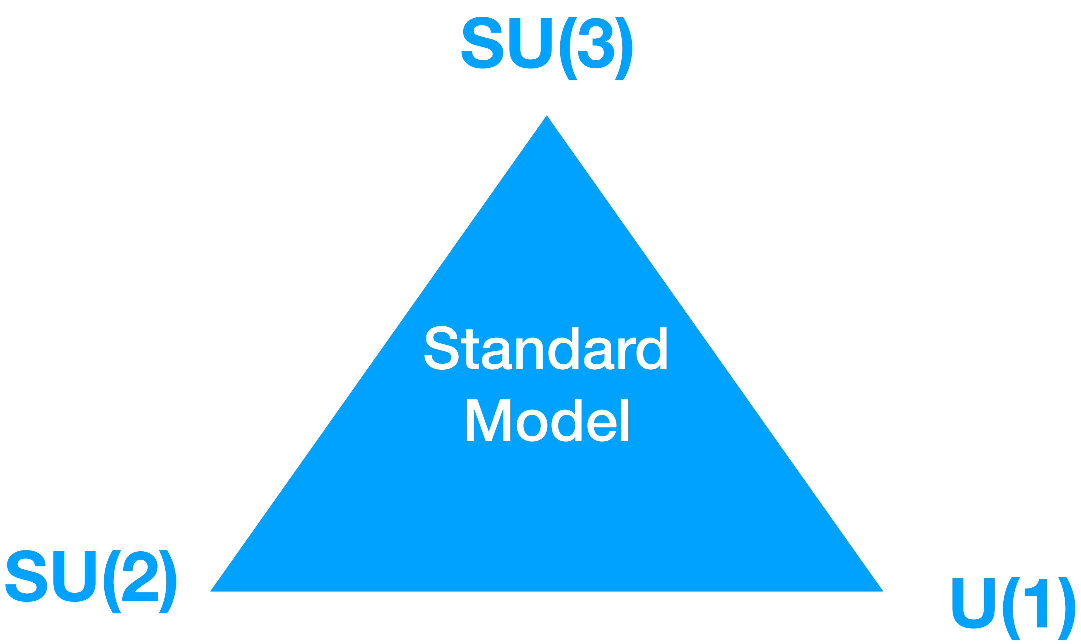
17 elementary particles

19 parameters, all measured but

$$+\tilde{\theta}G^{a}_{\mu\nu}\tilde{G}^{a}_{\mu\nu}$$

Note that $\theta_B B_{\mu\nu}\tilde{B}_{\mu\nu}$ is not physical, while $\theta_W W^k_{\mu\nu}\tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

Symmetry of the SM



Particle(s)	Field(s)	Content	Charge	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	Q_i	$(u,d)_L$	$(2/3, -1/3)$	1/2	3	2	1/6
(Three generations)	u_{Ri}	u_R	2/3	1/2	$\bar{3}$	1	4/3
	d_{Ri}	d_R	−1/3	1/2	$\bar{3}$	1	−2/3
Leptons	L_i	$(\nu_e, e)_L$	$(0, -1)$	1/2	1	2	−1
(Three generations)	l_{Ri}	e_R	−1	1/2	1	1	−2
Gluons	G^a_μ	g	0	1	8	1	0
W bosons	$W^{1,2}_\mu$	$W^\pm$	±1	1	1	3	0
Photon, Z boson	W^3_μ, B_μ	γ, Z^0	0	1	1	3, 1	0
Higgs boson	ϕ	H	0	0	1	2	1

SO(3,1)

$$(M_{12}, M_{23}, M_{31}) = (J_3, J_1, J_2)\,, \qquad \tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2\,,$$
$$(M_{01}, M_{02}, M_{03}) = (K_1, K_2, K_3)\,, \quad \tfrac{1}{2}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J}\cdot\mathbf{K}\,,$$

$$[M_{\lambda\rho}, M_{\mu\nu}] = -i(g_{\lambda\mu}M_{\rho\nu} + g_{\rho\nu}M_{\lambda\mu} - g_{\lambda\nu}M_{\rho\mu} - g_{\rho\mu}M_{\lambda\nu})$$

$$M_i = \tfrac{1}{2}(J_i + iK_i)\,,$$
$$N_i = \tfrac{1}{2}(J_i - iK_i)\,,$$

$$\begin{aligned} [M_i, M_j] &= i\epsilon_{ijk}M_k\,, \\ [N_i, N_j] &= i\epsilon_{ijk}N_k\,, \\ [M_i, N_j] &= 0\,. \end{aligned}$$

SU(2) x SU(2)

$$H_i \in (0,0)\,, \quad H^{\dagger i} \in (0,0)\,,$$

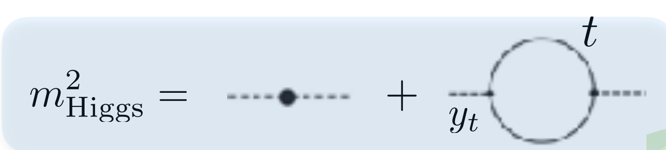
$$\psi_\alpha \in (1/2, 0), \quad \psi^\dagger_{\dot\alpha} \in (0, 1/2),$$

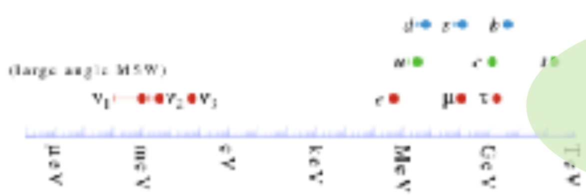
$$F_{\mathrm{L}\alpha\beta} = \frac{i}{2}F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0), \quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2}F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1).$$

