



新物理模型

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提纲

- 轴子类轴子模型
- 暗光子模型
- 中微子模型

提纲：轴子

- The axion and strong CP problem
- Visible and invisible axion models
- Axion dark matter, ALP and experimental searches
- Summary

The QCD axion and the Strong CP problem

$$\mathcal{L} \supset -\frac{\theta g_s^2}{32\pi^2} G\tilde{G} - (\bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \text{h.c.})$$

- The CKM matrix from $M_{u,d}$
 - CP violating phase $\theta_{\text{CP}} \sim 1.2$ radian
- QCD induced CP violating phase, $\bar{\theta}$

$$\bar{\theta} = \theta + \arg [\det [M_u M_d]]$$

- $\bar{\theta}$ is invariant under quark chiral rotation
- According to neutron EDM experiment

$$d_{\text{EDM}}^n \sim \theta \times 10^{-16} \text{ e cm}$$

$$d_{\text{exp}}^n < 10^{-26} \text{ e cm}$$

$$\bar{\theta} \lesssim 1.3 \times 10^{-10} \text{ radian}$$

The Peccei-Quinn solution to Strong CP problem

- Experiment requires $\bar{\theta} = \theta + \arg [\det [M_u M_d]] \lesssim 10^{-1} \text{rad}$
- PQ: promote the constant $\bar{\theta}$ to a dynamical field, a
- Vafa-Witten theorem: vector-like theory (QCD) has ground state $\langle \theta \rangle = 0$
- Introduce a *global* PQ-symmetry $U(1)_{\text{PQ}}$, *anomalous* under the QCD
- The massless Goldstone boson a is called *axion*

- $a \rightarrow a + \kappa f_a \Rightarrow \mathcal{S} \rightarrow \mathcal{S} + \frac{\kappa}{32\pi^2} \int d^4x G\tilde{G}$, cancels $\bar{\theta}$

- Low energy:
$$\mathcal{L} = \sum_q \bar{q} \left(iD_\mu \gamma^\mu - m_q \right) q - \frac{1}{4} GG + \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G\tilde{G} + \frac{1}{2} \left(\partial_\mu a \right)^2 + \mathcal{L}_{\text{int}}[\partial_\mu a]$$

Model independent visible axion properties

- For two flavor QCD, $q = (u, d)^T$
- $\mathcal{L} \supset \frac{1}{2} \left(\partial_\mu a \right)^2 + \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G\tilde{G} + \frac{1}{4} g_{a\gamma}^0 a F\tilde{F} - \bar{q}_L M_q q_R + \frac{\partial_\mu a}{2f_a} \bar{q} c_q^0 \gamma^\mu \gamma_5 q + h.c.$
- The three QCD related terms can be eliminated to 2 d.o.f.
- Choose to eliminate $G\tilde{G}$ term by quark field redefinition in a-related chiral rotation

- A new quark field: $q' = \exp \left(i \frac{a}{2f_a} \gamma_5 Q_a \right) q$

- $\mathcal{L} \supset \frac{1}{2} \left(\partial_\mu a \right)^2 + \frac{1}{4} g_{a\gamma} F\tilde{F} - \bar{q}'_L M_{q'} q'_R + \frac{\partial_\mu a}{2f_a} \bar{q}' c_{q'} \gamma^\mu \gamma_5 q' + h.c.$

Model independent visible axion properties

- In the new basis

- $\mathcal{L} \supset \frac{1}{2} \left(\partial_\mu a \right)^2 + \frac{1}{4} g_{a\gamma} a F \tilde{F} - \bar{q}'_L M_a q'_R + \frac{\partial_\mu a}{2f_a} \bar{q}' c_q \gamma^\mu \gamma_5 q' + h.c.$

- $c_q = c_q^0 + Q_q, \quad g_{a\gamma} = g_{a\gamma}^0 - 2N_c \frac{\alpha_{em}}{2\pi f_a} \text{Tr}[Q_a Q^2]$

- Quark mass is complex: $M_a = \exp \left(i \frac{a}{2f_a} Q_a \right) M_q \exp \left(i \frac{a}{2f_a} Q_a \right)$

- Subtlety: fix the basis (gauge) and follow through the whole calculation

- E.g. $\text{BR}(K^- \rightarrow \pi^- a)$ was overestimated by 40 times for ~35 years

Bauer, Neubert et al: 2102.13112 (PRL)

- The auxiliary chiral rotation should not affect physical observables

The axion and Chiral Lagrangian

- The generic low energy Lagrangian is

$$\mathcal{L} \supset \frac{1}{2} \left(\partial_\mu a \right)^2 + \frac{1}{4} g_{a\gamma} F \tilde{F} - \bar{q}_L M_a q_R + \frac{\partial_\mu a}{2f_a} \bar{q} c_q \gamma^\mu \gamma_5 q + h.c.$$

- Two flavor quarks $q = (u, d)$; quark mass term $M_a = e^{i\frac{aQ_a}{2f_a}} M_q e^{i\frac{aQ_a}{2f_a}}$
- Below the QCD scale, one needs the chiral axion Lagrangian

$$\mathcal{L}_a^{\chi PT} = \frac{f_\pi^2}{4} \text{Tr} \left[(D^\mu U)^\dagger D_\mu U + 2B_0 (UM_a^\dagger + M_a U^\dagger) \right] + \frac{\partial^\mu a}{4f_a} \text{Tr}[c_q \sigma^a] J_\mu^a$$

$$U \equiv e^{i\pi^a \sigma^a / f_\pi}$$

$$J_\mu^a \equiv e^{i\pi^a \sigma^a / f_\pi}$$

Axion mass and interaction with pions

$$\mathcal{L}_a^{\chi PT} = \frac{f_\pi^2}{4} \text{Tr} \left[(D^\mu U)^\dagger D_\mu U + 2B_0(UM_a^\dagger + M_a U^\dagger) \right] + \frac{\partial^\mu a}{4f_a} \text{Tr}[c_q \sigma^a] J_\mu^a$$

- Axion mass: $m_a^2 \simeq (Q_u + Q_d)^2 \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$
- Axion- π^0 mixing: $\theta_{a\pi} \simeq \frac{(Q_d m_d - Q_u m_u)}{(m_u + m_d)} \frac{f_\pi}{f_a}$
- Axion-pion couplings: $-\frac{3}{2} \frac{\epsilon}{f_a f_\pi} \partial_\mu a \left(2\partial^\mu \pi^0 \pi^+ \pi^- - \pi^0 \partial^\mu \pi^+ \pi^- - \pi^0 \pi^+ \partial^\mu \pi^- \right)$
- Coefficient: $\epsilon = -\frac{1}{2} \left(\frac{Q_d m_d - Q_u m_u}{m_u + m_d} + c_d^0 - c_u^0 \right) \frac{f_\pi}{f_a}$

提纲：轴子

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- Visible and invisible axion models
- Axion dark matter, ALP and experimental searches

The invisible axion models

- SM particles does not directly charge under $U(1)_{\text{PQ}}$
 - KSVZ model:
 - Heavy vector-like quark: $Q_{L,R}$
 - Q_L and Q_R has different charge under $U(1)_{\text{PQ}}$
 - A heavy complex scalar $\Phi = re^{ia}$ charge under $U(1)_{\text{PQ}}$
 - Yukawa: $y\Phi\bar{Q}_L Q_R \supset \frac{yf_a}{\sqrt{2}}e^{ia/f_a}\bar{Q}_L Q_R$
 - Low energy: $\mathcal{L} \supset \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G}$

The invisible axion models

- Induce a complex scalar with very large vev
- DFSZ model:
 - Two Higgs doublet $H_{u,d}$ and a complex singlet Φ charged under $U(1)_{PQ}$, with phase factor $e^{i\phi_{u,d,0}}$
 - Similar to previous UV model, but $\langle \Phi \rangle \gg v_h$
 - Yukawa: $(\bar{Q}Y_u H_u u_R + \bar{Q}Y_d H_d d_R + \bar{L}Y_e H_d e_R) + h.c.$
 - Potential term: e.g. $H_u H_d \Phi^2$,
 - Axion mode: $a = \frac{1}{f_a} \sum_{i=u,d,0} Q_i v_i \phi_i$
 - Low energy: $\mathcal{L} \supset \frac{\alpha_s}{8\pi^2} \frac{a}{f_a} G\tilde{G} + \frac{\alpha_{em}}{8\pi} \frac{E}{N} \frac{a}{f_a} F\tilde{F} - \bar{f}_L M_f f_R + \frac{\partial_\mu a}{2f_a} \bar{f} c_f \gamma^\mu \gamma_5 f$

The visible QCD axion constraints

- The visible axion is highly constrained by pion measurements

experimental limit on observable	translates into bound on model	expectation for generic MeV axions (LO in χ PT)	expectation for MeV axion variant in (3.1) (NLO in χ PT)	reason
beam dumps (see Fig.1)	$\tau_a \lesssim 10^{-13}$ s (see Fig.1)	$\tau_a < 10^{-12}$ s if $ Q_e^{\text{PQ}} \sim \mathcal{O}(1)$ $\tau_a \gtrsim 10^{-11}$ s if $Q_e^{\text{PQ}} = 0$	$ Q_e^{\text{PQ}} \sim \mathcal{O}(1)$ $\tau_a \lesssim 10^{-13}$ s for $m_a \gtrsim 5 - 10$ MeV	MB
$\text{Br}(\pi^+ \rightarrow e^+ \nu(a \rightarrow e^+ e^-)) \lesssim 10^{-10}$ $\Delta \text{Br}(\pi^0 \rightarrow e^+ e^-) \lesssim 2 \times 10^{-8}$	$ \theta_{a\pi} \lesssim (0.5 - 0.7) \times 10^{-4}$ $Q_e^{\text{PQ}} \times \theta_{a\pi} \lesssim 1.6 \times 10^{-4} \left(\frac{f_a}{\text{GeV}} \right)$	$ \theta_{a\pi} \sim \mathcal{O}(0.01 - 0.1) \left(\frac{\text{GeV}}{f_a} \right)$	$\theta_{a\pi} \sim (0.2 \pm 3) \times 10^{-3} \left(\frac{\text{GeV}}{f_a} \right)$	PP, NLO
$\text{Br}(K^+ \rightarrow \pi^+(a \rightarrow e^+ e^-)) \lesssim 10^{-5} - 10^{-6}$	$ \theta_{a\eta_{ud}} \lesssim 10^{-4}$ if octet enhanced $ \theta_{a\eta_{ud}} \lesssim 10^{-2}$ if not	$ \theta_{a\eta_{ud}} \sim \mathcal{O}(10^{-3} - 10^{-2}) \left(\frac{\text{GeV}}{f_a} \right)$	$\theta_{a\eta_{ud}} \sim (-2 \pm 8) \times 10^{-3} \left(\frac{\text{GeV}}{f_a} \right)$	NLO
$\text{Br}(K^+ \rightarrow \pi^+(a \rightarrow \gamma\gamma)) \lesssim 10^{-9}$ $\text{Br}(\Phi \rightarrow \gamma(a \rightarrow e^+ e^-)) \lesssim 5 \times 10^{-5}$	$ \theta_{a\eta_s} \lesssim \mathcal{O}(10^{-1})$	$ \theta_{a\eta_s} \sim \mathcal{O}(10^{-2}) \left(\frac{\text{GeV}}{f_a} \right)$	$\theta_{a\eta_s} \sim (1 \pm 2) \times 10^{-2} \left(\frac{\text{GeV}}{f_a} \right)$	NLO
$\text{Br}(K^+ \rightarrow \pi^+(a \rightarrow \text{inv})) \lesssim 0.5 \times 10^{-10}$	$\text{Br}(a \rightarrow \text{inv}) \lesssim \mathcal{O}(10^{-4})$ (assuming $\tau_a < 10^{-12}$ s)	$\text{Br}(a \rightarrow \nu\bar{\nu}) \sim (Q_\nu^{\text{PQ}} m_\nu / Q_e^{\text{PQ}} m_e)^2$	$\text{Br}(a \rightarrow \chi\bar{\chi}) \sim (Q_\chi^{\text{PQ}} m_\chi / Q_e^{\text{PQ}} m_e)^2$ $\chi = \nu_{(e,\mu,\tau,s)}$, sub-MeV DM, ...	MB

The pionphobic axion and Atomki experiment

- Old solution: pion-phobic axion (not elegant but can work)

$$\theta_{a\pi} \simeq \frac{(Q_d m_d - Q_u m_u) f_\pi}{(m_u + m_d) f_a} \Rightarrow \theta_{a\pi} \lesssim 10^{-4}$$

$$m_u/m_d \simeq 0.47 \pm 0.06, \text{ \& } Q_u \simeq 2Q_d$$

Can be tested by 3-body decay: $K \rightarrow \pi\pi a$
Pospelov et al: 2002.04623

- Atomki anomaly see bumpy feature from $^8\text{Be}^*$ and $^4\text{He}^*$ decays

$$^8\text{Be}^*(17.64) \rightarrow ^8\text{Be}(0) + e^+e^-, \quad \Delta E = 17.64 \text{ MeV}, \quad \Delta I \approx 1,$$

$$(E0) \quad ^4\text{He}^*(20.49) \rightarrow ^4\text{He}(0) + (\gamma^* \rightarrow e^+e^-), \quad \Delta E = 20.49 \text{ MeV}, \quad \Delta I = 0,$$

$$^8\text{Be}^*(18.15) \rightarrow ^8\text{Be}(0) + e^+e^-, \quad \Delta E = 18.15 \text{ MeV}, \quad \Delta I \approx 0.$$

$$(M0) \quad ^4\text{He}^*(21.01) \rightarrow ^4\text{He}(0) + (a \rightarrow e^+e^-), \quad \Delta E = 21.01 \text{ MeV}, \quad \Delta I = 0,$$

- It might be connect to 17MeV pion-phobic axion which couples to electrons.

The UV model building

- The visible axion receives stringent constraint, but pion-phobic axion can be safe
- It has to couple to 1st gen fermions with $Q_u = 2Q_d$
- Atomki experiments and electron g-2 can be explained if axion couples to electron
- The requirements of 1st gen quarks coupling, CKM matrix and quark mass generation are non-trivial: fully determines the UV Yukawa
- SM Higgs can decay to axion pair, resulting lepton-jet features

The UV model building

- Pion-phobic QCD axion with coupling to electron

$$\mathcal{L}_{int} = \sum_{f=e,u,d} m_f e^{iQ_f a/f_a} \bar{f}_L f_R + h.c. \approx \sum_f g_a^f i a \bar{f} \gamma_5 f, \quad g_a^f = \frac{Q_f m_f}{f_a}.$$

$$g_a^{\gamma\gamma} = \frac{\alpha}{4\pi f_\pi} \left(\theta_{a\pi} + \frac{5}{3} \theta_{a\eta_{ud}} + \frac{\sqrt{2}}{3} \theta_{a\eta_s} \right),$$

- The goal: couples to 1st gen fermions, CKM matrix and quark mass generation

$$\mathcal{L}_{PQ}^{Yuk} \supset - \sum_{i=1,2,3} \left(\bar{Q}^i Y_u^{i1} H_u u_R^1 + \bar{Q}^i Y_d^{i1} H_d d_R^1 + \bar{L}^i Y_e^{i1} H_e e_R^1 \right) + h.c.$$

$$\mathcal{L}_{SM}^{Yuk} \supset - \sum_{i=1,2,3} \sum_{j=2,3} \left(\bar{Q}^i Y_u^{ij} \tilde{H} u_R^j + \bar{Q}^i Y_d^{ij} H d_R^j + \bar{L}^i Y_e^{ij} H e_R^j \right) + h.c.$$

Particles	H	H_u	$H_{d,e}$	u_R	d_R	e_R	ϕ_f
$SU(2)_L$	2	2	2	1	1	1	1
$U(1)_Y$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1	0
$U(1)_{PQ}$	0	$-Q_u$	$-Q_{d,e}$	Q_u	Q_d	Q_e	$-Q_f$

$$Q_u = 2, Q_d = 1; Q_e = 1/2 \text{ or } 1/3$$

Quark mass and CKM matrix

- Up and Down quark mass diagonalization ($V_R = 1$)

$$M_u \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} Y_u^{11} v_u & Y_u^{12} v & Y_u^{13} v \\ Y_u^{21} v_u & Y_u^{22} v & Y_u^{23} v \\ Y_u^{31} v_u & Y_u^{32} v & Y_u^{33} v \end{pmatrix} = V_{u_L} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} V_{u_R}^\dagger \quad V_{\text{CKM}} = V_{u_L}^\dagger V_{d_L}.$$

- The specific Higgs H_u couples to 1st gen up-quark only

$$\bar{Q}^i Y_u^{i1} H_u u_R^1 = \bar{Q}^{\bar{m}i} \left(V_{u_L}^\dagger \right)^{ij} Y_u^{j1} H_u u_R^{1,m} = \sqrt{2} \frac{m_u}{v_u} \bar{Q}^{\bar{m}1} H_u u_R^{1,m}, \quad \Rightarrow \left(V_{u_L}^\dagger \right)^{ij} Y_u^{j1} = \sqrt{2} \frac{m_u}{v_u} (1, 0, 0)^T.$$

- The unitary matrix can be decomposed into

$$V_{u_L}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_x & s_x \\ 0 & -s_x & c_x \end{pmatrix} \begin{pmatrix} (Y_u^{j1})_\parallel^T \\ (Y_u^{j1})_{\perp 1}^T \\ (Y_u^{j1})_{\perp 2}^T \end{pmatrix} \quad Y_u^{j1} = \sqrt{2} \frac{m_u}{v_u} \begin{pmatrix} c_{13} \\ 0 \\ s_{13} \end{pmatrix} \quad x = \theta_{23}$$

- The Yukawa coupling is fully fixed by requirements: CKM and 1st gen only coupling

The axion couplings

- The scalar potential

$$V_{\text{PQ}} = \left(A_u \phi_u^* H \cdot H_u + A_d \phi_d^* H^\dagger H_d + A_e \phi_e^* H^\dagger H_e + A_\phi \phi_u^* \phi_d^2 + B_\phi \phi_d^* \phi_e^n \right) + h.c.,$$

$$V_{\text{dia}} = \sum_{\Phi} -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2,$$

- The axion as the Goldstone mode

$$\vec{G}_{\text{SM}} = \frac{1}{\sqrt{v^2 + v_u^2 + v_d^2 + v_e^2}} \begin{pmatrix} v, & v_u, & v_d, & v_e & 0, & 0, & 0 \end{pmatrix},$$

$$\vec{G}_{\text{PQ}} \approx \frac{1}{\sqrt{\sum_f Q_f^2 (v_f^2 + v_{\phi_f}^2)}} \times$$

$$\begin{pmatrix} -\frac{\sum_f (-1)^f Q_f v_f^2}{v}, & -Q_u v_u, & Q_d v_d, & Q_e v_e, & Q_u v_{\phi_u}, & Q_d v_{\phi_d}, & Q_e v_{\phi_e} \end{pmatrix}$$

- The axion coupling to SM

$$\sum_{f=u,d,e} \frac{m_f}{f_a} Q_f i a \bar{f} \gamma_5 f + Q_{F,\text{eff}}^{\text{PQ}} \sum_{F=2\text{nd},3\text{rd}} \frac{m_F}{f_a} i a \bar{F} \gamma_5 F,$$

$$Q_{F,\text{eff}}^{\text{PQ}} \equiv -\frac{\sum_f (-1)^f Q_f v_f^2}{v^2}.$$

Outline

- The axion and strong CP problem
- Visible and invisible axion models
- Axion dark matter, ALP and experimental searches

