$SU(3) \times SU(2) \times U(1)$

	$SU(3)$	$SU(2)_L$	$U(1)_Y$
q	3	2	$\frac{1}{6}$
u	3	1	$2/3$
d	3	1	$-1/3$
l	1	2	$-1/2$
e	1	1	-1
(ν)	1	1	0
H	1	2	$+1/2$

$Q = T_3 + Y$

×

$$\varphi \to \exp(\omega_a \lambda^a + \alpha_i T^i + \theta)\varphi$$

center of the group

$SU(3)$ $\quad \underline{\mathbb{1}}, \quad \underline{\omega}, \quad \underline{\omega^2}$

$$\omega = \begin{pmatrix} e^{\frac{2\pi i}{3}} & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \end{pmatrix} = e^{\frac{2\pi i}{3}}\mathbb{1}. \quad \omega^2 = e^{\frac{4\pi i}{3}}\mathbb{1}.$$

$SU(2)$ $\quad \underline{\mathbb{1}}. \quad \underline{-\mathbb{1}}.$

6.

$\mathbb{Z}_6$

$\to U(1)$

monopole.

$su(3) \times su(2) \times u(1)$

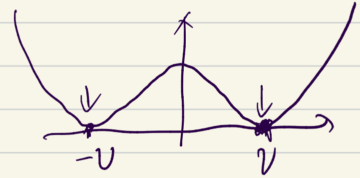
$SU(3) \times SU(2) \times U(1) / \Gamma$

$\Gamma = \mathbb{1}. \ \mathbb{Z}_6, \ \mathbb{Z}_2, \ \mathbb{Z}_3$

$\underline{SU(5)?}$

EWSB

SSB: 小振动 $V(x)$ $V(x_0)'=0$



$$V(x)=(x^2-v^2)^2$$

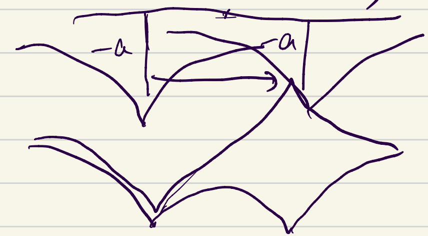
$$L=\frac{1}{2}m\dot{x}^2-(x^2-v^2)^2 \qquad \underline{x\to -x}$$

$-x(t)\neq x(t)$

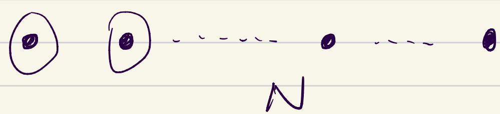
$x(t) \doteq v+a\cos\omega t \cdots$

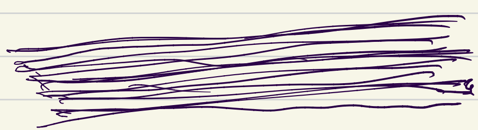
QM: 基态是唯一的 $H \xrightarrow{M} H$ $M|0\rangle \to |0\rangle$

$\delta(x-a)\neq\delta(x+a)$



能带





$N\left\{\Delta E\right.$ $N\to\infty$ $\Delta E\to 0$

$\sim e^{-\frac{2\pi^2}{x^2}}$

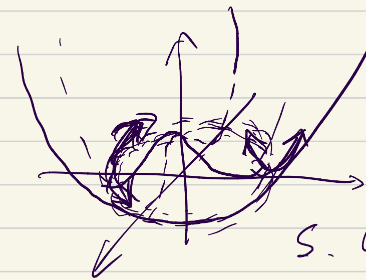
QFT $N=+\infty$

Goldstone $D \geq 2+1$

QFT. $\underline{1+1}$

例: $U(1)$. $\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - V(\varphi)$ $V(\varphi) = \frac{\lambda}{4}(\varphi^*\varphi)^2 + m^2 \varphi^*\varphi$

$m^2 < 0$ $\mu^2 \equiv -m^2 > 0$



S. Coleman.

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \mu^2 \varphi^*\varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2$$