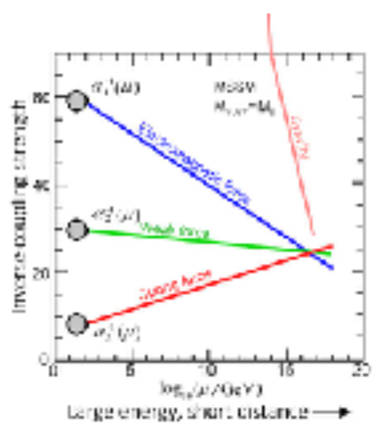
New Physics (NP) Models

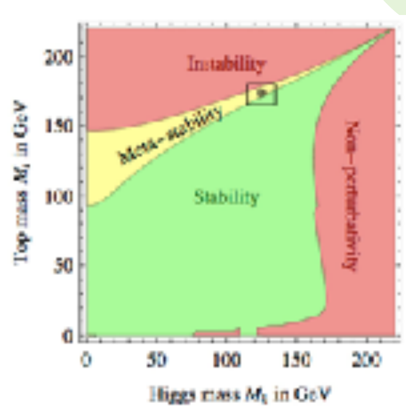
theoretical motivation

experimental challenges









Higgs mass

Flavor Hierarchy

Gauge Unification

Vacuum Stability

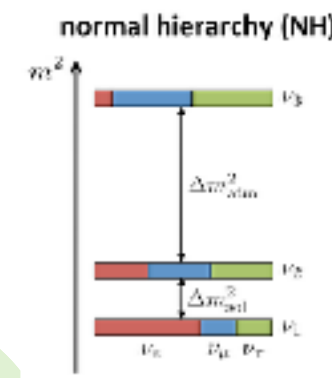
New Physics

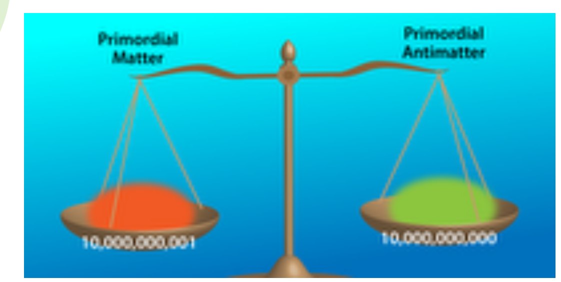
Neutrino

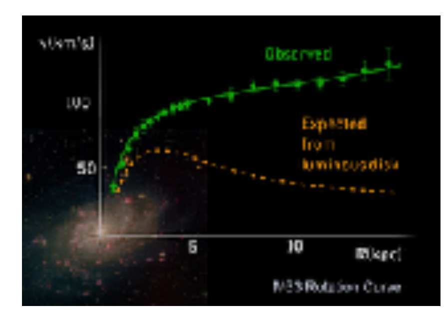
Baryon Asymmetry

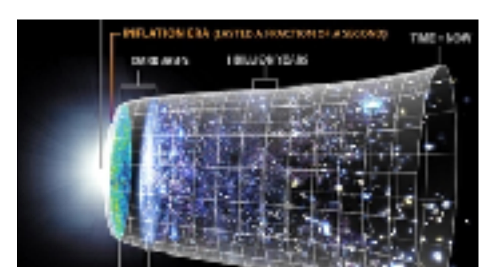
Dark matter

Inflation

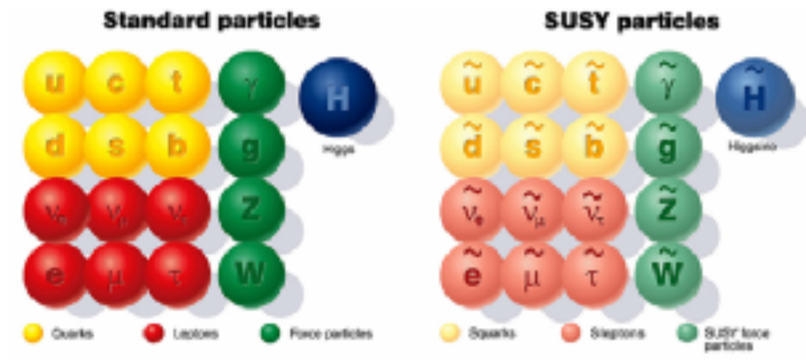




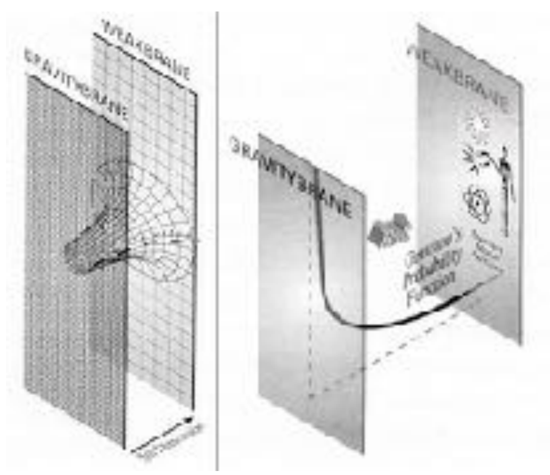




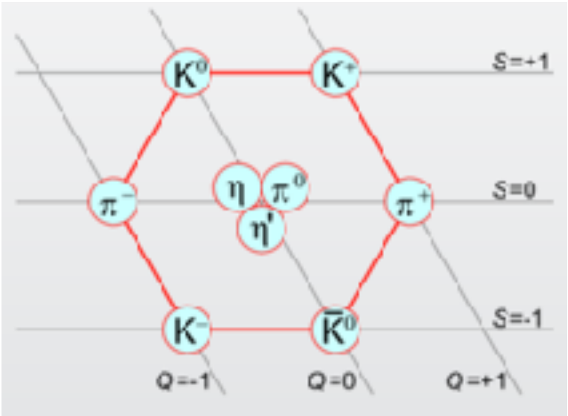
NP Motivated Simplified Model



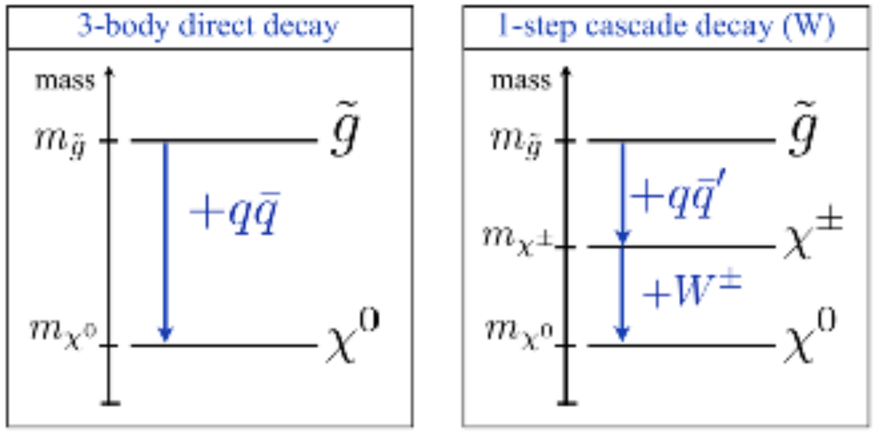
SUSY

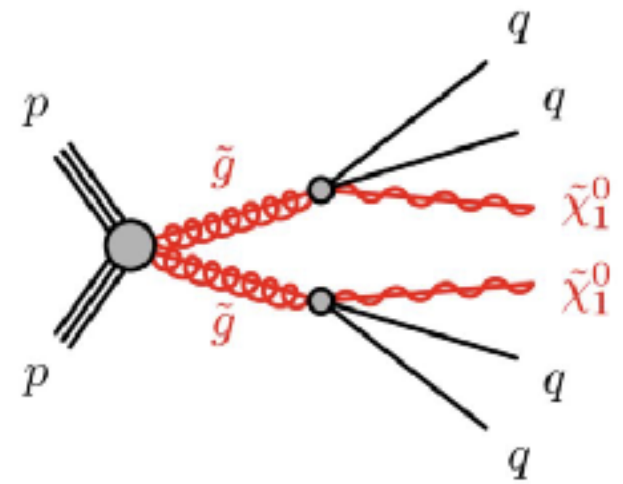


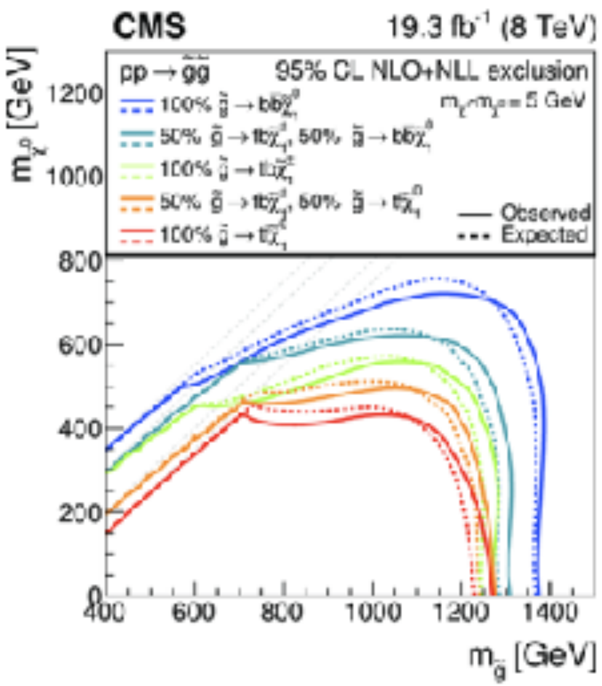
Extra Dimension



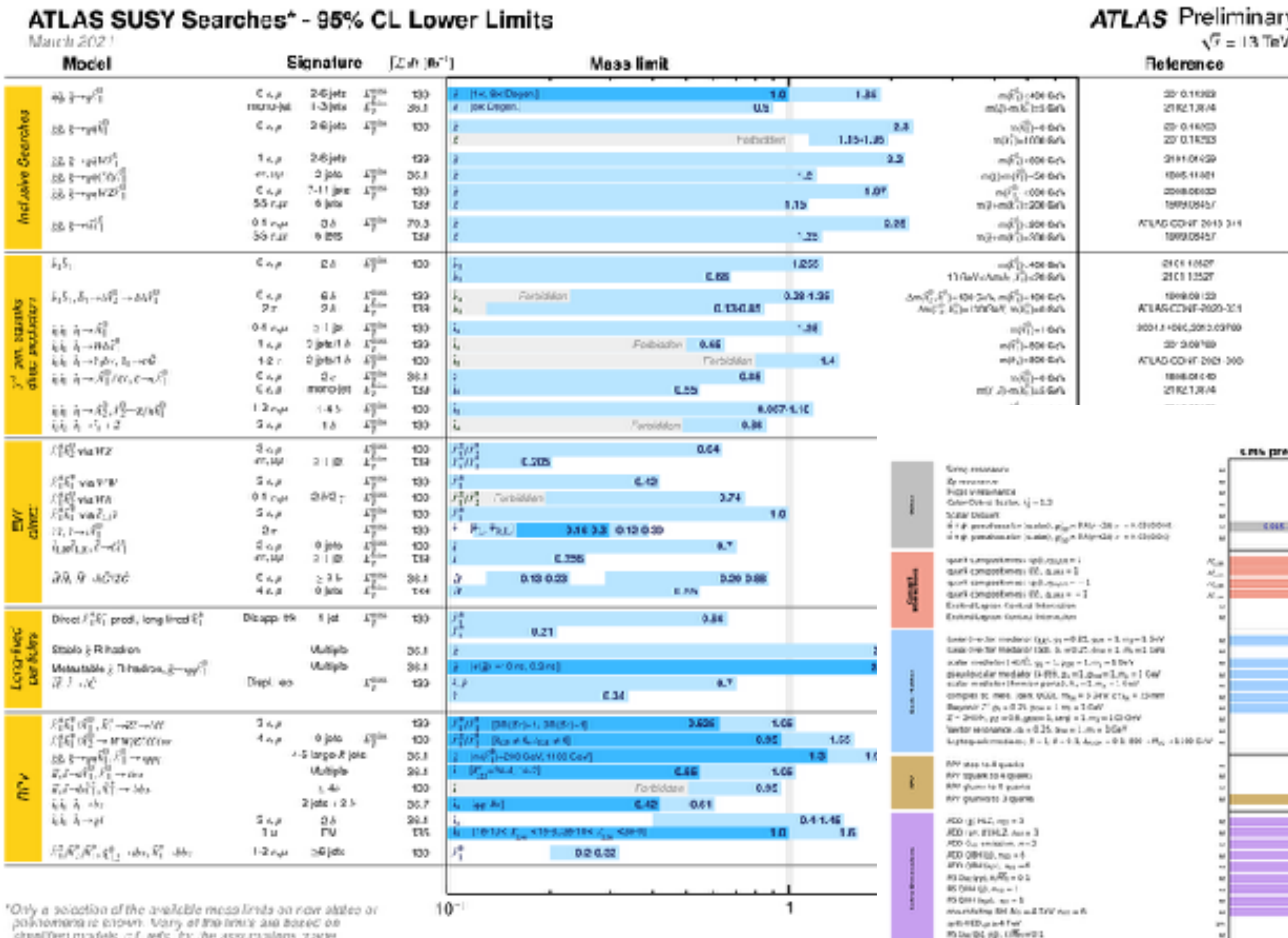
Composite Dynamics

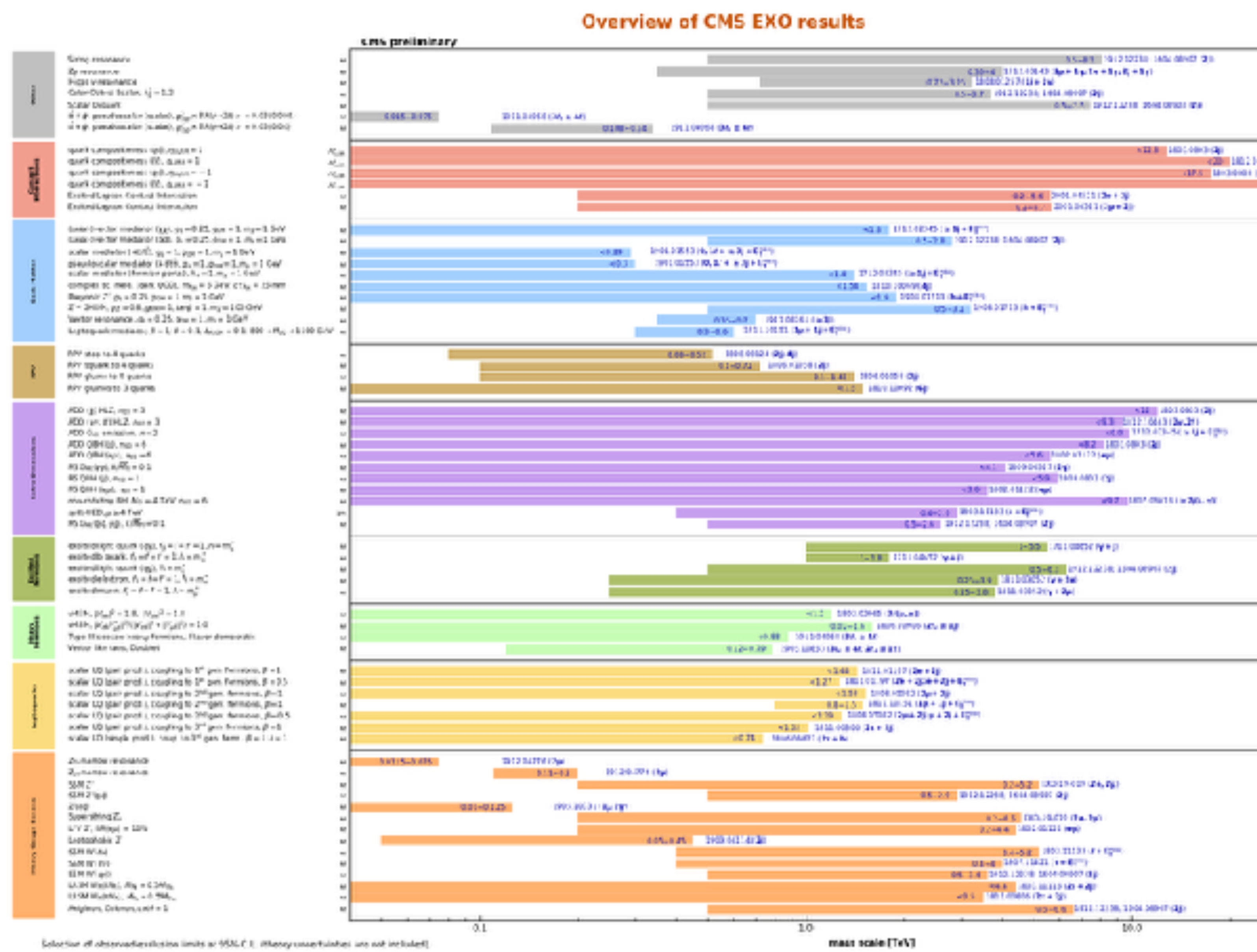






New Physics @ LHC



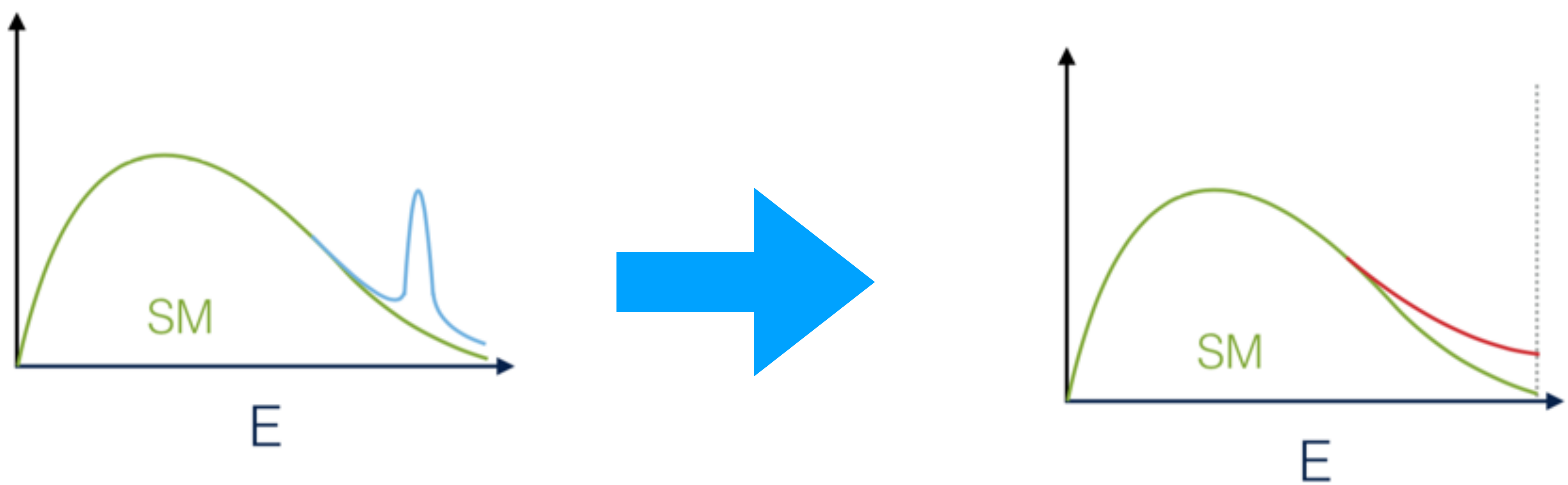


Paradigm Shift

New physics beyond the LHC threshold: paradigm shift for BSM searches

Direct signature

Indirect searches



resonance bump hunting at the LHC

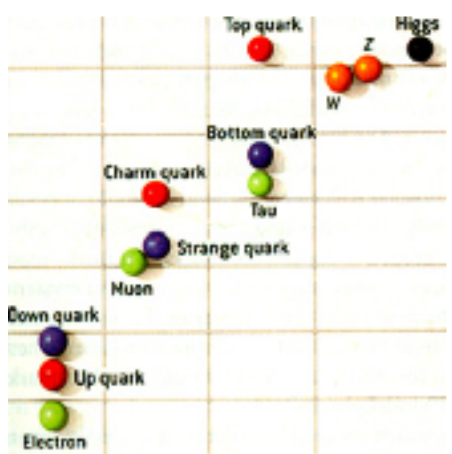
distribution tail deviation at the LHC

Top-Down EFT

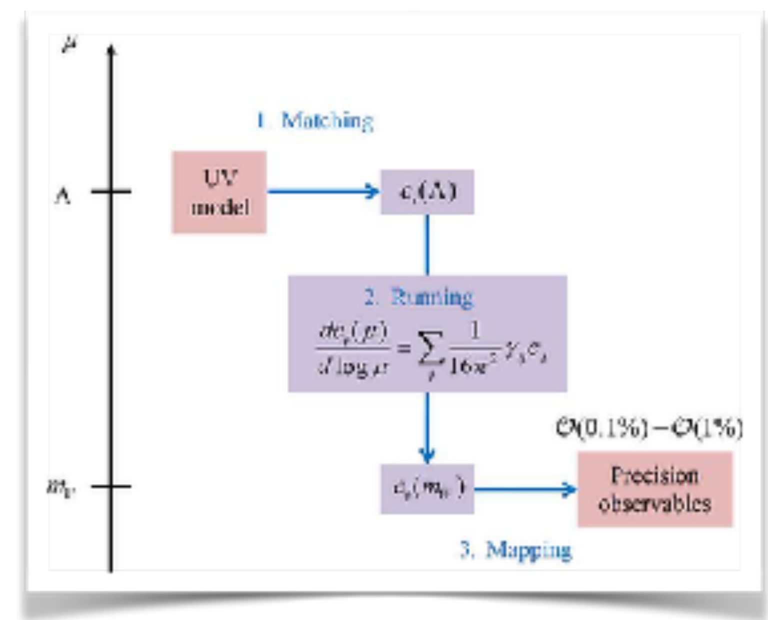
Given new physics models, integrate out heavy particles and match to SMEFT

New Physics Models

Integrate out and matching/running



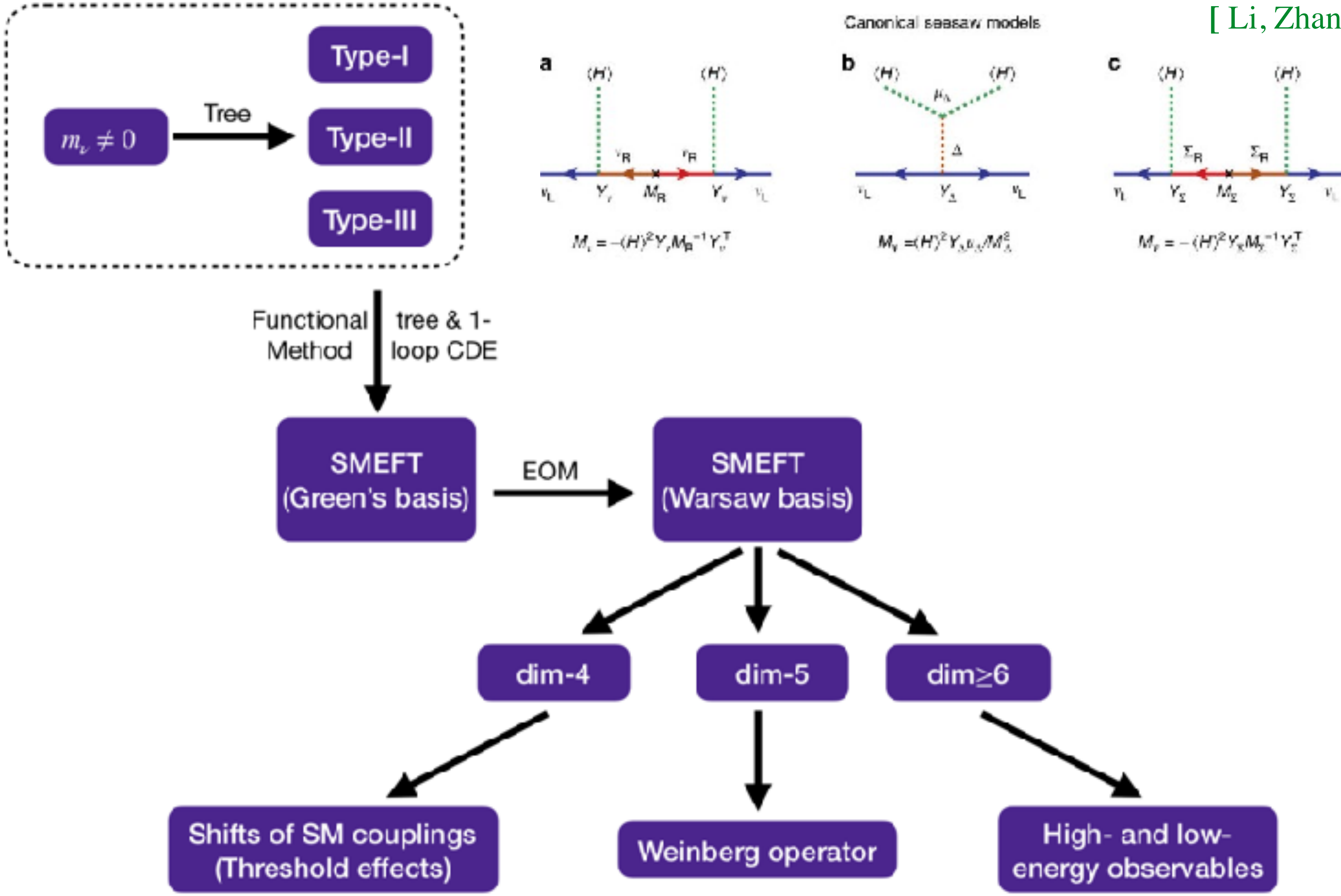
EFT Operators



Decoupling theorem

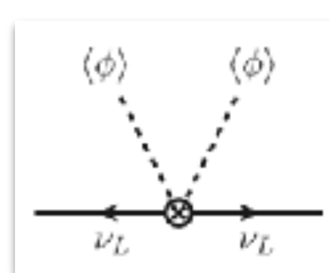
Canonical Seesaw Models

[Du, Li, **Yu**, 2201.04646]
[Li, Zhang, Zhou, 2021,2022]



$\mathcal{O}^{(1)}_{lequ}, \mathcal{O}_{eB}, \mathcal{O}_{ll}, \mathcal{O}_{HWB}, \mathcal{O}_{W}$

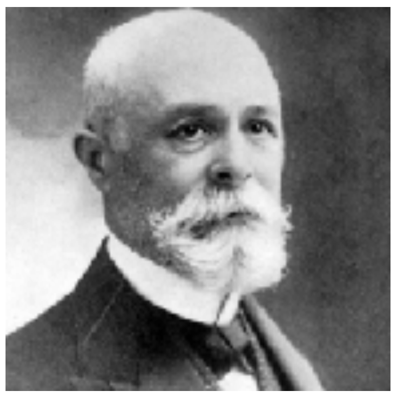
Radiative symmetry breaking



dim-6 operators

History of Weak Theory

Looking back to the past, we did not know the UV theory ahead

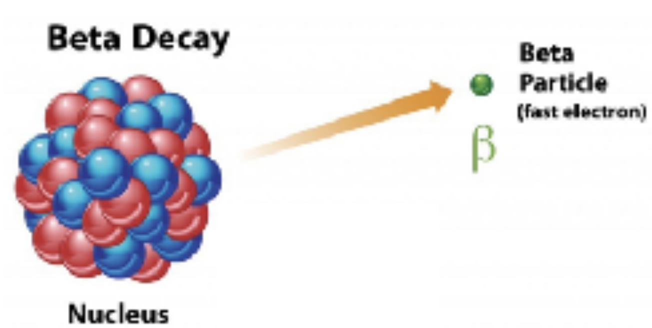


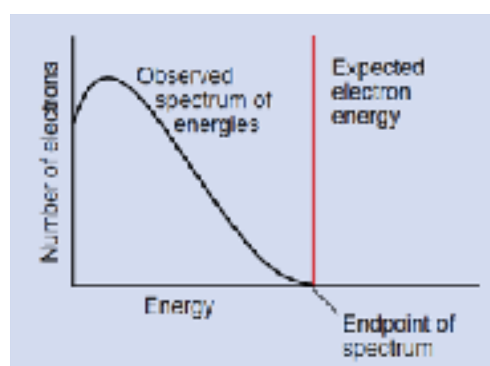
Becquerel
1896

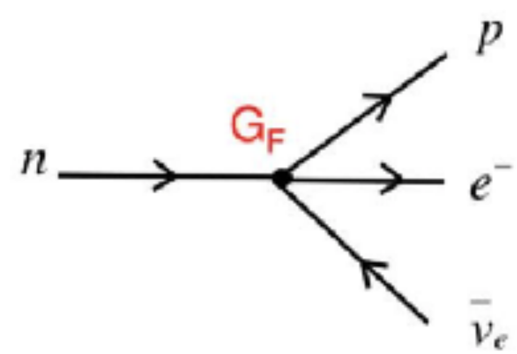
Pauli
1933

Fermi
1934

Gamov-Teller 1936
Fierz 1937







$$\mathcal{L}_i = \sum_{i=1}^{5} g_i \{\bar{\psi}_1 \mathcal{O}^i \psi_2\} \{\bar{\psi}_3 \mathcal{O}_i \psi_4\}$$

$$\mathcal{O}_i = (\mathbf{1},\ \gamma_\mu,\ \sigma_{\mu\nu},\ i\gamma_5\gamma_\mu,\ \text{or}\ \gamma_5$$

$$M_{fi} = G_F \left[\bar{\psi}_n \gamma^\mu \psi_p\right] \left[\bar{\psi}_e \gamma^\mu \psi_\nu\right]$$

vector current to
Fermi(V/S),
GT(A/T), P

Four-fermi EFT

Four-Fermi EFT

With parity violation, Lee and Yang wrote the most general 4-fermi operators

Lee-Yang 1956
Wu 1956

$$\vec{\sigma}_{\mathrm{Co}} \cdot \vec{p}_e$$

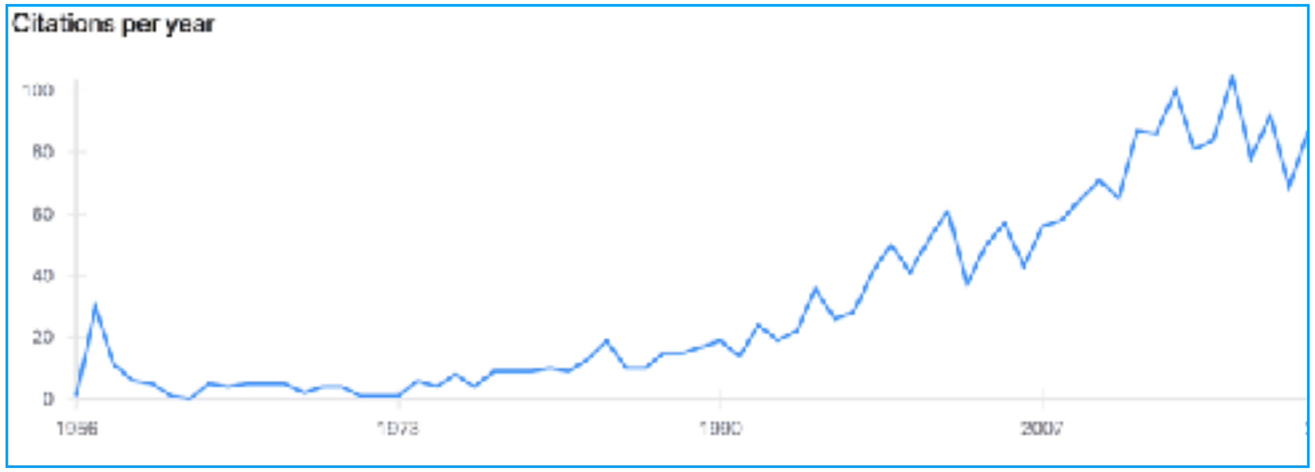
Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*
AND
C. N. Yang,† *Brookhaven National Laboratory, Upton, New York*
(Received June 22, 1956)

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\mathrm{int}} = & (\psi_p^\dagger \gamma_4 \psi_n)(C_S \psi_e^\dagger \gamma_4 \psi_\nu + C_S' \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_\mu \psi_n)(C_V \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu + C_V' \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu) \\ & + \tfrac{1}{2} (\psi_p^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_n)(C_T \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_\nu \\ & + C_T' \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \gamma_5 \psi_\nu) + (\psi_p^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_n) \\ & \times (-C_A \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu - C_A' \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_5 \psi_n)(C_P \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu + C_P' \psi_e^\dagger \gamma_4 \psi_\nu), \qquad \text{(A.1)} \end{aligned}$$

Complete charge current LEFT operators



Comprehensive analysis of beta decays within and beyond the Standard Model

[Falkowski, et.al 2021]

energies. The general EFT Lagrangian describing these interactions at the leading order was written more than 60 years ago by Lee and Yang [6]:

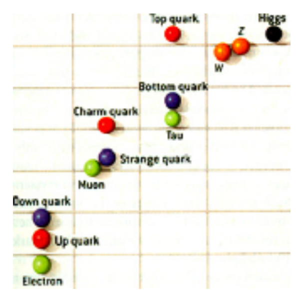
$$\begin{aligned} \mathcal{L}_{\text{Lee-Yang}} = & -\bar{p}\gamma^\mu n \left(C_V \bar{e}\gamma_\mu \nu - C_V' \bar{e}\gamma_\mu \gamma_5 \nu\right) + \bar{p}\gamma^\mu \gamma_5 n \left(C_A \bar{e}\gamma_\mu \gamma_5 \nu - C_A' \bar{e}\gamma_\mu \nu\right) \\ & - \bar{p} n \left(C_S \bar{e}\nu - C_S' \bar{e}\gamma_5 \nu\right) - \frac{1}{2}\bar{p}\sigma^{\mu\nu} n \left(C_T \bar{e}\sigma_{\mu\nu}\nu - C_T' \bar{e}\sigma_{\mu\nu}\gamma_5 \nu\right) \\ & - \bar{p}\gamma_5 n \left(C_P \bar{e}\gamma_5 \nu - C_P' \bar{e}\nu\right) + \text{h.c.} \qquad (1.1) \end{aligned}$$

Bottom-up Approach

PV Lesson: start from the complete bottom-up operators

?

UV unknown



EFT Operators

[Weinberg 1933 - 2021]

a folk theorem: "if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

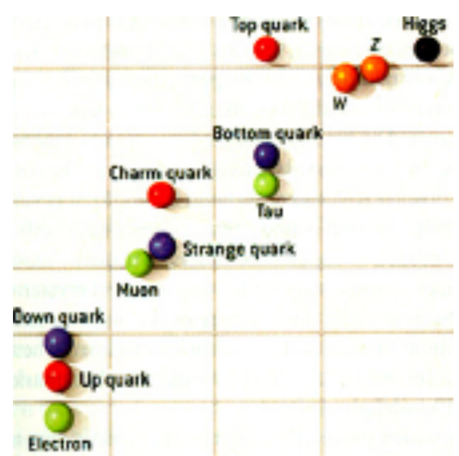
Weinberg's Folk theorem, 1979

SMEFT

Standard model effective field theory (SMEFT)

UV model unspecified

LHC probing scale



SMEFT

Most general Lagrangian with Lorentz and SM gauge symmetry

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots$$

power counting: canonical dim.

SMEFT provides systematic parametrization of

… all possible Lorentz inv. new physics!

SMEFT Operators

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

$$\mathscr{L}_{D=2} = \mu_H^2 H^\dagger H$$

$$\mathscr{L}_{D=3} = 0$$

$$\mathscr{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in q,u,d,L,e} i\bar{f}\gamma^\mu D_\mu f - (\bar{u}Y_u QH + \bar{d}Y_d H^\dagger Q + \bar{e}Y_e H^\dagger L + \mathrm{h.c.}) + D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2 + \bar{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

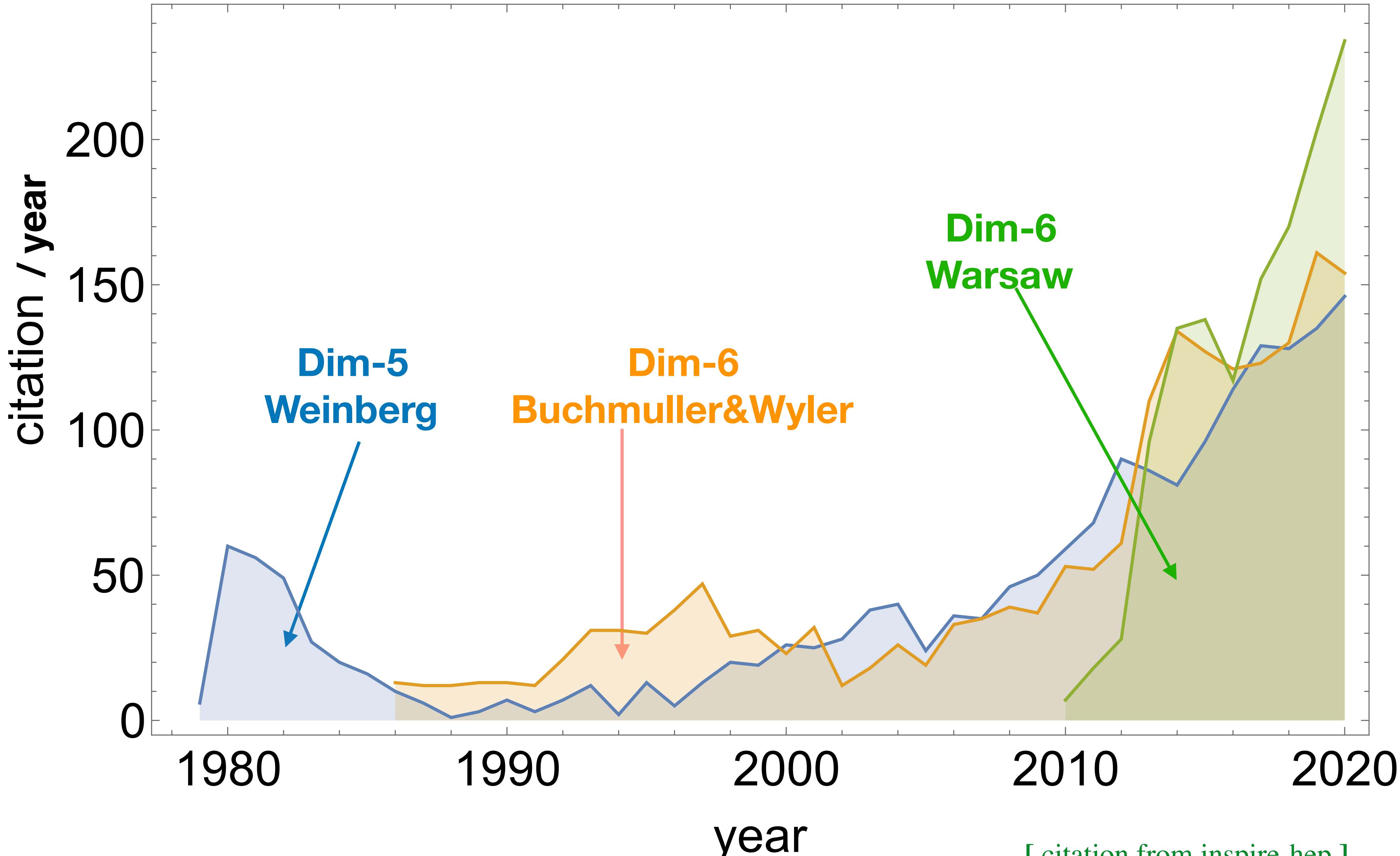
d.o.f: SM fields

symmetry: gauge/Lorentz

power counting: canonical dim.