Misalignment and Axion Dark Matter

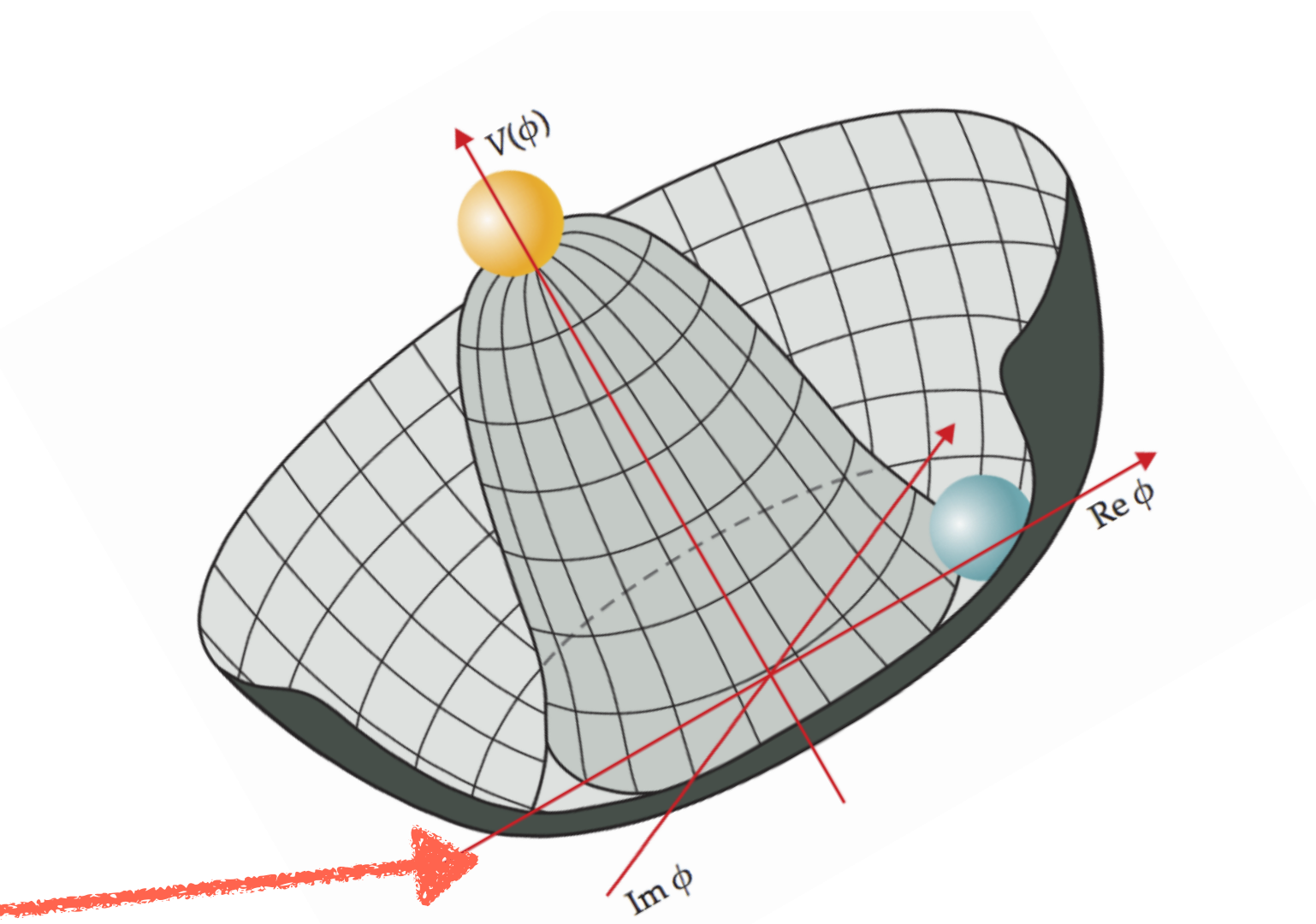
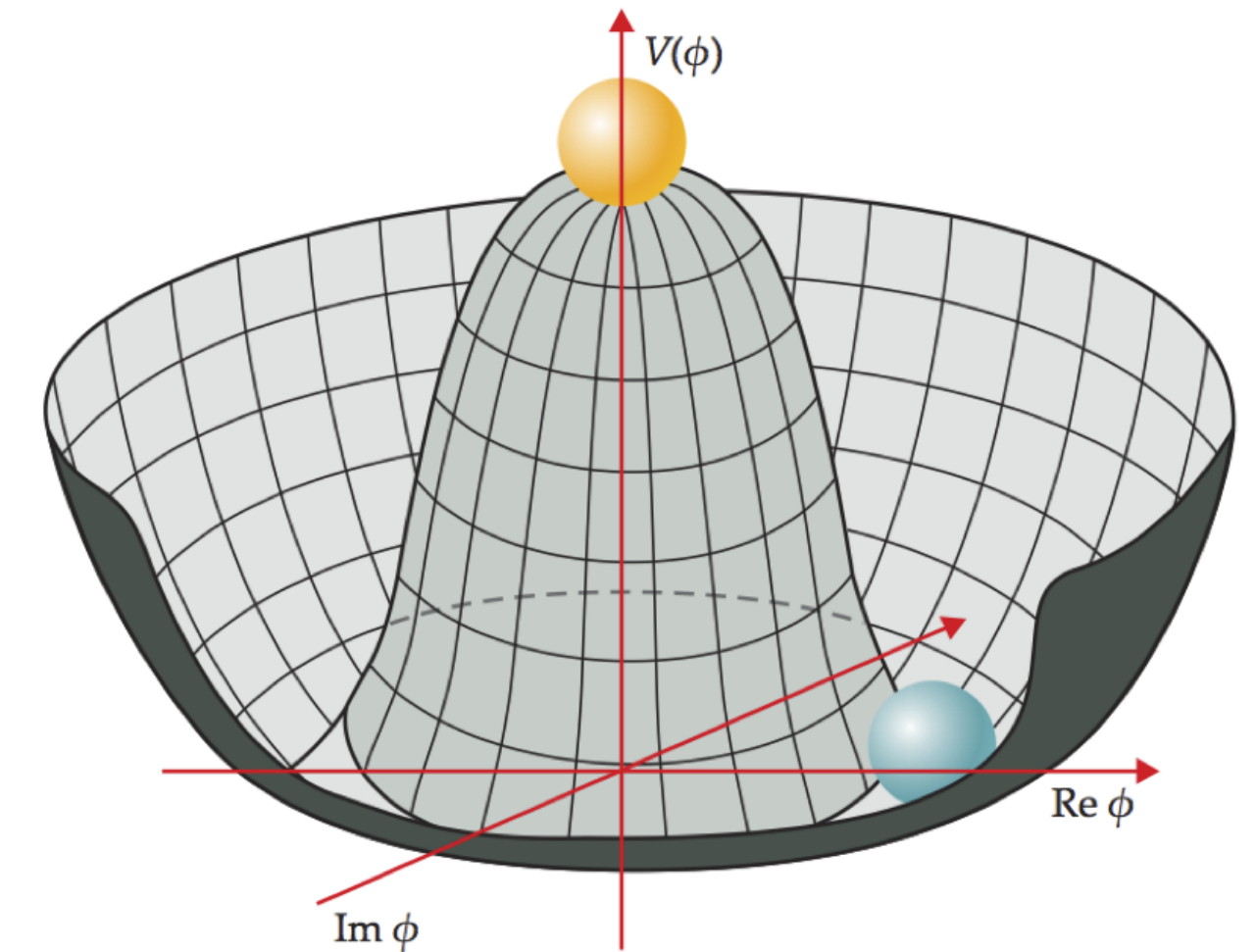
- Global $U(1)_{PQ}$ symmetry
- Spontaneous broken leads to massless goldstone (**Axion**)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

- At QCD scale $\sim O(1)$ GeV,
- Potential from Chiral Lagrangian explicitly breaks the symmetry leads to massive axion
- Energy stored in coherent oscillation of axion field

- When $m_\phi \sim \frac{\Lambda^2}{f_\phi} \sim H$, misalignment happens and the fields turns into particles: **cold dark matter**

- QCD vacuum picks $\Theta = \theta_{QCD} + \xi \langle a \rangle / f_a = 0$



Experimental searches for Axion-Like Particles axion

$$\mathcal{L}_{\text{ALP}} = g_{ag} \frac{a}{f_a} G\tilde{G} + g_{a\gamma} \frac{a}{f_a} F\tilde{F} + g_{af} \frac{\partial_\mu a}{2f_a} \bar{f}\gamma^\mu\gamma_5 f$$

- Sources of ALP:

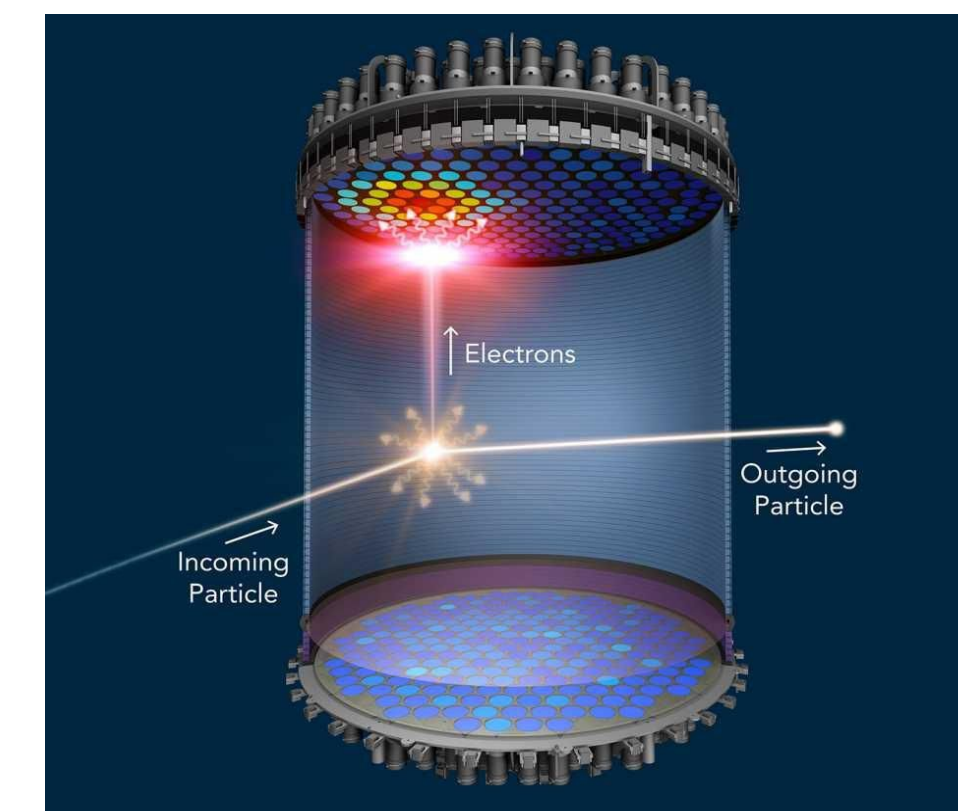
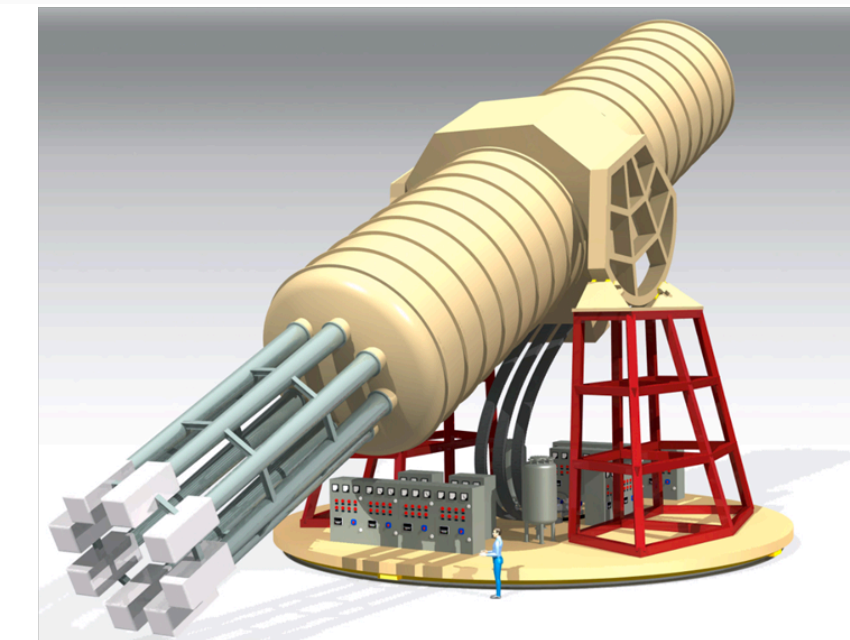
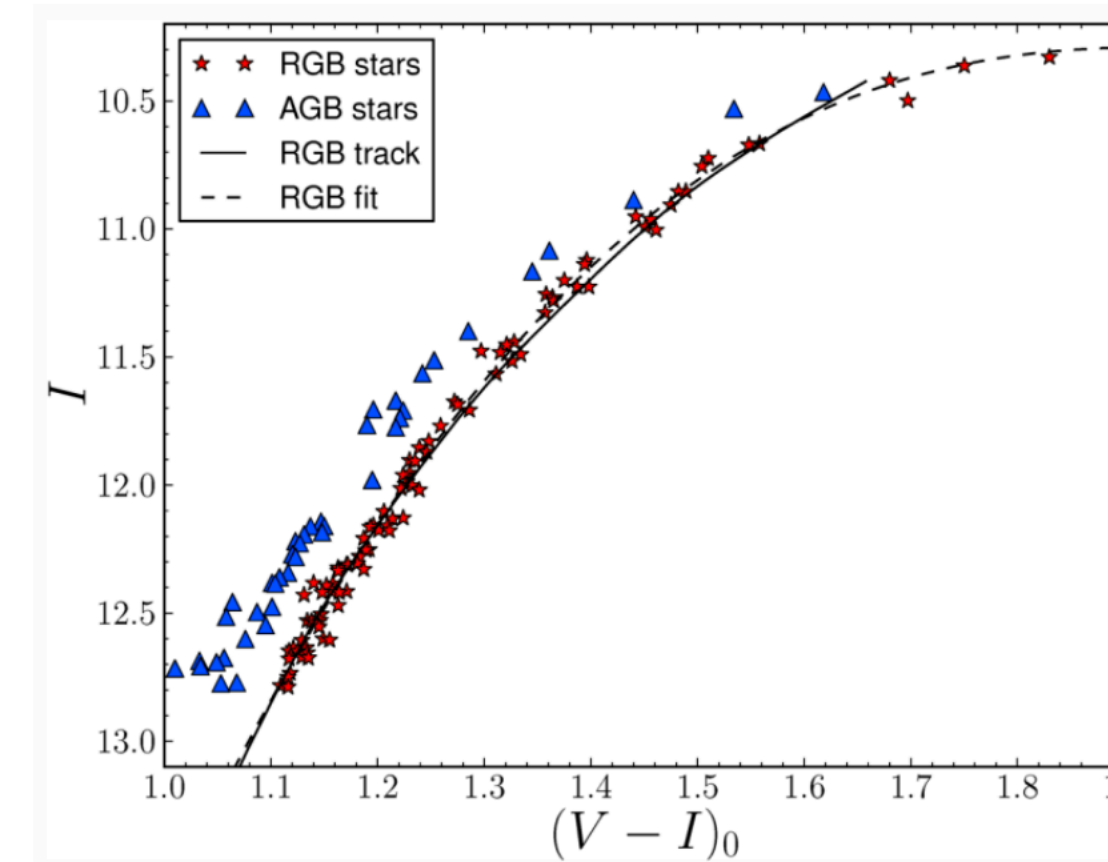
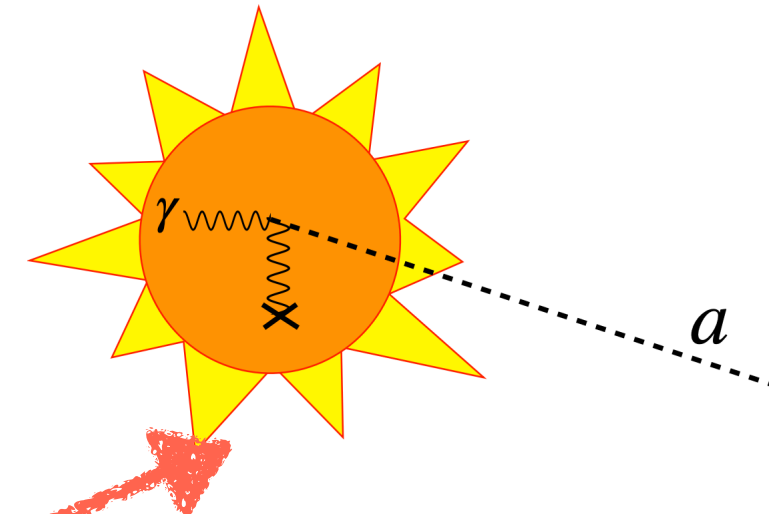
m_a and f_a are not fixed by QCD scale

- Astrophysical objects: Sun, Horizontal Branch, Neutron Star, SuperNova etc...
- ALP dark matter from misalignment
- ALP produced inside laboratory: laser conversion, beam dump, ee collider, pp collider
- ALP as force mediator

Experimental searches for Axion-Like Particles axion

Methodology:

- Dark Matter Axion: haloscopes ..
- Axion independent searches:
 - Rare meson decays
 - **Stellar cooling**
 - Supernova
 - **Helioscopes: solar axion (CAST, IAXO, or DM direct detection searches)**
 - Light shining through walls
 - Polarization
 - Fifth force
 - Radio wave detection