$V(\varphi)_{min} = ?$

$$\langle \varphi^*\varphi \rangle = 2\mu^2/\lambda \equiv v^2$$

$$\langle \varphi \rangle = v e^{i\alpha} \qquad \varphi(x) = (\rho(x)+v)\, e^{i(\alpha+\theta(x))}$$

$$\mathcal{L} = \partial_\mu \rho \partial^\mu \rho + (\rho+v)^2 \partial_\mu\theta \partial^\mu\theta + \frac{\lambda}{4}v^4 - \lambda^2 v^2 \rho^2 - \lambda v \rho^3 - \frac{\lambda}{4}\rho^4$$

$$\rho \to \frac{1}{\sqrt{2}}\rho(x) \qquad a = \frac{1}{\sqrt{2}} v\theta(x)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - \frac{1}{2}(\lambda v)^2\rho^2 - \frac{\lambda v}{2\sqrt{2}}\rho^3 - \frac{\lambda}{16}\rho^4$$
$$+ \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\rho}{v}\partial_\mu a\partial^\mu a + \frac{\rho^2}{2v^2}\partial_\mu a\partial^\mu a$$

ρ — 实标量场 ($m = \lambda v$)

a — 赝标粒子 ($m=0$)

Gauge Symmetry (Higgs mechanism)

SU(2) doublet. $H(x) = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$ $SU(2) \times U(1)$. $W^{\pm}, Z^0, \gamma$.

$$H \to e^{i\alpha_i \cdot \frac{\sigma_i}{2}} e^{i\alpha \frac{I}{2}} H.$$

σ_1 σ_2 σ_3 is Pauli matrix

$$V(H) = \frac{\lambda}{4} (H^+H)^2 - \mu^2 H^+ H \qquad \langle H^+H \rangle = 2\mu^2/\lambda = v^2$$

$$\langle\varphi_1\rangle^2 + \langle\varphi_2\rangle^2 + \langle\varphi_3\rangle^2 + \langle\varphi_4\rangle^2 = v^2 \qquad \underline{\partial_\mu \langle\varphi_i\rangle \equiv 0}$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad H = \begin{pmatrix} \varphi_1(x) + i\varphi_2(x) \\ v + \varphi_3(x) + i\varphi_4(x) \end{pmatrix}$$

$SU(2) \times U(1)$ $\alpha_i(x)$ $\alpha(x)$ $H = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$$\mathcal{L} = (D_\mu\varphi)^*(D^\mu\varphi) + \mu^2\varphi^*\varphi - \frac{\lambda}{2}(\varphi^*\varphi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad D_\mu\varphi = \partial_\mu\varphi - igA_\mu\varphi$$

$\varphi(x) \to (v+\rho(x))\,e^{i\alpha(x)}$ U(1) is gauged. $x. \to e^{-i\alpha(x)}$

$\cancel{\alpha(x)}$ $$D_\mu[(v+\rho)e^{i\alpha}] = e^{i\alpha}\left[\partial_\mu\rho - ig\left(A_\mu + \frac{\partial_\mu\alpha}{g}\right)(\rho+v)\right]$$

~~规范自由~~ → 幺正规范

$$\varphi = (v+\rho)e^{i\alpha} \qquad \varphi \to v+\rho \qquad A_\mu \to A_\mu + \frac{\partial_\mu\alpha}{g}$$

$$C_\mu \equiv A_\mu + \frac{\partial_\mu\alpha}{g} \qquad \partial_\mu C_\nu - \partial_\nu C_\mu = F_{\mu\nu}$$

$$(D_\mu\varphi)^*(D^\mu\varphi) = (\partial_\mu\rho + igC_\mu(\rho+v))(\partial^\mu\rho - igC^\mu(\rho+v))$$

$$= \partial_\mu\rho\,\partial^\mu\rho + 2vg^2\rho C_\mu C^\mu + g^2\rho^2 C_\mu C^\mu + \boxed{g^2v^2 C_\mu C^\mu}$$

$$v^2\left(A_\mu + \frac{\partial_\mu\alpha}{g}\right)\left(A^\mu + \frac{\partial_\mu\alpha}{g}\right) \qquad m = gv$$

$m \to 0$

$$H = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \qquad \langle H^\dagger H \rangle = v^2 \qquad H = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$e^{\frac{i}{2}\alpha_i \sigma_i} = \begin{pmatrix} \cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\cos\theta & ie^{-i\phi}\sin\frac{\alpha}{2}\sin\theta \\ ie^{i\phi}\sin\frac{\alpha}{2}\sin\theta & \cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}\cos\theta \end{pmatrix}$$

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2} \qquad \sin\theta = \sqrt{\alpha_1^2 + \alpha_2^2}/\alpha \qquad \cos\theta = \alpha_3/\alpha.$$

$$e^{i\phi} = (\alpha_1 + i\alpha_2)/\sqrt{\alpha_1^2 + \alpha_2^2}$$

$$e^{\frac{i}{2}\alpha_i \sigma_i} H \to \begin{pmatrix} \varphi_1 \cos\frac{\alpha}{2} - \cdots \\ -\varphi_1 \sin\frac{\alpha}{2}\sin\theta\sin\phi \cdots \end{pmatrix} \qquad \varphi(x) = \sqrt{\varphi_1(x)^2 + \cdots + \varphi_4(x)^2}$$