In the spirit of EFT, each $\mathscr{L}_D$ should include a complete and non-redundant set of interactions

SMEFT Operators



[citation from inspire-hep]

Dim-5 Weinberg Operator

The only operator at the dimension 5

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.} \rightarrow c_{ij}\frac{v^2}{\Lambda}\nu_i \nu_j + \text{h.c.}$$

$$H \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad L_i \rightarrow \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$$

Lepton number violation **Neutrino Majorana mass!**

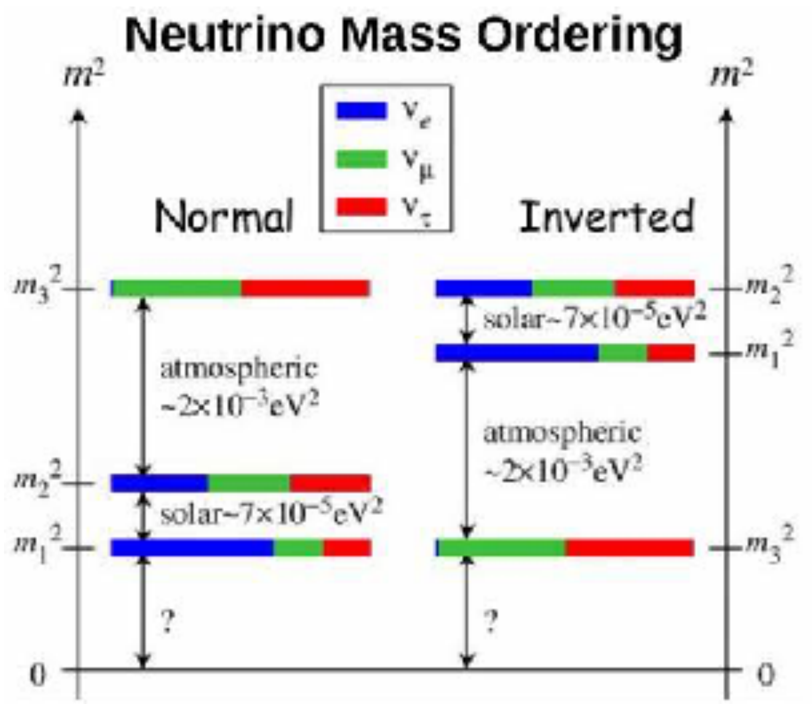
$N+h$ \ $N-h$	1	3	5
1		$\psi^2 D^2,\ F\phi D^2$	$F^2\phi,\ F\psi^2$
3	$\bar{\psi}^2 D^2,\ \bar{F}\phi D^2$	$\phi^3 D^2,\ \bar{\psi}\psi\phi D$	$\psi^2\phi^2$
5	$\bar{F}^2\phi,\ \bar{F}\bar{\psi}^2$	$\bar{\psi}^2\phi^2$	ϕ^5

Odd power of scalar, and SU(2)L transformation $\bar{\psi}_L \sigma^{\mu\nu}\psi_R$

Red color: eliminated by equation of motion

Dim-5 Operator

Dim-5 neutrino masses predicted by SMEFT and later observed!



$$\mathscr{L}_{\text{SMEFT}} \supset c_{ij}\frac{v^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$

0.01 eV - 0.1 eV

$$\frac{\Lambda}{c_{ij}} \sim 10^{15}\ \textbf{GeV}$$

Naively: $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **and then** $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$, **and so on**

It is however possible that Λ is not far from TeV, but instead $c_{ij} \ll 1$

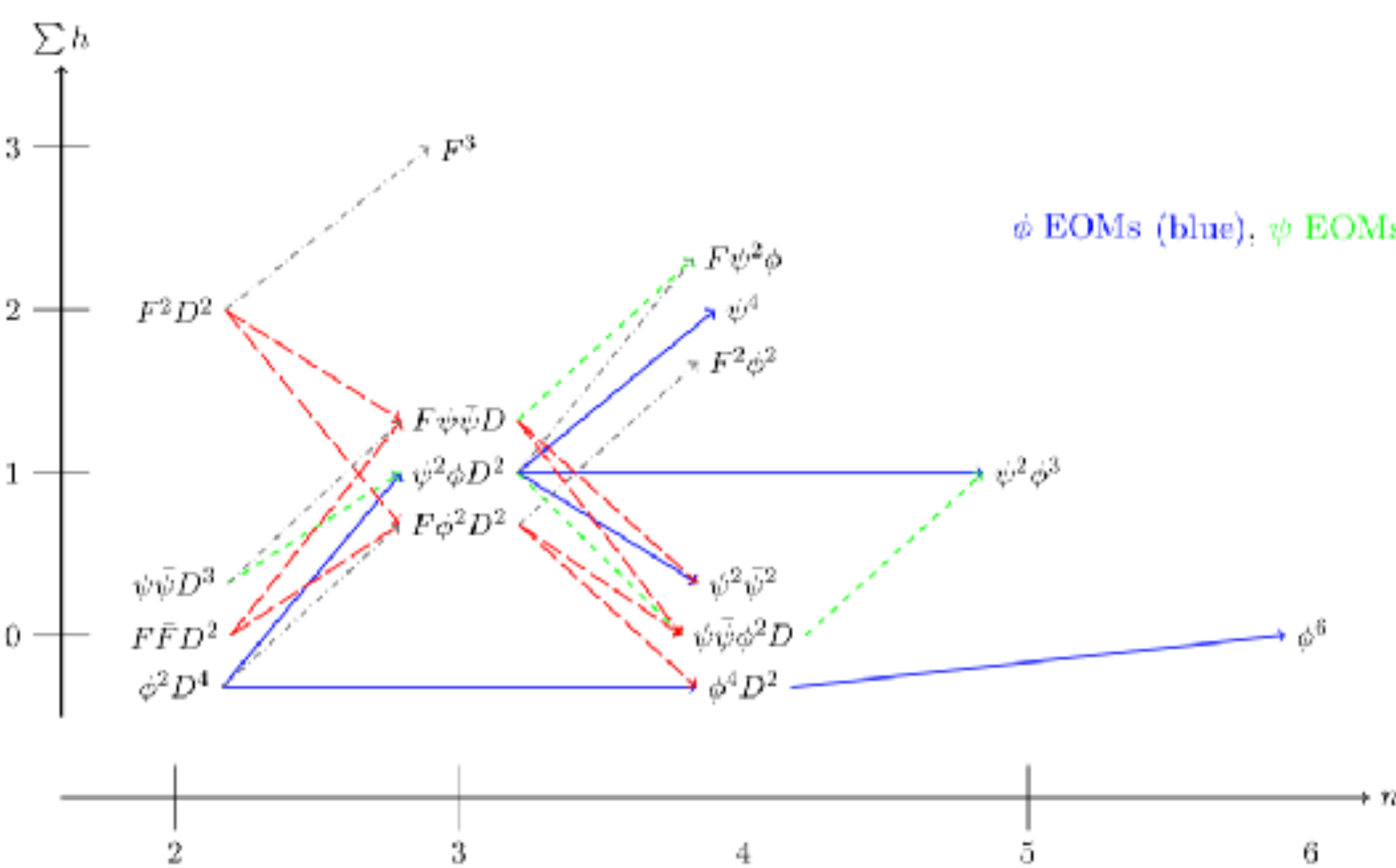
Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

$$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L},\ \mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2},\ \mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3},\ \mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4},$$ **and so on**

Dim-6 Operators

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$N+h$ \ $N-h$	0	2	4	6
0			F^2D^2	F^3
2		$\bar\psi\psi D^3$, $\bar F F D^2$, $\phi^2 D^4$	$F\bar\psi\psi D$, $\psi^2\phi D^2$, $F\phi^2 D^2$	$F^2\phi^2$, $F\psi^2\phi$, ψ^4
4	F^2D^2	$F\bar\psi\psi D$, $\bar\psi^2\phi D^2$, $\bar F\phi^2 D^2$	$\bar\psi\psi\phi^2 D$, $\bar\psi^2\psi^2$, $\phi^4 D^2$	$\psi^2\phi^3$
6	$\bar F^3$	$\bar F^2\phi^2$, $\bar F\bar\psi^2\phi$, $\bar\psi^4$	$\bar\psi^2\phi^3$	ϕ^6



ϕ EOMs (blue), ψ EOMs (green, short dashed), F EOMs (red, long dashed)

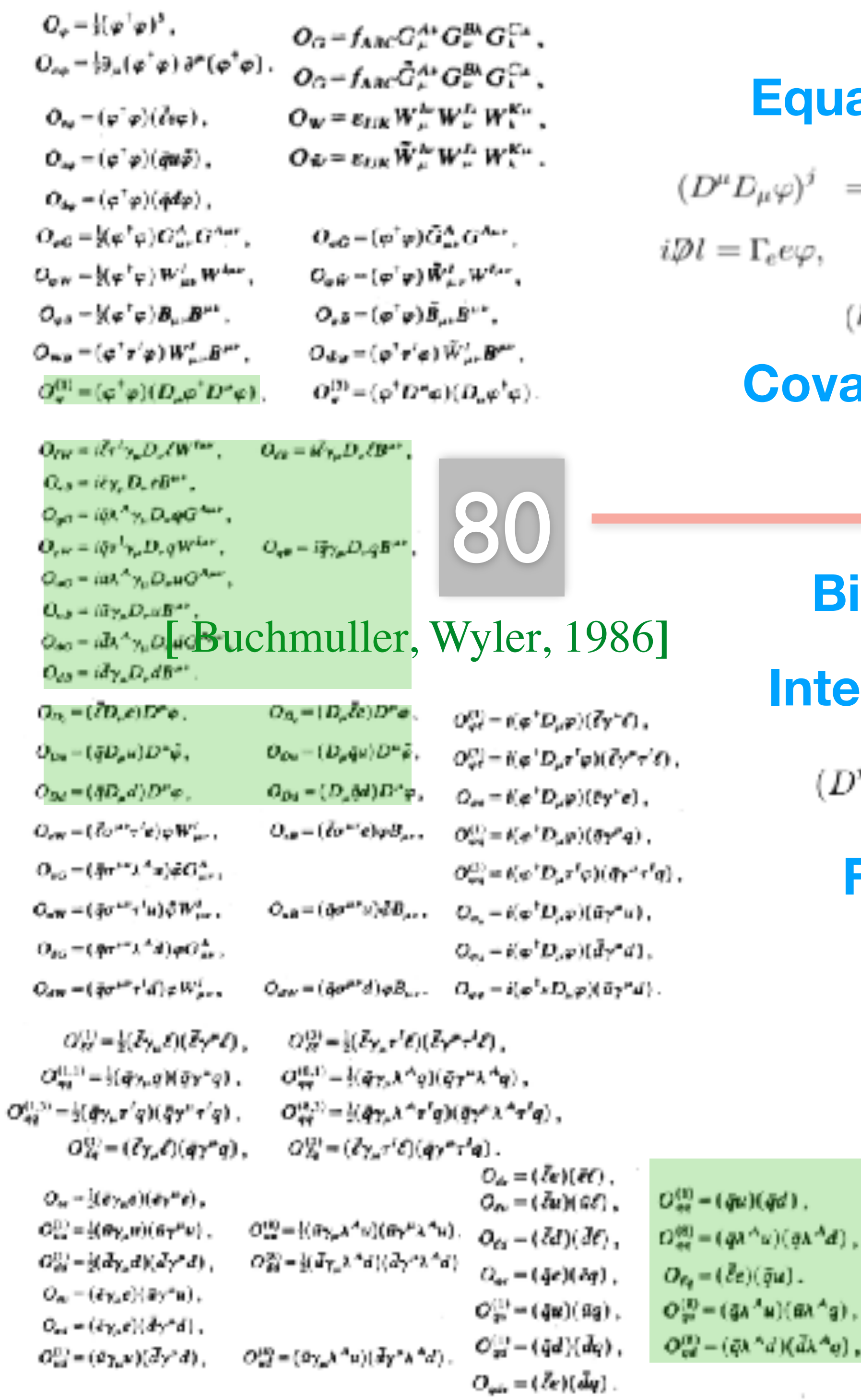
field strengths (grey, dash-dotted)

- 59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)
  Buchmuller-Wyler, Grzadkowski-Iskrzynski-Misiak-Rosiek
- 5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)
  Weinberg, Wilczek-Zee, Abbott-Wise
- 3 generations: 1350 CP-even and 1149 CP-odd operators with D=6
  Alonso-Jenkins-Manohar-Trott

Dim-6 Operators

Why completing dim-6 took more than 25 years?

tedious and prone-to-error



80

[Buchmuller, Wyler, 1986]

Equation of motion (Field redefinition)

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda (\varphi^\dagger \varphi) \varphi^j - \bar{e} \Gamma_e^\dagger l^j + \epsilon_{jk} \bar{q}^k \Gamma_u u - \bar{d} \Gamma_d^\dagger q^j$$

$$i \not{D} l = \Gamma_e e \varphi, \quad i \not{D} e = \Gamma_e^\dagger \varphi^\dagger l, \quad i \not{D} q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i \not{D} u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q,$$

$$(D^\rho W_{\rho\mu})^I = \frac{g}{2} \left(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi + \bar{l} \gamma_\mu \tau^I l + \bar{q} \gamma_\mu \tau^I q \right),$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

Bianchi identity $D_{[\rho} X_{\mu\nu]} = 0$

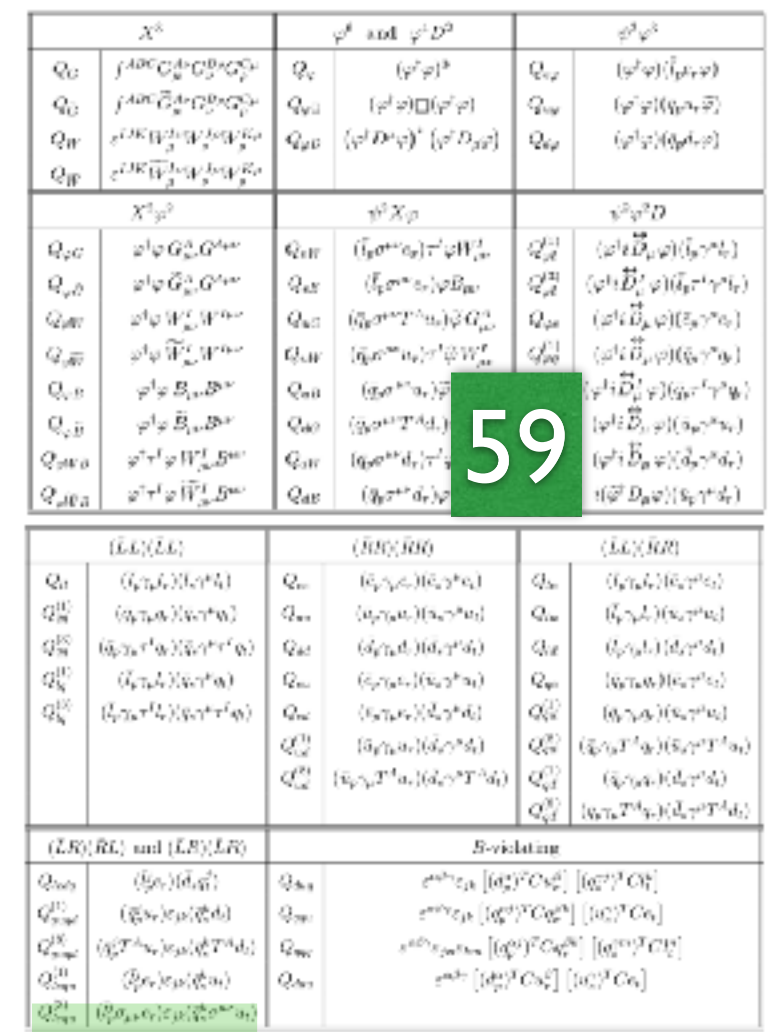
Integration by part (total derivatives)

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1} \varphi)^\dagger (D^{m-1} \varphi) + \partial \left[(D^n \varphi)^\dagger (D^{m-1} \varphi) \right]$$

Fierz identity

$$T^A_{\alpha\beta} T^A_{\kappa\lambda} = \frac{1}{2} \delta_{\alpha\lambda} \delta_{\kappa\beta} - \frac{1}{6} \delta_{\alpha\beta} \delta_{\kappa\lambda}$$

$$\tau^I_{jk} \tau^I_{mn} = 2 \delta_{jn} \delta_{mk} - \delta_{jk} \delta_{mn}$$



59

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

80-1-16-5+1 = 59

EOM Redundancy

$$i\not{D}l = \Gamma_e e\varphi, \quad i\not{D}e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\not{D}q = \Gamma_u u\widetilde{\varphi} + \Gamma_d d\varphi, \quad i\not{D}u = \Gamma_u^\dagger \widetilde{\varphi}^\dagger q, \quad i\not{D}d = \Gamma_d^\dagger \varphi^\dagger q. \tag{6.2}$$

$$\gamma_\mu\gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}, \qquad \gamma_\mu\gamma_\nu\gamma_\rho = g_{\mu\nu}\gamma_\rho + g_{\nu\rho}\gamma_\mu - g_{\mu\rho}\gamma_\nu - i\varepsilon_{\mu\nu\rho\sigma}\gamma^\sigma\gamma_5. \tag{6.3}$$

$$\begin{aligned}
(D^\mu D_\mu\varphi)^j &= m^2\varphi^j - \lambda(\varphi^\dagger\varphi)\varphi^j - \bar{e}\Gamma_e^\dagger l^j + \varepsilon_{jk}\bar{q}^k\Gamma_u u - \bar{d}\Gamma_d^\dagger q^j, \\
(D^\rho G_{\rho\mu})^A &= g_s\left(\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d\right), \\
(D^\rho W_{\rho\mu})^I &= \frac{g}{2}\left(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi + \bar{l}\gamma_\mu\tau^I l + \bar{q}\gamma_\mu\tau^I q\right), \\
\partial^\rho B_{\rho\mu} &= g' Y_\varphi\, \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi + g' \sum_{\psi\in\{l,e,q,u,d\}} Y_\psi\, \bar{\psi}\gamma_\mu\psi.
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
\bar{\psi}\psi D_\mu D^\mu\varphi &\overset{(5.1)}{=} \boxed{\psi^4} + \boxed{\psi^2\varphi^3} + m^2\boxed{\psi^2\varphi} + \boxed{E}, \\
\varphi\bar{\psi} D_\mu D^\mu\psi &\overset{(6.3)}{=} \varphi\bar{\psi}\not{D}\not{D}\psi + \boxed{\psi^2 X\varphi} \overset{(6.2)}{=} \boxed{\psi^2 X\varphi} + \boxed{\psi^2\varphi^2 D} + \boxed{E}, \\
(D_\mu\varphi)\bar{\psi}\sigma^{\mu\nu}D_\nu\psi &= \frac{i}{2}(D_\mu\varphi)\bar{\psi}\left(\gamma^\mu\not{D} - \not{D}\gamma^\mu\right)\psi = i(D_\mu\varphi)\bar{\psi}\gamma^\mu\not{D}\psi - i(D^\mu\varphi)\bar{\psi}D_\mu\psi \\
&\overset{(6.2)}{=} -i(D^\mu\varphi)\bar{\psi}D_\mu\psi + \boxed{\psi^2\varphi^2 D} + \boxed{E}, \\
2(D^\mu\varphi)\bar{\psi}D_\mu\psi &= (D^\mu\varphi)\bar{\psi}(\gamma_\mu\not{D} + \not{D}\gamma_\mu)\psi \\
&= (D^\mu\varphi)\bar{\psi}\gamma_\mu\not{D}\psi - \bar{\psi}\overleftarrow{\not{D}}\gamma_\mu\psi\, D^\mu\varphi - \bar{\psi}\gamma^\nu\gamma^\mu\psi\, D_\nu D_\mu\varphi + \boxed{T} \\
&\overset{(6.2)}{=} \boxed{\psi^2\varphi^2 D} + \boxed{\psi^4} + \boxed{\psi^2\varphi^3} + m^2\boxed{\psi^2\varphi} + \boxed{\psi^2 X\varphi} + \boxed{E} + \boxed{T}, \\
X^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi &= \frac{1}{2}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} + \gamma_\mu\not{D}\gamma_\nu)\psi = \frac{1}{2}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi + X^{\mu\nu}\bar{\psi}\gamma_\nu D_\mu\psi \\
&\overset{(*)}{=} \frac{1}{4}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi = \frac{1}{4}X^{\mu\nu}\bar{\psi}\gamma_\mu\gamma_\nu\not{D}\psi + \frac{1}{4}\bar{\psi}\overleftarrow{\not{D}}\gamma_\mu\gamma_\nu\psi\, X^{\mu\nu} \\
&+ \frac{1}{4}\bar{\psi}\gamma_\rho\gamma_\mu\gamma_\nu\psi\, D^\rho X^{\mu\nu} + \boxed{T} \overset{(6.2)}{=} \boxed{\psi^2 X\varphi} + \boxed{\psi^2\varphi^2 D} + \boxed{\psi^4} + \boxed{E} + \boxed{T}.
\end{aligned} \tag{6.6}$$

$$\begin{aligned}
(\varphi^\dagger\tau^I\varphi)\left[(D_\mu\varphi)^\dagger\tau^I(D^\mu\varphi)\right] &\overset{(4.3)}{=} 2\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right) - (\varphi^\dagger\varphi)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right], \\
(\varphi^\dagger\varphi)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right] &\overset{(5.1)}{=} \frac{1}{2}(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi) + \boxed{\psi^2\varphi^3} + \boxed{\varphi^6} + m^2\boxed{\varphi^4} + \boxed{E}.
\end{aligned}$$

$$\overset{(\sim)}{X}{}^{\mu\nu} D^\rho D_\rho X_{\mu\nu} = -\overset{(\sim)}{X}{}^{\mu\nu}\left(D^\rho D_\mu X_{\nu\rho} + D^\rho D_\nu X_{\rho\mu}\right) = \boxed{X^3} + \boxed{\varphi^2 X D^2} + \boxed{\psi^2 X D} + \boxed{E}.$$

Fierz Identity

$$
\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}
$$

$$(\bar\psi_L\gamma_\mu\psi_L)(\bar\chi_L\gamma^\mu\chi_L) = (\bar\psi_L\gamma_\mu\chi_L)(\bar\chi_L\gamma^\mu\psi_L) \tag{4.1}$$

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn} \tag{4.3}$$

$$T^A_{\alpha\beta}T^A_{\kappa\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}\,, \tag{7.3}$$

$$(\bar l_p\gamma_\mu\tau^I l_r)(\bar l_s\tau^I\gamma^\mu l_t) = 2(\bar l_p^j\gamma_\mu l_r^k)(\bar l_s^k\gamma^\mu l_t^j) - Q_{ll}^{prst} \;=\; 2Q_{ll}^{ptsr} - Q_{ll}^{prst}$$