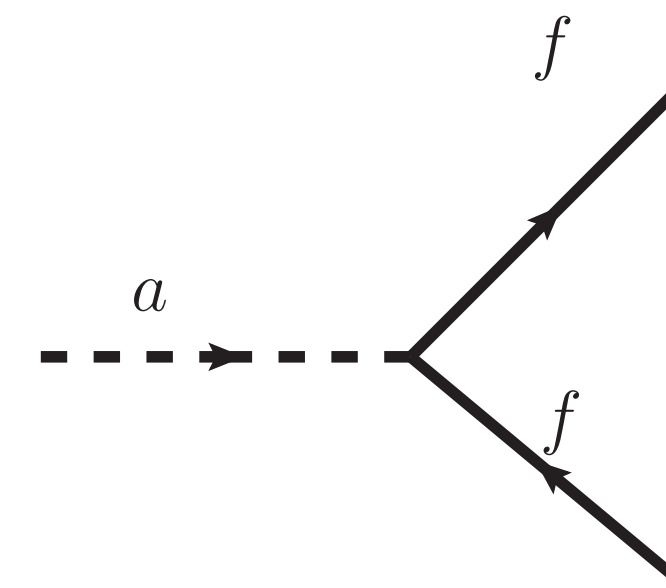
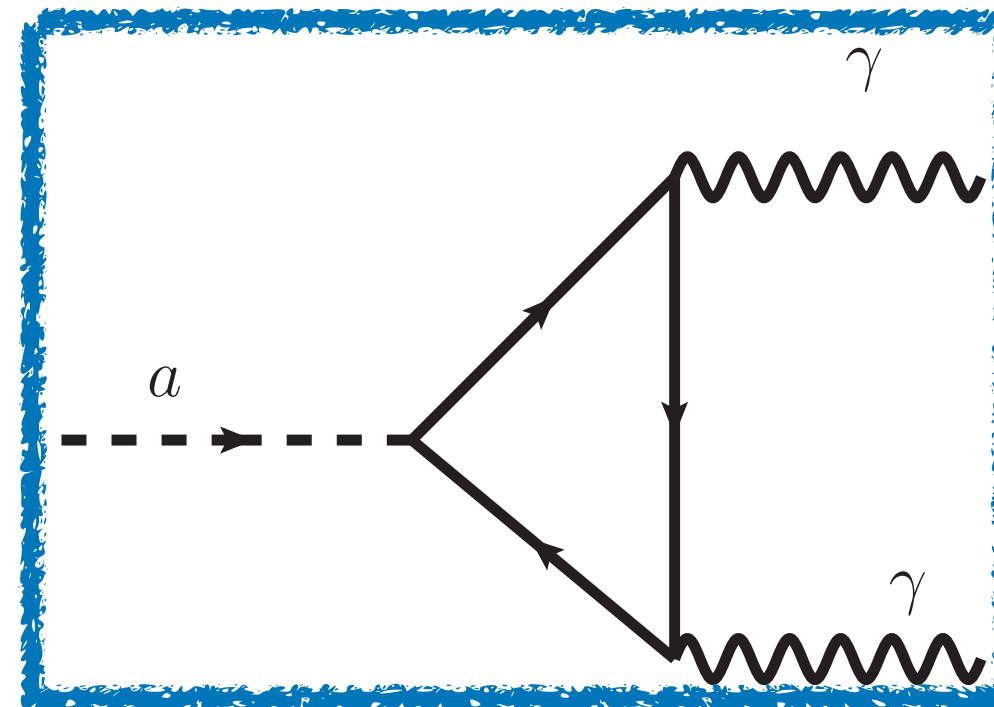
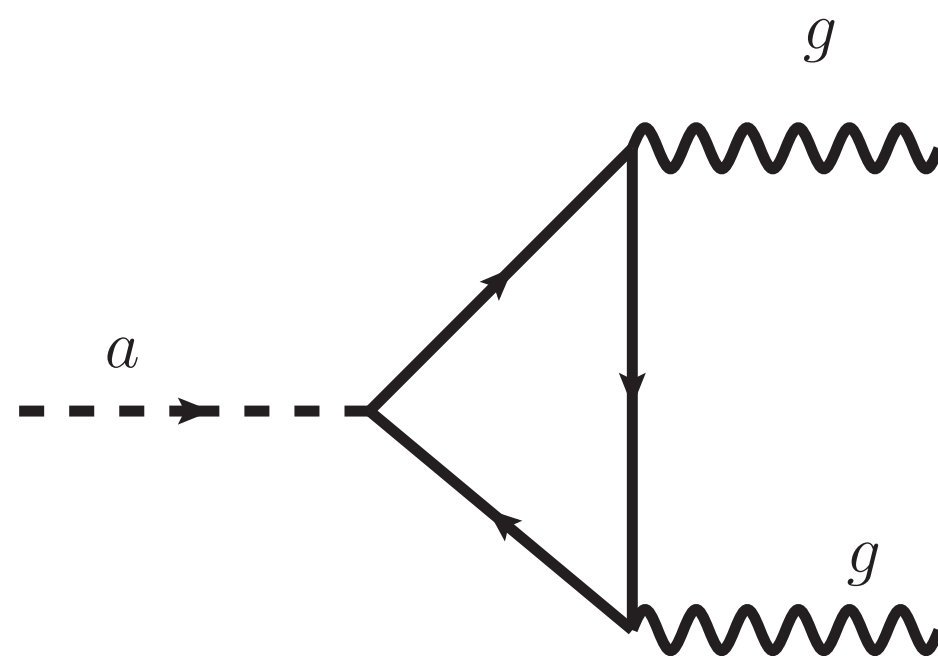


Experimental searches for Axion-Like Particles axion

$$\mathcal{L}_{\text{ALP}} = g_{ag} \frac{a}{f_a} G \tilde{G} + g_{a\gamma} \frac{a}{f_a} F \tilde{F} + g_{af} \frac{\partial_\mu a}{2f_a} \bar{f} \gamma^\mu \gamma_5 f$$

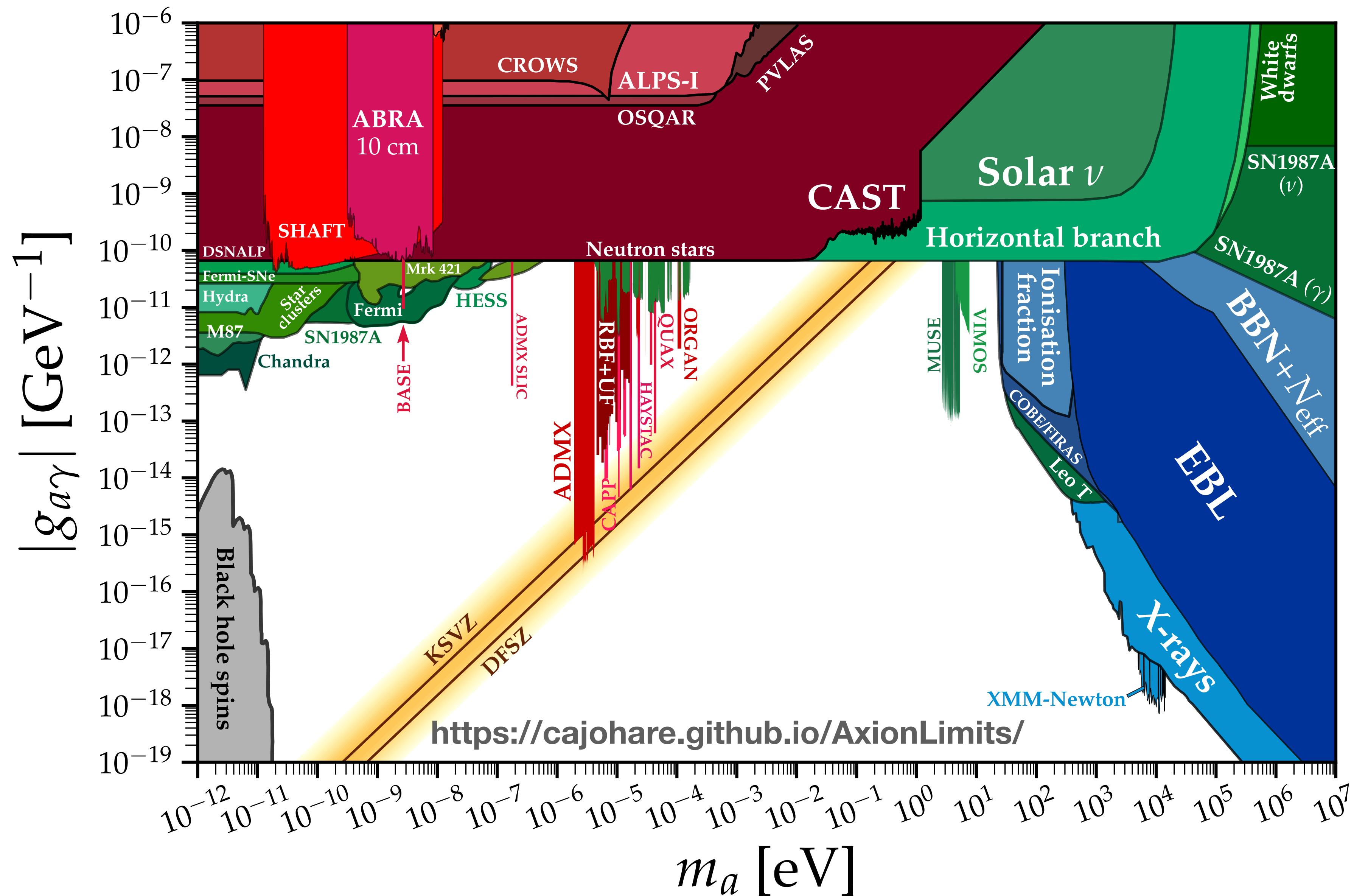
- ALP couplings:

$$g_{a\gamma\gamma} a F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \sim g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

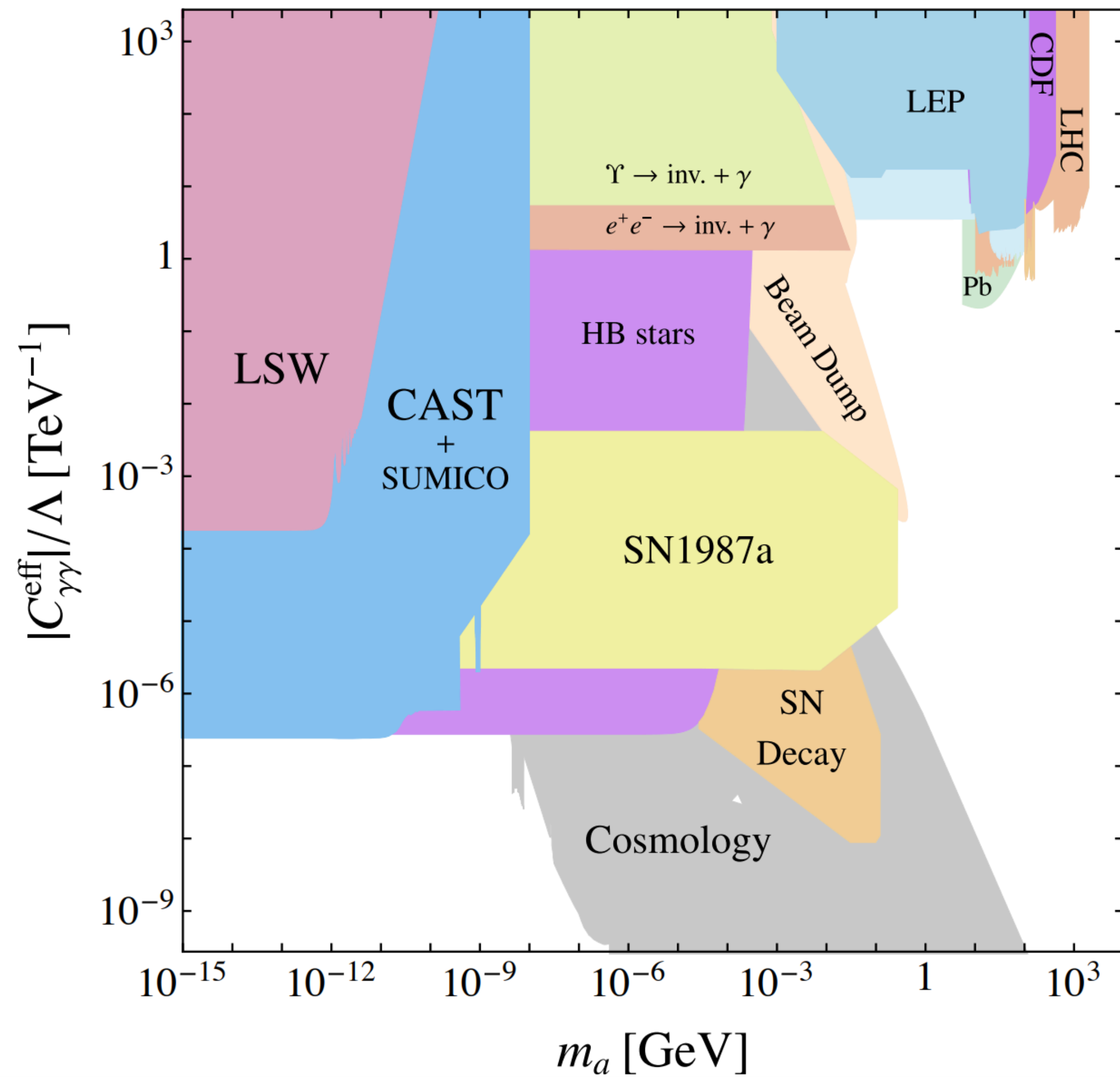


$$f = q, \ell, N$$

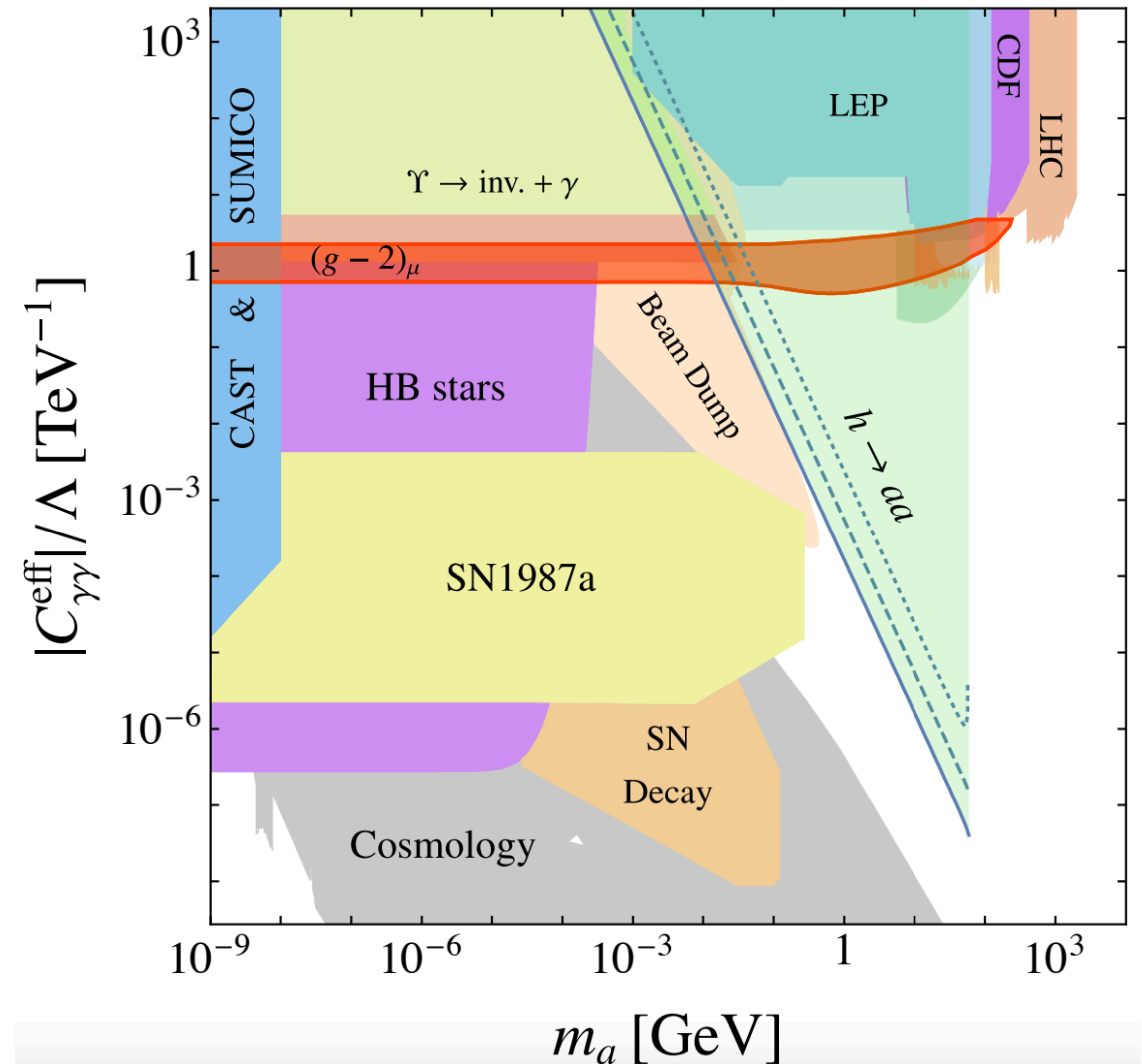
The current status for light/ultralight ALP photon couplings



The current status for heavy ALP photon couplings

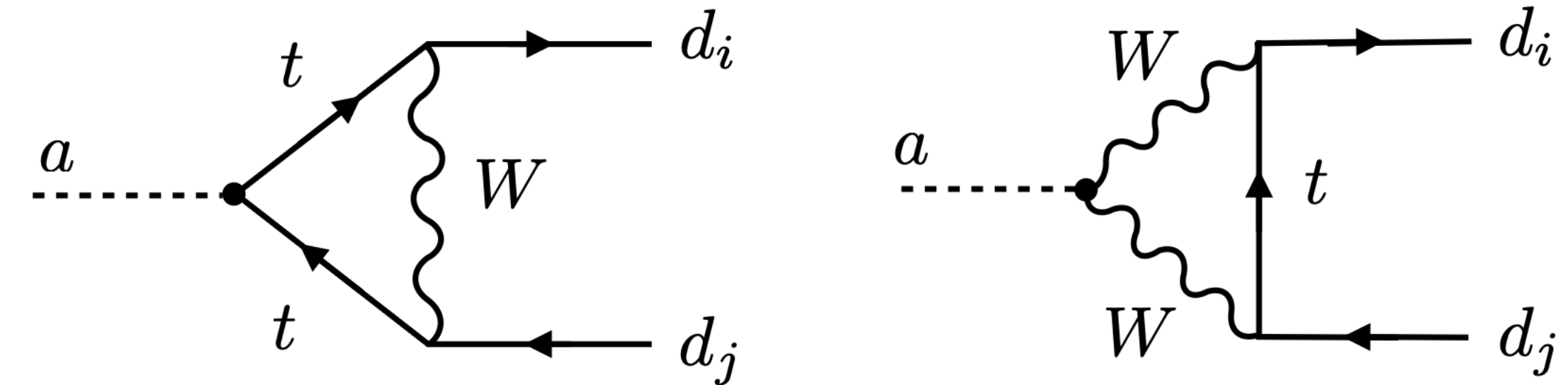
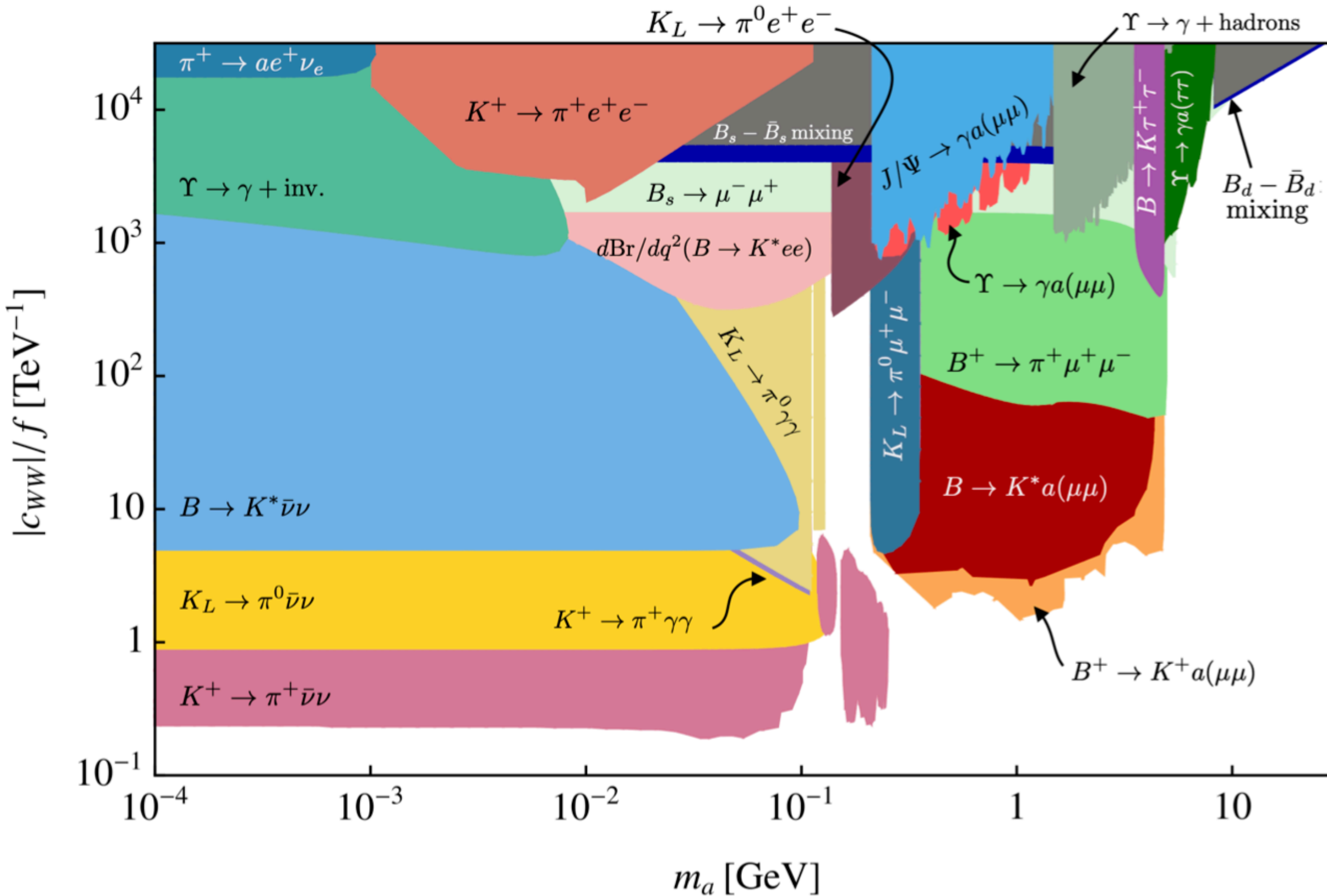