α_i

$$\cos\frac{\alpha}{2} = \varphi_3(x)/\varphi(x) \qquad \sin\frac{\alpha}{2}\cos\theta = \varphi_4(x)/\varphi(x)$$

$$e^{i\phi}\sin\frac{\alpha}{2}\sin\theta = -\varphi_2(x) - i\varphi_1(x)$$

$$H(x) = \begin{pmatrix} 0 \\ \sqrt{\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2} \end{pmatrix} = \begin{pmatrix} 0 \\ v+\frac{1}{\sqrt{2}}h(x) \end{pmatrix}$$ 幺正规范

$$D_\mu H = \partial_\mu H - ig'\frac{y}{2}B_\mu H + ig\frac{\sigma_i}{2}W^i_\mu H$$

$$= \begin{pmatrix} \partial_\mu - ig'\frac{y}{2}B_\mu + \frac{i}{2}gW^3_\mu & \frac{ig}{2}(W^1_\mu - iW^2_\mu) \\ \frac{ig}{2}(W^1_\mu + iW^2_\mu) & \partial_\mu - ig'\frac{y}{2}B_\mu - \frac{i}{2}gW^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v+\frac{1}{\sqrt{2}}h \end{pmatrix}$$

$$(D_\mu H)^\dagger (D^\mu H) = \frac{g^2}{2}\left(v+\frac{1}{\sqrt{2}}h\right)^2 W^+_\mu W^{-\mu} + \frac{1}{2}\partial_\mu h \partial^\mu h$$
$$+ \frac{1}{4}\left(v+\frac{1}{\sqrt{2}}h\right)^2 (yg'B_\mu + gW^3_\mu)(yg'B^\mu + gW^{3\mu})$$

$$\left(W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)\right)$$

$$Z_\mu = (yg'B_\mu + gW^3_\mu)/\sqrt{y^2g'^2+g^2}$$

$$A_\mu = (gB_\mu - yg'W^3_\mu)/\sqrt{y^2g'^2+g^2} \qquad m_A = 0$$

$$m_W = gv \qquad m_Z = v\sqrt{y^2g'^2+g^2} \qquad m_W = m_Z\cos\theta_W$$

$$A_\mu \neq B_\mu \qquad U(1)_{em} \neq U(1)_Y$$

$$D_\mu \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \to e Q_f A_\mu + \left(\frac{e\sin\theta_W}{2\cos\theta_W}\, y_f - \frac{e\cos\theta_W}{\sin\theta_W}\, I_3\right) Z_\mu$$

$$\to eQ_f A_\mu - \frac{g}{\cos\theta_W}\left(I_3 - Q_f \sin^2\theta_W\right) Z_\mu$$

$$-\frac{g}{\sqrt{2}}\bar\psi_1\gamma^\mu P_L\psi_2 W^+_\mu - \frac{g}{\sqrt{2}}\bar\psi_2\gamma^\mu P_L \psi_1 W^-_\mu \qquad Q_2 = Q_1 - 1$$

$$\mathcal{L}_f \to -\frac{g}{2\cos\theta_W}\sum_{i=1,2} Z_\mu \bar\psi_i \gamma^\mu\left[\left(I_{3i} - 2Q_i\sin^2\theta_W\right) - I_{3i}\gamma_5\right]\psi_i$$

$m_W = m_Z\cos\theta_W$ $\quad$ $m_W = gv$ $\quad$ $m_Z \sim \cancel{g}\, v\sqrt{g^2+g'^2}$ $\quad$ tree-level.