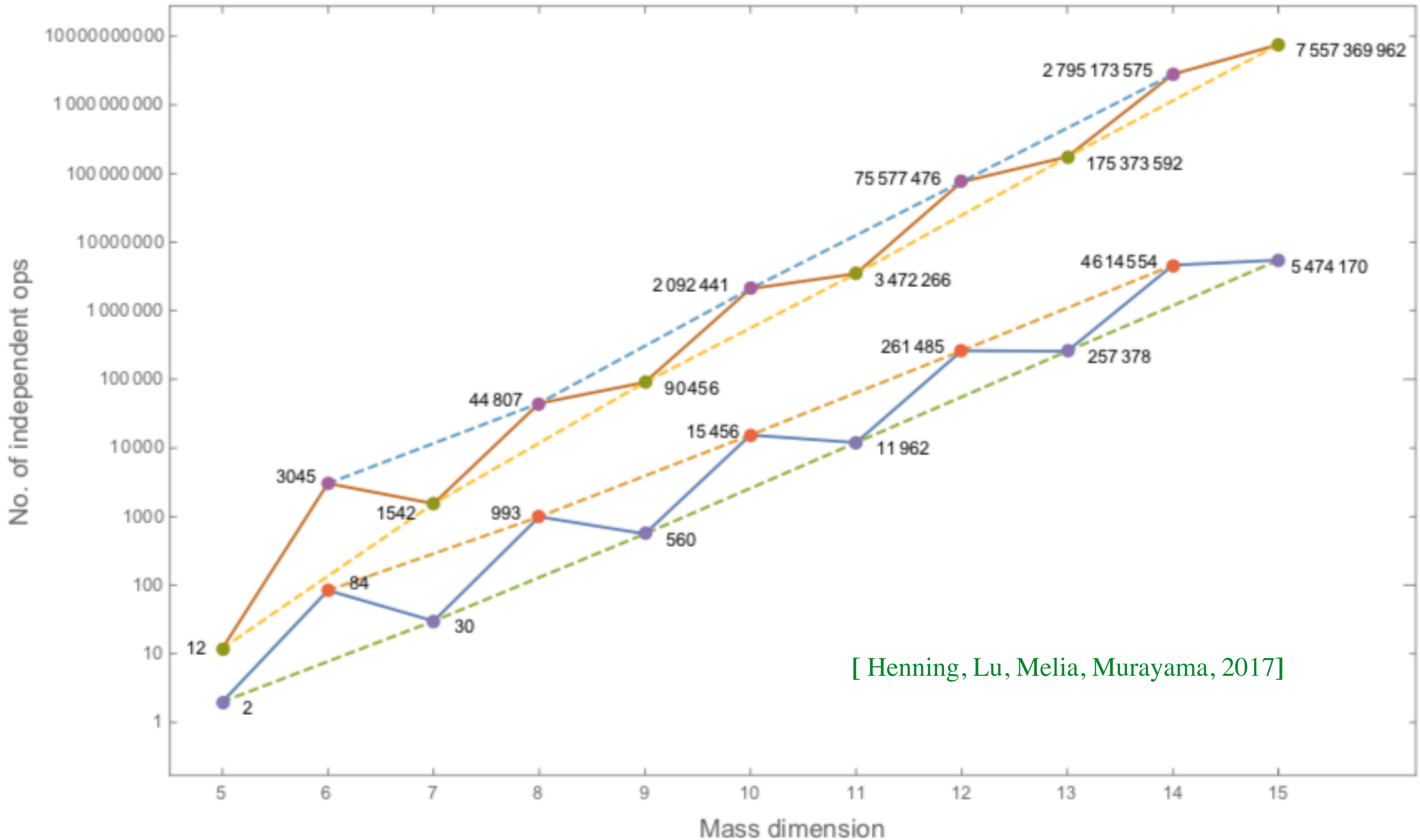
$$(\bar u_p\gamma_\mu T^A u_r)(\bar u_s T^A\gamma^\mu u_t) \overset{(7.3)}{=} \frac{1}{2}(\bar u_p^\alpha\gamma_\mu u_r^\beta)(\bar u_s^\beta\gamma^\mu u_t^\alpha) - \frac{1}{6}Q_{uu}^{prst} \;=\; \frac{1}{2}Q_{uu}^{ptsr} - \frac{1}{6}Q_{uu}^{prst}, \tag{7.4}$$

$$(\bar d_p\gamma_\mu T^A d_r)(\bar d_s T^A\gamma^\mu d_t) \overset{(7.3)}{=} \frac{1}{2}(\bar d_p^\alpha\gamma_\mu d_r^\beta)(\bar d_s^\beta\gamma^\mu d_t^\alpha) - \frac{1}{6}Q_{dd}^{prst} \;=\; \frac{1}{2}Q_{dd}^{ptsr} - \frac{1}{6}Q_{dd}^{prst}, \tag{7.5}$$

$$
\begin{aligned}
(\bar q_p\gamma_\mu T^A q_r)(\bar q_s T^A\gamma^\mu q_t) &\overset{(7.3)}{=} \frac{1}{2}(\bar q_p^{\alpha j}\gamma_\mu q_r^{\beta j})(\bar q_s^{\beta k}\gamma^\mu q_t^{\alpha k}) - \frac{1}{6}Q_{qq}^{(1)prst} \\
&\overset{(4.1)}{=} \frac{1}{2}(\bar q_p^{\alpha j}\gamma_\mu q_t^{\alpha k})(\bar q_s^{\beta k}\gamma^\mu q_r^{\beta j}) - \frac{1}{6}Q_{qq}^{(1)prst} \\
&\overset{(4.3)}{=} \frac{1}{4}Q_{qq}^{(3)ptsr} + \frac{1}{4}Q_{qq}^{(1)ptsr} - \frac{1}{6}Q_{qq}^{(1)prst},
\end{aligned} \tag{7.6}
$$

$$
\begin{aligned}
(\bar q_p\gamma_\mu T^A\tau^I q_r)(\bar q_s T^A\tau^I\gamma^\mu q_t) &\overset{(7.3)}{=} \frac{1}{2}(\bar q_p^\alpha\gamma_\mu\tau^I q_r^\beta)(\bar q_s^\beta\gamma^\mu\tau^I q_t^\alpha) - \frac{1}{6}Q_{qq}^{(3)prst} \\
&\overset{(4.3)}{=} (\bar q_p^{\alpha j}\gamma_\mu q_r^{\beta k})(\bar q_s^{\beta k}\gamma^\mu q_t^{\alpha j}) - \frac{1}{2}(\bar q_p^{\alpha j}\gamma_\mu q_r^{\beta j})(\bar q_s^{\beta k}\gamma^\mu q_t^{\alpha k}) - \frac{1}{6}Q_{qq}^{(3)prst} \\
&\overset{(4.1)}{=} Q_{qq}^{(1)ptsr} - \frac{1}{2}(\bar q_p^{\alpha j}\gamma_\mu q_t^{\alpha k})(\bar q_s^{\beta k}\gamma^\mu q_r^{\beta j}) - \frac{1}{6}Q_{qq}^{(3)prst} \\
&\overset{(4.3)}{=} -\frac{1}{4}Q_{qq}^{(3)ptsr} + \frac{3}{4}Q_{qq}^{(1)ptsr} - \frac{1}{6}Q_{qq}^{(3)prst}.
\end{aligned} \tag{7.7}
$$

SMEFT Operators



Main Difficulties

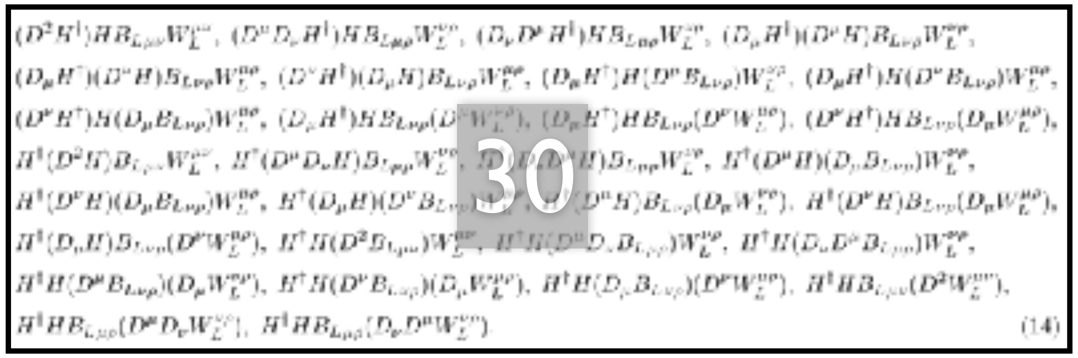
How about higher dimensional operators?

difficult to write down explicit form of operators

Derivatives

$BWHH^\dagger D^2$

2



Which 2 should be picked up?

Repeated fields

$QQQL$

57

$$Q^{qqq\ell}_{prst} = C^{prst} \begin{matrix} \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \end{matrix} \qquad p, r, s, t = 1, 2, 3$$

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What flavor relations should be imposed?

Operator as Spinor Tensor

Operator has more symmetries than what we expect

SO(3,1)	SL(2,C) $SU(2)_l \times SU(2)_r$	Spinor-helicity
ϕ	$\phi \in (0,0)$	
ψ	$\psi_\alpha \in (1/2, 0)$	λ_α
	$\psi^\dagger_{\dot\alpha} \in (0, 1/2)$,	
$F_{\mu\nu}$	$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu} \sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$	$\lambda_\alpha \lambda_\beta$
	$F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu} \bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1)$.	
$R_{\mu\nu\rho\sigma}$	$\mathcal{C}_{\alpha\beta\gamma\delta} - C_{\mu\nu\rho\sigma} \sigma^{\mu\nu}_{\alpha\beta} \sigma^{\rho\sigma}_{\gamma\delta} \in (2,0)$	$\lambda_\alpha \lambda_\beta \lambda_\gamma \lambda_\delta$
D_μ	$D_{\alpha\dot\alpha} = D_\mu \sigma^\mu_{\alpha\dot\alpha} \in (1/2, 1/2)$,	$\lambda_\alpha \tilde\lambda_{\dot\alpha}$

Operator with explicit spinor indices

$$W_{\mu\lambda} \left(e_{cp} \sigma^{\nu\lambda} D^\mu L_r \right) D_\nu H^\dagger$$

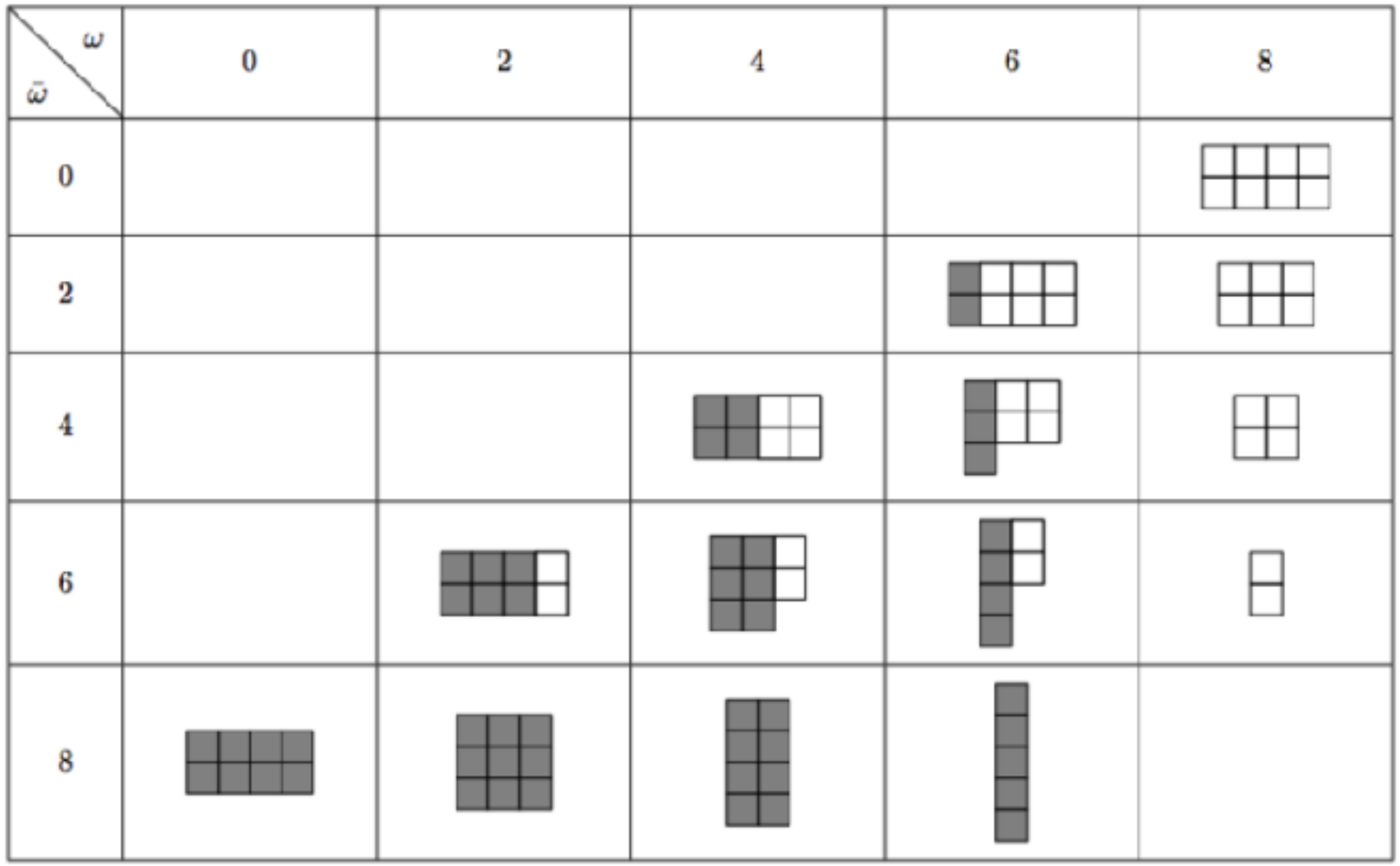
$$F_1^{\alpha\beta} \psi_2^{\gamma} (D\psi_3)_{\alpha\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$\epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_2\alpha_4} \epsilon^{\dot\alpha_3\dot\alpha_4} F_1^{\alpha_1^2} \psi_2^{\alpha_2} (D\psi_3)^{\alpha_3^2}_{\dot\alpha_3} (D\phi_4)^{\alpha_4}_{\dot\alpha_4}$$

Easier to find more symmetries of the operator with spinor indices

Operator as Spinor Tensor

Dim-8 operators: 993 (44807) operators for 1 (3) generations



Unified construction of Lorentz & gauge structures by Young Tableau

$$\left(\begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline 2&3&3&4\\\hline\end{array} + \begin{array}{|c|c|c|c|}\hline 1&1&1&3\\\hline 2&2&3&4\\\hline\end{array}\right)\times \begin{array}{|c|c|}\hline i&j\\\hline k&l\\\hline\end{array} = \boxed{(\tau^I)^i_j W^I_{\mu\nu}(e_{\mathbb{C}p} D^\mu L_{ri}) D^\nu H^{\dagger j}} + \boxed{(\tau^I)^i_j W^I_{\mu\lambda}(e_{\mathbb{C}p}\sigma^{\nu\lambda}L_{ri}) D^\mu D_\nu H^{\dagger j}}$$

Operator as Spinor Tensor

Young tensor method (No need EOM&IBP)

[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]
[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]

$\boxed{BWHH^\dagger D^2}$ $\quad \#1=3, \#2=3, \#3=1, \#4=1$

2

$$\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 3 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 3 & 4 \\ \hline \end{array}$$

$$\tilde{\epsilon}_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4} \qquad \tilde{\epsilon}_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$$

$$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^\dagger\right)^\gamma{}_{\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}, \qquad B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\left(DH^\dagger\right)_{\beta\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$$

$$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34] \qquad \langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$$

All Things EFT...seminar series

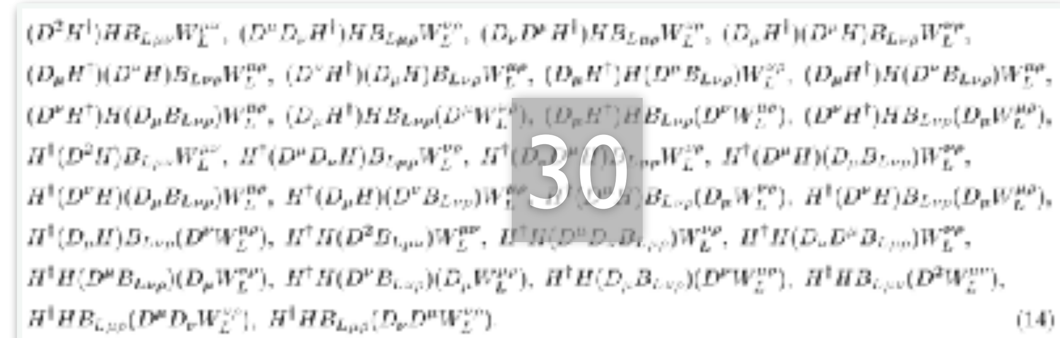
EFT Operator Bases for Standard Model and Beyond
报告时间：2021-06-09
报告人：于江浩

https://www.koushare.com/video/videodetail/12645

Traditional method

$\boxed{BWHH^\dagger D^2}$

[Hays, Martin, Sanz, Setford, 2018]



EOM

7

$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\tfrac{1}{2}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\delta\xi}(\epsilon^{\alpha\gamma}\epsilon^{\beta\eta}+\epsilon^{\beta\gamma}\epsilon^{\alpha\eta})$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$(DH^\dagger)_{\alpha\dot\alpha}\,H\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,(DB_L)_{\{\beta\gamma\delta\},\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger\,(DH)_{\alpha\dot\alpha}\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$$
$$H^\dagger H\,(DB_L)_{\{\alpha\beta\gamma\},\dot\alpha}(DW_L)_{\{\xi\eta\delta\},\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\xi}\epsilon^{\beta\eta}\epsilon^{\gamma\delta}$$

IBP

2

$$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^\dagger\right)^\gamma{}_{\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$$
$$B_L{}^{\alpha\beta}W_{L\alpha}{}^\gamma\left(DH^\dagger\right)_{\beta\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$$

Repeated Fields with Flavor

Another difficulty to write down the independent EFT operators

B-violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

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$$Q_{prst}^{qqq\ell\,(1)} = -(Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell})$$
$$Q_{prst}^{qqq\ell\,(3)} = -(Q_{prst}^{qqq\ell} - Q_{rpst}^{qqq\ell})$$

[Grzadkowski, et.al. v3 2017]

B-violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

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[Alonso, Chang, Jenkins, Manohar, Shotwell 2014]

$$Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell} = Q_{sprt}^{qqq\ell} + Q_{srpt}^{qqq\ell}$$

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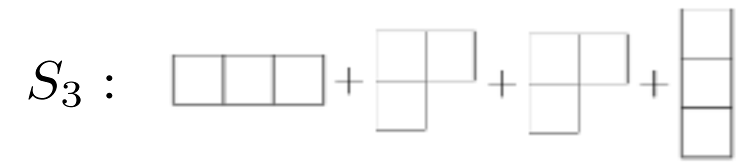
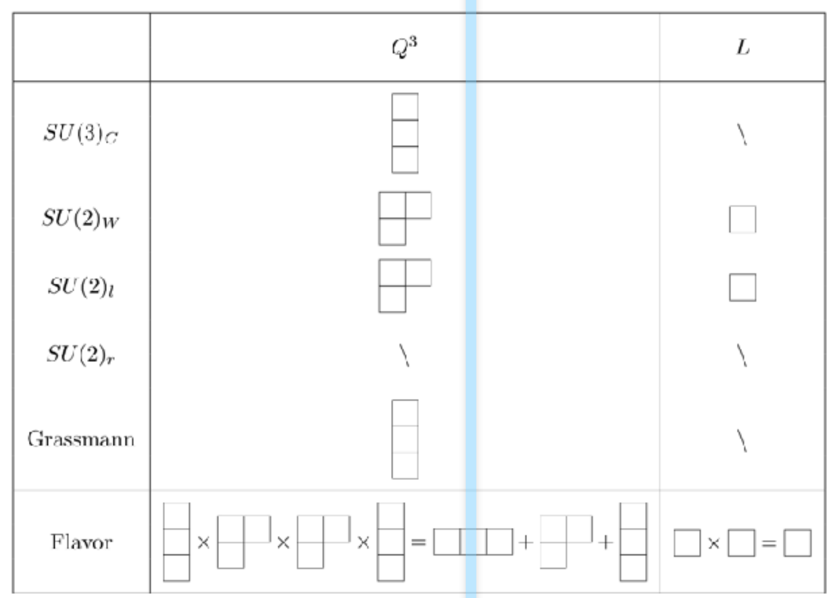
Flavor relations not easy task!