Bauer, Neubert, Thamm 1708.00443 [JHEP]



Similar for $h \rightarrow Za$

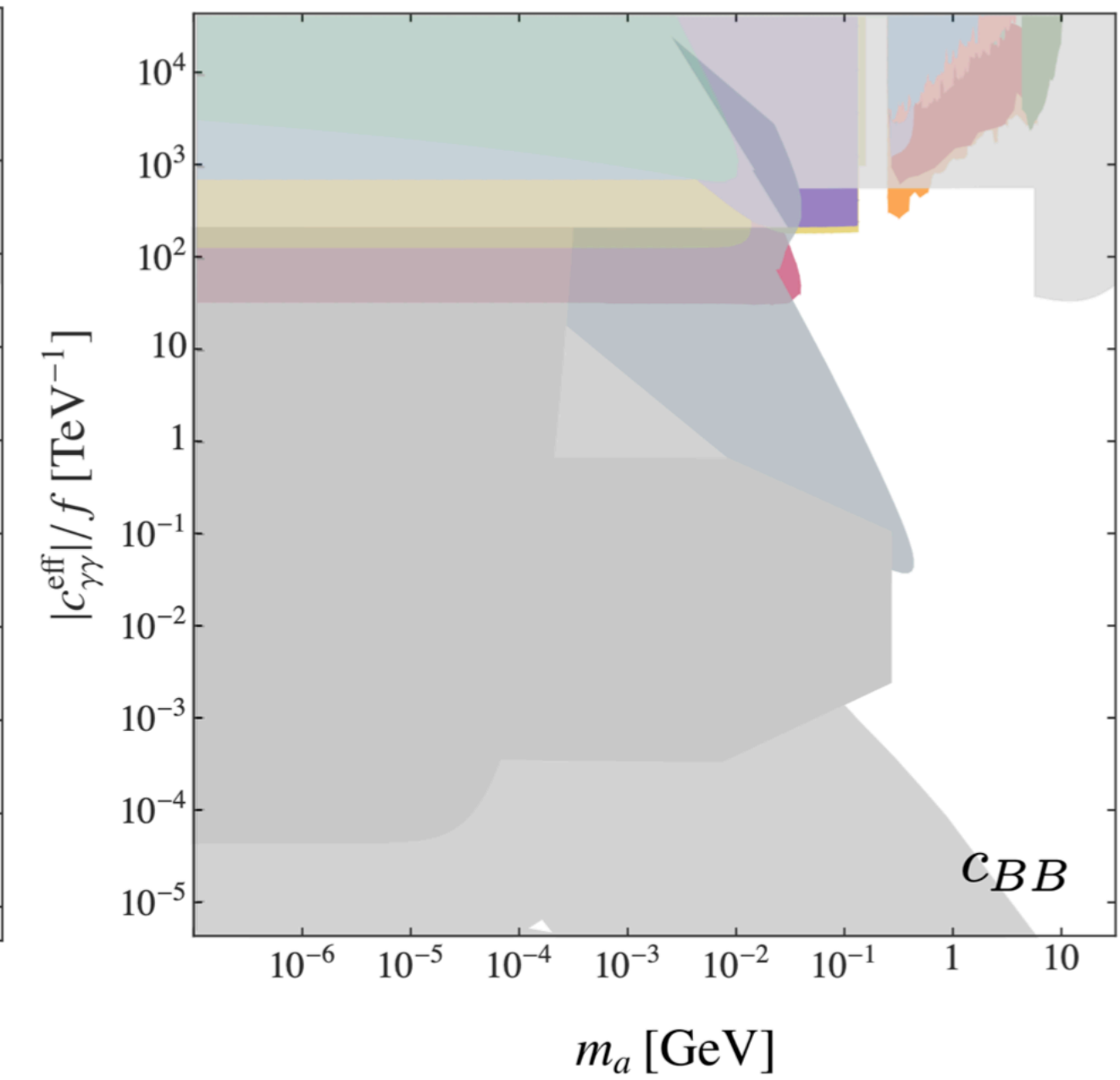
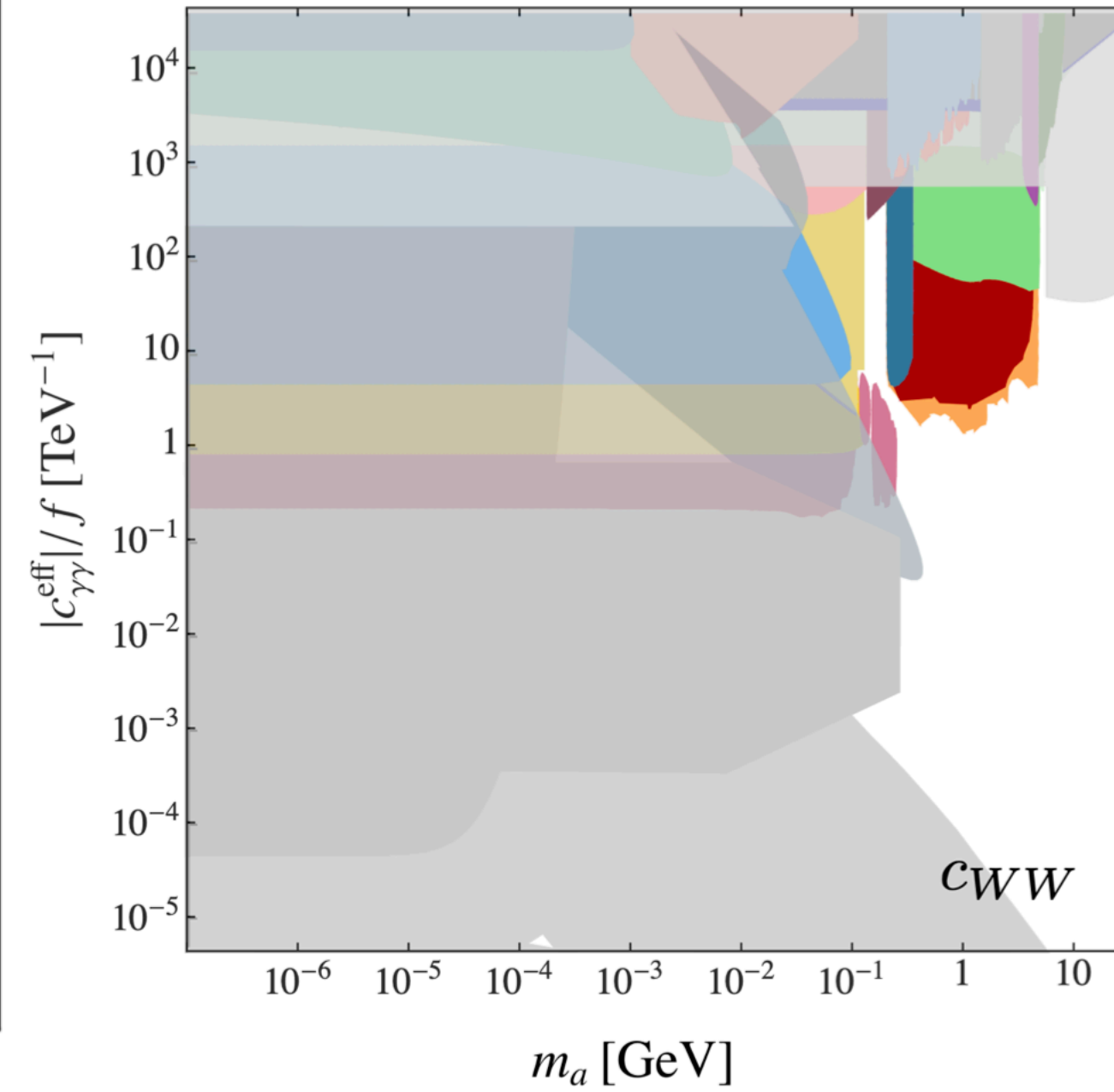
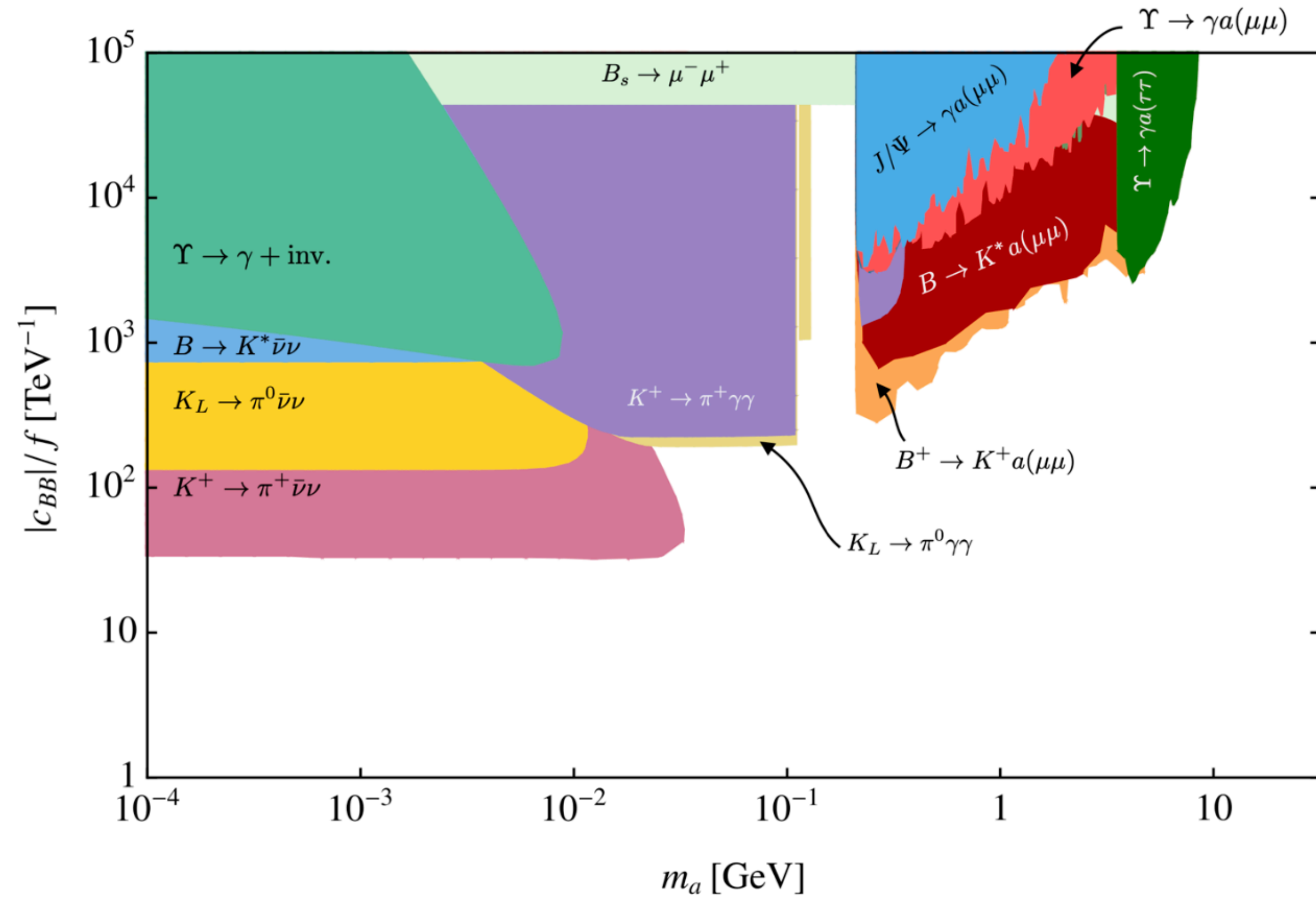
The RG evolution and flavor probe



Bauer, Neubert, Renner, Schnubel, Thamm, 2110.10698 [JHEP]

- ALP couplings to all other Wilson coefficients set to zero at $\Lambda = 4\pi f$, $f = 1$ TeV
- RG running will generate flavor violating Wilson coefficients

The RG evolution and flavor probe



Bauer, Neubert, Renner, Schnubel, Thamm, 2110.10698 [JHEP]

- ALP couplings to all other Wilson coefficients set to zero at $\Lambda = 4\pi f$, $f = 1$ TeV
- RG running will generate flavor violating Wilson coefficients

提纲

- 轴子类轴子模型
- 暗光子模型
- 中微子模型

暗光子模型

- SM direct-charged U(1), e.g. B-L

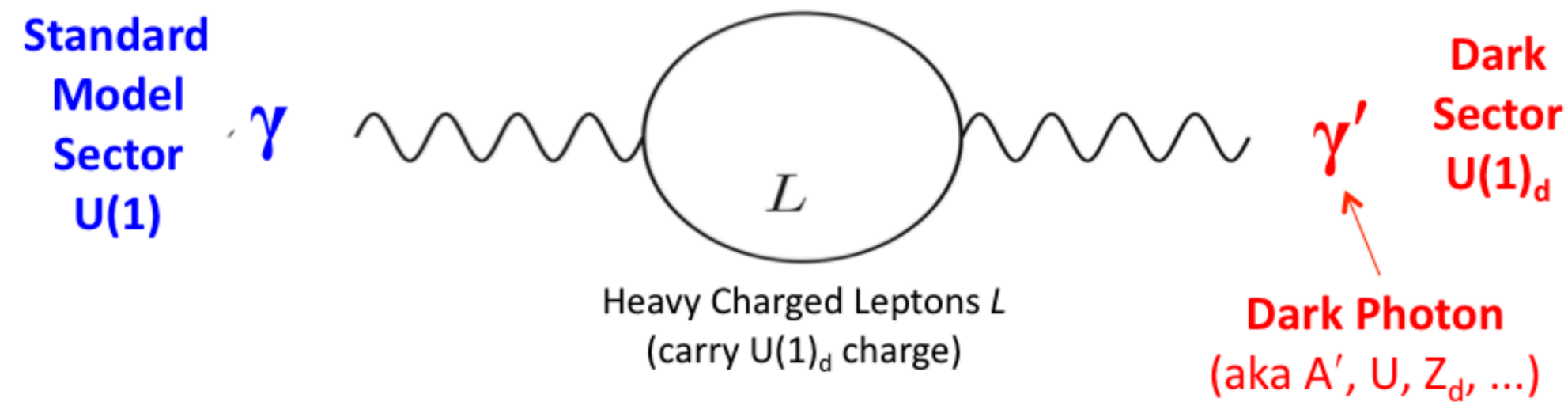
$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu - g' A'_\mu j_{B-L}^\mu$$

- 特点:
 - 具有质量，直接与SM 费米子有相互作用
 - $p + 1; e - 1; n + 1$
 - 电中性的原子核：带电数正比于中子数
 - 可以通过fifth force 来探测

暗光子模型

- Kinetic mixing dark photon

$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu - \frac{1}{2}\epsilon F_{\mu\nu}F'^{\mu\nu}$$



- 重新定义并正则化动量项

$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu - e\epsilon A'_\mu j_{\text{em}}^\mu$$

- 特点:

- 具有质量，与光子类似的微弱相互作用

- 电中性的原子核：不带电

动量混合的暗光子

- 在电弱破缺以前，与Hypercharge场混合

JL, Wang, Yu, 1704.00730 [JHEP]

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^iW^{i\mu\nu} - \frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{\epsilon}{2\cos\theta_W}B_{\mu\nu}K^{\mu\nu} \\ + |D_\mu H|^2 + |D_\mu \Phi|^2 + \mu_H^2|H|^2 - \lambda_H|H|^4 + \mu_D^2|\Phi|^2 - \lambda_D|\Phi|^4 - \lambda_{HP}|H|^2|\Phi|^2$$

- 在电弱破缺以后，质量项

$$\Phi = \frac{1}{\sqrt{2}}(v_D + \phi) , \\ H = \frac{1}{\sqrt{2}}(v_H + h) ,$$

$$\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} W^{\mu 3} & B^\mu & K^\mu \end{pmatrix} \begin{pmatrix} g^2 \frac{v_H^2}{4} & -g'g \frac{v_H^2}{4} & 0 \\ -g'g \frac{v_H^2}{4} & g'^2 \frac{v_H^2}{4} & 0 \\ 0 & 0 & g_D^2 v_D^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \\ K_\mu \end{pmatrix}$$

- 先进入标准模型的基

$$R_{\theta W} = \begin{pmatrix} c_W & s_W & 0 \\ -s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{L} \supset \frac{-1}{4} \begin{pmatrix} Z_{\text{SM}}^{\mu\nu} & A_{\text{SM}}^{\mu\nu} & K^{\mu\nu} \end{pmatrix} \begin{pmatrix} 1 & 0 & \epsilon t_W \\ 0 & 1 & -\epsilon \\ \epsilon t_W & -\epsilon & 1 \end{pmatrix} \begin{pmatrix} Z_{\mu\nu, \text{SM}} \\ A_{\mu\nu, \text{SM}} \\ K_{\mu\nu} \end{pmatrix} \\ + \frac{1}{2} \begin{pmatrix} Z_{\text{SM}}^\mu & A_{\text{SM}}^\mu & K^\mu \end{pmatrix} \begin{pmatrix} m_{Z, \text{SM}}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_K^2 \end{pmatrix} \begin{pmatrix} Z_{\mu, \text{SM}} \\ A_{\mu, \text{SM}} \\ K_\mu \end{pmatrix} ,$$