PDG. $\quad$ θ_W

$$\mathcal{L}_Y = \underbrace{Y_d \, \bar{Q}_L \cdot H \, d_R = Y_d \, (\bar{u}_L \; \bar{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} d_R} = Y_d v \, \bar{d}_L d_R$$

$$Y_e \, \bar{L}_L H e_R \to Y_e v \bar{e}_L e_R. \qquad m_e \sim Y_e v.$$

$m_u = ?$

$$\bar{Q}_L \cdot H$$

$2^* = 2 \qquad SU(2).$

$$\boxed{(\eta_1 \cdot \eta_2) \cdot \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}}$$

$$\boxed{\det \begin{pmatrix} \eta_1 & \xi_1 \\ \eta_2 & \xi_2 \end{pmatrix}}$$

$$-Y_u \det \begin{pmatrix} \bar{u}_L & \bar{d}_R \\ 0 & v \end{pmatrix} u_R. \to -Y_u v \bar{u}_L u_R.$$

$$-Y_u \, \varepsilon^{\alpha\beta} \, \bar{Q}_{L\alpha} H^*_\beta \, u_R$$

$$\mathcal{L}_{Yukawa} = -Y_d \bar{Q}_L H d_R - Y_e \bar{L}_L H e_R - Y_u \varepsilon^{\alpha\beta} \bar{Q}_{L\alpha} H_\beta u_R + h.c.$$

$$= -\underbrace{(\bar{d}_L \;\; \bar{s}_L \;\; \bar{b}_L)}_{\bar{d}_{1L}\ \bar{d}_{2L}\ \bar{d}_{3L}} Y_d v \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \begin{matrix} d_{1R} \\ d_{2R} \\ d_{3R} \end{matrix} \quad (e \cdots\cdots) -$$

$$- (\bar{u}_L \;\; \bar{c}_L \;\; \bar{t}_L) Y_u v \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} + h.c.$$

$M_d = Y_d v \qquad M_u = Y_u v \qquad M_e = Y_e v$

SVD $\qquad n \times m$, M. $\qquad U, V$. $\qquad UMV = \mathrm{diag}(m_1 \cdots m_n, 0)$

$m_i \geq 0$

$$U \begin{pmatrix} & & \end{pmatrix} V \to \begin{pmatrix} m_1 & & 0 & \\ & \ddots & & 0 \\ 0 & & m_n & \end{pmatrix}$$

$U = V^\dagger$

$V_u^\dagger M_u U_u \qquad V_d^\dagger M_d U_d \qquad V_\ell^\dagger M_\ell U_\ell$

$$\begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} \qquad \begin{pmatrix} m_d & & 0 \\ & m_s & \\ 0 & & m_b \end{pmatrix} \qquad \begin{pmatrix} m_e & & 0 \\ & m_\mu & \\ 0 & & m_\tau \end{pmatrix}$$

$$F_L = V_F^\dagger \tfrac{1}{2}(1-\gamma_5) f$$

$$F_R = U_F^\dagger \tfrac{1}{2}(1-\gamma_5) f$$

$$\bar{\psi}_{iL} \underbrace{V V^\dagger}_{} \psi_{iL} \qquad \bar{\psi}_{iR} \underbrace{U U^\dagger}_{} \psi_{iR}$$

$$u_i \xrightarrow{\gamma/Z} u_i$$

$$u \xrightarrow{\gamma/Z} u \qquad \text{GIM}$$

$$c \qquad\qquad c$$

$$(\bar{u}_{1L} \;\; \bar{u}_{2L} \;\; \bar{u}_{3L}) \gamma^\mu \begin{pmatrix} d_{1L} \\ d_{2L} \\ d_{3L} \end{pmatrix} W_\mu^+ + h.c.$$

$$(\bar{u} \;\; \bar{c} \;\; \bar{t}) \underline{V_u V_d^\dagger} \gamma^\mu (1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_u \neq V_d$$

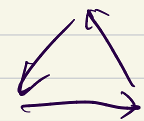
$$V_{CKM} \equiv V_u V_d^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo – Kabayoshi – Maskawa 矩阵

V_u V_d 幺正 V_{CKM} 3×3 幺正矩阵.

$$V_{CKM}^\dagger V_{CKM} = I.$$

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$$



三个幺正三角形

V_{CKM} $\quad (\bar{u} \quad \bar{c} \quad \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$u \to u e^{i\alpha_u}$ $\quad \boxed{\delta_{ud}} = \alpha_u - \alpha_d$

$$V_{CKM} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \cos\theta_3 & -\sin\theta_1 \sin\theta_3 \\ \sin\theta_1 \cos\theta_2 & V_{cs} & V_{cb} \\ \sin\theta_1 \sin\theta_2 & V_{ts} & V_{tb} \end{pmatrix}$$

$\sum_j V_{1j}^* \cdot V_{2j} = 0$ $\quad V_{cb} = \frac{c_1 c_2}{s_3} - \frac{c_3}{s_3} V_{cs}$

V_{cs}

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\not{C}\not{P} \sim 1‰$ $\qquad \delta \sim 70^\circ$ $\qquad s_{13} \ll 1.$

J