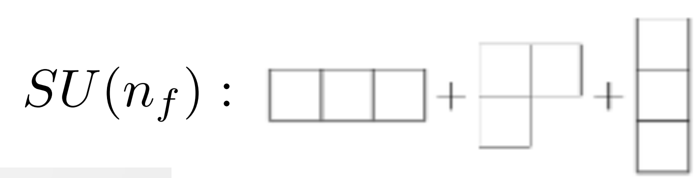
Flavor Symmetry

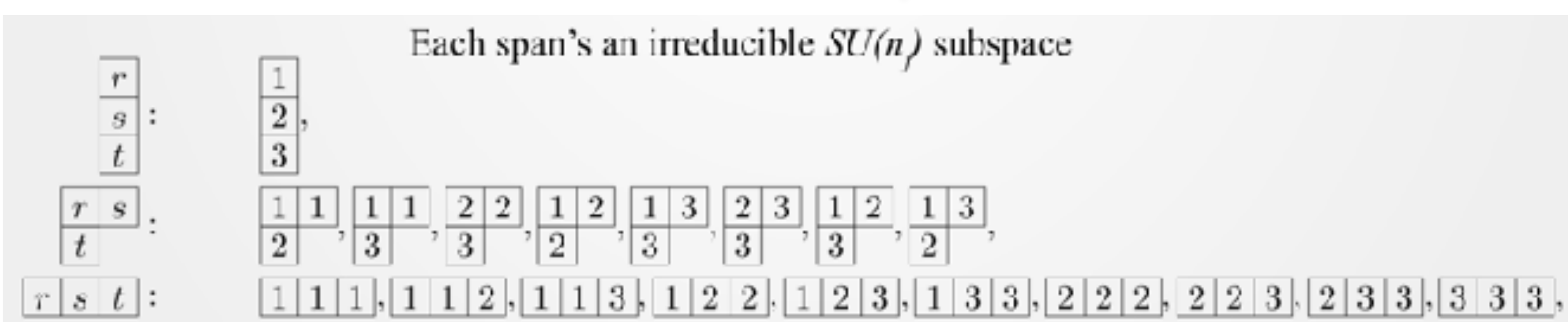
According to Schur-Weyl theorem, flavor tensor decomposed via S(nf) symmetry

$$O^{p,rst}_{qqql}\,\epsilon^{abc}\epsilon_{ji}\epsilon_{km}[(q^{aj}_r)^T C q^{bk}_s][(q^{cm}_t)^T C l^i_p]$$









19 x 3 = 57

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

SMEFT Operators

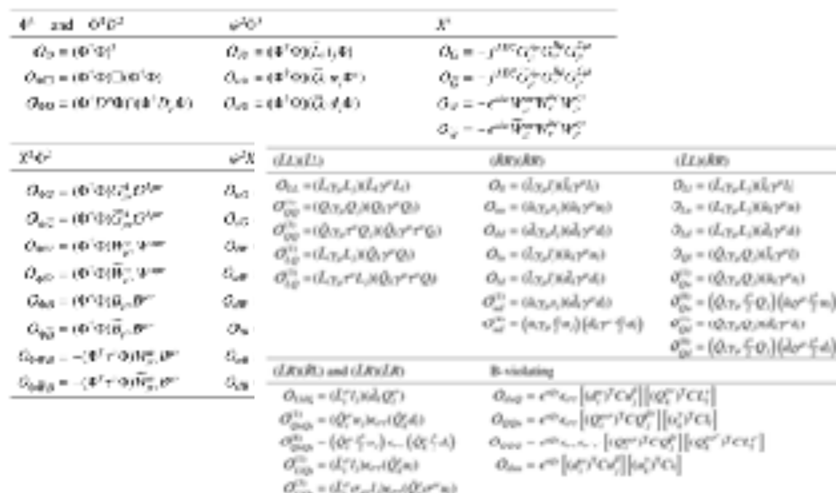
Dimension-5

$$\epsilon_{ij}\epsilon_{mn}(L^i C L^m) H^j H^n$$

[Weinberg, 1979]

2

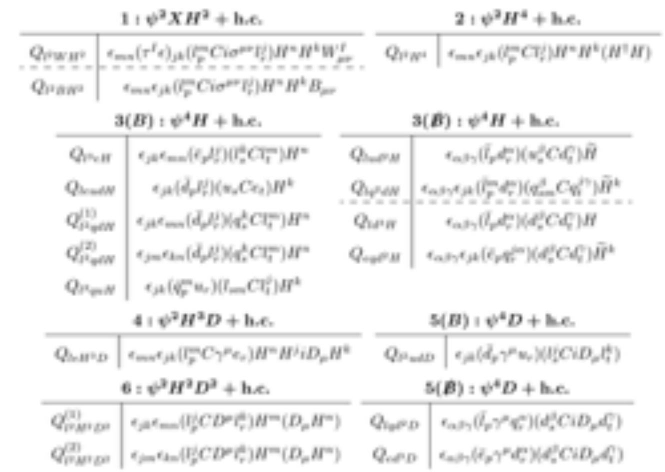
Dimension-6



84

[Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dimension-7



[Lehman, 2014]
[Henning, Lu, Melia, Murayama, 2015]
[Liao, Ma, 2016]

Dimension-8

[Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**]

[Murphy, 2020]

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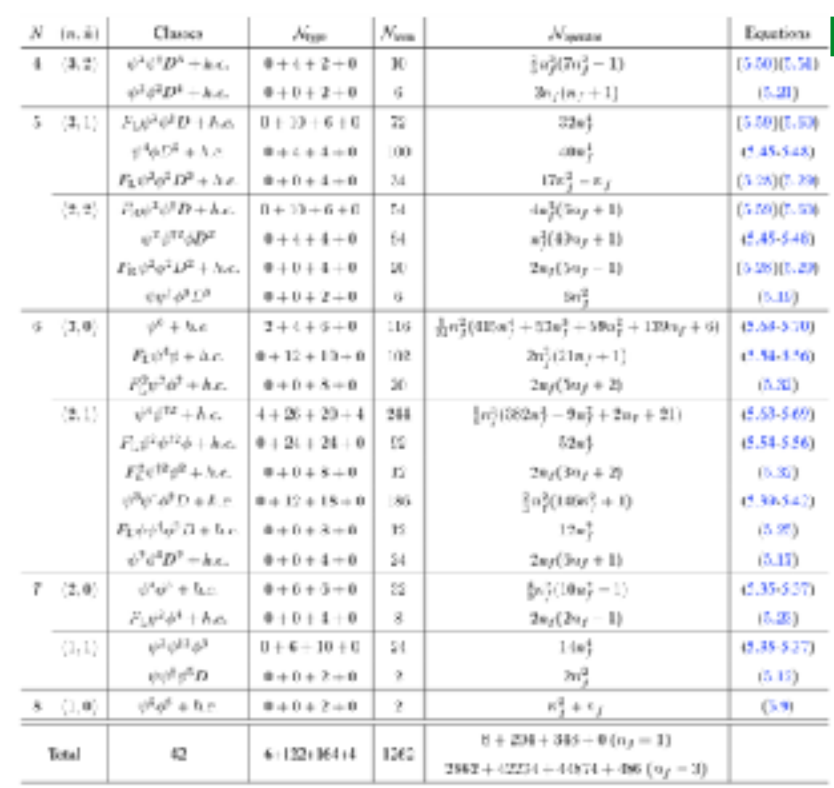
Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]



[Liao, Ma, 2020]

LEFT Operators

Dimension-5

Dim-5 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$
3	(2,0)	$F_L\psi_L^2 + h.c.$	$10+0+2+0$

10

Dimension-6

Dim-6 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	(3,0)	$F_L^3 + h.c.$	$2+0+0+0$	2
4	(2,0)	$\psi_L^4 + h.c.$	$14+12+8+2$	78
	(1,1)	$\psi_L^2\psi_R^2$	$40+20+12+0$	84
Total		5	$56+32+20+2$	164

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[Jenkins, Manohar, Stoffer, 2017]

Dimension-7

Dim-7 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3,0)	$F_L^2\psi_L^2 + h.c.$	$16+0+4+0$	32
	(2,1)	$F_L^2\psi_R^2 + h.c.$	$16+0+4+0$	24
		$\psi_L^3\psi_R D + h.c.$	$50+32+22+$	
Total		6	$82+32+30+$	

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[Liao, Ma, Wang, 2020]

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

N	$(n,\tilde{n})$	Subclasses	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	(4,0)	$F_L^4 + h.c.$	14	26	26	(4.15)
	(3,1)	$F_L^2\psi\psi^\dagger D + h.c.$	22	22	$22n_f^2$	(4.21)
		$\psi^4 D^2 + h.c.$	[illegible]	[illegible]	[illegible]	(4.75, 4.78, 4.80)
		[illegible]	16	32	$32n_f^2$	(4.44)
		$F_L^2\phi^2 D^2 + h.c.$	8	12	12	(4.14)
	(2,2)	$F_L^2F_R^2$	14	17	17	(4.15)
		[illegible]	27	35	$35n_f^2$	(4.36, 4.21)
		$\psi^2\psi^{\dagger 2}D^2$	17+4	34+8	[illegible]	(4.74, 4.79-4.81)
		[illegible]	16	16	$16n_f^2$	(4.44)
		$F_LF_R\phi^2D^2$	5	4	4	(4.14)
		[illegible]	7	15	$15n_f^2$	(4.31, 4.32)
		$\phi^4 D^4$	1	2	2	(4.8)
5	(3,0)	[illegible]	[illegible]	[illegible]	[illegible]	(4.90, 4.98, 4.95, 4.100)
		$F_L^2\psi^2\phi + h.c.$	32	50	$50n_f^2$	(4.47, 4.48)
		$F_L^3\phi^2 + h.c.$	6	4	4	(4.16)
	(2,1)	[illegible]	[illegible]	172+32	[illegible]	(4.58-4.63), (4.88-4.92)
		$F_R^2\psi^2\phi + h.c.$	32	[illegible]	[illegible]	(4.43, 4.49)
		[illegible]	[illegible]	[illegible]	[illegible]	(4.66, 4.69-4.72)
		[illegible]	38	30	$30n_f^2$	(4.35, 4.40)
		[illegible]	6	30	$30n_f^2$	(4.28)
		[illegible]	4	6	6	(4.16)
6	(2,0)	[illegible]	[illegible]	[illegible]	[illegible]	(4.72, 4.76, 4.83, 4.84)
		[illegible]	16	22	$22n_f^2$	(4.50)
		$F_L^2\phi^4 + h.c.$	8	10	10	(4.12)
	(1,1)	[illegible]	[illegible]	[illegible]	[illegible]	(4.54, 4.55, 4.56-4.63)
		[illegible]	7	15	$15n_f^2$	(4.24, 4.27)
		$\phi^6 D^2$	1	3	3	(4.8)
7	(1,0)	[illegible]	6	6	$6n_f^2$	(4.53)
8	(0,0)	ϕ^8	1	1	1	(4.3)
Total		48	[illegible]	[illegible]	[illegible]	

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Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	(3,2)	[illegible]	$0+4+2+0$	10	[illegible]	(5.50)(5.51)
		[illegible]	$0+0+2+0$	6	[illegible]	(5.21)
5	(3,1)	[illegible]	$0+33+6+0$	72	$32n_f^4$	(5.50)(5.53)
		[illegible]	$0+4+4+0$	100	[illegible]	(5.45-5.48)
		[illegible]	$0+0+4+0$	34	[illegible]	(5.28)(5.29)
	(2,2)	[illegible]	$0+33+6+0$	54	[illegible]	(5.50)(5.53)
		[illegible]	$0+4+4+0$	64	[illegible]	(5.45-5.46)
		[illegible]	$0+0+4+0$	20	[illegible]	(5.28)(5.29)
		[illegible]	$0+0+2+0$	6	[illegible]	(5.11)
6	(3,0)	$\psi^6 + h.c.$	$2+4+6+0$	116	[illegible]	(5.64-5.70)
		[illegible]	$0+12+13+0$	102	[illegible]	(5.54-5.56)
		[illegible]	$0+0+8+0$	20	[illegible]	(5.32)
	(2,1)	[illegible]	$4+26+20+4$	244	[illegible]	(5.53-5.69)
		[illegible]	$0+24+24+0$	52	$52n_f^4$	(5.54-5.56)
		[illegible]	$0+0+8+0$	12	[illegible]	(5.35)
		[illegible]	$0+12+18+0$	186	[illegible]	(5.30-5.42)
		[illegible]	$0+0+8+0$	15	[illegible]	(5.27)
		[illegible]	$0+0+4+0$	24	[illegible]	(5.13)
7	(2,0)	[illegible]	$0+0+5+0$	22	[illegible]	(5.35-5.37)
		[illegible]	$0+0+4+0$	8	[illegible]	(5.28)
	(1,1)	[illegible]	$0+6+10+0$	24	$14n_f^4$	(5.35-5.27)
		[illegible]	$0+0+2+0$	2	[illegible]	(5.15)
8	(1,0)	[illegible]	$0+0+2+0$	2	[illegible]	(5.9)
Total		42	[illegible]	1262	[illegible]	

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vSMEFT and vLEFT

Dimension-5

Dim-5 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$
3	(2, 0)	$F_L\psi^2+h.c.$	$0+0+2+0$	2
4	(1, 0)	$\psi^2\phi^2+h.c.$	$0+0+2+0$	2
Total		4	$0+0+4+0$	4

2

Dimension-6

Dim-6 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$
4	(2, 0)	$\psi^4+h.c.$	$4+2+0+2$	14
		$F_L\psi^2\phi+h.c.$	$4+0+0+0$	4
	(1, 1)	$\psi^2\psi^{\dagger 2}$	$10+2+0+0$	12
		$\psi\psi^\dagger\phi^2 D$	$3+0+0+0$	3
5	(1, 0)	$\psi^2\phi^3+h.c.$	$2+0+0+0$	2
Total		8	$23+4+0+2$	35

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[Aguila, Bar-Shalom, Soni, Wudka, 2009]

Dimension-7

Dim-7 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$
4	(3, 0)	$F_L^2\psi^2+h.c.$	$0+0+6+0$	6
	(2, 1)	$F_L^2\psi^{\dagger 2}+h.c.$	$0+0+6+0$	6
		$\psi^3\psi^\dagger D+h.c.$	$0+4+20+0$	24
		$F_L\psi\psi^\dagger\phi D+h.c.$	$0+0+8+0$	8
		$\psi^2\phi^2 D^2+h.c.$	$0+0+4+0$	6
5	(2, 0)	$\psi^4\phi+h.c.$	$0+2+10+0$	24
		$F_L\psi^2\phi^2+h.c.$	$0+0+6+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi$	$0+4+22+0$	30
		$\psi\psi^\dagger\phi^3 D$	$0+0+2+0$	4
6	(1, 0)	$\psi^2\phi^4+h.c.$	$0+0+2+0$	2
Total		18	$0+10+56+0$	116

[Bhattacharya, Wudka, 2016]

[Liao, Ma, 2017]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$
4	(3, 1)	$\psi^4 D^2+h.c.$	$4+0+2+2$	22
		$F_L\psi^2\phi D^2+h.c.$	$4+0+0+0$	8
	(2, 2)	$F_L F_R\psi\psi^\dagger D$	$3+0+0+0$	3
		$\psi^2\psi^{\dagger 2}D^2$	$10+2+0+0$	24
		$F_R\psi^2\phi D^2+h.c.$	$4+0+0+0$	4
		$\psi\psi^\dagger\phi^2 D^3$	$3+0+0+0$	4
5	(3, 0)	$F_L\psi^4+h.c.$	$10+4+0+2$	50
		$F_L^2\psi^2\phi+h.c.$	$8+0+0+0$	12
	(2, 1)	$F_L\psi^2\psi^{\dagger 2}+h.c.$	$42+12+0+0$	58
		$F_L^2\psi^{\dagger 2}\phi+h.c.$	$8+0+0+0$	8
		$\psi^3\psi^\dagger\phi D+h.c.$	$24+6+0+2$	108
		$F_L\psi\psi^\dagger\phi^2 D+h.c.$	$12+0+0+0$	16
		$\psi^2\phi^3 D^2+h.c.$	$2+0+0+0$	12
6	(2, 0)	$\psi^4\phi^2+h.c.$	$8+2+0+2$	30
		$F_L\psi^2\phi^3+h.c.$	$4+0+0+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^2$	$16+4+0+2$	28
		$\psi\psi^\dagger\phi^4 D$	$3+0+0+0$	3
7	(1, 0)	$\psi^2\phi^5+h.c.$	$2+0+0+0$	2
Total		31	$167+30+2+10$	398

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Dimension-9

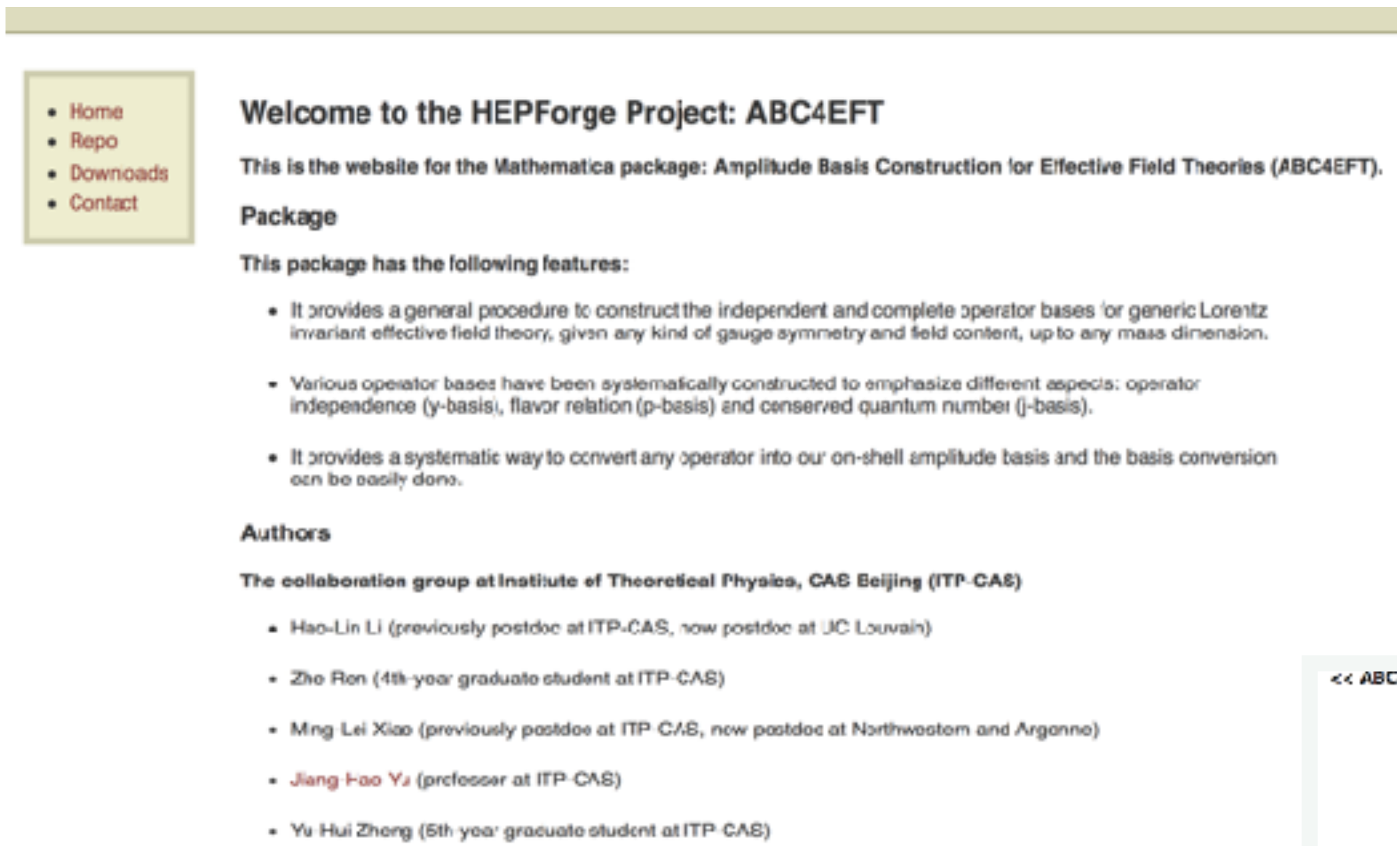
[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$
4	(4, 1)	$F_L^2\psi^2 D^2+h.c.$	$0+6+0+0$	12
	(3, 2)	$F_L F_R\psi^2 D^2+h.c.$	$0+6+0+0$	6
		$F_L^2\psi^{\dagger 2}D^2+h.c.$	$0+6+0+0$	6
		$\psi^3\psi^\dagger D^3+h.c.$	$4+20+0+3$	45
		$F_L\psi\psi^\dagger\phi D^2+h.c.$	$0+8+0+0$	15
		$\psi^2\phi^2 D^4+h.c.$	$0+1+0+0$	8
5	(4, 0)	$F_L^3\psi^2+h.c.$	$0+10+0+3$	15
	(3, 1)	$F_L^3\psi^{\dagger 2}+h.c.$	$0+4+0+0$	4
		$F_L\psi^3\psi^\dagger D+h.c.$	$10+42+8+0$	222
		$F_L^2\psi\psi^\dagger\phi D+h.c.$	$0+10+0+3$	32
		$\psi^4\phi D^2+h.c.$	$9+10+0+3$	130
		$F_L\psi^2\phi^2 D^2+h.c.$	$0+8+0+0$	42
	(2, 2)	$F_L F_R^2\psi^2+h.c.$	$0+19+0+3$	19
		$F_R\psi^3\psi^\dagger D+h.c.$	$10+42+8+0$	166
		$F_L F_R\psi\psi^\dagger\phi D$	$0+10+0+3$	26
		$\psi^2\psi^{\dagger 2}\phi D^2$	$4+22+0+3$	210
		$F_R\psi^2\phi^2 D^2+h.c.$	$0+8+0+0$	26
		$\psi\psi^\dagger\phi^3 D^3$	$0+2+0+0$	20
6	(3, 0)	$\psi^6+h.c.$	$6+10+6+2$	130
		$F_L\psi^4\phi+h.c.$	$6+26+0+3$	110
		$F_L^2\psi^2\phi^2+h.c.$	$0+12+0+3$	18
	(2, 1)	$\psi^4\psi^{\dagger 2}+h.c.$	$40+106+14+0$	474
		$F_L\psi^2\psi^{\dagger 2}\phi+h.c.$	$24+116+0+0$	176
		$F_L^2\psi^{\dagger 2}\phi^2+h.c.$	$0+10+0+3$	10
		$\psi^3\psi^\dagger\phi^2 D+h.c.$	$10+44+8+0$	268
		$F_L\psi\psi^\dagger\phi^3 D+h.c.$	$0+8+0+0$	32
		$\psi^2\phi^4 D^2+h.c.$	$0+1+0+0$	20
7	(2, 0)	$\psi^4\phi^3+h.c.$	$2+12+0+3$	28
		$F_L\psi^2\phi^4+h.c.$	$0+6+0+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^3$	$4+22+0+3$	36
		[illegible]	[illegible]	[illegible]

1358

Mathematica Code: ABC4EFT

Amplitude Basis Construction for Effective Field Theory



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Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Theories (ABC4EFT).

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic Lorentz invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dimension.
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis conversion can be easily done.