动量混合的暗光子

• 动量项正则化

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\epsilon^2 t_W & 1 & \epsilon \\ -\epsilon t_W & 0 & 1 \end{pmatrix}, \quad U_2 = \begin{pmatrix} \sqrt{\frac{1-\epsilon^2}{1-\epsilon^2 c_W^{-2}}} & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-\epsilon^3 t_W}{\sqrt{(1-\epsilon^2)(1-\epsilon^2 c_W^{-2})}} & 0 & \frac{1}{\sqrt{1-\epsilon^2}} \end{pmatrix}$$

• 质量项对角化

$$R_M = \begin{pmatrix} c_M & 0 & s_M \\ 0 & 1 & 0 \\ -s_M & 0 & c_M \end{pmatrix} \quad \tan \theta_M = \frac{1}{\beta \pm \sqrt{\beta^2 + 1}}$$

$$\beta \equiv \frac{m_{Z, \text{SM}}^2(1-\epsilon^2)^2 - m_K^2(1-\epsilon^2 c_W^{-2}(1+s_W^2))}{2m_K^2 \epsilon t_W \sqrt{1-\epsilon^2 c_W^{-2}}}$$

$$\begin{aligned} \mathcal{L} \supset & \frac{-1}{4} \begin{pmatrix} Z_{\text{SM}}^{\mu\nu} & A_{\text{SM}}^{\mu\nu} & K^{\mu\nu} \end{pmatrix} (U_1^T)^{-1} (U_2^T)^{-1} \mathbb{I}_3 U_2^{-1} U_1^{-1} \begin{pmatrix} Z_{\mu\nu, \text{SM}} \\ A_{\mu\nu, \text{SM}} \\ K_{\mu\nu} \end{pmatrix} \\ & + \frac{1}{2} \begin{pmatrix} Z_{\text{SM}}^\mu & A_{\text{SM}}^\mu & K^\mu \end{pmatrix} (U_1^T)^{-1} (U_2^T)^{-1} \begin{pmatrix} \frac{m_{Z, \text{SM}}^2(1-\epsilon^2)^2 + m_K^2 \epsilon^2 t_W^2}{(1-\epsilon^2)(1-\epsilon^2 c_W^{-2})} & 0 & \frac{-m_K^2 \epsilon t_W}{(1-\epsilon^2) \sqrt{1-\epsilon^2 c_W^{-2}}} \\ 0 & 0 & 0 \\ \frac{-m_K^2 \epsilon t_W}{(1-\epsilon^2) \sqrt{1-\epsilon^2 c_W^{-2}}} & 0 & \frac{m_K^2}{1-\epsilon^2} \end{pmatrix} \\ & \times U_2^{-1} U_1^{-1} \begin{pmatrix} Z_{\mu, \text{SM}} \\ A_{\mu, \text{SM}} \\ K_\mu \end{pmatrix}, \end{aligned} \quad (8)$$

$$\begin{pmatrix} \tilde{Z}_\mu \\ \tilde{A}_\mu \\ \tilde{K}_\mu \end{pmatrix} = R_M^T U_2^{-1} U_1^{-1} \begin{pmatrix} Z_{\mu, \text{SM}} \\ A_{\mu, \text{SM}} \\ K_\mu \end{pmatrix} = \begin{pmatrix} Z_{\mu, \text{SM}} - \frac{t_W m_K^2}{m_{Z, \text{SM}}^2 - m_K^2} \epsilon K_\mu - \frac{m_{Z, \text{SM}}^4 t_W^2}{2(m_{Z, \text{SM}}^2 - m_K^2)^2} \epsilon^2 Z_{\mu, \text{SM}} \\ A_{\mu, \text{SM}} - \epsilon K_\mu \\ K_\mu + \frac{t_W m_{Z, \text{SM}}^2}{m_{Z, \text{SM}}^2 - m_K^2} \epsilon Z_{\mu, \text{SM}} - \left(\frac{1}{2} + \frac{m_K^4 t_W^2}{2(m_{Z, \text{SM}}^2 - m_K^2)^2} \right) \epsilon^2 K_\mu \end{pmatrix}$$

动量混合的暗光子

- 最终结果 $\epsilon \ll 1$
 - 质量修正 $\mathcal{O}(\epsilon^2)$

$$m_{\tilde{K}}^2 = m_K^2 + \frac{m_K^2 c_W^{-2} \epsilon^2 (m_{Z, \text{SM}}^2 c_W^2 - m_K^2)}{m_{Z, \text{SM}}^2 - m_K^2}, \quad m_{\tilde{Z}}^2 = m_{Z, \text{SM}}^2 + \frac{m_{Z, \text{SM}}^4 t_W^2 \epsilon^2}{m_{Z, \text{SM}}^2 - m_K^2}$$

- 相互作用修正 $\mathcal{O}(\epsilon^1)$

$$\begin{aligned} \mathcal{L} &\supset g Z_{\mu, \text{SM}} J_Z^\mu + e A_{\mu, \text{SM}} J_{\text{em}}^\mu + g_D K_\mu J_D^\mu \\ &= \tilde{Z}_\mu \left(g J_Z^\mu - g_D \frac{m_{Z, \text{SM}}^2 t_W}{m_{Z, \text{SM}}^2 - m_K^2} \epsilon J_D^\mu + g \frac{m_{Z, \text{SM}}^2 (m_{Z, \text{SM}}^2 - 2m_K^2) t_W^2}{2(m_K^2 - m_{Z, \text{SM}}^2)^2} \epsilon^2 J_Z^\mu - e \frac{m_{Z, \text{SM}}^2 t_W}{m_{Z, \text{SM}}^2 - m_K^2} \epsilon^2 J_{\text{em}}^\mu \right) \\ &\quad + \tilde{K}_\mu \left(g_D J_D^\mu + g \frac{m_K^2 t_W}{m_{Z, \text{SM}}^2 - m_K^2} \epsilon J_Z^\mu + e \epsilon J_{\text{em}}^\mu + g_D \frac{(m_{Z, \text{SM}}^4 c_W^2 - 2m_K^2 m_{Z, \text{SM}}^2 + m_K^4) c_W^{-2}}{2(m_{Z, \text{SM}}^2 - m_K^2)^2} \epsilon^2 J_D^\mu \right) \\ &\quad + \tilde{A}_\mu e J_{\text{em}}^\mu. \end{aligned} \tag{15}$$

质量混合的暗光子

- 在电弱破缺以前，与Hypercharge场混合

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{\epsilon}{2\cos\theta_W}B_{\mu\nu}K^{\mu\nu} \times$$
$$+ |D_\mu H|^2 + |D_\mu \Phi|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \mu_D^2 |\Phi|^2 - \lambda_D |\Phi|^4 - \lambda_{HP} |H|^2 |\Phi|^2$$

- 在电弱破缺以后，质量项

$$\Phi = \frac{1}{\sqrt{2}}(v_D + \phi) ,$$
$$H = \frac{1}{\sqrt{2}}(v_H + h) ,$$

$$\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} W^{\mu 3} & B^\mu & K^\mu \end{pmatrix} \begin{pmatrix} g^2 \frac{v_H^2}{4} & -g' g \frac{v_H^2}{4} & 0 \\ -g' g \frac{v_H^2}{4} & g'^2 \frac{v_H^2}{4} & 0 \\ 0 & 0 & g_D^2 v_D^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \\ K_\mu \end{pmatrix}$$

通过非阿贝尔项混合的暗光子模型

- Non-abelian 暗光子模型

$$\mathcal{L} = -\frac{1}{4}K_{\mu\nu}^a K_{\mu\nu}^a + (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

$$\mathcal{L}_{\text{mix}} = \frac{1}{\Lambda^2} (\Phi^\dagger T^a \Phi) K_{\mu\nu}^a B_{\mu\nu}$$

- 自发破缺之后

$$\mathcal{L}_{\text{mix}} \supset \frac{\varepsilon}{2} \left(1 + \frac{\phi}{v_d}\right)^2 [\partial_\mu K_\nu^3 - \partial_\nu K_\mu^3 + g_d(K_\mu^1 K_\nu^2 - K_\mu^2 K_\nu^1)] \frac{1}{\cos \theta_w} B_{\mu\nu}$$

- 所需要转动

$$K_\mu^3 \rightarrow \frac{1}{\sqrt{1 - \frac{1}{4} \frac{\varepsilon^2}{\cos^2 \theta_w}}} K_\mu^3$$
$$B_\mu \rightarrow B_\mu - \frac{\varepsilon}{\cos \theta_w} \frac{1}{2\sqrt{1 - \frac{1}{4} \frac{\varepsilon^2}{\cos^2 \theta_w}}} K_\mu^3,$$

通过非阿贝尔项混合的暗光子模型

- 最终结果 $\epsilon \ll 1$

- 质量修正 $\mathcal{O}(\epsilon^2)$

$$m_{K_3}^2 = (m_k - \Delta)^2 = m_k^2 \left(1 + \frac{\epsilon^2}{\cos^2 \theta_w} \frac{(m_k^2 - \cos^2 \theta_w m_{Z,\text{SM}}^2)}{m_k^2 - m_{Z,\text{SM}}^2} \right)$$

- 相互作用修正 $\mathcal{O}(\epsilon^1)$

$$\mathcal{L} \supset K_3^\mu \left(\epsilon e J_{\text{em}}^\mu - \epsilon g \tan \theta_w \frac{m_k^2}{m_k^2 - m_Z^2} J_Z^\mu \right)$$

- 更多的 $\mathcal{O}(\epsilon^1)$ 相互作用

$$\mathcal{L}_{\text{mix}} \supset \frac{\epsilon}{2} \left(1 + \frac{\phi}{v_d} \right)^2 [\partial_\mu K_\nu^3 - \partial_\nu K_\mu^3 + g_d (K_\mu^1 K_\nu^2 - K_\mu^2 K_\nu^1)] \frac{1}{\cos \theta_w} B_{\mu\nu}$$

通过非阿贝尔项混合的暗光子模型

- 质量混合的模型

$$\mathcal{L} = -\frac{1}{4}K_{\mu\nu}^a K_{\mu\nu}^a + (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$

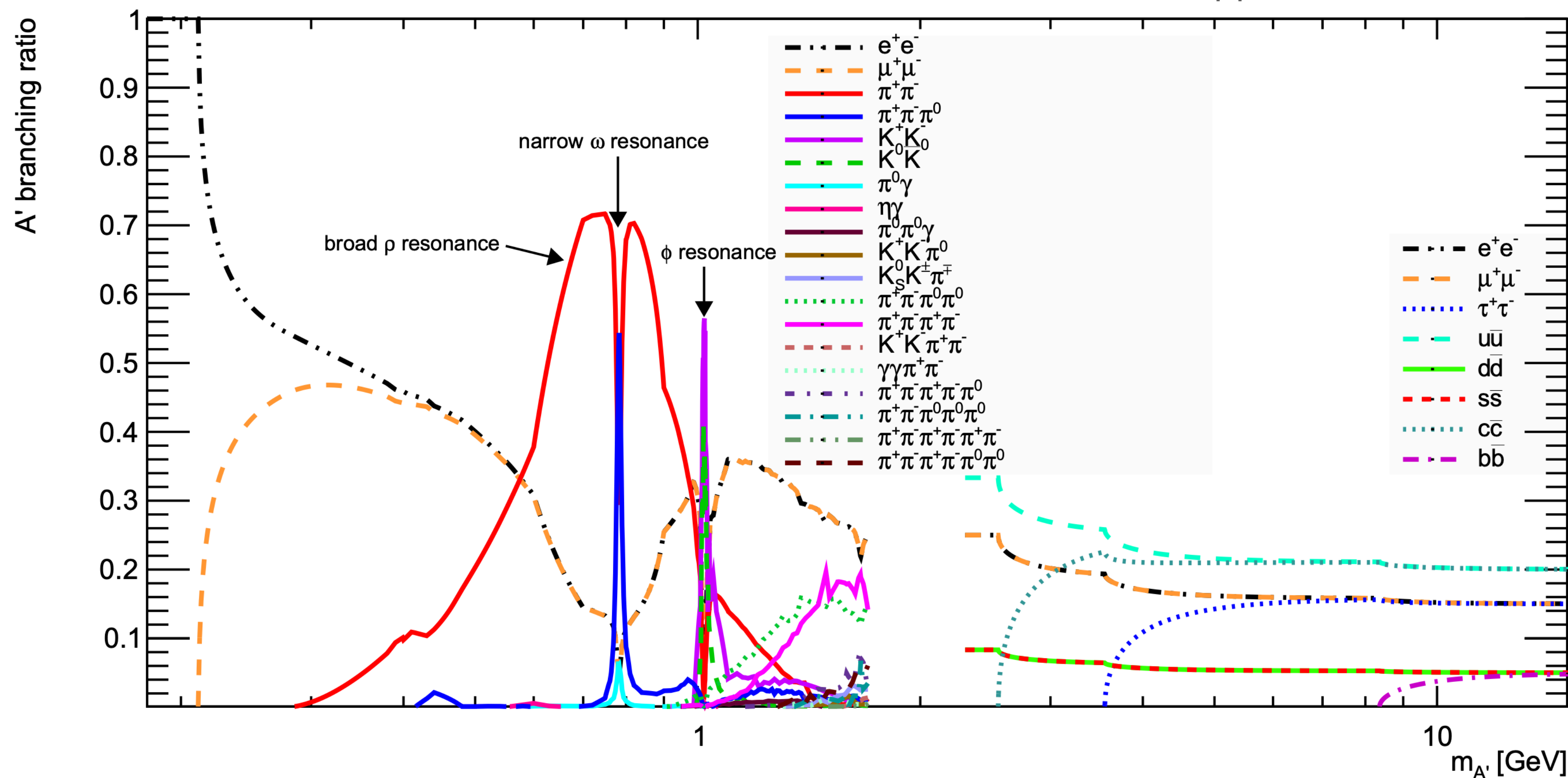
$$\mathcal{L} \supset \frac{(\Phi_D^\dagger D^\mu \Phi_D)(\Phi^\dagger D_\mu \Phi)}{\Lambda^2}$$

动量混合暗光子模型的实验限制

- 含有2个自由参数
- 可见衰变分支比

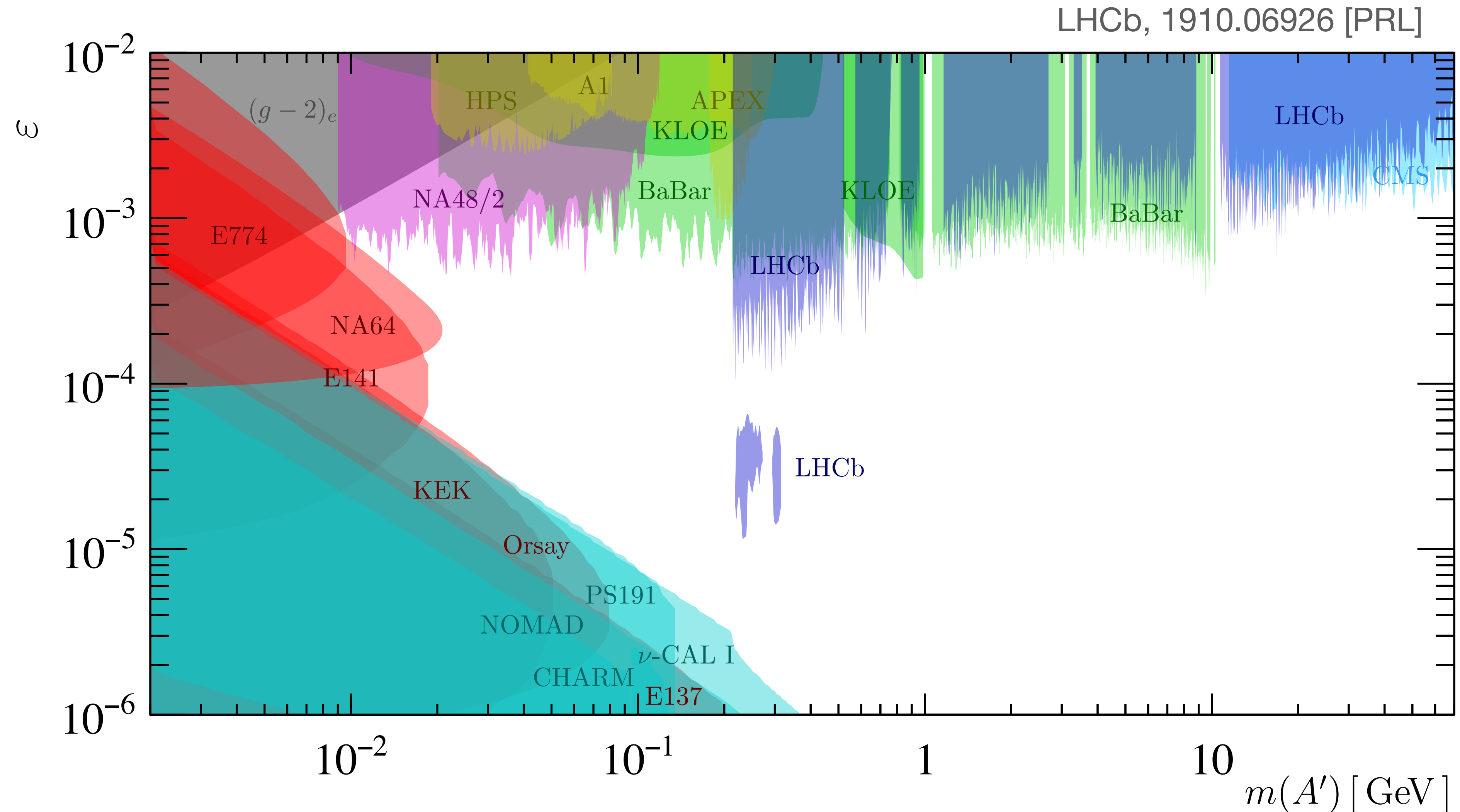
$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu - e\epsilon A'_\mu j_{\text{em}}^\mu$$

Buschmann, Kopp, JL, Machado, 1505.07459 [JHEP]