Authors

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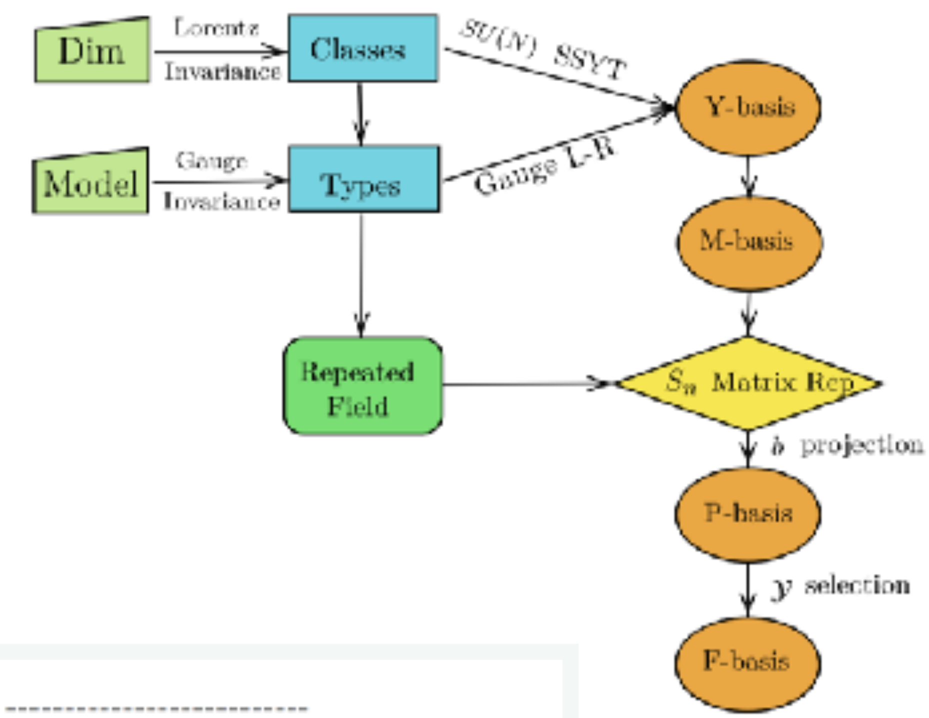
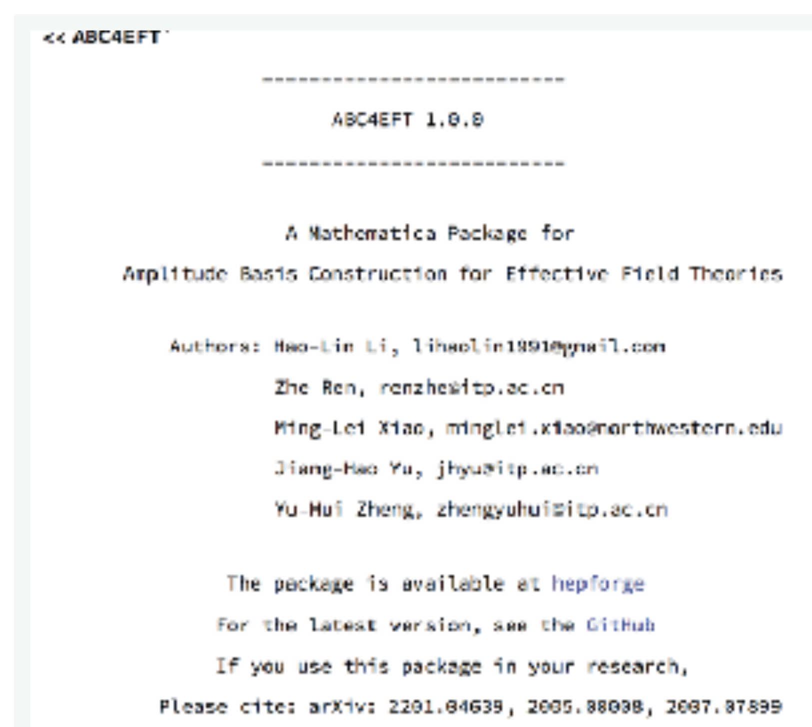


https://abc4eft.hepforge.org/

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]



<< ABC4EFT`

ABC4EFT 1.0.0

A Mathematica Package for

Amplitude Basis Construction for Effective Field Theories

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The package is available at hepforge

For the latest version, see the GitHub

If you use this package in your research,

Please cite: arXiv: 2201.04639, 2005.00008, 2007.07899

Dim-6 Operators

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^\star (\varphi^\dagger D_\mu\varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

59 independent operators

real parameters (degrees of freedom)

76: flavor universal — All fermion generations have the same coefficient

2499: flavor general — Independent coefficient for all flavor combinations

Dim-6 Operators

Dimension-6 operators of the SMEFT:	Interaction	Impact
$X^3 : \epsilon_{IJK} W^I_{\mu\nu} W^{J,\nu\rho} W^{K,\mu}_{\rho}$	gauge boson self-coupling	diboson
$H^6 : (\varphi^\dagger \varphi)^3$	Higgs potential, self-coupling	di-Higgs
$\psi^2 H^3 : (\varphi^\dagger \varphi)\ (\bar{q}_i\, u_j\, \tilde{\varphi})$	Higgs-fermion (Yukawa)	ttH, H→bb
$\psi^2 H^2 D : (\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{q}_i\, \gamma^\mu\, q_j)$	gauge-fermion (Z,W)	Z,W prod.
$X^2 H^2 : (\varphi^\dagger \varphi)\ G^a_{\mu\nu} G^{\mu\nu}_a$	gauge-Higgs	ggH, H→VV
$H^4 D^2 : (\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)$	Higgs-Z	m_Z (LEP)
$\psi^2 X H : (\bar{q}_i\, \sigma^{\mu\nu}\, u_j\, \tilde{\varphi}) B_{\mu\nu}$	dipole	ffV, ffVH
$\psi^4 : (\bar{q}_i\, \gamma^\mu\, q_j)(\bar{q}_k\, \gamma_\mu\, q_l)$	four fermion	ffff scattering

SM gauge group singlets

Dim-6 RGE

$$\mu \frac{\mathrm{d}C_i}{\mathrm{d}\mu} = \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j \quad \rightarrow \quad C_i(\mu) = C_i(\Lambda) - \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j(\Lambda) \log\left(\frac{\Lambda}{\mu}\right)$$

$$\gamma \sim O(1, g^2, \lambda, y^2, g^4, g^2\lambda, g^2y^2, \lambda^2, \lambda y^2, y^4, g^6, g^4\lambda, g^6\lambda)$$

Alonso–Jenkins–Manohar–Trott

	g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
g^3X^3	g^2	0	0	1	0	0	0	0
H^6	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

4-Fermi EFT: From Beta to NSI

Energy

UV models

many models …..

[Du, Li, Tang, Vihonen, **Yu, 2021**]

[Du, Li, Tang, Vihonen, **Yu, 2022**]

TeV scale

Standard model EFT

$$(\bar l^j_\alpha e_\beta)\epsilon_{jk}(\bar q^k_s u_t),\quad (\bar l^j_\alpha \sigma_{\mu\nu} e_\beta)\epsilon_{jk}(\bar q^k_s\sigma^{\mu\nu}u_t),\quad (\bar l^j_\alpha e_\beta)(\bar d_s q_{tj}),\quad (\bar l_\alpha\gamma^\mu\tau^I l_\beta)(\bar q_s\gamma_\mu\tau^I q_t)$$
$$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_\alpha\tau^I\gamma^\mu l_\beta),\quad (H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar q_p\tau^I\gamma^\mu q_r),\quad (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p\gamma^\mu d_r)$$
$$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_2\tau^I\gamma^\mu l_2),\quad (H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_1\tau^I\gamma^\mu l_1),\quad (\bar l_2\gamma^\mu l_1)(\bar l_1\gamma_\mu l_2),\quad (\bar l_1\gamma^\mu l_2)(\bar l_2\gamma_\mu l_1)$$

EW scale

Low energy EFT

$$\{[\epsilon_{L(R)}(\mu)]^{ij}_{\alpha\beta}\,(\bar u_i\gamma^\mu P_{L(R)}d_j)\,(\bar\ell_\alpha\gamma_\mu P_L\nu_\beta)\,,$$
$$[\epsilon_{S(P)}(\mu)]^{ij}_{\alpha\beta}(\bar u_i\mathbf{1}(\gamma_5)d_j)\,(\bar\ell_\alpha P_L\nu_\beta)\,,$$
$$[\epsilon_T(\mu)]^{ij}_{\alpha\beta}\,(\bar u_i\sigma^{\mu\nu}P_L d_j)\,(\bar\ell_\alpha\sigma_{\mu\nu}P_L\nu_\beta)\}$$

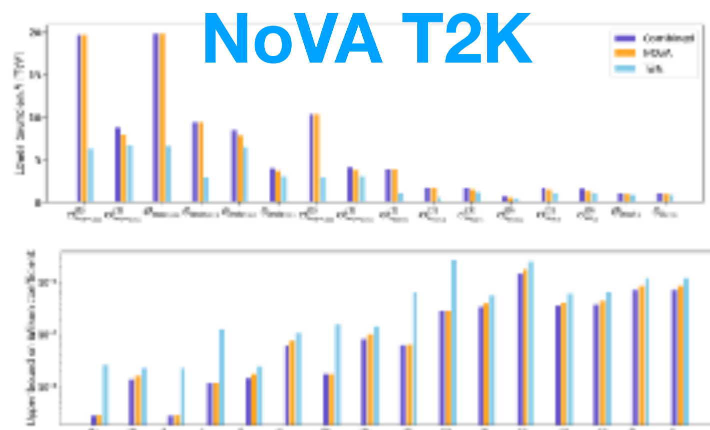
$\Lambda_{\rm QCD}$

Chiral Lagrangian

$$[\epsilon_{L(R)}]^{ij}_{\alpha\beta},\ [\epsilon_{S(P)}]^{ij}_{\alpha\beta},\ [\epsilon_T]^{ij}_{\alpha\beta}$$

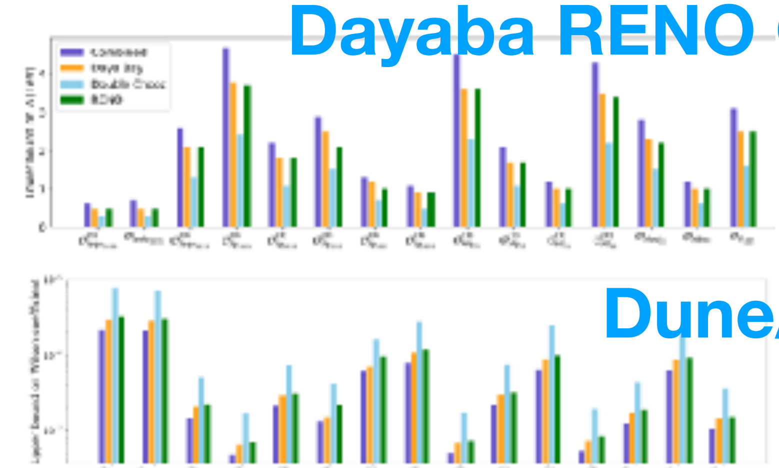
MeV scale

NoVA T2K



Dayaba RENO CHOOZ

Dune/HyperK



Complete Dim-6 UV Resonances

Scalar	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$S1\ (\mathbf{1},\mathbf{1},0)$	$B_L{}^2HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_CHH^{\dagger 2}Q[(F11),(F8)]$ $e_CHH^{\dagger 2}L[(F3),(F2)]$ $G_L{}^2HH^\dagger$ $H^2H^\dagger Qu_C[(S4),(F11),(F9)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S6),(S2),(S5),(S4,S6),(S2,S4),(S4,S5),(S4)]$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$
$S2\ (\mathbf{1},\mathbf{1},1)$	$d_CHH^{\dagger 2}Q[(S4),(F10),(F9)]$ $e_CHH^{\dagger 2}L[(S4),(F4),(F1)]$ $H^2H^\dagger Qu_C[(F8),(F12)]$ $L^2L^{\dagger 2}$ $H^3H^{\dagger 3}[(S4),(S5),(S5,S6),(S1),(S4,S5),(S1,S4),(S5,S6),(S4,S6)]$
$S3\ (\mathbf{1},\mathbf{1},2)$	$e_C{}^2e_C^{\dagger 2}$
$S4\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$d_C^\dagger e_CLQ^\dagger$ $d_CHH^{\dagger 2}Q[(S6),(S2)]$ $e_CHH^{\dagger 2}L[(S6),(S2)]$ $H^2H^\dagger Qu_C$ $H^2H^\dagger Qu_C[(S5),(S1)]$ $QQ^\dagger u_Cu_C^\dagger$ $H^3H^{\dagger 3}[(S6),(S2),(S5,S6),(S2,S5),$ $(S1,S6),(S1,S2),(S2,S6),(S5),(S1,S5),(S1)]$
$S5\ (\mathbf{1},\mathbf{3},0)$	$B_LHH^\dagger W_L$ $D^2H^2H^{\dagger 2}$ $d_CHH^{\dagger 2}Q[(F11),(F13)]$ $e_CHH^{\dagger 2}L[(F3),(F6)]$ $H^2H^\dagger Qu_C[(S4),(F11),(F14)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S7),(S6),(S2,S6),(S1),($ $e_CHH^{\dagger 2}L$ d_C
$S6\ (\mathbf{1},\mathbf{3},1)$	$d_CHH^{\dagger 2}Q[(S4),(F10),(F14$ $H^2H^\dagger Qu_C[(F1$ $H^3H^{\dagger 3}[(S7),(S4),(S$ $,(S5,S7),(S4,S5),($
$S7\ (\mathbf{1},\mathbf{4},\frac{1}{2})$	$H^3H^{\dagger 3}[(S$
$S8\ (\mathbf{1},\mathbf{4},\frac{3}{2})$	H^3
$S9\ (\mathbf{3},\mathbf{1},-\frac{4}{3})$	
$S10\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$Q^2Q^{\dagger 2}$ e_CLQu_C
$S11\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	
$S12\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	d
$S13\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	L
$S14\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	
$S15\ (\mathbf{6},\mathbf{1},-\frac{2}{3})$	
$S16\ (\mathbf{6},\mathbf{1},\frac{1}{3})$	d_CQ^2u
$S17\ (\mathbf{6},\mathbf{1},\frac{4}{3})$	
$S18\ (\mathbf{6},\mathbf{3},\frac{1}{3})$	
$S19\ (\mathbf{8},\mathbf{2},\frac{1}{2})$	Q

Fermion	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$F1\ (\mathbf{1},\mathbf{1},0)$	$DHH^\dagger LL^\dagger$ $e_CHH^{\dagger 2}L[(F3),$
$F2\ (\mathbf{1},\mathbf{1},1)$	$B_Le_CH^\dagger L$ $DHH^\dagger LL^\dagger$ e_CHH $e_CHH^{\dagger 2}L$
$F3\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$B_Le_CH^\dagger L$ $e_CHH^{\dagger 2}L[(F5),(F1),$
$F4\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$De_Ce_C^\dagger HH^\dagger$ $e_CHH^{\dagger 2}L[(F6),(F2),$
$F5\ (\mathbf{1},\mathbf{3},0)$	$DHH^\dagger LL^\dagger$ $e_CHH^{\dagger 2}L[(F3),$
$F6\ (\mathbf{1},\mathbf{3},1)$	$e_CH^\dagger LW_L$ $e_CHH^{\dagger 2}L[(F$
$F8\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$B_Ld_CH^\dagger Q$ $d_CG_LH^\dagger Q$ $DHH^\dagger QQ^\dagger$ d_C
$F9\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$DHH^\dagger QQ^\dagger$ B_LHQu_C G_LHQu_C $d_CHH^{\dagger 2}Q[(F11),(S2)]$ $H^2H^\dagger Qu_C$
$F10\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$Dd_Cd_C^\dagger HH^\dagger$ $d_CHH^{\dagger 2}Q[(F13),(F8),(S6),(S2)]$ $d_CHH^{\dagger 2}Q$
$F11\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$B_Ld_CH^\dagger Q$ B_LHQu_C G_LHQu_C $DHH^\dagger u_Cu_C^\dagger$ $d_CHH^{\dagger 2}Q[(F14),(F9),(F13),(F8),(S5),(S1)]$ $H^2H^\dagger Qu_C[(F14),(F9),(F13),(F8),(S5),(S1)]$
$F12\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	$DHH^\dagger u_Cu_C^\dagger$ $H^2H^\dagger Qu_C[(F14),(F9),(S6),(S2)]$ $H^2H^\dagger Qu_C$
$F13\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	$d_CH^\dagger QW_L$ $d_CHH^{\dagger 2}Q[(F10),(F11),(S5)]$ H^2
$F14\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	HQu_CW_L $d_CHH^{\dagger 2}Q[(F11),(S6)]$ $H^2H^\dagger Q$

Vector	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$V1\ (\mathbf{1},\mathbf{1},0)$	$d_C{}^2d_C^{\dagger 2}$ $d_Cd_C^\dagger e_Ce_C^\dagger$ $e_C{}^2e_C^{\dagger 2}$ $Dd_Cd_C^\dagger HH^\dagger$ $De_Ce_C^\dagger HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_Cd_C^\dagger LL^\dagger$ $e_Ce_C^\dagger LL^\dagger$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $d_Cd_C^\dagger QQ^\dagger$ $e_Ce_C^\dagger QQ^\dagger$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $e_Ce_C^\dagger u_Cu_C^\dagger$ $DHH^\dagger u_Cu_C^\dagger$ $LL^\dagger u_Cu_C^\dagger$ $QQ^\dagger u_Cu_C^\dagger$ $d_CHH^{\dagger 2}Q$ $e_CHH^{\dagger 2}L$ $H^2H^\dagger Qu_C$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$
$V2\ (\mathbf{1},\mathbf{1},1)$	$D^2H^2H^{\dagger 2}$ $Dd_CH^{\dagger 2}u_C^\dagger$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$ $d_CHH^{\dagger 2}Q$
$V3\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$e_Ce_C^\dagger LL^\dagger$
$V4\ (\mathbf{1},\mathbf{3},0)$	$D^2H^2H^{\dagger 2}$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$ $e_CHH^{\dagger 2}L$
$V5\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$d_C^\dagger e_CLQ^\dagger$
$V6\ (\mathbf{3},\mathbf{1},\frac{5}{3})$	$e_Ce_C^\dagger u_Cu_C^\dagger$
$V7\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$d_Cd_C^\dagger LL^\dagger$ $d_C^\dagger e_CLQ^\dagger$ $e_Ce_C^\dagger QQ^\dagger$ $d_CL^\dagger Q^\dagger u_C$ $e_CQ^{\dagger 2}u_C$ $QQ^\dagger u_Cu_C^\dagger$
$V8\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_Cd_C^\dagger QQ^\dagger$ $d_CL^\dagger Q^\dagger u_C$ $LL^\dagger u_Cu_C^\dagger$
$V9\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	$LL^\dagger QQ^\dagger$
$V10\ (\mathbf{6},\mathbf{2},-\frac{1}{6})$	$d_Cd_C^\dagger QQ^\dagger$
$V11\ (\mathbf{6},\mathbf{2},\frac{5}{6})$	$QQ^\dagger u_Cu_C^\dagger$
$V12\ (\mathbf{8},\mathbf{1},0)$	$d_C{}^2d_C^{\dagger 2}$ $d_Cd_C^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $QQ^\dagger u_Cu_C^\dagger$ $u_C{}^2u_C^{\dagger 2}$
$V13\ (\mathbf{8},\mathbf{1},1)$	$d_Cd_C^\dagger u_Cu_C^\dagger$
$V14\ (\mathbf{8},\mathbf{3},0)$	$Q^2Q^{\dagger 2}$

[Li, Ni, Xiao, **Yu**, Zheng, 2204.03660 **]**

[de Blas, Criado, Perez-Victoria, Santiago, 2017]

New LHC searches!

Complete Dim-7 UV Resonances

Scalar	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$S1\,(\mathbf{1},\mathbf{1},0)$	$H^3H^\dagger L^2[(S6),(S2),(F5),(F1),(S4,S6),(S2,S4),$ $(S4,F5),(S4,F1),(F3,F5),(F1,F3),(S6,F3),(S2,F3)]$
$S2\,(\mathbf{1},\mathbf{1},1)$	$D^2H^2L^2 \quad e_CHL^3[(S4),(F4),(F1)] \quad d_CHL^2Q[(S4),(F10),(F9)]$ $HL^2Q^\dagger u_C^\dagger[(S4),(F8),(F12)]$ $De_CH^{\dagger 3}L^\dagger[(F1),(F3),(V3)]$ $H^3H^\dagger L^2[(F1,F3),(S5,S6),(S1),(F5,F6),(F1,F2),(S4,S6),$ $(S4),(S5,S6),(S5),(S4,S5),(S1,S4),(S4,F5),(S4,F1),(F3,F5)$ $,(S5,F6),(S5,F2),(F3,F6),(F2,F3),(S5,F3),(S1,F3)]$ $H^2L^2W_L \quad B_LH^2L^2 \quad e_CHL^3$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q \quad De_C^\dagger H^3L$
$S4\,(\mathbf{1},\mathbf{2},\frac{1}{2})$	$H^3H^\dagger L^2[(S6),(S2,S6),(S2),(S5,S6),(S2,S5),(S1,S6),(S1,S2),$ $(S6,F5),(S6,F1),(S2,F5),(S2,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1)]$
$S5\,(\mathbf{1},\mathbf{3},0)$	$H^3H^\dagger L^2[(S6),(S2,S6),(F5)$ $(S2,S4),(S7,F5),(S4,F5),$ $(F1,F3),(S6,F7),$
$S6\,(\mathbf{1},\mathbf{3},1)$	$D^2H^2L^2 \quad e_CHL^3[(S4),(F4),$ $HL^2Q^\dagger u_C^\dagger[(S4$ $De_CH^{\dagger 3}L^\dagger[(F5),(F3),$ $H^3H^\dagger L^2[(F3,F5),(S5),(S1),(S2,$ $(S7),(S4),(S2,S4),(S8),(S5),(S2,S5),$ $(S4,F1),(F5,F7),(F1,F3),(S8,F6),$ $,(F2,F3),(S5,F7),$ $H^2L^2W_L \quad B_L$ $HL^2Q^\dagger u_C^\dagger \quad d_CH$
$S7\,(\mathbf{1},\mathbf{4},\frac{1}{2})$	$H^3H^\dagger L^2[(S6),(S5,S$
$S8\,(\mathbf{1},\mathbf{4},\frac{3}{2})$	$H^3H^\dagger L^2[(S$
$S10\,(\mathbf{3},\mathbf{1},-\frac{1}{3})$	$d_C^2HLu_C[(S12),(F10),(F1)]$ $d_Ce_C^\dagger HLu_C^\dagger[(S12),(F10),(F1)]$
$S11\,(\mathbf{3},\mathbf{1},\frac{2}{3})$	$d_C^3H^\dagger L[(S12),(F11),(F2)] \quad d_C^2HLu_C[(F11$
$S12\,(\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_C^3H^\dagger L[(S11),(F11)] \quad d_C^2$ $d_CHL^2Q[(S10),(S14),(F5),(F1),(F14)$
$S13\,(\mathbf{3},\mathbf{2},\frac{7}{6})$	$d_C^2HLu_C[(S11),(F10)]$
$S14\,(\mathbf{3},\mathbf{3},-\frac{1}{3})$	$d_CHL^2Q[(S12),(F10),(F5)]$

Vector	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$V2\,(\mathbf{1},\mathbf{1},1)$	$Dd_CL^2u_C^\dagger \quad D^2H^2L^2 \quad De_CH^{\dagger 3}L^\dagger[(F1),(V3),(F3)]$ $H^2L^2W_L \quad B_LH^2L^2 \quad e_CHL^3$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q \quad De_C^\dagger H^3L$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q$
$V3\,(\mathbf{1},\mathbf{2},\frac{3}{2})$	$De_CH^{\dagger 3}L^\dagger \quad d_Ce_C^\dagger HLu_C^\dagger[(F10),(F12)] \quad De_CH^{\dagger 3}L^\dagger[(V2),(V5),(S6),(S2)]$
$V5\,(\mathbf{1},\mathbf{3},1)$	$D^2H^2L^2 \quad De_CH^{\dagger 3}L^\dagger[(F3),(V3),(F5)]$ $H^2L^2W_L \quad B_LH^2L^2 \quad e_CHL^3$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q \quad De_C^\dagger H^3L$
$V5\,(\mathbf{3},\mathbf{1},\frac{2}{3})$	$Dd_C^2e_C^\dagger \quad HL^2Q^\dagger u_C^\dagger[(F1),(V8),(F12)]$ $d_Ce_C^\dagger HLu_C^\dagger[(F1),(V8),(F12)] \quad d_CHLQ^{\dagger 2}[(V8),(F10),(F8)]$
$V8\,(\mathbf{3},\mathbf{2},\frac{1}{6})$	$Dd_CL^2u_C^\dagger \quad Dd_C^2LQ^\dagger \quad HL^2Q^\dagger u_C^\dagger[(F5),(F1),(V9),(V5),(F13),(F8)]$ $d_CHLQ^{\dagger 2}[(F5),(F1),(V9),(V5),(F13),(F8)]$ $d_Ce_C^\dagger HLu_C^\dagger[(V5),(F10),(F3)] \quad d_C^2e_C^\dagger HQ^\dagger[(V5),(F10),(F3)]$ $d_CHLQ^{\dagger 2} \quad d_C^2e_C^\dagger HQ^\dagger \quad d_C^2HLu_C$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q$
$V9\,(\mathbf{3},\mathbf{3},\frac{2}{3})$	$HL^2Q^\dagger u_C^\dagger[(F5),(V8),(F12)] \quad d_CHLQ^{\dagger 2}[(V8),(F10),(F13)]$

Fermion	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$F1\,(1,\mathbf{1},0)$	$D^2H^2L^2 \quad e_CHL^3[(S4),(S2)] \quad d_CHL^2Q[(S4),(S10),(S12)]$ $HL^2Q^\dagger u_C^\dagger[(S4),(V5),(V8)] \quad De_CH^{\dagger 3}L^\dagger[(S2),(F3),(V2)]$ $d_C^2HLu_C[(S11),(S10)] \quad d_Ce_C^\dagger HLu_C^\dagger[(S10),(V5)] \quad d_CHLQ^{\dagger 2}[(S10),(V8)]$ $H^2L^2W_L[(F5)]$ $H^3H^\dagger L^2[(S2,F3),(S5,F5),(S1),(S6,F6),(S2,F2),(F5,F5),$ $(F3),(S4,S6),(S2,S4),(S5,F3),(S4,S5),(S1,S4),(F3,F6),$ $(F2,F3),(S5,F3),(S1,F3)]$ $H^2L^2W_L \quad B_LH^2L^2 \quad e_CHL^3$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q \quad De_C^\dagger H^3L$
$F2\,(1,\mathbf{1},1)$	$d_C^3H^\dagger L[(S11)] \quad H^3H^\dagger L^2[(S6,F5),(S2,F1),(F3,F5),$ $(F1,F3),(S5,S6),(S2,S5),(S6,F3),(S2,F3)]$
$F3\,(1,\mathbf{2},\frac{1}{2})$	$De_CH^{\dagger 3}L^\dagger[(F5),(F1),(S6),(S2),(V2),(V5)] \quad d_Ce_C^\dagger HLu_C^\dagger[(S12),(V8)]$ $d_C^2e_C^\dagger HQ^\dagger[(V8),(S11)]$
$F4\,(1,\mathbf{2},\frac{3}{2})$	$e_CHL^3[(S6),(S2)]$
$F5\,(1,\mathbf{3},0)$	$e_CHL^3[(S4),(S6)] \quad d_CHL^2Q[(S4),(S12),(S14)] \quad HL^2Q^\dagger u_C^\dagger[(S4),(V9),(V8)]$ $D^2H^2L^2 \quad De_CH^{\dagger 3}L^\dagger[(S6),(F3),(V5)] \quad d_CHLQ^{\dagger 2}[(S14),(V8)]$ $H^2L^2W_L[(F7),(F1)]$ $H^2L^2W_L \quad B_LH^2L^2 \quad e_CHL^3$ $HL^2Q^\dagger u_C^\dagger \quad d_CHL^2Q \quad De_C^\dagger H^3L$
$F6\,(1,\mathbf{3},1)$	$H^3H^\dagger L^2[(S5,F5),(S6,F1),(S2,F5),(F5,F7),(F3,F5),$ $(F1,F3),(S6,S8),(S5,S6),(S2,S5),(S5,F7),(S6,F3),(S2,F3)]$
$F7\,(1,\mathbf{4},\frac{1}{2})$	$H^2L^2W_L[(F5),(S6)] \quad H^3H^\dagger L^2[(F5),(S8,F5),(F5,F6),(S6,F6),(S5,F5),(S5,S6)]$
$F8\,(\mathbf{3},\mathbf{1},-\frac{1}{3})$	$HL^2Q^\dagger u_C^\dagger[(S2),(V8)] \quad d_CHLQ^{\dagger 2}[(V8),(S12),(V5)] \quad d_C^2e_C^\dagger HQ^\dagger[(V5),(S11)]$
$F9\,(3,\mathbf{1},\frac{2}{3})$	$d_CHL^2Q[(S12),(S2)]$
$F10\,(\mathbf{3},\mathbf{2},-\frac{5}{6})$	$d_C^2HLu_C[(S12),(S10),(S13)] \quad d_CHL^2Q[(S10),(S6),(S$ $d_Ce_C^\dagger HLu_C^\dagger[(S10),(V3),(V8)] \quad d_CHLQ^{\dagger 2}[(S10),(S14),$
$F11\,(\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_C^3H^\dagger L[(S11),(S12)] \quad d_C^2HLu_C[(S11),(S12)$
$F12\,(\mathbf{3},\mathbf{2},\frac{7}{6})$	$HL^2Q^\dagger u_C^\dagger[(S6),(S2),(V9),(V5)] \quad d_Ce_C^\dagger HLu_C^\dagger[(V5),(S12),(V3)]$
$F13\,(\mathbf{3},\mathbf{3},-\frac{1}{3})$	$HL^2Q^\dagger u_C^\dagger[(S6),(V9)] \quad d_CHLQ^{\dagger 2}[(V8),(S12),(V9)]$
$F14\,(\mathbf{3},\mathbf{3},\frac{2}{3})$	$d_CHL^2Q[(S12),(S6)]$

[Li, Ni, Xiao, **Yu**, Zheng, 2204.03660]

More LHC searches!

Chiral Lagrangian

Effective Field Theory with Nambu-Goldstone Modes

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The Sigma Model

$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \mathbf{\Phi}^T \partial^\mu \mathbf{\Phi} \;-\; \frac{\lambda}{4}\,\left(\mathbf{\Phi}^T\mathbf{\Phi} - v^2\right)^2 \qquad\qquad \mathbf{\Phi}^T = (\phi_1, \cdots, \phi_N)$$

Global Symmetry: $\quad O(4) \sim SU(2) \otimes SU(2)$

SSB: $\quad O(4) \;\to\; O(3) \qquad\qquad [\frac{4\times 3}{2} - \frac{3\times 2}{2} \;=\; 3$ broken generators]

$$\mathcal{L}_\sigma = \frac{1}{2}\left\{\partial_\mu\hat\sigma\,\partial^\mu\hat\sigma + \partial_\mu\vec\pi\,\partial^\mu\vec\pi - M^2\hat\sigma^2\right\} - \frac{M^2}{2v}\,\hat\sigma\left(\hat\sigma^2+\vec\pi^2\right) - \frac{M^2}{8v^2}\left(\hat\sigma^2+\vec\pi^2\right)^2$$

1) $\mathbf{\Sigma}(x) \;\equiv\; \sigma(x)\,\mathbf{I}_2 \;+\; i\,\vec\tau\,\vec\pi(x) \qquad ; \qquad \langle\mathbf{A}\rangle \;\equiv\; \mathrm{Tr}\,(\mathbf{A})$

$$\mathcal{L}_\sigma \;=\; \frac{1}{4}\,\langle\partial_\mu\mathbf{\Sigma}^\dagger\,\partial^\mu\mathbf{\Sigma}\rangle \;-\; \frac{\lambda}{16}\,\left(\langle\mathbf{\Sigma}^\dagger\mathbf{\Sigma}\rangle - 2\,v^2\right)^2$$

$O(4) \sim SU(2)_L \otimes SU(2)_R$ **Symmetry:** $\quad \mathbf{\Sigma} \to g_R\,\mathbf{\Sigma}\,g_L^\dagger \quad ; \quad g_{L,R} \in SU(2)_{L,R}$

2) $\mathbf{\Sigma}(x) \;\equiv\; [v + S(x)]\;\mathbf{U}(x) \qquad ; \qquad \mathbf{U} \equiv \exp\left\{\frac{i}{v}\,\vec\tau\,\vec\phi\right\} \to g_R\,\mathbf{U}\,g_L^\dagger$

$$\mathcal{L}_\sigma = \frac{v^2}{4}\,\left(1+\frac{S}{v}\right)^2\,\langle\partial_\mu\mathbf{U}^\dagger\partial^\mu\mathbf{U}\rangle + \frac{1}{2}\,\left(\partial_\mu S\,\partial^\mu S - M^2S^2\right) - \frac{M^2}{2v}\,S^3 - \frac{M^2}{8v^2}\,S^4$$

$E \ll M \sim v:$ $\quad \boxed{\mathcal{L}_\sigma \;\approx\; \frac{v^2}{4}\,\langle\partial_\mu\mathbf{U}^\dagger\,\partial^\mu\mathbf{U}\rangle}$

Symmetry Realization

Symmetry $\mathbf{G}$ $\{T_a\}$ ⟷ **Conserved charges** $\mathcal{Q}_a$

Noether Theorem: $\partial_\mu j_a^\mu = 0 \quad ; \quad \mathcal{Q}_a = \int d^3x \, j_a^0(x) \quad ; \quad \frac{d}{dt}\mathcal{Q}_a = 0$

Wigner–Weyl	**Nambu–Goldstone**
$\mathcal{Q}_a \lvert 0 \rangle = 0$	$\mathcal{Q}_a \lvert 0 \rangle \neq 0$
• Exact Symmetry	• Spontaneously Broken Symmetry
• Degenerate Multiplets	• Massless Goldstone Bosons
• Linear Representation	• Non-Linear Representation

$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \mathbf{\Phi}^\mathsf{T} \partial^\mu \mathbf{\Phi} \;-\; \frac{\lambda}{4}\,\left(\mathbf{\Phi}^\mathsf{T}\mathbf{\Phi} - v^2\right)^2$$

$\langle 0|\sigma|0\rangle = v$

$m_\Phi^2 = \lambda\, v^2$ $\qquad$ $M^2 = 2\,\lambda\, v^2 \quad m_\pi = 0$

Chiral Symmetry

$$\mathcal{L}^0_{QCD} = -\frac{1}{4} G^{\mu\nu}_a G^a_{\mu\nu} + \bar{\mathbf{q}}_L \, i\gamma^\mu D_\mu \, \mathbf{q}_L + \bar{\mathbf{q}}_R \, i\gamma^\mu D_\mu \, \mathbf{q}_R \qquad \mathbf{q}^T \equiv (u\,,\,d\,,\,s)$$

$$q'_L = V_L \cdot q_L$$
$$q'_R = V_R \cdot q_R$$

- $\mathcal{L}^0_{QCD}$ **invariant under** $\mathbf{G \equiv SU(3)_L \otimes SU(3)_R}$**:**

$$\bar{\mathbf{q}}_L \to g_L \, \bar{\mathbf{q}}_L \quad ; \quad \bar{\mathbf{q}}_R \to g_R \, \bar{\mathbf{q}}_R \quad ; \quad (g_L, g_R) \in \mathbf{G}$$

- **Only** $\mathbf{SU(3)_V}$ **in the hadronic spectrum:** $(\pi, K, \eta)_{0^-}\,;\,(\rho, K^*, \omega)_{1^-}\,;\,\cdots$

$$M_{0^-} < M_{0^+} \quad ; \quad M_{1^-} < M_{1^+}$$

- **The** $\mathbf{0^-}$ **octet is nearly massless:** $\mathbf{m_\pi \approx 0}$

The vacuum is not invariant (SSB): $\langle 0 | \, (\bar{\mathbf{q}}_L \mathbf{q}_R + \bar{\mathbf{q}}_R \mathbf{q}_L) \, | 0 \rangle \neq 0$

$$G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{SCSB} H \equiv SU(3)_V \quad \Longrightarrow \quad \mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\,\mathbf{\Phi}/f \right) \right\}_{ij} \qquad \mathbf{U} \longrightarrow g_R \, \mathbf{U} \, g_L^\dagger$$

$$\mathbf{\Phi} = \frac{\vec{\lambda}}{\sqrt{2}} \, \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}$$

Goldstone Theorem

Noether QCD Currents: $G \equiv SU(3)_L \otimes SU(3)_R$

$$J_X^{a\mu} = \bar{\mathbf{q}}_X \gamma^\mu \frac{\lambda^a}{2} \mathbf{q}_X \quad ; \quad \mathcal{Q}_X^a = \int d^3x \, J_X^{a0}(x) \qquad (a = 1, \cdots, 8 \, ; \, X = L, R)$$

Current Algebra ('60) : $\left[\mathcal{Q}_X^a , \mathcal{Q}_Y^b\right] = i\,\delta_{XY}\, f^{abc}\, \mathcal{Q}_X^c$

"Chiral symmetry" of massless QCD $[Q_i^V, H_0] = 0 \qquad [Q_i^A, H_0] = 0$

Vafa and Witten 1984: $Q_i^V |0\rangle = 0$

Axial charges ? $Q_i^A |0\rangle = ?$

- $\mathcal{Q}_A^a = \mathcal{Q}_R - \mathcal{Q}_L \quad ; \quad \mathcal{O}^b = \bar{\mathbf{q}}\, \gamma_5\, \lambda^b\, \mathbf{q}$

$$\langle 0| \left[\mathcal{Q}_A^a , \mathcal{O}^b\right] |0\rangle = -\frac{1}{2} \langle 0| \bar{\mathbf{q}} \left\{\lambda^a , \lambda^b\right\} \mathbf{q} |0\rangle = -\frac{2}{3} \langle 0| \bar{\mathbf{q}}\, \mathbf{q} |0\rangle$$

$Q_i^A \vert 0\rangle = 0$	$Q_i^A \vert 0\rangle \neq 0$
Wigner-Weyl realization of G ground state is symmetric $\langle 0 \vert \bar{q}_R q_L \vert 0\rangle = 0$ ordinary symmetry spectrum contains parity partners degenerate multiplets of G	Nambu-Goldstone realization of G ground state is asymmetric $\langle 0 \vert \bar{q}_R q_L \vert 0\rangle \neq 0$ "order parameter" spontaneously broken symmetry spectrum contains Goldstone bosons degenerate multiplets of $SU(3)_V \subset G$

$\langle 0| \bar{u} u |0\rangle = \langle 0| \bar{d} d |0\rangle = \langle 0| \bar{s} s |0\rangle \neq 0$

Goldstone Theorem

$H_0\, Q_i^A |0\rangle = Q_i^A\, H_0 |0\rangle = 0$

spectrum must contain 8 states

$Q_1^A |0\rangle, \ldots, Q_8^A |0\rangle$ with $E = 0$,

spin 0, negative parity, octet of $SU(3)_V$

Goldstone bosons

EFT Ladder

M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Big\downarrow$ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)},\ \mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Big\downarrow$ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

Chiral Lagrangian

- $SU(3)_L \otimes SU(3)_R$ **invariant**

$\mathbf{U} \rightarrow g_R\,\mathbf{U}\,g_L^\dagger \quad ; \quad g_{L,R} \in SU(3)_{L,R} \rightarrow \boxed{\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U}\rangle}$ Derivative Coupling

$$\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U}\rangle = \partial_\mu \pi^- \partial^\mu \pi^+ + \frac{1}{2}\partial_\mu \pi^0 \partial^\mu \pi^0 + \cdots$$
$$+ \frac{1}{6f^2}\left\{\left(\pi^+ \overleftrightarrow{\partial}_\mu \pi^-\right)\left(\pi^+ \overleftrightarrow{\partial}^\mu \pi^-\right) + 2\left(\pi^0 \overleftrightarrow{\partial}_\mu \pi^+\right)\left(\pi^- \overleftrightarrow{\partial}^\mu \pi^0\right) + \cdots\right\}$$
$$+ O\left(\pi^6/f^4\right)$$

- **Expansion in powers of momenta** $\leftrightarrow$ **derivatives**

Parity $\rightarrow$ even dimension ; $\mathbf{U}\,\mathbf{U}^\dagger = 1 \rightarrow 2n \geq 2$

$$\mathcal{L}_4 = L_1 \langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U\rangle \langle D^\mu U^\dagger D^\nu U\rangle + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle$$

Explicit Symmetry Breaking

$$\mathcal{L}_{QCD} \equiv \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}(\not{v} + \not{a}\gamma_5)\mathbf{q} - \bar{\mathbf{q}}(\mathbf{s} - i\gamma_5\mathbf{p})\mathbf{q}$$
$$= \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}_L \not{l}\mathbf{q}_L + \bar{\mathbf{q}}_R \not{r}\mathbf{q}_R - \bar{\mathbf{q}}_R(\mathbf{s} + i\mathbf{p})\mathbf{q}_L - \bar{\mathbf{q}}_L(\mathbf{s} - i\mathbf{p})\mathbf{q}_R$$

$$\mathbf{l}_\mu \equiv \mathbf{v}_\mu - \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots$$
$$\mathbf{r}_\mu \equiv \mathbf{v}_\mu + \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots \qquad \mathcal{Q} \equiv \tfrac{1}{3}\,\mathrm{diag}(2,-1,-1)$$
$$\mathbf{s} = \mathcal{M} + \cdots \quad ; \quad \mathcal{M} \equiv \mathrm{diag}(m_u, m_d, m_s)$$

Local $SU(3)_L \otimes SU(3)_R$ Symmetry:

$$\mathbf{q}_L \to g_L\,\mathbf{q}_L \qquad \mathbf{l}_\mu \to g_L\,\mathbf{l}_\mu\,g_L^\dagger + i\,g_L\,\partial_\mu g_L^\dagger$$
$$\mathbf{q}_R \to g_R\,\mathbf{q}_R \qquad \mathbf{r}_\mu \to g_R\,\mathbf{r}_\mu\,g_R^\dagger + i\,g_R\,\partial_\mu g_R^\dagger$$
$$(\mathbf{s} + i\,\mathbf{p}) \to g_R\,(\mathbf{s} + i\,\mathbf{p})\,g_L^\dagger$$

$$\boxed{\mathcal{L} = \frac{f^2}{4}\langle D_\mu\mathbf{U}\,D^\mu\mathbf{U}^\dagger + \chi\,\mathbf{U}^\dagger + \mathbf{U}\,\chi^\dagger\rangle}$$

$$D_\mu\mathbf{U} = \partial_\mu\mathbf{U} - i\,\mathbf{r}_\mu\,\mathbf{U} + i\,\mathbf{U}\,\mathbf{l}_\mu$$
$$\chi \equiv 2\,B_0\,(\mathbf{s} + i\,\mathbf{p})$$

Pseudo-Goldstone Boson

$$\boxed{\mathcal{L} = \frac{f^2}{4}\langle D_\mu \mathbf{U}\, D^\mu \mathbf{U}^\dagger + \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle}$$

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i\, \mathbf{r}_\mu\, \mathbf{U} + i\, \mathbf{U}\, \mathbf{l}_\mu$$

$$\chi \equiv 2\, B_0\, (\mathbf{s} + i\, \mathbf{p})$$

Currents:

$$\mathbf{J}^\mu_{L} = \frac{\partial}{\partial \mathbf{l}_\mu}\mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}^\dagger\, \mathbf{U} = \frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\mathbf{J}^\mu_{R} = \frac{\partial}{\partial \mathbf{r}_\mu}\mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}\, \mathbf{U}^\dagger = -\frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\langle 0 |\, (J^\mu_A)_{12}\, | \pi^+(p)\rangle = i\sqrt{2}\, f\, p^\mu \quad \rightarrow \quad \boxed{f = f_\pi \approx 92.2 \text{ MeV}}$$

$(\pi^+ \to \mu^+ \nu_\mu)$

$$\bar{\mathbf{q}}^j_L\, \mathbf{q}^i_R = -\frac{\partial \mathcal{L}_2}{\partial(\mathbf{s} - i\,\mathbf{p})^{ji}} = -\frac{f^2}{2} B_0\, \mathbf{U}^{ij} \quad \rightarrow \quad \boxed{\langle 0 | \bar{\mathbf{q}}^j \mathbf{q}^i | 0 \rangle = -f^2\, B_0\, \delta_{ij}}$$

$$\frac{f^2}{4}\langle \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle \quad \rightarrow \quad \mathcal{L}_m = -B_0\, \langle \mathcal{M}\, \Phi^2 \rangle \quad + \frac{B_0}{6 f^2}\langle \mathcal{M}\, \Phi^4 \rangle + \cdots$$

$$= B_0 \frac{f^2}{2}\langle s(U^\dagger + U)\rangle$$

$$\mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\, \mathbf{\Phi}/f \right) \right\}_{ij}$$

$$= -B_0 \Big\{ (m_u + m_d) \left[\pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 \right] + (m_u + m_s)\, K^+ K^- + (m_d + m_s)\, K^0 \bar{K}^0 + \frac{1}{6}(m_u + m_d + 4\, m_s)\, \eta^2 + \frac{1}{\sqrt{3}}(m_u - m_d)\, \pi^0 \eta \Big\}$$

Gell-Mann–Okubo: $4\, M_K^2 = M_\pi^2 + 3\, M_\eta^2$

Dashen Theorem $\boxed{(M^2_{K^0} - M^2_{K^\pm})_{em} = (M^2_{\pi^0} - M^2_{\pi^\pm})_{em} + \mathcal{O}(e^2 p^2)}$

Chiral Lagrangian at p4

$\mathbf{U}$	$\mathcal{O}(p^0)$
$D_\mu\mathbf{U}\,,\ \mathbf{l}_\mu\,,\ \mathbf{r}_\mu$	$\mathcal{O}(p^1)$
$\chi\,,\ \mathbf{F}^{\mu\nu}_{L,R}$	$\mathcal{O}(p^2)$

$$\mathbf{F}_L^{\mu\nu} \equiv \partial^\mu \mathbf{l}^\nu - \partial^\nu \mathbf{l}^\mu - i\,[\mathbf{l}^\mu, \mathbf{l}^\nu]$$