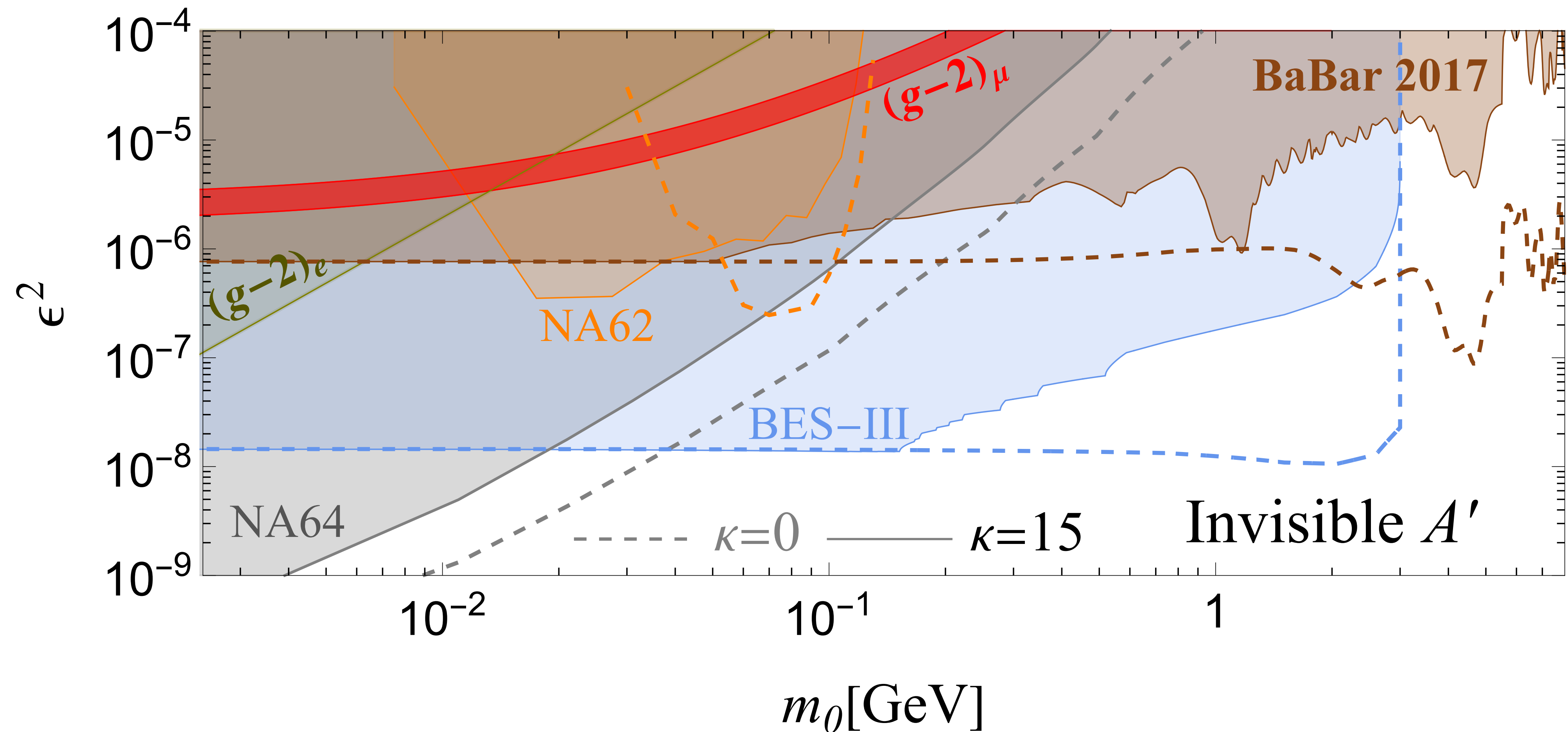
动量混合暗光子模型的实验限制

- 可见 A' 的限制



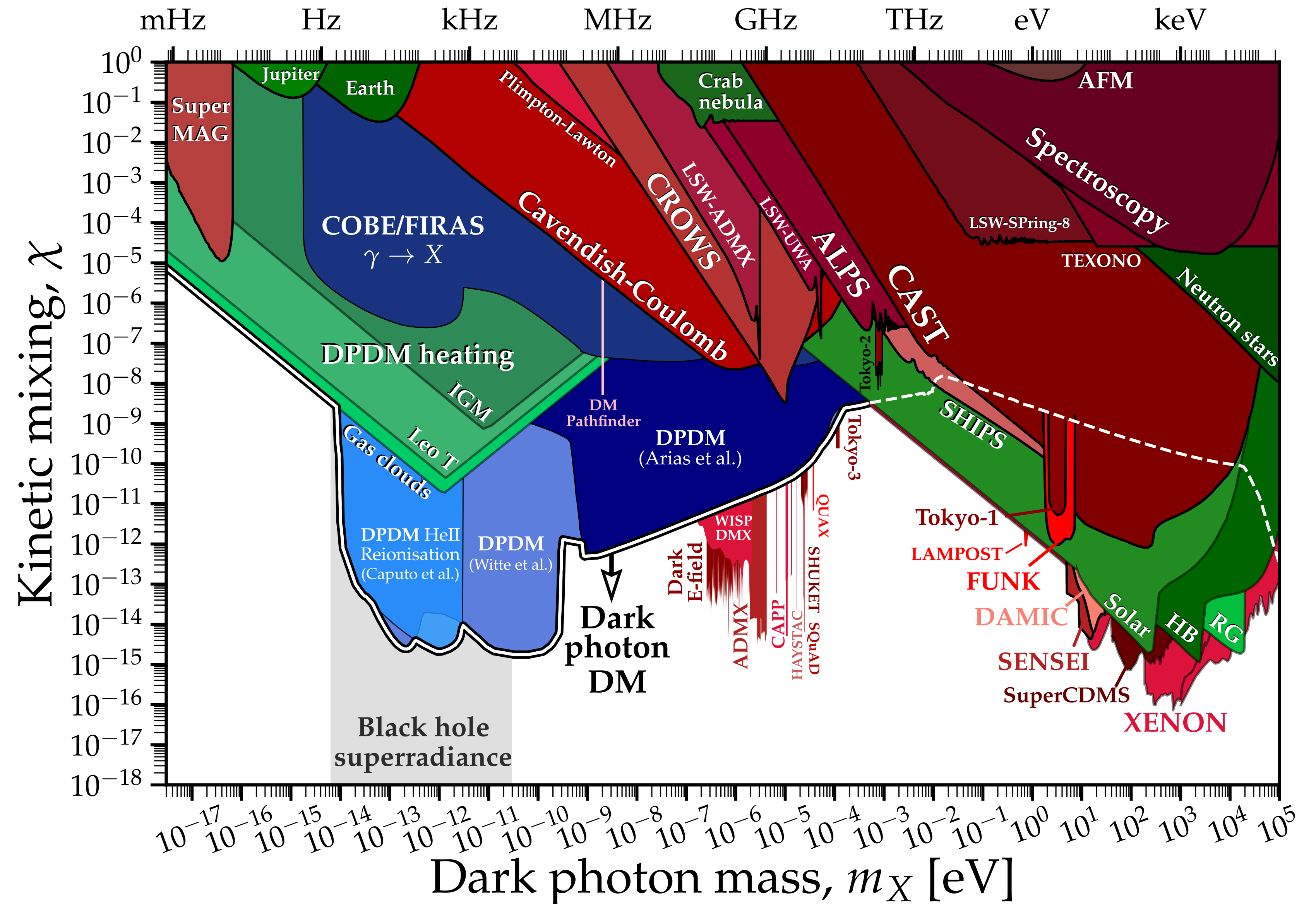
动量混合暗光子模型的实验限制

- 不可见 A' 的限制



低质量动量混合暗光子模型的实验限制

- A' mediator
- A' DM



超轻矢量玻色暗物质的残余丰度

- 超轻质量：Stuckelberg mechanism

$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{2}\partial^\mu A^\nu \partial_\mu A_\nu + \frac{1}{2}m^2 A^\mu A_\mu + \frac{1}{2}\partial^\mu B \partial_\mu B - \frac{1}{2}m^2 B^2$$

$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 \left(A^\mu - \frac{1}{m}\partial^\mu B \right)^2 - \frac{1}{2}(\partial_\mu A^\mu + m B)^2$$

- 规范变换

$$\delta A_\mu(x) = \partial_\mu \Lambda(x)$$

$$\delta B(x) = m \Lambda(x)$$

- 等效于decoupled Higgs and large λ
- Stuckelberg case occurs naturally in large volume string compactifications

- 残余丰度：

- Non-minimal enhanced modified Misalignment

- Homogeneous solution $\partial_i X^\mu = 0 \rightarrow X_0 = 0$

- Invariant term $X^\mu X_\mu = -a^{-2}(t)X_i X_i \equiv -\bar{X}_i \bar{X}_i$

- Vector field cosmological evolution

$$\ddot{\bar{X}}_i + 3H\dot{\bar{X}}_i + \left(m_{\gamma'}^2 + (1 - \kappa)(\dot{H} + 2H^2) \right) \bar{X}_i = 0$$

- The energy density

$$\rho(t) = T_0^0 = \frac{1}{2} \left(\dot{\bar{X}}_i \dot{\bar{X}}_i + m_{\gamma'}^2 \bar{X}_i \bar{X}_i + (1 - \kappa)H^2 \bar{X}_i \bar{X}_i + 2(1 - \kappa)H\dot{\bar{X}}_i \bar{X}_i \right)$$

- non-minimal coupling to gravity

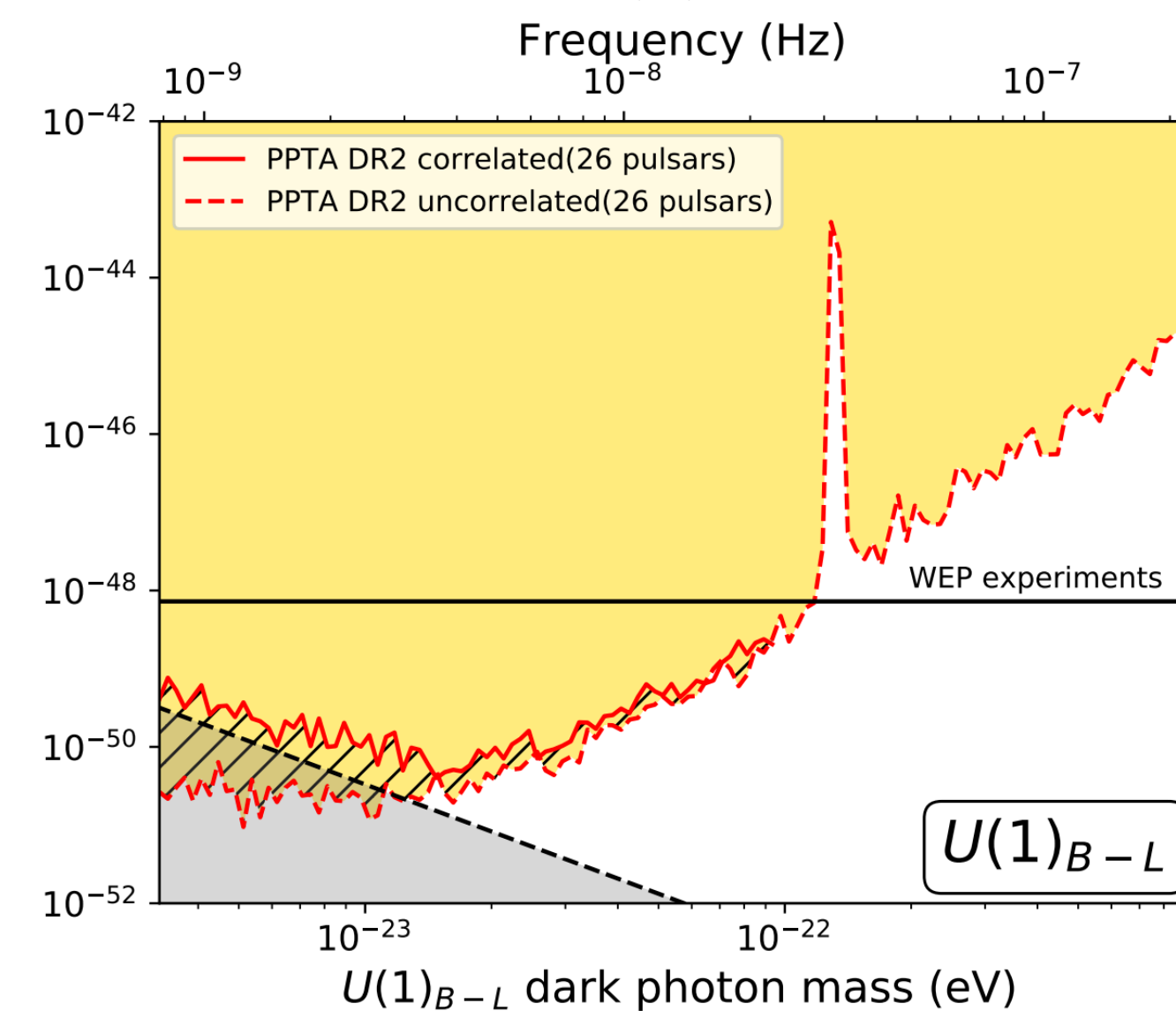
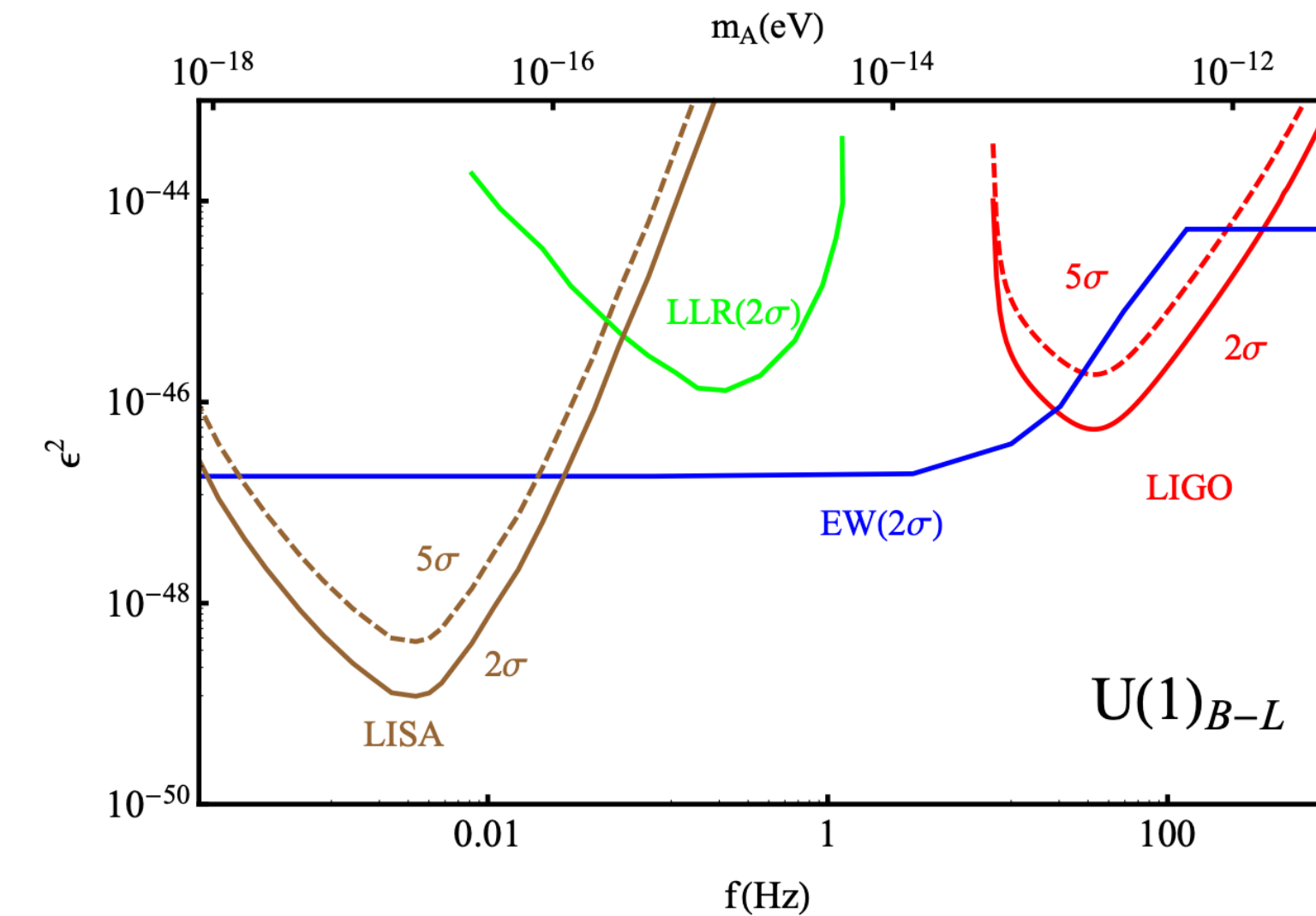
- $\frac{\kappa}{12} R X_\mu X^\mu$, with $\kappa = 1$ back to scalar case

- 其它机制：inflationary fluctuations, parametric resonances, cosmic strings

$U(1)_{B-L}$ 暗光子暗物质

Yue Zhao etc, 1801.10161 (PRL)

- 极小质量：长程相互作用
- 宏观物体：具有极大电荷
- 观测宏观物体的反常位移
- 第5种力
- LIGO 引力波探测器的镜子
- 行星之间的位移
- Pulsar Timing
- Gaia ...
- 超轻暗标量粒子暗物质也部分适用



提纲

- 轴子类轴子模型
- 暗光子模型
- 中微子模型

翘翘板模型 Type-I

- 增加Sterile Neutrino

[Minkowski, 1977]

$$\Delta\mathcal{L}_\nu = -\lambda_\nu \bar{L} \tilde{H} N - \frac{m_N}{2} \bar{N}^c N + h.c.$$

- 质量矩阵

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \quad \sin \theta = m_D / m_N$$

- 质量本征值

$$m_\nu \equiv m_1 \simeq \frac{m_D^2}{m_N}, \quad m_2 \simeq m_N + \frac{m_D^2}{m_N} \simeq m_N$$

- 限制条件

$$\sin^2 \theta \simeq \frac{m_\nu}{m_N}$$

- 规范相互作用

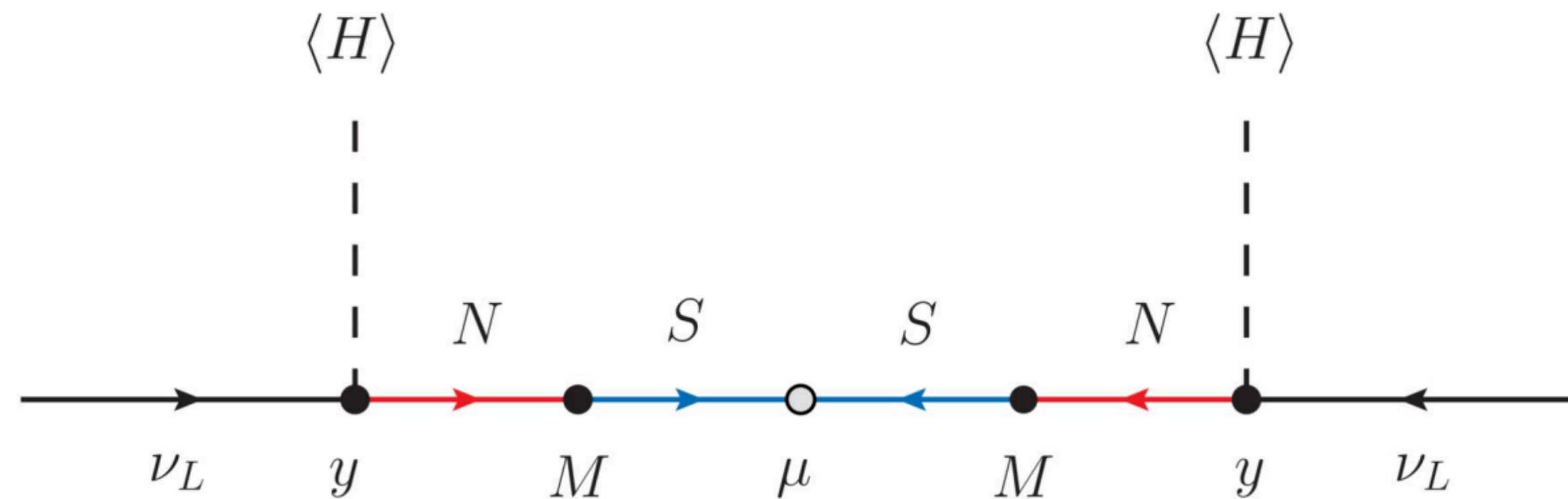
$$\mathcal{L} = \frac{g \sin \theta}{\sqrt{2}} (W_\mu \bar{\ell}_L \gamma^\mu N + h.c.) - \frac{g \cos \theta \sin \theta}{2 \cos \theta_w} Z_\mu (\bar{\nu}_L \gamma^\mu N + \bar{N} \gamma^\mu \nu_L) + \frac{g \sin^2 \theta}{2 \cos \theta_w} Z_\mu \bar{N} \gamma^\mu P_L N$$

翘翘板模型 Type-I: inverse seesaw

- 额外增加Sterile Neutrino

$$\mathcal{L} = y\bar{L}NH + M\bar{N}S + \frac{1}{2}\mu\bar{S}^cS + h.c.$$

[Mohapatra, Valle, 1986]



$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

- 混合角 ($\mu \ll m_D \ll M$)

$$\sin^2 \theta = \frac{m_\nu}{\mu} \quad (\text{inverse seesaw})$$

- 质量

$$m_\nu = \mu \left(\frac{m_D}{m_N} \right)^2$$

翘翘板模型 Type-I: linear seesaw

- 额外增加Sterile Neutrino (lepton number?)

$$\mathcal{L}_Y = Y_D \bar{L} H^c N + Y_\epsilon \bar{L} H^c S + M_R \bar{N}^c S + \text{h.c.}$$

$$M_\nu = \begin{pmatrix} 0 & m_D & M_\epsilon \\ m_D^T & 0 & M_R \\ M_\epsilon^T & M_R^T & 0 \end{pmatrix}$$

- 混合角

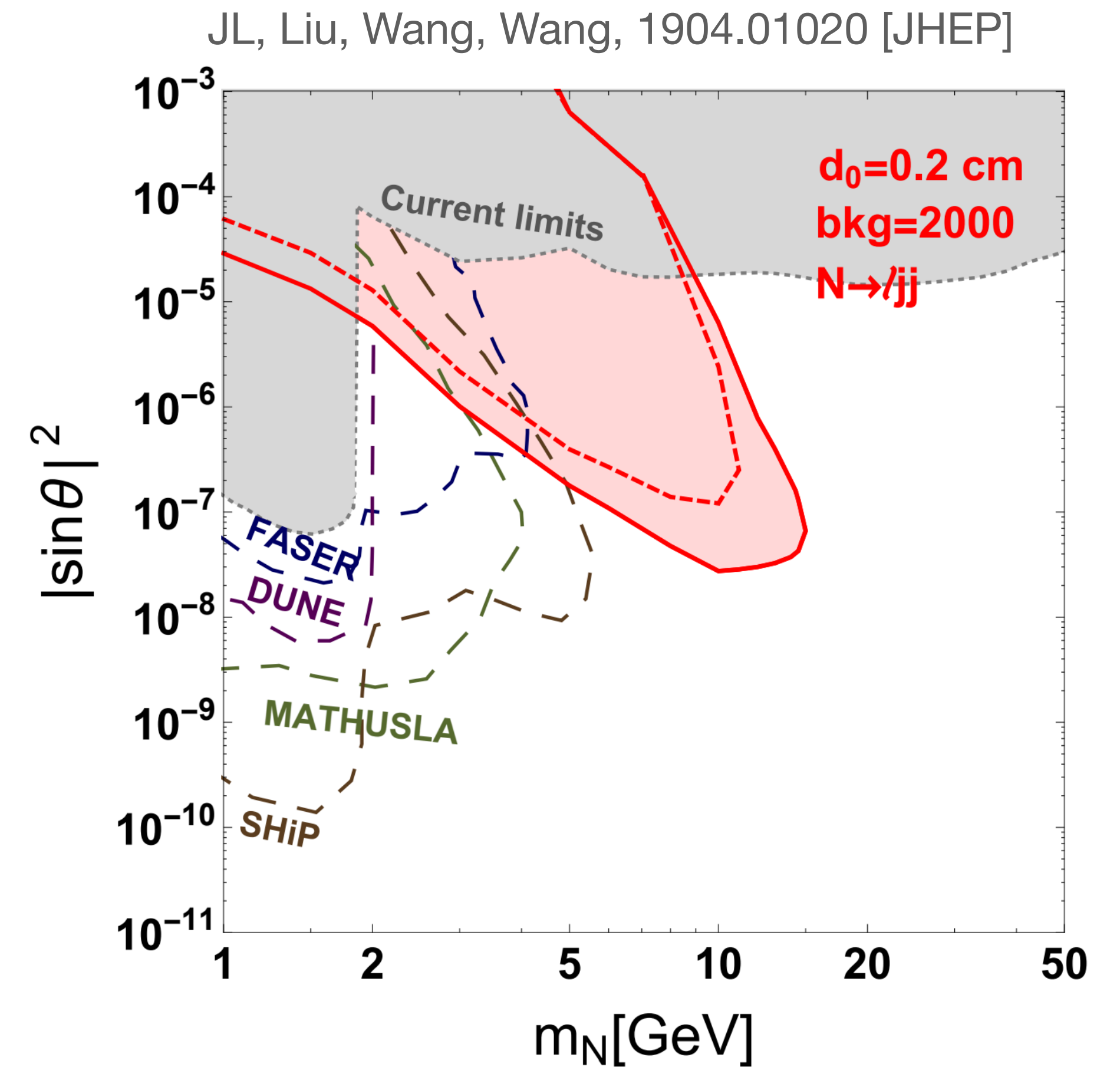
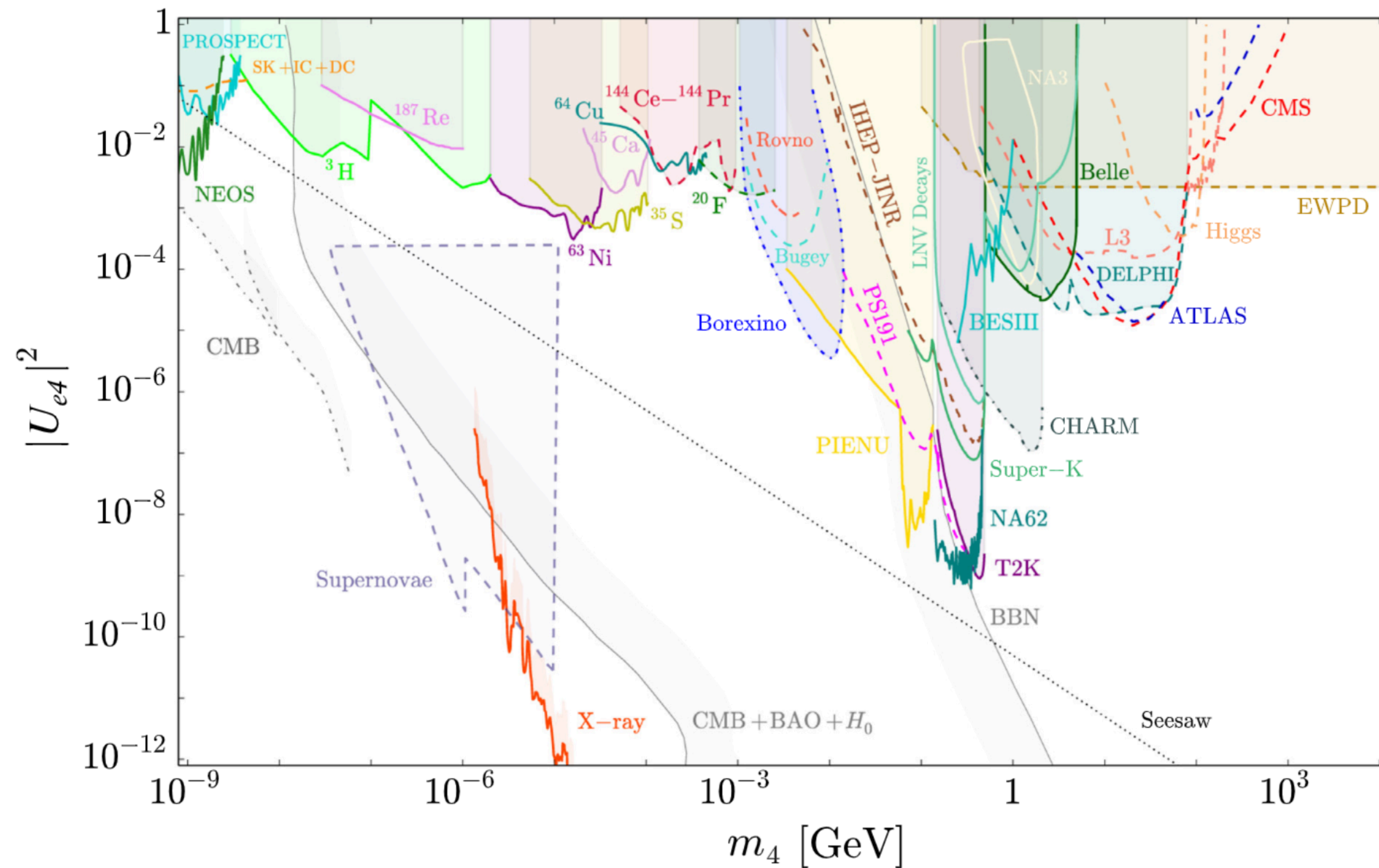
$$\sin \theta = \frac{m_\nu}{m_\psi} \quad (\text{linear seesaw})$$

- 质量

$$m_\nu = m_D M_R^{-1} M_\epsilon^T + M_\epsilon M_R^{T^{-1}} m_D^T$$

翘翘板模型 Type-I

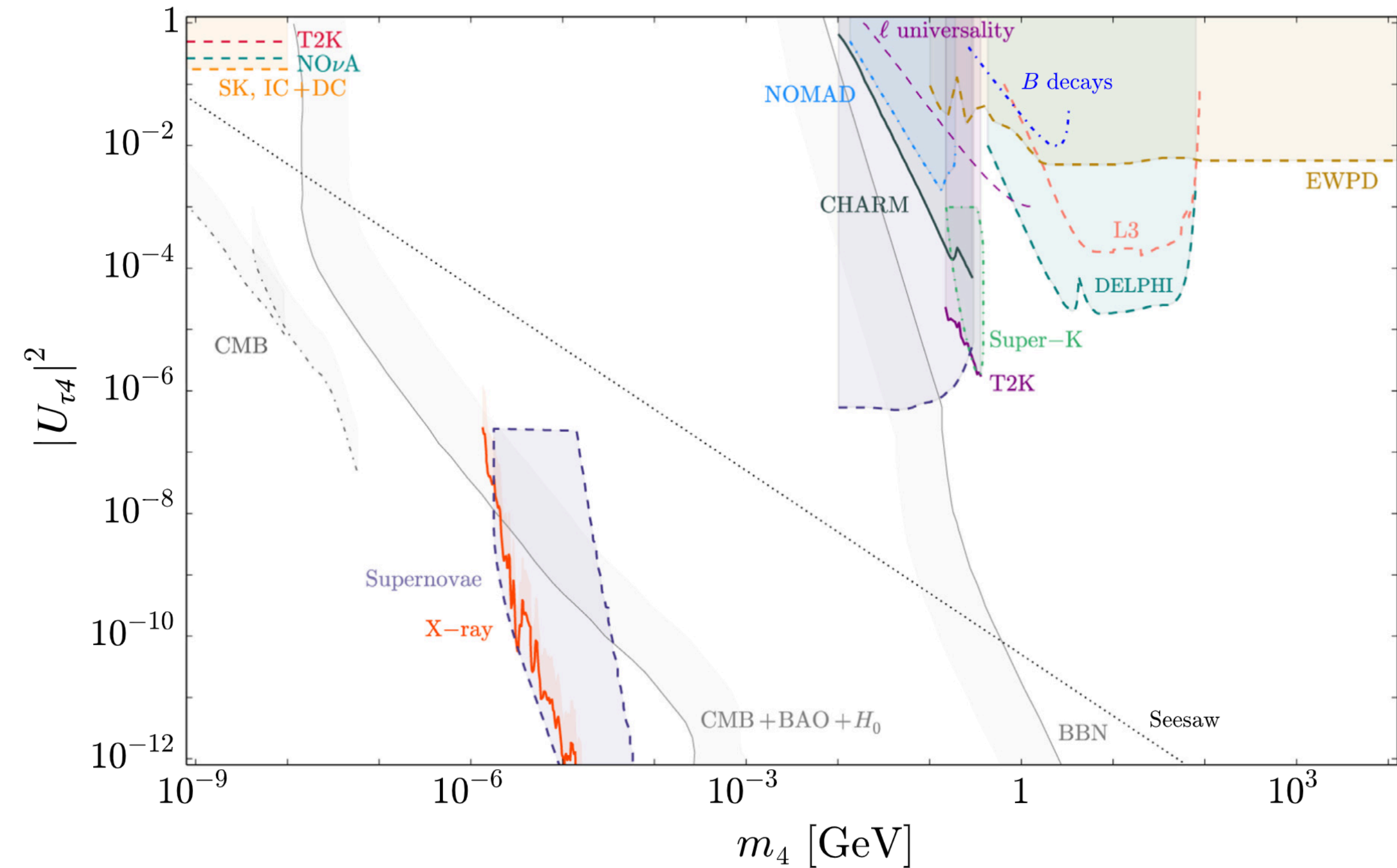
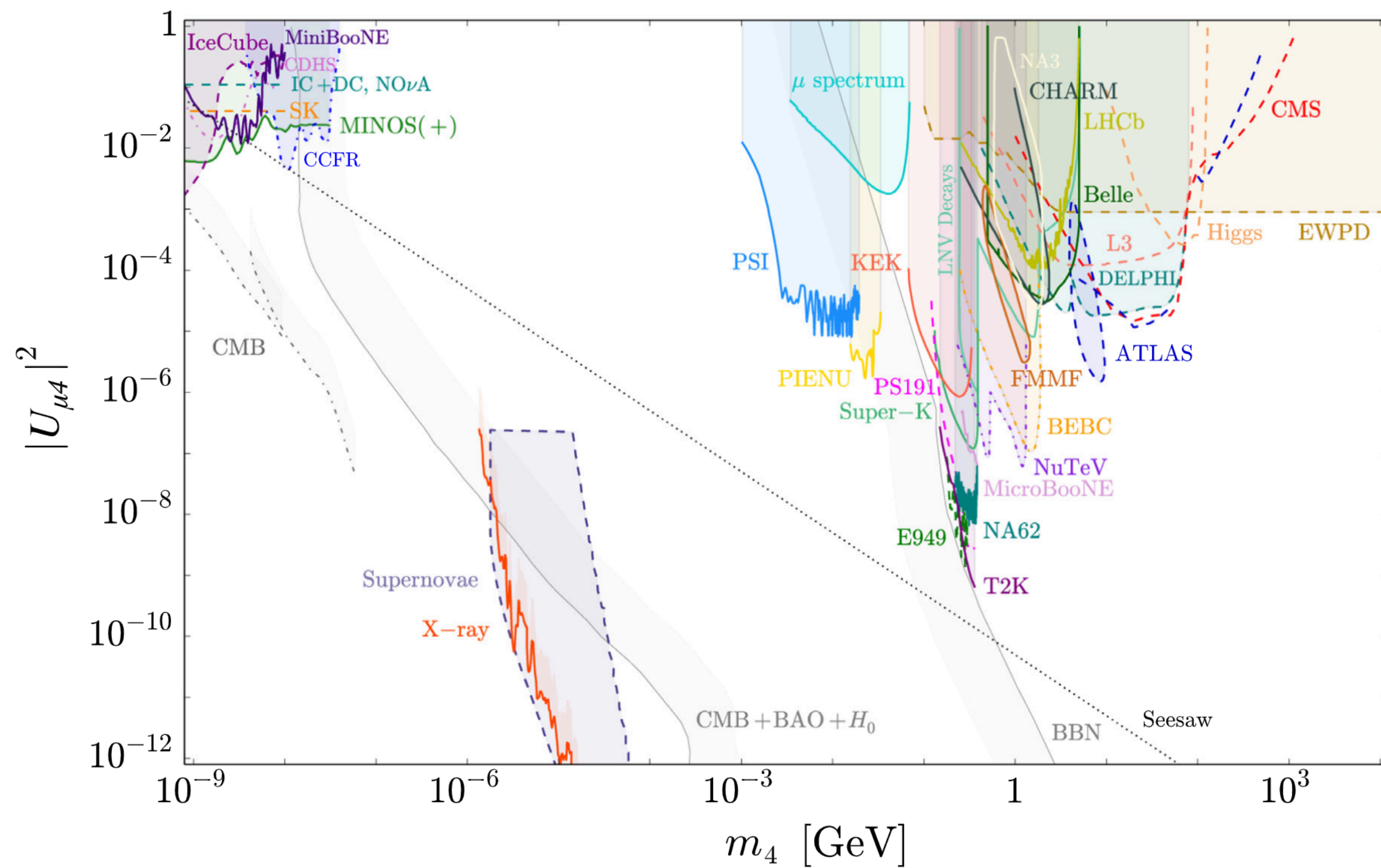
- 两个自由参数模式 $\{m_N, \sin \theta\}$