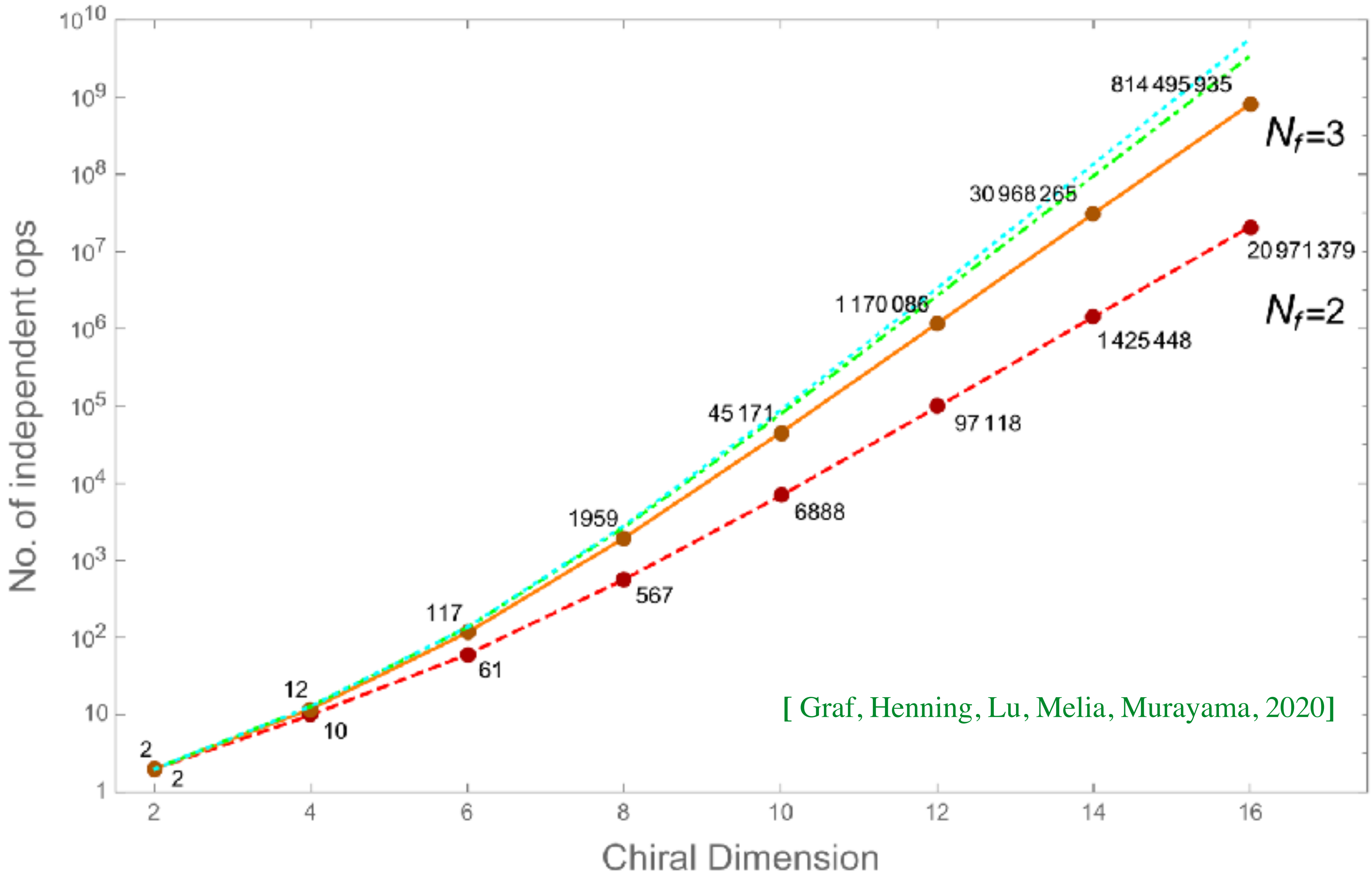
$$\mathbf{F}_R^{\mu\nu} \equiv \partial^\mu \mathbf{r}^\nu - \partial^\nu \mathbf{r}^\mu - i\,[\mathbf{r}^\mu, \mathbf{r}^\nu]$$

General connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L loops:

$$D = 2L + 2 + \sum_d N_d\,(d-2) \qquad \text{Weinberg}$$

$$\begin{aligned}
\mathcal{L}_4 = {} & L_1\,\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\,\langle D_\mu U^\dagger D_\nu U\rangle\,\langle D^\mu U^\dagger D^\nu U\rangle \\
& + L_3\,\langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle + L_4\,\langle D_\mu U^\dagger D^\mu U\rangle\,\langle U^\dagger\chi + \chi\dagger U\rangle \\
& + L_5\,\langle D_\mu U^\dagger D^\mu U\,(U^\dagger\chi + \chi^\dagger U)\rangle + L_6\,\langle U^\dagger\chi + \chi^\dagger U\rangle^2 \\
& + L_7\,\langle U^\dagger\chi - \chi^\dagger U\rangle^2 + L_8\,\langle\chi^\dagger U\chi^\dagger U + U^\dagger\chi U^\dagger\chi\rangle \\
& - i\,L_9\,\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10}\,\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle
\end{aligned}$$

Chiral Lagrangian Counting



Custodial Symmetry

Consider the Higgs sector (gauge-less limit)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_\Phi &= (D_\mu \Phi)^\dagger D^\mu \Phi - \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2} \mathrm{Tr}\left[(D^\mu \Sigma)^\dagger D_\mu \Sigma \right] - \frac{\lambda}{4} \left(\mathrm{Tr}\left[\Sigma^\dagger \Sigma \right] - v^2 \right)^2 \end{aligned}$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\qquad \Sigma \to g_L \, \Sigma \, g_R^\dagger$

EW chiral Lagrangian

$$\Phi = \frac{1}{\sqrt{2}} (v + H) \, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \Sigma \equiv (\Phi^c, \Phi) \equiv \frac{1}{\sqrt{2}} (v + H) \, U(\vec{\varphi})$$

$$\mathcal{L}_{\text{Higgs}} = \frac{(v+H)^2}{4} \mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U \right] - \frac{\lambda}{4} (H^2 - v^2)^2$$

EW Chiral Lagrangian

Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \quad , \quad \vec{\pi} \to \vec{\varphi} \to W_L^{\pm}, Z_L$$

- Goldstone Bosons

$\langle 0| \bar{q}^j_R q^i_L |0\rangle$ **(QCD)**, Φ **(SM)** ➡ $U_{ij}(\phi) = \{\exp(i\vec{\sigma}\cdot\vec{\varphi}/v)\}_{ij}$

- Expansion in powers of momenta ⟷ derivatives

Parity ➡ even dimension ; $U U^\dagger = 1$ ➡ $2n \geq 2$

- $\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R}$ invariant

U ➡ $g_L\, U\, g_R^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

$$\boxed{\mathcal{L}_2 = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)}$$

Derivative Coupling

HEFT Lagrangian

$$\boxed{\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle}$$

$$U(\varphi) = \exp\left\{\frac{i\sqrt{2}}{v}\Phi\right\} \quad , \quad \Phi \equiv \frac{1}{\sqrt{2}}\vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\varphi^0 \end{pmatrix}$$

$$D^\mu U = \partial^\mu U - i\hat{W}^\mu U + i U\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i U^\dagger \hat{W}^\mu - i\hat{B}^\mu U^\dagger \quad , \quad \langle A\rangle \equiv \mathrm{Tr}(A)$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i[\hat{B}^\mu, \hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$ Symmetry: $U(\varphi) \rightarrow g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \rightarrow g_L \hat{W}^\mu g_L^\dagger + i\, g_L \partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \rightarrow g_R \hat{B}^\mu g_R^\dagger + i\, g_R \partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\hat{W}^\mu = -\frac{g}{2}\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2}\sigma_3 B^\mu$

$$\mathcal{L}_2 = \frac{v^2}{4}\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

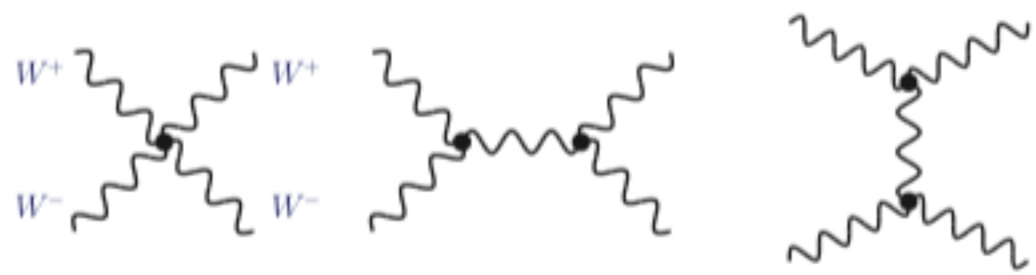
$$M_W = M_Z \cos\theta_W = \frac{1}{2} g v$$

- **EW Goldstones are responsible for** $M_{W,Z}$ **(not the Higgs!)**

Unitarity Violation

$$\frac{v^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U \rangle = \partial_\mu \varphi^- \partial^\mu \varphi^+ + \frac{1}{2}\partial_\mu \varphi^0 \partial^\mu \varphi^0$$

$$+ \frac{1}{6v^2}\left\{\left(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi^-\right)\left(\varphi^+ \overleftrightarrow{\partial}^\mu \varphi^-\right) + 2\left(\varphi^0 \overleftrightarrow{\partial}_\mu \varphi^+\right)\left(\varphi^- \overleftrightarrow{\partial}^\mu \varphi^0\right)\right\}$$



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$\boxed{\begin{aligned} T(W_L^+ W_L^- \to W_L^+ W_L^-) &= \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right) \\ &= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right) \end{aligned}}$$

The scattering amplitude grows with energy

Goldstone dynamics $\longleftrightarrow$ derivative interactions

Tree-level violation of unitarity

Higgs EFT

$$\Delta\mathcal{L}_2^{\text{Bosonic}} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}m_h^2 h^2 - V(h/v) + \frac{v^2}{4}\mathcal{F}_u(h/v)\,\langle (D^\mu U)^\dagger D_\mu U\rangle$$

Assumptions:

- **Spontaneous Symmetry Breaking:** $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$
- **h(125)** **is an** $\mathrm{SU}(2)_{\mathrm{L+R}}$ **scalar singlet**

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \rightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h/v)\,\mathcal{O}_X$$

$$\mathcal{F}_X(h/v) = \sum_{n=0} c_n^{(X)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the **scalar potential**:

$$V(h/v) = v^4 \sum_{n=2} c_n^{(V)} \left(\frac{h}{v}\right)^n$$

SMEFT vs HEFT

SU(3)$_C$ x SU(2)$_L$ x U(1)$_Y$

SU(3)$_C$ x U(1)$_{em}$

$L \in SU(2)_L \quad R \in U(1)_Y$

$$H \to LH$$

$$U \to LUR^\dagger \qquad h \to h$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix}$$

125 GeV Higgs boson

Goldstone bosons eaten by W and Z

$$U = \exp\left(\frac{i\pi^a \sigma^a}{\mathrm{v}}\right)$$

In general, the two formulations lead to two distinct effective theories

Higgs VEV
$v \approx 246$ GeV

$\boxed{\textbf{SMEFT}} \subset \boxed{\textbf{HEFT}}$

Expansion parameter
$v \approx 246$ GeV

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} - \frac{c_6}{\Lambda^2}|H|^6 + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{m_h^2}{2\mathrm{v}}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8\mathrm{v}^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{\mathrm{v}}h^5 - \frac{\lambda_6}{\mathrm{v}^2}h^6$$

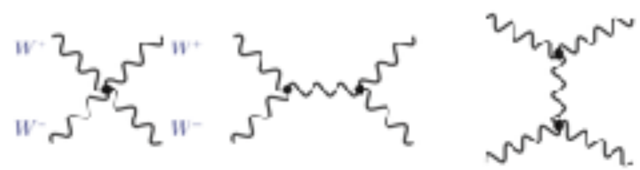
$$\delta\lambda_3 = \frac{2c_6\mathrm{v}^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6\mathrm{v}^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6\mathrm{v}^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6\mathrm{v}^2}{8\Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as Λ >> v

$$\mathscr{L}_{\text{HEFT}} \supset -c_3\frac{m_h^2}{2\mathrm{v}}h^3 - c_4\frac{m_h^2}{8\mathrm{v}^2}h^4 - \frac{c_5}{\mathrm{v}}h^5 - \frac{c_6}{\mathrm{v}^2}h^6 + \dots$$

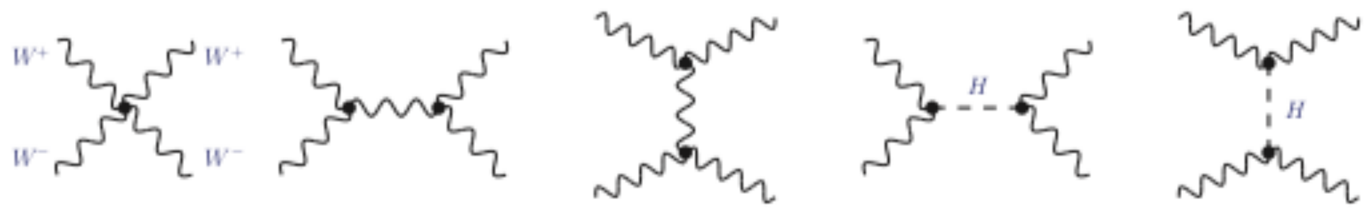
HEFT: no correlations between self-couplings

Unitarity Restoration



Gauge
Cancelation

$$T(W_L^+W_L^- \to W_L^+W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$
$$= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$



$$T_{\rm SM} = \frac{1}{v^2}\left\{s+t-\frac{s^2}{s-M_H^2}-\frac{t^2}{t-M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s-M_H^2}+\frac{t}{t-M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

When $s \gg M_H^2$, $\quad T_{\rm SM} \approx -\frac{2M_H^2}{v^2} \quad , \quad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\frac{M_H^2}{8\pi v^2}$

Perturbative unitarity:

Lee–Quigg–Thacker

$$|a_0| \le 1 \quad \Rightarrow \quad M_H < \sqrt{8\pi}v \underbrace{\sqrt{2/3}}_{W^+W^-,\ ZZ,\ HH} \approx 1 \text{ TeV}$$

Unitarity Restoration

- QCD satisfies unitarity (it is a renormalizable theory)
- Pion scattering unitarized by exchanges of resonances (composite objects):
  - P-wave ($J = 1$) unitarized by ρ exchange
  - S-wave ($J = 0$) unitarized by σ exchange
- The σ meson is the QCD equivalent of the SM Higgs
- BUT, the σ is an 'effective' object generated through $\pi\pi$ rescattering (summation of pion loops)

Yukawa Sector

$$\boxed{\Delta\mathcal{L}_2^{\mathrm{Ferm.}} = -v\left\{\bar{Q}_L\, U(\varphi)\left[\hat{\mathrm{Y}}_{\mathrm{u}}\,\mathcal{P}_+ + \hat{\mathrm{Y}}_{\mathrm{d}}\,\mathcal{P}_-\right] Q_R \,+\, \bar{L}_L\, U(\varphi)\,\hat{\mathrm{Y}}_\ell\,\mathcal{P}_+\, L_R \,+\, \mathrm{h.c.}\right\}}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

- **Symmetry Breaking:** $\mathcal{P}_\pm = \frac{1}{2}\left(\mathrm{I}_2 \pm \sigma_3\right)$
- **Flavour Structure:** $\hat{\mathrm{Y}}_{\mathrm{u,d},\ell}$ 3×3 **matrices in flavour space**
- **Higgs field:** $\hat{\mathrm{Y}}_{\mathrm{u,d},\ell}(h/v) = \sum_{n=0} \hat{\mathrm{Y}}^{(\mathrm{n})}_{\mathrm{u,d},\ell}\left(\frac{h}{v}\right)^n$

HEFT Lagrangian

(i) SM content:

- Bosons χ: Higgs h + gauge bosons W^a_μ, B_μ *(and QCD)*
  + EW Golsdtones $\omega^\pm$,z [non-linearly realized via $U(\omega^a)$]
- Fermions ψ: (t,b)-type doublets

(ii) Symmetries:

- **SM symmetry:** Gauge sym. group $G_{SM} = SU(2)_L \times U(1)_Y$ *(and QCD)*
  Spont. Breaking (EWSB) $G_{SM} \rightarrow H_{SM} = U(1)_{EM}$
- **Symmetry of the SM scalar sector:**
  Global CHIRAL sym. $G = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \supset G_{SM}$
  Sp.S.Breaking to Cust.sym. $G \rightarrow H = SU(2)_{L+R} \times U(1)_{B-L} \supset H_{SM}$
  Explicit Breaking: L⇔R asymmetry of the gauge sector ($g, g' \neq 0$)
  t⇔b splitting ($\lambda_t \neq \lambda_b$)

(iii) Chiral power counting:

[boson]	⇔	order 0	($\sim p^0$)
$[g W^\mu] = [g' B^\mu] = [d_\mu] = [g] = [\lambda_\psi] = [m_{\chi,\psi}] = [\cancel{\psi\psi}]$	⇔	order 1	($\sim p^1$)
weak SM fermion coupling [$\psi\psi$]	⇔	order 2	($\sim p^2$)

- $U(\varphi)$, φ , h $\sim O(p^0)$
  $D_\mu U$, $\hat{W}_\mu$, $\hat{B}_\mu \sim O(p^1)$, $\hat{W}_{\mu\nu}$, $\hat{B}_{\mu\nu} \sim O(p^2)$

HEFT NLO Operators

$$\mathcal{L}_{\rm EW}^{(4)}\Big|_{\rm Bosonic} = \sum_i \mathcal{F}_i(h/v)\,\mathcal{O}_i \qquad\qquad \mathcal{F}_i(h/v) = \sum_{n=0} \mathcal{F}_{i,n}\left(\frac{h}{v}\right)^n$$

Appelquist-Bernard, Longhitano, Buchalla et al, Alonso et al, Pich et al. . .

$\mathcal{O}(p^4)$ $\mathcal{P}$-even bosonic operators A.P., Rosell, Santos, Sanz-Cillero

$$\mathcal{O}_1 = \langle\, U^\dagger \hat{W}_{\mu\nu}\, U\, \hat{B}^{\mu\nu}\,\rangle \qquad \mathcal{O}_2 = \langle\, \hat{W}_{\mu\nu}\hat{W}^{\mu\nu} + \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\,\rangle$$

$$\mathcal{O}_3 = i\,\langle\, \hat{W}_{\mu\nu} D^\mu U\, D^\nu U^\dagger + \hat{B}_{\mu\nu} D^\mu U^\dagger D^\nu U\,\rangle \qquad \mathcal{O}_4 = \langle\, D_\mu U^\dagger D_\nu U\,\rangle\,\langle\, D^\mu U^\dagger D^\nu U\,\rangle$$

$$\mathcal{O}_5 = \langle\, D_\mu U^\dagger D^\mu U\,\rangle^2 \qquad \mathcal{O}_6 = \frac{1}{v^2}\,(\partial_\mu h)(\partial^\mu h)\,\langle\, D_\nu U^\dagger D^\nu U\,\rangle$$

$$\mathcal{O}_7 = \frac{1}{v^2}\,(\partial_\mu h)(\partial_\nu h)\,\langle\, D^\mu U^\dagger D^\nu U\,\rangle \qquad \mathcal{O}_8 = \frac{1}{v^4}\,(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$$

$$\mathcal{O}_9 = -\frac{i}{v}\,(\partial^\mu h)\,\langle\, \hat{W}_{\mu\nu} D^\nu U\, U^\dagger + \hat{B}_{\mu\nu} D^\nu U^\dagger\, U\,\rangle$$

Complete NLO Lagrangian is written recently!

$$\mathcal{O}_{33}^{Uh\psi^4} = (Q_{Ls}^\dagger\sigma_\mu\tau^I \mathbf{T} Q_{Lp})(Q_{Rr}^\dagger\sigma^\mu \mathrm{U}^\dagger\tau^I \mathrm{U} Q_{Rt})\mathcal{F}_{33}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{34}^{Uh\psi^4} = (Q_{Ls}^\dagger\sigma_\mu\lambda^A\tau^I \mathbf{T} Q_{Lp})(Q_{Rr}^\dagger\sigma^\mu\lambda^A \mathbf{U}^\dagger\tau^I \mathbf{U} Q_{Rt})\mathcal{F}_{34}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{89}^{Uh\psi^4} = (L_{Ls}^\dagger\sigma_\mu\tau^I L_{Lp})(L_{Rt}^\dagger\sigma^\mu\tau^I \mathrm{U}^\dagger \mathrm{T}\mathrm{U} L_{Rs})\mathcal{F}_{89}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{107}^{Uh\psi^4} = (L_{Ls}^\dagger\sigma_\mu\tau^I \mathbf{T} L_{Lp})(Q_{Lt}^\dagger\sigma^\mu\tau^I Q_{Lr})\mathcal{F}_{107}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{113}^{Uh\psi^4} = (L_{Rs}^\dagger\sigma_\mu\tau^I \mathrm{T} L_{Rp})(Q_{Rt}^\dagger\sigma^\mu\tau^I Q_{Rr})\mathcal{F}_{113}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{119}^{Uh\psi^4} = (L_{Rs}^\dagger\sigma_\mu \mathrm{U}^\dagger\tau^I \mathrm{T}\mathrm{U} L_{Rp})(Q_{Lt}^\dagger\sigma^\mu\tau^I Q_{Lr})\mathcal{F}_{119}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{125}^{Uh\psi^4} = (L_{Ls}^\dagger\sigma_\mu\tau^I \mathbf{T} L_{Lp})(Q_{Rt}^\dagger\sigma^\mu \mathbf{U}^\dagger\tau^I \mathbf{U} Q_{Rr})\mathcal{F}_{125}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{140}^{Uh\psi^4} = \mathcal{Y}\left[\begin{smallmatrix} r & s \\ t & \end{smallmatrix}\right]\epsilon^{abc}\epsilon^{ln}\epsilon^{km}((\mathbf{T}L_L)_{pm}(\mathbf{T}Q_L)_{ran})(Q_{Lrak}Q_{Ltcl})\mathcal{F}_{159}^{Uh\psi^4}(h),$$
$$\mathcal{O}_{160}^{Uh\psi^4} = \mathcal{Y}\left[\begin{smallmatrix} r & s \\ t & \end{smallmatrix}\right]\epsilon^{abc}\epsilon^{km}\epsilon^{ln}((\mathbf{T}L_R)_{pm}(\mathbf{T}Q_R)_{ran})(Q_{Rsbk}Q_{Rtcl})\mathcal{F}_{160}^{Uh\psi^4}(h).$$

[Sun, Xiao, **Yu**, 2206.07722]

SMEFT vs HEFT

- EFT with non-linearly realized electroweak symmetry (aka HEFT) is equivalent to EFT with linearly realized electroweak symmetry but whose Lagrangian is a non-polynomial function of the Higgs field that is non-analytic at H=0

- This non-analyticity leads to explosion of multi-Higgs amplitudes at the scale $4 \pi v$. For this reason, the validity regime of HEFT is limited below the scale of order $4 \pi v \sim 3$ TeV

- HEFT is useful to approximate BSM theories where new particles' masses vanish in the limit $v \to 0$, e.g. SM + a 4th generation of chiral fermions or when most of the new particle mass comes from EW symmetry breaking

- On the other hand, an EFT with linearly realized electroweak symmetry and the Lagrangian polynomial in the Higgs field (aka SMEFT) is useful to approximate BSM theories where new particles' masses do not vanish in the limit $v \to 0$, and are parametrically larger than the electroweak scale, e.g. SM + vector-like fermions

Other EFTs

Heavy Quark Effective Theory

Soft Collinear Effective Theory

Gravity EFT

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Summary

Take home message 1:

Core of EFTs: d.o.f separation, symmetry, power counting, decoupling

Take home message 2:

All QFTs are EFT, and EFT is renormalizable and predictive

Take home message 3:

EFT would reproduce full theory results with matching and running

Take home message 4:

SMEFT (NP), Chiral Lagrangian (QCD), EW Chiral Lagrangian

Thanks for your attention!