翘翘板模型 Type-I

- 两个自由参数模式 $\{m_N, \sin \theta\}$

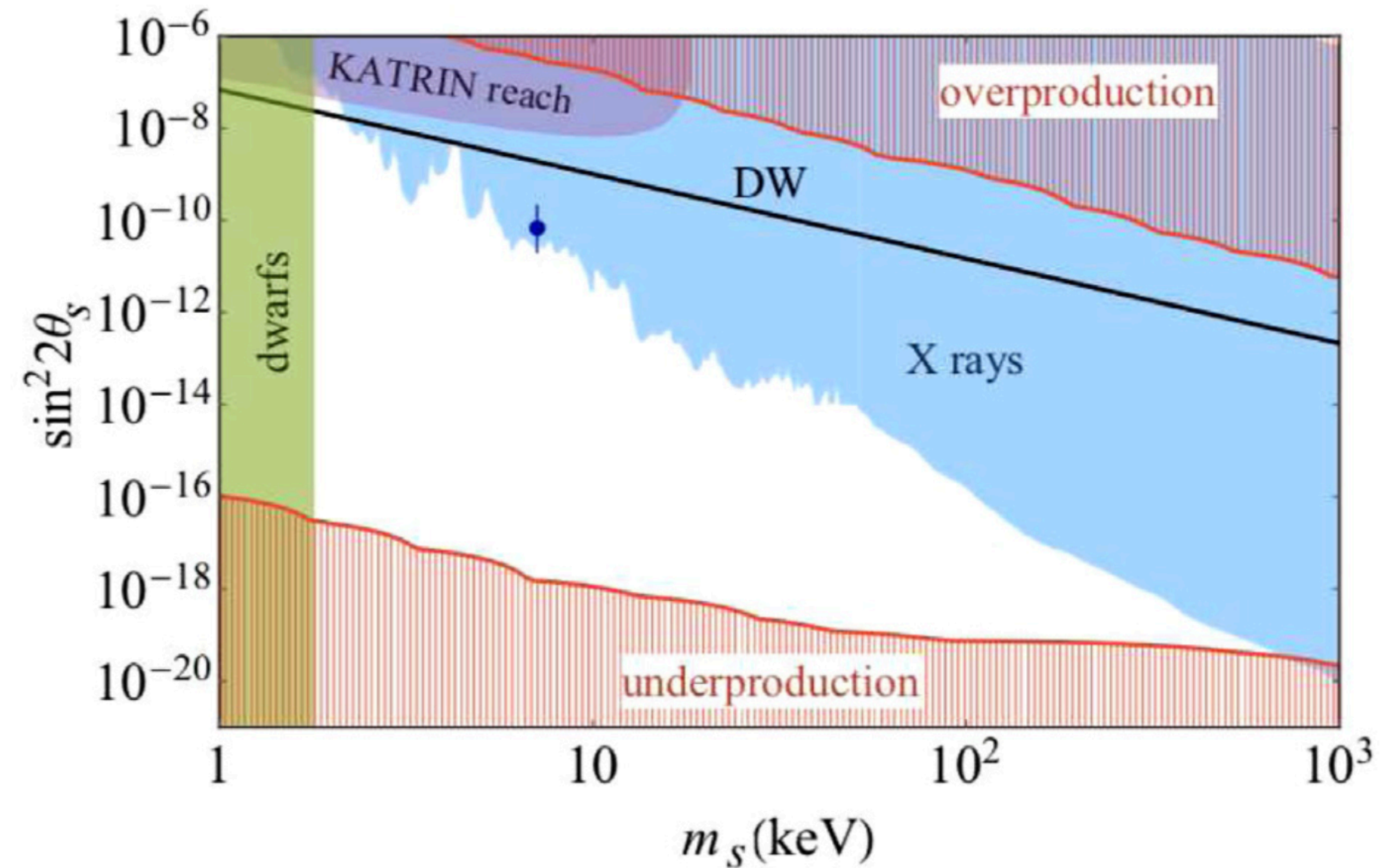
Dasgupta, Kopp, Physics Report 2021



翘翘板模型 Type-I

- Sterile neutrino DM
- Shi-Fuller mechanism

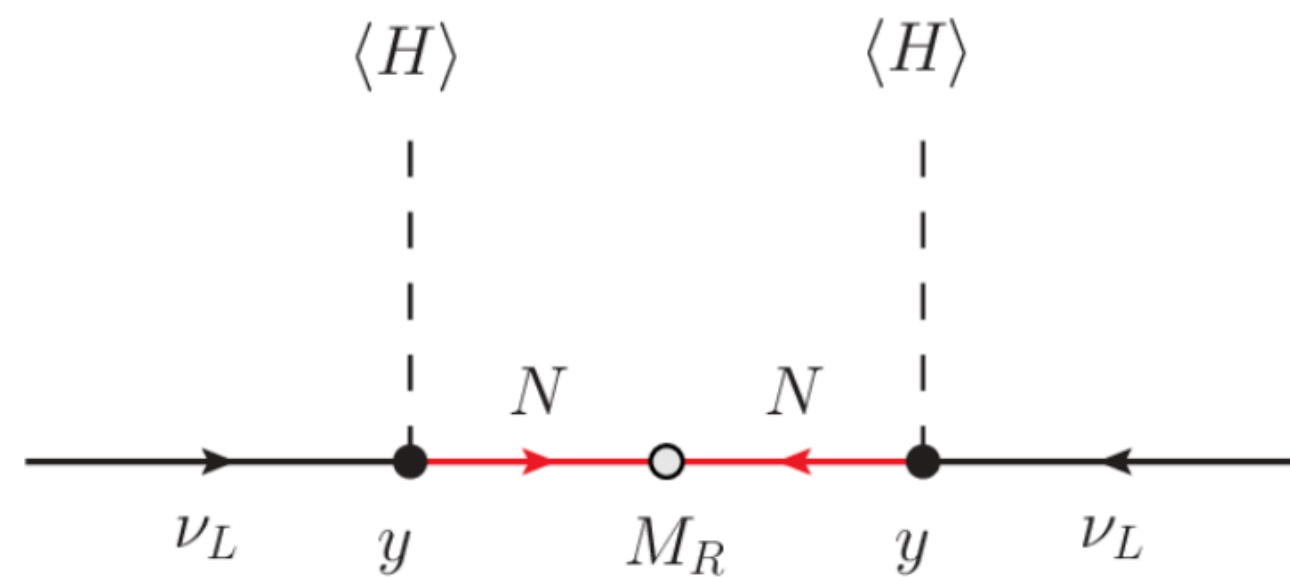
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\left(\cos 2\theta + \frac{2E_\nu}{\Delta m^2} V_{\text{eff}}\right)^2 + \sin^2 2\theta}$$



翘翘板模型 其它

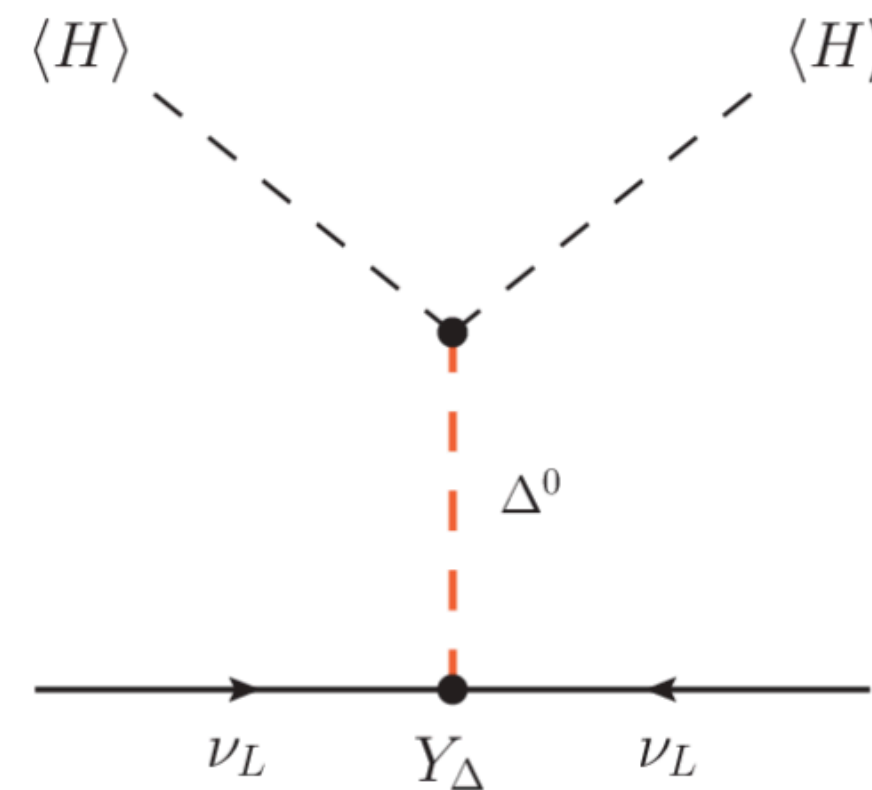
- Type-I, II, III

Type-I



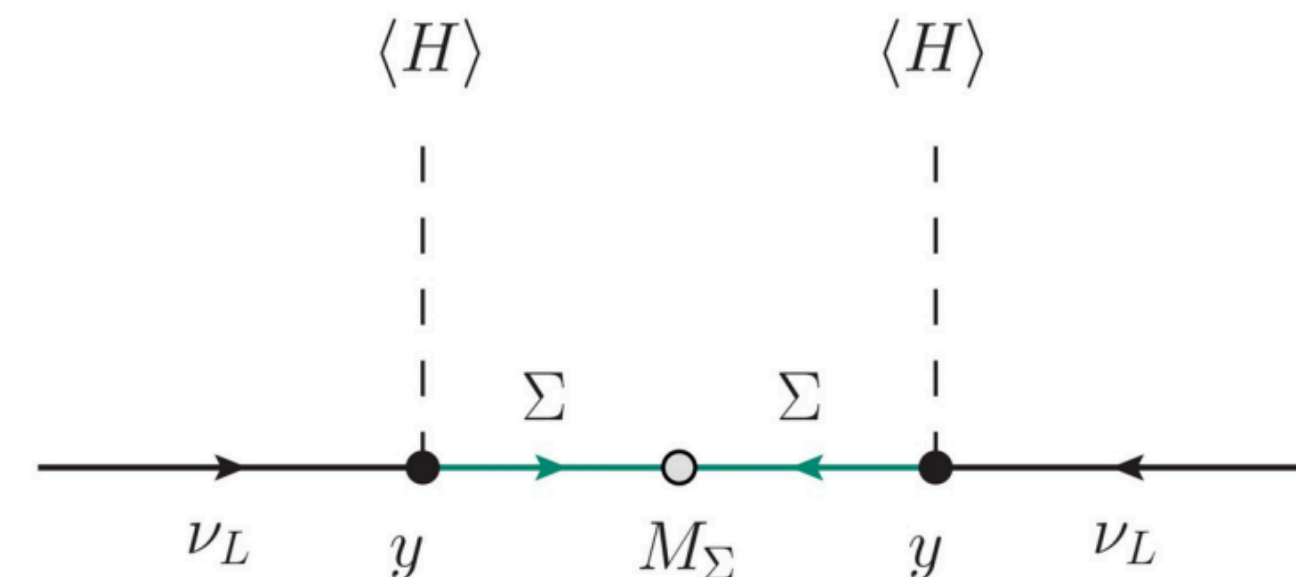
[Minkowski, Yanagida,
Mohapatra, Senjanovic,
Gell-Mann, Ramond, Slansky,
Schechter, Valle]

Type-II



[Schechter, Valle,
Mohapatra, Senjanovic]

Type-III



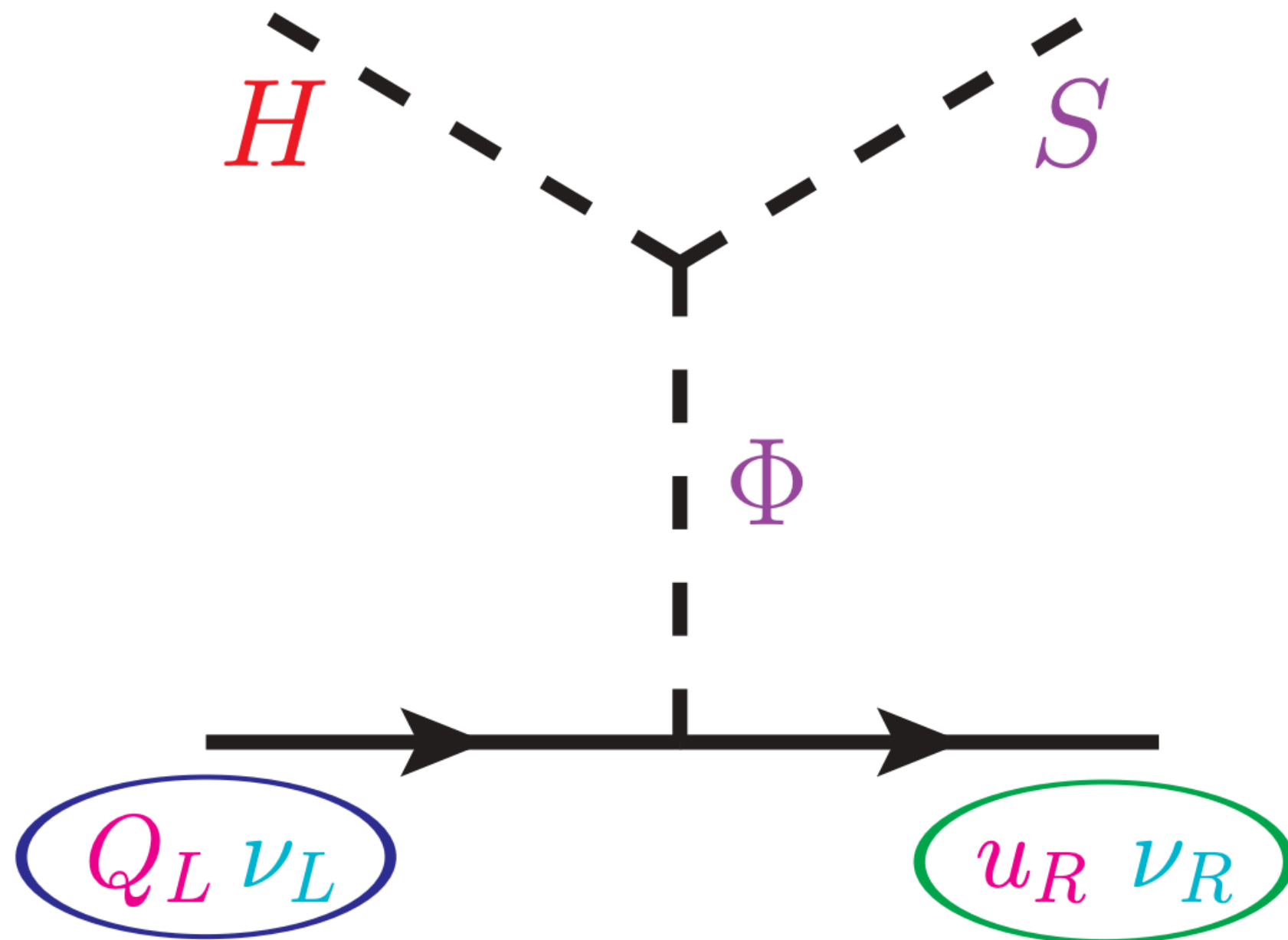
[Foot, Lew, He, Joshi]

Courtesy of Avelino Vicente

Dirac翘翘板模型与强CP问题

P.H. Gu, H.J. He JCAP 2006

- Dirac Seesaw



- Strong CP solution with small up mass

$$\text{Im}[m_u m_d m_s \exp(i\theta)] = |m_u m_d m_s| \exp(i\theta_{\text{QCD}})$$

$$\theta_{\text{QCD}} = \arg[m_u]$$

$$\text{Im}[m_u(1\text{GeV})] < 10^{-3}\text{eV}$$

- 具体参数

$$m_u = m_u^H + m_u^{\text{inst}}$$

$$|m_u^H| \ll |m_u| \simeq |m_u^{\text{inst}}|$$

$$\theta_{\text{QCD}} \simeq \sin \theta^H \frac{|m_u^H|}{|m_u|} (1\text{GeV})$$

$$|m_u^H|(1\text{ GeV}) \sin \theta^H < 6.5 \times 10^{-4}\text{eV}$$

Dirac翘翘板模型与强CP问题

- UV 模型构造

$$\mathcal{L} = Y_\nu \bar{L}_L \bar{\Phi} \nu_R + Y_u \bar{Q}_L \bar{\Phi} u_R + h.c., \quad \bar{\Phi} = i\sigma_2 \phi^*$$

$$V = m_\Phi^2 \Phi^\dagger \Phi + (\rho S H^\dagger \Phi + h.c.) + \dots$$

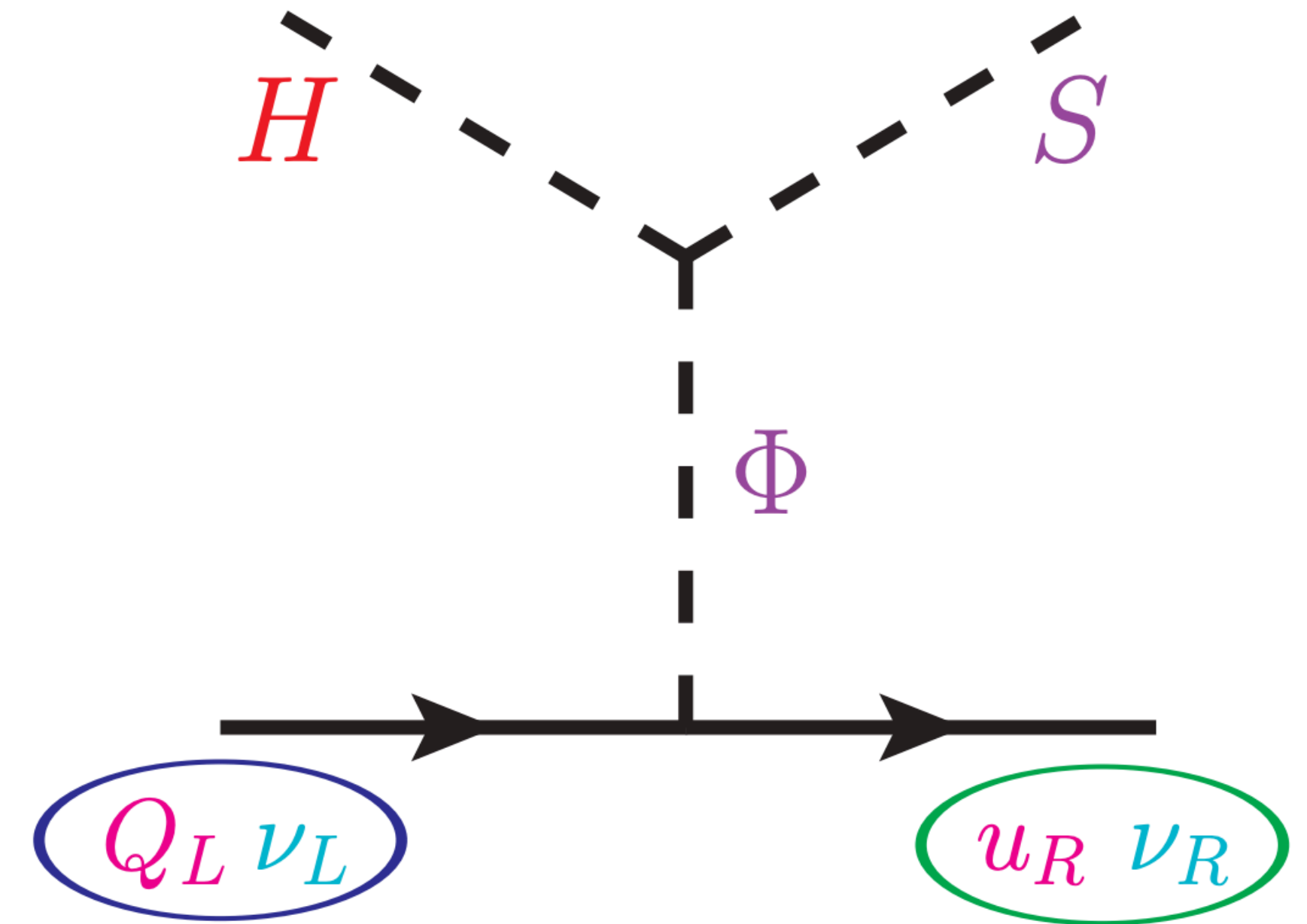
- 质量项

$$m_\nu \simeq Y_\nu \frac{\rho v_s v}{2 m_\Phi^2}, \quad m_u^H \simeq Y_u \frac{\rho v_s v}{2 m_\Phi^2}$$

- 超重的希格斯粒子

$$m_\Phi \simeq 6 \times 10^{12} \text{GeV} \left(\frac{Y_\nu}{0.1} \right) \left(\frac{\rho}{0.1 m_\Phi} \right) \left(\frac{v_s}{v} \right) \left(\frac{0.05 \text{eV}}{m_\nu} \right)$$

- Yukawa关系 $Y_u(M_Z) < 0.1 Y_\nu \left(\frac{0.1}{\sin \theta_H} \right), \quad \text{with } \text{Im}[m_u^H] = |m_u^H| \sin \theta_H$



总结

- 新物理模型多种多样, 只需要满足量子场论要求
 - 轴子类轴子模型
 - 暗光子模型
 - 中微子模型
 - Vector-like fermion, lepto-quark etc...

谢谢观看!

Backup slides